Bachelor's thesis in theoretical physics

On the possibility of massive gluons in QCD

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1 Abstract

Quarks and gluons are known to possess an additional internal quantum number in comparison with other fundamental particles, they have a color charge which is brought out by eight generators. The Lagrangian describing these particles obeys a $SU(3)$ color symmetry and as a consequence of this gauge invariance, the gluons would theoretically be predicted to be massless. However, the difference between $m_g = 0$ and $m_g \approx 0$ turns out to be hardly distinguishable in experiments. Similarly, there exist theoretical mechanisms which are potentially capable of attributing mass to gluons. The exact consequences of $m_g \neq 0$ are extensively discussed throughout the paper. By analyzing theoretical and phenomenological models whether it is plausible that the mass of gluons is nonzero, and if so, what their upper bound would be. The experimental upper bound that has been found is $m_g < \mathcal{O}(1)$, however, the exact mechanism though which mass would be acquired is not known.

2 Preliminaries

Among the desiderata of quantum field theory (QFT) is the ability to describe the fundamental forces of nature.[1] Electromagnetic processes are described by quantum electrodynamics (QED) and weak processes by the intermediate vector boson theory (IVB theory). These forces can be described within a single framework at high energies i.e. the electroweak theory [[6],p.419]. The strong interactions are understood within a separate theory, they are described within the context of quantum chromodynamics (QCD), which is the topic of this thesis.

Some basic characteristics of QCD will be discussed before the field-theoretic aspects of this theory will be examined. Subsequently, the theoretical models regarding how these gluons could acquire mass in quantum field theory will be discussed and the question is analyzed in a phenomenological manner. Throughout this article, the $(+,-,-,-)$ signature for the metric $\eta^{\mu\nu}$ has been chosen and Greek letters correspond to the indices {0,1,2,3}. The index 0 corresponds to the temporal direction and 1,2,3 to the spatial directions. The time dependence is absent when Latin letters have been employed. Other notable conventions are that natural units have been employed, so we have $\hbar = c = 1$, and that summation is understood over alle possible values of recurring indices, i.e. the Einstein summation convention.

2.1 Quarks

In the 1960's, physicists started to explore the possibility that hadrons could consist of more fundamental particles. Before the MIT-SLAC experiments took place in the Stanford Linear Accelerator Center, the consensus among most particle physicists was that quarks have no innate physical reality as objects of experience. They might nevertheless still be useful mathematical constructs in order to understand strong interactions [[1]]. By examining deep inelastic electron scattering experiments, the MIT-SLAC experiments have proven that these particles actually exist. Nowadays, these particles play a pivotal role in the Standard Model.

Quarks have been classified as spin-1/2 fermions. This implies that they are subject to the Dirac equation, given by

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi, \tag{1}$$

[1] Gravity is not described within QFT, this belongs to the realm of General Relativity.

where the slashed notation $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ has been used. The usual expansion of the fields in the Heisenberg picture can be utilized, i.e.

$$\psi(x) = \psi^+(x) + \psi^-(x) \tag{2a}$$

$$= \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b^s_{\vec{p}} u^s(\vec{p}) e^{-ip\cdot x} + c^{s\dagger}_{\vec{p}} v^s(\vec{p}) e^{ip\cdot x} \right], \tag{2b}$$

where s denotes the possible spin states, thus $m_s = \pm 1/2$ for these fermions [[2]]. From this expression we can immediately obtain

$$\psi(x)^\dagger = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b^{s\dagger}_{\vec{p}} u^s(\vec{p})^\dagger e^{ip\cdot x} + c^s_{\vec{p}} v^s(\vec{p})^\dagger e^{-ip\cdot x} \right] \tag{3}$$

and hence we have obtained the expansion for the conjugate Dirac field as $\bar{\psi}(x) = \psi^\dagger(x)\gamma^0$. The plane wave expansion for the conjugate field can now be written as

$$\bar{\psi}(x) = \sum_{s=1}^{2} \int \frac{d^3p}{(2\pi)^3} \frac{1}{\sqrt{2E_{\vec{p}}}} \left[b^{s\dagger}_{\vec{p}} \bar{u}^s(\vec{p}) e^{ip\cdot x} + c^s_{\vec{p}} \bar{v}^s(\vec{p}) e^{-ip\cdot x} \right]. \tag{4}$$

The terms $c^{s\dagger}_{\vec{p}}$ and $c^s_{\vec{p}}$ correspond, respectively, to the creation and annihilation operators of antiparticles. Therefore it can be expected that antiquarks should be existent as well, and this has been verified experimentally. Antiquarks have been found to be consituents of mesons and antibaryons. Let us denote quarks by q and antiquarks by $\bar{q}$ henceforth.

Six non-equivalent quarks are known to exist. They differ in their respective masses and their flavor quantum number, of which the latter have been given the names *up (u), charm (c), top (t)* and *down (d), strange (s), bottom (b)*.[2] The former three particles share the property of having a fractional positive charge $Q = \frac{2e}{3}$, $(e > 0)$ being the elementary unit of charge, whereas the latter three have $Q = -\frac{1}{3}e$. Every antiquark has an electric charge whose magnitude is identical to that of the quark, however, with an opposite sign, thus e.g. $Q_{s\bar{s}} = +e/3$ and $Q_{u\bar{u}} = -2e/3$. Furthermore, there are notable differences in mass between the quarks, the lightest quark (up) weighs a couple of MeV and the heaviest quark (top) has been measured to be 175 GeV. Although their numerical values are not required for the purposes of this paper, it is worth noting that $m_u < m_c < m_t$ and the same hierarchy applies to d, s, b. We will see that heavy particles can be integrated out of low-energy effective QCD theories.[3]

These pieces of information allow us to rewrite the Dirac equation as

$$\mathcal{L} = \sum_{f=1}^{6} \bar{\psi}^f (i\partial\!\!\!/ - m)\psi^f, \tag{5}$$

where the index f corresponds to the flavor of the particle. The spin-statistics theorem entails that every field which describes fermions uses anticommutation relations and obeys the conditions set

[2]However, top and bottom are sometimes can be referred to by *truth* and *beauty*, respectively, in textbooks.

[3]The mass of a quark is actually a parameter occurring in the Lagrangian of the theory, which describes the self-interaction of the quark, and is not directly observable. As such, the mass parameter is much like a coupling constant in quantum field theory. It is technically dependent on the momentum scale and scale-dependent [[23],2].

by the Pauli exclusion principle, both consequences of the Fermi-Dirac statistics. Quarks and their antiparticles are the elementary constituents of hadrons and hence they are the subatomic particles occurring in baryons (qqq), antibaryons ($\bar{q}\bar{q}\bar{q}$) and mesons ($q\bar{q}$).[4] It is believed that solely these bound states of quarks are allowed, whereas e.g. singlet quark states cannot occur in nature. The Pauli exclusion principle has played a crucial role in the discovery that our current quark model is unsufficient to account for all observed particles - some are incompatible with the statistics.

In this version of the quark model, several anomalies appear. Despite the fact that this model enables us to understand what the constituents are of, say, protons (uud) and neutrons (udd), baryonic matter has been encountered that cannot be understood likewise. Several particles are composed of three identical quarks, e.g.

$$\Omega^- = sss \qquad \text{and} \qquad \Delta^{++} = uuu, \tag{6}$$

both of which are interpreted as particles containing subatomic particles with parallel aligned spin states $m_s = +1/2$ [[4],p.2] as the particle itself has $S = 3/2$. This clearly violates the Pauli exclusion principle, because the total wavefunction corresponding to $\Omega^- = u_\uparrow u_\uparrow u_\uparrow$ is symmetric under the exchange of identical quarks, an argument which holds for Δ^{++} as well.

Greenberg recognized this problem in 1964 and argued that these subatomic particles ought to obey parastatistics of rank three [[3]]. Each parafermionic quark was assigned three labels that exhibit the required antisymmetric properties under permutations of quarks. Nambu and Han contrived another solution to the deficiency in the quark model in the subsequent year, wherein the Fermi-Dirac statistics could still be employed [[14]]. Besides the ordinary observable quantum numbers, these physicists argued, there exists an additional quantum number which belongs to quarks only, also with three degrees of freedom. These additional degrees of freedom have been given the name of *color* and these became a crucial element in the paradigm of strong interactions.[5] We can now decompose the wavefunction of a quark as

$$\psi_{quarks} = \psi(\mathbf{r})\psi(spin)\psi(color). \tag{7}$$

Fritzsch, Leutwyler and Gell-Mann have finalized the theory of color charges in 1972 [[25]]. The assumption that these physicist brought into the framework of strong interactions is that every quark can exist in one of three possible color states, often abbreviated by r, g, b, corresponding to red, green and blue. The color vectors are represented in the *fundamental representation* here, this entails that

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \; g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \; b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \tag{8}$$

This representation has dim $= \mathbf{3}$ and it will be shown that this naturally fills out the color symmetry $SU(3)_c$ - a cornerstone of QCD.

Rather ad hoc, the restriction that baryon wavefunctions ought to be totally antisymmetric in color quantum numbers as well as that color charges do not commute with other charges were

[4]There have been indications that a pentaquark bound state had been found. Although this state is allowed by Pauli's exclusion principle, the 2006 Particle Data group published that '[t]he conclusion that pentaquarks in general, and the θ^+, in particular, do not exist, appears compelling," which leaves us with the three allowed bound states mentioned in this article.

[5]Quantum chromodynamics even attributes its name to it as chroma is the literal translation of $\mathcal{X}\rho\acute{\omega}\mu\alpha$ - the Greek word for color.

postulated. These assumption were soon to be confirmed. Due to this we are now able to ascribe the following interpretation to the formerly forbidden particle Ω^- and Δ^{++}, of which the latter is now described by

$$\sum \epsilon_{ijk}(u^i_\uparrow, u^j_\uparrow, u^k_\uparrow). \tag{9}$$

The color indices are given by i, j, k and the Levi-Civita symbol ensures that color fulfills the condition of being completely antisymmetric. Similarly, we can now represent the well-known proton and negatively charged kaon particle as

$$p^+ = \sum \epsilon_{ijk}(u^i_\uparrow, d^j_\uparrow, u^k_\downarrow) \ , \qquad K^- = \sum \bar{u}^{\bar{i}} \gamma_5 s^i, \tag{10a}$$

where $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$. We see that the quark has a color and the antiquark has its anticolor in the kaon, leading to a bound color singlet state. Anticolours lie in the complex conjugate representation of $SU(3)_c$, which is conveniently referred to as $\bar{\mathbf{3}}$. After the discovery of gluons it was predicted these particles have color as well, and only particles interacting with gluons have a nonzero color. Leptons, other force mediators and the Higgs boson do not have this additional quantum number. Consequently, the quarks and gluons are unique in this aspect within the Standard Model.

The strong force describes the interactions between quarks. This force is mediated by gluons and it will be discussed in the next section why we need to introduced eight nonequivalent gluon fields. Besides these interactions, quarks are involved in electromagnetic ($Q_q \neq 0$) and weak interactions (for instance β-decay, where the process $d \to u$ is required) as well. This entangles the possibility of analyzing solely strong interactions.

2.2 Color confinement

The requirement that every possibly existing combination of quarks ought to be a color singlet plays a crucial role in QCD. This so-called color confinement hypothesis will be discussed in this section, however, firstly there will be a mathematical elucidation on the description of colors.

The recently introduced color wavefunctions χ^c require color operators which act upon them. The Gell-Mann matrices are closely related to these operators and these are given by [[5],p.1]

$$\left.\begin{aligned}
&\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \\
&\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.
\end{aligned}\right\} \tag{11}$$

These matrices provide a basis for an $SU(3)$ symmetry. It can immediately be noticed that the Gell-Mann matrices are traceless and exhibit the property of being hermitian ($\lambda^\dagger = \lambda$). This is a general property of generators of $U(n)$ and hence of $SU(n)$ matrices. The interested reader is referred to Appendix 2, where the derivation of the Gell-Mann matrices are presented. This sheds light on the issue as to why generators of $SU(n)$ ought to possess these properties. A relation that holds specifically for the Gell-Mann matrices is that

$$\mathrm{Tr}(\lambda_i \lambda_j) = 2\delta_{ij}. \tag{12}$$

Strictly speaking, we are not dealing with the color operators yet, in order to obtain them we merely need to divide the matrices by 2. the Gell-Mann operators are given by

$$\hat{F}_i = \frac{\lambda_i}{2} \ (i = 1, 2, \cdots, 8) \ . \tag{13}$$

Hence, the traceless and hermitian properties have not been lost. This is very similar to the case or $SU(2)$, whose generators are given by $\hat{T}_i = \sigma_i/2$, where i are the Pauli spin matrices. The Gell-Mann matrices are not a unique representation of the symmetry group, but Gell-Mann chose them because they are generalizations of the spin matrices, which turns out to be a convenient representation.[6] The Lie brackets (commutation relations) of the $SU(3)$ group are given by [[23]]

$$[\hat{F}_i, \hat{F}_j] = i f_{ijk} \hat{F}_k, \tag{14}$$

where the *structure constants* f_{ijk} are completely antisymmetric,

$$f_{ijk} = f_{jki} = f_{kij} = -f_{ikj} = -f_{jik} = -f_{kji}. \tag{15}$$

The value of the structure constant, however, depends on the choice of indices ijk.[7] For $SU(2)$ on the other hand, one finds that the structure constant is completely antisymmetric, i.e. $f_{ijk} = \epsilon_{ijk}$, because this group has exactly three indices. The latter symmetry becomes relevant when analyzing the symmetry breaking $SU(3) \rightarrow SU(2)$ in Section 5.1. The reader who is (relatively) unfamiliar with group theory can find information on these group in this Intermezzo.

Intermezzo

Let us look at some generalities of $SU(n)$ and $U(n)$ symmetries. SU stands for Special Unitary and the special unitary group is a subgroup of the unitary group U, i.e.

$$SU(n) \subset U(n). \tag{16}$$

$U(n)$ corresponds to the group of all $n \times n$ matrices, satisfying the unitarity condition

$$UU^\dagger = U^\dagger U = I_n. \tag{17}$$

In group theory, every element of a group has an inverse. The inverse of an element a is the element that counteracts the transformation that a causes, hence we know that

$$UU^{-1} = I_n \ , \qquad \therefore U^{-1} = U^\dagger, \tag{18}$$

We obtain

$$\det\Big(UU^\dagger\Big) = \det(I_n) = \underbrace{1 \cdot 1 \cdots 1}_{n \text{ times}} = 1. \tag{19}$$

[6] The Pauli matrices σ_i can still be recognized in λ_i, $i \in \{1, 2, 3\}$. The other matrices appear because of the generalization of the matrices to higher dimensions. The sets $\{\lambda_1, \lambda_2, \lambda_3\}$, $\{\lambda_4, \lambda_5, \frac{1}{2}(\lambda_3 + \sqrt{3}\lambda_8)\}$, $\{\lambda_6, \lambda_7, \frac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8)\}$ have identical algebraic properties to the Pauli matrices and form $\mathfrak{su}(2)$ subgroups of the $SU(3)$ group [[5],p.1].

[7] Exemplary values are $f_{123} = 1$ and $f_{147} = 1/2$.

We know from linear algebra that

$$\det(A) = \det\left(A^{\dagger}\right), \tag{20}$$

leading us to the conclusion that

$$\det(U) = \det\left(U^{\dagger}\right) = \pm 1. \tag{21}$$

Because $U(n)$ is a continuous group, it belongs to the family of Lie groups. The $U(n)$ group has dimension n^2.

The special unitary group consists of the matrices that satisfy the 'special' property $\det(U) = +1$. The matrix with $\det(U) = -1$ is thereby excluded, hence this group has a dimensionality of

$$\dim\big(SU(n)\big) = n^2 - 1. \tag{22}$$

The symmetry of our particular interest in this thesis is $SU(3)$, this group requires 8 generators $T^i, i \in \{1, 2, \cdots, 8\}$. They form the Lie-algebra of this group, $\mathfrak{su}(3)$. $SU(n)$ groups with $n \geq 2$ are non-Abelian, which means that at least 2 elements are non-commutative. A homomorphism can be constructed from $SU(2)$ to $SO(3)$, meaning that its algebraic structure is preserved [[7]]. The latter group is the threedimensional rotation group and it can easily be shown that this has a non-Abelian character, i.e. the sequence in which operations are performed influences the outcome. The non-Abelian character of this group is a well-known phenomenon from everyday life, as has been illustrated in Figure 1.

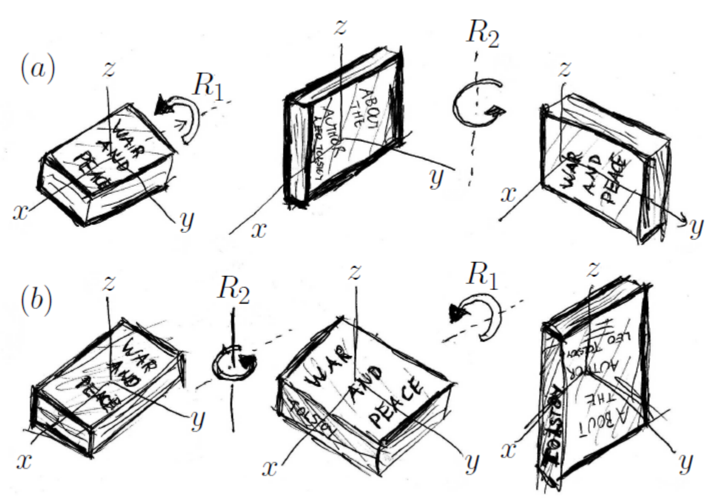

Figure 1: *An illustrative means to show that SO(3) is non-Abelian.* $O(R_1)O(R_2) \neq O(R_2)O(R_1)$.

Analysis of the Gell-Mann matrices teaches that λ_3 and λ_8 commute, as can quickly be shown by

$$\frac{1}{\sqrt{3}}\left[\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}-\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\right]=\frac{1}{\sqrt{3}}\left[\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}-\begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix}\right]=0\,,$$

whereas all other operators do not commute. As a consequence thereof, the possible color states $\chi^c = r, g, b$ are eigenstates of $\hat{F}_3$ and $\hat{F}_8$ simultaneously. This does not hold for the other operators. By applying $\hat{F}_3$ and $\hat{F}_8$ to the color states we easily obtain the result of Table. 1. All eigenvalues are additive quantities, so we immediately see that the combination rgb has $\hat{F}_3 = \hat{F}_8 = 0$ These conserved color charges are called color isospin (I_3^C) and color hypercharge (Y^C) [[31]]. The other

Color	$\hat{F}_3$	$\hat{F}_8$
r	$1/2$	$1/(2\sqrt{3})$
g	$-1/2$	$1/(2\sqrt{3})$
b	0	$-1/(\sqrt{3})$

Table 1: *Quarks and their corresponding color charges for $\hat{F}_3$ and $\hat{F}_8$. For antiquarks, these values undergo a sign-flip.*

operators change the color state of quarks when acting on them, e.g. we could encounter

$$\hat{F}_1 g = \frac{1}{2}\begin{pmatrix}0&1&0\\1&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\0\\0\end{pmatrix} = \frac{1}{2}r. \tag{23}$$

Similarly, $\hat{F}_1 r = \frac{1}{2} g$ and $\hat{F}_1 b = 0$. Complex eigenvalues are not excluded, e.g. we have

$$\hat{F}_7 g = ib, \qquad \hat{F}_7 r = 0, \qquad \hat{F}_7 b = -ig. \tag{24}$$

It will appear later that the color state of quarks can indeed change and it are the gluons that are capable of inducing the effect that color states are changed.

Now let us return to the hypothesis of color confinement. This claim can now be formulated as the requirement that hadrons ought to fulfill

$$\hat{F}_i \chi_h^c = 0\,, \; (i = 1, 2, 3, \cdots, 8), \tag{25}$$

The requirement that $\hat{F}_3$ and $\hat{F}_8$ cancel for every hadron is of our particular interest. An analysis of Table 2.2 tells that this corresponds to the possible combinations $rgb, r\bar{r}, g\bar{g}$ and $b\bar{b}$, whereas for instance quarks and diquarks are prohibited according to the theory because the eigenvalue of at least one color operator is nonzero. This is consistent with observations of existing particles, which confirm that the possible combinations consist of baryons, antibaryons and mesons. The requirement of colorless states corresponds to the claim that all physical hadrons are singlets (i.e. scalars) under rotations in the color space. This is what the color $SU(3)$ symmetry originates in.

The color charge of particle is conveniently denoted by the dimension of the $SU(3)$ representation. The 27-dimensional ($\mathbf{3}\times\mathbf{3}\times\mathbf{3}$) representation for mesons and (anti-)baryons, respectively, are thus reduced to merely the singlet state. From group theory, we know that the group multiplication can be represented as

$$\mathbf{3}\times\mathbf{3}\times\mathbf{3} = \mathbf{1}+\mathbf{8}+\mathbf{8}+\mathbf{10}. \tag{26}$$

The outcome with dimension 1 is the only allowed combination for baryons, as this is a color singlet. The group multiplication for mesons ($\mathbf{3} \times \bar{\mathbf{3}}$) differs from the one presented in Eq. (26) but it similar in the aspect that only the color singlet is allowed.

Color confinement states that physically realized particle are always color singlets, thus free quarks and gluons cannot occur in nature. The basic degrees of freedom cannot be observed and we therefore obtain a relatively complex world of hadronic and nuclear physics [[13],p.1].

When dealing with baryons, we know from Eq. (9) that the color wavefunction can be represented as

$$\chi_B^c = \alpha_1 r_1 g_2 b_3 + \alpha_2 b_1 r_2 g_3 + \alpha_3 g_1 r_2 b_3 + \alpha_4 r_1 b_2 g_3 + \alpha_5 b_1 g_2 r_3 + \alpha_6 g_1 b_2 r_3, \tag{27}$$

where he quarks have been labelled by index $i = 1, 2, 3$. Hence the first term corresponds to quark 1 being in the red color state. The α-constants can to be determined from the remaining conditions, i.e.

$$\hat{F}_j \chi_B^c = 0 \ , \qquad j \in \{1, 2, 4, 5, 6, 7\}. \tag{28}$$

After all these mathematical requirements are combined, one finds that they can be added in the uniquely determined, elegant formula for the wavefunction

$$\chi_B^c = \epsilon_{ijk} r_i g_j b_k, \tag{29}$$

where ϵ_{ijk} is the usual Levi-Civita symbol. The Levi-Civita symbol ensures that the wavefunction of color charge is completely antisymmetric, as was required by Fritzsch, Leutwyler and Murray Gell-Mann who hypothesized the color charges. The interested reader is referred to Appendix A for more information on this result. When ijk refers to the color states instead of particles, we can rewrite our result as

$$\boxed{\chi_B^c = \epsilon_{ijk} q^i q^{j\prime} q^{k\prime\prime}}, \tag{30}$$

where the Einstein summation convention has been used. Henceforth this will always be done in this text.

Because of color confinement, allowed bound quark states ought to be able of undergoing rotations through color space while the color state of the particle is not altered. This constraint is satisfied iff no change in color occurs under any possible rotation, in other words, these particles should be *color singlets*. Due to this, all hadronic matter obeys a color invariance for

$$\psi_i(x) \to U_{ij} \psi_j(x) \tag{31a}$$

and

$$\bar{\psi}_i(x) \to \bar{\psi}_i(x) U_{ij}^\dagger. \tag{31b}$$

The symmetry occurring in the Lagrangian for the colored quarks is given by

$$UU^\dagger = U^\dagger U = 1. \tag{32}$$

The unitary matrices are 3x3 matrices, because we have a three dimensional color space. Due to this we see that there is a global $SU(3)$ color invariance, denoted by $SU(3)_c$. We have discussed before that $SU(3)$ has 8 generators. After this symmetry has been made *local*, as will be done in Chapter 3.1, we find that we need to postulate eight additional colored vector fields. These correspond to the gluon fields.

2.3 Gluons

One might ask what causes all quarks to possess the additional internal quantum number. Soon after the discovery of the existence of color it was realized that the gauge bosons of the strong interactions play a crucial role in that phenomenon. Because of gauge invariance, we need to postulate 8 new fields in QCD. These are the following vector fields

$$A_a^\mu, \qquad a \in \{1, 2, \cdots, 8\}. \tag{33}$$

These fields are associated with spin-1 bosons called gluons, where a is the color index. Gluons are the quanta of QCD and they are the force carriers of quarks. As quarks cannot be colorless we expect gluons to be colored as well. This has been confirmed experimentally, however, they do not possess the same color eigenvalues as quarks. The eight possible gluon colors are directly related to the Gell-Mann matrices which have been introduced in Chapter 2.2. The difference is that every entry of these matrices carries an additional color element, as given by

$$\begin{pmatrix} r\bar{r} & r\bar{b} & r\bar{g} \\ b\bar{r} & b\bar{b} & b\bar{g} \\ g\bar{r} & g\bar{b} & g\bar{g} \end{pmatrix}, \tag{34}$$

which leads to different normalization factors as well. As an example, a gluon can have a color charge

$$O_4 = \frac{r\bar{g} - g\bar{r}}{\sqrt{2}}. \tag{35}$$

This leads to an allowed color octet for gluons, every member of which consists of a linear combination of color-anticolor pairs. In the process of a one-gluon-exchange, the gluon is capable of changing the color states of the involved quarks. A gluon with color O_4 would make the green quark red by applying $r\bar{g}$ and the green quark red in a similar fashion. It is worth noting that the color singlet

$$\frac{1}{3}(r\bar{r} + g\bar{g} + b\bar{b})$$

can indeed not be obtained for gluons. A color singlet gluon would be capable of mediating a strong force to orders significantly exceeding a fermimeter, as this particle would satisfy the color confinement hypothesis. An elaboration and mathematical description of this color octet can be found in Appendix A.

What is important for our discussion is that color is a *conserved charge*, i.e. a Noether's current, of quantum chromodynamics. This means that it is always conserved in strong interactions. Let us briefly have a look what happens in the interactions of QCD. The gluon turns out to *couple* to the color charge of quarks, a concept which is analogous to photons which couple to electic charge in quantum electrodynamics. In the latter theory we deal with the fine-coupling constant[8]

$$\alpha = \frac{e^2}{4\pi} \approx \frac{1}{137}, \tag{36}$$

[8]This value has been presented in S.I. units. Furthermore, the fine-coupling constant becomes bigger at a higher momentum scales due to screening effects, so we effectively have $\alpha = \frac{e^2(\mu^2)}{4\pi}$. These effects will be discussed in Section 3.2, and we have e.g. $\alpha_{eff}(\mu = m_Z) = \frac{1}{128}$[[28],].

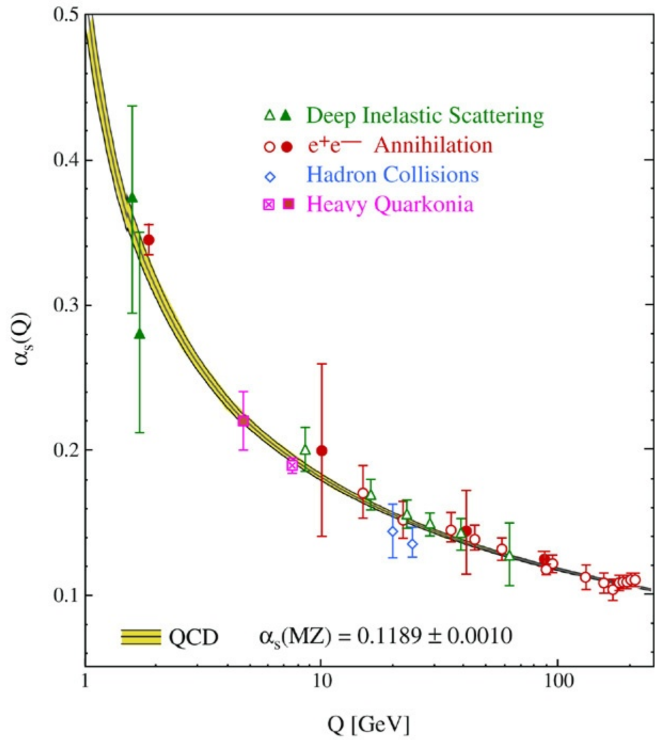

Figure 2: *Graph of the strong coupling constant. It can be seen that the coupling constant decreases with energy, which means that particles are weakly coupled when they are close to one another, i.e. at $r < 1$ fm.*

whereas in QCD the strong coupling constant naturally appears. This coupling constant - or interaction strength - is described by [[23],p.9]

$$\alpha_s(\mu) \equiv \frac{g_s(\mu)^2}{4\pi} \approx \frac{2\pi}{\beta_0 \ln(\mu/\Lambda_{QCD})} \tag{37}$$

and occurs for every vertex in Feynman diagrams. The β_0-function has been determined by Gross, Politzer and Wilczek, all of whom have been awarded a Nobel Prize for their discovery of asymptotic freedom in 1973 [29].[9] The gluons couple to color, how this can be recognized in the running coupling constant will be shown in the Sections 2.4 and 3.2, where asymptotic freedom and screening effects are discussed. The coupling constant has been plotted against energy in Figure 2.

A fundamental property of QCD is that the coupling of gluons to quarks is independent of the flavor of the quark. This flavor independence is a consequence of an isospin symmetry of QCD, however, corrections need to be imposed due to mass differences between quarks and electromagnetic interactions [[34]]. The quarks u and d have nearly identical masses and hence the flavor independence has its most notable effects for the proton and neutron.

We cannot perform direct measurements on gluons. These particles are also subject to color confinement, entailing that they cannot be separated from hadrons. So, analogously to the reason why we cannot encounter free quarks, there is an impossibility of performing experiments on free

[9] $\beta_0 = \frac{33-2n_f}{3}$, where n_f denotes the amount of involved quark flavors [[23],p.8].

gluons. In this article, *quasiconfining* quantum chromodynamics theories will be discussed. These theories predict that free particles can exist in the high-energetic regime. Quasiconfining theories can only exist in QCD when $m_g = 0$.

In spite of the practical impossibility of performing experiments on gluons, QCD makes the prediction that these particles ought to be massless. Let us take this as the point of view to start with and analyze the prediction that are entailed by setting its mass to zero. Similarly to photons, the gluons have three degrees of freedom for the polarization (2x transverse and 1x longitudinal), but gauge invariance excludes the occurrence of the longitudinal mode. However, in the case of massive bosons, longitudinal modes are allowed by gauge invariance. This implies that one additional degree of freedom will occur for every gluon which has undergone an acquisition of mass. These longitudinal modes play a crucial role in the argumentation why we cannot add a bare gluon mass term, as will be discussed in Section 4.

2.4 Asymptotic freedom

Asymptotic freedom is the phenomenon that the strength of the strong force decreases for smaller distances between quarks. As a consequence of the relatively weak force, quarks are free to move around in the core of hadrons. [[12]] This is what the name ‘asymptotic freedom’ can be ascribed to - the force of containment asymptotically approaches zero for quarks that are in close vicinity. This can be seen in the expression for the coupling constant, in the limit $\mu \to \infty$ we obtain $\alpha_s(\mu) \to 0$. Large energy is transferred at small distances, so therefore the coupling constant decreases for small separations between quarks. In the case of QED, the coupling constant becomes larger at small distances, because the Coulomb potential decreases with $V \propto 1/r^2$.

The opposite effect occurs at bigger distances than the equilibrium distance. The attractive color force increases and thereby tries to pull the quarks tighter together. These phenomena are reflected in the QCD potential, which is given by

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \sigma r. \tag{38}$$

At short distances of $r \sim 0.5$ fm the potential has a negative sign, and the dominant process is a single gluon exchange between quarks. The first term has a Coulombic form and is therefore QED-like, except for the factor of $4/3$ which arises becauses the gluons carry color indices [[17],p.5]. The gluon self-interactions are responsible for completely different behavior than can be found in QED at $r > 0.1$ fm. It can be seen that the potential increases linearly for large separations between quarks, the factor σ in the second term represents the string tension of the flux tubes. This is known to be $\sigma = 0.9$ GeV fm^{-1} and is independent of the length of the separation distance [[8],p.6]. This is understood as being a consequence of a chromoelectric flux tube that forms between the quarks, which consists of gluon fields [[12],[17]]. This reveals that it would cost infinitely much energy to separate quarks completely, i.e. making them free particles without a flux tube. When spatial separations between interacting quarks become larger than 1 fm, approximately the diameter of a nucleus of an atom, the gluon string is energetic enough to break. In order to retain color confinement, a phenomenon called *fragmentation* occurs, which corresponds to the creation of a quark-antiquark pair. for instance when the constituents of kaons (let us take $\bar{d}s$) become too widely separated we can expect either an $s\bar{s}$ or $d\bar{d}$ pair to appear, leading to two kaons which individually satisfy color confinement. With enough energy it is possible to create multiple mesons during a fragmentation process.

Although QCD is known to exhibit the property of being asymptotically free, the mathematical derivation thereof has not been provided up to date. Many physicists have vainly attempted to solve this mathematical problem, nowadays it has become one of the Millennium Prizes of the Clay Mathematics Institute [[11]]. Physicists consider it to be a core property of QCD, and adjustments to the theory (e.g. the Higgs mechanism discussed in Section 5.2) which render the theory not (entirely) asymptotically free are thereby regarded fallacious.

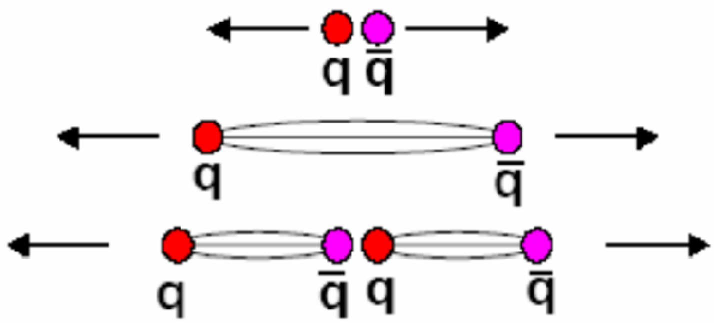

Figure 3: *When quarks become too widely separated, it is energetically favorable within QCD to produce a quark-antiquark pair in order to satisfy the condition of color confinement. The string that connects the quarks needs to be broken for this purpose, this phenomenon is known as fragmentation. A particle can decay into several hadrons, e.g. a charmonium state (J/ψ) could decay into three pions, $c\bar{c} \to \pi^+\pi\pi^-$.*

Before the regime where string are capable of breaking, thus $r > 1$ fm, we can apply perturbative QCD, abbreviated as pQCD. At bigger distances - which correspond to higher energies - the disturbances are too big to be examined with perturbative methods. A nonperturbative field-theory that has turned out to be effective for these cases is *lattice field theory*, although this still requires a low-energy regime [[13],p.3-4].

3 Field theories

What we have found up until now is that the following particles and constraints must be incorporated in QCD theories.

- Quarks with 6 different flavors whose existence has been confirmed. They exist while possessing one of three possible colors, referred to by $\chi^c(x)$. Quarks occur in the combinations $q^i\bar{q}^{\bar{i}}$, $q^iq^jq^k$ and $\bar{q}^i\bar{q}^j\bar{q}^k$.
- Gluons which occur in eight gluon fields, $A^i(x)$. All fields should be color non-singlets.

The Lagrangian density for quantum chromodynamics, where quarks are bound by massless gluons, is given by [[6],p.229]

$$\mathcal{L} = \underbrace{\bar{\Psi}^f(x)\left[i\not{D} - m_f\right]\Psi^f(x)}_{\text{Quarks}} - \underbrace{\frac{1}{4}G_{i\mu\nu}(x)G_i^{\mu\nu}(x)}_{\text{Gluons}}, \tag{39}$$

where a is the color index of the gluon. D_μ Is the covariant derivative, whose purpose is to restore gauge invariance in the Lagrangian.[10] The $SU(3)_c$ gauge symmetry which the Lagrangian obeys, implies the existence of additional terms. They are a consequence of the employment of the partial derivative in the Dirac field, which will act on the exponential function of the symmetry group as well due to the product rule. The covariant derivative cancels exactly those additional terms and needs eight vector fields for that purpose, as will be shown in Section 3.1.

3.1 Lagrangian for quarks

We recognize that the term of the Lagrangian which describes the quarks looks similar to the Dirac equation, when written in slashed notation. This is exactly what one would expect to be the proper relativistic description for spin-1/2 particles, such as quarks. Let us start with the ordinary Dirac equation and see why we obtain a triplet Ψ^f for quarks and why the covariant derivative D needs to be employed. We have

$$\mathcal{L} = \underbrace{\bar{\psi}^f(x)\big[i\partial\!\!\!/ - m_f\big]\psi^f(x)}_{\text{Quarks}} \tag{40a}$$

$$= \bar{\psi}^f_r(x)\big[i\partial\!\!\!/ - m_f\big]\psi^f_r(x) + \bar{\psi}^f_g(x)\big[i\partial\!\!\!/ - m_f\big]\psi^f_g(x) + \bar{\psi}^f_b(x)\big[i\partial\!\!\!/ - m_f\big]\psi^f_b(x), \tag{40b}$$

where it must be kept in mind that we actually have 18 terms, as the flavor summation is implicitly assumed. It is convenient to gather the different color fields in a triplet, i.e.

$$\Psi^f = \begin{pmatrix} \psi^f_r \\ \psi^f_g \\ \psi^f_b \end{pmatrix} \tag{41a}$$

since now we have combined the colors in one term. Correspondingly we have

$$\bar{\Psi} = \begin{pmatrix} \bar{\psi}^f_r & \bar{\psi}^f_g & \bar{\psi}^f_b \end{pmatrix}. \tag{41b}$$

These are exactly the terms that occur in the QCD Lagrangian of Eq. (39), now we can rewrite Eq. (40a) as

$$\mathcal{L} = \underbrace{\bar{\Psi}^f(x)\big[i\partial\!\!\!/ - m_f\big]\Psi^f(x)}_{\text{Quarks}}. \tag{42}$$

The difference between the expressions lies in covariant derivative which has been utilized, rather than the partial derivative. In order to understand why this is a necessary substitution, let us analyze what the SU(3) transformations amount to. The global $SU(3)$ invariance is introduced by Eqs. (31) and we will exploit that the generators can be represented as

$$U = \exp(i\theta_a F^a), \tag{43a}$$

θ_a being infinitesimal and $\theta_a \in$ R. The Gell-Mann color operators are given by F^a, henceforth without the hat. From this expression we immediately obtain [[23],p.3]

$$U^\dagger = \exp(-i\theta_a F^a). \tag{43b}$$

[10] More gauge invariant terms could be added to the Lagrangian, for instance $\mathcal{L} = ig_s\bar{\Psi}^f(x)\Psi^f(x)G_{a\mu\nu}(x)G^{\mu\nu}_a(x)$. However, the criterion that QCD should be renormalizable excludes additional terms like these [[6],p.229].

This implies that our $SU(3)$ transformations are given by

$$\Psi^f(x) \to \Psi^{f\prime}(x) = \Psi^f(x)e^{i\theta_a F^a}, \tag{44a}$$

$$\bar{\Psi}^f(x) \to \bar{\Psi}^{f\prime}(x) = e^{-i\theta_a F^a}\bar{\Psi}^f(x). \tag{44b}$$

We analyze the infinitesimal transformations, thus we expand Eq. (43a) as

$$\exp(i\theta_a F^a) \approx 1 + i\theta_a F^a + \mathcal{O}(\alpha^2), \tag{45}$$

Eq. (43b) can be expanded likewise and we hence obtain

$$\Psi^f(x) \to \Psi^f(x)(1 + i\theta_a F^a), \tag{46a}$$

$$\bar{\Psi}^f(x) \to (1 - i\theta_a F^a)\bar{\Psi}^f(x). \tag{46b}$$

The partial derivative occurring in the Lagrangian

$$\mathcal{L} = \underbrace{\bar{\Psi}^f(x)\left[i\partial\!\!\!/ - m_f\right]\Psi^f(x)}_{\text{Quarks}} = \bar{\Psi}^f(x)\left[i\gamma^\mu\partial_\mu - m_f\right]\Psi^f(x), \tag{47}$$

acts in the following manner after an infinitesimal global $SU(3)$ transformation

$$\left.\begin{aligned} \partial_\mu\Psi^f e^{i\theta_a F^a} &\approx \partial_\mu\Psi^f(1 + i\theta_a F^a) \\ = \partial_\mu\Psi^f + i(\partial_\mu\Psi^f)\theta_a F^a &+ i\Psi^f F^a \underbrace{(\partial_\mu\theta_a)}_{=0} \\ &= (\partial_\mu\Psi^f)\Big(1 + i\theta_a F^a\Big) \end{aligned}\right\}. \tag{48}$$

This has as a consequence that

$$\mathcal{L} \to \mathcal{L}' = \Big(1 - i\theta_a F^a\Big)\bar{\Psi}^f(x)[i\gamma^\mu\partial_\mu - m_f]\Psi^f(x)\Big(1 + i\theta_a F^a\Big) = \mathcal{L} + \mathcal{O}(\theta_a^2). \tag{49}$$

As we have infinitesimal θ_a, our Lagrangian is invariant under global $SU(3)_c$ transformations, thus $\delta\mathcal{L} = 0$. By Noether's theorem this leads to the following eight conserved currents

$$J^\mu = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi_a)}\delta\psi_a - \underbrace{F^\mu}_{=0} = \frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)}\delta\psi + \underbrace{\frac{\partial\mathcal{L}}{\partial(\partial_\mu\bar{\psi})}\delta\bar{\psi}}_{=0}. \tag{50}$$

After inserting

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\psi)} = i\gamma^\mu\bar{\Psi}^f \tag{51a}$$

and

$$\delta\psi = i\theta_a F^a\psi \tag{51b}$$

we obtain the following conserved current

$$J_i^\mu = \bar{\Psi}^f\gamma^\mu F_i\bar{\Psi}^f. \tag{52}$$

We see that there is one conserved current for every generator of the gauge symmetry. They lead to the conserved charges

$$Q_i = \int d^3\mathbf{x}\ J_i^0 = \frac{1}{2}\int d^3\mathbf{x}\ \underbrace{\bar{\Psi}^f\gamma^0\lambda_i\bar{\Psi}^f}_{\bar{\Psi}^f=\Psi^{f\dagger}\gamma^0} \tag{53a}$$

$$= \frac{1}{2}\int d^3\mathbf{x}\ \Psi^{f\dagger}\underbrace{\gamma^0\gamma^0}_{(\gamma^0)^2=1}\lambda_i\bar{\Psi}^f = \frac{1}{2}\int d^3\mathbf{x}\ \Psi^{f\dagger}\lambda_i\bar{\Psi}^f. \tag{53b}$$

This corresponds to conservation of color charge. We fill in λ_3 to analyze what such a charge looks like,

$$Q_3 = \frac{1}{2}\int d^3\mathbf{x}\begin{pmatrix}\bar{\psi}_r^f & \bar{\psi}_g^f & \bar{\psi}_b^f\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}\psi_r^f\\ \psi_g^f\\ \psi_b^f\end{pmatrix} = \frac{1}{2}\int d^3\mathbf{x}\big[\psi_r^{f\dagger}\psi_r^f - \psi_g^{f\dagger}\psi_g^f\big]. \tag{54}$$

After expanding the fields as in Eqs. (2b) and (4) we obtain

$$Q_3 = \frac{1}{2}\big[N_r - \bar{N}_r - N_g - \bar{N}_g\big], \tag{55}$$

where N_c is the number operator for quarks having color index c, which sums over all allowed flavor, spin and momentum states. When the conserved charges are combined, it turns out there every color ought to be conserved. [11]

The Lagrangian of quarks should be invariant after the global symmetry has been changed to be a local symmetry. The difference between global and local transformations is that in the latter case, the transformation is not identically performed at all points of interest, i.e. it becomes dependent on the spacetime co-ordinate that it operates on. By doing so, we are creating a gauge theory. In order to obtain this, we upgrade θ_a to eight arbitrary differentiable functions $\theta_a(x)$ and consequently substitute our unitary matrices by

$$U = \exp[ig_sF_a\theta_a(x)]\ , \qquad U^\dagger = \exp[-ig_sF_a\theta_a(x)]. \tag{56}$$

These arbitary functions cause additional term in the Lagrangian when partial derivative operate on them as

$$\partial_\mu\theta_a(x) \neq 0\ , \tag{57}$$

since $\theta(x)$ differs from one spacetime co-ordinate to the other. Furthermore, $g_s \in \mathrm{R}$ and will later be identified as the coupling constant [[6],p.227]. After the symmetry has been made local, the phase transformation of Eq. (48) becomes

$$\left.\begin{aligned} \partial_\mu\Psi^f e^{ig_s\theta_a(x)F^a} &\approx \partial_\mu\Psi^f(1 + ig_s\theta_a(x)F^a)\\ = \partial_\mu\Psi^f + ig_s(\partial_\mu\Psi^f)\theta_a(x)F^a &+ ig_s\Psi^f F^a(\partial_\mu\theta_a(x))\\ = (\partial_\mu\Psi^f)\Big(1 + ig_s\theta_a(x)F^a\Big) &+ ig_s\Psi^f F^a(\partial_\mu\theta_a(x)) \end{aligned}\right\}. \tag{58}$$

[11] There are many more conserved charges in QCD. Some authors refer to these as *accidental symmetries*, i.e. "symmetries that are an automatic consequence of the assumed gauge invariance" [[13],2]. Among these accidental symmetries are the discrete symmetries of parity (P) and charge conjugation (C), conservation of all flavor (thus also stangeness number $S \equiv -N_s$, as opposed to in weak interactions) and an approximate symmetry of $SU(2)$-isospin, leading to flavor independence as discussed in Section 2.3.

The term $ig_s\Psi^f F^a(\partial_\mu\theta_a(x))$ is a consequence of the local symmetry, hence we need a covariant derivative to restore Lorentz invariance. Inserting the additional term into the Lagrangian leads to the transformation

$$\mathcal{L}_0 \to \mathcal{L}_0' = \mathcal{L}_0 + \bar{\Psi}^f\Big(i\gamma^\mu\big(ig_s\Psi^f F^a(\partial_\mu\theta_a(x))\big)\Big)\Psi^f = \mathcal{L}_0 - g_s\gamma^\mu\bar{\Psi}^f F^a(\partial_\mu\theta_a(x))\Psi^f. \tag{59}$$

This leads one to make the substitution to the *covariant derivative* in order to obtain gauge invariance

$$\partial_\mu \to D_\mu = \partial_\mu + \frac{ig_s}{2}\lambda_j A_{\mu j}(x) = \partial_\mu + ig_s\hat{F}_j A_{\mu j}(x) \tag{60}$$

in the Dirac equation. It can be noted that we needed to postulate eight vector fields A_j^μ to restore gauge invariance. These turn out to describe the gluon fields, the gauge bosons of QCD. Thus the existence of the gluon fields is a consequence of the local $SU(3)_c$ gauge invariance of the QCD Lagrangian.

One often encounters covariant derivatives in QFT, because all particles in the Standard Model are related to a unitary group, $U(1)$, $SU(2)$ or $SU(3)$. These symmetries, when made local, usually give rise to additional terms in the Lagrangian. For the electroweak theory with a local $SU(2)\times U(1)$ symmetry, we will find the covariant derivative

$$D_\mu = \partial_\mu + i\frac{g_w}{2}\vec{\sigma}\cdot\vec{W}_\mu + i\frac{g_b}{2}B_\mu\ , \tag{61}$$

which will be used in Section 5.1 of this paper, where spontaneous symmetry breaking is discussed. Also here, the existence of the force carriers (4 in this case: $B_\mu, W_\mu^i, i\in\{1,2,3\}$) is a consequence of the gauge invariance.

3.2 Lagrangian for gluons

Now let us look at the gluon part of the Lagrangian, i.e.

$$\mathcal{L} = \underbrace{-\frac{1}{4}G^a_{\mu\nu}(x)G_a^{\mu\nu}(x)}_{\text{Gluons}}. \tag{62}$$

The spacetime dependence of the fields is taken to be implicit from now on. This Lagrangian density is quite reminiscent of the electromagnetic field strength in QED. In this theory we find the lagrangian term

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{63}$$

The difference lies in the formulation for the tensor which represents the field strength. In the case of photons we obtain the electromagnetic tensor, i.e.

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \partial^{[\mu}A^{\nu]} = \begin{array}{cc} \xrightarrow{\quad\quad\nu\quad\quad} & \\ \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & -B_3 & B_2 \\ E_2 & B_3 & 0 & -B_1 \\ E_3 & -B_2 & B_1 & 0 \end{pmatrix} & \Big\downarrow\ \mu, \end{array} \tag{64}$$

which is gauge invariant under the transformation

$$A^\mu \to A^{\mu\prime} = A^\mu + \partial_\mu f(x), \qquad f(x) \in \mathrm{R}. \tag{65}$$

The expression for the field strength belonging to gluons contains an extra term. The *gluon field strength* of Eq. (62) is described by the formula [[6],p.229]

$$G_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial^\nu A_i^\mu + g f_{ijk} A_j^\mu A_k^\nu, \tag{66a}$$

where f_{ijk} are the $SU(3)$ structure constants which have been introduced in Section 2.2 and the indices i, j, k denoting the color charges. Our expression can be formulated more compactly by

$$G_i^{\mu\nu} = \partial^{[\mu} A_i^{\nu]} + g f_{ijk} A_j^\mu A_k^\nu. \tag{66b}$$

The difference between the field strengths of photons and gluons is that the latter expression, thus Eqs. (66) are not gauge invariant [[18], p.16]. We will see in this section that it therefore leads to self-interaction gluons. This phenomenon is held responsible for the fact that QCD exhibits the properties of, among others, asymptotic freedom and color confinement [[31],§5.4]. Before discussing this in more detail, let us first investigate what the Lagrangian term for gluons originates in.

Both QED and QCD have spin-1 gauge bosons, thus it is not surprising that the Lagrangians for gluons and photons do not differ significantly. Both expressions are derived from the Yang-Mills Lagrangian. Yang Mills theory deals with non-Abelian gauge-theories and is therefore applicible within the context of $SU(2) \times U(1)$ for photons and in our case - within the context of an $SU(3)_c$ gauge theory. For gluons we have

$$\mathcal{L} = -\frac{1}{2}\,\mathrm{Tr}\left\{G^2\right\} = -\frac{1}{2}\,\mathrm{Tr}\left\{G^i_{\mu\nu} G^{j\mu\nu}\right\} = -\frac{1}{4} G^i_{\mu\nu} G^{i\mu\nu}, \tag{67}$$

where the trace disappeared because of identity Eq. (12), which can be rewritten as

$$\mathrm{Tr}\Big(\hat{F}^i \hat{F}^j\Big) = \frac{\delta^{ij}}{2}.$$

The reason why we have an additional term for gluons is that they obey the non-abelian symmetry of SU(3), whereas photons obey the Abelian symmetry of U(1) of electromagnetism. In the Yang-Mills Lagrangian we obtain cubic and quartic terms from gluons,

$$\mathrm{Tr}\left(ig(\partial_\mu A_\nu - \partial_\nu A_\mu)[A^\mu, A^\nu]\right)\Big\}\,\text{Cubic term} \tag{68a}$$

and

$$\mathrm{Tr}\left(-g^2[A_\mu, A_\nu][A^\mu, A^\nu]\right)\Big\}\,\text{Quartic term.} \tag{68b}$$

The brackets denote commutation relations. For the expression without the trace the reader is referred to Appendix C, where the complete Lagrangian for gluons is mentioned. These cubic and quartic terms imply that the gluons are self-interacting [[18], p.16]. Self-interaction is possible in QCD because the gauge bosons carry charge (color for gluons), whereas photons have no electric charge in QED. This allows gluons to couple to - and hence interact with - other gluons. It can be stated in a broader sense, that one of the major differences between Abelian and non-Abelian theories in QFT lies in the fact that self-interactions are allowed in the non-Abelian cases [[35],p.1]

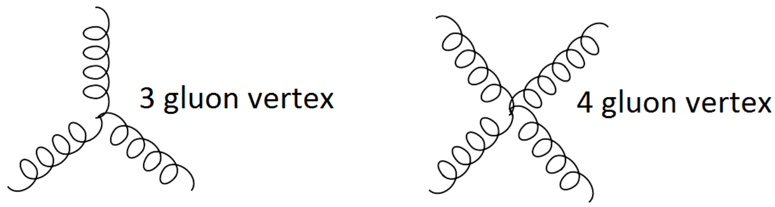

Figure 4: *Self-interactions of gluons in Feynman diagrams. The first interaction is known as one-gluon exchange when one of the created gluons is absorbed by another gluon, the second is known as contact interaction. These processes are the leading lowest-order contributions of gluon-gluon scattering processess. The scattering processes are held responsible for many features that QCD exhibits, such as asymptotic freedom and color confinement. Another consequence of the these self-interactions is chiral symmetry breaking, however, this phenomenon is irrelevant for further purposes of this paper* [[23]].

Quarks are surrounded by virtual gluons, leading to screening effects on the color charge. This is analogous to electric charge screening effects which occur for photons. The vacuum is not an empty space in quantum field theory, rather it is the state with the lowest possible energy configuration. The vacuum can have electromagnetic fields which pop into and out of existence. Photons which propagate through a vacuum are capable of producing a pair of positive and negative energy state particles - most notably, an electron-positron pair - which subsequently annihilates. The positive and negative elementary charge of these virtual particles are 'screened' by the photon, which results in a charge and interaction strength that appear altered from a distance. In the case of screening effects, fewer charge is registered by the particle and the coupling constant decreases. This second-order electromagnetic process $\gamma \to \gamma$ leads to a *vacuum polarization* in the presence of an external electric field [[6],p.109], therefore these quantum fluctuations have a notable effect on their surrounding.

The same process occurs for gluons, but rather than electric charge it is the color charge of the temporarily existing particles that is being screened. The lowest order screening process is where one $q\bar{q}$-pair is created. There is one major difference with QED, processes which are known as *anti-screening* processes are allowed here as well, which is a consequence of the gluon's self-interactions. Antiscreening occurs when $g \to gg$ followed by $gg \to g$. In these processes it is the case that the effective color charge decreases with diminution of the distance between quarks [[17],8]. This is analogous to the phenomenon that the coupling constant becomes small for quarks in close vicinity. Anti-screening is the dominant effect in QCD at the low energies, which is reflected in the running coupling constant of QCD

$$\alpha_s(\mu) = \frac{2\pi}{\beta_0 \ln\left(\frac{\mu}{\Lambda_{QCD}}\right)} \ , \tag{69}$$

from which it can be deduced that

$$\lim_{\mu\to\infty} \alpha_s(\mu) \to 0 = 0. \tag{70}$$

The fact that quarks become strongly coupled at larger distances is a consequence of the antiscreening processes. At larger distances between quarks, a bigger 'cloud' of charge is perceived and the coupling constant thereby increases. High values of μ are associated with small distances between quarks. Therefore antiscreening leads to the effect that the coupling constant decreases for small distances between quarks. Deep inelastic scattering experiments have confirmed this statement. Due to asymptotic freedom, we can only apply pQCD meaningfully when quarks are close to one

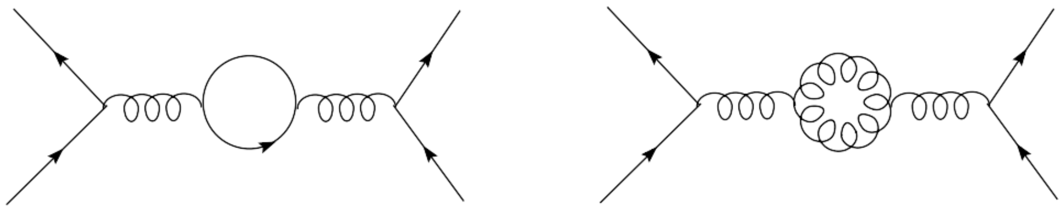

Figure 5: *The first process is similar to the screening process in QED. Charge of the $q\bar{q}$ is screened and the interaction strength thereby increases at smaller distance. The second process occurs solely in QCD and represents an anti-screening effect. This leads to a charge whose screened value is larger than its actual value, causing the coupling to increase at increasing separation distances between quarks. This is a direct consequence of the nonlinear interactions beteen gluons.*

another. Perturbation theory works up to separation distances of $r \approx 1$ fm. For bigger separations we need to employ nonperturbative field-theoretic approaches such as *lattice QCD*, which will be done in Section 5.2 for the Higgs mechanism.

3.3 Massive gluons

The $SU(3)_C$ symmetry which is present in the Lagrangian density of QCD requires the gluons to be massless. As we know from the electroweak theory, wherein $m_\gamma = 0$, gauge particles with exactly zero mass mediate a force with infinite range. This is consistent with what experiments reveal about the behavior of photons, e.g. the Cosmic Microwave Background can still be observed from the Big Bang. On the other hand we have the massive force carriers of the weak force, their finite range (up to $10^{-16}m$) is attributed to their property of being massive [[31]]. Physicists are convinced that gluons cannot mediate a force with an infinitely extended range, however, they also theoretically predict that gluons ought to be massless.

Asympotic freedom poses constraints which ensure that range of the strong force is not infinite, because quarks cannot be separated up to arbitrarily large distance. The QCD string breaks when the quarks are too widely separated, at this point quark-antiquark pairs are created and new flux tubes are thereby created. Thus the range of the strong force is definitely not infini

So why exactly should gluons be massless? Generically the mass of a particle is related to its Lagrangian the following way. Given the Lagrangian term

$$\mathcal{L} = C(A)^2 = CA_\mu A^\mu, \tag{71a}$$

the mass of the particle is related by the formula [[19]]

$$m_A = -\sqrt{2C}. \tag{71b}$$

The $SU(3)$ color symmetry enabled us to obtain the Yang-Mills Lagrangian for gluons

$$\left.\begin{aligned}\mathcal{L}_{\text{gluons}} = -\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial^\mu A^\nu_i - \partial^\nu A^\mu_i) \\ +\frac{g_s}{2} f_{ilm}(\partial_\mu A^i_\nu) A^\mu_l A^\nu_m \\ +\frac{g_s}{2} f_{ijk} A^j_\mu A^k_\nu (\partial^\mu A^\nu_i) \\ -\frac{g_s^2}{4} f_{ijk} f_{ilm} A^\mu_j A^\nu_k A^l_\mu A^m_\nu)\end{aligned}\right\}, \tag{72}$$

i, j, k being the color charges. Because there is no term of the form $A^i_\mu A^{i\mu}$, it follows directly that gluons are massless in QCD.

An illustrative means would be to look at the Klein-Gordon-Fock equation. This describes relativistic spin-0 bosons and should therefore be applicable to the Higgs particle. In its free theory, Higgs is described by the following Lagrangian density

$$\mathcal{L}^H_{free} = (\frac{1}{2}\Box - m^2)h^2(x) = \frac{1}{2}\partial_\mu h(x)\partial^\mu h(x) - m^2 h^2(x), \tag{73a}$$

where the last term contains the scalar product of $h(x)$, i.e.

$$\mathcal{L}^H_{free} = \frac{1}{2}\partial_\mu h(x)\partial^\mu h(x) - m^2 h_\mu(x) h^\mu(x). \tag{73b}$$

The mass of the Higgs scalarfield $h(x)$ is solely determined by the factor of $(-m^2h^2)$ and is thus given by

$$m_H = -(-\sqrt{2m^2}) = \sqrt{2}m. \tag{73c}$$

The fact that the gluons naturally come out without mass terms in QCD does not necessarily reveal their true nature. In the electroweak theory the $W^\pm$ and Z-bosons appear to be massless as well. In this particular case, the effect of spontaneous symmetry breaking caused by the Higgs particle still needs to be included in order for the particles to become massive. The goal of the following sections is to examine what mechanisms would lead to massive gluons - and correspondingly in what order of magnitude these masses ought to lie. A problem which is bound to occur is that the $SU(3)_c$ is lost as soon as $m_g > 0$, which has as a consequence that quarks and gluons are not exactly confined. Chapters 4 and 5 discuss the theoretical aspects of possible mechanisms that allow (several or all) gluons to be massive. Experimental data on an upper bound of the mass are discussed in Chapter 6.

4 Adding a bare mass term

An attempt to attain a nonzero mass of gluons could be to analyze a proper modification to the Lagrangian of QCD. Our Lagrangian for gluons and quarks is given in Eq. (39) and we know that it describes massless gluons. An additional term can be postulated which contains a nonzero bare mass of gluons, denoted by mg. The bare mass is not identical to the invariant mass $m_{inv} = (k_0 k^0)^{1/2}$. The invariant mass is equal to the bare mass plus mass terms resulting from the effects of force fields around the particle, these give contributions to the particles *self-energy*,

$$m_{\rm inv} = m_{bare} + m_{\substack{self- \\ energy}} \tag{74}$$

These fields can create virtual particles which exist temporarily and alter the effective mass of particles. In the case of an electron, the mass of a bare electron differs from the mass of a physical electron, i.e. an electron which is surrounded by its photon cloud [[31],p.108]. As discussed in the context of anti-screening effects, gluons are capable of undergoing self-interactions due to their nonzero color charge, for instance where a gluon creates multiple virtual gluons or a $q\bar{q}$ pair. In the Higgs mechanism in the electroweak theory we will encounter particles that acquire mass solely through interactions with the Higgs field, hence these particles have a $m_{bare} = 0$. In this section, we analyze the mass of gluons which do not undergo such interactions.

Let us analyze the following addition to $\mathcal{L}_{QCD}$ of Eq. (39)

$$\mathcal{L} = -\frac{m_g^2}{2}\sum_i A^i_\mu A^{i\mu}, \tag{75}$$

The choice of the prefactor can be understood by recognizing that the mass of fields appears in front of the squared free fields in QFT, as discussed in Eq. (71). In our expression for the bare gluon mass, Eq. 75, it can be checked that we have added exactly a mass m_g.[12] The additional Lagrangian term in Eq. (75) is uniquely determined so we could not add another additional term in the Lagrangian in order to obtain a bare mass term.

The effect of adding the bare gluon mass term is that the $SU(3)_c$ gauge invariance is broken. The Lagrangian still exhibits an $SU(3)_c$ symmetry, however, it now has the property of being a global invariance. Gauge theories are always local invariant, which ensures that quantities are conserved in every point of spacetime. This is done by means of the covariant derivative D_μ presented in Eq. (60) and the gluon transformation law [[18],20]

$$A_\mu \rightarrow A'_\mu = U A_\mu U^\dagger + \frac{i}{g_s} U(\partial_\mu A_\mu)U^\dagger. \tag{76}$$

In the case of a global symmetry we do not need to include the second term Eq. (76) and do not need to employ the covariant derivative. It are the global symmetries that lead to conserved currents and therefore we know that the conserved current of QCD is a consequence of the global transformation [32]. When the local gauge invariance is reduced to a global invariance, we thus do not lose the essential property of colors being conserved in QCD. Nevertheless, there is another objection to the method of postulating a bare mass term.

Unfortunately, if this were to be regarded a viable method of obtaining a nonzero mass, some severe objections to this procedure will need to be overcome. This additional term brings along that the theory of QCD is rendered nonrenormalizable by perturbative methods, thus in pQCD. Renormalization is an essential tool in QFT to obtain physically relevant information from infinities

[12] This model presupposes that the bare mass of a gluon is independent of its color, i.e. all 8 gluon fields have an identical mass. Nussinov & Shrock argue that it certainly is possible to write the more general gluon mass terms, which would be given by $\mathcal{L}_{add} = -(1/2)\sum_a m^2_{g,a} A^i_\mu A^{i\mu}$. For our discussion this would deteriorate the amount of involved mathematics, whereas it does not lead to additional relevant information, hence we will employ Eq. (75).

arising in quantum field theories. Many integrals in the framework lead to infinities - amongst others those corresponding to loop divergences occurring in QED and QCD - and renormalization eliminates these unphysical results by analyzing the effective results. For instance, in QED we have a divergent integral for the vacuum polarization processes discussed in Section 3.2, which is eliminated by renormalization [6], p.102]. Also for electrons we find an infinite bare mass, although its invariant mass is finite.[13] Infinitudes arising in self-energy diagrams disappear when self-energy effects are incorporated into the properties of the bare electron. It is due to mass renormalization that we can eliminate such infinities, because they naturally cancel one another. If QCD were to become nonrenormalizable, the theory of strong interactions would be left with senseless infinities. An example thereof would be the infinities resulting from loop divergences of the gluon self-interaction processes.

So what exactly makes the renormalization problematic when the adjusted $\mathcal{L}_{QCD}$ is employed? Longitudinally polarized gluons play a role in this question. A massive vector bosons can always be decomposed like W-bosons can be decomposed, i.e. [30]

$$W_\mu = W_\mu^\perp + \partial_\mu W^L. \tag{77}$$

Where W^L denotes the longitudinally polarized bosons, which are forbidden in the low-energetic regime but whose occurence increases linearly with four-momentum. In the ultra-relativistic limit one finds that

$$\frac{\partial_\mu W^L}{W^\perp} \gg 1, \tag{78}$$

where the partial derivative scales with $|k^\mu|$. This implies that at very high energies the longitudinal mode dominates. This is not problematic, as massive gauge bosons are allowed to have longitudinal modes.

However, in the case of gluons we face unphysical results when computing Feynman amplitudes for processes involving these polarization modes. A possible process is the scattering of 2 longitudinally polarized gluons to multigluon final states. Partial wave amplitudes of such processes involve terms of s/m_g^2, s being the 4-momentum, and would thus violate unitarity when $\sqrt{s} \geq m_g$. As has been discussed before, m_g is known to have an upper limit of a few MeV, a value that the momentum of the gluons easily surpasses. The energy scale (Λ) of QCD lies around 300 MeV, this is the region where the coupling constant α_s grows to $\mathcal{O}(1)$. It is known that perturbative methods are not valid when applied to mass regions far below Λ_{QCD}, because the coupling between these particles becomes too strong to be examined by perturbative methods. In our current theory, we cannot conclude whether violation of unitarity actually has occurred for the longitudinally polarized gluons.

The inapplicability of perturbation theory to the modified QCD Lagrangians (Eq. (75) renders the theory nonrenormalizable. The nonrenormalizability of the theory needs to be overcome. An ultraviolet completetion is a commonly used method to make field-theories renormalizable. An upper energy bound (in the UV-regime) needs to be found such that a cut-off can be imposed at that particular value for the energy. The theory would then only be valid up to that energy scale.

[13] Steven Weinberg wrote about this the following; "the bare mass and charge are not the same as the measured electron mass and charge that are listed in tables of elementary particles. In fact, in order to account for the observed values (which of course are finite) of the mass and charge of the electron, the bare mass and charge must themselves be infinite. The total energy of the atom is thus the sum of two terms, both infinite: the bare energy that is infinite because it depends on the infinite bare mass and charge, and the energy shift (...) that is infinite because it receives contributions from virtual photons of unlimited energy [[33]]."

Despite the fact that we cannot use this nonrenormalizable theory to obtain an upper mass bound for gluons, there is a conceptual issue that we should be aware of. Since the $SU(3)_c$ symmetry is broken when adding the additional Lagrangian term is, we cannot expect quarks and gluons to be confined anymore [[8],p.1-2]. Free quarks are heavier than bound quarks, because they are surrounded by a cloud a virtual particles leading to a large positive self-energy mass. The adjusted Lagrangian is defined to be a *low-energy effective field theory*, and the masses of these particles exceed the QCD-scale significantly. Therefore, these particles are integrated out of the theory when energy scale μ runs down to the regime where pQCD is operable. Therefore these particles do not give effective contributions to the theory. However, this mechanism would imply that free quark states exist and should be existent in the high-energy regime, such theories are known as quasiconfining theories. Free quarks or gluons have not been experimentally discovered hitherto, but this does not exclude that they exist. Free quark searches, discussed in Section 6, have set the upper limit for the mass of bound quarks on $m_g < \mathcal{O}(1)$ MeV.

We will examine alternative schemes for gluon mass acquisitions, however, we have already found a potential problem concerning the mass of gluons. Regardless of our employed methodology to examine the mass of gluons, we already know that their upper bound is way below the QCD-scale, $m_g \ll QCD$, entailing that we can impossibly perform measurements which are sensitive to such finite, nonzero masses. As Nussinov & Shrock wrote in Physical Review, it would be the case that "the physical meaning of this [nonzero value of m_g] is not completely clear [[8],2]," which will be elucidated for $m_g < \mathcal{O}(1)$ at the end of Section 6. Let us first analyze whether Higgs mechanisms might me responsible for mass generation of gluons, they have the virtue of being renormalizable [[8],p.2].

5 Spontaneous symmetry breaking

As mentioned before, the four particles corresponding to the $SU(2) \times U(1)$ symmetry of the electroweak theory started massless. The Higgs mechanism revealed that all particles, except for the photon, actually do have a mass. In this section, the Higgs mechanism will be applied to the $SU(3)$-theory of QCD, but first we will analyze how this mechanism works in the Standard Model to see how Higgs can give mass to particles in the first place. Afterward, several Higgs mechanisms will be presented which are capable of breaking the $SU(3)_c$ symmetry of QCD. They will be discussed in a comparative manner to the Higgs mechanism occurring in the Standard Model (SM).

5.1 Higgs mechanism in the Standard Model

In the Standard Model we encounter spontaneous symmetry breaking in the electroweak theory. We know from group theory that an $SU(2) \times U(1)$ symmetry corresponds to $3 + 1$ generators, hence we expect 4 particles to occur within this theory. Theoretically speaking, there was a missing particle. Another particle would be responsible for mass acquisition of the gauge bosons, because gauge invariance would be broken if the gauge bosons themselves were naturally massive [[9], p.717-719]. The existence of the so-called Higgs boson H^0 was predicted in 1967, when Weinberg formulated the electroweak theory. This model became known as the Weinberg-Salam model, or the Glashow-Weinberg-Salam model [[7],p.432, [9],p.700]. In this framework, the natural starting point is where the four involved gauge particles are defined to be massless.

The Higgs particle is responsible for an acquisition of mass for the mediators of the electroweak force, with the photon exluded. Let us have us look which mechanism is capable of generating relatively large masses of the W^+, W^- and Z^0, which have been experimentally measured to be around 80 GeV for $W^\pm$ bosons and around 90 GeV for the Z-boson. The crux of this mechanism lies in a nonzero vacuum expectation value for the Higgs boson. In field theories, the vacuum state is defined to be the state which has the lowest possible energy configuration.

Nambu and his co-workers discovered that spontaneous symmetry breaking does not have any effect when the vacuum is non-degenerate, and it has turned out that it is indeed non-unique [[6],p.404]. This implies that at least one quantity in the vacuum must be non-vanishing. The Higgs particle is attributed the property that it generates a nonzero expectation value in the vacuum. We do not expect the vacuum expectation value (VEV) to be affected by Lorentz translations and rotations, therefore we postulate a scalar Higgs field obeying the relation

$$\langle 0| \phi(x) |0\rangle = C \neq 0. \tag{79}$$

On the other hand, the expectation value of spinor and vector fields should vanish, i.e.

$$\langle 0| \psi(x) |0\rangle = 0 \qquad \langle 0| V^\mu |0\rangle = 0. \tag{80}$$

The Goldstone model satisfies these conditions and is given by [[6],p.405-405]

$$\mathcal{L} = [\partial^\mu \phi][\partial_\mu \phi] - \mu^2 |\phi^\dagger \phi| - \lambda |(\phi^\dagger \phi)^2| = [\partial^\mu \phi][\partial_\mu \phi] - \mu^2 |\phi|^2 - \lambda |(\phi)^4|. \tag{81}$$

This scalar potential has the virtue that it has proven itself renormalizable [[35],p.4]. Spontaneous symmetry breaking occurs for $\mu < 0$ only and we need to set $\lambda > 0$ in order to have a lower potential energy bound. The specific potential energy density, which has obtained a rather informal name in the terminology of the electroweak theory - the *Mexican hat*, is now obtained. A potential energy density plot has been provided in Figure 5.

It must be kept in mind that lagrangians are represented in the form $L = T - V$, in this case we thus have a potential energy of[14]

$$V = \mu^2 |\phi|^2 + \lambda |\phi|^4 \ . \tag{82}$$

Due to the condition

$$\mu^2 < 0, \tag{83}$$

we have shifted the ground state from the origin to a circle whose centre is located in the origin. As every point on this circle is an eigenstate of the ground state there is no unique eigenstate anymore, the proper parameterization for this state is

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda} \right)^{1/2} e^{i\theta} \ , \qquad 0 < \theta \leq 2\pi. \tag{84}$$

The unique ground state does not exist anymore, instead, it is represented by several quantum alignments. Due to the invariance of $\mathcal{L}$ under phase rotations, we can choose our vacuum state to correspond to $\theta = 0$, without loss of generality. The fact that we can arbitrarily pick a ground state is essential for spontaneous symmetry breaking to occur. In a degenerate ground state we select one of the states to depict the vacuum state, after which this state does not share symmetries, e.g.

[14]We are actually, however, talking about a potential energy density, denoted by $\mathcal{V}(\phi)$.

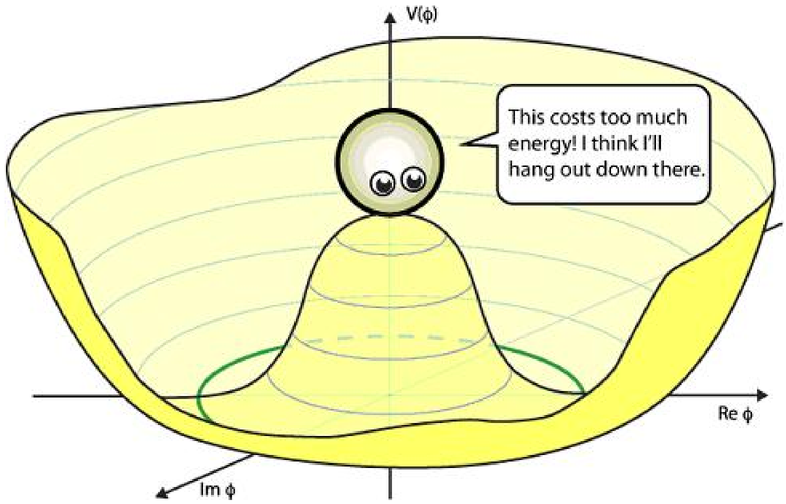

Figure 6: *The potential energy plot belonging to the Higgs field. The ground state is reprented by the green line encircling the origin, clearly we have obtained a degenerate ground state. This is what Nambu had discovered to be a requirement for spontaneous symmetry breaking to occur. Due to a spherical symmetry, the Higgs particle has no preferred direction where it falls down to the ground state.*

rotational ones, with the other possible states anymore [[6],p.404]. Such an arbitrary choice of the ground state leads to an asymmetric ground state, and this is thus not a consequence of asymmetric term in the Lagrangian.[15] The arbitrariness in the Goldstone model lies in the fact that the Higgs boson falls down from the origin to the ground state, but has no preferred orientation so it ends up in a 'randomly chosen' ground state. In our model, the Higgs particle moves from the origin towards the ground state represented by $\theta = 0$.

In the Goldstone model with $= 0$ we obtain

$$\phi_0 = \left(\frac{-\mu^2}{2\lambda}\right)^{1/2}, \tag{85a}$$

which can be related to the VEV (ν) by [[8],p.414]

$$\phi_0 = \frac{\nu}{\sqrt{2}} > 0\,, \qquad \phi_0 \in \mathbb{R}, \tag{85b}$$

$$\therefore \nu = \left(\frac{-\mu^2}{\lambda}\right)^{1/2}. \tag{85c}$$

This leads to the result

$$\langle 0|\, \phi(x)\, |0\rangle = \phi_0, \tag{86}$$

which ensures that the vacuum expectation value of the scalar field is exactly the value belonging to the minimum of the potential energy density in the Mexican hat.

[15] A familiar example is ferromagnetism. The spins in a ferromagnetic material are aligned in a certain direction, where they cause a nonzero magnetization $\mathbf{M}$. The Hamiltonian $\mathcal{H}$, which includes the electronic spin coupling terms, exhibits a rotational invariance however. On basis of the Hamiltonian the magnetization could be oriented towards any direction, and spontaneous symmetry breaking occurs because one of these orientations is arbitrarily chosen.

When two real fields are now incorporated into the Higgs field in the following fashion

$$\phi(x) = \frac{1}{\sqrt{2}}[\nu + \sigma(x) + i\eta(x)]. \tag{87}$$

where $\sigma(x)$ and $\eta(x)$ are fields which measure deviations of the Higgs field from its equilibrium position. More precisely, $\eta(x)$ corresponds to rotations through the minima of the potential energy density, i.e. it represents displacements in the valley of potential energy minima. The other field, , registers deviations which are the result of displacements in the radial plane

Methodological remark

The condition $\mu^2 < 0$ implied that the ground state had a degeneracy. This means that we are dealing with an unstable solution of the ground state, which means that we should be aware that perturbation theory maybe cannot be applied here. It turns out that the quartic term $\lambda|\phi(x)|^4$ cannot be treated as a perturbation of bilinear terms of ϕ and ϕ^*. In this particular case, application of perturbative methods leads to particles with imaginary mass, which finite orders of perturbation theory cannot counteract [[6],p.407].

Now we can already conclude what the masses of the η and σ particle should be. By employing the Higgs field, Eq. 87, and inserting this in the Lagrangian density of the Goldstone model, Eq. 81, we obtain

$$\mathcal{L}(x) = \frac{1}{2}\Big(\underbrace{[\partial^\mu\nu][\partial_\mu\nu]}_{=0} + \frac{1}{2}[\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] + \frac{1}{2}[\partial^\mu\eta(x)][\partial_\mu\eta(x)]\Big) - \mu^2|\phi|^2 - \underbrace{\lambda|\phi|^4}_{\text{omit this term}} . \tag{88}$$

In the term $|\phi(x)|^2$, the only term of interest is $\sigma^2(x)$. This is because its prefactor reveals the mass of this particle. Crossterms are consequently not relevant when searching for the mass of particles. From Eq. 85b we know that

$$\mu^2 = 2\lambda\nu^2. \tag{89}$$

Hence we obtain

$$\mathcal{L} = \frac{1}{2}[\partial^\mu\sigma(x)][\partial_\mu\sigma(x)] + \frac{1}{2}[\partial^\mu\eta(x)][\partial_\mu\eta(x)]\Big) - \frac{1}{2}(2\lambda\nu^2)\sigma^2(x), \tag{90}$$

from which it follows that we have

$$\mathcal{L} = 0\eta^2(x) \ , \qquad \therefore m_\eta = 0, \tag{91a}$$

$$\mathcal{L} = -\frac{1}{2}(2\lambda\eta^2)\sigma^2(x) \ , \qquad \therefore m_\sigma = \sqrt{2\lambda\eta^2}. \tag{91b}$$

The η-bosons move through a valley with constant potential energy and the fact that their mass is zero is a consequence of this vacuum degeneracy [[6],p.407]. These massless spin-0 pseudoscalars play a role all spontaneous symmetry mechanisms, they are referred to by *Goldstone bosons.*[16]

[16]The name Nambu-Goldstone bosons can be encountered as well in the literate.

Although Goldstones theorem states that they always occur in spontaneous symmetry breaking processes, these massless spin-0 particles have never been observed [[35]]. They always appear to be unphysical particles in spontaneous symmetry breaking processes, as will be illustrated below in the section on symmetry breaking in an $SU(2) \times U(1)$ framework. However, the degrees of freedom belonging to the Goldstone bosons will be transferred to the vector fields of the gauge bosons, leading to longitudinal polarization modes [[6],p.410]. The Goldstone bosons are said to be 'eaten' by the gauge bosons. We start with four massless bosons, the W^+, W, Z^0 and the γ.[17] Due to the massless characters of these particles, they cannot have a longitudinal polarization. Therefore two transversal degrees of freedom for the polarization are exhibited by the massless particles. The Higgs particle makes three particles massive and correspondingly has a minimum of three degrees of freedom. Because it is a spin-0 boson, and such particles ought to have one degree of freedom for $m_s = 0$, we know that Higgs begins with 4 degrees of freedom.[18] Putting all this together, we know that

Particle	$W^\pm$	+	Z^0	+	H^0	+	γ		
Degrees of freedom	4	+	2	+	4	+	2	=	12

,

Table 2: *The amount of degrees of freedom that are involved before spontaneous symmetry breaking*

from which we can conclude that 12 degrees of freedom are involved, this number needs to be conserved after symmetry breaking.

After the symmetry breaking of the Higgsfield has occurred we obtain three massive particles, which means that the degrees of freedom have been increased by a number of 1. Electroweak theory consists of the following amount of degrees of freedom.

Particle	$W^\pm$	+	Z^0	+	H^0	+	γ		
Degrees of freedom	6	+	3	+	1	+	2	=	12

.

Table 3: *The amount of degrees of freedom that are involved after spontaneous symmetry breaking*

We need to start with a Higgs particle which has exactly 4 degrees of freedom. The simplest multiplet which constists of 4 degrees of freedom is a *Higgs doublet.* Now we can start the procedure of symmetry breaking to find the masses of the gauge bosons.

We have the following term in the Klein-Gordon equation for Higgs

$$D_\mu \Phi^\dagger D^\mu \Phi, \tag{92}$$

where Φ is the Higgs doublet in electroweak theory and D_μ is the covariant derivative of the electroweak theory. The Higgs doublet is given by

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \nu + \sigma(x) + i\eta_3(x) \end{pmatrix}. \tag{93}$$

[17]The information that has been provided here is physically speaking not entirely correct. The Higgs mechanism start with the following 4 fields; $B_\mu, W_\mu^i,\ i \in \{1,2,3\}$. Linear combinations of them correspond to the force mediators of the electroweak theory, the $W^\pm$, γ and Z^0 particles. The amount of degrees of freedom is not altered by this, for clarity the degrees of freedom of the force mediators are counted.

[18]m_s denotes the quantum number of the spin projection here, not the mass of some particle s.

The Unitary Gauge (or Unitarity gauge), which has been discovered by 't Hooft, shows that some of these fields are unphysical. Three of the involved fields turn out to be unphysical. In the Unitary Gauge we want to go through all minima of the potential, as they all correspond to the same VEV. They are all located in a circle (η-dependence) around the origin, thus the contribution of η should be zero. Therefore

$$\eta_i \, , i \in \{1.2.3\}$$

can be omitted from the Higgs doublet. As physics is independent of gauge fixing we know that Higgs has one degree of freedom, independent of how the field is mathematically described. In other words, this particular gauge reveals that some degrees of freedom are actually unphysical. The degrees of freedom of these fields are transferred to the massless particles, which are capable of having longitudinal polarizations after acquiring mass, Our doublet maintains one degree of freedom,

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + \sigma(x) \end{pmatrix}. \tag{94}$$

The expression for the contravariant derivative for our $SU(2) \times U(1)$ gauge theory is given by [[6],413]

$$D^\mu = \partial^\mu + ig\sigma^j W_j^\mu/2 + ig'YB^\mu \tag{95}$$

We start by inserting the hypercharge, $Y = 1/2$ and by expanding the inner product of Pauli spin matrices and the W-fields. The spin matrices appear, because they correspond to the generators of $SU(2)$ after having been divided by 2. The Pauli spin matrices are given by

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; , \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \; , \; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{96}$$

So we need to investigate the expression

$$D^\mu = \partial^\mu + i\frac{g_w}{2}\vec{\sigma} \cdot \vec{W}^\mu + i\frac{g_b}{2}B^\mu, \tag{97}$$

where the abbreviations $g_b = g'$ and $g_w = g$ have been used. Similarly, the covariant derivative can be written in the following way

$$D_\mu = \partial_\mu + i\frac{g_w}{2}\vec{\sigma} \cdot \vec{W}_\mu + i\frac{g_b}{2}B_\mu. \tag{98}$$

We know that the fields W_μ^1 and W_μ^2 are electrically charged, whereas W_μ^3 is electrically neutral. Hence we want to combine the former two fields into

$$W^+ = \frac{W_\mu^1 - iW_\mu^2}{\sqrt{2}}, \tag{99a}$$

$$W^- = \frac{W_\mu^1 + iW_\mu^2}{\sqrt{2}}. \tag{99b}$$

They turn out to be useful in this procedure later on, because these linear combinations correspond to the W bosons of electroweak theory. We need to calculate $D_\mu \Phi^\dagger D^\mu \Phi$ and will use the property that

$$D_\mu\Phi^\dagger D^\mu\Phi = (D_\mu\Phi)^\dagger D^\mu\Phi$$

We first express the term inside parantheses and will later take the dagger thereof

$$D_\mu\Phi = \underbrace{\partial_\mu\Phi}_{\text{first term}} + \underbrace{i\frac{g_b}{2}B_\mu\Phi}_{\text{seond term}} + \underbrace{i\frac{g_w}{2}\vec{\sigma}\cdot\vec{W}_\mu\Phi}_{\text{third term}}. \tag{100}$$

The first term denotes the partial derivative of the Higgs field. We have

$$\partial_\mu\Phi = \partial_\mu\begin{pmatrix}0\\ \nu+\sigma(x)\end{pmatrix} = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\partial_\mu\sigma(x)\end{pmatrix}. \tag{101}$$

The third term contains a factor which includes the Pauli matrices, this can be written fully as

$$\vec{\sigma}\cdot\vec{W}_\mu = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}W_\mu^1 + \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}W_\mu^2\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}W_\mu^3 = \begin{pmatrix}W_\mu^3 & W_\mu^1 - iW_\mu^2\\ W_\mu^1 + iW_\mu^2 & -W_\mu^3\end{pmatrix}, \tag{102}$$

which is a result wherin combinations of real W_μ^1 and complex W_μ^2 occur. We will utilize Eqs. (99) to rewrite these terms.

$$i\frac{g_w}{2}\vec{\sigma}\cdot\vec{W}_\mu\Phi = i\frac{g_w}{2}\begin{pmatrix}W_\mu^3 & \sqrt{2}W_\mu^+\\ \sqrt{2}W_\mu^- & -W_\mu^3\end{pmatrix}\begin{pmatrix}0\\ \nu+\sigma(x)\end{pmatrix} = i\frac{g_w}{2}\begin{pmatrix}\sqrt{2}W_\mu^+\\ -W_\mu^3\end{pmatrix}\Big(\nu+\sigma(x)\Big). \tag{103}$$

Now we can write the covariant derivative as

$$D_\mu\Phi = \partial_\mu\Phi + i\frac{g_b}{2}B_\mu\Phi + i\frac{g_w}{2}\vec{\sigma}\cdot\vec{W}_\mu\Phi = \begin{pmatrix}0\\ \frac{1}{\sqrt{2}}\partial_\mu\sigma(x)\end{pmatrix} + i\frac{g_b}{2}B_\mu\begin{pmatrix}0\\ \nu+\sigma(x)\end{pmatrix} + i\frac{g_w}{2}\begin{pmatrix}\sqrt{2}W_\mu^+\\ -W_\mu^3\end{pmatrix}\Big(\nu+\sigma(x)\Big). \tag{104}$$

$(D_\mu\Psi)^\dagger$ changes the column vectors to row vectors and changes $i \leftrightarrow -i$, hence also $W_\mu^+ \leftrightarrow W_\mu^-$ occurs after applying the dagger

$$D_\mu\Phi^\dagger = (D_\mu\Phi)^\dagger = \Big(0, \quad \tfrac{1}{\sqrt{2}}\partial_\mu\sigma(x)\Big) - i\frac{g_b}{2}B_\mu\left(0, \quad \nu+\sigma(x)\right) - i\frac{g_w}{2}\left(\sqrt{2}W_\mu^-, \quad -W_\mu^3\right)\Big(\nu+\sigma(x)\Big). \tag{105}$$

Now we can calculate

$$D_\mu\Phi^\dagger D^\mu\Phi. \tag{106}$$

In this expression, nine terms will occur. However, we do not have to calculate all of them for the purpose of this question. In Lagrangian densities, it is always only the factor standing in front of the squared field that has a relation to the mass of the field, as discussed in Section 4. For W and Z-bosons we have the following term in the Lagrangian density [[6], Eq. (17.61)]

$$\mathcal{L} = m_W^2 W_\mu^\dagger(x)W^\mu(x) + \frac{1}{2}m_Z^2 Z_\mu(x)Z^\mu(x). \tag{107}$$

Analysis of the terms occurring in the contravariant and covariant derivative tells us that we need only to multiply the second and third terms with one another, because they lead to squared fields. All these terms carry factors of $(\nu+\sigma(x))$, hence we are looking for an answer in the form

$$[\cdots]\Big(\nu+\sigma(x)\Big)^2. \tag{108}$$

Four terms need to be considered, i.e.

1. First quadratic term

$$-i\frac{g_w}{2}\left(\sqrt{2}W^-_\mu, \quad -W^3_\mu\right)\bigg(\nu+\sigma(x)\bigg)i\frac{g_w}{2}\begin{pmatrix}\sqrt{2}W^{+\mu}\\ -W^{3\mu}\end{pmatrix}\bigg(\nu+\sigma(x)\bigg) \tag{109a}$$

$$=\frac{g_w^2}{4}\left(\sqrt{2}W^-_\mu, \quad -W^3_\mu\right)\begin{pmatrix}\sqrt{2}W^{+\mu}\\ -W^{3\mu}\end{pmatrix}\bigg(\nu+\sigma(x)\bigg)^2 \tag{109b}$$

$$=\frac{g_w^2}{2}(W^-_\mu W^{\mu+})\bigg(\nu+\sigma(x)\bigg)^2+\frac{g_w^2}{4}W^3_\mu W^{3\mu}\bigg(\nu+\sigma(x)\bigg)^2. \tag{109c}$$

From now on we will abbreviate the Higgs-field by

$$\bigg(\nu+\sigma(x)\bigg)\equiv\phi_0\ ,$$[19]

hence the first quadratic term becomes

$$\frac{g_w^2}{4}\Big(2W^-_\mu W^{\mu+}+W^3_\mu W^{3\mu}\Big)\phi_0^2\ . \tag{109d}$$

2. First cross term

$$-i\frac{g_w}{2}\left(\sqrt{2}W^-_\mu, \quad -W^3_\mu\right)\bigg(\nu+\sigma(x)\bigg)i\frac{g_b}{2}B^\mu\begin{pmatrix}0\\ \nu+\sigma(x)\end{pmatrix} \tag{110a}$$

$$=-\frac{g_w g_b}{4}W^3_\mu B^\mu\phi_0^2\ . \tag{110b}$$

3. Second cross term

$$-i\frac{g_b}{2}B_\mu(0,\nu+\sigma(x))i\frac{g_w}{2}\begin{pmatrix}\sqrt{2}W^{\mu+}\\ -W^{\mu 3}\end{pmatrix}\bigg(\nu+\sigma(x)\bigg)$$

$$=-\frac{g_b g_w}{4}B_\mu W^{3\mu}\phi_0^2\ . \tag{111}$$

4. We have one more quadratic term that we want to consider

$$-i\frac{g_b}{2}B_\mu(0,\nu+\sigma(x))i\frac{g_b}{2}B^\mu\begin{pmatrix}0\\ \nu+\sigma(x)\end{pmatrix}$$

$$=\frac{g_b^2}{4}B_\mu B^\mu\phi_0^2\ . \tag{112}$$

In the end we want to have the fields $A^\mu, Z^\mu, W^\mu, W^{\mu\dagger}$, we can obtain these fields by utilizing the Weinberg angle θ_W that relates the B^μ and W^3_μ fields to A_μ and Z_μ by [[6],p.397]

$$\begin{pmatrix}B_\mu\\ W^3_\mu\end{pmatrix}=R(-\theta_W)\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}=\begin{pmatrix}\cos\theta_W & -\sin\theta_W\\ \sin\theta_W & \cos\theta_W\end{pmatrix}\begin{pmatrix}A_\mu\\ Z_\mu\end{pmatrix}. \tag{113a}$$

[19] This is not the same that occurred in $\langle 0|\,\phi(x)\,|0\rangle=\phi_0$. Henceforth in this section, solely the new definition will be used for this term.

The following relations are its straightforward consequences

$$B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W, \tag{113b}$$

$$W^3_\mu = A_\mu \sin\theta_W + Z_\mu \cos\theta_W. \tag{113c}$$

The first term in the enumeration of Lagrangian terms can be written as

$$\frac{g_w^2}{2} W^-_\mu W^{\mu+}\phi_0^2 + \frac{g_w^2}{4} W^3_\mu W^{3\mu}\phi_0^2 \tag{114a}$$

$$= \frac{g_w^2}{2} W^-_\mu W^{\mu+}\phi_0^2 + \frac{g_w^2}{4}(A_\mu \sin\theta_W + Z_\mu \cos\theta_W)(A^\mu \sin\theta_W + Z^\mu \cos\theta_W)\phi_0^2 \tag{114b}$$

$$= \frac{g_w^2}{4}\big(2W^-_\mu W^{+\mu} + A_\mu A^\mu \sin^2\theta_W + A_\mu Z^\mu \sin\theta_W \cos\theta_W + Z_\mu A^\mu \sin\theta_W \cos\theta + Z_\mu Z^\mu \cos^2\theta_W\big)\phi_0^2 \tag{114c}$$

The second term as

$$-\frac{g_w g_b}{4} W^3_\mu B^\mu \phi^2 = -\frac{g_w g_b}{4}(A_\mu \sin\theta_W + Z_\mu \cos\theta_W)(A^\mu \cos\theta_W - Z^\mu \sin\theta_W)\phi_0^2 \tag{115a}$$

$$= -\frac{g_w g_b}{4}(A_\mu A^\mu \sin\theta_W \cos\theta_W - A_\mu Z^\mu \sin^2\theta_W + Z_\mu A^\mu \cos^2\theta_W - Z_\mu Z^\mu \cos\theta_W \sin\theta_W)\phi_0^2 \tag{115b}$$

The third term as

$$-\frac{g_b g_w}{4} B_\mu W^{3\mu}\phi^2 = -\frac{g_b g_w}{4}(A_\mu \cos\theta_W - Z_\mu \sin\theta_W)(A^\mu \sin\theta_W + Z^\mu \cos\theta_W)\phi_0^2 \tag{116a}$$

$$= -\frac{g_b g_w}{4}(A_\mu A^\mu \sin\theta_W \cos\theta_W + A_\mu Z^\mu \cos^2\theta_W - Z_\mu A^\mu \sin^2\theta - Z_\mu Z^\mu \sin\theta_W \cos\theta_W)\phi_0^2 \tag{116b}$$

$$= -\frac{g_b g_w}{4}(A_\mu A^\mu \sin\theta_W \cos\theta_W + A_\mu Z^\mu \cos^2\theta_W - Z_\mu A^\mu \sin^2\theta - Z_\mu Z^\mu \sin\theta_W \cos\theta_W)\phi_0^2 \tag{116c}$$

The fourth term

$$\frac{g_b^2}{4} B_\mu B^\mu \phi_0^2 = \frac{g_b^2}{4}(A_\mu \cos\theta_W - Z_\mu \sin\theta_W)(A^\mu \cos\theta_W - Z^\mu \sin\theta_W)\phi_0^2 \tag{117a}$$

$$= \frac{g_b^2}{4}(A_\mu A^\mu \cos^2\theta_W - A_\mu Z^\mu \sin\theta_W \cos\theta_W - Z_\mu A^\mu \sin\theta_W \cos\theta_W + Z_\mu Z^\mu \sin^2\theta_W)\phi_0^2 \tag{117b}$$

Now we can combine these terms. We are merely interested in the mass of the fields, thus we need to consider the terms $A_\mu A^\mu, Z_\mu Z^\mu, W_\mu W^\mu, W^\dagger_\mu W^{\mu\dagger}$ and therefore the interaction fields will be omitted. The *Weinberg angle* or *weak mixing angle* measures the angle between g_b and g_w in the Weinberg-Salam model,

hence we obtain the following equations

$$\boxed{\cos\theta_W = \frac{g_w}{\sqrt{g_w^2 + g_b^2}} \text{ and } \sin\theta_W = \frac{g_b}{\sqrt{g_w^2 + g_b^2}}}\,. \tag{118}$$

These will be used to obtain the masses of the involved fields [[6],p.398 & p.423]

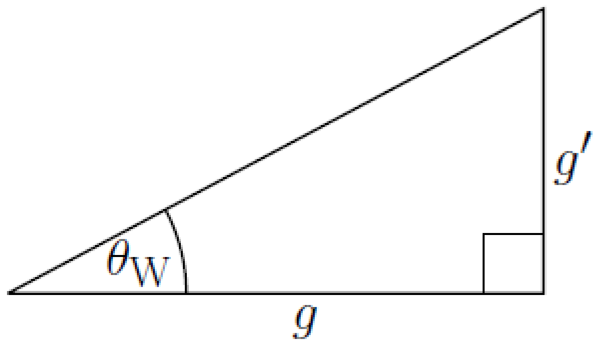

Figure 7: *The Weinberg angle θ_W is the angle between g_b and g_w. In this figure $g' = g_b$ and $g = g_w$.*

- Photon field

  In the expression for the photon field, i.e.

$$A_\mu A^\mu \Big[\frac{g_b^2}{4}\cos^2\theta_W + \frac{g_w^2}{4}\sin^2\theta_W - \frac{g_b g_w}{2}\sin\theta_W\cos\theta_W\Big]\phi_0^2, \tag{119}$$

  the term inside parantheses represents the squared mass, thus we will simplify this expression by using the Weinberg angle.

$$A_\mu A^\mu \Big[\frac{g_b^2}{4}\frac{g_w^2}{g_w^2+g_b^2} + \frac{g_w^2}{4}\frac{g_b^2}{g_w^2+g_b^2} - \frac{g_b g_w}{2}\frac{g_b}{\sqrt{g_w^2+g_b^2}}\frac{g_w}{\sqrt{g_w^2+g_b^2}}\Big]\phi_0^2 \tag{120}$$

$$= A_\mu A^\mu \Big[\frac{1}{4}\frac{g_b^2 g_w^2}{g_w^2+g_b^2}\Big(1+1\Big) - \frac{1}{2}\frac{g_b^2 g_w^2}{g_w^2+g_b^2}\Big] = 0 A_\mu A^\mu. \tag{121}$$

  Hence the photon field remains massless after spontaneous symmetry breaking. In other words, the Higgs field does not interaction with the $U(1)$ symmetry of the general electroweak theory $U(1) \times SU(2)$.

  All involved fields which obey an $SU(2)$ symmetry do acquire mass through this mechanism. Let us start by analysing the Z^0 gauge boson.

- Z^0 field

$$Z_\mu Z^\mu \Big[\frac{g_w^2}{4}\cos^2\theta_W + \frac{g_b^2}{4}\sin^2\theta_W + \frac{g_b g_w}{2}\sin\theta_W\cos\theta_W\Big]\phi_0^2 \tag{122}$$

$$= Z_\mu Z^\mu \Big[\frac{1}{4}\frac{g_w^4}{g_w^2+g_b^2} + \frac{1}{4}\frac{g_b^4}{g_w^2+g_b^2} + \frac{g_b g_w}{2}\frac{g_b}{\sqrt{g_w^2+g_b^2}}\frac{g_w}{\sqrt{g_w^2+g_b^2}}\Big]\phi_0^2$$

$$= \frac{1}{4(g_w^2+g_b^2)} Z_\mu Z^\mu \Big[g_w^4 + g_b^4 + 2 g_b^2 g_w^2\Big]\phi_0^2$$

  Now we use that

$$g_w = \cos\theta_W \sqrt{g_w^2+g_b^2} \tag{123a}$$

and similarly

$$g_b = \sin\theta_W \sqrt{g_w^2 + g_b^2} \tag{123b}$$

to rewrite our expression as

$$\begin{aligned}
&\frac{g_w^2 + g_b^2}{4} Z_\mu Z^\mu \Big[\cos^4\theta_W + \sin^4\theta_W + 2\cos^2\theta_W \sin^2\theta_W\Big]\phi_0^2 \\
&= \frac{g_w^2 + g_b^2}{4} Z_\mu Z^\mu \Big[\cos^2\theta_W (1 - \sin\theta_W^2) + \sin^4\theta_W + 2\cos^2\theta_W \sin^2\theta_W\Big]\phi_0^2 \\
&= \frac{g_w^2 + g_b^2}{4} Z_\mu Z^\mu \Big[\cos^2\theta_W + \sin^4\theta_W + \cos^2\theta_W \sin^2\theta_W\Big]\phi_0^2 \\
&= \frac{g_w^2 + g_b^2}{4} Z_\mu Z^\mu \Big[\cos^2\theta_W + \sin^2\theta_W (\underbrace{\sin^2\theta_W + \cos^2\theta_W}_{=1})\Big]\phi_0^2 \\
&= \frac{g_w^2 + g_b^2}{4} Z_\mu Z^\mu \Big[\underbrace{\cos^2\theta_W + \sin^2\theta_W}_{=1}\Big]\phi_0^2 \\
&= Z_\mu Z^\mu \Big[\frac{g_w^2 + g_b^2}{4}\Big]\phi_0^2
\end{aligned} \tag{124}$$

Hence by comparing this result with Eq. 107 and using that $\phi_0^2 = 2^{-1}\nu^2$ we find that the mass is given by

$$M_Z = \frac{\sqrt{g_w^2 + g_b^2}}{2}\nu \tag{125}$$

- $W^\pm$ fields

The $W^\pm$ fields are the remaining fields to be considered. The terms that carry corresponding massfactors are

$$\frac{g_w^2}{4}(2W_\mu^- W^{+\mu}) = \frac{g_w^2}{2} W_\mu^- W^{+\mu}$$

We recognize that W_μ^- corresponds to annihilation of the W-boson, thus to W_μ and W_μ^+ corresponds to the creation $W^\dagger$ Hence from

$$\frac{g_w^2}{2} W W^\dagger$$

It follows from Eq. 107 that the mass is given by

$$M_W^\pm = \left[\frac{g_w^2 \phi_0^2}{2}\right]^{1/2} = \left[\frac{g_w^2 \nu^2}{2 \cdot 2}\right]^{1/2} = \frac{|g_w|}{2}\nu \tag{126}$$

In the expression for ϕ_0 it is ν, the generic VEV of the Higgsfield, that gives the contribution to the mass. Hence the final results are

$$M_A = 0 \tag{127}$$

$$M_Z = \frac{\sqrt{g_w^2 + g_b^2}}{2}\nu \tag{128}$$

$$M_W^\pm = \frac{|g_w|}{2}\nu \tag{129}$$

5.2 Higgs mechanism in QCD

We need to have a Higgs potential which is capable of obtaining a vacuum expectation value (VEV) for at least one color-nonsinglet Higgs field, in order for a breaking in the $SU(3)_c$ symmetry to occur.

First, let us analyze how many degrees of freedom are involved in the symmetry breaking of QCD. Initially the gluons have $8 \times 2 = 16$ degrees of freedom. No additional degrees of freedom appear for the quarks after the symmetry breaking, but obviously the gluons gain one degree of freedom due to the formerly forbidden longitudinal polarization. The extra degrees of freedom are denoted by d, where $d \in \{1, 2, 3, \cdots, 8\}$ is the amount of gluons that have acquired mass. How many gluons become massive is dependent on the kind of symmetry breaking which occurs, and it is possible that all gluons become massive,

In this paper a mechanism will be analyzed wherein $SU(3)_c$ is broken to an $SU(2)$ symmetry. This new theory has $2^2 - 1 = 3$ generators and it are these gluons that remain massless. The same applies when $SU(3)$ would be broken to $SO(3)$. In general. $SO(n)$ has $\frac{n}{2}(n-1)$ generators, thus the special orthogonal group that we are considering has 3 generators. Only the gluons which occur in the *coset* of both symmetries will acquire mass. In another type of spontaneous symmetry breaking that will be examined, the color symmetry will be completely broken, implying that all eight gluons occur in the coset and hence acquire mass.

Our initital group is represented by

$$G = SU(3),$$

and we require the symmetry to be broken to another group H. These group satisfy the relation

$$H \subset G.$$

We have $g \in G$ and obtain two cosets, given by

$$gH = \{gh \ : h \in H\} \ , \qquad Hg = \{hg \ : h \in H\}. \tag{130}$$

The former is called the left coset and the latter the right coset. Only for non-Abelian groups it is the case that

$$gH \neq Hg,$$

thus we will obtain a left and right coset when breaking $SU(3)_c$.

We can obtain symmetry breaking when the baryon wavefunction is retained. This requires that an $SO(3)$ subgroup of $SU(3)_c$ remains preserved and that all quarks will transform as the vector presentation of this $SO(3)$ group. We have found before, in Eq. (30), that baryons which satisfy the constraints set by $SU(3)$ color symmetry, are described by the wavefunction

$$\epsilon_{ijk} q^i q'^j q''^k. \tag{131}$$

The vector presentation of $SO(3)$ would for baryons be given by

$$\vec{q} \cdot (\vec{q}\,' \times \vec{q}\,''). \tag{132}$$

We need to find the conditions under which these expressions are identical in order to break the symmetry to SO(3). In this mechanism, we will obtain five massive gluons and three massless gluons. This implies that we have five additional degrees of freedom in the final stage, hence these should be present in the Higgs field. A Higgs triplet contains enough degrees of freedom, this would be given by

$$\Psi = \begin{pmatrix} \eta_1 + i\eta_2 \\ \eta_3 + i\eta_4 \\ \eta_5 + i\eta_6 + \nu \end{pmatrix}. \tag{133}$$

This SSB scheme requires that five Goldstone bosons occur in this triplet, whose degrees of freedom can be transferred to the gluons.

When the $SO(3)$ is a subgroup of $SU(3)_c$ we find that 5 gluons exist which obey both symmetries, i.e. they occur in the coset space $SU(3)_c/SO(3)$. It is exactly these gluons that will acquire nonzero masses, whereas the remaining 3 gluons remain massless. These massless gluons correspond to the three generators of the $SO(3)$ symmetry. These gluons would occur naturally in bound states which would make them color singlets, i.e. $\vec{q} \cdot \vec{q}$. Hence, they are the singlets of the $SO(3)$ group and thus we are looking for a Higgs field which contains a component obeying the transformation laws of singlets under the $SO(3)$ subgroup. Because we are considering three quarks, the lowest-dimensional representation that we can utilize is the $\mathbf{3} \times \mathbf{3} \times \mathbf{3} = 27$-dimensional $SU(3)_c$ representation. There exists such a representation which satisfies the recently introduced criteria and leads to $m_g \sim g_s|\nu|$.

A second Higgs mechanism makes use of two Higgs fields. These fields transform according to the fundamental representation of $SU(3)_c$, which has been introduced in Eq. (8). As in the latter Higgs mechanism, five gluons will pick up masses. The procedure itself, however, is completely different. Instead of $SU(3) \to SO(3)$, we now impose

$$SU(3) \to SU(2) \to 1,$$

thus we break the symmetry entirely in two steps. In both of these processes, acquisition of mass occurs, for five and three gluons respectively. Hence, two scales of masses appear for the gluons and these are not necessarily equally heavy. As all gluons acquire mass we obtain eight extra degrees of freedom, which should be enabled by the Higgs fields.

We employ the same Higgs potential as in the electroweak theory, stated in Eq. (82), together with the constraint of Eq. (83). This leads to the same vacuum expectation value as mentioned in Eq. (85c). However, instead of a Higgs triplet, we postulate Higgs singlets now.

Our obtained VEV has a rotational invariance, thus we can set

$$<\psi_0> = \begin{pmatrix} 0 & 0 & \nu \end{pmatrix}^T. \tag{134}$$

This reduces the $SU(3)$ symmetry to one of its subgroups, the $SU(2)$ group. This can be understood geometrically by imagining rotations in the $(x, y)-$plane, thus around the z-axis. In this case the physics is unchanged, thus invariant. An example of an $SU(2)$ matrix is

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \qquad \det(A) = +1, \tag{135}$$

which is one of such 2-dimensional rotations. As a consequence of the VEV, the $SU(3)$ has been broken to $SU(2)$. $SU(3)$ has three independent $SU(2)$ subgroups, it has been broken to the group whose space is spanned by $\{\lambda_1, \lambda_2, \lambda_3\}$, which are the spin matrices. These have no $\hat{z}$-components so we know that the rotation takes place in the proper plane for our new basis for $SU(3)$.

Because the $SU(2)$ symmetry group is homomorphic to the rotation group $SO(3)$ we know that this group has three generators as well. The generators of this group are related to the Pauli spin matrices, as given in Eq. (96). The generators are given by

$$\hat{T}_i = \frac{\sigma_i}{2}, \tag{136}$$

these provide the complete $\mathfrak{su}(2)$ Lie algebra and their structure constant is identical to the Levi-Cevita tensor. The three gluons which obey this algebra are prevented from mass acquisition due to this symmetry. In the Goldstone model, we need to work out the term $D_\mu \Phi^\dagger D^\mu \Phi$, just as in the case of electroweak symmetry. However, we now need to employ the covariant derivative of QCD, which has been presented in Eq. (60). This leads to

$$\big(\partial^\mu - ig_s \hat{F}_a A^\mu_a(x)\big)\Phi^\dagger(x)\big(\partial_\mu + ig_s \hat{F}_a A_{\mu a}(x)\big)\Phi(x), \tag{137}$$

where the relevant term from the Higgs field is the VEV and we search the quadratic gluon terms. This leads to

$$\big(-ig_s \hat{F}_a A^\mu_a(x)\big)\Phi^\dagger(x)\big(ig_s \hat{F}_a A_{\mu a}(x)\big)\Phi(x) = g_s^2 \hat{F}_a^2 \Phi^2 A^\mu_a A_{a\mu} \tag{138}$$

Again, the contribution of the Higgs field is a factor of $2^{-1}\nu^2$ This has as a consequence that the five gluons occurring in the coset $SU(3)_c/SU(2)_c$ gain masses of $m_g = g_s|\nu|$.

5.3 The aspects which make the Higgs mechanism problematic for SU(3)

The advantage of this procedure over the addition of a bare mass term to the Lagrangian is that the renormalizability of the theory has not been lost. In many aspects, the Higgs mechanism resembles the mechanism that gave mass to the gauge bosons in the electroweak theory. However, there are theoretical arguments supporting the claim that we cannot use a similar approach in the case of strong interactions.

Nusseniv and Shrock claim that gluons have a mass whose upper bound is a couple of MeV, satisfying $m_g \ll \Lambda_{QCD}$ [[8],p.5].[20] This has as a consequence that the Higgs fields themselves are as well involved in strong interactions. Perturbation theory is meaningful when small perturbations are analyzed, which is definitely not the case for these strongly coupled Higgs fields. Now, if the condition $\mu^2 < 0$ is imposed in the Goldstone model, Eq. (81), we cannot determine anymore whether we obtain a nonzero vacuum expectation value for our Higgs field [[16]]. Thus, although renormalizability is retained, we now have to ensure that we find a solution to our new problem.

There is a second problem as well. Due to the inapplicability of perturbation theory, we cannot know whether the involved Higgs field have a mass which exceeds or is actually less than the perturbative expression for its mass,

$$m_H \sim \sqrt{\lambda}|\nu|. \tag{139}$$

In the case of relatively light Higgs boson they might be unobservable by experiments, just as small nonzero values of mg would be unobservable. Fortunately, field-theoretic approaches exist for strongly coupled gauge systems.

The Higgs and fermion fields are allowed to be strongly coupled in the lattice QCD formulation. The calculations in this theory lie outside the scope of this paper, however, the virtues of this theory

[20] We will analyze in this paper what this upper bound is based on. This can be found in the chapter where experimental results are discussed, see Section 6.

will be discussed. Lattice QCD is a nonperturbative field theory. A discrete set of spacetime points is chosen where the equations of motions of the involved fields are computed. Lattice QCD is exact local gauge invariant, which has consequences for the Higgs mechanism. The VEV caused by the Higgs particle breaks the $SU(3)$ symmetry, but this quantity vanishes in a local gauge invariant theory. The theory calculates the gauge-invariant quantities, such as the masses of fermions and of the Higgs particle.[21] A problem which is encountered when employing a discrete set of spacetime points is where to set the continuum limit so all relevant lattice parameters are present and the UV-completion is present in the model. Lattice theories are capable of finding the ratio of occurrence of the involved gauge bosons.

'[T]hese lattice studies tended to find ratios of Higgs to gauge bosons masses which did not differ strongly from unity.' [[8],2] As the Higgs fields are color non-singlets, they are expected to create bound states with the quarks or amongst themselves, i.e. $H \cdot q$ and $H \cdot H$. This prediction raised scepticism among many physicists on this version of the Higgs mechanism, because the existence of these predicted bound states has never been experimentally confirmed. Therefore we are led to the conclusion that nonperturbative studies reveal that there are problems with this color-nonsinglet Higgs mechanism. The gluon masses would satisfy $m_g \ll \Lambda_{QCD}$ whereas the Higgs boson would be in the regime $m_H \gg \Lambda_{QCD}$. This leads to a hierarchy problem in the Higgs mechanism. Either the model needs to be reformulated or alternatives need to be sought.

Another problem with this method which has been discovered is that this coupling mechanism undermines the asymptotic freedom of QCD. It are the quartic coupling terms where this phenomenon manifests itself. These terms are not asymptotically free, which means that increasing the energy scale leads to an increase in the coupling constant. This is incompatible with the pivotal role played by asymptotic freedom in QCD, a phenomenon which, although its mathematical derivation has not been provided up to date, is unanimously believed to hold by physicists. Furthermore, because the quartic terms have lost their property of being asymptotically free, another potential problem is faced. The coupling constant can become infinite at a finite energy, a phenomenon which can be attributed to a *Landau pole*. These poles occur only in theories that are not asymptotically free, so every model in QCD wherein gluons become massive risk to be struck by a Landau pole. Although this seems a mathematical inconsistency, this is not necessarily a problem. It is possible that physical processes counteract this divergence or that an energy (UV) cut-off could be introduced.

These problematic entailments activate one's scepticism as to whether a Higgs mechanism can be used at all in quantum chromodynamics. The discussed mechanisms, where the gauge symmetry was broken to a nontrivial subroup gauge symmetry as well as where it was completely broken, were not capable of obtaining a gluon mass which lies in the range of a few MeV.

6 Experimental upper bound on the mass of gluons

Both the procedures which have been discussed above have severe limitations. In their current formulations, they are insufficiently powerful in order to determine an upper bound on the gluon's masses. This does not imply that gluons cannot have a nonzero mass, it is e.g. still not mathematically understood why asymptotic freedom occurs but experiments confirm that it does. Hence it is desirable to examine what experimental results tell about the allowed order of magnitude of m_g. Especially the so-called bag model turns out to be of our interest for this purpose. In order to

[21] As this is the invariant mass, this differs from the bare mass which was introduced in Section 4

understand the experimental data we, nevertheless, need quite an elaborate theoretical framework. Firstly, let us retrieve the Lagrangian containing the bare mass term as given in Eq. (75).

In the current paradigm regarding QCD it is believed that $m_g \ll \Lambda_{QCD}$.[22] The mathematical reason for this is that gluons would otherwise be intergrated out of the theory, a statement which will be discussed henceforth. The Born-Oppenheimer approximation can be employed for the wavefunctions of quarks and gluons. The standard Born-Oppenheimer approximation tells that the wavefunctions of nuclei and their surrounding electrons can be separated. The electrons are relatively light - $m_n/m_e \approx m_p/m_e \approx 1836$ - and therefore the electrons move significantly faster than the components of a nucleus. The latter can be regarded stationary and hence the wavefunctions of the nucleus and the electronic wavefunctions can be separated. Afterward corrections can be added to account for the fact that the nucleus is not completely stationary. In QCD, the same situation occurs. The mass of gluons is relatively small compared to the masses of quarks and therefore we can analyze the wavefunctions of gluons separately from those of the quarks.

A particle with mass m cannot play a role in the effective theory which is valid at energies scales far below the value m [[8], 5]. We know well for QCD what the area is in which perturbative QCD is valid, it is set by the scale parameter of QCD, $m_g \simeq 300$ MeV. Take a gluon with mass m_g and let us examine the case where $m_g > \Lambda_{QCD}$. These particles would be integrated out of the theory as the energy scale μ decreases to Λ_{QCD}, which for QCD would imply that the running coupling constant would never grow to values of $\mathcal{O}(1)$.

The question has now shifted to how much smaller it is than $E = \Lambda_{QCD}$. As has been discussed before, the introduction of a nonzero gluon mass causes a broken color symmetry. Consequently, color confinement does not necessarily hold and separated quarks would not be infinitely energetic anymore. The Higgs particle can be used as an examplory means, this created a nonzero vacuum expectation value and the Higgs was a color nonsinglet particle. A consequence of the broken symmetry is that color confinement is removed from the theory and consequently isolated quarks and gluons should be expected, provided they are energetic enough to induce string breaking. These free particle states are forbidden for the case where $m_g = 0$ and therefore it is interesting to see what happens when we examine

$$\lim_{m_g \to 0} \Big(\frac{m_g}{\Lambda_{QCD}}\Big). \tag{140}$$

Before the limit is reached, we have free particles in our theory. Such color-nonsinglet states would have to disappear when $m_g = 0$ is reached in Eq. (140). These states are not necessarily forbidden in nature, however, they are forbidden in our low-energy effective field theory. Therefore color non-singlet particles would have to have masses which significantly exceed the scale parameter of QCD, in order for them to be integrated out of our modified QCD. This signifies that QCD does not strictly obey color confinement. Such a theory, in which massive particle are not excluded from being free particles, is said to exhibit the property of being quasiconfining, in the same sense as in the context of the bare mass term. We need to investigate whether we can find the dependency of the mass of a free gluon or quark on the mass of a usual gluon, which is subject to color confinement.

The scenario where the masses of free quarks are dependent on the mass of gluons has been confirmed by experiment. This has been borne out in the *bag model* of the Massachusetts Institute of Technology (MIT). The bag model is a covariant model which describes hadron structures, while

[22] Hence we know that $m_g \ll 300$ MeV. Convering to SI-units, this means that $m_g \ll \frac{\Lambda_{QCD}}{c^2} = \frac{3 \cdot 10^8 eV}{9 \cdot 10^{16} (m/s)^2} = 3.33 \cdot 10^{-7}$ kg. Also intuitively one might predict that it is indeed smaller.

it 'retains an elegant simplicity' [[26],226]. In the MIT model, the field variables are hung solely on the spacetime points where either (anti)quarks are present or the fieldstrength of gluons is nonzero. This set of spacetime points has been called a *bag*, hence the name of the model [[27]]. The bag simulates nonperturbative effects and the definition of the bag ensures that no currents or energy-momenta leave the bag [[4],§6.6]. For the boundary conditions of the model, the *dual Meissner effect* has been employed.

This choice can be understood by an examining the similarities between the Meissner effect and the chromoelectric flux which is present between quarks. Mandelstam and t Hooft have shown in 1975 that color confinement could be understood as being described by the dual Meissner effect [[22]]. The pivotal aspects of this effect will briefly be discussed to make the reader familiar with them, after which the analogy with the chromoelectric flux can be recognized.

In superconductors which are cooled below their critical temperature, the phenomenon has been observed that these objects try to repel external magnetic fields.[23] This has been confirmed by conservation of flux, when the superconductor is cooled below its critical temperature the internal magnetic field decreases whereas its exteral field increases, leaving the sum of the magnitude of the fields invariant. A second discovery was made that magnetic field lines which are *forced* to run through the superconductor are compressed to *magnetic flux tubes* [**??**]. These tubes have a uniform energy density and exist for the complete length of the superconductor without undergoing any form of damping. Particularly the existence of these flux tubes is what motivated t Hooft and Mandelstam to suggest an analogy between the Meissner effect and QCD [[26]].

We assumed the superconductor to be a dual superconductor. This means that the electric and magnetic fields can be interchanged and therefore electric field lines are also expelled by the object. Superconductivity is described by the BCS-theory, which has been proposed by Cooper, Schrieffer and Bardeen, all of whom have been awarded a Nobel Prize for this theory [[15]]. In this model, superconductivity is found to be a consequence of the condensation of electrons into Cooper pairs, i.e. bound pairs of electrons and hence electric dipoles. Because of the dual character of the theory, the electric dipoles create an electric field which can be described identically to the magnetic field of a magnetic dipoles, thus of magnets with a north and south pole.

We will discuss the case of mesons here as these particles solely contain one flux tube and the antiquark possesses exactly the anticolor of the quark. The quark and antiquark are regarded as electric monopoles and therefore we expect magetic monopoles to play a role in this theory. We are considering their color charges now instead of electric charge, but the conjecture presented by Mandelstam and t Hooft entails that they can be described likewise. Now the equivalent (dual) field would be that of magnetic monopoles and we take the *vacuum* to be the superconductor for the color charge. The electric color field lines run from the quark to the antiquark and are compressed within a flux tube. As explained for the Meissner effect, the energy stored in this flux tube increases linearly with distance. Due to this, we can obtain infinitely energetic free particles when their separation distance keeps on increasing, i.e. color confinement. This is attributed to the presence of the color magnetic monopoles in this model. Its existence for the case electromagnetism has never been found, however, it has been proven that the magnetic monopole would exist in non-Abelian theories, thus definitely for colors in QCD [[21]].

This interpretation has the advantage that it exhibits crucial features of QCD. It was exactly for this reason that the dual superconductor effect has been incorporated in the bag model. The

[23]It does not hold for arbitrarily large fields of $|\vec{\mathbf{B}}|$. Above a certain threshold, the magnetic field penetrates the superconductor and the latter loses its superconductivity.

MIT has found the following relation [[10]]

$$m_{q_{dr}} = \frac{\sigma}{m_g}\left[1 + \mathcal{O}\left(\left(\frac{m_g^2}{\sigma}\right)^{\frac{1}{6}}\right)\right] = \frac{0.18}{m_g} \text{ GeV}. \tag{141}$$

Sigma represents the string tension in the chromoelectric flux tube. As mentioned in the section on Asymptotic Freedom (2.4), the energy stored per area in the flux tube ought to be constant. This is what is implemented in the bag model. Experimentally the Regge slope α has has been determined, this value satisfies $\alpha \sim \sigma^{-1}$ and led to the value of[24]

$$\sigma = 0.90 \text{ GeV fm}^{-1}. \tag{142}$$

Furthermore, this string tension scales with the squared QCD-scale. Now we employ another formula, also found by MIT, which relates the mass of gluons to the mass of quarks [[10]],

$$m_{q_{dr}} = \frac{2}{3} m_{g_{dr}}. \tag{143}$$

Combining these pieces of information leads to

$$m_{q_{dr}} = \frac{2}{3} m_{g_{dr}} = C\frac{\Lambda_{QCD}^2}{m_g}\left[1 + \mathcal{O}\left(\left(\frac{m_g}{\sqrt{\sigma}}\right)^{\frac{1}{3}}\right)\right], \tag{144}$$

where C is the constant resulting from the proportionality $\sigma \propto \Lambda_{QCD}^2$. The mass of the quark is now written in terms of the mass of the gluon. Let us analyze what happens when the limit $m_g/\Lambda_{QCD} \to 0$ is examined. Firstly, it must be noted that this limit is smooth in the sense of low-energy effective field theory. We now that the following holds,

$$\lim_{m_q \to 0} \frac{m_q}{\Lambda_{QCD}} = 0 \;\leftrightarrow\; \lim_{m_q \to 0} \frac{\Lambda_{QCD}}{m_q} = \infty, \tag{145}$$

and we can see from Eq. (144) that either the mass of the free gluon or of the free quark diverges. Due to this mass divergence, these particles are integrated out of the theory, which is valid for low energies only. As they are integrated out of the theory, this model cannot provide information on the masses of the gluons. Thus this is consistent with our knowledge that the only physical states in this low-energy theory are color singlets and the theory can still be quasiconfining. The free particles are integrated out while the limit is approached and hence there are nonzero values of m_g for which the free particles are already integrated out, emphasizing the possibility of nonzero gluon masses.

Free quarks searches have been carried out. These particles have never been observed, but we can obtain an upper mass bound for the bound gluons if we can find a lower mass bound for the free gluons, which is actually what happens if we cannot observe particles at energies below the value of the particle's mass. As mentioned in Section 2.1, quarks are involved in weak and electromagnetic processes. These interactions are understood in the covering electroweak theory, as presented in Section 5.1, but we need to make assumption about their decays and transformation properties [[24]]. Three quarks exist with $Q = 2/3$ and the same amount with $Q = -1/3$, the lower limits of their masses resulting from this lie in the range $[200, 340]$ GeV. We are looking for lower limits

[24] $\sigma = \frac{1}{2\pi\alpha}$.

here, because this results in an upper bound for gluons due to Eq. (144), wherein $m_g \propto m_q^{-1}$. When 300 GeV is taken as an exemplary value, we obtain the constraint $m_g \leq 0.5$ MeV. Since the lower bound is dependent on model-dependent factors of the MIT bag model - and 300 GeV is not the lowest value that has been obtained in this model - we ought to incorporate a theoretical uncertainty in our upper bound for the gluon mass. Therefore we are led to the conclusion that the mass is in the neighborhood of, or smaller than, 1 MeV, i.e.

$$m_g < \mathcal{O}(1) \text{ MeV}. \tag{146}$$

This limit satisfies the criterion of being small in comparison to Λ_{QCD}.

What expectations do we have about free color-nonsinglet particles and can they teach us anything about upper limits of gluons? One expectation is that the size of deconfined, dressed elementary particles exceeds the typical size of 1 fm of hadrons. This follows rather straightforwardly from our bag model, presented in Eq. (144). We want to calculate the the radius of such particles and hence we can employ the relationship

$$m = V\rho, \tag{147}$$

where ρ represents the energy density which we know to be constant, i.e. independent of spacetime co-ordinates. Furthermore, we know that [[8],6]

$$\rho = \Lambda_{QCD}^4, \tag{148a}$$

and because we are dealing with spherically symmetrical elementary particles

$$V = \frac{4\pi}{3} r^3. \tag{148b}$$

Now we obtain the following relation,

$$\frac{4\pi\Lambda_{QCD}^4}{3} r^3 = m_{q_{dr}} \sim \frac{\Lambda_{QCD}^2}{m_g}\Big[1 + \mathcal{O}(\Big(\frac{m_g}{\sqrt{\sigma}}\Big)^{\frac{1}{3}})\Big], \tag{149a}$$

from which it follows that

$$r^3 \sim \underbrace{\frac{3}{4\pi\Lambda_{QCD}^4}\frac{\Lambda_{QCD}^2}{m_g}\Big[1 + \mathcal{O}(\Big(\frac{m_g}{\sqrt{\sigma}}\Big)^{\frac{1}{3}})\Big]}_{\text{constants will be omitted for the proportionality}}, \tag{149b}$$

$$r^3 \sim \frac{1}{m_g\Lambda_{QCD}^2}. \tag{149c}$$

$$\therefore\ r \sim \Big(\frac{1}{m_g\Lambda_{QCD}^2}\Big)^{1/3} = \frac{1}{\Lambda_{QCD}}\Big(\frac{\Lambda_{QCD}}{m_g}\Big)^{1/3} \sim 1 \text{ fm} \cdot \Big(\frac{\Lambda_{QCD}}{m_g}\Big)^{1/3}. \tag{149d}$$

It follows from this that a small nonzero m_g leads to radii belonging to dressed quarks and gluons which are substantially larger that the 1 fm which is typically taken to be the radius of hadrons. If free quarks and/or gluons would be observed, measurements on the radii of free color-nonsinglet states would indicate the order of magnitude of m_g.

Another methodology can be used in order to investigate the upper limit on the gluon's mass more accurately. As discussed in Section 2.4, the short-distance potential is dominated by the processes of single-gluon exchanges. The short-distance potential can consequently be formulated in the Coulombic form

$$V_{q\bar{q}} = \frac{4\alpha_s(\mu)}{3}\frac{1}{r}. \tag{150}$$

It has been discussed that small distances correspond to large energy transfers μ. A nonzero gluon mass would not affect the potential significantly and therefore we can set $m_g \neq 0$ and employ Eq. (150) for our calculation. For distances bigger than the size of hadrons the potential is dominated by energy of the chromoelectric flux tube, i.e. the second term in Eq. (38), thus

$$V_{q\bar{q}} = \sigma r \qquad \text{for } r \geq \Lambda_{QCD}^{-1} \sim 1 \text{ fm}. \tag{151}$$

This term *is* affected by setting $m_g \neq 0$ because the mass causes a damping factor in the flux tube. The Yukawa potential is identical to the Coulomb potential for $m_g = 0$, but in the case of nonzero m_g the damping factor of $e^{-km_g r}$ becomes manifest. Instead of propagating without bound, the flux tube will vanish for a sufficiently large $\mathbf{r}$. The potential can now be represented as [[8],6]

$$V_{q\bar{q}} = \underbrace{\frac{4\alpha_s(\mu)}{3}\frac{1}{r}}_{\text{coulombic}} + \underbrace{\sigma r \begin{cases} 1 & \text{for } 1 \text{ fm} < r < m_g^{-1} \\ \exp\{-m_g r\} & \text{for } r \geq m_g^{-1} \end{cases}}_{\text{chromoelectric tube}}. \tag{152}$$

The consequence that this has, is that the potential $V_{q\bar{q}}$ has gotten an upper bound, rather than undergoing a continual increase with distance. Both terms of the lagrangian decrease for $r \gg m_g^{-1}$ and we find that the maximum of the potential lies at $r \sim m_g^{-1}$. As usual, the force depends on the potential by

$$\vec{F} = -\vec{\nabla} V_{q\bar{q}}(r). \tag{153}$$

We see that there has to be a distance where the force vanishes, i.e. at the maximum of the potential. In the case where the potential is near its maximum but there is an energy deficit to cross the maximum. It is energetically favorable (and possible) for quarks to tunnel through this potential barrier. This would give information on the damping factor, which contains the factor of our interest - m_g. We need heavy quarks for this phenomenon to be observable - a more elaborate discussion of which can be found below - because the damping factor would only be measurable in that case. However, QCD consists of quarks which are too light for that purpose and this leads to further complications.

Due to the property of quarks of being light, quarks move at relativistic velocities. The potential energy which we associated with the string is nonrelativistic and thus cannot be interpreted as capable of giving reliable results in this case. Furthermore it can be shown that the string will break and induce a process of hadronization before the damping factor plays a role. We have found that $m_g < \mathcal{O}(1)$ and we know that the following relation holds

$$(1 \text{ MeV})^{-1} = 200 \text{ fm}. \tag{154}$$

It follows from pQCD with $m_g = 0$ that string breaking can occur from $r = 1$ fm. The nonzero mass of the gluons does not play a role in the potential before $r = 200$ fm and therefore we can conclude that the string breaks at the same typical distances as when the force mediator was massless. The flux tube will never maintain intact up to 200 fm, instead it will have produced $q\bar{q}$ particles with additional gluons. The chromoelectric flux tube is also capable of producing hadronic states, for instance 4π, $2K$ and 2ρ states. This process is called hadronization and this phenomenon ensures that the damping factor of $e^{-m_g r}$ does not play a physical role in nature.

Therefore the best experimental that we have obtained thusfar came from the MIT bag model, in combination with the free quark searches. Theoretical uncertainty has been incorporated in this theory and this led to the upper limit $m_g < \mathcal{O}(1)$. This is the most stringent upper limit that can be obtained with our employed models. Some authors have claimed to have found smaller values for the upper bound, e.g. Ynduráin. He has employed Eq. (152), which we know to be invalid for nonrelativistic quarks, and found the mass bound $m_g < 2 \cdot 10^{-10}$ MeV [[4],[8],p.1]. Furthermore, he made the assumption that the string would not break up to values of m_g^{-1}. When more stringent upper bounds than 1 MeV of the gluon mass are inserted in the potential, the distance that it takes before the damping factors were to play a physical role would even increase - due to $r = E^{-1}$ in natural units. So there is no justification for the methodology where m_g is determined from this damping factor.

A conceptual issue will need to be taken into account for gluon mass in our order of magnitude. As discussed before, gluons with small nonzero masses are quasiconfined. This leads us to principles of ordinary quantum mechanics, the principle that plays a crucial role here is Heisenberg's uncertainty principle, i.e.

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}. \tag{155}$$

For the temporal components, this implies that

$$\Delta(t)\Delta(E) \geq \frac{\hbar}{2}, \tag{156}$$

and we know the energy to be defined by the relation

$$E = \sqrt{k_0 k^0 + |\vec{k}_i \vec{k}^i|} = \sqrt{m^2 + |\vec{k}_i|^2}. \tag{157}$$

We cannot define an upper limit for m_g when $m_g \ll \Lambda_{QCD}$, as setting this mass one order of magnitude smaller than this scale already becomes problematic. The gluons are effectively in the volume belonging to the size of hadrons, i.e. with radius $1/\Lambda_{QCD} \sim 1$ fm. The three-momentum can vary as

$$\Delta|\vec{k}_g| \sim \Lambda_{QCD}, \tag{158}$$

and the energy of the gluon cannot be measured to a better accuracy. Therefore we are unable to distinguish the case where

$$E_g = \sqrt{|\vec{k}_g|^2 + m_g^2} \tag{159}$$

from

$$E_g = \sqrt{|\vec{k}_g|^2} = |\vec{k}_g|, \tag{160}$$

because of the allowed fluctuations of the energy. Only if the mass of gluons were in the area of - or bigger than - Λ_{QCD} would the mass be measurable. This introduces the philosophical issue that gluons might have a positive nonzero mass, but nature has made it impossible to experimentally confirm a nonzero value and hence determine whether this is the case. This is reflected by the fact that the confined gluon propagator does not contain a pole, and hence a contour integral in the complex p^0-plane does not pick up contribution from any residues. This emphasizes that the gluon does not have a well-defined mass.

7 Concluding remarks

Theoretically it is predicted that the mass of all eight gluons should be exactly zero. We have discussed in this paper that the $SU(3)_c$ symmetry is broken when $m_g \neq 0$, which has led to quasiconfining theories in the section on the bare mass Lagrangian term as well as in the MIT bag model. The fact that free quarks and gluons have never been observed and that the particles come out naturally massless make it plausible that they are indeed massless. On the other hand, we know from the formula which relates the masses belonging to (1) ordinary gluons and (2) free gluons and quarks, presented in Eq. (141), that the latter are inversely propertional to the former. Taking $m_g = \mathcal{O}(1)$ MeV, we find that the masses of the free particles are too heavy to occur in pQCD. This implies that it costs lots of energy to create these particles from bound quark or gluon states. If these particles were to be observed in nature once, this would immediately imply that color is not an exact conserved quantity and this signifies that a gluon mass term breaks the $SU(3)_c$ symmetry, which becomes apparent for high energies. The upper mass bound has been determined in a phenomenological framework and the exact mechanism responisble for mass acquisition should be examined if free quarks were observed.

In the theoretical models discussed in this paper, severe problems occurred. The Lagrangian $\mathcal{L}_{QCD}$ became nonrenormalizable when adding a bare mass term and the Higgs mechanism makes the prediction that bound Higgs states exist, which have not been experimentally confirmed. A troublesome aspect that is encountered when analyzing theoretical model which would be capable of a mass generation of $m_g < \mathcal{O}(1)$ MeV, is that these gluons become strongly coupled, therefore nonperturbative studies need to be utilized in order to find theoretical mass acquisition models. There are models which might still be capable of giving gluons a nonzero mass, one of them is the *Dynamical breaking of* $SU(3)_c$ which can be found in [[8]]. Furthermore, $SU(3)$ has many nontrivial subgroups and another scheme can be found in [32]. There might be Higgs mechanisms wherein no predictions are made that are in conflict with the experimental data.

Appendices

1 Appendix A: The antisymmetric color wavefunction of baryons

$$\hat{F}_1\chi_B^c = \hat{F}_1\big(\alpha_1 r_1 g_2 b_3 + \alpha_2 b_1 r_2 g_3 + \alpha_3 g_1 r_2 b_3 + \alpha_4 r_1 b_2 g_3 + \alpha_5 b_1 g_2 r_3 + \alpha_6 g_1 b_2 r_3\big). \tag{161}$$

Working out the first and third term leads to

$$\alpha_1\bigg((\hat{F}_1 r_1)g_2 b_3 + r_1(\hat{F}_1 g_2)b_3 + r_1 g_2(\hat{F}_1 b_3)\bigg) + \alpha_2\bigg((\hat{F}_1 g_1)r_2 b_3 + g_1(\hat{F}_1 r_2)b_3 + g_1 r_2(\hat{F}_1 b_3)\bigg)$$

$$= \alpha_1\left(\frac{1}{2}g_1 g_2 b_3 + \frac{1}{2}r_1 r_2 b_3\right) + \alpha_2\left(\frac{1}{2}r_1 r_2 b_3 + g_1 g_2 b_3\right),$$

where we can recognize the terms inside parentheses to be equal. The same occurs when we apply to $\hat{F}_1$ to the second and fifth term (leading to a term of $\frac{b_1}{2}(g_2 g_3 + r_2 r_3)$ and to the fourth and sixth terms, leading to $\frac{1}{2}(g_1 b_2 g_3 + r_1 b_2 r_3)$. As the sum of all these terms needs to cancel out, this immediately leads to the additional information that

$$\alpha_1 = -\alpha_3,\ \alpha_2 = -\alpha_5,\ \alpha_4 = -\alpha_6. \tag{162}$$

Now let us analyze the action of $\hat{F}_7$, for which we have found the following relations; $\hat{F}_7 g = ib$, $\hat{F}_7 b = -ig$ and $\hat{F}_7 r = 0$. By emplying the same proceduce, we obtain

$$i\alpha_1 r_1(b_2 b_3 - g_2 g_3) + i\alpha_4 r_1(-g_2 g_3 + b_2 b_3) = 0, \tag{163a}$$

$$\therefore \alpha_1 = -\alpha_4. \tag{163b}$$

When all these conditions are combined, they can be described by

$$\boxed{\chi_B^c = \epsilon_{ijk} r_i g_j b_k}. \tag{164}$$

Where ϵ_{ijk} is the usual Levi-Civita symbol. The cases $\alpha_1 = -\alpha_3 = -\alpha_4$ can easily be checked - they indeed require an odd amount of permutations, the same applied to the requirements that $\alpha_2 = -\alpha_5$ and $\alpha_4 = -\alpha_6$, whose validity is consistent with the Levi-Civita symbol as well.

2 Appendix B: Gell-Mann matrices and the color octet

Gell-Mann matrices

In this Appendix a derivation of the Gell-Mann matrices will be given soon. In the $SU(3)$-group we will examine elements close to the identity matrix, i.e. an infinitesimal expansion up to first order

$$\hat{U} = \hat{E} + \hat{A}. \tag{165}$$

A property of every special unitary group is that its elements satisfy the relation

$$\hat{U}\hat{U}^\dagger = +\hat{E}. \tag{166}$$

Hence we have

$$(\hat{E} + \hat{A})(\hat{E} + \hat{A})^\dagger = (\hat{E} + \hat{A})(\hat{A}^\dagger + \hat{E}^\dagger) = \hat{E}\hat{A}^\dagger + \hat{E}\hat{E}^\dagger + \hat{A}\hat{A}^\dagger + \hat{A}\hat{E}^\dagger,$$

$$= \hat{A}^\dagger + \hat{A}\hat{A}^\dagger + \hat{A} = \hat{E}. \tag{167}$$

We know as well that $\hat{A}\hat{A}^\dagger = \hat{E}$, hence we obtain

$$\hat{A} = -\hat{A}^\dagger, \tag{168}$$

which ensures that $\hat{A}$ contains no imaginary numbers on the diagonal. Furthermore, this implies that the most general form of the final matrix obeys the following biconditional

$$A_{ij} = a_{ij} + ib_{ij} \leftrightarrow A_{ji} = -a_{ij} + ib_{ij}. \tag{169}$$

We also obtain

$$\det\left(\hat{U}\right) = 1 \rightarrow \mathrm{Tr}\left\{\hat{A}\right\} = 0, \tag{170}$$

implying that in the case of $SU(3)$ symmetry the following formula has been satisfied

$$\sum_{i=1}^{3} A_{ii} = 0. \tag{171}$$

This can all be parameterized by

$$\hat{A} = \begin{pmatrix} a_3 + a_8 & a_1 - ia_2 & a_4 - ia_5 \\ a_1 + ia_2 & a_8 - a_3 & a_6 - ia_7 \\ a_4 + ia_5 & a_6 + ia_7 & -2a_8 \end{pmatrix}. \tag{172}$$

This is easily decomposed into the eight Gell-Mann matrices by using

$$\hat{A} = \sum_i a_i \hat{F}_i, \tag{173}$$

which can be checked for, e.g.

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \text{and} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{174}$$

The other matrices mentioned in Eq. (11) are obtainable in an identical fashion.

The color octet

The color octet of the gluons is given by

$$\left.\begin{array}{llll} O_1 = \dfrac{r\bar{b} - b\bar{r}}{\sqrt{2}}, & O_2 = \dfrac{r\bar{g} - g\bar{r}}{\sqrt{2}}, & O_3 = \dfrac{r\bar{g} - g\bar{r}}{\sqrt{2}}, & O_4 = \dfrac{r\bar{r} - b\bar{b}}{\sqrt{2}} \\ O_5 = -i\dfrac{r\bar{b} - b\bar{r}}{\sqrt{2}}, & O_6 = -i\dfrac{r\bar{g} - g\bar{r}}{\sqrt{2}}, & O_7 = -i\dfrac{b\bar{g} - g\bar{b}}{\sqrt{2}}, & O_8 = \dfrac{r\bar{r} + b\bar{b} - 2g\bar{g}}{\sqrt{6}} \end{array}\right\} \tag{175}$$

3 Appendix C: Lagrangian for gluons

$$\mathcal{L} = \underbrace{-\frac{1}{4}G^i_{\mu\nu}(x)G^{\mu\nu}_i(x)}_{\text{Gluons}}. \tag{176}$$

$$G^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu - g_s f_{ijk} A^j_\mu A^k_\nu, \tag{177a}$$

$$G^{\mu\nu}_i = \partial^\mu A^\nu_i - \partial^\nu A^\mu_i - g_s f_{ilm} A^\mu_l A^\nu_m. \tag{177b}$$

Thus our Lagrangian is represented by

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu - g_s f_{ijk} A^j_\mu A^k_\nu)(\partial^\mu A^\nu_i - \partial^\nu A^\mu_i - g_s f_{ijk} A^\mu_j A^\nu_k). \tag{178}$$

$$\left.\begin{aligned} &= -\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial^\mu A^\nu_i - \partial^\nu A^\mu_i) \\ &-\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(-g_s f_{ilm} A^\mu_l A^\nu_m) \\ &-\frac{1}{4}(-g_s f_{ijk} A^j_\mu A^k_\nu)(\partial^\mu A^\nu_i - \partial^\nu A^\mu_i) \\ &-\frac{1}{4}(-g_s f_{ijk} A^j_\mu A^k_\nu)(-g_s f_{ijk} A^\mu_j A^\nu_k) \end{aligned}\right\}. \tag{179}$$

We know that the second term can be written as

$$-\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(-g_s f_{ilm} A^\mu_l A^\nu_m) = \underbrace{\frac{1}{4} g_s f_{ilm} \partial_\mu A^i_\nu (A^\mu_l A^\nu_m)}_{:=\chi} - \frac{1}{4} g_s f_{ilm} \partial_\nu A^i_\mu (A^\mu_l A^\nu_m). \tag{180}$$

In the last term the substitution $\mu \leftrightarrow \nu$ is made, which corresponds to an interchange of $l \leftrightarrow m$ in color indices. Because of the antisymmetric character of the structure constant, we can rewrite this term as

$$-\frac{1}{4} g_s f_{ilm} \partial_\nu A^i_\mu (A^\mu_l A^\nu_m) \overset{\mu\leftrightarrow\nu}{=} -\frac{1}{4} g_s \underbrace{f_{iml}}_{=-f_{ilm}} \partial_\mu A^i_\nu (A^\nu_m A^\mu_l) = \frac{1}{4} g_s f_{ilm} \partial_\mu A^i_\nu (A^\mu_l A^\nu_m) = \chi. \tag{181}$$

Hence the complete term can be represented as

$$\frac{1}{2} g_s f_{ilm} \partial_\mu A^i_\nu (A^\mu_l A^\nu_m) \tag{182}$$

Similarly the third term in Eq. (179) can be written as

$$\frac{1}{2} g_s f_{ijk} A^j_\mu A^k_\nu \partial^\mu A^\nu_i \tag{183}$$

and $\mathcal{L}_{gluons}$ can be rewritten as

$$\left.\begin{aligned} &= -\frac{1}{4}(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu)(\partial^\mu A^\nu_i - \partial^\nu A^\mu_i) \\ &+\frac{1}{2} g_s f_{ilm} \partial_\mu A^i_\nu (A^\mu_l A^\nu_m) \\ &\frac{1}{2} g_s f_{ijk} A^j_\mu A^k_\nu \partial^\mu A^\nu_i \\ &-\frac{g_s^2}{4} f_{ijk} f_{ilm} A^\mu_j A^\nu_k A^l_\mu A^m_\nu) \end{aligned}\right\}. \tag{184}$$

This is identical to the Lagrangian mentioned in Chapter (3.2). We recognize the last three lines to correspond to cubic and quartic terms and the first terms correspond to the description of gluons which do not undergo interactions.

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