ATTRACTOR INFLATION AND MODULI STABILIZATION IN STRING THEORY

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ABSTRACT

A large class of inflation models called alpha-attractors predicts a tensor-to-scalar ratio and a spectral tilt that sits right at the observationally favored region found by Planck. These models exhibit an attractor structure due to a relation between the natural geometrically defined inflaton and the canonically defined inflaton, a relation which is analogous to the one between velocity and rapidity in special relativity. Recently, an alpha attractor model was constructed with a single chiral superfield, using Kähler- and superpotentials which appear remarkably natural from the perspective of string theory. It is therefore interesting to see whether it can consistently be coupled to the moduli of a string theory compactification. We find results that are to some extent in line with previous work on this subject. However, we find some interesting improvements on older models that we argue are generic for alpha-attractor constructions. The model performs especially well when it is coupled to a nilpotent chiral superfield for stability.

INTRODUCTION

In the world of high-energy physics, there are two speculative theories which have produced enormous excitement, among professionals working in the field as well as in popular science. The first of these is *string theory*. It is currently the only viable candidate for a theory of everything: a theory which provides a quantum description of gravity, and unifies it with the forces of the Standard Model in a single framework. It is a theory of extended 1-dimensional objects called strings, and higher-dimensional objects called branes. The vibrational modes of these strings are the point particles of particle physics. String theory requires the existence of six extra spatial dimensions. To make contact with the real world, these extra dimensions must be curled up into very small compact spaces called Calabi-Yau manifolds. Whereas the gauge theories of the Standard Model warn us of their own limited range of validity through the ultraviolet divergences that appear in loop diagrams, string theory is completely free of these divergences. Furthermore, string theory is formulated without any dimensionless free parameters, whereas the Standard Model requires 20. This suggests string theory is a fundamental theory of nature, or something very close to it.

The second theory is *cosmological inflation*. Inflation solves the naturalness problems of the Standard Model of cosmology, the ΛCDM-model. These problems are all cousins of the *horizon problem*. The Cosmological Microwave Background (CMB) is the left-over radiation from when the universe was hot and dense enough to be completely opaque to radiation. Photons scattered constantly off ionized nuclei and electrons, producing a radiation spectrum that is closer to the theoretical blackbody radiation than anything else ever observed. However, the CMB radiation is suspiciously uniform. The temperature fluctuates by only about 1 part in 10^5 in different spatial directions. This is a fine-tuning problem, since the CMB consists of some 10^4 patches which could never have been in causal contact with each other according to the ΛCDM-model. Inflation solves this problem by positing that there was a period of accelerated expansion in the early universe. This shrinks the size of the observable universe at early times down to such a small size that it was all in causal contact for a period of time. This allowed the observable universe to reach thermal equilibrium. Quantum fluctuations in the *inflaton field*, a scalar particle which drove inflation, then produced the visible thermal fluctuations present in the CMB, which later grew into the large-scale structure of the present-day universe.

It is extremely appealing to combine these two succesful and exciting theories in a single framework. This prospect is made even more attractive by the fact that inflation is more natural within a SUSY model. Inflation takes place at an energy scale which is relatively close to the Planck scale, which makes it sensitive to the details of high-energy physics. Supersymmetry can reduce the influence of the ultraviolet sensitivity. On the one hand, the sensitivity is a nuisance for model building as it may spoil interesting possibilities. On the other hand, the sensitivity of inflation may serve as a probe into the physics of energy scales which are completely inaccessible at modern particle colliders.

Whereas inflation is made more natural by SUSY, string theory absolutely requires it. However, string theory contains a large number of scalar fields called *moduli*. They appear as the "breathing modes" of the extra dimensions. These scalars are naturally massless, which is phenomenologically unacceptable. Furthermore, some of these moduli control the size and shape of the extra dimensions. If they are not dynamically constrained, string theory cannot be effectively four-dimensional. The moduli must be stabilized by some mechanism. However, it turns out that these stabilization mechanisms can easily spoil a model of inflation by gravitational interactions.

In the past year, a model of inflation was constructed which appeared remarkably natural from the perspective of string theory. This model was formulated in supergravity, the low-energy effective field theory of string theory. It was an example of a class of models called α-attractors. These models make predictions for the inflationary observables, the spectral tilt n_s and the tensor-to-scalar ratio r, which sit right at the experimentally favored region discovered by CMB measurements made by the Planck satellite. Furthermore, these predictions are not sensitive to the details of the model's formulation. This makes it an excellent candidate for a string theory model, since the gravitational interactions with the moduli may not be enough to significantly affect the observable predictions.

If we want to realize α-attractor models within string theory, we will first need to see if they can survive being coupled to string theory moduli. This will be the objective of this thesis. We will start by giving an introduction to the concepts described above. However, this will still not be entirely self-contained. We assume some familiarity with high-energy physics, including concepts from quantum field theory and general relativity, as well as knowledge of differential geometry, including concepts such as (almost) complex structures, (co)homology groups and differential forms. For a general treatment of the differential geometry, see [44].

A NOTE ON CONVENTIONS

Most of the thesis makes use of Planck units, which is the system of units in which $c = \hbar = G = 1$, so that $M_{Pl} = 1$ determines the mass scale. All equations and plots will use Planck units, unless otherwise indicated (sometimes we will restore M_{Pl} in equations, when it makes things more understandable).

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CONTENTS

Part I

BACKGROUND INTRODUCTION

1 INFLATIONARY COSMOLOGY

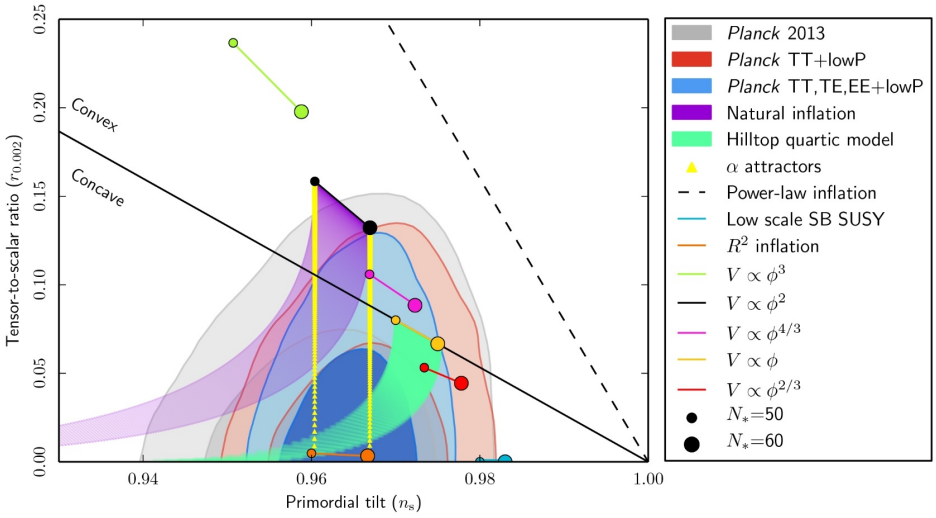
The theory of inflation was conceived to deal with a number of naturalness problems in cosmology. The most significant of these is the *horizon problem*: running back the clock in the ΛCDM model cosmology, we find that some points on the CMB could never have been in causal contact with each other. In fact, there is around 10^4 causally disjoint regions on the CMB. However, the CMB is remarkably uniform: it has the same temperature everywhere, up to inhomogeneities δT of order $\frac{\delta T}{T} = 10^{-5}$. There must be some explanation for this, unless we are to accept a remarkable deal of fine-tuning in the initial conditions of the universe. Another problem concerns the flatness of the universe. Current observations tell us that the universe is very close to being exactly flat, to about one part in a hundred, $|\Omega - 1| \simeq 0.01$ (where Ω is the density in units of the *critical density* Ω_c, which is the density required to make the universe exactly flat). If we run back the clock another time, we find that it must have been even closer to exactly flat in the past. By the time we reach GUT-scale energies, the density parameter Ω must be equal to unity up to about one part in 10^{-55}. This is an obscenely fine-tuned number. We require a mechanism that acts as an attractor toward flatness. Further naturalness problems in cosmology include the magnetic monopole problem and the entropy problem.

A solution to these problems was proposed in the early 1980s by Alan Guth [26]. His idea was to posit the existence of a period of accelerated expansion in the early universe, called *inflation*. Such an accelerated expansion drives the universe closer to flatness, and allows for patches of the CMB that were causally disconnected before to come to thermal equilibrium in the very early universe. In the early theories of inflation, the universe is stuck in a false vacuum with positive energy density for a while, before it decays by quantum tunneling to the true vacuum in a massive phase transition. However, such a model could not provide a *graceful exit* from inflation, and produced an unrealistic universe.

This issue was solved by Andrei Linde [39] and others [3]. Instead of a universe stuck in a false vacuum, they posited that a scalar field, called an *inflaton*, with a sufficiently flat potential could drive the accelerated expansion. The scalar field very slowly rolls down its potential, until it settles down into the minimum, providing a graceful exit from inflation.

Quantum fluctuations in this scalar field leave a very distinct signature on the CMB. It produces scalar and tensor perturbations on

Figure 1: Planck results on the inflationary observables



the metric which leave an almost *scale-invariant* power spectrum. The size of these perturbations may be quantified in the two parameters n_s (the scalar spectral tilt) and r (the tensor-to-scalar ratio, which measures the amount of primordial gravitational waves). We will henceforth refer to these parameters as the *inflationary observables*. The Planck satellite has in recent years provided very accurate measurements of the inflationary observables [16] (see Figure 1).

Right at the center of the observationally-favored region (the Planck "sweet spot") sits a class of models called *α-attractors*[36][37][30][31], developed by Renata Kallosh, Andrei Linde, Diederik Roest, and others. These models will be the focus of this thesis. They are called α-attractors for two reasons: firstly, each one of the constructions is a continuous family of models parametrized by the number α. α also completely determines the amount of observationally detectable primordial gravitational waves. In each α-attractor model, the scalar potential depends on some function of the inflaton which may be chosen almost at random. Every choice leads to the same universal prediction for n_s. This is why the models are called *attractors*.

In this chapter we will provide a short introduction to inflationary cosmology. In the next, we will look at inflation in string theory and supergravity. We will follow the discussions in [48][5][6] and a number of other sources. The emphasis on the importance of conformal time and the decreasing Hubble radius is due to [6]

1.1 THE FLRW METRIC

Cosmology is based on two fundamental assumptions: that at large scales the universe is spatially homogeneous (appears the same at

every point in space), and spatially isotropic (looks the same in every direction). Observationally, these assumptions appear to hold true at scales of about $100\,\mathrm{Mpc}$[48]. However, there is no such symmetry in the time direction. Distant galaxies appear to be receding from us, with a speed[1] that is proportional to their distance. We therefore assume that the spacetime $\mathbb{M}_4$ of the universe consists of a maximally symmetric spatial manifold Σ_3, and a time direction $\mathbb{R}$: $\mathbb{M}_4 = \mathbb{R} \times \Sigma_3$ [13].

The metric on such a space can be written in terms of *comoving coordinates*, in which the time-dependence of the metric is captured in a single *scale factor* $a(t)$:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{1}$$

The parameter k can take on the values $k = 0, \pm 1$. These dictate the spacelike curvature of the universe. For $k = 0$, it is flat; for $k = 1$ it is positively curved or spherical; for $k = -1$ it is negatively curved or hyperbolic. $d\Omega$ is a differential solid angle. This metric is called the *FLRW metric*, after its discoverers Friedmann, Lemaitre, Robertson and Walker.

The matter and energy content of the universe can be treated as a *perfect fluid*. A perfect fluid is isotropic in its own rest frame. This assumption of isotropy to the following energy-momentum tensor:

$$T_{\mu\nu} = (p + \rho)U_\mu U_\nu + p g_{\mu\nu} \tag{2}$$

where U_μ is the four-velocity of the fluid. The parameters p and ρ are called the *pressure* and the energy density of the perfect fluid. The perfect fluid is at rest in a frame defined by the comoving coordinates (since the fluid is isotropic in its rest frame, and the metric is isotropic in comoving coordinates). The energy-momentum tensor then becomes:

$$T^\mu_\nu = \mathrm{diag}(-\rho, p, p, p) \tag{3}$$

From the conservation of energy equation $\nabla_\mu T^\mu_\nu = 0$, we obtain:

$$\partial_0 \rho = -3\frac{\dot{a}}{a}(\rho + p) \tag{4}$$

The pressure and the energy density are connected by an equation of state:

$$p = w\rho \tag{5}$$

The equation of state of ordinary matter at non-relativistic energies becomes approximately $w = 0$. This kind of matter is called *dust*.

1 In the strictest sense, a relative velocity between distant galaxies cannot be defined in a curved universe, due to the ambiguity of parallel transporting velocity vectors between tangent spaces. The apparent recession is, at the most fundamental level, really nothing more than an observed increase in redshift with distance.

Relativistic matter (or *radiation*), such as a photon or a high-energy neutrino, satisfies $w = \frac{1}{3}$. Substituting the equation of state into (4) and integrating, we obtain:

$$\rho \propto a^{-3(1+w)} \tag{6}$$

We can give a very simple interpretation of this equation. For $w = 0$, we have $\rho \propto a^{-3}$. This is simply due to a decrease in the number density of matter particles as the scale factor increases. For relativistic matter, we obtain $\rho \propto a^{-4}$. Relativistic particles, such as photons, receive a redshift by a factor a in addition to the decrease in number density. We see that the universe inevitably has a period at low scale factors where it is radiation-dominated, and a period later on where it is matter-dominated.

There is another, more exotic kind of energy density called vacuum energy, which has $w = -1$. Vacuum energy may be associated with slowly-rolling scalar fields or cosmological constants. Vacuum energy satisfies $\rho \propto a^0$ - i.e. its energy density does not change as the universe expands or contracts. Vacuum energy, if it is there at all, inevitably comes to dominate the universe at large scale factors.

Substituting (2) into the Einstein equations, we obtain, in Planck units[2]:

$$\frac{\ddot{a}}{a} = \frac{1}{6}(\rho + 3p) \tag{7}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3}\rho - \frac{k}{a^2} \tag{8}$$

The second of these equations can be rewritten as:

$$\Omega - 1 = \frac{1}{H^2 a^2} \tag{9}$$

where $\Omega = \frac{\rho}{3H^2}$ is the density in units of the critical density $\rho_c = 3H^2$ required to make the universe flat. If $\Omega < 1$, we have $k = -1$, since $H^2 a^2 \geqslant 0$. Conversely, for $\Omega > 1$, we have $k = 1$. When $\Omega = 1$, the universe is flat and we have $k = 0$. The energy density of the universe determines its spatial curvature.

Specializing to the flat case $k = 0$, the Friedmann equations become:

$$H^2 = \frac{1}{3}\rho \tag{10}$$

$$\dot{H} + H^2 = -\frac{1}{6}(\rho + 3p) \tag{11}$$

where $H := \frac{\dot{a}}{a}$ is the Hubble parameter or *Hubble constant*. (7) are called the *Friedmann equations*. Substituting (6) and the equation of

2 *Planck units* refers to the choice of units in which $c = \hbar = G = 1$, so that $M_{Pl} = 1$. We will be working in this unit system unless otherwise indicated.

state into the Friedmann equations, we obtain that $a(t) \propto t^{\frac{1}{2}}$ for a radiation-dominated universe, and $a(t) \propto t^{\frac{2}{3}}$ for a matter-dominated universe. Furthermore, $a \propto e^{Ht}$ for a vacuum-dominated universe. This exponential expansion is the hallmark of inflation.

In the Standard Model of cosmology, the ΛCDM model, the universe is modeled by an FLRW spacetime with matter energy density $\Omega_m \simeq 0.3$, radiation energy density $\Omega_\gamma = \Omega_m/(1+z_{eq}) \simeq 0$, and vacuum energy density $\Omega_\Gamma \simeq 0.7$ (all given in units of the critical density). z_{eq} is the redshift at the time when there were equal amounts of radiation and matter in the universe. It is given by $z_{eq} \simeq 3400$[6]. The presence of the large vacuum energy, called *Dark Energy*, was discovered in 1998 by [46][45].

1.2 THE HORIZON PROBLEM AND DECREASING HUBBLE RADIUS

The cosmic microwave background is the leftover radiation from when the universe was extremely opaque. When the universe was dense and hot enough to be ionized to a very high degree, photons had a very short mean free path due to scattering off charged particles. As the universe cooled down to about 3700 Kelvin, electrons and nuclei became bound into atoms. This period in cosmological history is called *recombination*. Shortly thereafter, the universe became transparent to radiation as the mean free path of photons became larger than the scale defined by the spatial expansion, the Hubble scale c/H. This happened at a temperature of about 3000 K.[48] This event defines a surface in spacetime called the *last scattering surface*. The CMB is the thermal radiation emitted from the last scattering surface. The spatial expansion of the universe since the time of last scattering has cooled the CMB down to about 2.7 K at the present time.

The CMB is the most perfect example of a blackbody ever found. Furthermore, it looks almost exactly the same in every direction. The largest temperature fluctuations between different directions in the CMB are of order $\frac{\delta T}{T} \simeq 10^{-5}$. However, when we turn back the clock in the ΛCDM model, we find that regions of the CMB separated by more than about a degree must have been causally disconnected at the time of recombination. This means that there are some 10^4 causally disconnected patches on the CMB, all of almost exactly the same temperature. This requires an explanation, unless we are willing to accept that the initial conditions of the universe were tuned to make this happen.

To see why this is the case, let us calculate the size of the particle horizon at the time of last scattering[5]. A massless particle travels on a null geodesic, $ds^2 = 0$. We insert this into (1), and set $d\Omega = 0$. We then have:

$$dt = a(t)dr \tag{12}$$

So the proper distance $d_H(t_r)$ of the particle horizon, at the time t_r of recombination, becomes:

$$d_H(t_r) = a(t_r) \int_0^{t_r} \frac{dt'}{a(t')} \tag{13}$$

Inserting $a \propto t^{\frac{1}{2}}$ for a radiation-dominated universe, we obtain $d_H(t_r) = 2t_r$. We can assume radiation dominance since the time of matter-radiation equality and the time of recombination are not far removed from each other. As the universe expands, it cools down such that $a(t)T = constant$. Let $V_0(t_0)$ be the volume of the observable universe at the present time t_0, and $V_r(t_r)$ be the volume of the observable universe at the time of recombination. We have:

$$\frac{V_0(t_0)}{V_r(t_r)} = \frac{V_0(t_0)a^3(t_r)}{V_r(t_r)a^3(t_0)} = \frac{V_0(t_0)T_0^3}{V_r(t_r)T_r^3} = \left(\frac{t_0}{t_r}\right)^3 \left(\frac{T_0}{T_r}\right)^3 \tag{14}$$

After the time of recombination, the universe was matter-dominated, so $a(t) \propto a^{\frac{2}{3}}$, or $t \propto T^{-\frac{3}{2}}$. We obtain:

$$\frac{V_0(t_0)}{V_r(t_r)} \simeq \left(\frac{T_r}{T_0}\right)^{\frac{3}{2}} \simeq 3 \times 10^4 \tag{15}$$

where we have used $T_r \simeq 3000K$, $T_0 \simeq 2.7K$. Ten thousand patches distributed evenly over the entire night sky are about a degree in diameter. Any two points on the CMB separated more than a degree from each other must never have been in causal contact with each other. However, the CMB is correlated over much larger scales than that. This is the *horizon problem*. The other naturalness problems of cosmology that we mention above are closely related to the horizon problem (in the sense that any solution to the horizon problem will probably be a solution to the other problems as well [6]). Let us now examine how to solve it.

We can make a useful coordinate transformation to *conformal time* $\tau(t)$:

$$d\tau = \frac{dt}{a(t)} \tag{16}$$

The metric becomes:

$$ds^2 = a^2(\tau)[-d\tau^2 + dr^2 + r^2 d\Omega^2] \tag{17}$$

In these coordinates, the geodesic of a photon with $d\Omega = 0$ is given by:

$$r(\tau) = \pm\tau + c \tag{18}$$

where c is a constant. This means that light cones are given by straight lines at angles $\pm 45°$ in the τ-r plane.

The conformal time is equal to the comoving coordinate distance to the particle horizon:

$$\delta r = \delta\tau = \int_{t_i}^{t} \frac{dt'}{a(t')} \tag{19}$$

We can rewrite this integral in terms of the *Hubble radius* $(aH)^{-1}$:

$$\tau = \int \frac{dt'}{a(t')} = \int (aH)^{-1} d\ln a \tag{20}$$

Inserting (6) into the definition of the Hubble radius, we obtain:

$$(aH)^{-1} \propto a^{\frac{1}{2}(1+3w)} \tag{21}$$

We see that the Hubble radius increases as the universe expands as long as $w \geqslant -\frac{1}{3}$. Inserting the above into the integral gives:

$$\tau \propto \frac{2}{1+3w} a^{\frac{1}{2}(1+3w)} \tag{22}$$

As long as $w > -\frac{1}{3}$, we have $\tau \to 0$ as $a \to 0$ in the beginning of the universe. However, for $w \leqslant -\frac{1}{3}$, we find $\tau \to -\infty$ as $a \to 0$. Adding a period of vacuum domination adds conformal time below $\tau = 0$. We remarked earlier that the light cones of particles were given by straight lines at angles $\pm 45°$ in the τ-r plane. Adding a large amount of conformal time below $\tau = 0$ may therefore cause the light cones of two separated points on the last scattering surface to overlap at negative τ. This means that the two regions could come to thermal equilibrium in the very early universe. Before the negative conformal time was added by a period of shrinking Hubble radius, the light cones terminated on the initial singularity before they could cross each other. This is how we solve the horizon problem.

The condition that the Hubble radius is shrinking $\frac{d}{dt}(aH)^{-1} < 0$ is equivalent to the condition that the expansion is accelerating: $\frac{d^2a}{dt^2} > 0$, as we can see by writing out the differentiation explicitly. We have already seen that the shrinking of the Hubble radius requires the presence of a significant amount of vacuum energy (or, technically, another kind of energy with equation of state$w \leqslant -\frac{1}{3}$)).

We may rewrite $\frac{d}{dt}(aH)^{-1}$ in terms of the *slow roll parameter* ϵ:

$$\frac{d}{dt}(aH)^{-1} = -\frac{\dot{a}H + a\dot{H}}{(aH)^2} = -\frac{1}{a}(1-\epsilon) \tag{23}$$

where $\epsilon := -\frac{\dot{H}}{H^2} = -\frac{d\ln H}{dN}$. A decreasing Hubble radius requires $\epsilon < 1$. The condition $\epsilon < 1$ may only be satisfied for a long time if the following parameter η is $<< 1$:

$$|\eta| := \frac{|\dot{\epsilon}|}{H\epsilon} \tag{24}$$

1.3 SLOW-ROLL INFLATION

We now introduce *new inflation*, or *slow-roll inflation*, in which a scalar field slowly rolls down a potential, driving inflation, until it settles

down into a minimum of the potential. A scalar field that is minimally coupled to gravity has the following action:

$$S = \int d^4x\sqrt{-g}\left[\frac{\mathcal{R}}{2} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{25}$$

where the kinetic term takes the *canonical* form. From the above action, we obtain an equation of motion for ϕ:

$$\ddot{\phi} + 3H\dot{\phi} = -V' \tag{26}$$

where H is the Hubble constant. From the Friedmann equations, we find:

$$\epsilon = \frac{\frac{1}{2}\dot{\phi}^2}{H^2} \tag{27}$$

From the above and the Friedmann equation, we see that inflation can only occur when the potential energy V is larger than the kinetic energy $\frac{1}{2}\dot{\phi}^2$. Additionally, we can see this from the equations that determine the pressure and energy density contained within a scalar field:

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

We see that $w < \frac{1}{3}$ can only be satisfied if the potential energy dominates. When $\dot{\phi} = 0$, we obtain $w = -1$, and the scalar field can model late-stage dark energy or, in other words, a cosmological constant.

The condition $\epsilon < 1$ may only be satisfied for a long time if the scalar field does not accelerate too much. This is quantified by the parameter δ:

$$\delta := -\frac{\ddot{\phi}}{H\dot{\phi}} \tag{28}$$

which satisfies $\eta = 2(\epsilon - \delta)$. The conditions $\{\epsilon, |\eta|, \delta\} << 1$ are called the *slow-roll conditions*. If they are satisfied, inflation will persist for a long time. We can substitute the slow-roll conditions into the equations of motion to make things more simple. This is the *slow-roll approximation*. From (27) and the Friedmann equations we see that the potential energy V dominates over the kinetic energy $\frac{1}{2}\dot{\phi}^2$ when $\epsilon << 1$. Substituting this into the Friedmann equation, we find (in Planck units):

$$H^2 \simeq \frac{V}{3} \tag{29}$$

From the slow-roll condition $|\delta| << 1$, we can simplify the Klein-Gordon equation:

$$3H\dot{\phi} \simeq -V'(\phi) \tag{30}$$

We can substitute the simplified Friedmann and Klein-Gordon equations into the definition of ϵ to find:

$$\epsilon \simeq \frac{1}{2}\left(\frac{V'}{V}\right) := \epsilon_v \tag{31}$$

Similarly, we can find $\delta + \epsilon \simeq \frac{V''}{V} := \eta_v$. The parameters ϵ_v and η_v are called the *potential slow-roll parameters*. Slow roll inflation is defined by $\{\epsilon_v, \eta_v\} << 1$.

The amount of spatial expansion that takes place during inflation is usually reported in the number of *e-folds* N, defined by:

$$N := \int_{a_i}^{a_f} d\ln a = \int_{t_i}^{t_f} H(t)dt \tag{32}$$

where the subscripts i and f refer to the beginning and end of inflation, respectively. The ratio between the scale factor $a(t_f)$ at the end of inflation and the scale factor at the start of inflation $a(t_i)$ is the exponent of the number of e-folds N: $\frac{a(t_f)}{a(t_i)} = e^N$. We can simplify this integration in the slow-roll approximation:

$$Hdt = \frac{H}{\dot{\phi}}d\phi \simeq -\frac{3H^2}{V'}d\phi \simeq \frac{1}{\sqrt{2\epsilon_v}}d\phi \tag{33}$$

We can rewrite (32) as an integration over the inflaton field ϕ:

$$N = \int_{\phi_i}^{\phi_f} \frac{V}{V'}d\phi \tag{34}$$

We can then easily calculate the number of e-folds from the scalar potential.

To solve the horizon problem, we need at least around 60 e-folds of inflation.[6] This is the minimum amount needed to make the universe naturally flat and uniform, assuming inflation happens at an energy scale close to the GUT scale. [48]

1.4 QUANTUM FLUCTUATIONS AND INFLATIONARY OBSERVABLES

The way we can extract information about the inflationary dynamics from the CMB is the look at the power spectra of tensor and scalar fluctuations. Let us see how these appear in the CMB. The generic action for a single-field model of inflation looks like (in Planck units):

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}\mathcal{R} - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \tag{35}$$

There are 5 scalar modes of fluctuations around a uniform background, 4 of them associated with fluctuations in the metric, and one associated with fluctuations in the inflaton. However, time and spatial translation invariance removes two of these, and the Einstein *constraint equations* remove two others. There is a single physical scalar

degree of freedom that parametrizes the fluctuations. The tensor fluctuations are associated with transverse perturbatons of the metric, i.e. perturbations δg_{ij} of the form $\delta g_{ij} = a^2 h_{ij}$. What we can get from the CMB is the power spectra of these tensor and scalar fluctuations. We will not explain how these are obtained from the CMB, or how the fluctuations result from quantum dynamics. For details, see [6].

The observational results are typically reported in terms of two parameters. Firstly, the spectral tilt n_s, which measures the deviation from scale invariance in the CMB power spectrum:

$$n_s - 1 := \frac{d \ln \Delta_s^2}{d \ln k} \tag{36}$$

where Δ_s^2 is the power spectrum of scalar perturbations and the mode k is evaluated at the *pivot scale*. What is important to us is the relation between n_s and the slow-roll parameters:

$$n_s - 1 = 2\epsilon - 6\eta \tag{37}$$

The second *inflationary observable* is the tensor-to-scalar ratio r:

$$r := \frac{\Delta_t^2}{\Delta_s^2} \tag{38}$$

It is related to the Hubble slow-roll parameters by $r = 16\epsilon$.

Let us calculate n_s and r in a simple example, the Starobinsky model of inflation. This is a plateau inflation model derived from a specific choice of $f(R)$ gravity. It has a scalar potential:

$$V = \frac{3}{4}\left(1 - e^{-\sqrt{\frac{2}{3}}\phi}\right)^2 \tag{39}$$

In the slow-roll approximation at large field values of ϕ (i.e. on the plateau where inflation occurs), we find:

$$\epsilon \simeq \epsilon_v = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \tag{40}$$

$$= \frac{1}{2}\left[\frac{8e^{-2\sqrt{\frac{2}{3}}\phi}}{3\left(1 - e^{-2\sqrt{\frac{2}{3}}\phi}\right)^2}\right] \qquad \simeq \frac{4}{3}e^{-2\sqrt{\frac{2}{3}}\phi} \tag{41}$$

Simiarly, we find:

$$\eta \simeq -\frac{4}{3}e^{-\sqrt{\frac{2}{3}}\phi} \tag{42}$$

Using (34), we find that:

$$N = \frac{3}{4}e^{\sqrt{\frac{2}{3}}\phi_f} \tag{43}$$

This relation may be inverted to yield:

$$\epsilon = \frac{3}{4N^2} \tag{44}$$

Similarly, we find:

$$\eta = -\frac{1}{N} \tag{45}$$

In the large-N limit, the inflationary observables become:

$$n_s - 1 = \frac{2}{N}, \quad r = \frac{12}{N^2} \tag{46}$$

The α-attractor models we will examine in the coming chapter have very similar predictions for the inflationary observables, since their scalar potentials have the same structure as the Starobinsky model. If we take $N \simeq 60$, we obtain $n_s \simeq 0.03$ and $r \simeq 0.003$, which lies within the observationally favored region found by Planck[16]. (See Figure 1).

2 STRING THEORY, SUPERGRAVITY AND INFLATION

The α-attractor models which are the subject of this thesis are supergravity constructions, and we wish to study their interplay with the *moduli* (dynamically unconstrained scalar fields) of a string theory compactification. In this chapter, we will give a short overview of what we need to know about string theory, supersymmetry, and supergravity. Let us first take a moment to consider why one would wish to incorporate inflation models within these speculative high-energy theories. Firstly, if string theory is to provide a full description of Nature, it must have an answer to the horizon problem and its several cousins. We must be able to realize inflation within string theory for it to be a viable fundamental theory. Secondly, inflation models themselves are naturally very sensitive to corrections from high-energy physics, in a way that particle physics, for example, is not. The sensitivity is a problem for model building, as it may spoil interesting possibilities, but it also provides us with an observational window into high-energy physics.

To see why inflation is so sensitive to ultraviolet physics, we must think of inflation in an effective field theory context. Let us sketch the effective action of a scalar field with high-energy corrections, incorporating higher-derivatives and higher-order operators which are suppressed by the Planck scale Λ[6]:

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) + \sum_n c_n V(\phi) \frac{\phi^{2n}}{\Lambda^{2n}} + \sum_n d_n \frac{(\partial\phi)^{2n}}{\Lambda^{4n}} + \dots \tag{47}$$

This Lagrangian has been constructed with a $\mathbb{Z}_2$ symmetry $\phi \to -\phi$. This is just to demonstrate that in general the effective field theory of a more fundamental high-energy theory must contain every kind of operator consistent with the symmetries of the fundamental theory.[1]

Ordinarily, the Planck-scale suppression of the higher-order operators is enough to make the operators *irrelevant* at low energies. In particle physics, this is believed to be the case for the Standard Model. Another example is the Fermi theory of weak interactions, which is a low-energy effective field theory generated by integrating out the heavy W, Z gauge bosons of the more fundamental electroweak theory. In both of these cases, a cutoff-scale suppression gives the theo-

1 For more information, see [6].

ries a clear region of validity. However, in inflationary cosmology we have the following condition on the slow-roll parameter η:

$$\eta = \frac{m_\phi^2}{3H^2} << 1 \tag{48}$$

This constraint is easily upset by high-energy corrections. Let us consider that ϕ has order $O(1)$ couplings to massive fields with masses of order $O(\Lambda)$, where Λ is the effective theory cut-off. This cut-off is where degrees of freedom of the high-energy fundamental theory become important. Generically, it takes place below, but not far below, the Planck scale. Integrating out the massive fields yields the following corrections to the scalar potential[6]:

$$\delta V = c_1 V(\phi) \frac{\phi^2}{\Lambda^2} \tag{49}$$

where c_1 is the $O(1)$ coupling of the inflaton to the massive fields. η then receives a correction:

$$\delta\eta = \frac{M_{Pl}^2}{V}(\delta V)'' \simeq 2c_1 \left(\frac{M_{Pl}}{\Lambda}\right)^2 \tag{50}$$

We see that η may receive corrections of order $O(1)$, which upsets the constraint (48). This issue is called the η *problem*. We will see it appear in various guises throughout the rest of the thesis.

How does supersymmetry (SUSY) help with this η problem? SUSY is a symmetry that relates bosons and fermions. In many examples throughout particle physics, fermions with low masses appear quite naturally (the low mass of electrons being natural due to gauge symmetry, for example). When SUSY is not spontaneously broken, fermions and bosons that sit within a *multiplet* will have exactly the same mass. We see that SUSY provides us with a framework in which low-mass scalars are quite natural. Secondly, supersymmetry protects certain classical quantities from receiving quantum corrections due to cancellations between fermionic and bosonic loops within a multiplet. However, SUSY is generically broken spontaneously, and the η problem can persist, but SUSY can still dramatically improve the situation. It may help keep the mass of the inflaton around the energy scale of inflation, instead of at the Planck scale ($\sim 10^{16} GeV$), which is several orders of magnitude higher ($\sim 10^{19} GeV$).

Supersymmetry can be made a local (gauge) symmetry. When this is done, gravity enters into the theory. Theories of local supersymmetry are called *supergravities*. In this chapter, we provide an (impossibly brief) introduction to supersymmetry and supergravity. Then, we explain the α-attractor models of [36][37][30][31].

2.1 GLOBAL SUPERSYMMETRY

Supersymmetry is of interest for several reasons. Firstly, it provides an answer to many naturalness problems of the kind described above.

These appear in the Standard Model of particle physics as well. The Higgs mass is unreasonably low at 125GeV, since quantum corrections naturally push it to the cut-off scale of the Standard Model effective field theory. Supersymmetry protects the Higgs mass from quantum corrections by cancellations between fermionic and bosonic loop diagrams. Secondly, string theory cannot contain spacetime fermions unless it is made supersymmetric. Thirdly, supersymmetry provides (under certain asssumptions) the only possible extension to the Poincaré group that does not lead to non-trivial dynamics. Concretely, if the full symmetry of the S-matrix is a product group of the Poincaré group and an internal symmetry, then any non-trivial internal group renders the S-matrix non-analytic in some circumstances[2]. There is a catch, however. We can turn the Poincaré group into a graded Lie algebra, which means that we consider anti-commutation relations as well as commutation relations. This leads to a symmetry group that relates bosons to fermions, known as supersymmetry (SUSY) [50].

Specifically, SUSY adds spinorial generators Q_α, $Q^\dagger_{\dot\alpha}$ to the Poincaré algebra[3]. The undotted indices α imply that the spinors have a left-handed chirality, whereas the dotted indices $\dot\alpha$ imply that they have right-handed chirality. The spinorial generators have the following anti-commutation relations:

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \tag{51}$$

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \tag{52}$$

where $\sigma^\mu_{\alpha\dot\alpha} = (1, \sigma^i)$ and σ^i are the Pauli matrices. Since Q_α are spinors that generate a symmetry, they turn bosonic states into fermionic states and vice versa. These bosonic and fermionic states which are connected by symmetry generators form a *supermultiplet*. Each supermultiplet contains the same number of fermionic and bosonic degrees of freedom. The spinorial generators have the following commutation relations with the Poincaré translations P_μ:

$$[P_\mu, Q_\alpha] = [P_\mu, Q^\dagger_{\dot\alpha}] = 0 \tag{53}$$

Since $P_\mu P^\mu$ commutes with the supersymmetry generators, all particles within a supermultiplet have the same mass when SUSY is not spontaneously broken. The anti-commutation relations (51) show that the Hamiltonian of a SUSY theory is determined by the SUSY generators:

$$H = \frac{1}{4}\left(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2\right) \tag{54}$$

If supersymmetry is not spontaneously broken, the vacuum $|0\rangle$ necessarily has zero energy, since $Q_\alpha |0\rangle = 0$.

2 Proved by Coleman and Mandula [15].
3 We are working in four dimensions for the time being.

2.1.1 *Representations of Supersymmetry*

We can construct representations of the SUSY algebra by starting with a vacuum state and acting with the SUSY generators. If the representation is massive, we can boost to the rest frame. The supersymmetry algebra then becomes:

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2m\delta_{\alpha\dot\alpha} \tag{55}$$

and all other anti-commutators vanish. Starting with a vacuum state $|\Omega\rangle$ satisfying $Q_\alpha |\Omega\rangle = 0$, we can construct a massive representation using $Q^\dagger_{\dot\alpha}$ as a raising operator:

$$\begin{aligned} &|\Omega\rangle \\ &Q^\dagger_1 |\Omega\rangle, Q^\dagger_2 |\Omega\rangle \\ &Q^\dagger_1 Q^\dagger_2 |\Omega\rangle \end{aligned}$$

If we choose $|\Omega\rangle$ to be a spin-0 state, then this multiplet contains a complex scalar and a massive spin-$\frac{1}{2}$ fermion. It is called a *chiral* supermultiplet. Alternatively, we can start with a spin-$\frac{1}{2}$ vacuum. The supermultiplet then contains a spin-$\frac{1}{2}$ massive fermion and a massive vector. This supermultiplet is called the *vector multiplet*.

In the massless case, the anti-commutators are different. In the frame where $p_\mu = (E, 0, 0, E)$, we find:

$$\{Q_1, Q^\dagger_1\} = 4E \tag{56}$$

and all other anti-commutators vanish. There is only one raising operator in this case, since $Q^\dagger_2$ produces states with vanishing norm:

$$\langle\Omega| Q_2 Q^\dagger_2 |\Omega\rangle = 0 \tag{57}$$

We can produce a massless representation by taking a vacuum state $|\Omega\rangle$ and acting with $Q^\dagger_1$. The vacuum state must have a definite helicity λ, and the other state in the multiplet then obtains helicity $\lambda + \frac{1}{2}$. Starting with a spin-0 vacuum, we obtain a massless vector multiplet. Starting with spin-$\frac{1}{2}$, we obtain the massless chiral multiplet. The CPT theorem requires the existence of a multiplet with the inverse helicities $-\lambda$, $-\lambda - \frac{1}{2}$. We generate this multiplet by starting with a vacuum of helicity $-\lambda - \frac{1}{2}$.

2.1.2 *Extended Supersymmetry*

Instead of adding just a group of four supersymmetry generators, we can add any number $\mathcal{N}$ groups of four generators. Labeling each group by $a, b = 0, 1, 2, \ldots, \mathcal{N}$, the supersymmetry algebra becomes:

$$\{Q^a_\alpha, Q^\dagger_{\dot\alpha b}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu \delta^a_b \tag{58}$$

and all other anti-commutators vanish. In the massless case, this becomes:

$$\{Q^a_1, Q^\dagger_{1b}\} = 4E\delta^a_b \tag{59}$$

Starting with a vacuum of helicity λ, we can produce states of helicity up to $\lambda + \mathcal{N}/2$ by acting with $Q^\dagger_{1a}$. At each helicity level, there is a degeneracy associated with the freedom to change the labels a, b, etc. In theories of global (or "rigid") supersymmetry, we are interested in particles with helicity greater than 1. This means that for $\mathcal{N} = 1$ we cannot go lower in helicity than $-\frac{1}{2}$ and for helicity $\mathcal{N} = 2$, we must restrict ourselves to $\lambda > -1$. Taking the vacuum to have helicity -1, we obtain the massless $\mathcal{N} = 2$ vector supermultiplet. The choice $\lambda = 0$ gives the $\mathcal{N} = 2$ chiral multiplet, and $\lambda = -\frac{1}{2}$ gives the $\mathcal{N} = 2$ *hypermultiplet*.

2.2 SUPERGRAVITY

We have examined supersymmetry as a global symmetry of nature. However, most of the symmetries in particle physics are in fact local (gauge) symmetries. Furthermore, in the presence of supersymmetry breaking, gravitational effects often become important even when the SUSY theory is designed to describe physics at the electroweak scale[4]. Since the supersymmetry algebra contains the Poincaré generators of translations, any attempt to realize supersymmetry as a local symmetry will result in a theory that contains gravity as well (since we can think of General Relativity as the gauge theory of Poincaré invariance). A theory of local supersymmetry is called a *supergravity*. There is a second reason why these are of interest: they are the low-energy effective field theories of superstring theory. The unique 11-dimensional supergravity theory is thought to be the low-energy limit of M-theory, the "parent" theory of superstring theory. Most of what we call string theory in this thesis will in fact take place in the supergravity limit.

Since a gravity theory must contain a massless spin-2 graviton, a supergravity must contain at least a spin-$\frac{3}{2}$ fermion, called the *gravitino* Ψ. The fact that a SUSY theory contains fermions means that we must use the vielbein formalism of General Relativity. A reasonable place to start constructing a supergravity Lagrangian is to construct a globally supersymmetric action from the standard actions of spin-$\frac{3}{2}$ and spin-2 particles, the Rarita-Schwinger and Einstein-Hilbert actions, respectively:

$$S_{global} = S_{RS} + S_{EH} \tag{60}$$

However, obviously the Einstein-Hilbert action already contains a local symmetry, so we must use its *linearized* version S^L_{EH} instead:

$$S_{EH} \simeq S^L_{EH} = -\frac{1}{2}\int d^4x \left(R^L_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} R^L \right) h^{\mu\nu} \tag{61}$$

This action is constructed by taking the Einstein-Hilbert action, imposing $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, and truncating to first order in $h_{\mu\nu}$. Taking S_{RS} to be the standard Rarita-Schwinger action,

$$S_{RS} = -\frac{1}{2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu \gamma_5 \gamma_\nu \partial_\rho \Psi_\sigma \tag{62}$$

it can be shown that the action (60) is invariant under supersymmetry transformations provided that the gravitino Ψ and the graviton $h_{\mu\nu}$ transform as [4]:

$$h_{\mu\nu} \to h_{\mu\nu} + \delta_\xi h_{\mu\nu} = h_{\mu\nu} - \frac{i}{2}\bar{\xi}\Big(\gamma_\mu \Psi_\nu + \gamma_\nu \Psi_\mu\Big) \tag{63}$$

$$\Psi_\mu \to \Psi_\mu + \delta_\xi \Psi_\mu = \Psi_\mu - i\sigma^{\rho\tau}\partial_\rho h_{\tau\mu}\xi \tag{64}$$

where ξ is a spinorial parameter of the supersymmetry transformation. We can make the action locally supersymmetric by replacing the derivatives in the Rarita-Schwinger action by covariant derivatives and replacing the linearized Einstein-Hilbert action by the Einstein-Hilbert proper. Furthermore, a term quartic in the gravitino needs to be added to close the supersymmetry algebra. The action becomes (in Planck units):

$$S = -\frac{1}{2}\int d^4x |\det e| R - \frac{1}{2}\int d^4x \epsilon^{\mu\nu\rho\sigma}\bar{\Psi}_\mu \gamma_5 \gamma_\nu \tilde{D}_\rho \Psi_\sigma \tag{65}$$

where $\tilde{D}$ is the covariant derivative $\tilde{D}_\mu := \partial_\mu - i\tilde{\omega}_{\mu mn}\frac{\sigma^{mn}}{4}$ and $\tilde{\omega}$ is a modified spin-connection. $\det e$ refers to the determinant of the vielbein. For the full derivation of the above action using the *Noether procedure*, see [4]. The above action is *on-shell*. It is often useful to place an auxilliary field into the action, whose equations of motion can be substituted into the off-shell action to yield the on-shell action.

2.2.1 *11-dimensional Supergravity and Dimensional Reduction*

Now that we have moved into supergravity territory, we necessarily relax our condition on helicities $|\lambda| \leqslant 1$ because a gravitons are spin-2 particles. However, it is not known how to consistently describe particles of spin greater than 2 in field theory. We must now restrict ourselves to helicities $|\lambda| \leqslant 2$. We have seen that the $\mathcal{N}$ SUSY generators can lift the helicity to $\lambda + \mathcal{N}/2$. The maximum amount of SUSY we can have is therefore $\mathcal{N} = 8$, if we start with a $\lambda = -2$ helicity ground state. This is a $D = 4$ result. In four dimensions, a group of SUSY generators has 4 real components, so the number of *supercharges* N_c in $\mathcal{N} = 8$ is equal to $8 \times 4 = 32$. The constraint $N_c < 32$ in fact holds in any spacetime dimension. 32 is the maximum number of supercharges that can exist in any supergravity theory. Since the number of components in a spinor depends on the dimension of spacetime, there is a maximum dimension in which a supergravity

theory can exist. A spinor in 11 dimensions has 32 components. For $D = 11$, there is a unique supergravity theory that is simply called *11-dimensional Supergravity*. It holds a special importance among SUSY theories, since it is thought to be the low-energy limit of M-theory, and the Type II 10-dimensional supergravities can be derived from it by dimensional reduction.

We can guess the particle spectrum of 11-dimensional supergravity by a simple counting of degrees of freedom. Any SUSY theory has the same number of fermionic and bosonic degrees of freedom at each mass level (when SUSY is not spontaneously broken). A supergravity theory must contain at least a graviton (a vielbein e^a_μ) and its superpartner, the gravitino Ψ_M[4]. These are massless fields, so they transform as representations of the *little group*, which is $SO(9)$ in $D = 11$. The vielbein transforms as a symmetric tensor with $(D-1)(D-2)/2-1 = 44$ degrees of freedom. Ψ is a spin-$\frac{3}{2}$ *vector spinor*. The vector part carries $(D-2) = 9$ degrees of freedom, whereas the spinor part carries 32 real components. However, not all of these $\mathbf{9} \times \mathbf{16} = \mathbf{128} + \mathbf{16}$ degrees of freedom are physical. There is a gauge symmetry of the gravitino $\Psi_M \to \Psi_M + \partial_M \epsilon$ where ϵ is a Majorana spinor. This removes 16 degrees of freedom. We now have 44 bosonic and 126 fermionic degrees of freedom. The missing 84 bosonic degrees of freedom are exactly the right amount to be filled by a 3-form field A_3. This is the entire spectrum of 11-dimensional supergravity.

There are three supergravity theories in $D = 10$. They are called Type I (an $\mathcal{N} = 1$ theory which is the low-energy limit of heterotic and Type I string theory), Type IIA and Type IIB ($\mathcal{N} = 1$ theories which are the low-energy limits of Type IIA and Type IIB superstring theory). The Type IIA supergravity can be obtained from 11-dimensional supergravity by dimensional reduction. This means that we place 11-dimensional supergravity on a space $\mathbb{R}^9 \times S$, where $\mathbb{R}$ is an ordinary spacetime and S is a circle. We then integrate over the compact circular dimension to obtain an effectively 10-dimensional theory. This procedure is called *compactification*, and it will be the subject of our next chapter. Let us see how the 11-dimensional degrees of freedom translate to 10-dimensional degrees of freedom. This will give us the Type IIA particle spectrum.

The 11-dimensional gravitino spinor components Ψ_M with M on the non-compact $\mathbb{R}^9$ space split apart into two 10-dimensional Majorana spinors of opposite chirality. All of these spinors combine into the two gravitino vector-spinors of $\mathcal{N} = 2$ Type IIA. The last spinor component of the gravitino, Ψ_{11}, becomes two spinors $\lambda^\pm$ on the 10-dimensional space called the *dilatinos*. The massless Dirac equation reduces the degrees of freedom in each dilatino from 16 to 8.

The vielbein splits apart in the usual way of a Kaluza-Klein compactification (see the next chapter). In the 10D space, we find a viel-

4 We now use latin indices M to refer to the spacetime components in $D > 4$.

bein, a vector A_μ called the *graviphoton* and a complex scalar field λ called the axio-dilaton.

The 3-form (84 degrees of freedom) splits into a 10-dimensional 3-form C_3 (56 degrees) and a 10D 2-form B_2 (28 degrees).

Counting the fermionic and bosonic degrees of freedom will verify this decomposition. We may construct the spectrum by explicitly using the raising and lowering operators from the $\mathcal{N} = 2$ algebra on a helicity 2 ground state, however this is more difficult to do. For details, see [50].

The Type IIB supergravity, which will be the primary focus in the rest of the thesis, cannot be obtained from 11-dimensional supergravity by dimensional reduction (we can see this easily since Type IIB contains gravitinos with the same chirality, and dimensional reduction always produces spinors of opposite chirality). However, the Type IIA superstring can be considered a projection of the Type IIB superstring onto states which are invariant under a symmetry of Type IIB. This projection is called an *orbifold*. We will discuss a related concept, *orientifolds* in the following chapter. The orbifold that takes Type IIB to Type IIA is one example of the many dualities and projections that interrelate the superstring theories in an intricate web. This *web of dualities* is the reason that the non-uniqueness of ten-dimensional superstring theories is no longer considered to be the problem it once was.

2.2.2 *Type II and $\mathcal{N} = 1$ Supergravity Actions*

In this section we present the actions of Type IIA and Type IIB supergravity in ten dimensions. We will not show how they can be derived, because this is very hard to do. Let us first arange the Type IIA and Type IIB particle spectra into supergravity multiplets [43].

Multiplet	Bosonic content	Fermionic content
Vector	$\hat{A}_M$	$\lambda^\pm$
Graviton	$\hat{g}_{MN}$, $\hat{B}_2$, ϕ	Ψ^+_M, λ^+
Gravitino	l, $\hat{C}_2$, $\hat{A}_4$	Ψ^+_M, λ^+
Gravitino	$\hat{A}_1$,$\hat{C}_3$	Ψ^-_M, λ^-

where the hats on the n-forms now indicate that the field live in the ten-dimensional space, as opposed to the four-dimensional space that results from a compactification. The first choice for the gravitino multiplet leads to Type IIB. The theory is called *chiral* since the two gravitinos have the same chirality. The second choice leads to Type IIA, which is non-chiral.

The action for Type IIA reads in the "string frame"[5][43]:

$$S_{IIA} = \int \left[e^{-2\phi} \left(-\frac{1}{2}\hat{R} \star \mathbf{1} + 2d\phi \wedge \star d\phi - \frac{1}{4}\hat{H}_3 \wedge \star \hat{H}_3 \right) \right.$$
$$\left. - \frac{1}{2}\left(\hat{F}_2 \wedge \star \hat{F}_2 + \hat{F}_4 \wedge \star \hat{F}_4 \right) + \mathcal{L}_{top} \right] \quad (66)$$

This action is given in terms of differential forms, which is by far the most convenient formalism to use in this case. For an introduction to differential forms and complex geometry, see [44].

The n-forms which appear in this action are field strengths of the potentials B_2 and C_3 which appear in the multiplets. They are defined by:

$$\hat{F}_2 = d\hat{A}_1$$
$$\hat{F}_4 = d\hat{C}_3 - \hat{B}_2 \wedge d\hat{A}_1$$
$$\hat{H}_3 = d\hat{B}_2$$

The term $\mathcal{L}_{top}$ is a topological term defined by:

$$\mathcal{L}_{top} = -\frac{1}{2}\left[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \frac{1}{3}(\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1 \right] \quad (67)$$

The Type IIB action reads:

$$S = \int e^{-2*\hat{\phi}} \left(-\frac{1}{2}\hat{R} \star \mathbf{1} + 2d\hat{\phi} \wedge \star d\hat{\phi} - \frac{1}{4}\hat{H}_3 \wedge \star \hat{H}_3 \right)$$
$$- \frac{1}{2}\left(dl \wedge \star dl + \hat{F}_3 \wedge \star \hat{F}_3 + \frac{1}{2}\hat{F}_5 \wedge \star \hat{F}_5 + \hat{A}_4 \wedge \hat{H}_3 \wedge d\hat{C}_2 \right) \quad (68)$$

where the field strengths are defined by:

$$\hat{H}_3 = d\hat{B}_2 \quad (69)$$
$$\hat{F}_3 = d\hat{C}_2 - l d\hat{B}_2 \quad (70)$$

There is a condition on the field strength F_5 which cannot be imposed covariantly in the action. It must be self-dual under the Hodge operator: $\hat{F}_5 = \star \hat{F}_5$. This relation must be imposed upon the equations of motion.

2.2.3 $\mathcal{N} = 1$ *Scalar Potential in Four Dimensions*

All of the inflation models that we will consider are $\mathcal{N} = 1$ constructions in $D = 4$. We will have more to say about the action and field content of $\mathcal{N} = 1$ supergravity in the following chapter. For now, let us see what happens to the scalar sector of the theory.

5 This is jargon for two equivalent formulations of the supergravity actions. The other is called the Einstein frame. They can be transformed into each other by a field redefinition.

The $\mathcal{N} = 1$, $D = 4$ SUGRA theory is not as constrained as the 10D string theory supergravities. It allows for any number of scalars in chiral multiplets. The dynamics of these scalars is completely determined by two functions: the *Kähler potential* $K(\phi_i, \bar{\phi}_{\bar{j}})$ and the *superpotential* $W(\phi_i, \bar{\phi}_{\bar{j}})$. From the Kähler potential K we can define a metric $K^{i\bar{j}}$ which is the matrix inverse of $K_{i\bar{j}} := \partial_{\phi_i}\partial_{\bar{\phi}_{\bar{j}}}K$. This metric determines the kinetic terms in the $\mathcal{N} = 1$ action. The scalars then parametrize a complex manifold called the *Kähler manifold*. The Kähler manifold has a covariant derivative defined by:

$$D_{\phi_i}W = \partial_{\phi_i}W + W\partial_{\phi_i}K \tag{71}$$

The scalar potential V that appears in the $\mathcal{N} = 1$ action is given by:

$$V = e^K(K^{I\bar{J}}D_I W\overline{D_J W} - 3|W|^2) + \frac{1}{2}\mathrm{Re}(f^{-1})^{\kappa\lambda}D_\kappa D_\lambda \tag{72}$$

Constructing a supergravity model of inflation amounts to choosing K and W such that V is a viable inflation potential.

Instead of using the two functions K and W, we may instead combine them into the Kähler function $G(\phi_I, \bar{\phi}_{\bar{I}})$:

$$G(\phi_i, \bar{\phi}_{\bar{j}}) = K(\phi_i, \bar{\phi}_{\bar{j}}) + \ln\left(|W(\phi_i, \bar{\phi}_{\bar{j}})|^2\right) \tag{73}$$

In terms of the Kähler function G, the scalar potential becomes:

$$V = e^G\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right) \tag{74}$$

The formalism that uses the Kähler function is perhaps more natural than that which uses both the Kähler potential K and the superpotential W. We will use the Kähler function formalism to couple an inflaton to a string theory moduli sector in a way which minimizes the mixing between the two sectors.

2.3 INFLATION IN SUPERGRAVITY

The challenge in constructing supergravity inflation models lies in the e^K factor of the scalar potential (72). A "minimal" Kähler potential is one which produces canonical kinetic terms for the scalar fields, i.e. $K = \phi\bar{\phi}$. However, a Kähler potential like this makes the potential too steep for inflation due to the $e^{\phi\bar{\phi}}$ factor. A solution to this problem was found by [38]. We can posit the existence of a shift symmetry $\phi \to \phi + c$ where c is a real constant. An example of a Kähler potential that respects this shift symmetry is $K = (\phi - \bar{\phi})^2/2$, which does not depend on the real part of ϕ at all. The real part $\phi + \bar{\phi}$ can then become the inflaton. The potential is naturally quite flat in the inflaton direction since the Kähler potential does not depend on it at all.

The shift symmetry can be broken spontaneously in the superpotential. For example, we may take $W = mS\phi$, where S is a scalar

stabilizer field. A model of inflation can be provided by a Kähler potential of the generic form:

$$K((\phi - \bar{\phi})^2, S\bar{S}) \tag{75}$$

with superpotential:

$$W = Sf(\phi) \tag{76}$$

where $f(\phi)$ is a real *holomorphic* function which satisfies $\bar{f}(\bar{\phi}) = f(\phi)$. The Kähler potential and the absolute value squared of the superpotential are invariant under an exchange $\phi \to \bar{\phi}$. From (74) we see that this implies that $\phi = \bar{\phi}$ is an extremum of the potential. Additionally, we have the symmetry $S \to -S$, so $S = 0$ is another extremum. We can now truncate to $\phi = \bar{\phi}$, $S = 0$. The scalar potential then becomes simply [35]:

$$V = |f(\phi)|^2 \tag{77}$$

We see that we can generate an arbitrary non-negative scalar potential in this formalism. However, there is one caveat. For the truncations above to be consistent, the mass of the imaginary ϕ direction and the mass of S both need to be positive definite. If they are not, then the extrema are maxima or saddle points, not minima. Stabilizing each direction of every complex scalar field in the theory is often the hurdle in realizing inflation in supergravity, as we will soon discover.

2.3.1 *Attractor Inflation*

Let us now discuss the ideas behind the α-attractor concept using the models of [36][37][30][31], which were based on spontaneously broken conformal symmetry. Let us forget about supergravity for a moment and consider a simple single-field model of de Sitter space with a conformal symmetry. We take the following action, as was done in [31]:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial_\nu\chi g^{\mu\nu} + \frac{\chi^2}{12}R(g) - \frac{\lambda}{4}\chi^4\right] \tag{78}$$

The theory has a conformal symmetry given by the simultaneous transformations:

$$g_{\mu\nu} \to e^{-2\sigma(x)}g_{\mu\nu}, \quad \chi \to e^{\sigma(x)}\chi \tag{79}$$

The scalar χ is called a *conformon*. The conformal symmetry allows us to fix a gauge $\chi = \sqrt{6}$. The action then becomes:

$$\mathcal{L} = \sqrt{-g}\left[\frac{R(g)}{2} - 9\lambda\right] \tag{80}$$

which has a de Sitter solution for $\lambda > 0$. We can add a second scalar field ϕ to obtain a similar multi-field model:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{\lambda}{4}(\phi^2 - \chi^2)^2\right] \tag{81}$$

This has the same kind of conformal symmetry as the single-field model. However, it has an additional $SO(1,1)$ symmetry between ϕ and χ. We can choose a conformal gauge which respects the $SO(1,1)$ symmetry:

$$\chi^2 - \phi^2 = 6 \tag{82}$$

This equation defines a parabola in $\chi - \phi$ space which we can represent parametrically by a scalar field φ:

$$\chi = \sqrt{6}\cosh\frac{\varphi}{\sqrt{6}}, \quad \phi = \sqrt{6}\sinh\frac{\varphi}{\sqrt{6}} \tag{83}$$

It is easy to see that $\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}\partial_\mu\partial^\mu\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\varphi$, so φ is a canonically normalized scalar field. Furthermore, its coupling to the Ricci scalar is minimal. In conformal gauge, we again find an action with a completely flat scalar potential, which is de Sitter if the coupling constant $\lambda > 0$.

The equation (83) is analogous to the relation between rapidity θ and velocity v in special relativity:

$$\theta = \tanh^{-1}\frac{v}{c} \tag{84}$$

As the rapidity is increased to infinity, the velocity merely approaches c. The same is true for the scalar fields in our model: as the canonical field φ is increased to infinite field values, the conformon merely approaches a constant finite value. This naturally produces potentials with plateaus at large field values of the canonical field, which are quite suitable for inflation. Let us consider what happens when we deform the $SO(1,1)$ symmetry between ϕ and χ. We can add a function $F(\phi/\chi)$ to mediate the symmetry breaking:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{\chi^2}{12}R(g) - \frac{1}{2}\partial_\mu\partial^\mu\phi - \frac{\phi^2}{12}R(g) - \frac{1}{36}F(\phi/\chi)(\phi^2-\chi^2)^2\right] \tag{85}$$

In the gauge $\chi^2 - \phi^2 = 6$, and in terms of the canonically normalized scalar φ, the action becomes:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu - F(\tanh\frac{\varphi}{\sqrt{6}})\right] \tag{86}$$

Since $F(\tanh\frac{\varphi}{\sqrt{6}})$ is an arbitrary function of its argument, the scalar potential is in principle arbitrary as well. However, in light of the symmetry arguments given above, it seems more natural that $F(\tanh\frac{\varphi}{\sqrt{6}})$ should be given by an even monomial of its argument. This choice is analogous to what is the done in standard models of chaotic inflation. We then produce a potential $V(\phi) = \lambda_n \tanh^{2n}(\varphi/\sqrt{6})$, where λ_n is a constant. In the limit of large φ, the potential approaches a plateau:

$$V(\phi) \simeq \lambda\left(1 - 4ne^{-\sqrt{2/3}\varphi}\right) \tag{87}$$

This potential allows for inflation to occur with the following universal inflationary observables, in the large-N limit:

$$1 - n_s = \frac{2}{N}, \quad r = \frac{12}{N^2} \tag{88}$$

These inflationary observables do not depend on the value of the scalar potential parameters λ or $4n$. This is why the model described here, named the T-model in [31], is said to have an *attractor* structure. It "attracts" the inflationary observables to the values given above, regardless of the details of the scalar potential. However, the T-model is not an α-attractor. An α-attractor has an additional variable α which allows the model to cover a range of the $n_s - r$ plane.

2.3.2 *Attractor Inflation in Supergravity*

Let us see how the above concepts can be generalized to a supergravity setting. Following [36], we take a superconformal Kähler potential $\mathcal{N}(X, \bar{X})$. X stands for the scalar fields in the model: the conformon X^0 the inflaton $X^1 = \phi$, and a sGoldstino $X^2 = S$. $\mathcal{N}(X, \bar{X})$ becomes:

$$\mathcal{N}(X, \bar{X}) = -|X^0|^2 + |X^1|^2 + |S|^2 \tag{89}$$

The Kähler potential has an $SU(1,1)$ symmetry that mixes the inflaton and the conformon, and another $SU(1,1)$ which mixes the conformon and the sGoldstino. We take the following superconformal superpotential:

$$W = Sf(X^1, X^0)\left[(X^0)^2 - (X^1)^2\right] \tag{90}$$

The $SU(1,1)$ symmetry of the Kähler potential is only maintained when f is equal to a constant. In general, the action becomes:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - f^2(\tanh\frac{\varphi}{\sqrt{6}})\right] \tag{91}$$

which is equivalent to (86) and we find the same universal inflationary predictions as before.

We can generalize the above supergravity attractor model to an α-attractor model by modifying the Kähler- and superpotentials. We now choose:

$$\mathcal{N}(X, \bar{X}) = -|X^0|^2\left[1 - \frac{|X^1|^2 + |S|^2}{|X^0|^2}\right]^\alpha \tag{92}$$

The $SU(1,1)$ symmetries are only preserved in the Kähler potential when $\alpha = 1$. The superpotential becomes:

$$W = S(X^0)^2 f(X^1/X^0)\left[1 - \frac{(X^1)^2}{(X^0)^2}\right]^{(3\alpha-1)/2} \tag{93}$$

In the same way as before, the symmetry is only preserved when f is equal to a constant. After gauge fixing the conformon $X^0 = \bar{X}^0 = \sqrt{3}$, we obtain the following action, in terms of a canonically normalized scalar field φ:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\varphi)^2 - f^2\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)\right] \tag{94}$$

For α of order $O(1)$ and in the large-N limit, this model predicts the following inflationary observables:

$$n_s - 1 = \frac{2}{N}, \quad r = \frac{12\alpha}{N^2} \tag{95}$$

We will describe a α-attractor model based on a single chiral multiplet (constructed in [47])in chapter 5. This is the model we would like to couple to string theory moduli, since it has a Kähler potential which is wonderfully natural from the perspective of string theory.

2.4 STRING THEORY

Since most of our discussion will take place in the low-energy limit of string theory, in which a supergravity approximation is valid, we have begun our discussion by describing supergravity. However, in the following it will be necessary to understand a few, but not that many, things about string theory proper.

String theory is the quantum theory of relativistic 1-dimensional objects called *strings*. It was developed in the 1970s as a candidate theory for the strong interactions. In this aspect, it was quickly replaced by QCD. However, it was soon discovered that string theory held the potential to become a quantum theory of gravity, perhaps even a Theory of Everything (one that combines quantum gravity with the forces of the Standard Model in a single framework). We will be extremely brief here, focusing only on the things we need to know.[6]

String theory lives in 10 spacetime dimensions. The Type IIB and Type IIA superstring theories are (at the level of perturbation theory) defined by an action with a large amount of symmetry. We can use the symmetry to move to a *light-cone gauge*, which simplifies many calculations. Even though the theory lives in $D = 10$, after gauge fixing the group of rotations becomes $SO(8)$, with covering group $Spin(8)$. $Spin(8)$ has the vector representation $\mathbf{8v}$, and the spinor and conjugate spinor representations, $\mathbf{8s}$ and $\mathbf{8c}$ respectively. The defining action of Type IIB in light-cone gauge becomes [17]:

$$S = \frac{-1}{2\pi}\int d\sigma d\tau\left(\partial_+ X^i \partial_- X^i - iS^a\partial_- S^a - i\tilde{S}^a\partial_+\tilde{S}^a\right) \tag{96}$$

6 An accessible and brief talk by Edward Witten about why string theory provides a quantum gravity free from ultraviolet divergences can be found at https://www.youtube.com/watch?v=HojLDoPphTY

where X^i are bosonic spatial coordinates which represent the coordinates of a 1-dimensional string in spacetime. S^a are so-called left-moving fermions and $\tilde{S}^a$ are right-moving fermions. The subscripts $\pm$ on the partial derivatives are linear combinations of the two coordinates τ and σ. These coordinates parametrize the *worldsheet* of the string: the two-dimensional surface that the string sweeps out over time (think of the worldline that a point particle traces over time). These fermions have identical spacetime chirality (as we have indicated by using indices a instead of $\dot{a}$ for each of them). This makes Type IIB chiral in spacetime, like we have seen in the supergravity case. There is an equally valid action in which the left- and right-moving fermions take opposite chiralities, which leads to a theory which is non-chiral in spacetime. This is the Type IIA superstring. The Type II theories only include *closed strings*. This means that we specify periodic boundary conditions on the worldsheet coordinate σ:

$$X^i(\tau, \sigma) = X^i(\tau, \sigma + 2\pi) \tag{97}$$

$$S^a(\tau, \sigma) = S^a(\tau, \sigma + 2\pi) \tag{98}$$

Given these periodic boundary conditions, we can expand the functions X and S in Fourier modes:

$$X^i = x^i + \frac{1}{2}p^i\tau + \frac{i}{2}\sum_{n\neq 0}\frac{1}{n}\alpha^i_n e^{-in(\tau+\sigma)} + \frac{1}{n}\tilde{\alpha}^i_n e^{-in(\tau-\sigma)} \tag{99}$$

$$S^a = \frac{1}{\sqrt{2}}\sum_{-\infty}^{\infty} S^a_n e^{-in(\tau+\sigma)} \tag{100}$$

$$\tilde{S}^a = \frac{1}{\sqrt{2}}\sum_{-\infty}^{\infty} \tilde{S}^a_n e^{-in(\tau-\sigma)} \tag{101}$$

The theory is quantized by promoting the Fourier coefficients α^i_n, $\tilde{\alpha}^j_m$, S^a_n, $\tilde{S}^b_m$, and the zero modes x^i, p^i to quantum operators. The bosonic coefficients are given commutation relations, and the fermionic ones are given anti-commutation relations. The fermionic zero modes generate the following algebra:

$$\{S^a_0, S^b_0\} = \delta^{ab}, \quad \{\tilde{S}^a_0, \tilde{S}^b_0\} = \delta^{ab} \tag{102}$$

We require that the ground state of the theory provides a representation of this algebra. This requires that both the left- and right-moving ground states transform as the representation $\mathbf{8v} + \mathbf{8c}$ (for details of this group theory decomposition, see [17]). The full ground state of the theory then transforms as $(\mathbf{8v} + \mathbf{8c}) \otimes (\mathbf{8v} + \mathbf{8c})$. Had we begun with left- and right-movers of opposite chirality (i.e. had we used the Type IIA string), the ground state would have been $(\mathbf{8v} + \mathbf{8s}) \otimes (\mathbf{8v} + \mathbf{8c})$ instead. Decomposing the product rep into a direct sum of

irreps, we identify the particles that exist in the string specrum at the massless level.

The $\mathbf{8c} \otimes \mathbf{8c}$ part decomposes into a scalar l called the axion, a 2-form C_2, and a self-dual 4-form A_4. These particles are called R-R states. The decomposition of $\mathbf{8v} \otimes \mathbf{8v}$ yields a symmetric 2-tensor g_{mn}, a 2-form B_2 and a scalar ϕ called the dilaton. These particles are called NS-NS states. The $\mathbf{8v} \otimes \mathbf{8c}$ part yields two vector-spinors Ψ_M, the gravitinos. These particles are called NS-R or R-NS states. We see that we have exactly reproduced the massless particle spectrum we found in Type IIB supergravity! The fact that the spectrum contains a massless symmetric 2-tensor (i.e. a massless spin-2 particle) is how we know that string theory gives us a theory of gravity.[7].

2.4.1 *Perturbative Symmetries of Type IIB*

The Type IIB superstring theory has a number of symmetries which will be of interest to us in the future. These are so-called *perturbative symmetries*, which means that they appear at the level of perturbation theory, but are conjectured to hold in the full non-perturbative string theory (although non-perturbative string theory only has a full definition in a small number of cases, where the definitionis provided by the AdS/CFT correspondence).

The first symmetry of interest is *worldsheet parity*, represented by the symbol Ω. This reverses the orientation of the string: $\sigma \to 2\pi - \sigma$. This turns right-moving fermions into left-moving fermions. The NS-NS states from the symmetric parts of $\mathbf{8v} \otimes \mathbf{8v}$ are even under this exchange, and the anti-symmetric parts are odd. The R-R states behave similarly, except that we now have anti-commuting Grassmannian fields due to Fermi statistics. In short, the fields which are even under Ω are: g_{mn}, C_2, ϕ. The fields which are odd are l, B_2, A_4.

The second symmetry transformation is given by the operator $(-1)^{F_L}$, where F_L is the total number of left-moving fermions in the state which it acts on. This means that R-NS, and R-R states are odd, since they have an odd number of left-moving fermions. The NS-NS and NS-R states are even under $(-1)^{F_L}$.

We will use the transformation properties described here to make what is called an *orientifold projection* of Type IIB superstring theory. This will be necessary to obtain a consistent *flux compactification*. The next chapter is devoted to this subject.

7 Edward Witten gives a much better explanation of why this is the case in the talk that we reference above.

3 CALABI-YAU FLUX COMPACTIFICATION

In order for string theory to have anything to do with describing the real world, it needs to be reduced to an (effectively) four-dimensional theory. There are two common ways to achieve this[1]. The first and most straightforward method is to reduce six of string theory's nine spatial dimensions to a very small compact manifold. If this internal space is small enough, the theory can be treated as effectively four-dimensional. The second way to reduce string theory to four dimensions is called the braneworld scenario. In this model, the Standard Model lives on a 3-dimensional membrane inside a larger 10-dimensional space, the "bulk". However, gravity is allowed to propagate into the full space, which accounts for its weakness in a very natural way. We will focus exclusively on the compactification scenario.

Considerations of phenomenology pose very strict conditions on the possible internal manifolds. We need to leave the correct amount of supersymmetry in the effective theory. We would like to leave at least $\mathcal{N} = 1$ unbroken SUSY, in order to solve some of the problems of the Standard Model. This poses certain topological and differential conditions on the internal manifold. They imply that the internal space is a certain kind of complex manifold called a *Calabi-Yau 3-fold*. However, these manifolds typically have a large number of dynamically undetermined parameters, called moduli. These will determine physical properties like the volume of the internal space. In the four-dimensional effective theory, the moduli will manifest as scalar fields. These scalar fields are all *flat directions* of the scalar potential, which means that the scalar potential does not depend on them. The moduli can fluctuate without any energy cost. In fact, there is nothing stopping them from running off to infinite field values. If the modulus associated with the volume of the internal space can run off to infinity, the theory is not at all effectively four-dimensional! Furthermore, the classical mass of a scalar field is determined by the square term in the scalar potential. This means that all fields associated with flat directions are massless. Massless scalar fields are phenomenologically unacceptable. They transmit long-range interactions that couple differently to different types of matter, possibly violating the principle of equivalence [22], which has been tested to great accuracy. Quantum corrections could change some of these results, and generate a mass for the moduli. However, to make these quantum corrections large

1 A third possibility exists. The degrees of freedom that would normally provide the six extra dimensions may not form a geometrical target space at all. We will not consider this possibility. For some discussion, see [23].

enough, we have to go to the large-coupling limit. Unfortunately, nobody yet understands string theory at large coupling well enough to do the necessary calculations. We have to find a way to solve the problem within the supergravity approximation. This is called the *Dine-Seiberg problem* [20].

We can solve the problem partially by turning on *fluxes*. These are non-vanishing vacuum expectation values for some of the field strengths in the theory. Turning on fluxes will generate a scalar potential (and a classical mass) for some, but not all, of the moduli[2]. The presence of background fluxes will necessitate introducing a *warp factor* between the compact internal space and the Minkowski spacetime. As we will see, there is a no-go theorem in supergravity that renders Minkowski or de Sitter compactifications with warp factors impossible. However, in string theory there are extended objects called orientifold planes, whose presence will help us evade the no-go theorem.

The rest of this chapter will be devoted to making the above statements mathematically precise. Unfortunately, the subject of flux compactifications is a vast one, and we will have to severely limit the scope to just what is necessary for the following chapters. For an extensive review, see [22].

3.1 KALUZA-KLEIN THEORY

The idea of compactifying a higher-dimensional theory was first put forward by Theodor Kaluza. The original idea was to reduce five-dimensional general relativity to four-dimensional relativity coupled to a scalar and electromagnetism. Oskar Klein brought these ideas into quantum theory. The original motivation is now obsolete, but the central principle is still highly relevant.

Let us see how Kaluza-Klein theory works in a simple example [43]. Most of the strategy outlined here will carry over to our further discussion. Take the five-dimensional Einstein-Hilbert action:

$$S = -\int d^5x\sqrt{-g}\hat{R} \tag{103}$$

where the hat on $\hat{R}$ indicates that it lives within the five-dimensional space. We will compactify one of the coordinates, say y, on a circle. This means that we make the following identification:

$$y \sim y + 2\pi R \tag{104}$$

2 Technically, once a scalar potential has been generated for a modulus, it is no longer a modulus. Moduli by definition are fields which parametrize a degeneracy. However, we will not always bother to be accurate with this terminology.

where R is the radius of the compactification circle. The five-dimensional equation of motion is the Einstein equation: $\hat{R}_{mn} = 0$. An obvious solution is the following:

$$G_{mn} = \begin{bmatrix} \eta_{\mu\nu} & 0 \\ 0 & 1 \end{bmatrix} \tag{105}$$

where Greek indices refer to coordinates of the four-dimensional spacetime and $\eta_{\mu\nu}$ is the Minkowski metric. We now want to consider the dynamics of small fluctuations around this background value. It is useful to parametrize the metric in the following way:

$$G_{mn} = \phi^{-1/3} \begin{bmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{bmatrix} \tag{106}$$

where A_μ is a vector field in four dimensions and ϕ is a scalar. When $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, and $\phi = 1$, the metric reduces to the background value, (105). The equations of motion now state that each of the components of G_{mn} must be an eigenvector of the five-dimensional Laplacian. Let's expand the fields $g_{\mu\nu}$, ϕ, and A_μ into Fourier modes on the circle:

$$g_{\mu\nu} = \sum_{n=-\infty}^{\infty} g_{\mu\nu}^{(n)} e^{\frac{2\pi n i y}{R}} \tag{107}$$

$$A_\mu = \sum_{n=-\infty}^{\infty} A_\mu^{(n)} e^{\frac{2\pi n i y}{R}} \tag{108}$$

$$\phi = \sum_{n=-\infty}^{\infty} \phi^{(n)} e^{\frac{2\pi n i y}{R}} \tag{109}$$

The fields $\phi^{(n)}$, $A_\mu^{(n)}$, and $g_{\mu\nu}^{(n)}$ are the fields that appear in the four-dimensional dynamics. The equation of motion for ϕ splits into Fourier modes:

$$\partial_m \partial^m \phi = \partial_\mu \partial^\mu \phi + (\partial_y)^2 \phi = \sum_{n=-\infty}^{\infty} \left[\partial_\mu \partial^\nu - \left(\frac{2\pi n}{R}^2\right) \right] \phi^{(n)} e^{\frac{2\pi n i y}{R}} \tag{110}$$

This is, schematically, a Klein-Gordon equation for each of the Kaluza-Klein modes $\phi^{(n)}$. The mass generated for each field is of order n^2/R^2. When we consider physics at an energy scale which is very small compared to the radius of the circle, only the massless modes matter. The crucial step in the Kaluza-Klein reduction is that we truncate the fields to just their *massless modes* - i.e. we choose $\phi = \phi^{(0)}$ and do likewise for the other fields. We have to substitute this choice into the Einstein-Hilbert action, and then integrate over the internal coordinate y to obtain the low-energy effective four-dimensional action:

$$S = \int d^4x \sqrt{-g} \left[-R - \frac{1}{4} \phi F_{\mu\nu} F^{\mu\nu} - \frac{1}{6\phi^2} \partial_\mu \partial^\mu \phi \right] \tag{111}$$

The effective four-dimensional theory of 5D general relativity contains electromagnetism and a scalar field ϕ. General relativity has been unified with electromagnetism in a simple way. Unfortunately, this unification strategy cannot be pursued any further. In particular, it is not possible to generate non-Abelian gauge theories in this way. However, the strategy outlined here will still be useful to us. We will construct low-energy effective theories of compactified Type IIB string theories in exactly the same way: we select a background configuration analogous to (105) and (107), consider the dynamics of small fluctuations around it, and truncate the spectrum to just the massless Kaluza-Klein modes.

3.2 IDENTIFYING THE INTERNAL SPACE

Let us now move from the simple Kaluza-Klein example to the compactification of Type IIB superstring theory. We focus exclusively on Type IIB superstrings, because the moduli stabilization schemes that we will use (KKLT [29], KL [35]) are Type IIB constructions (the discussion can in principle be translated to Type IIA by mirror symmetry). Our goal is to obtain a four-dimensional low-energy effective theory. As such, we are mostly interested in Type IIB supergravity, the effective field theory of the massless sector of Type IIB superstring theory. This is a valid approximation scheme at the energy scale of inflation, around 10^{16}GeV. Furthermore, the model of inflation which we want to incorporate within string theory has a natural supergravity formulation.

We want to decompose the ten dimensional spacetime on which Type IIB lives into a four dimensional external spacetime and a six dimensional compact internal space:

$$\mathcal{M} = M_4 \times K_6 \tag{112}$$

For the metric on $\mathcal{M}$, we make the following Ansatz:

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{mn} dy^m dy^n \tag{113}$$

where we denote coordinates of $\mathcal{K}$ by Latin indices (i.e. y^m) and coordinates of M_4 by Greek indices (i.e. x^μ)[3].

We want the background $\mathcal{M}$ to retain at least some of the supersymmetry of the original theory. This way, supersymmetry may still provide a resolution to some of the naturalness problems that plague the Standard Model. A compactification on a generic manifold will explicitly break all of the supersymmetry. To retain some of it, the manifold must admit a globally well-defined supersymmetry spinor.

3 We will sometimes use the Latin indices to refer to coordinates of the entire ten-dimensional space.

Type IIB has two supersymmetry spinors of the same ten-dimensional chirality. We may decompose them in the following way:

$$\epsilon^A_{IIB} = \xi^A_+ \otimes \eta_+ + \xi^A_- \otimes \eta_- \tag{114}$$

where $A = 1, 2$. The spinors $\xi^A_\pm$ live in the external spacetime, and the spinors η_{pm} live in the internal space. The $+$ and $-$ subscripts on the spinors indicate six- or four-dimensional chirality. We have decomposed the spinors using only a single internal Weyl spinor η and its complex conjugate. We make this choice because supporting even one non-trivial supersymmetry spinor is quite a stringent requirement for the internal manifold, as we will see shortly. When the internal space admits a globally well-defined spinor, we see from the above decomposition that the compactification may preserve up to eight of the supercharges. In other words, the theory will have $\mathcal{N} = 2$ supersymmetry.

The existence of a globally well-defined spinor poses a topological restriction on the internal manifold. In technical terms: the manifold must have a reduced *structure group*[4] isomorphic to $SU(3)$. A generic Riemannian manifold in six dimensions has a structure group contained within $SO(6)$, whose Lie algebra is isomorphic to $SU(4)$. A spinor transforms according to the $\mathbf{4}$ irrep. The $\mathbf{4}$ has an $SU(3)$ decompostion: $\mathbf{4} \to \mathbf{3} + \mathbf{1}$. A globally well-defined spinor must sit in the $\mathbf{1}$ irrep [22].

We now impose that M_4 is a maximally symmetric space (Minkowski, de Sitter or anti-de Sitter). This means all the fields which transform non-trivially under the Lorentz group must vanish. This includes all of the fermions and all of the n-forms except the scalars. This requirement will be relaxed when we turn on fluxes, but to do so we will have to introduce a *warp factor* into the metric, which means that we must change the metric Ansatz. Most of our analysis will be about the dynamics of fluctuations around this background configuration.

We require that all fermions vanish in the background, so their supersymmetry variations must vanish as well. We have:

$$\delta_{\epsilon^A_{IIB}}(\text{fermions}) = 0 \tag{115}$$

If this requirement is not met, then M_4 cannot be both maximally symmetric and supersymmetric. The same statements apply to the supersymmetry tranformations of the bosonic fields, but this is a trivial matter since all terms in the transformation of a boson contain a fermionic field.

In particular, the supersymmetry transformations for the gravitinos can be written as [43]:

$$\delta_{\epsilon^A_{IIB}} \psi_M = \nabla_M \epsilon^A_{IIB} + \sum F_M \epsilon \tag{116}$$

4 We only bring up structure groups to distinguish the topological restrictions on the internal manifold from the differential ones. We will not mention structure groups again after this short discussion.

where F denotes a particular contraction of a field strength with a gamma matrix, and the summation is carried out over all the field strengths in the theory. These vanish by assumption, so the transformation becomes:

$$\delta_{\epsilon^A_{IIB}} \psi_M = \nabla_M \epsilon^A_{IIB} \tag{117}$$

This means that the geometry (112) must admit a covariantly constant spinor.

$$\nabla_M \epsilon^A_{IIB} = 0 \tag{118}$$

We can decompose ϵ^A_{IIB} into external spinors $\xi^A_\pm$ and internal spinors $\eta^A_\pm$. ξ_A and ϵ^A_{IIB} are physical spinors, so they are Grassmann odd (anti-commute with other Grassmann odd quantities). We see that η must be Grassmann even. The spacetime part of equation (118) is:

$$\nabla_\mu \xi^A_\pm = 0 \tag{119}$$

Such a relation leads to an integrability condition:

$$[\nabla_\mu, \nabla_\nu]\theta = \frac{1}{4} R_{\mu\nu\rho\sigma} \Gamma^{\rho\sigma} \theta = 0 \tag{120}$$

where Γ is a gamma matrix. Together with the assumption of maximal symmetry, the integrability condition implies that the external Ricci scalar vanishes. In other words: the spacetime must be Minkowski. Unfortunately, this result has nothing to say about the cosmological constant problem, as it only holds when supersymmetry is not spontaneously broken.

The internal part of equation (118) is:

$$\nabla_m \eta_\pm = 0 \tag{121}$$

This means that K_6 must admit a covariantly constant spinor by itself. This is a differential condition on the connection. It implies that the metric has $SU(3)$ *holonomy*, as we will see later. Every manifold with $SU(3)$ structure group admits a connection with $SU(3)$ holonomy. When this connection has no torsion (which we assume from now on), the manifold is called a *Calabi-Yau* manifold. We will study the properties of these manifolds extensively in the next section.

Every Calabi Yau manifold admits a unique metric with vanishing Ricci tensor. Therefore (112) (with M_4 Minkowski and K_6 Calabi-Yau) is a solution to the Einstein equations when all the fields vanish in the background. Unfortunately, string theory offers us no mechanism to dynamically select one Calabi-Yau over any other. We will have to make an Ansatz for the Calabi-Yau metric g_{mn} of the internal space[5], and then consider fluctuations around that background value. Since every Calabi-Yau manifold in principle has the same energy, these fluctuations can grow exceedingly large. This is the first hint at the appearance of *moduli* in the theory.

5 In an abstract sense - we will never construct the Calabi-Yau metric explicitly.

A Calabi-Yau 3-fold is a compact, complex Kähler manifold with six real dimensions that admits a covariantly constant spinor η:

$$\nabla_m \eta = 0 \tag{122}$$

Such a manifold must have a holonomy group isomorphic to $SU(3)$. Equivalently, one might say that a Calabi-Yau manifold is a compact, complex Kähler manifold on which a unique nowhere-vanishing holomorphic 3-form can be defined. A third definition is that a Calabi-Yau manifold is a compact, complex manifold whose Kähler metric has a vanishing Ricci tensor ($R_{\mu\nu} = 0$). Most of the following discussion is based on [7][43] and [23]. The analysis was first performed by Candelas, Horowitz, Strominger and Witten, in their seminal paper [11].

3.3.1 *The Holonomy Group*

A covariantly constant spinor is invariant under parallel transport around a closed loop. When a generic spinor on a manifold with six real dimensions is parallel transported around a closed loop, it receives a rotation by a subgroup of $Spin(6) = SU(4)$. A spinor on six dimensional manifold has eight real components, but it can be split into two $SU(4)$ irreps of definite chirality:

$$\mathbf{8} = \mathbf{4} \oplus \bar{\mathbf{4}} \tag{123}$$

If a covariantly constant spinor exists on the manifold, it must transform according to a trivial irrep of $Spin(6)$ under a parallel transport around a loop. The largest subgroup of $SU(4)$ for which the $\mathbf{4}$ contains a trivial part is $SU(3)$. Within $SU(3)$, there exists the following decomposition of the $\mathbf{4}$:

$$\mathbf{4} = \mathbf{3} \oplus \mathbf{1} \tag{124}$$

This explains why a manifold that admits a covariantly constant spinor has an $SU(3)$ holonomy group. The existence of the covariantly constant spinor is needed to preserve some of the supersymmetry after compactification, as we will explain in the next section.

3.3.2 *The Holomorphic 3-form*

Both the $\mathbf{4}$ and the $\bar{\mathbf{4}}$ in the decomposition of a spinor on K_6 contain a singlet part under the holonomy group $SU(3)$ (the $\bar{\mathbf{4}}$ has a decomposition just like its complex conjugate does (124)). Let's denote these definite-chirality covariantly constant spinors by $\eta_\pm$. We want to know what bilinears we can construct from these spinors. To this

end, let us work with a Dirac matrix basis which is fully antisymmetric. One choice is the following:

$$\sigma_2 \otimes 1 \otimes \sigma_{1,3} \quad \sigma_{1,3} \otimes \sigma_2 \otimes 1 \quad 1 \otimes \sigma_{1,3} \otimes \sigma_2 \tag{125}$$

The bilinears $\eta_-^T \gamma_a \eta_-$ and $\eta_-^T \gamma_{ab} \eta_-$ vanish. To see this, write out the product explicitly and interchange the order of indices in the Dirac matrices. Note that these spinors are Grassmann even (i.e, they commute with each other) by assumption, as opposed to physical spinors, which are Grassmann odd. The following bilinear, however, is nowhere-vanishing:

$$\Omega_{abc} = \eta_-^T \gamma_{abc} \eta_- \tag{126}$$

This is the case because η_- satisfies $\nabla_a \eta_- = 0$. Whenever η vanishes, the equation reduces to $\partial_a \eta = 0$, so we see that a covariantly constant spinor that vanishes somewhere must vanish everywhere. In other words, it is a trivial spinor, which contradicts our assumption. We can use this bilinear to define the following $(3,0)$-form:

$$\Omega = \frac{1}{6} \Omega_{abc} dz^a \wedge dz^b \wedge dz^b \tag{127}$$

This $(3,0)$-form is closed: $(\partial + \bar{\partial})\Omega = 0$. The first term vanishes because there are only three holomorphic coordinates on a complex manifold with six real dimensions, $\partial\Omega = 0$. To see that the second term vanishes, note that $\nabla_{\bar{a}} \Omega = 0$ because η is covariantly constant. On a complex Kähler manifold, the connection coefficients with mixed indices all vanish, so the equation reduces to $\partial_{\bar{a}} \Omega = \partial_{\bar{a}} \Omega_{cde} dz^c \wedge dz^d \wedge dz^e \wedge dz^a = 0$. Ω is a nowhere-vanishing holomorphic $(3,0)$-form. It is the only non-trivial $(3,0)$ form on a Calabi-Yau manifold, $h^{3,0} = 1$.

3.3.3 *The Ricci-flat Metric*

The existence of a covariantly constant spinor requires the Ricci curvature to vanish. In order to see this, we need the identity:

$$[\nabla_m, \nabla_n]\eta = \frac{1}{4} R_{mnpq} \Gamma^{pq} \eta \tag{128}$$

This can be shown by writing out the commutator explicitly using the definition of the covariant derivative of a spinor:

$$\nabla_n \eta = \partial_n \eta + \frac{1}{4} \omega_{npq} \gamma^{pq} \eta \tag{129}$$

where ω_{npq} is the spin connection and γ is a Dirac matrix. The result is:

$$\frac{1}{4}(\partial_m \omega_{nrs} - \partial_n \omega_{mrs} + \omega_{mrp} \omega^p_{ns} - \omega_{nrp} \omega^p_{ms} \gamma^{rs})\eta = \frac{1}{4} R_{mnrs} \gamma^{rs} \eta \tag{130}$$

When the spinor η is covariantly constant, the commutator of course vanishes: $R_{mnpq}\gamma^{pq}\eta = 0$. It turns out that this implies that $R_{mn} = 0$ (for details, see Chapter 9 of [7]), i.e. the manifold must be Ricci flat.

Shing-Tung Yau, half namesake of Calabi-Yau manifolds, proved that compact n-dimensional Kähler manifolds with vanishing *first Chern class* always admit a Kähler metric with holonomy contained in $SU(n/2)$ (and therefore admit a covariantly constant spinor, a unique nowhere-vanishing holomorphic $(3,0)$-form, and a Ricci-flat metric). We will now show the converse: that a manifold which admits a Ricci-flat metric always has vanishing first Chern class. In doing so, we establish the equivalence between the three definitions of Calabi-Yau manifolds we have discussed.

The Chern classes of a manifold are the coefficients in the expansion of the quantity $\det(1+\mathcal{R})$, where $\mathcal{R}$ is a matrix-valued 2-form defined from the Riemann tensor:

$$\mathcal{R} = R^k_{li\bar{j}} dz^i \wedge d\bar{z}^{\bar{j}} \tag{131}$$

More concretely, the j-th Chern class c_j is defined by:

$$\det(1+\mathcal{R}) = 1 + \sum_{j=1} c_j = 1 + \mathrm{tr}\mathcal{R} + \mathrm{tr}(\mathcal{R}\wedge\mathcal{R} + \mathrm{tr}(\mathcal{R}\wedge\mathcal{R} - 2(\mathrm{tr}\mathcal{R})^2)) + \dots \tag{132}$$

The trace of $\mathcal{R}$ is equal to the Ricci tensor. So we see that a manifold with vanishing Ricci tensor has a vanishing first Chern class.

Ricci flatness of the internal manifold, more than merely being a phenomenological restriction, is actually required for the consistency of string theory in the limit in which we are working. The bosonic part of the string theory action on the six-dimensional internal target space is [23]:

$$S = \int dz d\bar{z} \big[g_{mn}(\partial X^m \bar{\partial} X^n + m \leftrightarrow n) + B_{mn}(\partial X^m \bar{\partial} X^n + m \leftrightarrow n) \big] \tag{133}$$

String theory may be formulated as a *superconformally* invariant quantum field theory on a string worldsheet. This is a supersymmetric generalization of ordinary conformal field theory. This superconformal symmetry is crucial to the theory and must persist even after quantization (in other words, it must not be an anomalous symmetry), much like gauge symmetry in the Standard Model. This is because the superconformal symmetry ensures that string theory does not contain ultraviolet divergences. The requirement of non-anomalous superconformal symmetry is actually what requires superstring theory to be ten-dimensional. It can be shown that the above action can only be superconformally invariant if the metric g_{mn} on the six-dimensional target space is Ricci flat to first order in the stringy parameter α'/R^2 (the limit $\alpha'/R^2 \to 0$ is the regime in which we do all

our analysis, because the validity of all our approximations will rely on it). The parameter α' controls the string tension.

3.4 THE MODULI SPACE OF CALABI-YAU MANIFOLDS

In general, it is not possible to construct the Ricci-flat metric of a Calabi-Yau manifold explicitly. Fortunately, the massless spectrum in the four-dimensional low-energy effective theory is determined entirely by topological quantities, namely the Hodge numbers $h^{(p,q)}$. These count the number of non-trivial independent harmonic (p,q)-forms that can exist on the manifold, or in other words the dimension of the cohomology group $H^{(p,q)}$. Calabi-Yau manifolds with a given set of Hodge numbers will in general form a multi-dimensional, continuous family of manifolds, distinguished by parameters which may have a physical meaning, the *moduli*. Furthermore, even within a set of Calabi-Yau manifolds with the same Hodge numbers, not all manifolds have to be topologically equivalent. The Hodge numbers do not determine the topology completely. There is at this moment no complete classification of all Calabi-Yau manifolds in three complex dimensions (in two dimensions, there is only one Calabi-Yau manifold, known as K3). In fact, it is not even known whether or not the number of topologically distinct Calabi-Yau 3-folds is finite.

3.4.1 *Metric deformations*

To see how the Hodge numbers relate to the physics in the four-dimensional effective theory, let's consider continuous deformations of the metric on the internal space which leave it Ricci flat, i.e. deformations δg such that:

$$R_{mn}(g+\delta g) = R_{mn}(g) = 0 \tag{134}$$

We can expand this equation to first order in δg to obtain:

$$\nabla_p \nabla^p \delta g_{mn} + 2R_{mnpq}\delta g^{pq} = 0 \tag{135}$$

where the indices of δg^{pq} have been raised using the original inverse metric g^{mn}. We have made a choice of gauge here ($\nabla^m \delta g_{mn} = \frac{1}{2}\nabla_n \delta g^m_m$) to elinate metric deformations which are simply coordinate transformations.

Thanks to the simple index structure of the Riemann tensor on a Kähler manifold (all of the components vanish except $R_{a\bar{b}c\bar{d}}$), the equations for δg_{ab} and $\delta g_{a\bar{b}}$ decouple. We can define the following $(1,1)$-form:

$$\delta g_{a\bar{b}} dz^a \wedge dz^{\bar{b}} \tag{136}$$

Using the Lichnerowicz equation, it can be shown that this $(1,1)$ form is harmonic. We can define a basis ω_i for the harmonic $(1,1)$-forms

on the internal space, and a basis $\tilde{\omega}^i$ for the harmonic $(2,2)$-forms in the following way:

$$\int \omega_i \wedge \tilde{\omega}^j = \delta_i^j \tag{137}$$

where the integration is carried out over the entire Calabi-Yau space. Using this basis, we can expand the mixed-index metric deformations $\delta g_{a\bar{b}}$ as follows:

$$\delta g_{a\bar{b}} = v^i (\omega_i)_{\alpha\bar{\beta}} \tag{138}$$

In a geometry of type (112), the expansion coefficients v^i may depend on the coordinates of the external space. This means that in the four-dimensional effective theory, the v_i are dynamical scalar fields. They are called *Kähler moduli*, and the deformations they parametrize are called *Kähler class deformations*.

The only non-vanishing components of a Kähler metric are those with mixed type indices, i.e. $g_{a\bar{b}}$. We can however still deform the pure type components: $g_{ab} \to g_{ab} + \delta g_{ab}$ (and likewise for the pure anti-holomorphic components). The resulting metric is no longer Hermitian with respect to the original complex structure. A holomorphic change of coordinates cannot undo this, so we must define a new complex structure. For that reason, the deformations of pure type indices are called *complex structure deformations*. We can use $\delta g_{\bar{a}\bar{b}}$ to define a $(2,1)$-form as follows:

$$\Omega_{abc} g^{c\bar{d}} \delta g_{\bar{d}e} dz^a \wedge dz^{\bar{b}} \wedge dz^{\bar{e}} \tag{139}$$

Again, we can use the Lichnerowicz equation to show that this $(2,1)$-form is harmonic, and expand $g_{\bar{a}\bar{b}}$ in a basis of the $(2,1)$ cohomology space. The expansion coefficients again are dynamical scalar fields in the four-dimensional effective theory. These are called the *complex structure moduli*. Let's denote them z^a (where $a = 0, \ldots, h^{(2,1)}$ for future reference)

3.4.2 *The massless Kaluza-Klein modes*

The second reason that the Hodge numbers affect the four-dimensional physics is that they count the number of massless Kaluza-Klein modes. We are working in the approximation that the internal space is quite small compared to the energy scale of inflation. We will truncate each p-form field to just its massless modes, and hope that the massive modes consistently disappear from the dynamics. When the internal space is flat, the massless Kaluza-Klein mode of a scalar is just the zero mode of the operator $\partial_m \partial^m$. On a curved internal space, the massless modes of a (p,q)-form are given by the zero modes of the Laplacian on the internal space, i.e. the (p,q) harmonic forms. For example, the decomposition of the 2-form $\hat{B}_2$ is given by:

$$\hat{B}_2 = B_2 + b^i \omega_i \tag{140}$$

where the hat on $\hat{B}_2$ denotes that it lives in the full ten-dimensional space, and B_2 is a 2-form on spacetime. There are no $(2,0)$- or $(0,2)$-forms in the expansion because a Calabi-Yau manifold does not admit harmonic forms of such type, as we will see shortly.

3.4.3 *The Hodge diamond*

Now that we know how the cohomology of the internal manifold influences the four-dimensional physics, let us examine the structure of the Calabi-Yau Hodge numbers in more detail. First of all, there exists the symmetry

$$h^{(p,q)} = h^{(n-p,n-q)} \tag{141}$$

where n is the complex dimension of the Calabi-Yau. To show this, define the harmonic (p,q)-form ω and the harmonic $(n-p,n-q)$-form ψ. The integral $\int_M \omega \wedge \psi$ defines a non-singular map from the Dolbeault cohomology groups $H^{p,q}_{\bar{\partial}} \times H^{n-p,n-q}_{\bar{\partial}}$ to the complex numbers (for details, see [44]). This means that the two Dolbeault cohomology groups must be isomorphic, so the dimensionality of their vector spaces must be equal: $h^{(p,q)} = h^{(n-p,n-q)}$.

By complex conjugating a harmonic (p,q)-form, we obtain a harmonic (q,p) form. We thus obtain a second symmetry:

$$h^{(p,q)} = h^{(q,p)} \tag{142}$$

Furthermore, a compact simply-connected manifold always satisfies $h^{(1,0)} = 0$. A compact, connected Kähler manifold satisfies $h^{(0,0)} = 1$. We have already seen that there is a unique harmonic holomorphic 3-form on each Calabi-Yau: $h^{(3,0)} = 1$. These numbers, along with the above symmetries, are enough to specify most of the Hodge numbers of the Calabi-Yau. The only unspecified ones are $h^{(2,1)} = h^{(1,2)}$ and $h^{(1,1)} = h^{(2,2)}$. These are the only Hodge numbers that vary between different Calabi-Yau manifolds. We have already seen that $h^{(1,1)}$ specifies the number of Kähler moduli, and $h^{(2,1)}$ specifies the number of complex structure moduli.

3.5 THE EFFECTIVE ACTION OF TYPE IIB SUPERGRAVITY ON CALABI-YAU MANIFOLDS

We now know enough about Calabi-Yau manifolds and their topological properties to derive the Type IIB low-energy effective action.

At the energy scale of inflation, we assume that only the massless Kaluza-Klein modes of the form fields are relevant. The massive states are too heavy to be excited, because the internal space is assumed to be quite small.

The Type IIB supergravity spectrum contains the following n-forms: the 2-forms B_2 and C_2, a 4-form A_4 with self-dual field strength, and

two real scalar fields l (the axion) and ϕ (the dilaton). After performing the truncation to the massless Kaluza-Klein modes, the n-forms can be expanded:

$$\hat{B}_2 = B_2 + b^i \omega_i \tag{143}$$

$$\hat{C}_2 = C_2 + c^i \omega^i \tag{144}$$

$$\hat{A}_4 = D_2^i \wedge \omega_i + V^A \alpha_A - U_B \beta^B + \rho_i \tilde{\omega}^i \tag{145}$$

where the hats again denote n-forms living in the ten-dimensional space. The parameters b^i, c^i, and ρ_i are scalar fields. B_2 and D_2^i are 2-forms in spacetime. V^A, U_B are vector fields. ω_i and $\tilde{\omega}^i$ are respectively the harmonic $(1,1)$- and $(2,2)$-forms whose basis is defined by (137). α_A and β^A together provide a basis for all the harmonic 3-forms on the Calabi-Yau (that is, the $(3,0)$-, $(0,3)$-, $(1,2)$-, and $(2,1)$-forms). Their normalization is given by[6]:

$$\int_{K_6} \alpha_A \wedge \beta^B = \delta_A^B \tag{146}$$

$$\int_{K_6} \alpha_A \wedge \alpha_B = \int_{K_6} \beta^A \wedge \beta^B = 0 \tag{147}$$

where $A, B = 0, \ldots, h^{(2,1)}$.

As discussed previously, we need to incorporate the dynamics of metric fluctuations as well. The fluctuations of the pure spacetime components $g_{\mu\nu}$ are gravitons. The components with mixed indices $g_{\mu N}$ have only massive fluctuations, because there are no harmonic $(1,0)$-forms on Calabi-Yau manifolds. The fluctuations of the purely internal components g_{ab} and $g_{\bar{a}\bar{b}}$ are discussed above. They lead to $h^{(2,1)}$ real scalars z^a and $h^{(1,1)}$ real scalars v^i.

The field strength $\hat{F}_5$, defined by:

$$\hat{F}_5 \equiv d\hat{A}_4 - \frac{1}{2}\hat{C}_2 \wedge d\hat{B}_2 + \frac{1}{2}\hat{B}_2 \wedge d\hat{C}_2 \tag{148}$$

has a self-duality condition, $\hat{F}_5 = \star\hat{F}_5$. This implies that not all the degrees of freedom in A_4 are physical. We can eliminate the 2-forms D_2^i and the vectors U_B as they become dual to the scalars ρ_i and the vectors V^A, respectively. The field content in the four-dimensional effective theory is exactly the same as an $\mathcal{N} = 2$ supergravity with the following multiplets:

- $h^{(1,1)}$ hypermultiplets with bosonic content (ρ^i, v^i, b^i, c^i)
- $h^{(2,1)}$ vector multiplets with bosonic content (V^a, z^a)
- A gravitational multiplet with bosonic content $(g_{\mu\nu}, V^0)$
- A double-tensor multiplet with bosonic content (B_2, C_2, ϕ, l)

6 These relations only determine the basis up to a transformation in the symplectic group $Sp(2h^{(2,1)} + 2, \mathbb{Z})$.

The strategy now is to insert the various expansions (the field expansions 143, and the fluctuations of the metric), into the Type IIB supergravity action (68), and then to integrate over the internal space. The resulting action is an $\mathcal{N} = 2$ supergravity in four dimensions. The algebra of the full computation is a little involved and not illuminating for our purposes. We reproduce a part of it in Appendix A (for details, see [43]).

The result of the computation is:

$$S = \int_{M_4} \Big[-\frac{1}{2} R \star \mathbf{1} - g_{ab} dz^a \wedge \star d\bar{z}^b \\ - h_{uv} dq^u \wedge \star dq^v + \frac{1}{2} \mathrm{Im}\mathcal{M}_{IJ} F^I \wedge \star F^J + \frac{1}{2} \mathrm{Re}\mathcal{M}_{IJ} F^I \wedge F^J \Big] \quad (149)$$

The definition of the matrix $\mathcal{M}_{IJ}$ can be found in the literature [43]. It relates to Abelian gauge fields and will be of no further concern to us. The matrix h_{uv} is likewise defined in [43]. It contains kinetic terms and scalar potential terms for the dilaton and the dual scalars of the 2-forms. The most important thing to note here is that we have *not generated a scalar potential for the Kähler and complex structure moduli*! This presents a big problem for phenomenology, as we have explained several times before. We will devote most of remaining chapter as well as the next to explaining the solution to this *moduli stabilization* problem.

3.6 THE METRIC ON MODULI SPACE

The matrices g_{ij} and g_{ab} that appear in (336) have a natural geometric interpretation as the metrics of Kähler manifolds. The full geometry of the moduli space has a product structure:

$$\mathcal{M}_m = \mathcal{M}^{2,1} \times \mathcal{M}^{1,1} \quad (150)$$

The coordinates on the $\mathcal{M}^{2,1}$ part are the deformations of the complexified Kähler form, and the ones on the $\mathcal{M}^{1,1}$ part are spanned by the complex structure deformations. A natural metric is given by:

$$ds^2 = \frac{1}{2V} \int g^{a\bar{b}} g^{c\bar{d}} [\delta g_{ac} \delta g_{\bar{b}\bar{d}} + (\delta g_{a\bar{d}} \delta g_{c\bar{d}} - \delta B_{a\bar{d}} \delta B_{c\bar{b}})] \sqrt{g} d^6 x \quad (151)$$

where the integration is carried out over the Calabi-Yau. The first term and the second term respectively correspond to the metrics g_{ij} and g_{ab} that appear in the four-dimensional action.

3.6.1 *The complexified Kähler moduli*

Let's focus on the $h^{(1,1)}$ piece first. It defines the following inner product on the $(1,1)$-forms ρ and σ:

$$G(\rho,\sigma) = \frac{1}{2V}\int \rho_{a\bar{d}}\sigma_{\bar{b}c}g^{a\bar{b}}g^{c\bar{d}}\sqrt{g}d^6x = \frac{1}{2V}\int \rho\wedge\star\sigma \tag{152}$$

Using the following cubic form κ:

$$\kappa(\rho,\sigma,\tau) \equiv \int \rho\wedge\sigma\wedge\tau \tag{153}$$

we can rewrite the metric as follows:

$$g(\rho,\sigma) = -\frac{1}{2V}\kappa(\rho,\sigma,J) + \frac{1}{8V^2}\kappa(\rho,J,J)\kappa(\sigma,J,J) \tag{154}$$

where $J = ig_{a\bar{b}}dz^a d\bar{z}^b$ is the Kähler form on the Calabi-Yau manifold. The complexified Kähler form $\mathcal{J}$ is defined by

$$\mathcal{J} = B + iJ = (b^i + iv^i)\omega_i = t^i\omega_i \tag{155}$$

We can use the coordinates t^i to write the metric as:

$$g_{ij} = \frac{1}{2}g(\omega_i,\omega_j) = \frac{\partial}{\partial t^i}\frac{\partial}{\partial \bar{t}^j}\log(-8\mathcal{K}) \tag{156}$$

where $\mathcal{K} \equiv \frac{1}{6}\int_{K_6} J\wedge J\wedge J = V$. We have shown that the natural metric defined on the $\mathcal{M}^{1,1}$ part of the moduli space is derived from a Kähler potential, i.e. it is a Kähler manifold. Let us now show that this metric coincides with the one that appears in the four-dimensional action. To this end, we define the following quantities:

$$\mathcal{K}_{ijk} = \int_{K_6}\omega_i\wedge\omega_j\wedge\omega_k \tag{157}$$

$$\mathcal{K}_{ij} = \int_{K_6}\omega_i\wedge\omega_j\wedge J = \mathcal{K}_{ijk}v^k \tag{158}$$

$$\mathcal{K}_i = \int_{K_6}\omega_i\wedge J\wedge J = \mathcal{K}_{ijk}v^j v^k \tag{159}$$

We then have $\mathcal{K} = \frac{1}{6}\mathcal{K}_{ijk}v^i v^j v^k$. A bit of algebra shows that:

$$g_{ij} = \partial_i\bar{\partial}_j(-\log\mathcal{K}) \tag{160}$$

$$= -\frac{1}{4}\left(\frac{\mathcal{K}_{ij}}{\mathcal{K}} - \frac{1}{4}\frac{\mathcal{K}_i\mathcal{K}_j}{\mathcal{K}^2}\right) \tag{161}$$

$$= \frac{1}{4\mathcal{K}}\int_{K_6}\omega_i\wedge\star\omega_j \tag{162}$$

So the natural metric on $\mathcal{M}^{1,1}$ coincides with the metric that appears in the four-dimensional action. The Kähler deformation moduli and the B_2 moduli (i.e. the complexified Kähler moduli) span a Kähler manifold.

3.6.2 *The complex structure moduli*

The manifold $\mathcal{M}^{2,1}$ spanned by the complex structure deformations is a Kähler manifold, just like $\mathcal{M}^{1,1}$. In fact, it is a so-called *special Kähler* manifold, because the Kähler potential $\mathcal{K}$ can be derived from a holomorphic prepotential (for details, see Chapter 9 of [7]) The complex structure part of the natural metric 151 is:

$$ds^2 = \int \sqrt{g} d^6x g^{a\bar{b}} g^{c\bar{d}} \delta g_{ac} \delta g_{\bar{b}d} \equiv 2 * g_{a\bar{b}} \delta t^a \delta \bar{t}^{\bar{b}} \tag{163}$$

where t^a are $h^{(2,1)}$ coordinates on the complex structure moduli space. We can write the metric in a simple way by using the following $(2,1)$-forms:

$$\chi_a = \frac{1}{2}(\chi_a)_{ab\bar{c}} dz^a \wedge dz^b \wedge d\bar{z}^{\bar{c}}$$
$$(\chi_a)_{ab\bar{c}} = -\frac{1}{2}\Omega^{\bar{d}}_{ab} \frac{\partial g_{\bar{c}\bar{d}}}{\partial t^a}$$

where the indices on the holomorphic 3-form Ω have been raised with the metric on the complex structure moduli space: $\Omega^{\bar{d}}_{ab} = g^{c\bar{d}}\Omega_{abc}$. We can invert the above relation to get an expression for $\delta g_{\bar{a}\bar{b}}$:

$$\delta g_{\bar{c}\bar{b}} = -\frac{1}{\|\Omega\|^2} \bar{\Omega}^{ed}_{\bar{c}} (\chi_a)_{ed\bar{b}} \delta t^a \tag{164}$$

where $\|\Omega\|^2 = \frac{1}{6}\Omega_{abc}\Omega^{abc}$. By substituting (164) into the metric, we get the following result:

$$g_{a\bar{b}} \delta t^a \delta \bar{t}^{\bar{b}} = -\left(\frac{\int \chi_a \wedge \bar{\chi}_{\bar{b}}}{\int \Omega \wedge \bar{\Omega}}\right) \delta t^a \delta \bar{t}^{\bar{b}} \tag{165}$$

When the complex structure defined on the Calabi-Yau fluctuates, Ω is no longer a purely $(3,0)$. Instead, it acquires a $(2,1)$ part. To see this, let us write down the definition of Ω explicitly:

$$\Omega = \frac{1}{6}\Omega_{\alpha\beta\gamma} dz^\alpha \wedge dz^\beta \wedge dz^\gamma \tag{166}$$

Taking a derivative with respect to the moduli space coordinate t_a, we obtain:

$$\partial_a \Omega = \frac{1}{6}\frac{\partial \Omega_{\alpha\beta\gamma}}{\partial t^a} dz^\alpha \wedge dz^\beta \wedge dz^\gamma + \frac{1}{2}\Omega_{\alpha\beta\gamma} dz^\alpha \wedge dz^\beta \wedge \frac{\partial (dz^\gamma)}{\partial t^a} \tag{167}$$

The term $\frac{\partial (dz^\gamma)}{\partial t^a}$ contains a $(0,1)$ part and a $(1,0)$ part. This means that $\partial_a \Omega$ has a $(3,0)$ part and a $(2,1)$ part. The exterior derivatives commutes with the derivative ∂_a, so $\partial_a \Omega$ is a closed form. It therefore lies in the space $H^{(3,0)} \oplus H^{(2,1)}$. By considering the structure of the dz^γ derivative, we can find the following (details in [7]):

$$\partial_a \Omega = K_a \Omega + \chi_a \tag{168}$$

Using this relation, we find that the metric (165) is given by $\partial_a \partial_{\bar{b}} K$, where K is the following Kähler potential:

$$K_{cs} = -\log\left(i \int \Omega \wedge \bar{\Omega}\right) \tag{169}$$

We can expand Ω into the symplectic basis (α_A, β^A):

$$\Omega = X^A \alpha_A - \mathcal{F}_A \beta^A \tag{170}$$

where $X^A = \int_{K_6} \Omega \wedge \beta^A$ and $\mathcal{F}_A = \int_{K_6} \Omega \wedge \alpha_A$. The $\mathcal{F}_A$ are actually functions of X^A:

$$\mathcal{F}_A = \frac{\partial F}{\partial X^A} \tag{171}$$

where $F(X)$ is a homogeneous function of degree two. In terms of the periods X^A and $\mathcal{F}_A$, the Kähler potential becomes:

$$K_{cs} = -\log\left(i \int \Omega \wedge \bar{\Omega}\right) = -\ln\left(i\bar{X}^A \mathcal{F}_A - X^A \bar{\mathcal{F}}_A\right) \tag{172}$$

3.7 FLUX COMPACTIFICATIONS

We have seen how various moduli arise in the four-dimensional effective theory from dynamically undetermined parameters of the internal manifold. As it stands, all the moduli are *flat directions* of the scalar potential. Changing a modulus from one value to another costs no energy, and consequently all the moduli are massless. This is called the *moduli stabilization problem*. As we have discussed earlier, trying to rely on quantum corrections to solve the problem is not helpful. We need to generate a *classical* mass for the moduli. This can be done by replacing the previous Ansatz that all of the field strengths and fermions vanish in the background. Instead, we will allow some of the field strengths to acquire a non-vanishing expectation value with a dependence on the coordinates of the internal space. This procedure is called "turning on *fluxes*". The fluxes are named in analogy with, for example, magnetic flux in electrodynamics. A surface encloses a magnetic flux when an amount of magnetic field passes through it. Similarly, when an n-form field strength acquires a flux, its integral over a non-trivial n-cycle does not vanish. We can see how, speaking qualitatively, turning on fluxes can change a theory by considering the following simple example [43].

3.7.1 *A simple example*

Take a theory on five dimensions of gravity coupled to a scalar field λ:

$$S = \int \left(-\hat{R} \star 1 - \frac{1}{2} \hat{d}\hat{\lambda} \wedge \star \hat{d}\hat{\lambda}\right) \tag{173}$$

Let us compactify it on a circle. This means that for one of the five directions, say y, we impose the following identification:

$$y + 2\pi R \sim y \tag{174}$$

where R is the radius of the circle. We now make the following Ansatz:

$$\lambda(x, y) = \lambda(x) + my \tag{175}$$

where x refers to the coordinates on the non-compactified space and m is a constant. The y component of the field strength becomes:

$$\hat{d}\hat{\lambda}_y = m dy \tag{176}$$

So we see that integrating the field strength over the compactification circle yields something non-vanishing. In other words, we have turned on a flux. Inserting the Ansatz into the action and integrating over the compact direction, we obtain the usual Kaluza-Klein action of a compactified scalar, but with some modifications. Firstly, the derivative in the kinetic term for λ is replaced by a covariant derivative: $D\lambda = d\lambda - mA$, where A is the Kaluza-Klein vector field. What has happened here is that the global symmetry $\lambda \to \lambda + a$ of the original action has now become a gauge symmetry. Secondly, a potential term proportional to m^2 is generated. The full action is:

$$S = \int \left(-R \star \mathbf{1} - \frac{1}{6\phi^2} d\phi \wedge \star\phi - \frac{\phi}{4} F \wedge \star F - \frac{1}{2} D\lambda \wedge \star D\lambda - \frac{m^2}{2\phi} \star \mathbf{1} \right) \tag{177}$$

One more thing to note is that this Ansatz is not compatible with a four-dimensional Minkowski vacuum. The flux contributes positively to the energy-momentum tensor, so a Minkowski spacetime no longer satisfies the Einstein equations.

3.7.2 *Fluxes on Calabi-Yau manifolds*

The qualitative results of the previous example continue to hold in the Calabi-Yau compactification of Type IIB. The fluxes generate potential terms for some of the scalars, and contribute to the energy-momentum tensor, changing the geometry. In Type IIB supergravity, the flux of an n-form field strength is a harmonic n-form. This is needed to satisfy the ten-dimensional equations of motion and Bianchi identity: $dF_n = d \star F_n = 0$. This means we can only give fluxes to 3-form field strengths, since $h^{(1,0)} = h^{5,0} = 0$ on a Calabi-

Yau. In the presence of fluxes, the field expansions (143) for F_3 and H_3 become:

$$\hat{F}_3 = d\hat{C}_2 - l d\hat{B}_2 + (2\pi)^2\alpha'(m^A_{RR}\alpha_A - e_{RRA}\beta^A) \tag{178}$$

$$= dC_2 - lH_3 + (dc^i - l db^i)\omega_i + (2\pi)^2\alpha'(m^A_{RR}\alpha_A - e_{RRA}\beta^A) \tag{179}$$

$$\hat{H}_3 = d\hat{B}_2 + (2\pi)^2\alpha'(m^A\alpha_A - e_A\beta^A) \tag{180}$$

$$= dB_2 + db^i\omega_i + (2\pi)^2\alpha'(m^A\alpha_A - e_A\beta^A) \tag{181}$$

where e_{RRA}, m^A_{RR}, m^A and e_A are constants. The subscripts in e_{RR} and m_{RR} refer to the fact that F_3 comes from the RR sector of the Type IIB superstring. The convention to include the factor $(2\pi)^2\alpha'$ is made so that the flux parameters m_K, e_K, etc. become integers. This is because in string theory fluxes must be *quantized*:

$$\frac{1}{(2\pi\sqrt{\alpha'})^{p-1}}\int_{\Sigma_p}\hat{F}_p \in \mathbb{Z} \tag{182}$$

This condition is exactly analogous to Dirac's quantization condition in electromagnetics with magnetic monopoles. It is derived in the same way, by requiring that the wavefunction of a p-brane minimally coupled to a $(p+1)$ gauge field should be globally well-defined. For details, see Chapter 6 of [7].

By defining the Poincaré-dual cycles of $\alpha_A = [K^A]$ and $\beta^A = [L_A]$, we obtain:

$$\frac{1}{(2\pi)^2\alpha'}\int_{L_A}\hat{H}_3 = m^A \tag{183}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{K^A}\hat{H}_3 = e_A \tag{184}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{L_A}\hat{F}_3 = m^K_{RR} \tag{185}$$

$$\frac{1}{(2\pi)^2\alpha'}\int_{K_A}\hat{F}_3 = e_{RRA} \tag{186}$$

In principle, the strategy now is the same as before. We should insert the above field expansions for F_3 and H_3 into the Type IIB action, impose self-duality of F_5, and integrate over the internal space to obtain the four-dimensional effective action. Unfortunately, the energy-momentum contained in the fluxes will make the previous Ansatz for the metric inconsistent. We need to replace it with the following:

$$ds^2 = e^{2A(y)}\tilde{g}_{\mu\nu}dx^\mu dx^\nu + g_{mn}dy^m dy^n \tag{187}$$

where Latin indices denote coordinates of the internal space, and Greek indices denote coordinates of the four-dimensional spacetime.

However, such compactifications are problematic in supergravity due to a well-known no-go theorem, which we will explain in the next section. Specifically, compactifications to de Sitter or Minkowski are not possible. Luckily, string theory contains extended objects which will allow us to evade the no-go theorem.

3.7.3 *Einstein equation no-go theorem*

The no-go theorem to be discussed here was first discovered by [41]. Giddings, Kachru, and Polchinksi (GKP) [21] showed how the no-go theorem could be avoided in string theory by using orientifold planes. The argument goes as follows. Take the trace of the Einstein equation:

$$\begin{aligned} \mathrm{Tr}(R_{mn} - \frac{1}{2} g_{mn} R) &= \mathrm{Tr}(T) \\ &= R - 5R = T \end{aligned}$$

Adding $-\frac{1}{2} g_{mn}$ times the above to the Einstein equation yields:

$$R_{mn} = T_{mn} - \frac{1}{8} g_{mn} T^l_l \tag{188}$$

This is called the trace-reversed Einstein equation. Substituting the Ansatz (187) into this equation yields:

$$R_{\mu\nu} = \tilde{R}_{\mu\nu} - \tilde{g}_{\mu\nu}(\nabla^2 A + 2(\nabla A)^2) = T_{\mu\nu} - \frac{1}{8} e^{2A} \tilde{g}_{\mu\nu} T^l_l \tag{189}$$

where $\tilde{R}_{\mu\nu}$ is the Ricci tensor formed from $\tilde{g}_{\mu\nu}$. Contracting both sides with $\tilde{g}^{\mu\nu}$ yields:

$$\tilde{R} + e^{2A}(-T^\mu_\mu + \frac{1}{2} T^l_l) = 4(\nabla^2 A + 2(\nabla A)^2) = 2e^{-2A} \nabla^2 e^{2A} \tag{190}$$

Multiplying the right-hand side with e^{2A}, we get $2\nabla^2 e^{2A}$. This total derivative term vanishes when integrated over the compact internal space. On the other hand, we will show that when we multiply the left-hand side by the same factor e^{2A}, we end up with something positive-definite. This is a contradiction, so we see that a flux compactification with metric (187) does not work. To this end, let us define the following quantity:

$$\hat{T} = \frac{1}{2}(-T^\mu_\mu + T^m_m) \tag{191}$$

The contribution to the energy-momentum tensor of an n-form flux F is given by:

$$T_{mn} = F_{mp_1 \ldots p_{n-1}} F_N^{p_1 \ldots p_{n-1}} - \frac{1}{2n} g_{mn} F^2 \tag{192}$$

Inserting this into (191), we obtain:

$$\hat{T} = -F_{\mu p_1 \ldots p_{n-1}} F^{\mu p_1 \ldots p_{n-1}} + \frac{n-1}{2n} F^2 \tag{193}$$

The purely internal components of the flux contribute:

$$\hat{T}_{int} = \frac{n-1}{2n} F^2 \geqslant 0 \tag{194}$$

This quantity is non-negative. It only vanishes when the flux is a 1-form, but there are no non-trivial harmonic 1-forms on a Calabi-Yau. We will not consider fluxes with spacetime components, but their contributions are similarly non-negative. Only those of 9- and 1-form field strengths vanish, the others are positive-definite.

We see that the left-hand side of (190) is non-negative when the scalar curvature of spacetime is either positive (de Sitter) or vanishing (Minkowski). We showed previously that the integral of the right-hand side over the internal space was vanishing, so we have arrived at the promised contradiction. Flux compactifications to de Sitter or Minkowski spacetime are ruled out in supergravity.

3.7.4 *Localized sources*

There exist extended objects in string theory besides the fundamental strings themselves. Firstly, there are hyperplanes to which the ends of open strings with Dirichlet boundary conditions may be attached. These hyperplanes are called Dirichlet p-branes, or *Dp-branes*. D-branes are dynamical objects: quantizing the theory of an open string attached to a Dp-brane shows that a number of excitation modes live on the world volume of the brane. Just as a 0-dimensional point particle may carry charge from a 1-form Maxwell field, a Dirichlet p-brane may carry charge from a $(p+1)$-form gauge field.

Secondly, there are *orientifold planes*, or Op-planes, where p is the dimension of the orientifold plane. They arise from making an *orientifold projection* of string theory. This amounts to the following: take a string theory A on a manifold $\mathcal{M}$. A generic symmetry of the theory may be written as a union of spacetime and worldsheet symmetries:

$$G = G_1 \cup \ \Omega G_2 \tag{195}$$

where G_1 is a symmetry of the manifold $\mathcal{M}$ and Ω refers to *worldsheet orientation reversal* [17]:

$$\Omega : \sigma \to -\sigma \tag{196}$$

We now gauge the symmetry group G. What this means for the string spectrum is that we truncate the states of A to only those that are invariant under G transformations. The fixed points of G_1 in the manifold $\mathcal{M}$ form a p-dimensional hyperplane called an *orientifold plane*. A p-dimensional orientifold plane, just like a Dp-brane, may carry $(p+1)$-form charges.

Orientifold planes and Dp-branes are physical objects which contribute to the energy-momentum tensor [7]. It can be shown that a p-brane[8] wrapped on a $(p-3)$-cycle Σ, and filling the three-dimensional

7 However, orientifold planes do not have physical excitations modes like D-branes do.

8 We now use the term p-brane to refer to either Dp-branes or p-dimensional orientifold planes.

external space, contributes the following action, to leading order in the string theory parameter α':

$$S_{loc} = -\int_{R^4\times\Sigma} d^{p+1}\xi T_p\sqrt{-g} + \mu_p \int_{R_4\times\Sigma} C_{p+1} \tag{197}$$

where T_p is the *tension* of the p-brane. The contribution of this action to the energy-momentum tensor is:

$$T^{loc}_{mn} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{loc}}{\delta g^{mn}} \tag{198}$$

The spacetime and internal parts become respectively [21]:

$$T^{loc}_{\mu\nu} = -T_p e^{2A}\tilde{g}_{\mu\nu}\delta(\Sigma) \tag{199}$$

$$T^{loc}_{mn} = -T_p \Pi^{\Sigma}_{mn}\delta(\Sigma) \tag{200}$$

where $\delta(\Sigma)$ is a delta function and Π^{Σ} is a projector on the cycle Σ. The contribution to the left-hand side of (190) becomes:

$$\hat{T}_{loc} = (T^m_m - T^\mu_\mu)^{loc} = (7-p)T_p\delta\Sigma \tag{201}$$

$Dp-branes$ with dimensionality $p \leqslant 7$ give a non-negative contribution to $\hat{T}$. However, orientifold planes have a *negative tension*, so their conttribution to $\hat{T}$ may cancel the contributions of the fluxes and the external Ricci scalar. This means that flux compactifications to de Sitter or Minkowski spacetime are possible in the presence of Op-planes with $p < 7$! As promised, adding stringy localized sources allows us to evade the no-go theorem that exists in the supergravity approximation.

3.7.5 *Tadpole cancellation*

In the previous sections, we saw how the Einstein equation placed some global constraints on the possibility of Minkowski or de Sitter vacua. We can find more global constraints by considering the integrated Bianchi identities. In Type IIB, the following equations hold for the field strengths F_3 and F_5:

$$d\hat{F}_3 = d(d\hat{C}_2 - l d\hat{B}_2) = -dl\wedge d\hat{B}_2 = \hat{H}_3\wedge\hat{F}_1 \tag{202}$$

$$d\hat{F}_5 = d(d\hat{A}_4 - \frac{1}{2}\hat{C}_2\wedge d\hat{B}_2 + \frac{1}{2}\hat{B}_2\wedge d\hat{C}_2) = d\hat{B}_2\wedge d\hat{C}_2 = \hat{H}_3\wedge\hat{F}_3 \tag{203}$$

The presence of localized sources modifies these equations:

$$d\hat{F}_n = \hat{H}_3\wedge\hat{F}_{n-2} + (2\pi\sqrt{\alpha'})^{n-1}\rho^{loc}_{8-n} \tag{204}$$

where ρ^{loc}_{8-n} is the charge distribution coming from localized sources. These sources are $(8-n)$-dimensional orientifold planes or D-branes,

that extend along spacetime and wrap $(5-n)$-cycles in the internal space. D5-branes and O3-planes wrapped on the internal 2-cycle $\tilde{\Sigma}_2$ contribute to the F_3 charge. Integrating the Bianchi identity for $\hat{F}_3$ over Σ_4 (the dual cycle of $\tilde{\Sigma}_2$), we obtain:

$$N_{D5}(\tilde{\Sigma}_2) - N_{05}(\tilde{\Sigma}_2) + \frac{1}{(2\pi)^2\alpha'}\int_{\Sigma_4} \hat{H}_3 \wedge \hat{F}_1 = 0 \tag{205}$$

Where N_{D5} and N_{05} refer to the number of D5-branes and O5-planes respectively that wrap the cycle Σ_2. We see that the 05-plane carries the opposite charge of the D5-brane.

D3-branes and O3-planes carry F_5 charge. A 3-dimensional hyperplane, extended along the three non-compact spatial dimensions, appears as a point in the 6-dimensional internal space. We therefore need to integrate the Bianchi identity over the entire internal manifold:

$$N_{D3} - \frac{1}{4}N_{03} + \frac{1}{(2\pi)^4\alpha'^2}\int \hat{H}_3 \wedge \hat{F}_3 = 0 \tag{206}$$

An O3-plane carries $-\frac{1}{4}$ times the charge of a D3-brane. The 3-form fluxes induce the following number of units of D_3 charge:

$$N_{flux} = \frac{1}{(2\pi)^4\alpha'^2}\int \hat{H}_3 \wedge \hat{F}_3 = (e_K m^K_{RR} - m^K_{e_{KRR}}) \tag{207}$$

Again we see how the presence of orientifold planes can help us satisfy a global constraint on compactifications. In the following section, we will discuss the low-energy effective action of Type IIB on a Calabi-Yau orientifold.

Let us take a moment to interpret the results of the current discussion. The physics contained within the integrated Bianchi identity is exactly analogous to the Gauss law in ordinary electrodynamics. The Gauss law tells us that the electric field lines coming from a charged particle have to either extend to infinity, or end on a particle of opposing charge. In a compact space, only the latter possibility exists. We see that a compact space must be globally electrically neutral. This expresses the same point as the integrated Bianchi identities.

3.8 LOW-ENERGY EFFECTIVE ACTION ON CALABI-YAU ORIENTIFOLDS

We have seen how a compactification of Type IIB on a Calabi-Yau 3-fold generates a four-dimensional effective theory with $\mathcal{N} = 2$ supersymmetry. Making an orientifold projection will explicitly break the supersymmetry to $\mathcal{N} = 1$. An orientifold group G consists of the union of a spacetime symmetry G_1 and a worldsheet symmetry ΩG_2: $G = G_1 \cup \Omega_p G_2$, where Ω is worldsheet orientation reversal. We represent the action of G_2 with the operator σ. We want to consider orientifolds of Type IIB where G_2 is an *involution* ($\sigma^2 = 1$) and an *isometry*.

The action of σ on differential forms is given by the pullback σ^*. In Type IIB, the left- and right-moving supersymmetry spinors both correspond to the holomorphic 3-form Ω (126). Worldsheet orientation reversal Ω_p exchanges left- and right-movers, so the pullback σ^* should act on the holomorphic 3-form as [1]:

$$\sigma^*\Omega = \pm\Omega \tag{208}$$

This makes σ a *holomorphic* involution. In Type IIA, on the contrary, the right-moving supersymmetry spinor corresponds to the *anti*-holomorphic 3-form $\bar{\Omega}$. The pullback in Type IIA should act as $\sigma^*\Omega = \bar{\Omega}$. This is an anti-holomorphic involution. However, as usual we will only consider Type IIB.

With this choice for σ, the two possible symmetry groups to orientifold with are generated by the operators[24]:

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\sigma^*, \quad \sigma^*\Omega = -\Omega \tag{209}$$

$$\mathcal{O}_{(2)} = \Omega_p\sigma^*, \quad \sigma^*\Omega = \Omega \tag{210}$$

where F_L is the number of left-moving spacetime fermions. Since the operator σ only acts on the internal space, the orientifold planes necessarily fill the entire external space. Because σ acts holomorphically on the coordinates of the internal space, the orientifold plane must be an O3-, O5-, O7- or O9-plane. With the operator $\mathcal{O}_{(1)}$, the possibilities are O3 and O7. With $\mathcal{O}_{(2)}$, they are O5 or O9. To see this, let us define a set of local coordinates at a point on the orientifold plane such that:

$$\Omega = dy^1 \wedge dy^2 \wedge dy^3 \tag{211}$$

When $\sigma^*\Omega = -\Omega$, the involution must leave either none of the directions invariant, which gives an O3-plane, or two of them, which gives an O5-plane. We now focus on orientifolds of Type IIB with $\mathcal{O}_{(1)}$. These have only O3-planes. We want to obtain the low-energy effective action for this model, by using the same Kaluza-Klein strategy that we have outlined several times in the previous discussion. We will neglect the warp factor $A(y)$ in the metric (187). This amounts to making a large-radius approximation. This is not consistent with the Calabi-Yau orientifolds we want to consider in the next chapter. However, some of the formulae we will discover continue to hold in more general orientifold setups than the ones we consider here. In particular, the effective theory will always have $\mathcal{N} = 1$ supersymmetry, and the flux-induced superpotential will always be given by the Gukov-Vafa-Witten equation[25].

3.8.1 *Field expansions and supersymmetry multiplets*

The fields ϕ, l, g_{mn}, and $\hat{A}_4$ are even under $(-1)^{F_L}\Omega_p$, and the two-forms B_2 and C_2 are odd. Orientifolding a theory truncates the string

spectrum to only those states which are invariant under a transformation of the orientifold group. We find that the two forms $\hat{B}_2$ and $\hat{C}_2$ must be odd under σ^*, while all the other fields are even. With this choice, all the fields in the massless spectrum are invariant under $\mathcal{O}_{(1)}$. The space of harmonic (p,q)-forms, $H^{(p,q)}$, splits up in two eigenspaces of σ^*:

$$H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)} \tag{212}$$

where $H_+^{(p,q)}$ is the space of harmonic (p,q)-forms ω which satisfy $\sigma^*\omega = \omega$, and $H_-^{(p,q)}$ is the space of harmonic (p,q)-forms ω which satisfy $\sigma^*\omega = -\omega$. We denote the dimensions of $H_+^{(p,q)}$ and $H_-^{(p,q)}$ by $h_+^{(p,q)}$ and $h_-^{(p,q)}$ respectively. The dimensions of the odd and even eigenspaces have the same symmetries as the Hodge numbers. Firstly, we have $h_\pm^{(p,q)} = h_\pm^{(n-p,n-p)}$, since the Hodge operator commutes with σ^*. Secondly, we have $h_\pm^{(p,q)} = h_\pm^{(q,p)}$ since σ acts holomorpically on the internal coordinates. The holomorphic 3-form Ω is in the odd eigenspace of σ^*, so $h_-^{(3,0)} = 1$ and $h_+^{(3,0)} = 0$. Since $h^{(3,0)} = 1$ on a Calabi-Yau, we find that $h_+^{(3,0)} = h_+^{(0,3)} = 0$.

We have shown that complex structure deformations are in correspondence with the $(1,1)$ harmonic forms. Since the metric is invariant under σ (i.e. σ is an isometry), the only complex structure moduli that remain are the ones v^α (where $\alpha = 1, 2, \ldots, h_+^{(1,1)}$) that correspond to forms in the even eigenspace of σ^*. Similarly, the Kähler class deformations correspond to harmonic $(2,1)$-forms. The only Kähler moduli z^k (where $k = 1, 2, \ldots, h_+^{(2,1)}$) that remain are the ones in the even $(2,1)$ eigenspace. We now define the following bases for the odd and even eigenspaces of σ^*.

σ eigenspace	Dimension	Basis
$H_+^{(1,1)}, H_+^{(1,1)}$	$h_+^{(1,1)}, h_-^{(1,1)}$	ω_α, ω_a
$H_+^{(2,2)}, H_-^{(2,2)}$	$h_+^{(1,1)}, h_-^{(1,1)}$	$\tilde{\omega}^\alpha, \tilde{\omega}^a$
$H_+^{(2,1)}, H_-^{(2,1)}$	$h_+^{(2,1)}, h_-^{(2,1)}$	χ_κ, χ_k
$H_+^{(3)}, H_-^{(3)}$	$2h_+^{(2,1)}, 2h_-^{(2,1)} + 2$	$(\alpha_\kappa, \beta^\lambda), (\alpha_{\hat{k}}, \beta^{\hat{l}})$

The bases of harmonic 3-forms $(\alpha_\kappa, \beta^\lambda)$ and $\alpha_{\bar{k}}, \beta^{\bar{l}}$ are symplectic, just like the basis we previously denoted by α_A, β^A:

$$\int \alpha_\kappa \wedge \beta^\lambda = \delta_\kappa^\lambda, \quad \int \alpha_{\bar{k}} \wedge \beta^{\bar{l}} = \delta_{\bar{k}}^{\bar{l}} \tag{213}$$

The 2-forms $\hat{B}_2$ and $\hat{C}_2$ are odd under σ^*. This means that they can now be expanded in a basis of the odd eigenspace. $\hat{A}_4$ is even,

so it must be expanded in a basis of the even eigenspace. The field expansions (143) become:

$$\hat{B}_2 = b^a(x)\omega_a \tag{214}$$

$$\hat{C}_2 = c^a(x)\omega_a \tag{215}$$

$$\hat{A}_4 = D_2^a(x) \wedge \omega_a + V^\kappa \wedge \alpha_\kappa + U_\kappa \wedge \beta^\kappa + \rho_\alpha \tilde{\omega}^\alpha \tag{216}$$

The four-dimensional 2-forms B_2 and C_2 have disappeared from the spectrum, since σ^* does not affect the external space. As usual, the self-duality condition of $\hat{F}_5$ must be imposed on the equations of motion. Just like before, this allows us to eliminate either V^κ or U_κ, and D_2^α or ρ_α. We choose to keep V and ρ. The spectrum now consists of the following $\mathcal{N} = 1$ multiplets:

- A gravity multiplet with bosonic content $g_{\mu\nu}$
- $h^{(2,1)}$ vector multiplets with bosonic content V^κ
- $h_-^{(2,1)}$ chiral multiplets with bosonic content z^k
- A chiral multiplet with bosonic content ϕ, l
- $h_-^{(1,1)}$ chiral multiplets with bosonic content (b^a, c^a)
- $h_+^{(1,1)}$ chiral multiplets with bosonic content (v^α, ρ_α)

If we had chosen to keep D_2^α instead of ρ_α, the spectrum would contain *linear multiplets* in the place of some of the chiral multiplets. The derivation of the effective low-energy action is perhaps more natural in the formulation with linear multiplets. For details, see [24].

3.8.2 *The effective action*

Let us now add fluxes to the above setup. Type IIB allows for 3-form fluxes, which are now parametrized by harmonic 3-forms in the odd eigenspace of σ^*. We can combine the $\hat{F}_3$ and $\hat{H}_3$ flux into a single complex 3-form G_3:

$$G_3 = F_3 - \tau H_3 \tag{217}$$

where τ is the *axio-dilaton*: $\tau = l + ie^{-\phi}$. Expanding G_3 in the $H_-^{(3)}$ harmonic forms, we obtain:

$$G_3 = m_G^{\hat{k}} \alpha_{\hat{k}} - e_{G\hat{k}} \beta^{\hat{k}} \tag{218}$$

where we define the flux parameters in analogy with (183):

$$m_G^{\hat{k}} = m^{\hat{k}} - \tau m_{RR}^{\hat{k}} \tag{219}$$

$$e_{G\hat{k}} = e_{\hat{k}} - \tau e_{RR\hat{k}} \tag{220}$$

The subscript G denotes that the parameter belongs to the combined flux in G_3, and the subscript RR denotes that $\hat{F}_3$ belongs to the RR sector of the Type IIB string.

The expansions for the field strengths become:

$$\begin{aligned}
\hat{H}_3 &= db^a \wedge \omega_a + m^{\hat{k}} \alpha_{\hat{k}} + e_{\hat{k}} \beta^{\hat{k}} \\
\hat{F}_3 &= dc^a \wedge \omega_a - l db^a \wedge \omega_a + m^{\hat{k}}_{RR} \alpha_{\hat{k}} + e_{RR\hat{k}} \beta^{\hat{k}} \\
\hat{F}_5 &= dD_2^\alpha \wedge \omega_\alpha + dV^\kappa \wedge \alpha_\kappa - dU_\kappa \wedge \beta^\kappa + d\rho_\alpha \wedge \tilde{\omega}^\alpha \\
&\quad - \frac{1}{2}(c^a db^b - b^a dc^b) \wedge \omega_a \wedge \omega_b
\end{aligned}$$

The effective action is obtained in exactly the same way as before[9]: we insert the above field expansions into the Type IIB action, then we impose self-duality of F_5 by adding a total derivative term, and integrate over the internal space. The result is [24]:

$$\begin{aligned}
\int_{\mathcal{M}_4} & -\frac{1}{2} R \star \mathbf{1} - g_{kl} dz^k \wedge \bar{z}^l - g_{\alpha\beta} dv^\alpha dv^\beta - \frac{1}{4} d\ln\mathcal{K} \wedge \star d\ln\mathcal{K} \\
& - \frac{1}{4} d\phi \wedge \star d\phi - \frac{1}{4} e^{2\phi} dl \wedge \star dl - e^{-\phi} g_{ab} db^a \wedge \star db^b \\
& - e^{\phi} g_{ab} (dc^a - l db^a) \wedge \star (dc^b - l db^b) \\
& - \frac{9 g^{\alpha\beta}}{4\mathcal{K}^2} \left(d\rho_\alpha - \frac{1}{2} \mathcal{K}_{\alpha ab} (c^a - b^a dc^b) \right) \wedge \star \left(d\rho_\beta - \frac{1}{2} \mathcal{K}_{\beta cd} (c^c db^d - b^c dc^d) \right) \\
& + \frac{1}{4} \mathrm{Im}\mathcal{M}_{\kappa\lambda} F^\kappa \wedge \star F^\lambda + \frac{1}{4} \mathrm{Re}\mathcal{M}_{\kappa\lambda} F^\kappa \wedge F^\lambda - V \star \mathbf{1}
\end{aligned} \tag{221}$$

where we have eliminated D_2^α and U_κ in favour of ρ_α and V^κ. The terms of interest to us are the ones that involve the moduli space metrics (g_{kl},g_{ab}, and $g_{\alpha\beta}$), and the scalar potential term V. The terms that involve the matrix $\mathcal{M}_{\kappa\lambda}$ are gauge kinetic factors and will be of no further concern.

The scalar potential V is contained within the following terms of the Type IIB action:

$$S = -\frac{1}{4} \int \left(e^{-\phi} \hat{H}_3 \wedge \star \hat{H}_3 + e^{\phi} \hat{F}_3 \wedge \star \hat{F}_3 \right) + \dots \tag{222}$$

The combined flux G_3 contributes the following to the above two terms:

$$\mathcal{L} = -\frac{1}{4} \int_{K_6} G_3 \wedge \star_6 \bar{G}_3 \tag{223}$$

where we have carried out the integration over spacetime, since the flux has no spacetime dependence. We can split the combined flux G_3 into a part G_3^+ which is imaginary self-dual (ISD) and a part G_3^- which imaginary anti-self-dual:

$$G_3^\pm = \frac{1}{2}(G_3 \pm i \star_6 G_3) \tag{224}$$

9 As promised, we neglect here the warp factor in the metric.

Substituting this decomposition into the action, we obtain the following:

$$\mathcal{L} = -\frac{1}{2}e^{\Phi}\int_{K_6} G_3^+ \wedge \star_6 \bar{G}_3^+ + \dots \tag{225}$$

where the dots stand for terms which are not relevant to the scalar potential. The action (221) has been *Weyl rescaled*: $g_{\mu\nu} \to \frac{\mathcal{K}}{6} g_{\mu\nu}$. This rescaling just multiplies the above by a factor $\frac{36}{\mathcal{K}^2}$. The scalar potential becomes:

$$V = \frac{18}{\mathcal{K}^2}e^{\Phi}\int_{K_6} G_3^+ \wedge \star_6 \bar{G}_3^+ \tag{226}$$

The ISD part of G_3 lies in the space $H_-^{(3,0)} \oplus H_-^{(1,2)}$. The only $(3,0)$-form in the odd eigenspace $H_-^{(3,0)}$ is Ω. A basis for $H_-^{(2,1)}$ is provided by the harmonic forms $\bar{\chi}_k$, which are defined in the table above. We can expand G_3^+ in a basis of these 3-forms:

$$G_3^+ = -\frac{1}{\Omega \wedge \bar{\Omega}}\left(\Omega \int \bar{\Omega} \wedge G_3 + g^{lk}\bar{\chi}_k \int \chi_l \wedge G_3\right) \tag{227}$$

where the integrations are all carried out over the Calabi-Yau. Substituting the above equation into the scalar potential, we obtain:

$$V = \frac{18ie^{\Phi}}{\mathcal{K}^2 \int \Omega \wedge \bar{\Omega}}\left(\int \Omega \wedge \bar{G}_3 \int \bar{\Omega} \wedge G_3 + g^{kl}\int \chi_k \wedge G_3 \int \bar{\chi}_l \wedge \bar{G}_3\right) \tag{228}$$

In Appendix B, we show that the effective action obtained here is of the standard $\mathcal{N} = 1$ form, just as the decomposition in $\mathcal{N} = 1$ multiplets has suggested.

The action of a four-dimensional $\mathcal{N} = 1$ supergravity (with the field content described above) is determined by:

- A Kähler metric $K_{I\bar{J}} = \partial_I \bar{\partial}_{\bar{J}} K(M, \bar{M})$ defined on the manifold spanned by all the scalar fields in the theory. M stands for all of the scalar fields collectively.
- A superpotential $W(M)$, a function of the scalar fields
- Gauge field kinetic constants $f_{\kappa\lambda}$
- A set of D-terms D_α

The action is:

$$-\int \frac{1}{2}R \star \mathbf{1} + K_{I\bar{J}} DM^I \wedge \star D\bar{M}^J + \frac{1}{2}\mathrm{Re} f_{\kappa\lambda} F^\kappa \wedge \star F^\lambda + \frac{1}{2}\mathrm{Im} f_{\kappa\lambda} F^\kappa \wedge F^\lambda + V \star \mathbf{1} \tag{229}$$

where M^I denotes each complex scalar in the theory, and the D's are covariant derivatives (not to be confused with the *D-terms* D_α) defined by:

$$D_I W = \partial_I W + W \partial_I K \tag{230}$$

The scalar potential V is:

$$V = e^{K}(K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2) + \frac{1}{2}\mathrm{Re}(f^{-1})^{\kappa\lambda} D_\kappa D_\lambda \tag{231}$$

We will not consider D-terms in the following. The Kähler- and superpotentials of the orientifold with fluxes are obtained in Appendix B. The punchline is the following: if we posit that $h_-^{(1,1)} = 0$, the Kähler potential for the Kähler modulus reduces to the form:

$$K = -3\ln\left[T + \bar{T}\right] \tag{232}$$

This is the case of interest in the KKLT scheme to be discussed in the next chapter.

The fluxes induce a superpotential W. It is given by the so-called *Gukov-Vafa-Witten* equation:

$$W = \int G_3 \wedge \Omega \tag{233}$$

This equation holds in more general orientifold setups than the ones considered here[25]. The superpotential depends on the axio-dilaton through the definition of G_3, and on the complex structure deformation through their connection with Ω. It does not depend on the Kähler moduli. To see that this superpotential generates the scalar potential (228), we need expressions for the Kähler covariant derivatives [22][24]:

$$D_\tau W = \frac{i}{2} e^{\phi} \int \Omega \wedge \bar{G}_3 + i g_{ab} b^a b^b W \tag{234}$$

$$D_{T_\alpha} = K_{T_\alpha} W = -2\frac{\nu\alpha}{\mathcal{K}} W \tag{235}$$

$$D_{G^a} W = 2 i g_{ab} b^b W \tag{236}$$

$$D_{z^k} W = i \int \chi_k \wedge G_3 \tag{237}$$

Inserting this into (231), we obtain (228). We have shown that the orientifold effective action has a scalar potential derived from a Kähler potential and a superpotential. The only thing that remains to show is that the $\mathcal{N} = 1$ gauge terms of (229) coincide with the gauge terms of the orientifold effective action (221). This is done in [24] and [22]. That completes the proof that the orientifold projection reduces the theory to four-dimensional $\mathcal{N} = 1$ supergravity with a vector multiplet and several chiral multiplets.

To find a supersymmetric Minkowski vacuum, we need to impose $W = D_I W = 0$. We can see from the above equations that this implies:

$$\int G_3 \wedge \Omega = \int \bar{G}_3 \wedge \Omega = \int G_3 \wedge \chi_k = 0 \tag{238}$$

The first of these relations says that G_3 contains no $(0,3)$ part, the second that it contains no $(0,3)$ part, the last that it contains no $(1,2)$

part. G_3 is restricted to be a $(2,1)$-form by supersymmetry conditions. Turning on a particular set of fluxes, G_3 will only be $(2,1)$ with respect to a certain complex structure and only at a certain field value of the axio-dilaton. We see that stabilization of the complex structure moduli and the axio-dilaton is a necessary condition for finding supersymmetric Minkowski solutions.

We have now shown that string theory contains in principle all the necessary machinery to construct a consistent compactification in which all of the complex structure moduli and the axio-dilaton are stabilized, but not the Kähler moduli. That is, we have shown how to generate a compactification with fluxes that induces a scalar potential which depends on each of these scalar fields. We have not shown how to explicitly construct such a solution. [22] contains some discussion on this subject. To stabilize the Kähler modulus as well, we need additional machinery: the KKLT mechanism.

4

MODULI STABILIZATION SCHEMES AND INFLATION

We have seen in the previous chapter that fluxes on Calabi-Yau orientifolds may generate superpotentials that stabilize a large portion of the moduli. The Kähler moduli, however, cannot be stabilized by fluxes. The Kähler moduli T_α correspond to the $h_+^{(1,1)}$-forms on the internal manifold. They parametrize the volume of the internal space. As such, a geometric construction of the kind (112) cannot be considered a true *compactification* until the T_α moduli are stabilized at values which make the internal space quite small. To achieve this, we will need to add some ingredients from string theory to our setup. The supergravity approximation scheme in which we have been working may be seen as the lowest order approximation in an expansion in the stringy parameters α' (string tension) and g_s (string coupling). The string coupling is controlled by the dilaton: $g_s = e^\phi$. The consistency of our approximation scheme requires that g_s is stabilized at a low value.

To stabilize the Kähler moduli, we need to go higher order in α' and g_s. There are both perturbative and non-perturbative effects that give corrections at higher order. Schematically, the action becomes:

$$S = S_{(0)} + \alpha' S_{(3)} + \ldots \alpha'^n S_{(n)} + \ldots + S^{CS}_{(0)} + S^{loc}_{(0)} + \alpha' S^{loc}_{(2)} \tag{239}$$

where S^{CS} is a *Chern-Simons* term, and the terms with superscript loc belong to localized sources such as D-branes. String theory loop diagrams give corrections that appear at order α'^3, and they are further suppressed by powers of g_s. We will consider the first-order α' corrections only. They may be represented as follows:

$$K = K_0 + K_p + K_{np} \tag{240}$$

$$W = W_0 + W_{np} \tag{241}$$

where the subscripts p and np stand for perturbative and non-perturbative, respectively. W_0 is the lowest-order superpotential (e.g. like the flux-induced Gukov-Vafa-Witten superpotential of a Calabi-Yau orientifold). There are no perturbative corrections to the superpotential. It happens to be the case that non-perturbative effects are understood better for the setups we consider. In the moduli stabilization mechanism that we will consider, only the non-perturbative corrections are taken into account. This must be done in a consistent way. The corrections to the scalar potential are, schematically:

$$V_p \sim W_0^2 K_p \qquad V_{np} \sim W_{np}^2 + W_0 W_{np} \tag{242}$$

We see that we can only neglect perturbative corrections when W_0 is not much larger than W_{np}. The perturbative corrections only disappear when W_0 vanishes.

Using non-perturbative corrections, Kachru, Kallosh, Linde, and Trivedi (KKLT [29]) managed to construct a mechanism by which the single Kähler modulus T of a Calabi-Yau orientifold with $h_+^{(1,1)} = 1$ may be stabilized. They found supersymmetric AdS minima, which they argue can be uplifted to de Sitter by adding anti-D3-branes. However, the validity of this mechanism has come under some scrutiny, as we will see later on[1].

The KKLT mechanism suffers from some phenomenological problems when it is incorporated in a model of inflation. There is a potential barrier separating the de Sitter minimum from the Minkowski minimum in the decompactification limit $T \to \infty$[2]. The height of this barrier is tied to the mass of the gravitino at the minimum. When inflation occurs, the Kähler modulus may fluctuate away from the minimum if the barrier is not higher than the scale of inflation (the Hubble constant H). This tells us that the Hubble constant may not be larger than the gravitino mass in a KKLT inflation scenario. This represents a problem in case the scale of SUSY breaking is supposed to be low enough to allow for the resolution of Standard Model naturalness problems. To solve this problem, Kallosh and Linde slightly altered the KKLT mechanism in such a way that the connection between the height of the barrier and the mass of the gravitino disappeared. This is called the *KL model*. We will consider inflation in both KL and KKLT setups.

4.1 THE KKLT MECHANISM

The success of the setup devised by KKLT was two-fold. Firstly, it was the first time that de Sitter vacua were found in string theory. Secondly, and perhaps more importantly, it was the first example of a *properly* effectively four-dimensional vacuum with a stabilized internal volume modulus. This vacuum is anti-de Sitter until it is *uplifted* by the addition of anti-D3-branes ($\bar{D}3$-branes). As we will see, this uplifting procedure is the most questionable part of the KKLT construction.

4.1.1 *Tree-level Ingredients*

The basic setup of the KKLT constructions is exactly what we discussed in the previous chapter: Type IIB on a Calabi-Yau orientifold

1 A good, not very technical review of the situation can be found at http://motls.blogspot.nl/2014/11/an-evaporating-landscape-possible.html

2 The minimum at infinite radius will always persist, as it is a general string theory feature [20].

with fluxes. The orientifold has $h^{(1,1)_+} = 1$, $h_-^{(1,1)} = 0$ and $h^{(2,1)}$ arbitrary (i.e. any number of complex structure moduli, but only a single Kähler modulus T). We have a Kähler potential (346) (350), where the Kähler moduli part reduces to $-3\ln[T + \bar{T}]$. The fluxes induce a superpotential (233). The scalar potential is of the standard $N = 1$ supergravity form:

$$V = e^K \left(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2 \right) = e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} \right) \tag{243}$$

where I, J run over all the scalar fields and i, j run over all the fields except the Kähler modulus. The equality results from the *no-scale* structure of the Kähler potential:

$$\partial_T K \partial_{\bar{T}} K K^{T\bar{T}} = 3 \tag{244}$$

The scalar potential stabilizes the complex structure moduli and the dilaton such that G_3 is imaginary self-dual. When we apply supersymmetry conditions, the G_3 must at this point be $(2,1)$. Adding non-perturbative corrections will allow for supersymmetric minima in the presence of a $(3,0)$ piece. We can see this from (234): normally the supersymmetry condition requires $W = 0$ in the vacuum (note that $h_-^{(1,1)} = 0$, so the G^a equations are not relevant). However, if we change the $D_T W$ part by adding non-perturbative corrections to the superpotential, the supersymmetry conditions derived from (234) become:

$$\int \bar{G}_3 \wedge \Omega = \int G_3 \wedge \chi_k = 0 \tag{245}$$

So G_3 may acquire a $(3,0)$ part in the presence of a non-perturbative superpotential. In this case, $W_0 \neq 0$ in the supersymmetric vacuum. In the KKLT construction, it must be negative and we must have $|W_0| << 1$ for consistency. The stabilized moduli acquire masses of the order $m \sim \frac{\alpha'}{R^3}$. From now on, we will set the complex structure moduli and the dilaton equal to their vacuum expectation values and only consider the dynamics of the Kähler modulus. This is a self-consistent approximation in that the mass generated for the Kähler modulus will be smaller than $\frac{\alpha'}{R^3}$.

4.1.2 *Non-perturbative Corrections*

KKLT argue that there are two significant sources of non-perturbative corrections to the superpotential. They are difficult to understand, so we will not go into detail, but their contribution to the superpotential is quite simple, and almost the same in each case. Firstly, there are contributions due to instantons called *Euclidean* D3-branes. When they wrap four-cycles in the internal manifold, they contribute the following to the superpotential:

$$W_{inst} = F(z) \exp(-2\pi T) \tag{246}$$

where $F(z)$ is a function of the complex structure moduli. We may treat $F(z)$ as a constant if we take the complex structure moduli to be equal to their vacuum expectation values.

Secondly, there may be new interactions generated in regions within the moduli space where there are geometrical singularities in the Calabi-Yau. At such points in the moduli space, the supergravity approximation can break down and corrections to the effective theory are inevitable. There may appear stacks of coincident D7-branes which generate non-Abelian gauge interactions. These can give the following correction to the superpotential:

$$W_{gauge} = Ae^{\frac{-2\pi T}{N}} \tag{247}$$

where N is the number of coincident branes. We see that both of these effects contribute an exponential term to the KKLT superpotential:

$$W_{KKLT} = W_0 + Ae^{-aT} \tag{248}$$

In principle, the parameters a and A are determined from analyzing the above effects, but we will take them to be free parameters. The scalar potential and the covariant derivative of W become:

$$V = \frac{aAe^{-a(T+\bar{T})}\left[3(e^{aT} + e^{a\bar{T}})W_0 + A\big(6 + a(T+\bar{T})\big)\right]}{3(T+\bar{T})^2} \tag{249}$$

$$D_T W = \left(-aAe^{-aT} - \frac{3(Ae^{-aT} + W_0)}{T+\bar{T}}\right) \tag{250}$$

We will truncate to $T = \bar{T}$ (i.e. $\mathrm{Im}(T) = 0$) and look for minima. The derivative with respect to T of the scalar potential becomes:

$$\partial_T V = -\frac{aAe^{-2aT}(2+aT)\big(A(3+2aT) + 3e^{aT}W_0\big)}{12T^3} \tag{251}$$

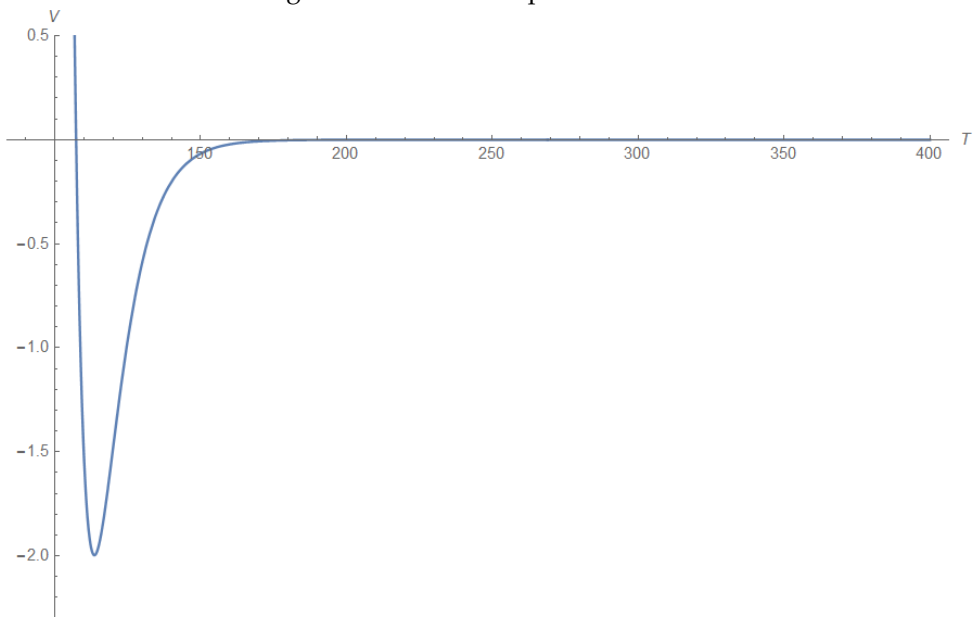
The equation that determines the minimum becomes:

$$W_0 = -Ae^{-aT_0}(1 + \frac{2}{3}aT_0) \tag{252}$$

where T_0 is the value of the Kähler modulus at the minimum. At the minimum, we have $D_T W = 0$, so that all solutions preserve supersymmetry. There are only solutions at positive values of the Kähler modulus when W_0 is negative. The integer G_3 fluxes must be fine-tuned to make this happen. Furthermore, we require that $T_0 >> 1$ for the consistency of the supergravity approximation. Lastly, we must have $aT_0 > 1$ so that the instanton contributions are well-approximated by the KKLT superpotential. The value of the potential at the minimum becomes:

$$V_{AdS} = -\frac{a^2A^2e^{-2aT}}{6T} \tag{253}$$

Figure 2: KKLT scalar potential



We find that all minima obtained in this setup are anti-de Sitter. The scalar potential is pictured in Figure 2 for the parameter choices $A = 1$, $a = 0.1$, and $W_0 = -0.0001$. These are the parameter choices of the KKLT paper. They are roughly $O(1)$, except for W_0 which must be tuned to small values in order for the consistency of the supergravity approximation. This would seem the most natural choice from the perspective of string theory. In the following chapters we will take these to be the "standard" parameter choices.

We see that the KKLT superpotential leads to a scalar potential which, in some region of parameter space, admits supersymmetric AdS minima separated by a large barrier from the Minkowski minimum at infinity.

4.1.3 *Uplifting to de Sitter*

The second achievement of the KKLT paper, besides obtaining stabilization of the volume modulus at a small value, was the discovery of the first de Sitter vacua in string theory. KKLT achieved this by adding $\bar{D}3$-branes to the configuration, which lifts the scalar potential to positive values. This *uplifting* procedure is the most questionable aspect of the KKLT paper. Recall that in a Type IIB Calabi-Yau orientifold we have the following global constraint (206)

$$N_{D3} - \frac{1}{4}N_{O3} + \frac{1}{(2\pi)^4\alpha'^2}\int \hat{H}_3 \wedge \hat{F}_3 = 0 \tag{254}$$

Previously, this equation was be satisfied by turning on the right amount of fluxes. We can, however, choose to add a $\bar{D}3$-brane which

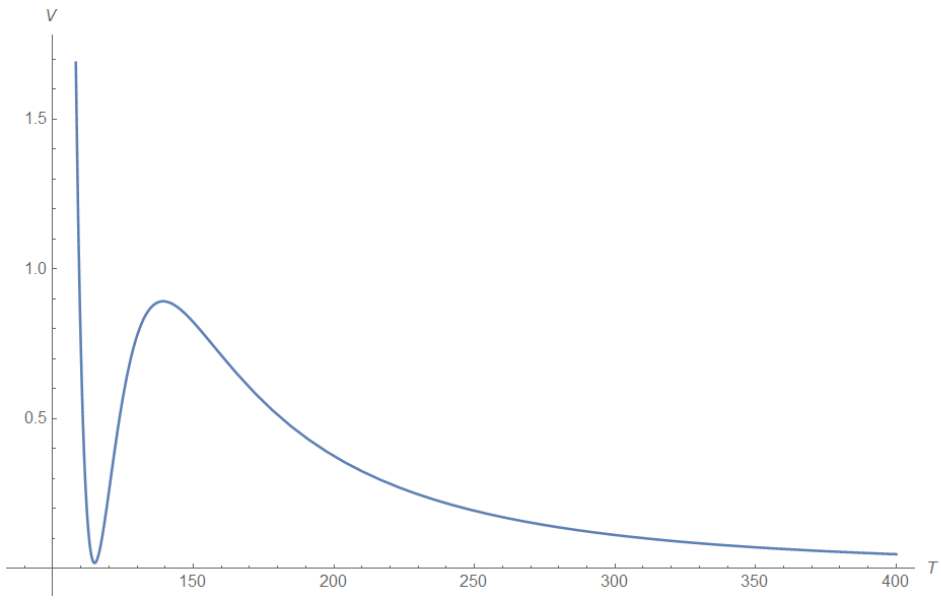
gives a negative contribution to (206). This requires that we turn on some extra flux to compensate the negative D3 charge from the $\bar{D}3$. The $\bar{D}3$-brane adds an energy density proportional to $1/T^3$ [27][3]:

$$\delta V = 2\frac{a_0^4 T_3}{g_s^4}\frac{1}{T^3} \tag{255}$$

We absorb the proportionality constants into a single constant D, which we take to be a free parameter, even though it should in principle be calculated explicitly. D is determined by the fluxes and the *warp factor* a_0 at the location of the $\bar{D}3$-brane. In geometries without warping, adding a $\bar{D}3$ results in additional moduli associated with the position of the $\bar{D}3$. However, a $\bar{D}3$ is gravitationally drawn to where the warp factor is minimal Ẇhen the background is of the type discussed in [21], the warp factor at the minimum is an exponentially small parameter. This allows us to tune D to very small values (in a discrete way, since fluxes in string theory are quantized). The scalar potential becomes:

$$V = \frac{aAe^{-2a(T)}\left[A\left(6+2aT\right)+6e^{aT}\right)\right]}{12T^2} + \frac{D}{T^3} \tag{256}$$

Since the anti-D3 brane contribution is not an F-term or a D-term, supersymmetry is now broken. Following KKLT, we take $D = 3 \times 10^{-10}$. The scalar potential (multiplied by 10^{15}) looks like:



There are de Sitter minima for small enough values of D. The barrier separating the dS minimum from the Minkowski minimum at infinity is of similar form to the AdS barrier we saw previously. As we will see later, the height of this barrier is tied to the gravitino mass at the dS minimum, and therefore to the scale of supersymmetry breaking. Lastly, KKLT show that the de Sitter minimum is quite

3 It was argued in [28] that the proportionality should be $1/T^2$ instead, however this does not change the qualitative results obtained here.

stable with respect to quantum tunneling to the Minkowski minimum at infinity. The tunneling time scale should always be larger than the age of the universe.

Let us take a moment to discuss the criticisms against the KKLT mechanism. The stabilization mechanism of the Kähler modulus has come under some scrutiny, but the most severe criticisms have concerned the uplifting procedure. Firstly, [14] have argued that integrating out the complex structure moduli after including non-perturbative corrections may not be consistent. There will be mixing terms between the heavy and light fields which change the overall results. [19] claim that the stabilization mechanism does not work out in the case $h^{(1,1)} = 1$ examined by KKLT.

The problem with the uplifting procedure concerns the reaction of the anti-D3-brane on the fluxes. The negative D3 charge of the anti-D3-brane draws the fluxes (which carry D3 charge of the opposite sign) towards the anti-D3. Moreover, there is a gravitational interaction as well. On the other hand, the charged interaction between fluxes and orientifold planes is exactly balanced by their gravitational repulsion (remember that O-planes carry *negative tension*). As the fluxes are drawn to the anti-D3, they pile up into a singularity. Schematically, we have $(H_3)^2 \to \infty$. This was worked out in [42] and [8]. This singular charge is not being sourced by any localized object, so this represents a real problem. Although the singularity has been the focus of much discussion, no definitive solution has been proposed[4].

4.2 MODULI STABILIZATION AND INFLATION: KL MODEL

The form of the KKLT potential around the minimum is very similar before and after uplifting. The depth of the supersymmetric AdS minimum before uplifting is almost equal, up to a factor of order one, to the height of the barrier in the uplifted dS potential [33]. Since the AdS minimum is supersymmetric, we have that $D_T W = 0$ there, and therefore:

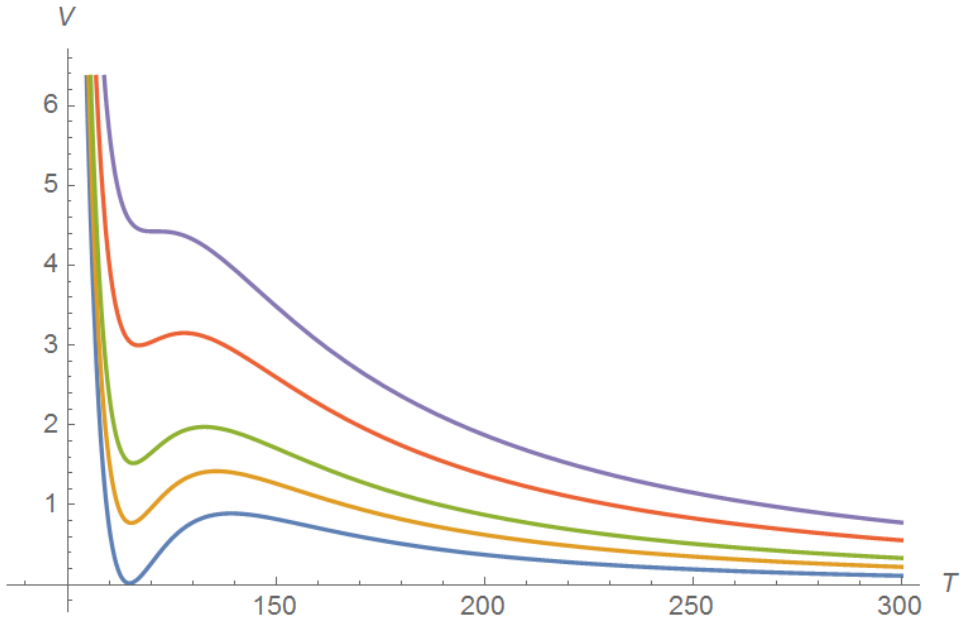
$$V_{AdS} = -3e^K |W|^2 \sim V_b \tag{257}$$

where V_b is the height of the de Sitter barrier in the uplifted potential. On the other hand, the gravitino mass in the minimum is given by $m_{3/2}^2 = e^{K(T_0)}|W(T_0)|^2 \simeq \frac{V_{AdS}}{3}$. We therefore obtain the connection $V_b \sim m_{3/2}^2$ between the height of the barrier and the gravitino mass. We can anticipate that adding an inflaton to this setup will remove the minimum from the potential, if it adds an energy density roughly equal to the height of the barrier. The inflationary Hubble constant H is roughly equal to the potential energy, $H^2 \simeq V_{inf}$. We

4 For a summary of the situation per November 2014, see http://motls.blogspot.nl/2014/11/an-evaporating-landscape-possible.html

anticipate the following generic constraint on KKLT inflation models: $H \lesssim m_{3/2}$. This presents a phenomenological problem: in most inflation models, the inflationary Hubble constant must be much larger than the TeV-scale gravitino mass needed to solve the Standard Model hierarchy problem. Furthermore, adding a Kähler modulus sector to an inflation setup typically results in $\mathcal{O}(m_{3/2}H)$ corrections to the inflationary scalar potential [18]. When the gravitino mass is large, the corrections may spoil the inflationary potential.

Let us visualize the problem for the simplest case of KKLT inflation: a D-term potential $V = V_{KKLT} + \frac{V(\phi)}{T^3}$ where $V(\phi)$ is an inflaton potential. This potential looks like this, for various values of $V(\phi)$:



We can see that the minimum disappears at larger values of $V(\phi)$. Moreover, if the mass of T at the minimum is not at least as large as the Hubble constant, it may fluctuate out of the minimum during inflation and decompactify the internal space.

To deal with these problems, Kallosh and Linde [35] devised a slightly modified version of the KKLT setup, called the KL model. Let us take the same Kähler potential as before $K = -3\ln(T + \bar{T})$, but change the superpotential to:

$$W_{KL} = W_0 + Ae^{-aT} - Be^{-bT} \tag{258}$$

where B and b are new free parameters. Such a superpotential might be generated in the rougly the same way as the KKLT superpotential. When we take

$$W_0 = -A\left(\frac{aA}{bB}\right)^{\frac{a}{b-a}} + B\left(\frac{aA}{bB}\right)^{\frac{b}{b-a}} \tag{259}$$

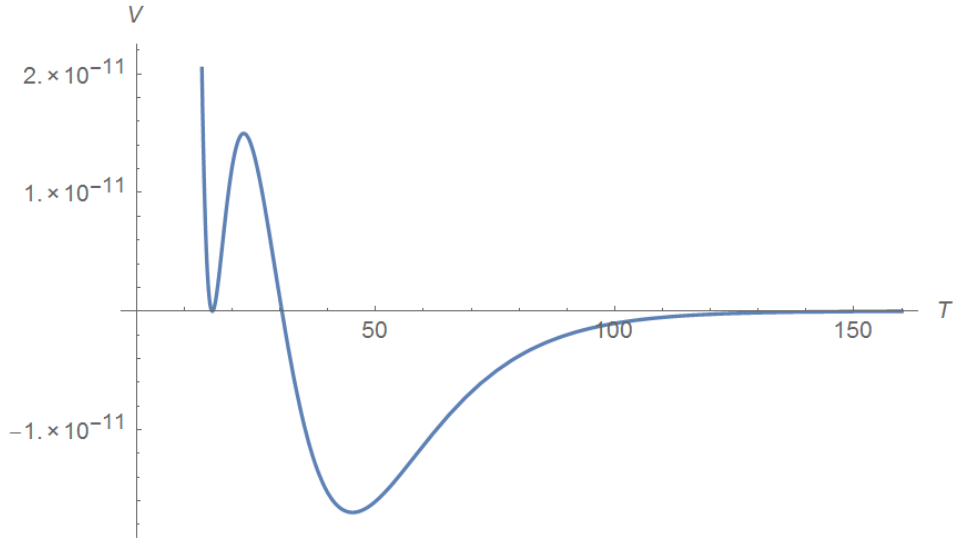
there is a supersymmetric Minkowski minimum at $T = T_0$ satisfying:

$$T_0 = \frac{1}{a-b}\ln\left(\frac{aA}{bB}\right) \qquad W(T_0) = D_T W(T_0) = V(T_0) = 0 \tag{260}$$

The scalar potential becomes:

$$V_{KL} = \frac{1}{6T^2} e^{-2(a+b)T} \left(bBe^{aT} - aAe^{bT}\right)\left(Be^{aT}(3+bT) - e^{bT}(A(3+aT)+3e^{aT})\right) \tag{261}$$

Taking $A = B = 1$, $a = 2\pi/100$,$b = 2\pi/99$ and W_0 as above, the potential looks like:



Since the minimum is supersymmetric Minkowski, the gravitino is massless, but there is still a potential barrier separating the Minkowski minimum from the AdS at larger field values and the Minkowski minimum in the decompactification limit. The masses associated with the imaginary and real directions of T are quite large, on the order 10^{-2} Planck masses - much larger than any inflaton. This means they may be consistently set equal to their vacuum expectation values during inflation. If we allow ourselves to freely tweak the parameters, a rescaling $A \to CA$, $B \to CB$ does not affect the position of the minimum but increases the mass of T by a factor C. There is ample opportunity here to obtain an inflation model with a large Hubble constant, a small supersymmetry breaking scale, and a comfortably stabilized Kähler modulus.

We may introduce supersymmetry breaking into the vacuum in the following way: perturb the superpotential by a small constant δW. This shifts the minimum from Minkowski to AdS:

$$V_{AdS}(\delta W) = -\frac{3(\delta W)^2}{8T_0^3} \tag{262}$$

and shifts the position of the minimum by a small amount $\delta T = \frac{3\delta W}{2T_0 W_{TT}}$. Then an uplifting term can take the minimum back to de Sitter, breaking supersymmetry. The gravitino mass becomes, up to an order one factor, $m_{3/2} \sim m_T|\delta W|$. For a very small δW, we may generate a gravitino mass of the order of a TeV, which is necessary to solve the hierarchy problem.

Let us now couple an inflaton to the Kähler modulus in a detailed setup. In the framework of $N = 1$ supergravity, there are two methods to couple an inflaton sector to a modulus sector. The first and most obvious method is to add their Kähler and superpotentials:

$$K_{coupled} = K_{inflaton} + K_{modulus} \tag{263}$$

$$W_{coupled} = W_{inflaton} + W_{modulus} \tag{264}$$

The second method (that was first explored in [2] and further developed in [18]) is to add Kähler potentials but *multiply* superpotentials:

$$K_{coupled} = K_{inflaton} + K_{modulus} \tag{265}$$

$$W_{coupled} = W_{inflaton} W_{modulus} \tag{266}$$

Multiplying superpotentials is equivalent to adding Kähler functions (the Kähler function G is defined by $G = K + \log W\bar{W}$ and is not the same thing as the Kähler *potential*). The $N = 1$ supergravity scalar potential is actually determined by the function G alone. In terms of the Kähler function, the scalar potential is:

$$V = e^{G}\left(G_i G^{i\bar{j}} G_{\bar{j}} - 3\right) \tag{267}$$

This suggests that adding Kähler functions is more natural than adding superpotentials. As we will see, for some models the former indeed works much better.

4.3.1 *Chaotic Inflation in KKLT*

[10] attempted to implement a chaotic inflation model in the KKLT setup[5]. They used a Polonyi field to uplift the AdS minimum of KKLT by means of an F-term, which is also responsible for most of the supersymmetry breaking. The Polonyi field X contributes the following to the Kähler and superpotentials:

$$K_{up} = k(|X|^2), \quad W_{up} = fX \tag{268}$$

$k(|X|^2)$ contains a quartic term which stabilizes X close to the origin, and f is a real constant. The Polonyi field then contributes a simple term to the scalar potential:

$$V_{up} = e^{K} f^2 \tag{269}$$

In the presence of the Polonyi uplifting, the condition $\partial_T V(T)|_{T=T_0} = 0$ leads to the following equation for T_0:

$$D_T W|_{T_0} = -\frac{3W}{4T_0}\left(1 \pm \sqrt{1 - \frac{2f^2}{(aT+2)W^2}}\right) \tag{270}$$

5 The impetus of the present research was partially to find some refutation of the results of this paper in an alpha attractor model.

Taking the negative sign, we obtain the position of the minimum T_0. The positive sign yields the position of the barrier separating the minimum from the runaway Minkowski minimum at infinite field values. We can insert the above equation for $D_T W$ at $T = T_0$ into the scalar potential, and use $W|_{T_0} \simeq W_0$ to find that the KKLT AdS minimum is uplifted to Minkowski when f is tuned to:

$$f = \sqrt{3} W_0 \left(1 - \frac{3}{2aT_0} + \mathcal{O}(T_0^{-2}) \right) \tag{271}$$

The gravitino mass at the uplifted Minkowski minimum is given by:

$$m_{3/2} = e^{K/2} W \simeq \frac{W_0}{(2T_0)^{3/2}} \tag{272}$$

As anticipated, the gravitino mass at the minimum is connected to the height of the barrier V_B:

$$V_B \simeq \frac{f^2}{(2T_B)^3} \sim m_{3/2}^2 \tag{273}$$

We now add the inflaton sector to the setup. We take the following Kähler and superpotentials:

$$W = W_0 + A e^{-aT} + fX + \frac{1}{2} m \phi^2 \tag{274}$$

$$K = -3 \ln\left(T + \bar{T}\right) + k(|X|^2) + \frac{1}{2}\left(\phi + \bar{\phi}\right)^2 \tag{275}$$

The imaginary part of ϕ, $\sqrt{2}\mathrm{Im}(\phi) = \varphi$, is the inflaton in this setup. As we explained in chapter 2, this Kähler potential has a *shift symmetry* $\phi \to \phi + ic$, where c is a real constant. This flattens out the potential in the φ direction. This is necessary because generically the e^K factor in the $\mathcal{N} = 1$ scalar potential renders the potential much too steep to support inflation[6]. On the other hand, the e^K term stabilizes the real direction of ϕ at the origin. The superpotential breaks the shift symmetry and generates a non-trivial potential for φ. Truncating ϕ down to its imaginary direction, we obtain the following superpotential:

$$W = W_0 + A e^{-aT} - \frac{1}{4} m \varphi^2 \tag{276}$$

The condition $\partial_T V = 0$ now leads to the following equation for T_0:

$$D_T W = \frac{-3W}{4T} \left[1 \pm \sqrt{1 - \frac{2}{(aT+2)W^2}\left(f^2 + \frac{1}{2} m^2 \varphi^2\right)} \right] \tag{277}$$

The negative sign corresponds to the minimum, and the positive sign corresponds to the barrier in the scalar potential, like before. Inserting the relations for $D_T W$ and W into the scalar potential, we obtain:

$$V(\varphi) = \frac{1}{2T^3} \left(f^2 + \frac{1}{2} m^2 \varphi^2 - 3W^2 + \mathcal{O}(T^{-3}) \right) \tag{278}$$

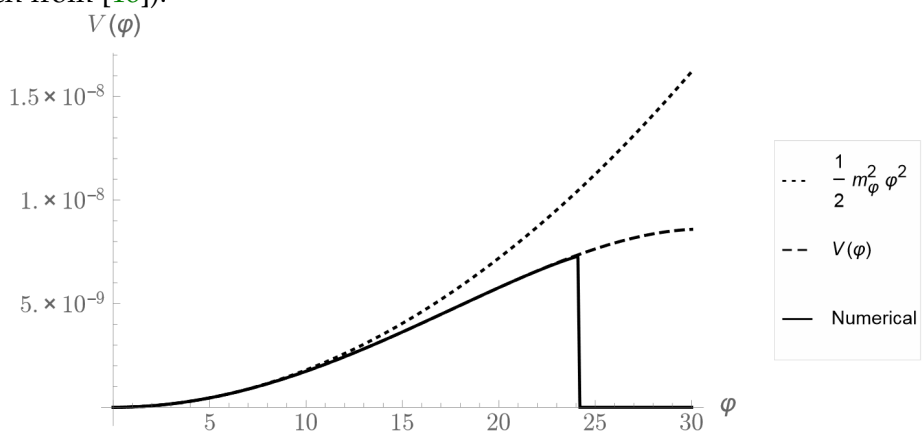
6 The issue with the e^K factor creating very steep scalar potentials is called the η-problem.

We may absorb the T^3 dependence into the constants, assuming that the Kähler modulus remains stabilized at approximately the same point. Including corrections associated with small shifts of the Kähler modulus, the result is a potential of similar form, but with rescaled coefficients.

The potential fails to have a minimum in the T direction when the term inside the square root in (277) becomes negative. At this point in field space, the Kähler modulus destabilizes and the internal space decompactifies. The condition that [10] discover is exactly as anticipated:

$$\tilde{m}\varphi^2 \lesssim 4m_{3/2} \tag{279}$$

where $\tilde{m}$ is the rescaled coefficient of the φ^2 term in the scalar potential. [10] ran a numerical test and found that the destabilization typically occurs slightly before the energy density in the inflaton field reaches the above bound. The scalar potential looks like this (picture taken from [10]):



The instant drop toward the origin is caused by the destabilization of the Kähler modulus. [10] succeeded in finding a region in parameter space (formed by the KKLT parameters A and a, the fine-tuned Polonyi field constant f, the inflaton mass constant m) where 60 e-folds of inflation may occur. However, there are two problems. Firstly, they conclude that the parameter choices needed to generate 60 e-folds are quite unnatural from the perspective of string theory. Secondly, the destabilization beyond the critical field value φ_c means that the initial conditions of inflation have to be finely tuned. If φ starts at a field value beyond φ_c, inflation cannot occur. The inflation model described here is no longer "chaotic"[7], in the sense that it is not insensitive to the initial conditions of the inflaton field, since the inflaton must be placed to the left of the instability. [10] repeat the same analysis with different moduli stabilization schemes (Kähler up-

7 Note that in this inflaton potential, there is a critical φ value beyond which no inflation can occur *even without* a Kähler destabilization. This is because the scalar potential contains a φ^4 term with a negative coefficient. One may generate purely quadratic potentials in supergravity by incorporating a stabilizer field [35].

lifting and the Large Volume Scenario, which are variants of KKLT), and find similar results.

4.3.2 *Hybrid Inflation in KKLT and KL: Adding Superpotentials*

[18] considered hybrid inflation within a KKLT and a KL setup. They found that hybrid inflation is not compatible with moduli stabilization when the inflaton and Kähler modulus are coupled by adding superpotentials. However, adding Kähler functions instead produced a viable model of inflation, due to the simpler mixing between the inflaton and modulus potentials, and the reduced backreaction of the inflaton energy density upon the Kähler modulus.

Let us give a short introduction to hybrid inflation[40]. It is a multi-field model in which one scalar field (ϕ) controls the slow-roll and two *waterfall fields* ($\phi^{\pm}$) abruptly trigger the end of inflation. It is called hybrid inflation since it incorporates elements from "new inflation" (the slowly-rolling scalar field), and from "old inflation". In old inflation, the universe sits in a false vacuum with positive energy during an inflationary period until a phase transition causes a decay to the true vacuum. We will give a formulation in a setup with global SUSY, without supergravity corrections (a supergravity setup of this kind only works when a field with no-scale Kähler potential is added, as we will see later). The superpotential is taken to be:

$$W_{inf} = \lambda\phi\left(\phi^{+}\phi^{-} - v^{2}\right) \tag{280}$$

where λ and v are constants which may be taken to be real without loss of generality. The scalar potential becomes:

$$V_{inf} = \lambda|\phi|^{2}\left(|\phi^{+}|^{2} + |\phi^{-}|^{2}\right) + \lambda^{2}|\phi^{+}\phi^{-} - v^{2}|^{2} \tag{281}$$

When $|\phi| > \lambda$, the waterfall fields are stabilized at the origin. Along this part of the trajectory, the scalar potential becomes a positive constant:

$$V = \lambda^{2}v^{4} \tag{282}$$

However, when $|\phi|$ reaches the critical value v, the waterfall fields become destabilized and abruptly put an end to inflation. To incorporate this model within supergravity, we need a Kähler potential with a shift symmetry in the direction of the inflaton, in order to solve the η problem. The simplest choice is:

$$K_{inf} = -\frac{(\phi - \bar{\phi})^{2}}{2} + |\phi^{+}|^{2} + |\phi^{-}|^{2} \tag{283}$$

The role of the inflaton is now played by the real direction of ϕ. However, this generates a scalar potential in which the imaginary direction of ϕ is tachyonic. Adding a field with no-scale Kähler potential can

solve the problem. We want to couple the model to a Kähler modulus, so we will take the Kähler modulus to be the no-scale field. The Kähler potential becomes:

$$K = -3\log\left(T + \bar{T}\right) + K_{inf} \tag{284}$$

This setup produces a hybrid inflation scalar potential in which the imaginary direction of ϕ is properly stabilized. [18] then proceed to add moduli stabilization of the Kähler modulus to the setup. They take the Kähler potential to be:

$$K = -3\log\left(T + \bar{T} + \frac{K_{O'}}{3}\right) + K_{inf} \tag{285}$$

with K_{inf} as above and $K_{O'}$ is an uplifting O'Raifeartaigh term [34]. It will not be important to consider this term in detail - we only need to know that it produces a SUSY-breaking F-term uplifting of the AdS KKLT minimum. The superpotential is:

$$W = W_{mod} + W_{inf} \tag{286}$$

where W_{mod} is the usual non-perturbative superpotential from either KKLT or KL, and W_{inf} is as above. The scalar potential can be written as:

$$\begin{aligned} V_F = e^{K_{inf}}\mathcal{V}_{KKLT} + \mathcal{V}_{lift} + \\ + e^{K}|\partial_i W_{inf} + K_i(W_{inf} + W_{mod})|^2 + V_{mix} \end{aligned} \tag{287}$$

where $\mathcal{V}_{KKLT}$ is the KKLT scalar potential, $\mathcal{V}_{lift}$ is the uplifting term, and V_{mix} is a mixing between the modulus and inflaton sectors:

$$\begin{aligned} V_{mix} = 2e^{K}Re\left[(K^{I\bar{J}}_{mod})D_I W_{mod} K_{mod,\bar{J}}\right] + \\ + e^{K}\left(K^{I\bar{J}}_{mod} K_{mod,I} K_{mod,\bar{J}} - 3\right)|W_{inf}|^2 \end{aligned} \tag{288}$$

In these expressions the indices I, J run over the moduli, and i runs over the inflaton scalar fields as indicated. The no-scale structure of the Kähler modulus Kähler potential makes sure that the second term in V_{mix} vanishes. The following things now need to be checked:

- The mixing term may not dramatically influence the scalar potential, since $\epsilon << 1$ is required for slow roll. Any correction that is too steep will render the potential unsuited for inflation. This requires that $Re(\partial_T W_{mod})$ becomes small.
- The imaginary direction of ϕ must remain stable.
- The masses of the waterfall fields must not be corrected too much, since they control the end of inflation.

[18] found that there is not enough room for fine-tuning within the KKLT parameters to satisfy all these requirements at the same time. Turning to the KL model for moduli stabilization, we acquire two extra parameters to fine-tune with. However, the Kähler modulus still acquires a ϕ-dependent shift, like we saw in the previous section. When this shift is substituted into the scalar potential, a large negative mass is generated for the inflaton, which makes η far too large: $\eta \simeq -3$.

4.3.3 *Hybrid Inflation in KKLT and KL: Adding Kähler Functions*

The innovation of the Postma and Davis paper was to consider adding Kähler functions instead of superpotentials, as we've explained above. The advantage of this method is two-fold.

Firstly, we can argue that corrections associated with inflaton-dependent shifts of the Kähler modulus minimum are much reduced. In the [10] chaotic inflation model, these corrections mostly rescaled the scalar potential coefficients, but did not affect the functional form of the potential (until, of course, the destabilization was reached). They did, however, have an effect on the inflationary observables. In the hybrid inflation model, the corrections were severe enough (in some regions of the parameter space) to render the potential unsuitable for inflation. Consider a supersymmetric minimum of the modulus sector scalar potential, which satisfies $\partial_T G_{mod}(T_0) = 0$. Incorporating an inflaton model by adding Kähler functions G_{mod} and G_{inf}, the full Kähler function becomes: $G = G_{mod} + G_{inf}$. We then still have $\partial_T G(T_0) = 0$. It is not hard to see from (267) that this is still an extremum of the scalar potential in the direction of T. In this setup, there are no inflaton-dependent corrections to the minimum of the Kähler potential at all. In the presence of supersymmetry breaking, we no longer have that the minimum occurs at T_0 given by $\partial_T G(T_0) = 0$. However, when supersymmetry breaking occurs at a low scale, we anticipate that the modulus shift corrections will be much reduced by comparison with the adding superpotentials case.

Secondly, there is much simpler mixing between the modulus sector and the inflaton sector in the scalar potential. The scalar potential can be written:

$$V = e^{K_{inf}}|W_{inf}|^2\mathcal{V}_F + e^{K_{mod}}|W_{mod}|^2 e^{K_{inf}}|\partial_i W_{inf} + K_i W_{inf}|^2 + \mathcal{V}_{lift} \tag{289}$$

where all the symbols are explained above. This has a much simpler structure than before. Generically, the most important effect of the coupling to the Kähler modulus is a rescaling of the potential coefficients. Postma and Davis found that in some cases, the inflaton now actually has a helpful effect on the moduli stabilization. They conclude that hybrid inflation may be implemented in both KKLT and KL models, when the Kähler functions are added.

In the next chapter, we will attempt to couple single-field alpha attractor models of inflation to a Kähler modulus, using both KKLT and KL setups, and both addition and multiplication of superpotentials.

Part II

OUR RESEARCH

5 THE SINGLE FIELD α-ATTRACTOR INFLATION MODEL

A family of attractor models with a single chiral superfield was constructed by [47] under the name *α-scale supergravities*. In this model the amount of primordial gravitational waves is related to the scalar curvature of the scalar manifold, and the tensor-scalar ratio assumes a universal value independent of input parameters. The model consists of the following Kähler and superpotentials:

$$W = \Phi^{n_-} - \Phi^{n_+} F(\Phi) K = -3\alpha \log\left(\Phi + \bar{\Phi}\right) \tag{290}$$

The function $F(\Phi)$ is an arbitrary Taylor expansion in Φ. The constants $n_\pm$ are given by:

$$n_\pm = \frac{3}{2}(\alpha \pm \sqrt{\alpha}) \tag{291}$$

The model realizes plateau inflation for a large class of choices for $F(\Phi)$. When $\alpha > 1$, the imaginary part of Φ is stabilized at the origin - $\Phi = \bar{\Phi} = \phi$ - and the real part can safely play the role of an inflaton in a single-field effective description. Along this trajectory, the connection between the geometric field Φ and the canonical scalar field φ is given by:

$$\Phi = \bar{\Phi} = e^{-\sqrt{\frac{2}{3\alpha}}\varphi} \tag{292}$$

Inflation always occurs in a very small range of field space close to the origin of the inflaton, $\phi = 0$. In this region both the scalar potential V and the function $F(\phi)$ are well approximated by the linear terms in their Taylor expansions around $\phi = 0$. The higher-order terms in the expansion of $F(\phi)$ only appear in the scalar potential at order ϕ^2 or higher, so they are irrelevant along the inflationary trajectory as long as their expansion coefficients are not too high. This is what gives the model its attractor structure. As long as the expansion coefficients of $F(\phi)$ have the right order of magnitude, inflation can occur with as many e-folds as required, and the inflationary observables r and n_s are not sensitive to the precise values of the coefficients.

The connection between the geometric field ϕ and the canonical field φ (292) assures that the scalar potential assumes the form common to all α attractor models, which is an exponential fall-off from a de Sitter plateau:

$$V = V_0 - V_1 e^{-\sqrt{\frac{2}{3\alpha}}\varphi} + \dots \tag{293}$$

In principle, one can choose to fine tune $F(\phi)$ to obtain a Minkowski vacuum at the minimum of the inflation potential. However, generically the minimum is AdS.

Polynomial superpotentials and logarithmic Kähler potentials appear quite often in explicit string theory compactifications. This model therefore seems wonderfully natural from the stringy perspective. It would be interesting to see if it can survive being coupled to a modulus sector. Doing so will introduce mixing terms which we will have to accommodate by slightly changing the structure of the inflaton sector superpotential, but the basic idea will be the same.

There are several reasons why one might expect that coupling an α-attractor model to a modulus sector would yield different results from the two models explored in the previous chapter. Firstly, Starobinsky-like inflation models occur at energy scales which are about a factor 10 lower than the chaotic inflation models of [10]. From the analysis of the previous chapter, it can be anticipated that the destabilization problem is somewhat improved. Secondly, the observable predictions of α-attractor models are insensitive to the precise value of the scalar potential coefficients. This indicates that the backreaction of the inflaton on the stabilization of the Kähler modulus does not have an impact on the inflationary observables. Any succesful realization of an α-attractor model coupled to a modulus will likely have inflationary observables that remain in the Planck "sweet spot".

5.1 ADDING SUPERPOTENTIALS AND KKLT MODULI STABILIZATION

As we have discussed in the previous chapter, there are two ways to couple an inflaton to a modulus sector. Firstly, one may add superpotentials and Kähler potentials. Secondly, we could add Kähler functions instead. Furthermore, there are several ways to achieve stabilization of the Kähler modulus. In the previous chapter, we have examined the KKLT and KL moduli stabilization schemes. In [10], Kähler uplifting and Large Volume Scenarios are explored as possibilities as well. The authors of that paper found no significant qualitative difference between the latter two methods and the KKLT scheme for chaotic inflation. We will consider both the KL model and the KKLT model.

In this section we construct a model based on adding superpotentials to a KKLT moduli stabilization scheme. As we explained previously, adding superpotentials produces many complicated mixing terms. These mixing terms will spoil the α-scale supergravity model. We will have to change the inflaton superpotential to accommodate the mixing terms. The simplest possibility is the following:

$$K = K_{mod} + K_{inf}, \quad W = W_{mod} + W_{inf} \tag{294}$$

$$K_{inf} = -3\alpha \log\left(\phi + \bar{\phi}\right), \quad K_{mod} = -3 \log\left(T + \bar{T}\right) \tag{295}$$

$$W_{inf} = \phi^{3\alpha}(c_0 + c_1\phi + \dots) \tag{296}$$

$$W_{mod} = W_{KKLT} = \omega + A\exp(-aT) \tag{297}$$

We require a scalar potential of the form (293), so we need all terms except the constant and linear (in ϕ) terms to be very small. The scalar potential defined by (294)-(297) has the following structure (after a truncation down to $T = \bar{T}$, $\phi = \bar{\phi}$):

$$V = \phi^{-3\alpha}T^{-3}\Big[f_0(T) + \phi^{3\alpha}\big(g_0(T) + g_1(T)\phi + \dots\big) + \dots$$
$$\dots + \phi^{6\alpha}\big(h_0(T) + h_1(T)\phi + \dots\big)\Big] \tag{298}$$

We will discuss the consistency of this construction extensively in what follows. The ellipses indicate terms of higher order in ϕ. The functions h_0, g_0, g_1, h_0, and h_1 implicitly depend on the model parameters (the ones from the superpotential: $c_0, c_1, \dots$; the curvature parameter of the scalar manifold: α; and the ones coming from the moduli stabilization mechanism). The stability of the truncation down to $\phi = \bar{\phi}$ has to be checked individually for every point in parameter space, and will depend on ϕ as well. The important part of the scalar potential is the $\phi^{3\alpha}$ term inside the brackets. If the Kähler modulus T is not shifted too much along the inflationary trajectory, it can be treated as a constant. Then this part of the scalar potential will give us (293). We are left with two troublemakers: the $\phi^{6\alpha}$ term and the ϕ^0 term inside the brackets. The $\phi^{6\alpha}$ term will be irrelevant as long as α is not much smaller than $\frac{2}{3}$. The function $f_0(T)$ must be made to vanish almost exactly. It causes a great deal of trouble especially close to $\phi = 0$. If it does not vanish, the scalar potential has a $\phi^{-3\alpha}$ term which grows exceedingly large at small ϕ, and makes the scalar potential too steep for inflation. $f_0(T)$ is given by

$$f_0(T) = \frac{8^{-1-\alpha}e^{-2aT}}{3T^3}\Big[A^2\big[9\alpha + 4aT(3 + aT)\big] + \dots$$
$$\dots + 6Ae^{aT}\big[3\alpha + 2aT\big]\omega + 9e^{2aT}\alpha\omega^2\Big] \tag{299}$$

When $\alpha = 1$, it becomes:

$$f_0(T)|_{\alpha=1} = \frac{e^{-2aT}\big[A(3 + 2aT) + 3e^{aT}\omega\big]^2}{192T^3} \tag{300}$$

The requirement $f_0(T) = 0$ is now just the equation that determines the minimum of the Kähler modulus in the KKLT model. If T is not shifted too far from this value by the presence of the inflaton field, $f_0(T)$ will vanish almost exactly (in part thanks to the e^{-2aT} suppression). When $\alpha < 1$, however, $f_0(T)$ becomes negative at the stabilization point $T = T_0$. We can solve this with an F-term uplifting, for example using a *Polonyi field* as was done in [10]. This will

add a factor $e^K f^2$ to the scalar potential (where f is a constant). Unfortunately, this uplifting procedure adds a degree of fine tuning to the model. $f_0(T)$ becomes positive for $\alpha > 1$. We cannot solve this with a Polonyi uplift. The model we have constructed is possibly an α-attractor in the region $\frac{2}{3} < \alpha \leqslant 1$ (where the lower limit is not well defined and may be a little bit lower). We still need to do a case-by-case check of two things. Firstly, the Kähler modulus T must not destabilize on the inflationary trajectory, and the backreaction of the inflaton on T must not shift the minimum too much even if T does remain stable. Secondly, the mass of the imaginary direction of ϕ must be positive definite everywhere on the inflationary trajectory, otherwise the truncation $\phi = \bar{\phi}$ is not consistent.

5.2 THE CASE $\alpha = 1$

When $\alpha = 1$, the model can yield a scalar potential in the form (293) for small positive values of ϕ (or large values of the canonical field φ) without any additional fine tuning. The relevant part of the scalar potential is

$$V = \frac{g_0(T) + g_1(T)\phi}{T^3} + \dots \tag{301}$$

$$g_0(T) = -\frac{c_0 e^{-aT}(3A - 2aAT + 3e^{aT}\omega)}{32T^3} \tag{302}$$

$$g_1(T) = -\frac{c_1 e^{-aT}(5A - 2aAT + 5e^{aT}\omega)}{32T^3} \tag{303}$$

We know that the equation $f_0(T) = 0$ determines the stabilization point of the Kähler modulus in the KKLT scheme. We can therefore expect that

$$g_0(T) \simeq -\frac{c_0 e^{-aT}(-aA)}{8T^2} \tag{304}$$

$$g_1(T) \simeq -\frac{c_1 e^{-aT}(-aA))}{6T^2} \tag{305}$$

So we require a positive c_0 and a negative c_1 to obtain a scalar potential in the form (293). Using the parameters $A = 1$, $a = 0.1$, $\omega = 10^{-4}$, as was done in [29], puts the Kähler modulus at around $T \simeq 113$ (the backreaction of ϕ will shift this upward). Plugging these values into (304), we see that c_0 should be roughly of $O(1)$ if we want a inflationary squared Hubble constant of order 10^{-10} on the inflationary plateau. This is around the order of magnitude required by COBE normalization.

The condition $\partial_T V = 0$ that determines the minimum of the Kähler modulus yields the following expression for $D_T W$ at the minimum:

$$D_T W = -\frac{3e-aT}{4T^2(2+aT)}\Bigg[AT + e^{aT}T + ae^{aT}T^2 + e^{aT}W_0 + \dots$$
$$\dots + \frac{1}{12}e^{aT}\sqrt{-288e^{-2aT}T^2(2+aT)(A+e^{aT})W_0(A(3+2aT)+\dots}$$
$$\overline{\dots + 3e^{aT}W_0) + (12T + 12aT^2 + 12T(Ae^{-aT}+W_0))^2}\Bigg] + O(\phi^3) \tag{306}$$

We can expand $D_T W$ in powers of ϕ and the small shift δT in the Kähler modulus (as was done in [10]), around $\phi = 0$ and $T = T_0$ (where T_0 is the location of the minimum at $\phi = 0$). We then substitute (306) into this expansion, and obtain that the shift δT is proportional to ϕ^3. Substituting this shift into the scalar potential, we conclude that small shifts in the Kähler modulus lead to ϕ^3 correction terms in the scalar potential (given by similarly huge expressions). These correction terms will have coefficients of smaller magnitude than the ϕ^3 terms already present in the scalar potential. Small shifts in the Kähler modulus leave the scalar potential untouched at very small ϕ, and only adjust coefficients already present in the scalar potential. We expect that Kähler modulus shifts have very little effect on the inflationary observables, as we had anticipated earlier.

5.2.1 *Stability*

We must still check two things: whether or not the truncation $\phi = \bar{\phi}$ is consistent and whether or not the Kähler modulus remains stabilized along the inflationary trajectory. These requirements are satisfied only on a partial region of the c_0-c_1 plane, and the specific location and shape of this region depends on the KKLT parameters A,a, and ω. We can address these questions numerically. The following is a plot of the scalar potential with $\alpha = 1$, $c_0 = 1$, $c_1 = -10$, and KKLT parameters $A = 1$,$a = 0.1$, $\omega = 4 \times 10^{-4}$. The first image displays V versus the canonical inflaton φ, the second displays V versus the geometric inflaton ϕ. All units are Planck. For each value of ϕ, the Kähler modulus has been placed at its minimum numerically. This value is then substituted into the scalar potential to yield the graph. We can see from the second graph that the linear approximation (293) is quite accurate up to where the minimum in the potential occurs. We expect the inflationary observables to be very well-approximated by the general α-attractor formulae (293). Behind the minimum (to the left of the minimum in φ-space and to the right in ϕ-space) a sharp drop in the scalar potential occurs due to a destabilization of the Kähler modulus. The minimum in the potential has disappeared. This is unfortunate, but the problem would be worse if the destabi-

Figure 3: Scalar potentials

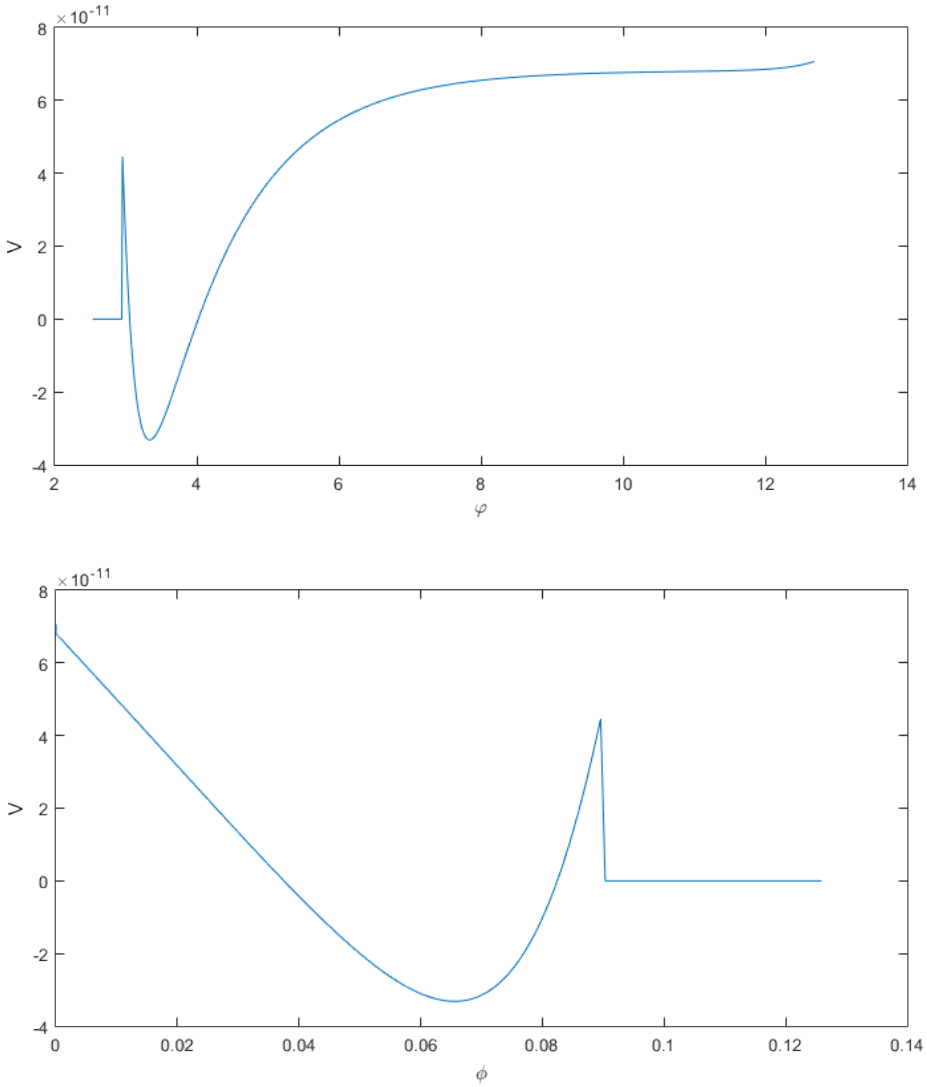
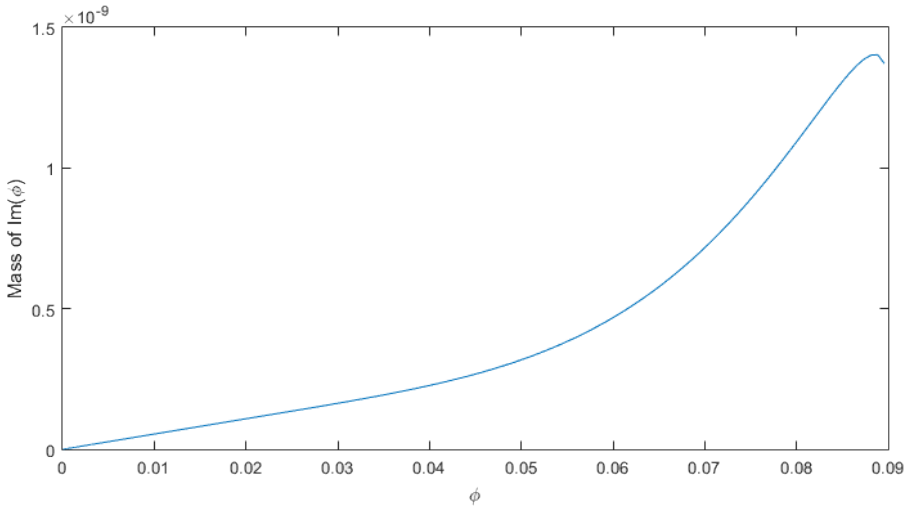


Figure 4: Mass of $\mathrm{Im}(\phi)$



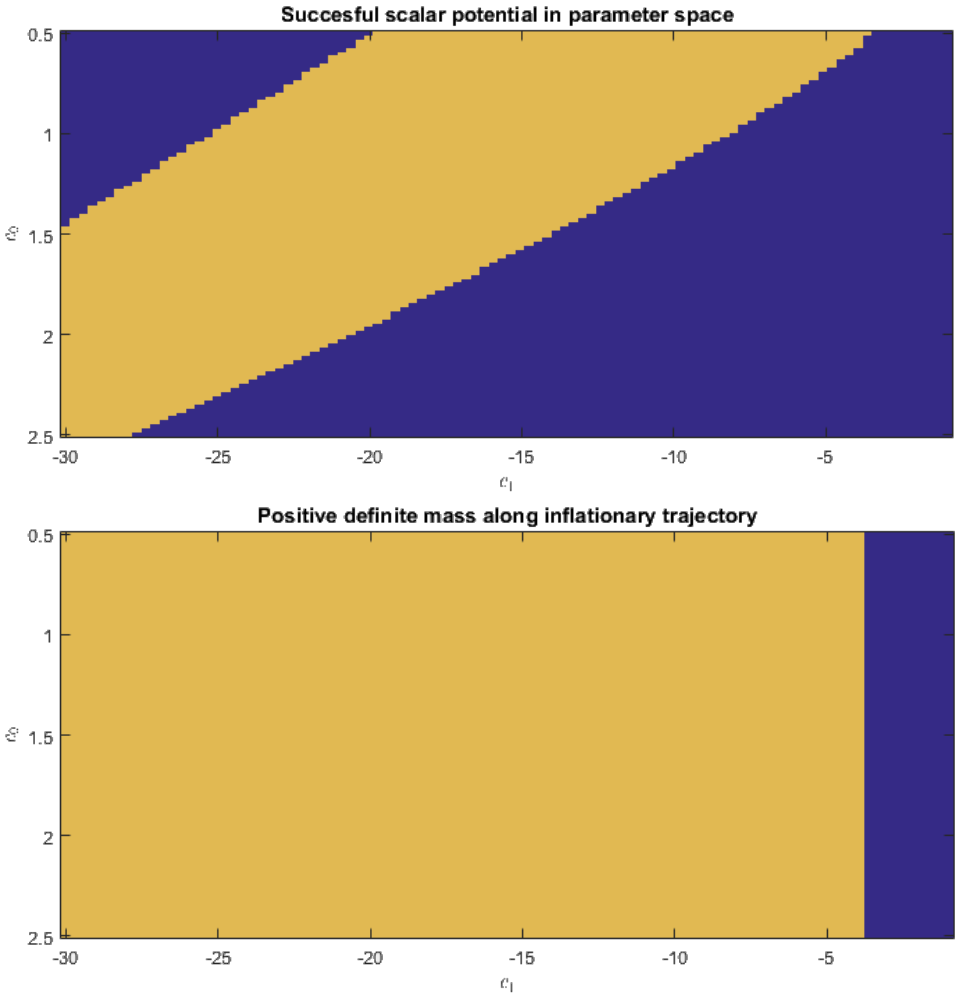
lization had occurred on the inflationary plateau instead, as is the case in the models examined in [10]. The inflaton will settle down in the minimum after inflation is done, and avoid the destabilization. The minimum achieved here is AdS, but it can be uplifted to de Sitter in the same way as the bare KKLT model: by a term $V_{up} = D/T^3$ generated by anti-D3 branes.

The mass squared of the imaginary direction of ϕ is positive definite over the entire field range examined here (including past the destabilization of T).

Again, this graph has been obtained by numerically placing the Kähler modulus at its minimum for each ϕ individually. Unfortunately, the mass squared of the imaginary direction is generically lower than the squared Hubble constant during inflation $H^2 \sim 10^{-10}$, on some part of the inflationary plateau. Quantum fluctuations will perturb the inflationary dynamics. It is reasonable to assume that the flatness of the imaginary direction is due to a shift symmetry $\mathrm{Im}(\phi) -> \mathrm{Im}(\phi) + c$ in the Kähler potential (294). This symmetry is broken by the superpotential. However, we may consider also breaking it slightly in the Kähler potential by adding minimal supergravity couplings, such as $\phi\bar{\phi}$ or $(\phi - \bar{\phi})^2$. These terms only add $O(\phi^2)$ corrections to the field redefinition (292), which connects the geometric field ϕ to the canonical field φ. These corrections are very small compared to $O(\log\phi)$ on the inflationary trajectory. We anticipate that such terms do not change the inflationary observables to leading order in $1/N$ (where N is the number of e-folds between the start and end of inflation). Furthermore, the change in the scalar potential from adding minimal terms to the Kähler potential is quite small over most of the field range.

Minimal Kähler terms shift the mass squared of the imaginary direction closer to the Hubble constant, as is necessary to neglect quan-

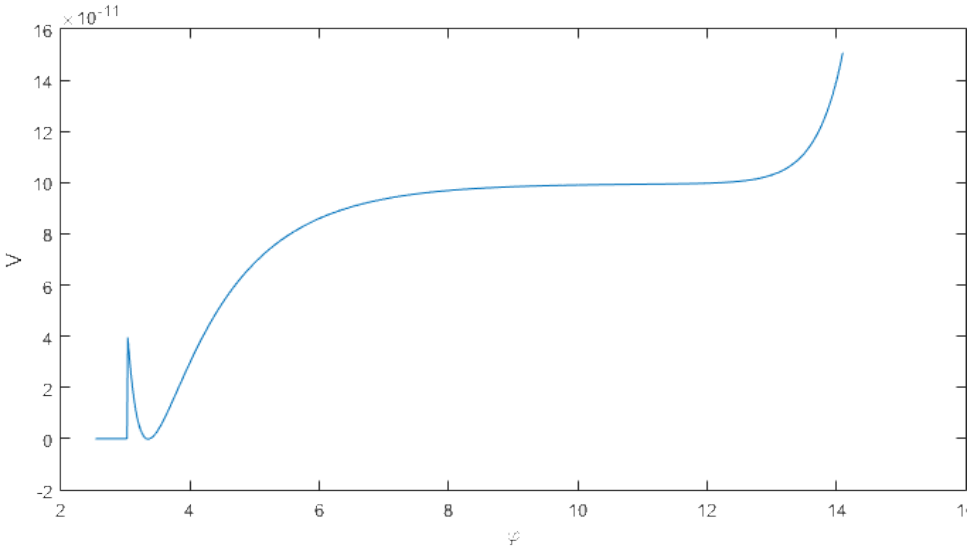
Figure 5: Parameter space check with KKLT parameters $A = 1$, $a = 0.1$, $W_0 = -4 \times 10^{-4}$. Dark regions indicate an unstable scalar potential or imaginary ϕ direction



tum fluctuation in the inflationary dynamics. However, there are now generically regions of field space on the inflationary trajectory where the mass squared becomes negative. Adding quartic terms (and so on) could remove these tachyonic regions, but this adds another level of fine-tuning to the model. Furthermore, adding corrections upsets the delicate balance between the order $\phi^{-3\alpha}$ terms in the scalar potential. We now require a Polonyi uplift even at $\alpha = 1$, or we must tune the coefficients of the correction terms to order $O(10^{-2})$ or smaller. All in all, shift symmetry-breaking terms in the Kähler potential do not seem to improve the model.

For this choice of input parameters, we have found a nice and stable scalar potential for inflation. We would like to see in what region of c_0-c_1 space we can find a stable scalar potential of the correct form (293). The result is pictured in Figure 5. The parameter c_0 is the ver-

Figure 6: Uplifted scalar potential, with $c_0 = 1$, $c_1 = -20$, $D = 0.00003$



tical axis, and c_1 is the horizontal axis. The first plot displays the stability of the Kähler modulus. In the light region, the Kähler modulus stays stable at least until the inflaton settles into a minimum after inflation. In the dark region, there is either a destabilization before a minimum, or the functions g_0 and g_1 do not have the right sign to produce the correct scalar potential (293). The second plot displays the stability of the imaginary direction of ϕ. In the light region, its mass squared is positive definite throughout the entire field range. In the overlap between the two plots, a stable inflationary scalar potential is achieved. The mass of the real direction of T is not checked specifically here, since it is generically at a much higher scale than the scale of inflation (when it remains stabilized). This mass is strongly enhanced by a factor $\phi^{-3\alpha}$. Repeating the same analysis for different KKLT parameters A, a, and W_0 gives a similar result. We conclude that the present model can realize a Starobinsky-like inflationary potential with a stabilized Kähler modulus, with only a moderate amount of tuning. To uplift the minimum to Minkowski takes fine-tuning of the same kind that it did in [10] or in the bare KKLT model.

5.2.2 *Uplifting*

We can uplift the minimum of the potential to de Sitter by the same way the KKLT minimum is uplifted: adding an anti-D3 brane term proportional to D/T^n, where $n = 2, 3$ and D is a fine-tuning constant. This has no effect on the mass of $\mathrm{Im}(\phi)$ except through the small shift in the Kähler modulus minimum. It is not hard to find regions of parameter space wherein the Kähler modulus remains stabilized on the

inflationary trajectory after uplifting. We attain potentials that look like Figure 6. We have not found points in the c_0-c_1 parameter space where a de Sitter minimum is reached without adding an uplift (anti-D3 or otherwise). Indeed, it may not be possible at all. It is hard to analyze this analytically since the minimum results from shifts in the Kähler modulus which are quite large and occur just before a destabilization. We have no further results to share on this point. In any case, the attractor structure of the model is lost by introducing fine-tuning on c_0 and c_1, so this option is not definitively more attractive than adding an anti-D3 brane or D-term uplift.

The gravitino mass in the uplifted minimum is given by:

$$m_{3/2}^2 = e^K W^2 = \frac{2^{-3-3\alpha}\phi^{-3\alpha}\left(Ae^{-aT} + (c_0 + c_1\phi + \ldots)\phi^{3\alpha} + \omega\right)^2}{T^3} \tag{307}$$

For the above choice of parameters, the minimum occurs at $T \simeq 97$, $\phi \simeq 0.0647$. The mass of the gravitino becomes $m_{3/2}^2 \simeq 3.7 \times 10^{-12} M_{Pl}^2$. This value is smaller than the squared Hubble constant during inflation by about an order of magnitude! We can find regions of field space where the regular KKLT bound $m_{3/2} > H$ does not seem to apply. This is due to the $\phi^{-3\alpha}$ enhancement of the Kähler modulus stabilization. We may consider replacing the KKLT bound by $\phi_0^{3\alpha}\phi_p^{-3\alpha} m_{3/2}^2 > H^2$, where ϕ_0 is the value of ϕ at the minimum, and ϕ_p is the approximate value of ϕ on the inflationary plateau. Inflation generically takes place around $\phi \sim 0.001$, and the minimum occurs around $\phi \sim 0.05$. This suggests that generating a gravitino mass on the order of a TeV ($\sim 10^{-16} M_{Pl}$) is not possible. It requires tweaking KKLT parameters to very different values from those we have been considering, and the delicate cancellation of the $O(\phi^{3\alpha})$ is easily upset by this. Furthermore, the KKLT parameters needed may be unnatural from the perspective of string theory. We can see from the gravitino mass formula that we may decrease the mass by: 1) pushing the minimum of ϕ to higher field values, 2) pushing the minimum of the Kähler modulus to higher field values, 3) decreasing A or ω. The minimum of ϕ is controlled by the superpotential parameters c_0, c_1, etc., but we cannot push the minimum to high enough field values using just these parameters (see Figure 5). We have not found any region of parameter space in which the gravitino mass of the minimum is of TeV scale, and H of the same order as before, but we cannot exactly prove that it is impossible. In any case, it requires tuning KKLT parameters, and cannot be done in a way that preserves the attractor structure. Generating a TeV mass in this way may be trading one fine-tuning problem for another. However, it is still interesting to note that we have found an inflation model in which the ordinary KKLT bound on the gravitino mass is changed.

As explained in the previous section, the $\alpha = 1$ scalar potential relies on a cancellation of the terms at $O(\phi^{-3\alpha})$ that is caused by the stabilization of the Kähler modulus. The cancellation condition

$$f_0(T)|_{\alpha=1} = \frac{e^{-2aT}\left[A(3+2aT)+3e^{aT}\omega\right]^2}{192T^3} = 0 \tag{308}$$

is equivalent to the equation $A(3+2aT)+3e^{aT}\omega = 0$ that determines the KKLT minimum. When the Kähler modulus minimum is not shifted too far away from this bare KKLT result, the $\phi^{-3\alpha}$ coefficient f_0 becomes very small, and it only causes trouble at large field values of the canonical field ($\varphi >\sim 12$), far away from the minimum φ_c required to obtain 60 e-folds of inflation. Restoring α in the above equation, we obtain:

$$f_0(T) = \frac{8^{-1-\alpha}e^{-2aT}}{3T^3}\Big[A^2\left[9\alpha+4aT(3+aT)\right]+\dots \\ \dots+6Ae^{aT}\left[3\alpha+2aT\right]\omega+9e^{2aT}\alpha\omega^2\Big] \tag{309}$$

We see that the cancellation is upset when $\alpha \neq 1$. When $\alpha > 1$, $f_0(T)$ becomes large and positive, when $\alpha < 1$, it becomes large and negative. This causes the scalar potential to look like a $\phi^{-3\alpha}$ asymptote at field values smaller than the minimum φ_c required for 60 e-folds of inflation. When $f_0(T)$ is large and negative, this also causes a serious initial value problem (even if $f_0(T)$ were small enough for 60 e-folds to occur). It makes the potential unbounded from below at large φ. A positive asymptote does not cause such a problem. Furthermore, when the initial value of the inflaton is such that it is placed on the asymptote instead of on the plateau, it is possible that Hubble friction slows it down fast enough to allow for 60 e-folds of inflation. However, this requires a calculation beyond the slow-roll approximation that we have not completed yet.

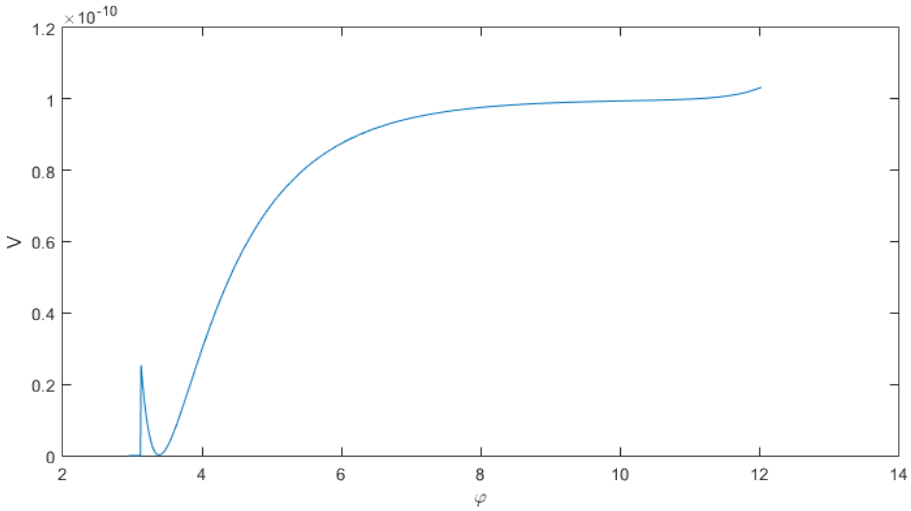
The large negative $f_0(T)$ for $\alpha < 1$ is a bigger problem than the large positive $f_0(T)$ for $\alpha > 1$, but it can be solved more easily. A straightforward way to deal with the large negative asymptote is to introduce a Polonyi field X as was done in [10]. This adds the following to the Kähler potential and the superpotential:

$$K_{up} = k(|X|^2) \qquad W_{up} = fX \tag{310}$$

where f is a constant. A quartic term in the function $k(|X|^2)$ with a very large coefficient serves to stabilize X at the origin with a very large mass. The Polonyi field adds the following term to the scalar potential:

$$e^K f^2 = \frac{1}{(2T)^3(2\phi)^{3\alpha}}f^2 \tag{311}$$

Figure 7: Polonyi-uplifted scalar potential for $c_0 = 1$, $c_1 = -20$, $\alpha = 0.9$, $A = 1$, $a = 0.1$, $\omega = -4 \times 10^{-4}$



This is a positive-definite term, since f must be real in order to keep the imaginary directions of T and ϕ at the origin. A Polonyi field can solve a large negative asymptote, but not a large positive asymptote.

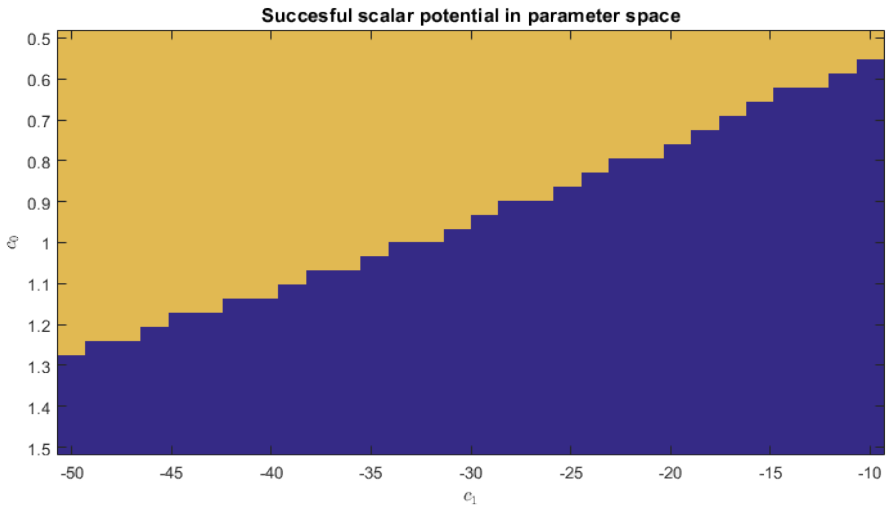
When the Kähler modulus is stabilized, using similar KKLT parameters as before, $f_0(T)$ is of order $O(10^{-14})$. At $\phi \simeq 0.01$ (which is approximately the maximum geometric field value where inflation can start and still generate 60 e-folds of expansion), $\phi^{-3\alpha} f_0(T)$ becomes $O(10^{-8})$. This must be cancelled by the Polonyi uplift to an accuracy of 10^{-11} in order not to interfere with the other terms in the scalar potential. We require fine-tuning to at least three significant digits. This is the bare minimum for 60 e-folds, however, and to generate a large plateau, we need far more accurate fine-tuning.

As we stated earlier in this chapter, we cannot generate a proper inflationary scalar potential when α becomes roughly equal to $\frac{2}{3}$ or smaller. At $\alpha = \frac{2}{3}$, $\phi^{3\alpha}$ terms in the scalar potential become larger than $O(\phi^2)$, and we can no longer neglect them.

In Figure 7, a Polonyi-uplifted scalar potential with $\alpha = 0.9$ is pictured.

We can produce inflationary potentials down to right around $\alpha \simeq \frac{2}{3}$. The region of c_0-c_1 space where inflation is succesful changes shape when α is changed, but we still do not require more than moderate tuning of c_0 and c_1, even down at $\alpha \simeq \frac{2}{3}$. This is demonstrated in Figure 8.

Figure 8: Succesful inflationary potential with $\alpha = 0.7$, $A = 1$, $a = 0.1$, $\omega = -4 \times 10^{-4}$, and c_0, c_1 as indicated



5.3.1 *Conclusion*

The model with added superpotentials and KKLT moduli stabilization can generate Starobinsky-like scalar potentials, i.e. a potential of the kind (293) with $\alpha = 1$. We can do this with only a small amount of tuning of the KKLT parameters and the inflaton parameters c_i (where $i = 0, 1, \ldots$). However, the mass squared of the imaginary direction of ϕ is generically lower than the squared Hubble constant during inflation, at least in some part of field space. We cannot solve this problem by adding corrections to the Kähler potential which approximately preserve (292). We can produce inflationary potentials with $\alpha < 1$, up to about $\alpha \simeq \frac{2}{3}$, by adding a Polonyi field. However, this requires fine-tuning of a single parameter in the Polonyi field Kähler potential. Adding any kind of fine-tuning goes against the spirit of the α-attractor concept. Ostensibly, we have constructed a model which realizes inflation with inflationary observables $n_s = 1 - \frac{2}{N}$, $r = \frac{12\alpha}{N^2}$ (with $\frac{2}{3} < \alpha \leqslant 1$). We say ostensibly because we have inconsistently ignored quantum fluctuations of the imaginary ϕ direction. We cannot claim that the present model is succesful. However, we have confirmed a few interesting things:

- The shifts in the Kähler modulus have little effect on the inflationary observables, even when they effectively modify coefficients in the scalar potential.
- Destabilization of the Kähler modulus is less of a problem than it is in the chaotic inflation models of [10]. We can easily construct scalar potentials in which destabilization only occurs behind a minimum, separated by a barrier.

- We can generate succesful potentials for a rather generic choice of KKLT parameters. We do not need to tune them to values which are unnatural from the perspective of string theory.
- The ordinary KKLT bound $m_{3/2} > H$ is altered to some extent. We were able to find stable potentials in which the squared gravitino mass at the minimum was lower than the squared Hubble constant by about one order of magnitude.

While the successes of this model were anticipated on the basis of qualitative arguments, it is hard to argue that the failures (the low mass scale of $\mathrm{Im}(\phi)$, the requirement of fine-tuning to deal with problematic asymptotes) are similarly generic features of α-attractors. The fine-tuned Polonyi uplift was necessary to cancel out a troublemaking term in the scalar potential. We expect that there are better models in which such terms do not appear. The low mass scale of $\mathrm{Im}(\phi)$ may be due to the shift symmetry of the Kähler potential. However, the α-attractor models of [47] and [49] do not suffer from this problem, even though they are constructed with the same shift symmetry.

So while this model fails, in our opinion these results allow us to be optimistic about implementing α-attractors in string theory.

5.4 ADDING SUPERPOTENTIALS AND KL MODULI STABILIZATION

A second possibility is to use the KL mechanism for moduli stabilization. If we choose to add superpotentials, the model is the following:

$$W = W_{KL} + W_{inf} = (Ae^{-aT} - Be^{-bT} + W_0) + \phi^{3\alpha}F(\phi) \tag{312}$$

$$K = -3\log\left[T + \bar{T}\right] - 3\alpha\log\left[\phi + \bar{\phi}\right] \tag{313}$$

where $F(\phi) = c_0 + c_1\phi + c_2\phi^2 + \dots$ is an arbitrary function. The scalar potential has a similar structure to the model examined in the previous section. That is, schematically it takes the following form:

$$V = \phi^{-3\alpha}T^{-3}\left[f_0(T) + \phi^{3\alpha}\big(g_0(T) + \phi g_1(T) + \dots\big) + \dots\right] \tag{314}$$

There is a pesky $\phi^{-3\alpha}$ term $f_0(T)$ due to the e^K factor of the scalar potential. We can tune between A and B to make the $\phi^{3\alpha}$ term smaller, but not small enough to make it disappear. These are small at large φ for $\alpha \simeq 1$, large and positive for $\alpha > 1$, large and negative for $\alpha < 1$. The situation is the same as before: we can generate Starobinsky-like scalar potentials at $\alpha = 1$, and use a Polonyi uplift to obtain plateau potentials for $\frac{2}{3} < \alpha < 1$.

However, it is harder than before to find regions of parameter space where the model produces a viable inflationary scalar potential. To see why this is the case, let us recall the formula that determines the location of the Kähler modulus minimum in the KL model[35]:

$$T_0 = \frac{1}{a-b}\ln\left(\frac{aA}{bB}\right) \tag{315}$$

Using a typical set of KL parameters (i.e. the $O(1)$ parameters used in [35])[1], we obtain a minimum at $T \simeq 25$. This is a much lower field value than in the KKLT model (where the minimum was placed at around $T \simeq 120$ for typical, $O(1)$ parameters). This means that the suppression factors in the scalar potential (T^{-3}, e^{-aT}, and so on) are much weaker than before. The inflationary plateau now sits at a scale $O(10^{-6}M_{Pl})$ instead of 10^{-10}, for $O(1)$ choices of the superpotential parameters c_0 and c_1. We can bring down the ϕ^0,ϕ^1 terms ($g_0(T)$, $g_1(T)$) to $O(10^{10})$ by reducing the magnitude of c_0 and c_1. However, the $\phi^{-3\alpha}$ term is not affected by this. The problem with large $\phi^{3\alpha}$ asymptotes[2] is enhanced by the KL model.

We can move the Kähler modulus minimum to larger field values by bringing a and b closer together. However, we can see from the KL mass formula:

$$m^2_{Re(T)} = \frac{2}{9}aAbB(a-b)\left(\frac{aA}{bB}\right)^{-\frac{a+b}{a-b}} \ln\left(\frac{aA}{bB}\right) \tag{316}$$

that bringing a and b closer together reduces the mass of the real direction of T. This makes the Kähler modulus more prone to destabilization. We can enhance the stability by a factor C by rescaling $A \to CA$, $B \to CB$. However, the parameters needed are no longer close to $O(1)$ and may be quite unnatural from the perspective of string theory.

We have found a slim region in parameter space where plausible scalar potentials may be generated, but in this region the imaginary direction of ϕ is tachyonic over the entire field range, making the truncation $\bar{\phi} = \phi$ inconsistent. It is hard to realize inflation in a KL setup, if we choose to maintain the fine-tuning on W_0 needed to generate a very small gravitino mass. Without this requirement, we can of course take $B = 0$, and the model will reduce to KKLT.

5.5 ADDING KÄHLER FUNCTIONS AND KKLT MODULI STABILIZATION

We will now turn to adding Kähler functions instead of adding superpotentials. This means that we *multiply* the superpotentials of the inflaton and moduli sectors, and add the Kähler potentials. Adding Kähler functions leads to a decreased mixing between the two sectors compared to adding superpotentials. The scalar potential takes the following form:

$$V = e^{K_{inf}}|W_{inf}|^2\mathcal{V}_F + e^{K_{mod}}|W_{mod}|^2 e^{K_{inf}}|\partial_i W_{inf} + K_i W_{inf}|^2 + \mathcal{V}_{lift} \tag{317}$$

where $\mathcal{V}_F$ refers to the moduli stabilization scalar potential, and $\mathcal{V}_{lift}$ is the contribution of an uplifting sector. Most of the mixing simply

1 $A = B = 1$, $a = \pi/25$, $b = \pi/10$.

2 This is another manifestation of the η problem

rescales the scalar potentials $\mathcal{V}_F$ and V_{inf}. When there is a supersymmetric minimum of the moduli sector potential at T_0 ($\partial_T G_{mod}(T_0) = 0$), then the inflaton sector will not shift the location T_0 of this minimum. However, an uplifting term generically breaks this supersymmetry at the Kähler modulus minimum. When it is broken, there may be inflaton-dependent shifts of the Kähler modulus. However, we can anticipate that they are very small since they equal zero in the supersymmetric limit.

Despite the dramatically simplified mixing, we still cannot simply use the inflaton superpotential of [47], since it relies too heavily on accidental cancellations. A potentially viable model consists of the following:

$$W_{inf} = \left(Ae^{-aT} + W_0\right)\left(\phi^{\frac{3\alpha}{2}} F(\phi)\right) \tag{318}$$

where $F(\phi) \simeq c_0 + c_1\phi + c_2\phi^2 + \ldots$.

The ϕ^0 term in the scalar potential becomes:

$$f_0(T) = \frac{2^{-1-3\alpha} aAe^{-2aT}\left(A(3+aT) + 3e^{aT}W_0\right)c_0^2}{3T^2} \tag{319}$$

At the minimum of the Kähler modulus, this becomes:

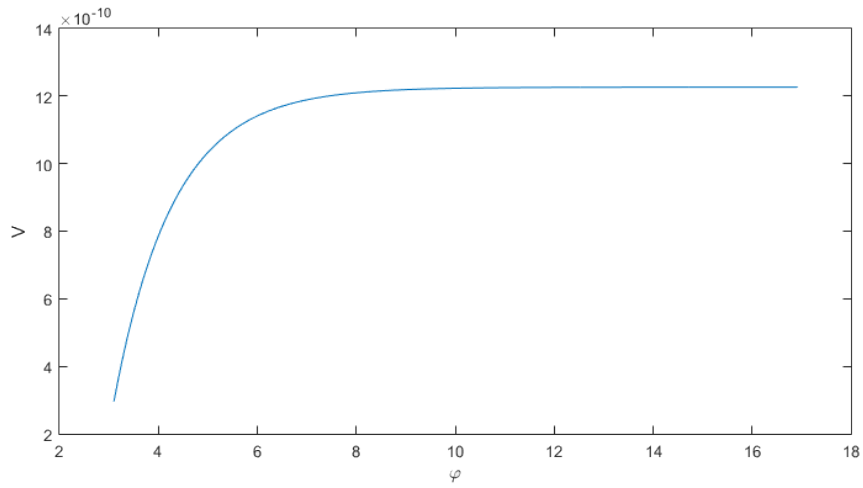
$$f_0(T_0) \simeq \frac{-2^{-1-3\alpha} a^2 A^2 e^{-2aT_0} c_0^2}{3T_0} \tag{320}$$

We see that the potential is always negative at small ϕ, regardless of the sign of c_0. We can add an anti-D3 brane uplift $V_{up} = D/T^3$ to solve this. However, we now require a moderate fine-tuning to realize an inflationary plateau, whereas previously this fine-tuning was only needed to generate a Minkowski minimum at the end of inflation. There may be a region of parameter space in which an inflationary plateau and a Minkowski minimum are realized with the same uplifting parameter D, but this destroys the attractor structure of the model.

On the other hand, the model generates a scalar potential which is essentially completely free from modulus destabilization (see Figure 9), even in the presence of a supersymmetry breaking uplifting term. The $F(\phi)$ coefficients c_0 and c_1 must be of order $O(1000)$ for the scalar potential to be $O(10^{(} - 10))$ when the KKLT parameters are close to the natural values $A = 1$, $a = 0.1$, $W_0 = -4 \times 10^{-4}$. Previously, they were $O(1) - O(10)$. However, we have no reason a priori to claim that $O(1)$ parameters are more natural than $O(1000)$ in this case.

Unfortunately, the scalar potential has a tachyonic $\mathrm{Im}(\phi)$ direction at $\phi = \bar{\phi}$. Again, we cannot solve this with corrections to the Kähler potential. Although one might expect that this would be possible in this case - especially because there is no delicate cancellation at $\phi^{3\alpha}$ that we need to be concerned with - all the corrections to the mass are of $O(\phi^2)$. The current model is plagued by too many problems to be viable.

Figure 9: Scalar potential generated with $A = 1$, $a = 0.1$, $c_0 = c_1 = 1000$, $W_0 = -0.0004$. The plot shows the stability of the Kähler modulus over a large field range, and the absence of a $\phi^{3\alpha}$ asymptote at large φ.



5.6 A NILPOTENT CHIRAL SUPERFIELD: CONTROL OF DARK ENERGY AND THE SCALE OF SUSY BREAKING

As we have seen, many of our models suffer from instability issues regarding the imaginary direction of the inflaton ϕ. A useful tool to deal with stability issues is a *nilpotent* chiral superfield S. Such a field is defined by:

$$S^2(x, \theta) = 0 \tag{321}$$

where x, θ are bosonic and fermionic coordinates, respectively. This is an equation that holds for the *superfield* S. We have not introduced the superfield formalism, so we will not go into detail about the interpretation of nilpotent fields, or how they may be generated in a string theory context. For some information, see [32]. The superfield $S(x, \theta)$ contains no propagating scalar degrees of freedom. However, there is a quantity S given by fermion bilinears which we may treat as if it were a proper scalar. The interesting thing about the nilpotent multiplet is that the "scalar" always takes the expectation value $< S >= 0$. This makes it an enormously useful tool for supergravity inflation constructions. The difficulty in realizing inflation in supergravity, as we have seen, lies in the stabilization of each real scalar field, of which there may be many. A nilpotent chiral superfield has an automatic stabilization $< S >= 0$.

The nilpotent chiral superfield was introduced into α-attractor models by [49][12]. The following model was constructed in [49]:

$$K = -3\alpha \log\left(\phi + \bar{\phi}\right) + S\bar{S} \tag{322}$$

$$W = \phi^{\frac{3\alpha}{2}}\left[f(\phi) + g(\phi)S\right] \tag{323}$$

At $\phi = \bar{\phi}$ and $S = 0$, the scalar potential becomes:

$$V = 8^{-\alpha}\left[g(\phi)^2 - 3f(\phi)^2 + \frac{4\phi^2 f'(\phi)^2}{3\alpha}\right] \tag{324}$$

We can control the location of the minimum, and the cosmological constant at the minimum, by tuning the functions $f(\phi)$ and $g(\phi)$. When we choose $f'(1) = g'(1) = 0$, the minimum is located at $\phi = 1$ ($\varphi = 0$) and the cosmological constant becomes:

$$\Lambda = \left(\sum_n b_n\right)^2 - 3\left(\sum_n a_n\right)^2 \tag{325}$$

where a_n and b_n are the Taylor coefficients of $f(\phi)$ and $g(\phi)$ respectively. The gravitino mass at the minimum, as well, is completely determined by these coefficients:

$$m_{3/2} = \sum_n a_n \tag{326}$$

The imaginary direction of ϕ has a positive-definite mass as long as $|b_0| > |a_0|$. This model gives essentially full control over dark energy and the scale of supersymmetry breaking, while at the same time its inflationary predictions are insensitive to the particulars of the superpotential, especially to the coupling between ϕ and S. The model realizes inflation with a purely logarithmic Kähler potential and a polynomial superpotential. These arise frequently in explicit string theory compactifications.[3] This model offers an even more exciting possibility of an explicit string theory realization than the model of [47]. The only price we have to pay is that we need to incorporate a nilpotent chiral superfield into the model. Let us see if it can survive being coupled to a string theory Kähler modulus.

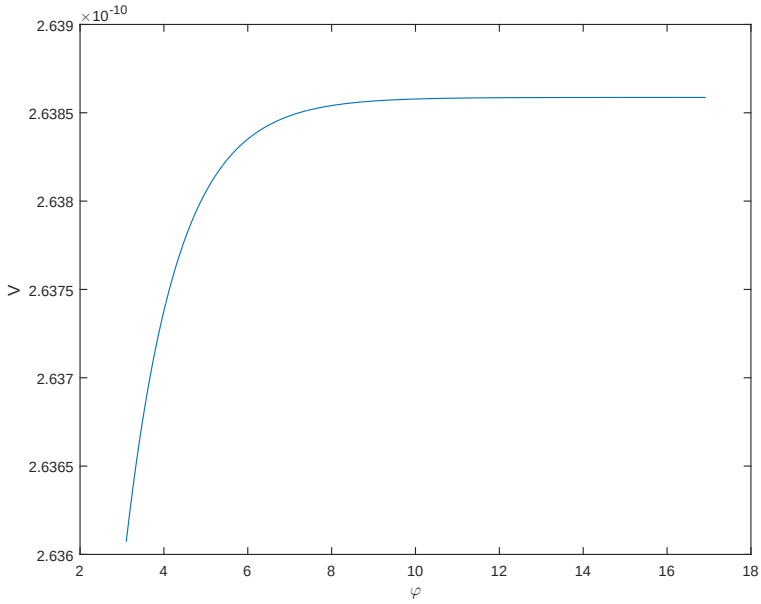
When we add superpotentials, the model suffers from the same problems as the one without the nilpotent field, namely that the $\phi^{3\alpha}$ asymptotes need to be cancelled by Polonyi uplifts for $\alpha < 1$. However, when we choose to multiply superpotentials, the additional freedom in specifying $G(\phi)$ allows us to generate a positive-definite mass squared for $\mathrm{Im}(\phi)$. Our model is the following:

$$K = -3\alpha \log\left(\phi + \bar{\phi}\right) - 3\log\left(T + \bar{T}\right) + S\bar{S}$$

$$W = \left(Ae^{-aT} + W_0\right)\left(\phi^{\frac{3\alpha}{2}}\left[F(\phi) + G(\phi)S\right]\right)$$

3 See for example the STU model of [9].

Figure 10: Scalar potential with $O(1000)$ coefficients in $F(\phi)$ and $G(\phi)$ and natural $O(1)$ KKLT parameters



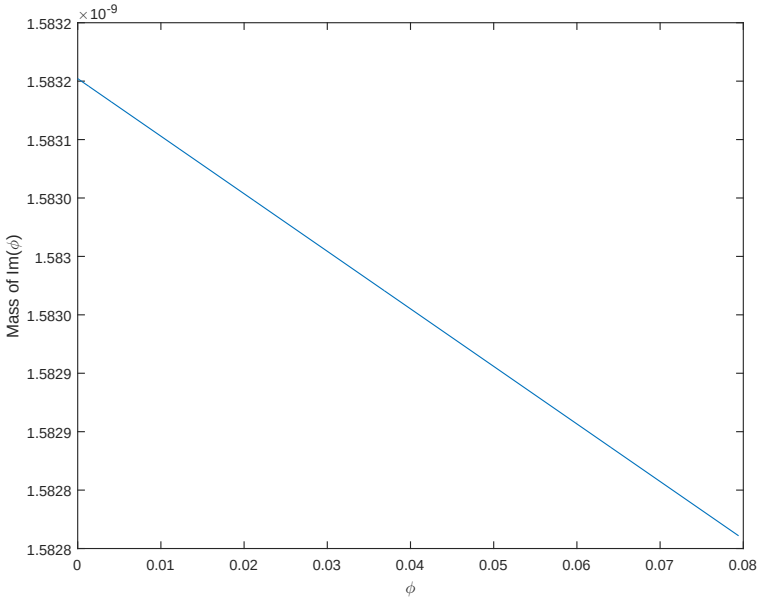
The scalar potential becomes, along the inflationary trajectory $\bar{\phi} = \phi$, $S = \bar{S} = 0$:

$$\frac{8^{-1-\alpha} e^{-2aT}}{3T^3\alpha}\Big[4aAT\alpha\big(A(3+aT)+3e^{aT}W_0\big)F(\phi)^2+\dots \\ \dots+\big(A+e^{aT}W_0\big)^2\big(3\alpha G(\phi)^2+4\phi^2F'(\phi)^2\big)\Big] \quad (327)$$

The Kähler modulus sector does not break supersymmetry, so the stability of the Kähler modulus is almost assured in this case. However, if we tune the functions $F(\phi)$ and $G(\phi)$ such that we produce a Minkowski minimum, the potential is flat in the T direction there. The model is not compatible with Minkowski minima. However, the model improves upon the previous one without a nilpotent field in the following way: the freedom in choosing $G(\phi)$ allows us to generate a positive mass squared for the imaginary direction of ϕ, while still maintaining a viable inflationary potential. Figures 10 and 11 display the scalar potential and the mass squared of $\mathrm{Im}(\phi)$ for $O(1000)$ values of the expansion coefficients of $F(\phi)$ and $G(\phi)$. With $b_0 > a_0$, the mass squared of $\mathrm{Im}(\phi)$ is positive-definite over the entire inflationary plateau.

The only remaining issue at this point is the allergy to stable Minkowski minima. Unforunately, coupling to the KL model does not solve this issue, and the results for the KL model are essentially the same as for KKLT.

Figure 11: Mass squared of $\mathrm{Im}(\phi)$ with $O(1000)$ coefficients in $F(\phi)$ and $G(\phi)$ and natural $O(1)$ KKLT parameters



6 CONCLUSIONS

In order to introduce our subject, we have provided an extensive introduction to inflationary cosmology, supersymmetry, supergravity, and string theory.

We have attempted to implement a single-field α-attractor model in a string theory moduli stabilization context. Our motivation for this task was two-fold: firstly, the observable predictions of α-attractor models have an appealing insensitivity to the details of their scalar potential. For that reason, we anticipated that α-attractors would not be sensitive to the specifics of the coupling to a string theory Kähler modulus. Secondly, we noted that certain examples of α-attractors have a structure that arises quite commonly in string theory. This makes it very natural to ask whether or not α-attractors can survive moduli stabilization.

This was to some extent unsuccesful when we chose to add superpotentials instead of Kähler functions. The inflationary scalar potentials were ruined by $\phi^{-3\alpha}$ terms from the e^K factor. This term could only be cancelled without additional fine tuning when $\alpha = 1$. Even then, the most succesful of our attempts suffered from a light $\mathrm{Im}(\phi)$ that could upset the inflationary dynamics. However, we were able to confirm some our initial suspicions with this simple model. Firstly, we saw that small Kähler moduli shifts produce no change to the inflationary observables, as they do in other models. Secondly, we saw that there is no destabilization of the Kähler modulus at large field values of the canonical inflaton field φ. This allows for some freedom in specifying the initial conditions of inflation. When a destabilization of the Kähler modulus occurs, it can be separated from the inflationary plateau by a potential energy barrier. The most notable success of this model was that we generated a viable inflationary scalar potential in a moduli stabilization setup, without introducing additional fine-tuning on top of that required by the KKLT construction.

When we multiplied superpotentials, we ran into trouble with tachyonic imaginary directions of the inflaton. However, adding a nilpotent chiral superfield into the setup solved this problem. The only issue remaining in this model is to generate a stable Minkowski (or very small de Sitter) minimum. In the current setup, the potential has a flat Kähler modulus direction in the Minkowski limit.

Our results indicate that further research into this topic is worthwhile. For instance, it may be possible to remove the steep asymptotes of the superpotential addition model by tweaking our Ansatzes a bit. A simple dependence on ϕ of the KKLT parameter A may be

enough to remove the α dependence of the $\phi^{-3\alpha}$ term. This would allow us to generate an inflationary scalar potential without Polonyi field fine-tuning for any value of α, whereas previously we were only able to do so at $\alpha = 1$.

Part III

APPENDIX

APPENDIX A: TYPE IIB EFFECTIVE ACTION WITH FLUXES

For easy reference, the Type IIB supergravity action in the string frame reads:

$$S = \int e^{-2*\hat{\phi}} \left(-\frac{1}{2}\hat{R} \star \mathbf{1} + 2d\hat{\phi} \wedge \star d\hat{\phi} - \frac{1}{4}\hat{H}_3 \wedge \star \hat{H}_3 \right) - \frac{1}{2}\left(dl \wedge \star dl + \hat{F}_3 \wedge \star \hat{F}_3 + \frac{1}{2}\hat{F}_5 \wedge \star \hat{F}_5 + \hat{A}_4 \wedge \hat{H}_3 \wedge d\hat{C}_2 \right) \quad (328)$$

where the field strengths are defined by (148) and:

$$\hat{H}_3 = d\hat{B}_2 \quad (329)$$

$$\hat{F}_3 = d\hat{C}_2 - l d\hat{B}_2 \quad (330)$$

The $\hat{H}_3 \wedge \star \hat{H}_3$ part is trivial: inserting (143), we see that $\hat{H}_3 = d\hat{B}_2 = dB_2$. This part of the action contributes a term

$$\int_{M_4} e^{-2\phi} \left(-\frac{1}{4} H_3 \wedge \star H_3 \right) \quad (331)$$

to the four-dimensional effective action.

The $\hat{F}_3 \wedge \star \hat{F}_3$ part reads:

$$\int_{\mathcal{M}} -\frac{1}{2} e^{-2\hat{\phi}} \left(\hat{F}_3 \wedge \star \hat{F}_3 \right) = -\frac{1}{2} \int_{\mathcal{M}} e^{-2\hat{\phi}} \left[dC_2 + dc^i \omega_i - l(dB_2 + db^i \omega_i) \right] \wedge \wedge \star \left[dC_2 + dc^j \omega_j - l(dB_2 + db^j \omega_j) \right] \quad (332)$$

Carrying out the integration over the Calabi-Yau, we obtain:

$$\int_{\mathcal{M}} -\frac{1}{2} e^{-2\hat{\phi}} \left(\hat{F}_3 \wedge \star \hat{F}_3 \right) = \int_{M_4} e^{-2\phi} \left(-2 * \mathcal{K} g_{ij} (dc^i - l db^i) \wedge \star (dc^j - l db^j) - \frac{1}{2}(dC_2 - l dB_2) \wedge \star (dC_2 - l dB_2) \right) \quad (333)$$

where $\mathcal{K}$ is the volume of the internal space, and g_{ij} is a metric defined on the Kähler moduli space:

$$g_{ij} \equiv \frac{1}{4\mathcal{K}} \int_{K_6} \omega_i \wedge \star \omega_j \quad (334)$$

The contribution of the terms in (68) that involve $\hat{A}_4$ is more difficult to obtain, since we need to impose the self-duality condition on $\hat{F}_5$.

This can be done by adding the following total derivative terms to the action:

$$\delta S = \frac{1}{2} F^A \wedge G_A + \frac{1}{2} dD^i \wedge d\rho_i \tag{335}$$

where $F^A \equiv dV^A$ and $G_A \equiv dU_A$. Adding this term to the action implements the self-duality condition as equations of motion for D_2^i and the vectors (again, for details see [43]). As promised, the self-duality condition has reduced the degrees of freedom in A_4 to the right amount needed for $\mathcal{N} = 2$ supergravity in four dimensions.

The compactification of the Ricci scalar contributes the following to the four-dimensional action:

$$\int_{K_6} -R^{(10)} \star \mathbf{1} \to -R^{(4)} \star \mathbf{1} - g_{ij} dv^i \wedge \star dv^j - g_{ab} dz^a \wedge \star d\bar{z}^b \tag{336}$$

where g_{ij} is the metric defined on the Kähler moduli space defined above, and g_{ab} is a metric defined on the space of complex structure deformations. We will examine the structure of these metrics in the following section.

The two 2-forms in the double-tensor multiplet can be dualized to two scalars, leaving behind a more familiar hypermultiplet. To see how this can be done, consider the following simplified action for a 2-form B_2:

$$S = -\int \left[\frac{g}{4} H_3 \wedge \star H_3 - \frac{1}{2} H_3 \wedge J_1 \right] \tag{337}$$

where J_1 is a generic 1-form and H_3 is the field strength for B_2, $H_3 = dB_2$. We can impose the latter condition as an equation of motion by adding a Lagrange multiplier term δS to the action:

$$\delta S = H_3 \wedge da \tag{338}$$

where a is a scalar field. The equation of motion for H_3 becomes:

$$\star H_3 = \frac{1}{g}(da + J_1) \tag{339}$$

which we can substitute back into (337) to obtain an action that depends only on a and J_1.

After dualizing the 2-forms to scalars, we are left with the following action in the four-dimensional effective theory:

$$\begin{aligned} S = \int_{M_4} \Big[& -\frac{1}{2} R \star \mathbf{1} - g_{ab} dz^a \wedge \star d\bar{z}^b \\ & - h_{uv} dq^u \wedge \star dq^v + \frac{1}{2} \mathrm{Im}\mathcal{M}_{IJ} F^I \wedge \star F^J + \frac{1}{2} \mathrm{Re}\mathcal{M}_{IJ} F^I \wedge F^J \Big] \end{aligned} \tag{340}$$

The definition of the matrix $\mathcal{M}_{IJ}$ can be found in the literature [43]. It relates to Abelian gauge fields and will be of no further concern to us. The matrix h_{uv} is likewise defined in [43]. It contains kinetic terms and scalar potential terms for the dilaton and the dual scalars of the 2-forms.

APPENDIX B: TYPE IIB EFFECTIVE ACTION ON ORIENTIFOLDS WITH FLUXES

In this Appendix, we show that the effective action of Type IIB on a Calabi-Yau orientifold with fluxes is a standard $\mathcal{N} = 1$ supergravity action. The action of a four-dimensional $\mathcal{N} = 1$ supergravity (with the field content described above) is determined by:

- A Kähler metric $K_{I\bar{J}} = \partial_I \bar{\partial}_{\bar{J}} K(M, \bar{M})$ defined on the manifold spanned by all the scalar fields in the theory. M stands for all of the scalar fields collectively.
- A superpotential $W(M)$, a function of the scalar fields
- Gauge field kinetic constants $f_{\kappa\lambda}$
- A set of D-terms D_α

The action is:

$$-\int \frac{1}{2} R \star 1 + K_{I\bar{J}} DM^I \wedge \star D\bar{M}^J + \frac{1}{2} \mathrm{Re} f_{\kappa\lambda} F^\kappa \wedge \star F^\lambda + \frac{1}{2} \mathrm{Im} f_{\kappa\lambda} F^\kappa \wedge F^\lambda + V \star 1 \tag{341}$$

where M^I denotes each complex scalar in the theory, and the D's are covariant derivatives (not to be confused with the *D-terms* D_α) defined by:

$$D_I W = \partial_I W + W \partial_I K \tag{342}$$

The scalar potential V is:

$$V = e^K (K^{I\bar{J}} D_I W \overline{D_J W} - 3|W|^2) + \frac{1}{2} \mathrm{Re}(f^{-1})^{\kappa\lambda} D_\kappa D_\lambda \tag{343}$$

We will not consider D-terms in the following.

Let us first examine the moduli space metrics which appear in (221). The metric on the complex structure moduli space was previously given by (165). After the orientifold projection, the holomorphic three-form Ω lies in $H^{(3)}_-$. This means that all the periods $(X^\kappa, \mathcal{F}_\kappa)$ which involve the basis of the even eigenspace $H^{(3)}_+$ vanish:

$$X^\kappa = \int_{K_6} \Omega \wedge \beta^\kappa = \mathcal{F}_\kappa = \int \Omega \wedge \alpha_\kappa = 0 \tag{344}$$

Therefore, Ω has the decomposition:

$$\Omega(z^k) = X^{\hat{k}} \alpha_{\hat{k}} - \mathcal{F}_{\hat{k}} \beta^{\hat{k}} \tag{345}$$

The metric g_{kl} remains Kähler with the following Kähler potential:

$$K_{cs} = -\ln\left[-i \int \Omega \wedge \bar{\Omega}\right] = -\ln\left[i \bar{X}^{\hat{k}} \mathcal{F}_{\hat{k}} - i X^{\hat{k}} \bar{\mathcal{F}}_{\hat{k}}\right] \tag{346}$$

The situation is different for the Kähler class deformations. The metric on this part of the moduli space appears twice in the effective action (221): once involving the $H_+^{(1,1)}$ indices $g_{\alpha\beta}$, and once involving the $H_-^{(1,1)}$ indices g_{ab}. The metric is given by:

$$g_{ij} = \frac{3}{2\mathcal{K}} \int_{K_6} \omega_i \wedge \star\omega_j = -\frac{3}{2}\left(\frac{\mathcal{K}_{ij}}{\mathcal{K}} - \frac{3}{2}\frac{\mathcal{K}_i\mathcal{K}_j}{\mathcal{K}^2}\right) \tag{347}$$

where the quantities $\mathcal{K}_{ij}$ and so forth are defined in (157). In the above equation we use again the convention that the indices ij of g_{ij} refer to elements of $H^{(1,1)}$ (that is, the full cohomology group and not one of its σ^* eigenspaces). The metric defined above differs from (160) by a rescaling convention. After the orientifold projection, all of the following quantities must vanish:

$$\mathcal{K}_{\alpha\beta c} = \mathcal{K}_{abc} = \mathcal{K}_{\alpha b} = \mathcal{K}_a = 0 \tag{348}$$

which may be checked by inspecting their symmetry properties under σ^* [1]. Substituting the above into (347), we arrive at:

$$g_{\alpha\beta} = -\frac{3}{2}\left(\frac{\mathcal{K}_{\alpha\beta}}{\mathcal{K}} - \frac{3}{2}\frac{\mathcal{K}_\alpha\mathcal{K}_\beta}{\mathcal{K}^2}\right), \quad g_{ab} = -\frac{3}{2}\frac{\mathcal{K}_{ab}}{\mathcal{K}} \tag{349}$$

and all the other components vanish: $g_{\alpha b} = g_{a\beta} = 0$. Unfortunately, this metric is not Kähler in the current coordinates. The correct set of Kähler coordinates is not easy to guess. They are defined by[24]:

$$\begin{aligned} G^a &= c^a - \tau b^a \\ \xi_\alpha &= -\frac{1}{2(\tau-\bar{\tau})}\mathcal{K}_{\alpha bc} G^b (G-\bar{G})^c \\ T_\alpha &= \frac{3i}{2}\rho_\alpha + \frac{3}{4}K_\alpha(v) - \frac{3}{2}\xi_\alpha(\tau,\bar{\tau},G,\bar{G}) \end{aligned}$$

In these coordinates, the Kähler deformation metric is Kähler, with the following Kähler potential:

$$K_k(\tau,T,G) = -\ln\left[-i(\tau-\bar{\tau})\right] - 2\ln\left[\frac{1}{6}\mathcal{K}(\tau,T,G)\right] \tag{350}$$

To be clear, the Kähler coordinates here (τ, T, G). The other quantities that appear in these formulae are functions of (τ, T, G). There is no general explicit solution for $\mathcal{K}$, but we can find one when $h_+^{(1,1)} = 1$ (i.e. only one Kähler modulus, $T_\alpha = T$). It reads [24]:

$$-2\ln\mathcal{K} = -3\ln\frac{2}{3}\left[T + \bar{T} - \frac{3i}{4(\tau-\bar{\tau})}\mathcal{K}_{1ab}(G-\bar{G})^a(G-\bar{G}^b)\right] \tag{351}$$

If we posit that $h_-^{(1,1)} = 0$, it reduces to the form:

$$K = -3\ln\left[T + \bar{T}\right] \tag{352}$$

1 $H_-^{(1,1)}$-forms ω_α change sign, the Kähler form J and $H_+^{(1,1)}$-forms ω_a are invariant.

This is the case of interest in the KKLT scheme to be discussed in the next chapter.

The Kähler potential (350) has a so-called *no-scale* structure:

$$\partial_I K \partial_{\bar{J}} K K^{IJ} = 3 \tag{353}$$

This means that the scalar potential will be positive semi-definite, $V \geqslant 0$, even in the presence of a flux-induced superpotential.

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