University of Groningen

Bachelor Research Project

Physics

Towards the detection of the primordial gravitational wave background

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Contents

1 Introduction

With the direct detection of gravitational waves by aLIGO [1] [2], the prediction [3] of these waves by Albert Einstein, after 100 years, was finally confirmed. In addition to providing the beautiful theory of General Relativity (GR) with yet another verification, the direct detection of gravitational waves opens up a new realm of physics as well. Where electromagnetic waves are confined by the limit imposed by the last-scattering surface[1] [4], gravitational waves are not. Gravitational waves can therefore give insight into phenomena of the very early universe[2], such as cosmic inflation and cosmic strings, which are predicted to have left behind a gravitational wave background. The detection of this background signal will reveal the possible existence and the larger than GUT energy-scale[3] physics of these processes. This paper will investigate what type of gravitational wave background is predicted to be left behind by these phenomena and how experiments such as aLIGO [5] and eLISA [6] can detect these backgrounds.

In order to investigate the detection of the primordial gravitational wave background this paper will have the following structure. First, the theory underlying gravitational waves will be outlined in section 2. After that, section 3 and 4 will describe the phenomena producing the primordial gravitational wave background and the gravitational wave imprint of these phenomena. Knowing the characteristics of these gravitational waves, section 5 will summarize the different detectors. Using the above, section 6 will show what steps have to be taken to detect the gravitational wave background, which is the aim of this paper. Now first the theory of gravitational waves will be outlined.

2 Gravitational waves

As mentioned above, Albert Einstein predicted gravitational waves as early as 1916 [3], one year after the publication of his field equations [7]. Before exploring the possibilities of detecting gravitational waves from early universe phenomena, the theory underlying these waves will be summarized. A logical starting point for this are these Einstein field equations (EFE)

$$G^{\mu\nu} + \Lambda g^{\mu\nu} = -\frac{8\pi G}{c^4} T^{\mu\nu}. \tag{2.1}$$

Where $G^{\mu\nu}$ denotes the Einstein tensor, which is a measure of the curvature of spacetime, $g^{\mu\nu}$ denotes the metric tensor, which is the form of spacetime itself, $T^{\mu\nu}$ denotes the energy-momentum tensor, c denotes the speed of light and G denotes the gravitational constant. Greek indices such as μ and ν run from 0 to 3. Finally, Λ is the cosmological constant initially introduced by Einstein [8] as an ad hoc modification of his equations to impose a static universe. However, after the discovery of the expansion of the universe by Hubble [9] the cosmological constant survived, now taking the role of the curvature of empty space[4].

[1] The last-scattering surface is at a redshift of $z \backsim 1000$.

[2] Up to the Planck time, $t_p \backsim 10^{-43}$ s

[3] $E \simeq 10^{16}$ GeV

[4] Note that a particle physicist might want to put the cosmological constant to the other side of the EFE. The cosmological constant then takes the role of the vacuum expectation value of $T^{\mu\nu}$.

The EFE thus state that mass and energy induce spacetime curvature, spacetime curvature then implies changes in mass and energy densities and so forth. The '=' sign therefore has a real physical meaning instead of simply equating two things.

The EFE are a summary of Einstein's theory of gravity, which states that gravity is a consequence of spacetime being globally curved. Locally spacetime is flat and a particle follows a geodesic, it is the global curvature of spacetime that makes a particle experience gravitational acceleration. The fact that spacetime is locally flat is shown by Einstein's famous lift experiment [4], which shows that one does not feel gravity in free fall. This thought experiment is shown in figure 1 below.

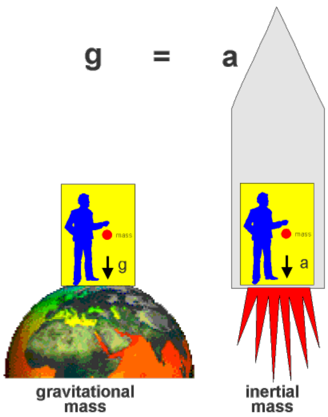

Figure 1: The Einstein Lift Experiment

The Einstein lift experiment not only shows that locally spacetime is flat, it shows that gravity just is an ordinary acceleration field as well. Such an accelerating frame of reference can be analyzed in infinitesimal steps using the framework of Special Relativity (SR) [10]. Spacetime curvature then is needed to connect all these infinitesimal slices of flat spacetime. This connection is mathematically made by the Christoffel symbols which are known as the affine connection as well, they are given by

$$\Gamma^{\alpha}_{\lambda\mu} = \frac{1}{2} g^{\alpha\nu} \left(\frac{\partial g_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial g_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial g_{\mu\lambda}}{\partial x^{\nu}} \right). \tag{2.2}$$

The changes in these connections make up the Riemann tensor $R^{\mu}_{\alpha\beta\gamma}$ given by

$$R^{\mu}_{\alpha\beta\gamma} = \frac{\partial \Gamma^{\mu}_{\alpha\gamma}}{\partial x^{\beta}} - \frac{\partial \Gamma^{\mu}_{\alpha\beta}}{\partial x^{\gamma}} + \Gamma^{\mu}_{\sigma\beta}\Gamma^{\sigma}_{\gamma\alpha} + \Gamma^{\mu}_{\sigma\gamma}\Gamma^{\sigma}_{\beta\alpha}. \tag{2.3}$$

The contracted forms[5] of the Riemann tensor show up in the EFE through the Einstein tensor, which is defined as

$$G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R. \tag{2.4}$$

[5] These contracted forms are the Ricci tensor, $R_{\alpha\beta} = R^{\mu}_{\alpha\beta\mu}$ and the curvature scalar, $R = R^{\mu}_{\mu} = g^{\mu\nu} R_{\mu\nu}$.

The metric thus is central to GR, it not only defines the structure of spacetime itself, it describes how particles move as well. Taking the weak field limit of these EFE gives rise to gravitational waves.

2.1 Weak field limit

Taking the weak field limit of the EFE means that there is little spacetime curvature. Following Peacock [4] and using perturbation theory in taking this limit, the metric takes the following form

$$g^{\mu\nu} = \eta^{\mu\nu} + h^{\mu\nu}, \qquad |h^{\mu\nu}| \ll 1. \tag{2.5}$$

This is the so called linearized theory of gravity [10], where $\eta^{\mu\nu}$ is the Minkowski metric of the flat spacetime of SR and $h^{\mu\nu}$ is a small perturbation yielding a small spacetime curvature. Taking the weak limit means that only first-order perturbation theory has to be considered. The merit of using first-order perturbation theory is that in manipulating indices of $h^{\mu\nu}$ the Minkowski metric $\eta^{\mu\nu}$ can be used instead of the general metric $g^{\mu\nu}$. This means that the scalar perturbation becomes

$$h = g_{\mu\nu}h^{\mu\nu} = \eta_{\mu\nu}h^{\mu\nu} = h^{00} - (h^{11} + h^{22} + h^{33}). \tag{2.6}$$

This allows for the definition of a new field

$$\bar{h}^{\mu\nu} \equiv h^{\mu\nu} - \tfrac{1}{2}\eta^{\mu\nu}h. \tag{2.7}$$

Furthermore, the affine connection and the Ricci tensor are now given by

$$\Gamma^{\alpha}_{\lambda\mu} = \frac{1}{2}\eta^{\alpha\nu}\left(\frac{\partial h_{\mu\nu}}{\partial x^{\lambda}} + \frac{\partial h_{\lambda\nu}}{\partial x^{\mu}} - \frac{\partial h_{\mu\lambda}}{\partial x^{\nu}}\right) \tag{2.8}$$

$$R_{\mu\nu} = \frac{\partial \Gamma^{\beta}_{\mu\beta}}{\partial x^{\nu}} - \frac{\partial \Gamma^{\beta}_{\mu\nu}}{\partial x^{\beta}}, \tag{2.9}$$

which allows the EFE to be written as the following wave equation

$$\Box \bar{h}^{\mu\nu} = -\frac{16\pi G}{c^4}T^{\mu\nu}. \tag{2.10}$$

To arrive at this wave equation a gauge condition analogous to the Lorentz gauge of electromagnetism is needed

$$\frac{\partial \bar{h}^{\mu\nu}}{\partial x^{\mu}} \equiv \bar{h}^{\mu\nu}_{\ \ ,\mu} = 0. \tag{2.11}$$

This Lorentz condition is needed due to the fact that the EFE, which are symmetric, have 10 independent equations for the 10 independent components of the metric $g^{\mu\nu}$, which is symmetric as well. With spacetime having 4 degrees of freedom this means that there are 6 excess, unphysical degrees of freedom. Introducing this gauge condition, which can be written (since we are dealing with weak fields) as the harmonic or Lorentz gauge as well

$$g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu} = 0, \tag{2.12}$$

fixes the 4 degrees of freedom associated with coordinate transformations. Leaving 2 initial conditions and 4 physical degrees of freedom. This allows for the wave equation form of the EFE. However since (2.11) is preserved under Lorentz transformations different gauges can be chosen. The gauge chosen for the purpose of gravitational waves is the Transverse Traceless gauge.

2.2 Transverse Traceless gauge

The Transverse Traceless (TT) gauge is analogous to the Coulomb gauge giving rise to electromagnetic radiation in electrodynamics [11]. In GR the TT gauge gives rise to gravitational radiation or gravitational waves. The definition of the TT gauge is

$$h_{\mu 0} = 0, \qquad h^{\mu}_{\mu} = 0 \tag{2.13}$$

therefore, $h^{\mu\nu} = \bar{h}^{\mu\nu}$. $\bar{h}^{\mu\nu}$ can now, using the TT gauge, be written as

$$\bar{h}^{\mu\nu} = A^{\mu\nu} e^{ik^{\sigma}x_{\sigma}}. \tag{2.14}$$

This clearly is a wave equation with amplitude $A^{\mu\nu}$ and wave number k^{σ}. The amplitude and wave number obey the following equations

$$k^{\sigma}k_{\sigma} = 0 \tag{2.15}$$

$$k^{\mu}A_{\mu\nu} = 0. \tag{2.16}$$

The first equation means that gravitational waves travel with the speed of light and the second means that since $A_{00} = 0$ in this gauge, gravitational waves are spatially transverse. Having shown that gravitational waves indeed occur in the weak field limit of GR, the physical characteristics of gravitational waves will be explored.

2.3 Characteristics of gravitational waves

Until now, the description of gravitational waves has been purely mathematical. However, this paper investigates the physical detection of gravitational waves, it is therefore important to know how an observer experiences gravitational waves. An observer experiences a gravitational wave as a relative acceleration with respect to another observer that cannot be removed by a change of coordinates, a so-called a tidal force [12]. These tidal forces are summarized by the tidal tensor

$$\Delta_{\mu\nu} \equiv R_{\mu\alpha\beta\nu}U^{\alpha}U^{\beta} \tag{2.17}$$

with U^{α} denoting four-velocity. Still using first-order perturbation theory the Riemann tensor in (2.17) becomes

$$R_{\mu\alpha\beta\nu} = \frac{1}{2}(-\partial_{\mu}\partial_{\alpha}h_{\nu\beta} - \partial_{\nu}\partial_{\beta}h_{\mu\alpha} + \partial_{\mu}\partial_{\beta}h_{\nu\alpha} + \partial_{\nu}\partial_{\alpha}h_{\mu\beta}). \tag{2.18}$$

In the rest frame, where $U^{\alpha} = (1, 0, 0, 0)$, the tidal tensor becomes

$$\Delta_{\mu\nu} = \frac{1}{2}\frac{\partial^2 h_{\mu\nu}}{\partial t^2}. \tag{2.19}$$

Remembering that in using the TT gauge $h^{\mu\nu} = \bar{h}^{\mu\nu}$ and that the gravitational waves are spatially transverse, a wave travelling in the $\hat{z}$-direction gives the following tidal tensor in the rest frame of an observer

$$\Delta_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \Delta_{+} & \Delta_{\times} & 0 \\ 0 & \Delta_{\times} & -\Delta_{+} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} e^{-ik^{\sigma}x_{\sigma}}. \tag{2.20}$$

A gravitational wave thus has two nodes Δ_+ and $\Delta_\times$. This is due to the quadrupole nature of gravitational waves, which is needed to have conservation of momentum, making a gravitational analogue of dipole radiation impossible. This quadrupole behavior is shown in figure 2 below.

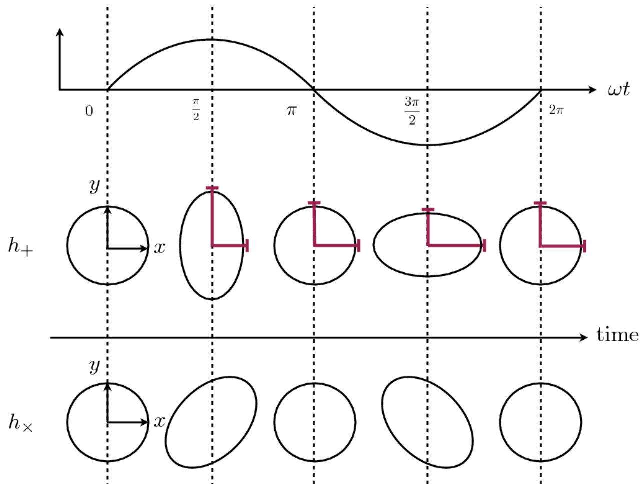

Figure 2: Quadrupole behaviour of a gravitational wave

The fact that gravitational waves are able to distort spacetime at a distance from their source through this tidal effect means they must carry energy, which must show up in the EFE. However the energy-momentum tensor of matter vanishes for vacuum radiation, therefore the EFE reduce to

$$G^{\mu\nu} = 0. \tag{2.21}$$

The energy of gravitational waves must thus be hidden within the Einstein tensor. To uncover this energy the Einstein tensor must be split into two parts. One part corresponds to the large-scale properties of spacetime, these background effects being on a much larger scale than the wavelength of gravitational waves. The other part corresponds to the energy-momentum tensor of the gravitational waves themselves. Since the linear Ricci tensor vanishes in vacuum, second order perturbation theory is needed here. The Einstein tensor is split in the following way

$$G_{\mu\nu} = G^{\text{BG}}_{\mu\nu} + G^{(2)}_{\mu\nu} \tag{2.22}$$

where $G^{\text{BG}}_{\mu\nu}$ denotes the contribution of the background to the Einstein tensor and $G^{(2)}_{\mu\nu}$ denotes the second order contribution the waves give to the Einstein tensor. The energy-momentum tensor for gravitational waves then becomes

$$T^{\text{GW}}_{\mu\nu} = \frac{c^4}{8\pi G} G^{(2)}_{\mu\nu}. \tag{2.23}$$

However, since gravitational waves obviously are a wave phenomenon, a time average of the energy density should be taken. This is the Landau-Lifshitz pseudotensor [13], which is analogous to the Poynting vector of electromagnetism [11]. In the TT gauge this pseudotensor is given by

$$T^{\text{GW}}_{\mu\nu} = \frac{c^4}{32\pi G} \left\langle \bar{h}_{\alpha\beta,\mu} \bar{h}^{\alpha\beta}_{,\nu} \right\rangle \tag{2.24}$$

where $\langle \quad \rangle$ denotes taking the time average. This energy-momentum carried by a gravitational wave will ultimately cause a gravitational strain on earth. Gravitational strain is the "fractional distortion in the length of objects induced by the fluctuating gravitational acceleration" [4]. Considering a test particle experiencing a plane polarized gravitational wave of frequency ω with equation of motion

$$\ddot{x}_i = \Delta_{ij} x_j. \tag{2.25}$$

Where $\ddot{x}$ is the second derivative with respect to time of x and a characteristic frequency for an object with $GM/c^2 r \sim 1$ [6] is

$$\omega \sim \frac{c^3}{GM} = \left(\frac{M}{10^{5.3} M_{\odot}} \right)^{-1} \text{Hz}. \tag{2.26}$$

The spatial strain corresponding to this motion is obtained by supposing only one mode is present. Since $\bar{h}_{\mu\nu} \propto \text{diag}(0, h_+/2, -h_+/2, 0)$ if only the Δ_+ node is considered, the tidal tensor becomes

$$\Delta_{\mu\nu} = \text{diag}(0, \ddot{h}_+/2, -\ddot{h}_+/2, 0) \equiv \text{diag}(0, \Delta_+, -\Delta_+, 0). \tag{2.27}$$

Now integrating (2.25) twice the amplitude of the spatial oscillation Δx is found to be

$$\left| \frac{\Delta x}{x} \right| = \frac{\Delta_+}{\omega^2} = \frac{h_+}{2}. \tag{2.28}$$

Using the same treatment for the mode $\Delta_\times$, the total strain h can be defined as

$$h \equiv \sqrt{h_+^2 + h_\times^2}. \tag{2.29}$$

Remembering that the total energy flux density of gravitational waves is

$$\rho^{GW} = T_{01}^{GW} = \frac{1}{32\pi G} \left\langle \bar{h}_{\alpha\beta,0} \bar{h}^{\alpha\beta}_{,1} \right\rangle = \frac{\omega^2}{32\pi G} h^2. \tag{2.30}$$

The total strain h is given by

$$h = \sqrt{\frac{32\pi G T_{01}}{c^3 \omega^2}}, \tag{2.31}$$

this total strain corresponds to the amplitude of gravitational waves. It is important to note that $h \propto \omega^{-1}$. The total strain h is what current and future detectors can measure, with aLIGO being the first and only detector in successfully doing so up until now. Before looking at these detectors the phenomena creating the primordial gravitational wave background will now be outlined, starting with inflation below.

3 Inflation

Inflation is a very powerful idea first proposed by Guth in 1981 [14] to solve the horizon and flatness problems first noted by Dicke in 1961 [15]. Inflation is the rapid "faster than light" expansion of the early universe that solves both of

[6] An object of mass M which has approximately the size, r, of its own Schwarzschild radius.

these problems. Although inflation has so far not been verified experimentally, the general consensus is that inflation did happen. Detection of gravitational waves left behind by inflation can therefore finally give inflation the experimental confirmation it still needs. Before looking at the gravitational wave imprint of inflation, the idea of inflation itself will be outlined, starting with the two problems it solves.

3.1 Horizon problem

When an observer looks at the Cosmic Microwave Background (CMB) sky[7] he or she sees, except for some anisotropies [18] of order $\frac{\delta T}{T} \backsim 10^{-5}$, a completely homogeneous universe. However, the CMB sky consists of a large number of causally disconnected regions that have not been able to communicate with each other since shortly after the big bang. The fact that these regions are homogeneous without being able to communicate this homogeneity in a causal manner is contradictory. This contradiction is called the horizon problem.

3.2 Flatness problem

In addition to the horizon problem the fact that the universe is almost flat today gives rise to another problem. If the universe is nearly flat today, this implies that it was finetuned to great precision at the moment that its initial conditions were set. Assuming the initial conditions of the universe were set at the Planck time $t_{\rm p} \sim 10^{-43}s$, the natural time unit of the universe, the Universe had to be finetuned to 1 part in 10^{60}, which is highly unlikely.

If the universe were to be exactly flat this finetuning is not necessary. However, the question then arises how the universe came to be exactly flat in the first place. There was thus either a mechanism that made the universe exactly flat in the first place or the universe was to know beforehand that it had to finetune itself up to a truly enormous precision. These two improbabilites together make up the flatness problem.

3.3 The inflationary universe

As mentioned above a period of rapid expansion in the early universe solves both these problems. If the size of the universe R expanded “faster than light” in the early universe ($R \propto t^{\alpha}$, $\alpha > 1$) this means that regions once causally connected, and thus able to communicate homogeneity, became causally disconnected. Hence inflation creates a universe that is homogeneous and has a large number of causally disconnected regions, therefore solving the horizon problem.

Furthermore, looking at the Friedmann equation [19] it can be seen how inflation solves the flatness problem. The Friedmann equation is the solution to the EFE for a homogeneous and isotropic universe and is given by

$$\dot{R}^2 = \frac{8\pi G\rho R^2}{3} - kc^2 \tag{3.1}$$

[7]The CMB is a black body radiation spectrum, first discovered by Penzias and Wilson in 1965 [16] with a temperature $T = 2.725$ K [17].

where ρ denotes the density of the universe and k refers to the topology of the universe. k can take the values -1, 0 and $+1$ corresponding to subsequently either a closed, flat or open universe. Due to the fact that during a rapid expansion like inflation matter and radiation are redshifted away, the universe becomes dominated by vacuum energy. The vacuum energy density ρ_{vac} is constant with respect to the size of the universe ($\rho_{\text{vac}} \propto R^0$), therefore the ρR^2 term can become arbitrarily large compared to the $-kc^2$ term. Inflation thus can make the curvature term as small as desired, solving the flatness problem [4].

For an inflationary phase to solve both these problems, the equation of state must be one with a negative pressure

$$\rho c^2 + 3p < 0, \tag{3.2}$$

where p denotes pressure. An equation of state that achieves a negative pressure is the equation of state of vacuum energy

$$p = -\rho c^2. \tag{3.3}$$

Now remembering that $\rho_{\text{vac}} \propto R^0$ and inflation makes the $-kc^2$ term negligible, the Friedmann equation becomes

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \rho_{\text{vac}}}{3}. \tag{3.4}$$

Solving this equation gives rise to the following equation for R

$$R \propto e^{Ht}, \quad H = \sqrt{\frac{8\pi G \rho_{\text{vac}}}{3}}. \tag{3.5}$$

This exponential, "faster than light" expansion means that the universe was in a de Sitter phase [20] during inflation. The universe being in a de Sitter phase had the following event horizon

$$r_{\text{EH}} = \int_{t_0}^{\infty} \frac{c\,dt}{R(t)} = \frac{c}{R_0 H} \tag{3.6}$$

where R_0 is the current size of the universe. The universe, like a black hole, thus had a finite event horizon at the time of inflation. This finite event horizon implies that the universe, like black holes, produced Hawking radiation [21] during its period of inflation. These thermal fluctuations of a de Sitter space are given by

$$kT_{\text{deSitter}} = \frac{\hbar H}{2\pi} \tag{3.7}$$

with k now denoting Boltzmann's constant. The quantum fluctuations originating from this Hawking radiation will exist in all fields, including gravitational waves. Inflation therefore leaves behind a gravitational wave background.

Besides an equation of state that has a negative pressure, a mechanism that allows the universe to exit its inflationary phase is needed as well. A cosmological phase transition to a state of close to zero vacuum energy will achieve this exit and reheat the universe in doing so [22]. Such a cosmological phase transition can be realised by quantum fields, especially scalar fields are a good canditate for doing so. Such a scalar field, which is called the inflaton and is denoted by ϕ has the following Lagrangian density

$$\mathcal{L} = \tfrac{1}{2}\partial_\mu \partial^\mu \phi - V(\phi) \tag{3.8}$$

where $V(\phi)$ denotes a general potential depending on the type of inflation. Using Noether's theorem [23] the following energy-momentum tensor is arrived at

$$T^{\mu\nu} = \partial^\mu \phi \, \partial^\nu \phi - g^{\mu\nu}\mathcal{L}. \tag{3.9}$$

From this energy-momentum tensor expressions for both density and pressure can be derived

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + V(\phi) + \tfrac{1}{2}(\nabla\phi)^2 \tag{3.10}$$

$$p = \tfrac{1}{2}\dot{\phi}^2 - V(\phi) - \tfrac{1}{6}(\nabla\phi)^2. \tag{3.11}$$

If both the $\dot{\phi}$ term and the $\nabla\phi$ term vanish due to the inflaton being spatially and temporally constant the equation of state becomes one with negative pressure since $p = -\rho$. Besides this negative pressure a mechanism that ends inflation and reheats the universe in doing so is needed as well. This mechanism is achieved by the following potential

$$V(\phi, T) = a|\phi|^4 - b|\phi|^3 + cT^2|\phi|^2. \tag{3.12}$$

This potential is shown in figure 3 below.

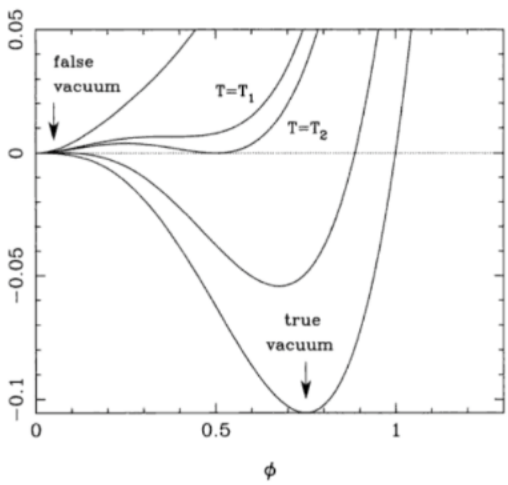

Figure 3: Plot of $V(\phi, T)$ for different temperatures, where $T_1 > T_2$.

From this figure it can be clearly seen that $V(\phi, T)$ (for certain temperatures) has both a false and a true vacuum. The universe initially being in the false vacuum[8] during the beginning of inflation, expands rapidly and redshifts away

[8]Assuming that the cosmological phase transition happens at GUT-scale energies this vacuum has an energy density of $\rho_{\text{vac}} = \dfrac{(10^{15}\text{GeV})^4}{\hbar^3 c^5} \simeq 10^{80}\text{kg m}^{-3}$.

matter and radiation, making the temperature of the universe drop. As the temperature of the universe T drops the barrier between the false vacuum and the true vacuum becomes smaller. This smaller barrier allows the inflaton ϕ to quantum tunnel through this barrier and to start to roll down the potential $V(\phi, T)$ towards the true vacuum. The inflaton will roll down the potential slowly due to the fact that $\dot{\phi}$ and $\nabla\phi$ have to be small for inflation to happen in the first place.

Instead of inflation ending when ϕ reaches the bottom of the potential, ϕ overshoots the minimum of the potential. The inflaton will therefore oscillate towards the minimum, in doing so slowly terminating inflation. Due to the coupling of the scalar field to matter fields, the vacuum energy the inflaton loses is converted into particles during this oscillatory phase. It is the creation of these particles that reheats our universe. Furthermore it is during these final stages of inflation that different regions of the universe inflate on points of the potential $V(\phi, T)$ that are perturbed by $\delta\phi$ due to the quantum fluctuations mentioned above. These perturbations

$$\delta\phi = \frac{\delta\phi}{\delta t}\delta t = \dot{\phi}\,\delta t \tag{3.13}$$

will induce a horizon-scale amplitude by the end of inflation of

$$\delta_{\mathrm{H}} \simeq H\delta t = \frac{H^2}{2\pi\dot{\phi}}. \tag{3.14}$$

For the last step of (3.14) quantum field theory is used to find the root mean square (rms) value of $\delta\phi$ to be $\frac{H}{2\pi}$. This horizon-scale amplitude yields the upper limit for gravitational waves from inflation.

3.4 Gravitational wave imprint from inflation

As mentioned above quantum fluctuations induced by Hawking radiation will in addition to inducing a horizon-scale amplitude δ_{H} lead to a constant background of gravitational waves today. The spectrum of these waves has a break between metric fluctuations observable via CMB anisotropies on super-horizon scales and density fluctuations on small scales. These metric fluctuations can be measured by observing the tensor-to-scalar ratio with experiments such as BICEP2 [24], Planck [25] and WMAP [26]. It is the density fluctuations on small scales that can be directly detected as gravitational waves.

The linearized contribution of these gravitational waves to the Lagrangian is denoted by $h_{\mu\nu}$ and looks like a scalar field Φ given by

$$\Phi = \frac{m_p}{4\sqrt{\pi}}h_{\mu\nu}, \tag{3.15}$$

where m_p denotes the Planck mass [9]. Therefore the expected rms gravity-wave amplitude is given by

$$h_{\mathrm{rms}} \sim \frac{H}{m_p}. \tag{3.16}$$

[9] The natural mass unit of the universe, $m_p \simeq 10^{19}$ GeV.

This rms gravity-wave amplitude is limited by the horizon scale amplitude

$$\left(\frac{\delta T}{T}\right)_{\rm GW} \sim h_{\rm rms} \lesssim \delta_{\rm H} \sim 10^{-5} \tag{3.17}$$

due to the fact that gravitational waves can never induce more than 100% of the measured CMB anisotropies. This rms gravity wave amplitude leads to the following energy-density for the gravitational waves left behind by inflation

$$\rho_{\rm GW} \sim m_p^2 h_{\rm rms}^2 H^2 \tag{3.18}$$

Now introducing the density parameter Ω which is given by

$$\Omega = \frac{8\pi\rho}{3H^2 m_p^2}, \tag{3.19}$$

it can be clearly seen that $\Omega_{\rm GW} \sim h_{\rm rms}^2$. At the time of birth of these gravitational waves, the universe was radiation dominated ($\Omega_r = 1$). Since gravitational waves and radiation are both waves and therefore redshift in the same way $\Omega_{\rm GW} \sim \Omega_r h_{\rm rms}$. Plugging in values for Ω_r and $h_{\rm rms}$ today the density parameter expected for inflation is $\Omega_{\rm GW} \sim 10^{-14}$.

From (2.30) and (3.19) the following expression for $\Omega_{\rm GW}$ can be derived as well

$$\Omega_{\rm GW} = \frac{\omega^2 h^2}{12 H_0^2}, \tag{3.20}$$

where H_0 denotes the current value of $\frac{\dot{R}}{R}$. The characteristic strain is thus given by

$$h \simeq \sqrt{\frac{H_0^2 \Omega_{\rm GW}}{\omega^2}}. \tag{3.21}$$

Plugging in all the values in to this equation gives the following characteristic strain for kHz waves from inflation

$$h_{\rm inf} \sim 10^{-27.5}. \tag{3.22}$$

This is the strain detectors need to measure in order to verify inflation. Before looking whether detectors can achieve the detection of such a signal another source of primordial gravitational waves will be looked at: cosmic strings.

4 Cosmic strings

Cosmic strings are a type of topological defect that might arise from discontinuities created by phase transitions in the early universe. Due to the size of the horizon at the time of these phase transitions, different regions within the universe might not be able to causally communicate their field configurations with each other. As these regions come into causal contact with each other, their field configurations will try to align. However, some field configurations might fail to align, leaving behind topological defects. The creation of topological defects in this way is known as the Kibble mechanism [27].

This mechanism might create cosmic strings which have a very large tension leading to the following energy-momentum tensor

$$T^{\mu\nu} = \text{diag}(\rho, -\rho, 0, 0). \tag{4.1}$$

Inserting this energy-momentum tensor into the EFE gives rise to the following metric as a solution

$$d\tau^2 = dt^2 - dz^2 - dr^2 - r^2(1 - \epsilon/\pi)d\theta^2 \tag{4.2}$$

with ϵ being the deficit angle and the string extending in the z direction. This metric is a simple flat cylindrical Minkowski metric where a slice of angle ϵ has been taken out. The spacetime is then essentially 'glued back to together' forming a cosmic string. The deficit angle ϵ is given by

$$\epsilon = \frac{8\pi G\mu}{c^2} \tag{4.3}$$

where μ denotes the string parameter. This string parameter is of the following order of magnitude

$$\frac{G\mu}{c^2} \sim \left(\frac{m}{m_p}\right)^2 = \left(\frac{E}{E_p}\right)^2 \tag{4.4}$$

where m is the characteristic mass of the phase transition the cosmic strings were created in. Due to the fact that cosmic strings can intercommute and form string loops, a network of cosmic strings can be formed [28]. The way in which cosmic strings intercommute is shown in figure 4.

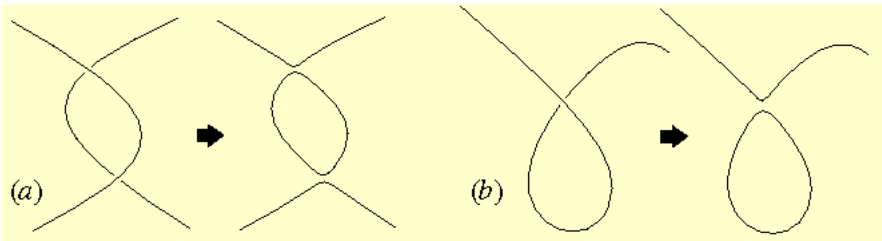

Figure 4: The way in which cosmic strings intercommute

Assuming this network of strings evolves until there is one cosmic string left of the size of the horizon at a given time, the magnitude of the density fluctuations produced by strings can then be deduced. These are of the following magnitude

$$\frac{\delta\rho}{\rho} \sim 30\frac{G\mu}{c^2}. \tag{4.5}$$

These density fluctuations, like the density fluctuations produced by inflation, give rise to primordial gravitational waves as well. The characteristic density of the gravitational waves produced by cosmic strings is

$$\Omega_{\text{GW}} \sim 100\frac{G\mu}{c^2}\Omega_r. \tag{4.6}$$

Now plugging in the current day value of Ω_r and assuming cosmic strings were created during the GUT phase transition, $\Omega_{\text{GW}} \sim 10^{-7}$. Using (3.21) the characteristic strain of cosmic strings for kHz gravitational waves is of the order

$$h_{\text{CS}} \sim 10^{-24}. \tag{4.7}$$

This is the characteristic strain detectors need to measure in order to verify the existence of cosmic strings. It is important to note that $h_{\text{CS}} \gg h_{\text{inf}}$. Now knowing the characteristic strains of both inflation and cosmic strings the outline of current and planned detectors will be described.

5 Detectors

As mentioned above aLIGO is the first and so far the only detector that has directly measured gravitational waves. However, there are many different detectors currently trying to measure gravitational waves or planned to measure gravitational waves in the future [29]. Among other things these detectors vary in their sensitivity, frequency range they operate in, what they aim to measure and how they operate. The sensitivity and frequency range of different detectors is shown in figure 5 below.

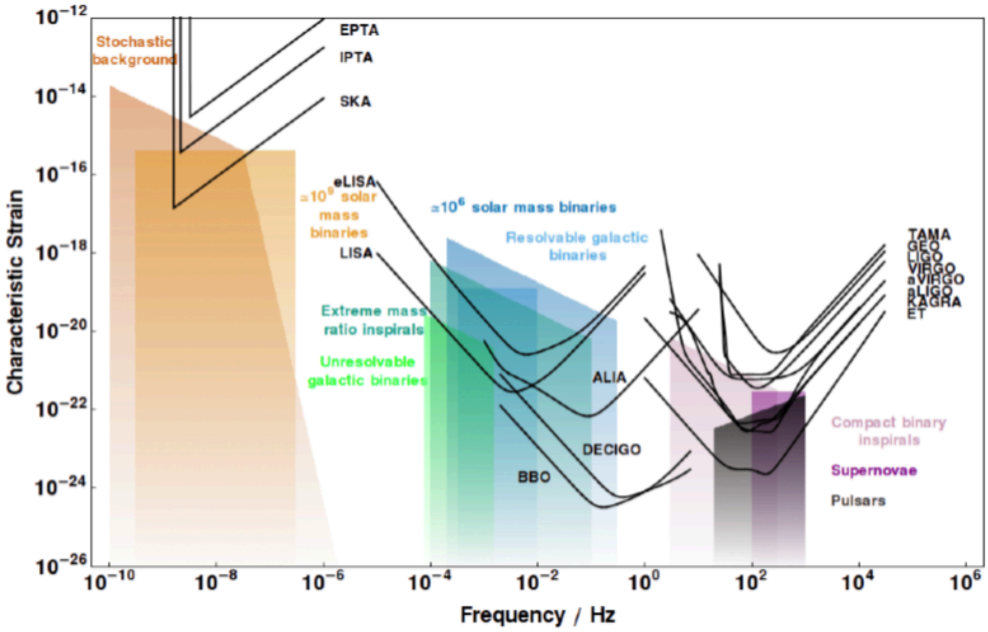

Figure 5: Different detectors with the sensitivity they can measure in the frequency range in which they operate.

From this figure it can be seen that there are currently 3 different groups of detectors. These are the ground-based interferometers, space-based interferometers and Pulsar Timing Arrays (PTAs). Due to the low frequencies PTAs look at, they are only capable of measuring characteristic strains of the order of

$10^{-16} - 10^{-18}$. They are thus unlikely to measure either inflation ($h \sim 10^{-27.5}$) or cosmic strings ($h \sim 10^{-24}$) in the near future. Therefore this paper will focus on both ground-based and space-based interferometers. These two types of interferometers are based on the Michelson interferometer [30], which creates an interference pattern by splitting a light beam and then reflecting these split light beams back towards the source. These interference patterns can be used to very precisely measure the movement of a test mass. If this test mass is distorted by a passing gravitational wave this will therefore show up in the interference pattern. This is the principle that both space-based and ground-based interferometers rely on. Having outlined this principle, now first the ground-based interferometers will be elaborated on.

5.1 Ground-based interferometers

Since earlier efforts by e.g. Weber [31] and Weiss & Block [32] failed to detect gravitational waves, ground-based interferometers, which surpassed the sensitivities of these experiments in 2003 [33], became the most important prospect to directly measure gravitational waves. Whereas the first generation of these detectors provided important test runs for the future development of ground-based interferometers. It was the second generation detector aLIGO that managed to achieve direct detection of gravitational waves for the first time.

5.1.1 aLIGO

The advanced laser interferometer gravitational wave observatory, known as aLIGO, consists of two L-shaped detectors. This L-shape is needed to verify the quadrupole nature of the measured gravitational wave. Both detectors use a Michelson Interferometer with 4 km long Fabry-Perot [34] cavity arms and power recycling mirrors. This allows aLIGO to look for gravitational waves with frequencies ranging from 10 Hz to up to several kHz with a characteristic strain of up to $h \sim 10^{-22}$ [33]. A schematic overview of this detector setup is shown in figure 6 below.

These features allowed aLIGO to unambigously[10] measure gravitational waves from 2 binary black hole (BBH) mergers during its first run from September 12, 2015 to January 19, 2016 [35]. Furthermore, it recorded another signal at a lower significance ($\lesssim 2\sigma$), that has a larger chance of being a BBH gravitational wave than being signal noise as well. This promising first run was the start of the field of gravitational wave astronomy, which will continue to grow as planned detectors will be commisioned in the future.

5.1.2 Future ground-based interferometers

Next to aLIGO, other ground-based interferometers will start measuring gravitational waves in the near future. First of all, the second part of the LIGO VIRGO collaboration advanced Virgo (AdV) is being tested and will start datataking alongside aLIGO in 2016 [36]. In addition to AdV another LIGO detector, IndIGO [37], will be built in India adding to this collaboration. These additional

[10]With a significance larger than 5σ

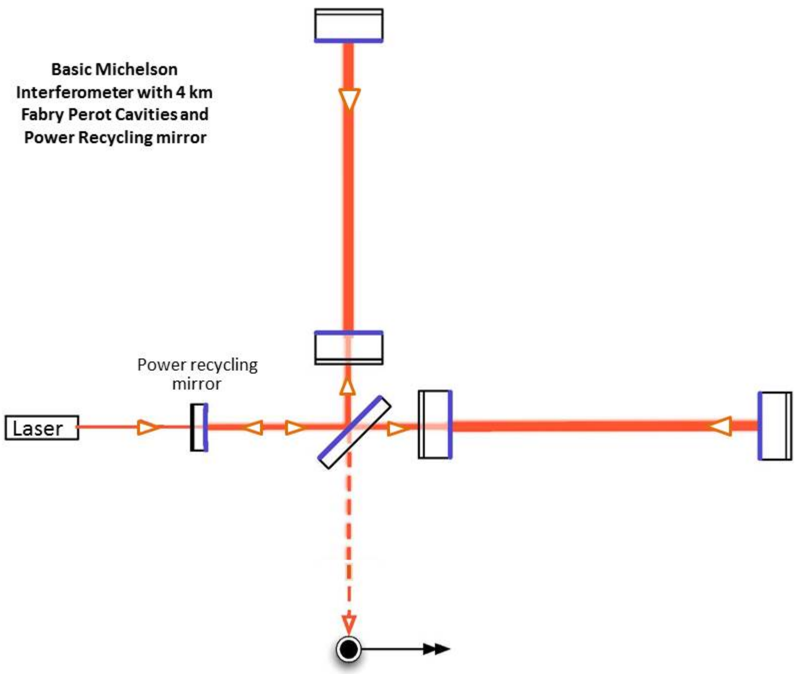

Figure 6: The setup of aLIGO

detectors will allow for better identification of BBH events, however for an increase in sensitivity a new generation of ground-based interferometers is needed.

These third generation ground-based interferometers such as LIGO Voyager [38] and the Einstein Telescope (ET) [39] will provide a significant increase in the sensitivity for direct gravitational wave measurements. These detectors are currently being planned and set up, with the ET planned to be operational in 2025 and the LIGO Voyager in 2027 or 2028. Now another type of planned gravitational wave detectors will be looked at: space-based interferometers.

5.2 Space-based interferometers

Whereas ground-based interferometers are impeded by the constraints given by the earth, e.g. size and seismic noise, space-based observatories are not. Therefore, they will be able to measure lower frequency waves at a higher sensitivity than current detectors. Currently, there are multiple detectors being planned such as ALIA, BBO and DECIGO [29]. However, presently there is only one detector for which prototypes are being tested and which therefore will be operational in the near future, the evolving laser interferometer space antenna known as eLISA.

5.2.1 eLISA

The eLISA detector will be a detector consisting of 3 spacecrafts that together will make a V-shaped form with arms of 10^9 m (descoped from the originally envisioned 5×10^9 m) at an angle of 60°. This V-shaped detector will follow the earth in its orbit at a distance of up to 7×10^{10} m. eLISA will, once operative, measure gravitational waves using Michelson interferometery as well. A picture of the eLISA setup can be found in figure 7 below.

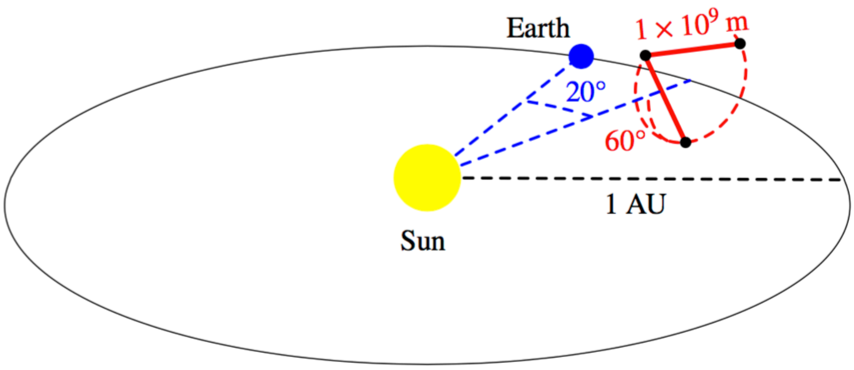

Figure 7: The position of the eLISA detector with respect to the earth and the sun

In order for eLISA to succesfully start measuring gravitational waves by 2028, LISA Pathfinder is currently testing multiple subsystems of eLISA in outer space. The first results of LISA Pathfinder [40] are very promising, especially the accelerator noise performance for the test mass. When implemented in eLISA, these subsystems will provide a higher sensitivity than currently planned. Now having outlined these detectors, their possible detection of the primordial gravitational wave background will be looked at.

6 Detection of the primordial gravitational wave background

Despite the fact that aLIGO managed to directly measure gravitational waves, which is a very impressive feat, aLIGO and other second generation ground-based detectors unfortunately will not be able to measure the primordial gravitational wave background. For detecting this background, future facilities such as space-based detectors and third generation ground-based detectors are required. Due to the fact that the characteristic strains of the possible signals left behind by cosmic strings and inflation differ significantly, these phenomena will be treated separately, starting with cosmic strings.

6.1 Cosmic strings

Cosmic strings, if created by phase transitions in the early universe, should have left behind a gravitational wave background with a characteric strain $h \sim 10^{-24}$ as mentioned above. Since the ET [41] and the LIGO Voyager [38] are planned to reach a peak sensitivity of $h \sim 10^{-25.5}$ and $h \sim 10^{-25}$ respectively. These detectors will be able to detect a possible gravitational wave background from GUT phase transition cosmic strings.

Furthermore, as the first generation detectors of the LIGO-Virgo collaboration already managed to put some constraint on the size of the string parameter $\frac{G\mu}{c^2}$ [42], it can therefore be expected that as second generation detectors start to collect more data, the string parameter can be constrained further.

Once operational, eLISA will reach a peak sensitivity of $h \sim 10^{-22.5}$ [43], which will be insufficient to measure a possible gravitational wave background from cosmic strings. However, due to its frequency range eLISA will provide new constraints for the cosmic string parameter. Furthermore, with the LISA Pathfinder mission obtaining results that exceed expectation, the sensitivity of eLISA will probably move up towards the characteristic strain of GUT-scale cosmics strings. Having looked at the possible detection of cosmic strings, now the possible detection of inflation will be elaborated on.

6.2 Inflation

Due to the fact that the possible gravitational wave background from inflation has a characteristic strain of $h \sim 10^{-27.5}$ third generation ground-based detectors and eLISA will be unable to detect this signal. However, there are some exotic inflation models, with a characteristic strain of $h \sim 10^{-23}$ [44], that ET, LIGO Voyager and even eLISA might be able to measure [39] [43]. However for the near future it seems very unlikely that gravitational waves from cosmic inflation will be discovered.

7 Conclusion

This paper aimed to investigate whether the gravitational wave background left behind by early universe phenomena can be directly detected in the near future. It first outlined the theory underlying gravitational waves themselves and then the theory underlying these phenomena. In outlining these phenomena it described their primordial gravitational imprint as well. Knowing this imprint current and future detectors and their characteristics were summarized. The characteristics of these detectors were then used to determine whether or not the primordial gravitational wave background can be detected.

It was found that possible evidence for GUT-scale cosmic strings is expected to be detected by the third generation ground-based detectors. Furthermore, it was found that direct detection of gravitational waves from cosmic inflation is very unlikely in the next decade. Nonetheless gravitational waves will provide us with new and valuable insights into early universe physics.

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