University of Groningen

# Trapping and cooling silica nanospheres

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# 1 Introduction

This thesis is about the mechanisms and limits of trapping and cooling levitated nanospheres. A silica nanosphere is essentially a tiny ball of glass. Their radii differ in size from a few nanometer to a few micrometer. Recently, there has been a lot of experiments that try to trap these nanospheres and cool them. There are different techniques for trapping nanoparticles, for example using optical forces and traps that make use of electric fields. In this thesis, only trapping with use of optical forces are considered. A nanosphere can be trapped in a laser beam and then levitates in this beam. A levitated and cooled nanosphere has almost no mechanical interaction with its environment at all. This makes the nanosphere an ideal candidate for experiments regarding the transition of quantum behaviour and classical behaviour. Furthermore, as we will see, cooled nanospheres are a good instrument for ultrasensitive force sensing.

The field that investigates the interaction between light and mechanical properties of a system is called optomechanics. Quantum optomechanics investigates the same objects, but then under conditions where the quantization of the electromagnetic field and the quantization of the mechanical degrees of freedom become relevant [1]. This field is developing rapidly and continues to push the boundary of quantum science and measurement technology. There is not only a lot of experimental progress going on, but the theoretical framework is investigated as well. Besides experiments done all over the world, there are proposals for experiments with levitated nanoparticles in space [2]. Space offers an unique combination of microgravity, low pressure and low temperatures and this could even give a stronger decoupling of the nanosphere from its environment.

This thesis is a prestudy for possible future experiments in Groningen. The aim is to give an overview of the motivations and techniques (by describing the optical forces of light on nanospheres) and stydy cavity trapping and compare different cooling mechanisms with each other. The composition of this thesis is as follows. First, a motivation for the experiments are given. Different possible research areas will be shortly addressed and a short introduction of quantum wavefunction collapse models are given. After that, the basic physics of light forces on nanoscale objects is discussed. It is explained how we can trap a nanosphere and which forces it encounters. The principles are not the same for all sizes of nanospheres, so we will distinguish three different regions of sizes of the nanosphere in comparison to the laser wavelength.
The fourth third chapter is about the properties of cavities. Finally, the different cooling methods for nanosphere are explained and compared to each other.

# 2 Motivation

It is important to think about possible future applications of the new techniques for trapping and cooling nanospheres. It turns out that there are quite a few research opportunities with trapped nanospheres. As mentioned in the introduction, levitated nanospheres are a perfect candidate for testing the poorly understood transition between quantum and classical behaviour. Furthermore, it turns out that they can be used as a very sensitive force detector. In this part, some research topics with optically levitated nanospheres are given. For some applications, a very global set-up is described. It requires knowledge of cavities, that one can find in chapter 4 of this thesis. Of course, it is possible to skip these parts and just scan the chapter to get a feeling of what is possible.

## 2.1 Ultrasensitve force detection

An important feature of levitated cooled nanospheres is their ability to be used for ultrahigh-precision force detection. Conventional high force sensitivity sensors have typically consisted of solid-state mechanical resonators such as cantilever or membranes. In these systems, the internal material losses and clamping mechanisms are the chief sources of dissipation and responsible for limiting the quality factor of the oscillator. The center of mass (C.O.M.) motion of optically trapped dielectric objects is immune to these sources of dissipation. In 2016, one group in the USA [3] managed to measure forces in the zeptonewton ($10^{-21}$N) regime. Furthermore, the recoil force was measured for the first time using levitated cooled nanospheres [4] and the Brownian motion investigated [5]. This ultrasensitive force detection has some interesting applications, a few of them will be discussed here.

*1. Non-Newtonian gravity measurements at short distances*

Isaac Newtons universal law for gravity is more than 300 years old. During these centuries, his theory of gravity has been tested many time. In the last decade, there has been a new interest in testing non-Newtonian gravity at sub-millimeter scales. This is based on theoretical predictions of modifications to gravity in this regime. Non-Newtonian gravity-like forces can be tested by monitoring the displacement of the sphere as a mass is brought behind the cavity mirror. Generally, corrections to Newtonian gravity are parametrized by a Yukawa-type potential [6]

$$V = -\frac{Gm_1m_2}{r}(1 + \alpha e^{-r/\lambda}) \tag{1}$$

where $\alpha$ is the strength of the correction to gravity and $\lambda$ is the range of the interaction.The force sensitivity of the trapped spheres is small enough to explore the limits of $\alpha$.

*2. Gravitational wave detection*

The detection of gravitational waves is another application of ultransensitve force sensing [7]. The laser-interferometers that exists at the moment, like LIGO, are optimized in the frequency range of $10 - 10^4$ Hz. Due to photon shot noise, their sensitivity decreases at higher frequency. An optically trapped sensor could be used to attain improved sensitivity

in the frequency range of 50-300 kHz. At such high frequencies, there may be sources of gravitational radiation from physics beyond the standard model. In ref [7] they propose the following approach. In order to detect gravitational waves, a dielectric nanosphere is trapped in an antinode of a cavity at a position close to the input mirror. A second light field with two different frequency components is used to cool and read out the position of the nanosphere. When a gravitational wave passes, the nanosphere is displaced from its equilibrium position. This displacement is big enough to be measured.

*3. Casimir force sensing*

Casimir force sensing [8] is possible as well. The Casimir force is a attractive force due to quantum fluctuations of vacuum. Quantum electrodynamics predicted that vacuum has an energy. The Casimir interaction is completely unexplored in the regime where the size of the sphere is on the order of the sphere-surface separation (the distance from the sphere to the mirror). Previous measurements only have been performed in the limit that their separation distance is small compared with the sphere radius [6] An measurement of the Casimir effect could be done by trapping a sphere close to the a mirror of the cavity. The Casimir force between the mirror and the nanosphere will strongly affect its the dynamics of the center of mass motion of the sphere and can be studied [6].

Besides all the research opportunities that arise from ultrasensitive force sensing, one of the most attractive goals is creating macroscopic Schrödinger's cat states. In mechanical operators, the main decoherence source is the mechanical contact with the environment.
The absence of mechanical contact of levitated nanoparticle make them an ideal candidate for the study of macroscopic quantum mechanics [6]. In the next section it is explained how nanospheres can be used to test different theories about wave function collapse.

## 2.2 Quantum physics and wave function collapse models

Quantum mechanics continue to amaze us. The difference between what we perceive in daily life and the predictions of quantummechanics couldn't be bigger. Nevertheless, quantum theory is one of the most successful theories known today [9]. The basic principles of quantum mechanics that were discovered en described almost 100 years ago, have withstood every experimental test and lies at the heart of important technologies like semiconductors, computer memories an superconductors for example[2].

The couterintuitive nature of quantum phenomena, like quantum superposition and entanglement, has given rise to scientific and philosophical discussions since the theory was formed. When a system is in quantum superposition, it is in principle impossible to distinguish whether the system is in one or another multiple possible states. Quantum entanglement is a direct consequence of quantum superposition when there are two composite systems where at least one of the subsystems initially is in a quantum superposition [2]. The question is why we do not observe these quantum phenomena for macroscopic objects. Schrodinger was the first

to notice that we cannot simply ignore the weirdness of these quantum concepts as being restricted to abstract things that only happen on very small scales. He came up with a thought experiment, that is well known as the paradox of Schrödinger's cat now. Schrödinger imagined a cat with an radioactive atom, a Geiger teller and a bottle of poison in a box that has no interaction with its environment. If the atom decays, the Geiger teller will trigger release of the poison and the cat dies. In this thought experiment, the state of the cat is entangled with that of the atom and the cat is in a superposition of dead and alive. Instead of a cat, we can try to prepare a simpler, but still macroscopic system in different distinct states at the same time. Typically the coupling of a physical system to its environment leads to decoherence. This is because one can determine the state of the physical system by looking at its environment most of the time. With increasing size of an object, decoupling from its environment becomes more difficult [2].

Quantum mechanics explains the motion of microscopic systems quite successfully. It is a probabilistic theory. It is not possible to predict the outcome of an experiment, but it is possible to calculate how probable each outcome is. On the other hand, classical mechanics is a very good theory for macroscopic systems that are deterministic and predictable if the initial sates of the system are determined. The key difference between the two theories is that the classical degrees of freedom such as momentum and location are described by wave functions in quantum mechanics. These wavefunctions contain information about the different states a particle is in, and the probability of finding a particle somewhere [6]. The big question however is why macroscopic objects do not display quantum behaviour. They are never found in a superposition of position states. The traditional explanation is the wave function collapse. One way or another, the wavefunction that originally explains the behaviour of a particle and is a superposition of different eigenstates, appears to reduce to a single eigenstate.

The Copenhagen interpretation postulates an artificial divide between the micro- and macroscopic world. It is not specified at what mass scale this divide should be. During a measurement, in other words when a microscopic system interacts with a macroscopic system the wave function collapses and the measured observable is no longer in a superposition of the eigenstates, but just in one. According to this theory, before and after the measurement the system evolves in a deterministic wat, but during a measurement there is nondeterministic evolution. This interpretation gives no explanation of how the collapse takes place. A big problem of this interpretation is that there is no precise definition of a classical measuring apparatus.

As an alternative for the Copenhagen interpretation, the many-worlds interpretation was developed by Everett. According to this theory, evolution during a measurement is also deterministic. During a measurement, it appears as if only one of the two outcomes has been realized, but actually the state continues to be in a superposition. Only one is seen by the observer, the other part exists in a different branch of the universe [10].

The Copenhagen interpretation this does not explain how the superposition of quantum mechanics are destroyed. It only changes the question to why the wave functions collapse. One of the simplest descriptions of the quantum-classical world transition is that this transition is determined by the de Broglie wavelength $\lambda = h/p$. For macroscopic objects, the momentum is very large and the de Broglie wavelength is too small to be observed. This explanation predicts that if we can reduce the momentum enough, for example by cooling and thus decreasing the velocity of the system, quantum superpositions of large objects could be observed.

Most theories about quantum wave function collapse are about environment-induced decoherence: they use the system-environment interaction to explain the transition between classical and quantum mechanics. The phenomenon of decoherence highlights the role played by the environment when a quantum system interacts with a measuring apparatus. The environment could be defined as the collection of particles which is present within a certain radius ($c\Delta t$) of the apparatus during the measurement and can causally interact with it [10]. The decoherence that is due to interaction with the environment (such as collisions with gas molecules, scattering blackbody radiation or coupling via mechanical suspension) is called quantum decoherence. If one only considers quantum decoherence as the source of the wavefunction collapse, it means that we could make arbitrarily large superpositions of states if we manage to decouple a system enough from its environment.

However, besides models and theories that only consider quantum decoherence due to the environment, there are other models that suggest a modification of quantum theory. They predict that even for completely isolated systems, quantum behaviour does not occur for macroscopic systems, because of other decoherence mechanisms that typically depend on the mass or size of the quantum system and are not dependent on the environment of the object.

One of these kinds of theories is developed by Roger Penrose. He came up with the idea that the conflict between quantum mechanics and general relativity leads to gravity induced collapses of superposition stats. According to his theory the self-gravitational energy of the quantum state determines the time-scale at which superpositions decay. Other theories propose that the collapse might intrinsically be due to quantum mechanics not being complete yet [6].

So the time-scale of the collapse of the wavefunction is different for environmentally-induced dechoherence theories and gravitationally induced state collapse. To compare these two theories , an experiment can be set up in which a superposition of position should be created with a large gravitational self-energy and a small environmental deocherence rate. This would require a mechanical system with a large mass and a weak coupling to the environment. It is not well known at which time and energy scales the gravitational effects become important. An experiment with a levitated particle could possibly detect the gravitational effects or put

constraint on these theories. [9] In order to observe quantum effects with a nanosphere, the sphere should be thoroughly decoupled of the environment. Normally, macroscopic systems are strongly coupled to their environments and behave classically. A possible experiment is to measure the expansion of hte wavefucntio as a function of time and to study possible deviations from the predicitions of quantum mechanics due to the influence of macroreaslistic decoherence emchanisms.A typical version of such an experiment consists of the following steps[1]:

1. Trap a nanoparticle into an optical trap in an optical cavity
2. Cool the motion of the nanosphere to quantum ground state.
3. Switch off the trap and let the particle's wave function expand freely for a time t.
4. Measure the position of the particle.
5. Trap the particle and repeat the steps.

Afeter repeating this procedure many times, the width of distribution of the particle positions can be determined. This can be performed for various values of t, to determine the time-dependence of the expansion of the wave-function.

Furthermore, it is really interesting to see if we can create a macroscopic object in a superposition (like Schrödinger's cat). If that is achievable, the double slit experiment with large mechanical resonators could be performed. This can be realized by applying the following steps[11]:

1. Trap a massive object into an harmonic potential to prepare a mechanical resonator.
2. Cool the C.O.M. to the ground state (along one direction).
3. Switch off the trap and let the wave function expand freely.
4. Get the state in a superposition by measuring $x^2$. The state collapses into superpositions of being at $+x$ and $-x$.
5. After letting the state evolve freely for a short time, measure the C.O.M. position x.
6. Repeat this experiment.

One way of preparing a spatial quantum superposition (step 4) is using two cavities. One for ground state cooling and one for preparing the superposition state with a squared position measurement when the nanoparticle falls through it (see figure 1) [6]. In this way, quantum physics with macroscopic objects can be used to shine a little bit more light on the transition of quantum to classical behaviour. The objective of this experiment is to determine the features of the interference pattern and to compare the interference visibility with the prediction of quantum theory [1].

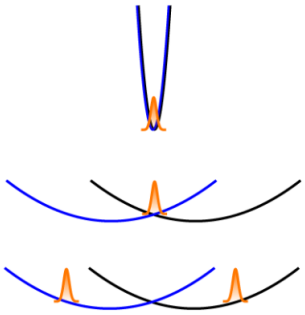

Figure 1: Bringing a nanosphere in a position superposition by using two cavities. Time evolution goes from top to bottom. Figure from ref [6].

In ref [12], quantum behaviour on a relatively large scale is demonstrated. In the experiment, they shoot soccer-ball-shaped $C_{60}$ cages through a double slit and obtain an interference pattern. This is the largest object with which quantum behaviour is demonstrated until now. The diameter of these bucky balls is approximately 1 nm. All the particles discusses in this thesis are ten to a hundred times larger. The boundaries of quantum sciences are pushed further and further.

# 3 Physics of optical forces

It is well known that light can exert forces on object. James C. Maxwell was the first to deduce theoretically the radiation pressure of light. Although light has no mass, an electromagnetic wave carries momentum. Every photon carries momentum $p = h/\lambda$. If an object absorbs or reflects light, there is transfer of momentum. When light is reflected, the transferred momentum is twice as high as when it is absorbed. The radiation force is really small, in daily life you never notice the light 'pushing' at you. However, for very small particles, the radiation force can be much stronger than the gravitational force and have a huge effect on the particle. Arthur Ashkin was the first person to demonstrate that the optical forces of light were strong enough to levitate micron-sized dielectric particles. In 1970, he used strongly focused laser beams to move the particles around and he developed a stable optical trap [13]. His work led to the development of a single beam optical trap and radiation pressure was soon used to cool and trap atoms and later even viruses and bacteria [14]. This led to breakthroughs in atomic and molecular physics. Further technological development attributed to the recent progress in optical trapping.

The simplest form of an optical trap makes use of only one strongly focused laser beam that traps the particle at the focus. This single gradient force dipole trap is also called an optical tweezer. On the hand of this example, the concepts of optical trapping are explained in more detail and it is showed that the forces that are exerted on a nanosphere really are much bigger than the gravitational force [5].

An optical trap is formed by a strong laser beam that is highly focused with an objective lens of high numerical aperture. A high numerical aperture focuses the beam very strong, so that the intensity in the focus is really high. Here, it is assumed that a the trapping laser beam is a Gaussian beam. Before explaining the forces in more detail, first an very short introduction of Gaussian beam optics is given.

## 3.1 Gaussian beam optics in a nutshell

A Gaussian beam is an electromagnetic beam with field amplitude profiles given by the Gaussian function, see figure 2. The electromagnetic field of a Gaussian is a radially symmetrical distribution whose electric field is given by:

$$E = E_0 \exp\left(-\frac{r^2}{w_0^2}\right) \tag{2}$$

The intensity distribution of the Gaussian beam is defined as follows:

$$I(x, y) = I_0 e^{-2(x^2+y^2)/w(z)^2} \tag{3}$$

where $I_0$ is:

$$I_0 = \frac{2P}{\pi w(z)^2} \tag{4}$$

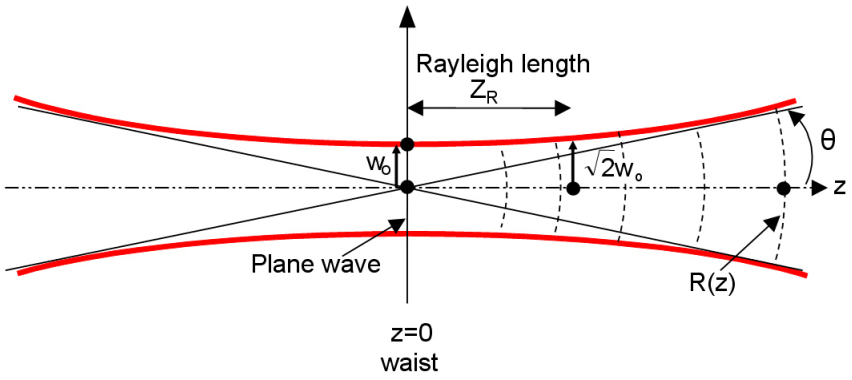


Figure 2: Profile of a Gaussian beam. Figure from ref [15]

In this formula P is the power of the laser beam. The beam has no 'sharp'edge, so the radius of the beam is defined as the distance from the axis at which the electric field has dropped by a factor e. The beam radius $\omega_z$ as a function of of the distance $z$ from the focus is:

$$w(z) = w_o\sqrt{1 + (\frac{z}{z_0})^2} \tag{5}$$

At the focus, the beam radius takes its minimum value $w_0$ (the waist). The beam waist is defined as follows:

$$w_0 = \sqrt{\frac{z_0 * \lambda}{\pi}} \tag{6}$$

where $z_0$ is called the Rayleigh range. It is the distance at which the cross-sectional area has doubled from its minimum value at the focus. The radius of curvature is given by:

$$R(z) = z(1 + (\frac{z_0}{z})^2) \tag{7}$$

For now, this is all the information needed to examine the forces on the nanoparticles.

### 3.2 Optical force

For convenience, we can divide the optical force in two parts: the scattering and gradient force. Intuitively, the scattering force is the easiest to understand. Light consists of photons and when a nanosphere is placed in a laser beam, a lot of photons collide with it. Every photon 'pushes' a bit on the sphere and exerts momentum. The light is scattered with a component in every direction, as can be seen in figure 3. The resulting force vanishes in all directions except the direction of the propagation of the light. In most situation, the scattering force dominates over the gradient force. Only when there is a very steep intensity gradient, the gradient force must be considered and can be stronger. The direction of the gradient force is towards the region with highest intensity. The way that the gradient force arises is different for different sizes of nanoparticles. It is good to keep in mind that this composition of the total optical force in two parts is merely an intuitive means of discussing the total force. Both components arise from the same underlying physics.

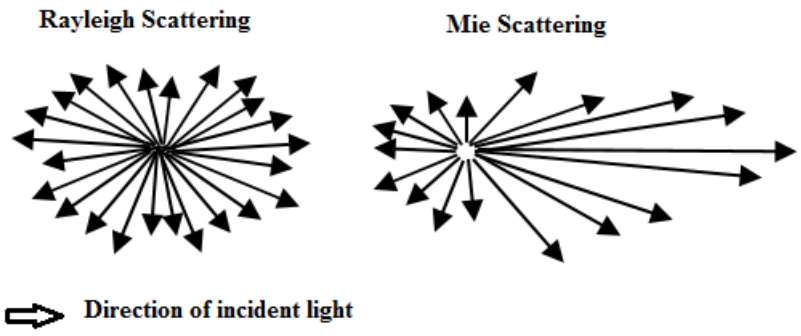


Figure 3: Scattering of light from a nanoparticle in the Rayleigh ($r << \lambda$) and Mie ($r >> \lambda$) limit. Picture from reference [16]

Generally, all the forces can be summarised by the Maxwell stress tensor ($T_{ij}$). It represents the interaction between electromagnetic forces and the mechanical momentum. It is a matrix with components that describe the electric and magnetic components of the field and the tensor is related to the force on an object in the following way:

$$F = \int_{\partial V} T * N(r) da \tag{8}$$

In this equation, N is the outgoing normal vector to the sphere surface. The elements of the Maxwell stress tensor give the relation between the incident and scattered light field. his equation is of general validity and can be used to calculate the mechanical force acting on an arbitrary body. This force is entirely determined by the electric and magnetic fields on the surface $\partial V$. It is interesting to notice that no material properties enter in this equation, the only constraint is that the body should be should be rigid [17].

Calculation of the optical forces using this equation can be really computational. Luckily there are some regimes in which the forces are easier to express. We can distinguish between three different situations. If the size of the particle is much smaller than the wavelength of the light ($r << \lambda$), it is possible to approximate the particle as a point dipole and calculate the optical forces exactly. This is called the Rayleigh limit. If the particle is much bigger than the wavelength of the light($r >> \lambda$), it is possible to use ray physics. This is also called geometrical optics and describes light in propagation of rays. Unfortunately, for particles that are about the same order of size as the wavelenght of the light, there is no easy approximation that can be made and it is required to use the full theory of electromagnetic waves to calculate the forces. Theoretical progress in calculating the forces has been made recently [13].

### 3.3 Ray optics

A single beam optical trap was originally designed for particles in the Rayleigh limit. Experimentally, it was found that these traps could also trap and manipulate micron-sized particles.

A qualitative view is easily given. The gradient force arises from the refraction of light. When a light ray travels through a nanosphere, the direction and magnitude of the momentum of the wave changes. This change in momentum is transferred to the nanosphere. If the microsphere is trapped at the focus of the laser, it will only experience the scattering force. If the microsphere moves to the left of the focus, it deflects the laser beam to the left and the photon momentum will increase to the left. The counter force will push the microsphere back to the focus. The intensity of the light on opposite sides of the sphere is not equal in a high gradient intensity region. If $n_{sphere} > n_{medium}$, this difference leads to a resultant force that points towards the highest intensity and thus forms a trapping potential for the sphere [18]. If the refracting index of the surrounding medium is higher, the sphere will be pushed away from the point of highest intensity. In figure 4 it is illustrated that for every arbitrary displacement, the netto restoring force points back to the focus.

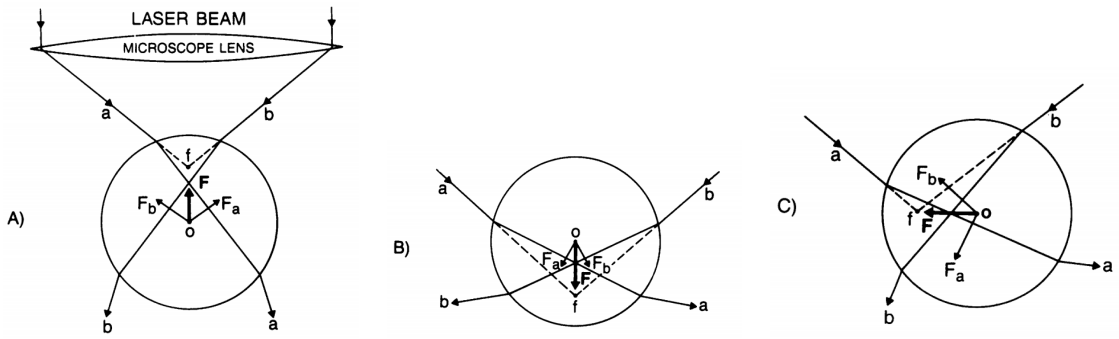


Figure 4: Gradient force in the ray approximation. Fig from ref [18]

Because the spheres we consider for the experiments are smaller than the wavelength of the light, this is all we mention about Ray optics.

### 3.4 Dipole limit

It is convenient to consider a small nanosphere as a point dipole. We can make this approximation when $\alpha = \frac{2\pi r}{\lambda} << 1$, where r is the radius of the particle and $\lambda$ is the scattering wavelength. The radius of the nanosphere has to be about a factor of 10 smaller than the wavelength of the trapping laser. To trap a nanoparticle, the gradient force should be larger than the scattering force. Rayleigh derived an approximation for scattering in the small-particle limit by assuming an internal electrostatic field that is homogeneous. The gradient force on a nanosphere is [19]:

$$F_{grad}(r,z) = \frac{2\pi n_s a^3}{c}\frac{m^2-1}{m^2+2}\nabla I(r,z) \tag{9}$$

Here, $m = n_p/n_s$ is the relative index ($n_p$ is the refracting index of the particle and $n_s$ of the surrounding medium), $a$ is the radius of the particle. The gradient force forms a trapping

potential:

$$V(\bar{r}) = -\frac{2\pi n_s a^3}{c}\frac{m^2-1}{m^2+2}I(\bar{r}) \tag{10}$$

The scattering force on the nanosphere is[5]:

$$F_{scat} = \hat{z}\frac{n_s}{c}C_{scat}I(r) \tag{11}$$

C is the cross-section:

$$C_{scat} = \frac{8}{3}\pi(ka)^4a^2(\frac{m^2-1}{m^2+2})^2 \tag{12}$$

and

$$k = \frac{2\pi n}{\lambda} \tag{13}$$

we can rewrite this as[5]:

$$F_{scat} = \hat{z}\frac{128\pi^5a^6n_s^5}{3c\lambda^4}(\frac{m^2-1}{m^2+2})^2I(r) \tag{14}$$

We see that the scattering force is proportional to the intensity of the laser and the gradient force is proportional to the gradient of intensity of. To form a stable trap, the gradient force should be larger than the scattering force. If that is not the case, the nanosphere will be pushed away from the beam. In air, the refractive index $m$ is larger than in water for example. Therefore, the scattering force gets larger in air. The gradient force gets larger as well, but the effect is less due to the extra square in the formula for the scattering.

The total optical force is a sum of the scattering and gradient force. The minimum value of the force must be negative in order to form a stable trap. If the force becomes negative, the direction changes and therefore there is a certain position where the nanosphere is trapped. The scattering force is proportional to $r^6$ and the gradient force to $r^3$. So the scattering force increases much faster than the gradient force when the nanospheres get bigger. Consequently, it is easier to achieve a negative minimum force for smaller nanospheres. In order to create a stable trap for the nanosphere, the well depth should be at least 10 times larger than the average kinetic energy of the particle [5]. Its average kinetic energy is $k_BT/2$.

In figure 5 and 6 the optical forces and potential wells of particles of different sizes are plotted using the formulas given before. The axial (z) direction is the direction of the propagating laser light. In the plotted pictures, the beam power is 0.2 W, the waist of the beam 1.5$\mu$m and the density of the sphere is 2200 kg/$m^3$. It is very clear that the scattering force becomes significantly bigger when the radius of the particle increases. So larger particles are more difficult to trap with an optical tweezer. The scattering force moves the position of where the particle is trapped as well. Furthermore it is obvious that the optical force is much larger than the gravitational one.

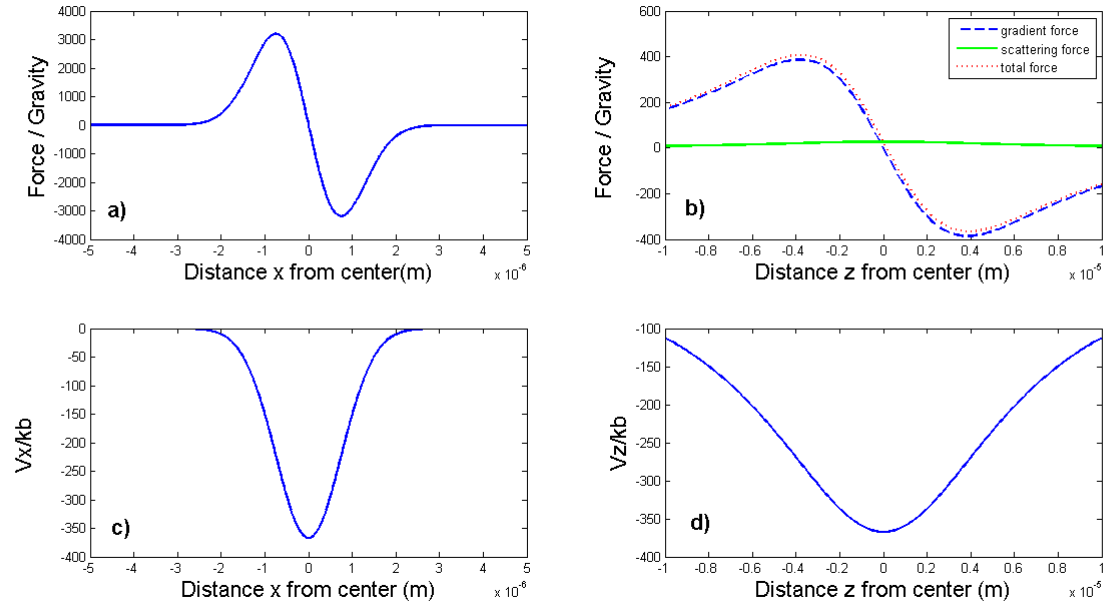


Figure 5: Optical forces for a particle with a radius of 25 nm, beam power P=0.2 W and waist 1.5$\mu$m. Figure a) is the gradient force in the radial direction, b) is the total optical force (consisting of the gradient and scattering force) in the axial direction, in c) the potential well in the radial direction is plotted and in d) the potential well in the axial direction.

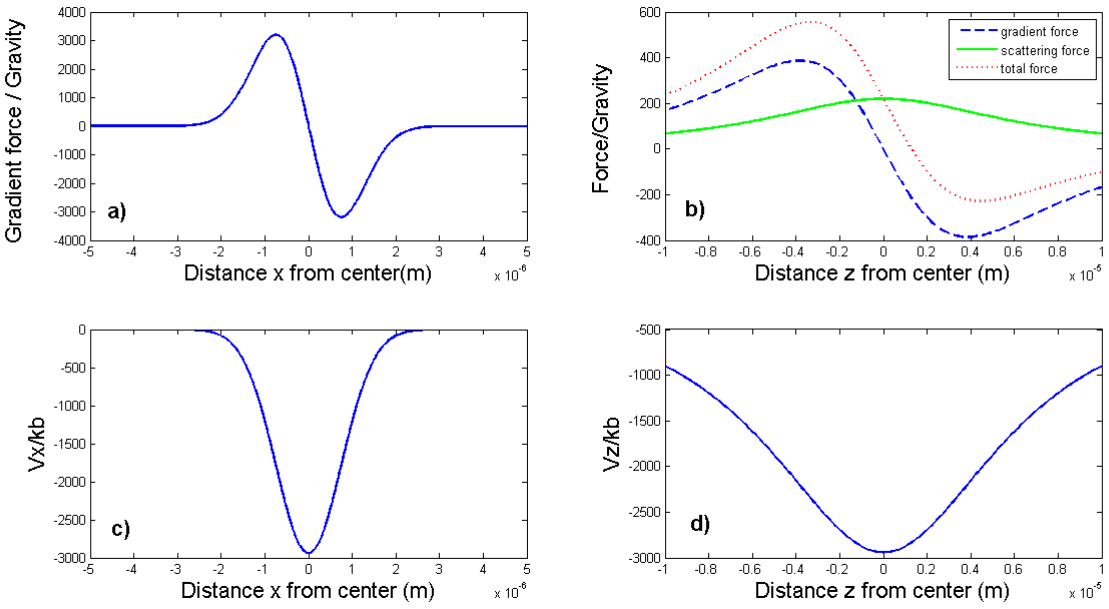


Figure 6: Forces for a particle with a radius of 50 nm, beam power P=0.2 W and waist 1.5$\mu$m. Figure a) is the gradient force in the radial direction, b) is the total optical force (consisting of the gradient and scattering force) in the axial direction, in c) the potential well in the radial direction is plotted and in d) the potential well in the axial direction.

### 3.5 Lorentz-Mie theory

If the size of the dielectric nanoparticle is comparable with the size of the wavelength, neither Ray optics nor the dipole approximation is usable. To calculate of $F_{rad}$ one must know the electromagnetic field in the presence of the particle. Calculation requires knowledge of both $E_1$ and $B_1$ (the incident fields) and $E_s$ and $B_S$ (scattered by the particles). There are several different methods to calculate these fields. One of them is the expansion of the electromagnetic field in a series of vector fields in a transition matrix approach [20]. The optical force is obtained by calculating the change of momentum caused by electric field scattering of the sphere. So, the problem of calculating optical forces is a problem of computational light scattering. For homogeneous isotropic spheres, an analytical solution is available: the Lorenz-Mie solution. However, this theory was originally developed for scattering of plane waves. Because in optical tweezers the beam is tightly focused, this is not the case. The extension of the original Lorentz-Mie theory is called the generalised Lorenz-Mie theory. In this theory, the relation between the expansion coefficients for the incident and scattered wave are linearly related: $p_k = \sum_n^\infty T_{kn} a_n$ where $p_k$ are the coefficients of the scattered wave, $a_k$ those of the incident wave and $T_{kn}$ elements of the T-matrix. The T-matrix depends only on the properties of the particle and the laser wavelength. It is independent of the incident field [21] [5].

The force on the particle is obtained by the change in momentum of the scattered electric field by the particle. In ref [5] a the effect of changing the numerical aperture (NA) is analyzed. A higher NA corresponds to a smaller beam waist. It turns out that only the forces in the radial direction are similar for different NA's, and forces in the axial direction are very dependent of the waist of the beam. This is due to the fact that the scattering force only exists in the axial direction. As mentioned earlier, the particle is trapped at position where the force changes sign, and this place depends on the strength of the scattering force. However, for some set-ups, the minimum of the force doesn't get negative and it is not possible to create a trap. In fig 7 the minimum of the axial force for different sizes of nanospheres is plotted. Because of the interference between scattered and non-scattered light, the minimum force oscillates as the diameter changes. It changes the trapping place as well. This oscillation period is about half of the wavelength of the laser beam in the sphere. We see that since nanosphere can not be trapped if the minimum force is positive, only specific sizes of nanospheres can be trapped as can be seen in picture 7.

### 3.6 Conclusion

We want to create a potential well deep enough to trap the particle. There are different ways to do that. One can either increase the laser power or decrease the waist of the trap. A single-beam trap requires a lens with high numerical aperture. The working distance of such a lens is really short, which can cause problems integrating some cooling methods. Also, the scattering force is a big obstacle in stably trapping of nanospheres. It is clear that if the light

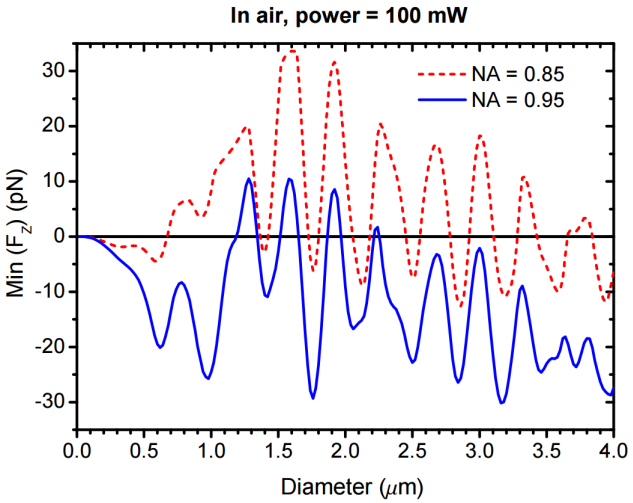


Figure 7: The minimum value of the optical force as a function of the diameter. Only sphere with a negative minimum force can be trapped. NA is the numerical aperture of the lens, it is a measure for the waist size of the beam. Picture from ref [5]

would come from two opposite directions, the scattering cancels out. One way to do that is with a dual beam trap. In a dual beam trap, two counter propagating lasers are aligned to form a trap. However, it is very difficult to make a precise alignment. Even small differences in the placement of the focus will let the particles move around. An optical cavity is a device in which the alignment of beams from different directions is inherently stable. . Furthermore, in an optical cavity the laser power is increased as well. In the next section, it is explained what an optical cavity is and how it works.

# 4 Laser cavity

Traditionally, optical trapping experiments involve freely propagating laser beams. These characteristics are described in the previous chapter. A more recent candidate for the source of optical forces are optical cavities (also called optical resonators).

An optical cavity is a circuit in which light is confined. Light circulates or is repeatedly reflected within a cavity, without escaping. This is done by placing mirrors in a geometrical arrangement. For example, you can have two mirrors between which the light keeps reflecting or a ring of three or more mirrors as shown in fig 8. The light enters through one of the mirrors.

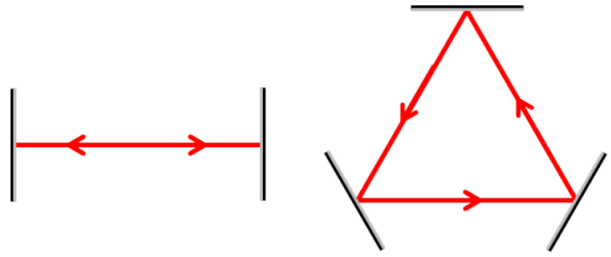

Figure 8: Two different configurations of an optical cavity. Figure from ref [22]

We will consider the simplest resonator, a cavity consisting of just two mirrors placed opposite to each other. This is called a Fabry-Perot cavity. For a stable cavity, it is unfavorable to use planar mirrors. The input field would have to be a perfectly aligned wave, every deviation causes the light to escape from the cavity. So we use a cavity with curved mirrors.
Because the light is constantly reflected, a standing wave will form in an optical cavity. In the cavity, only a discrete number of half-wavelength will fit exactly. The frequencies are restricted to discrete values and given by:

$$v_q = q\frac{c}{2d} \tag{15}$$

where $q$ is the mode number and $d$ the distance between the mirrors. The frequency difference between two cavity modes is called the free spectral range (FSR) and given by $v_{FSR} = c/(2d)$. In a longer cavity, more frequencies can resonate and thus is the FSR smaller. The reflectivity of the mirrors never is 100%. So every round trip, some light is lost. The amplitude of a wave decreases with a factor $\sqrt{R}$ each reflection, R is the reflectivity of the mirrors. For simplicity it is assumed that both mirrors have the same reflectivity for now. A measure of the quality of the cavity is the finesse. The higher the finesse, the lower the losses are. So a high finesse means a high number of round trips for each photon in the cavity. The finesse is defined as follows [23]:

$$F = \frac{\pi\sqrt{\sqrt{R}}}{1-\sqrt{R}} \tag{16}$$

In this equation $R$ is the reflectivity of the mirrors. The finesse corresponds to the average amount of round trips a photon performs in the cavity before it is lost. The amplitude cavity decay rate, also called the linewidth of the cavity is:

$$\kappa = \frac{FSR}{F} = \frac{c}{2LF} \tag{17}$$

which corresponds to the full width at half maximum (FWHM). For a larger finesse, the linewidth is smaller, which means that only a small range of frequencies is allowed in the resonator.

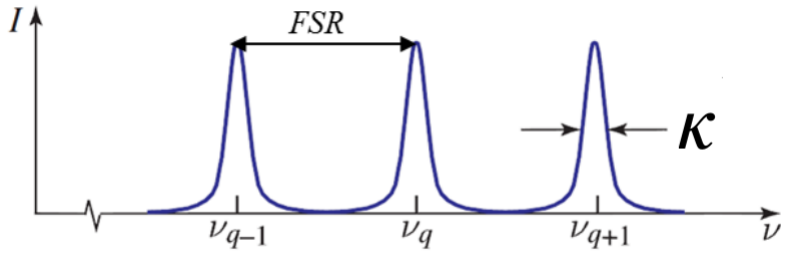


Figure 9: Free spectral range and linewidth. Figure from ref [22]

The successive waves all contribute to the final intensity. This is given by:

$$I = \frac{I_{max}}{1 + (2F/\pi)^2 sin^2(\pi F/FSR)} \tag{18}$$

$$I_{max} = \frac{I_0}{(1-\sqrt{R})^2} \tag{19}$$

So the intensity distribution is given by one over sine squared. There are multiple regions with an intensity maximum, so more than one potential well where a particle can be trapped as well. The intensity reaches a maximum at the resonance frequencies. This intensity is much higher than the incoming light intensity due to the many round-trips inside the cavity. To calculate the intensity enhancement (or the power enhancement) we define A:

$$A = \frac{4}{1-\sqrt{R}} \tag{20}$$

In figure 10 the power enhancement and finesse are plotted as a function of the reflectivity. It can be seen that for higher reflectivity, the finesse increases really fast. In equation (20) losses are neglected and the factor 4 is due to the constructive interference in the cavity. If different reflectivities of the mirrors are considered and including losses one finds:

$$A = \frac{R_1}{(1-\sqrt{R_1 R_2 exp(-2\alpha d)})^2} \tag{21}$$

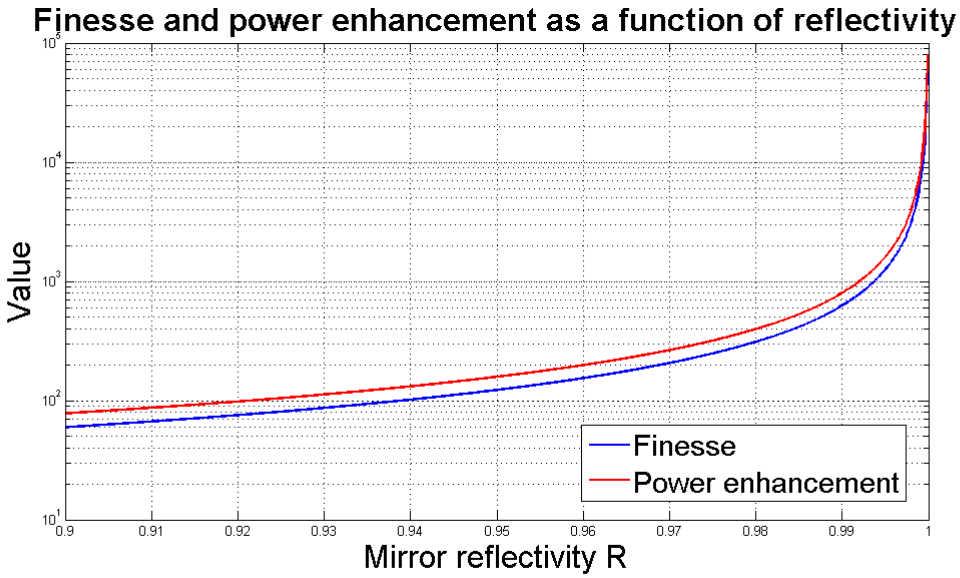


Figure 10: Finesse and power enhancement

The factor $e^{-2\alpha d}$ is the factor with which the intensity drops for each round trip. Its is caused by absorption and scattering by the medium between the mirrors. The waist size of a cavity is given by:

$$\omega_0 = \sqrt{\frac{\lambda}{2\pi}\sqrt{d(2r-d)}} \tag{22}$$

where d is the distance between the mirror of the cavity and r the radius of curvature. The waist size is maximal at d=r. The mean photon number inside a cavity is given by:

$$\overline{n}_c = \frac{\kappa}{\Delta^2 + \kappa^2/4}\frac{P_{in}\eta^2}{\hbar\omega_l} \tag{23}$$

and $\Delta = \omega_l - \omega_c$ is the laser detuning and $\kappa$ is the FWHM [24] [23].

### 4.1 Trapping potential in cavity

The trapping potential in a cavity is given by [25]:

$$U(r,z) = -\frac{2U_0}{1+(z/z_0)^2} exp(-2r^2/\omega(z)^2) cos^2(kz) \tag{24}$$

where $U_0 = -2\alpha P_0/(\pi\epsilon_0 c\omega_0^2)$. The real part of the polarizability is $\alpha = 3\epsilon_0 V\frac{\epsilon-1}{\epsilon+2}$, combined this gives [17]:

$$U_0 = \frac{3I_0 V}{c} Re\frac{\epsilon-1}{\epsilon+2} \tag{25}$$

$I_0$ is the intracavity intensity, V is the volume of the sphere, related to the power as in equation (4) and $\epsilon$ is the dielectric constant of the particle. The trap frequencies are given by [25] [22]:

$$\omega_{axial} = \frac{2\pi}{\lambda}\sqrt{-\frac{2U_0}{m}} \tag{26}$$

$$\omega_{radial} = \frac{2}{\omega_0}\sqrt{-\frac{U_0}{m}} \tag{27}$$

The axial length of the traps is much smaller than the radial length. Therefore, $\omega_{axial}$ is a lot larger than $\omega_{radial}$. Without a cavity, this is the other way around. In figure 11 the potential well in the axial direction of the cavity is plotted. Here we considered an intracavity power of 10000W, a waist $\omega_0 = 200\mu$m and a nanosphere with a radius of 25nm. It is clear that the particle can be trapped at different positions. A lattice structure is formed with a lot of potential minima.

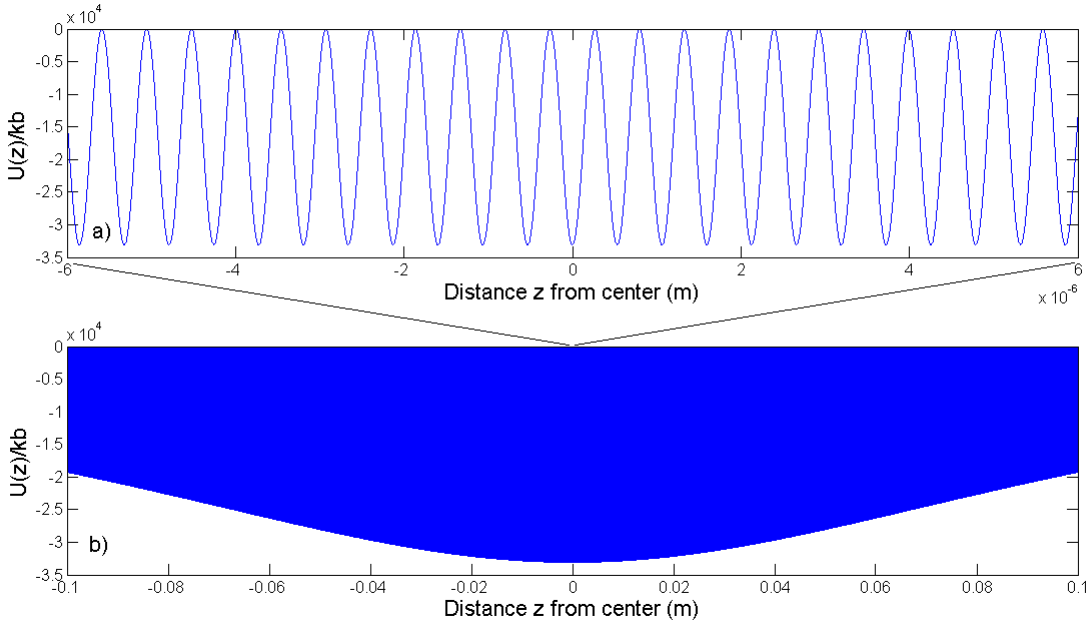


Figure 11: The potential well in an optical cavity. In figure b), the total lenght of the cavity is plotted and in figure a only a few micrometers at the center of the cavity.

With a single focused beam, the dipole trap is cigar-shaped like the focus of the beam and the confinement in the radial direction is stronger than in the axial direction. In a cavity, a standing wave is formed. So the nanoparticle can be trapped at the anti-nodes of the standing wave, because they have highest intensity.
This dipole approximation for cavity is unfortunately only usable for really small particles. For bigger nanospheres, the nanosphere itself changes the length of the cavity. For further information about optical cavities, see reference [22].

## 4.2 Optomechanical coupling strength

By introducing an nanosphere into an optical cavity, the resonant frequency of a cavity mode is shifted by an amount $\delta\omega$. We again consider the nanosphere to be much smaller than the wavelength. Taking a mode profile $E \propto cos(kx - \phi)$ and applying perturbation theory one finds:

$$\delta\omega = -\frac{3V}{4V_c}\frac{\epsilon - 1}{\epsilon + 2}cos(2kx - 2\phi)\omega \tag{28}$$

The optomechanical coupling strength is a measure for the interaction between the sphere and the optical field and is defined as [26] $g = \frac{3V}{4V_c}\frac{\epsilon-1}{\epsilon+2}\omega$.

### 4.2.1 Internal temperature rise of the nanosphere

The imaginary part of the dielectric constant $\epsilon$ causes a non-zero absorption cross-section. The absorbed power for a point-like dipole is:

$$P_{abs} = 12\pi V\frac{I_0}{\lambda}Im\frac{\epsilon - 1}{\epsilon + 2} \tag{29}$$

In this formula V is the volume of the sphere and $\epsilon$ the dielectric constant. The internal energy of the sphere increases due to the absorbed power. This is balanced by thermalization with a background gas and blackbody radiation. In ultrahigh vacuum, the energy absorbed is reradiated as blackbody radiation. This results in a temperature rise of the sphere. The internal temperature $T_{int}$ and the temperature of the center of mass $T_{c.o.m.}$ are not significantly coupled.

If the nanosphere is trapped under good vacuum conditions, effect of the bakcground gas is negligible and the main component of power dissipation aobsorbed by the sphere is balckbody radiation. The radii of the nanospheres we are considering are much smaller than the absorption lengths at and thus the usual formulas for blackbody radiated power don't apply. The cooling rate for blackbody radiation is:

$$\frac{dE}{dt} = -\frac{72\zeta(5)}{\pi^2}\frac{V}{c^3\hbar^4}Im\frac{\epsilon_{bb} - 1}{\epsilon_{bb} + 2}(k_bT_{int})^5 \tag{30}$$

In this formula T is the internal temperature and $\zeta(5) \approx 1.04$ is the Riemann zeta function. For silica, $\frac{\epsilon_{bb}-1}{\epsilon_{bb}+2} = 0.1$ and the dielectric constant is given by $\epsilon = \epsilon_1 + i\epsilon_2$ with $\epsilon_1 = 2$ and $\epsilon_2 = 1.0 \times 10^{-7}$ [27]. For a silica nanosphere with a radius of 50 nm, the intensity can be $10W/\mu m^2$ before the melting point is exceeded [17].

# 5 Cooling

When the sphere is trapped, we can start cooling it. There are a few different cooling methods. In this chapter, cavity cooling, (laser) feedback cooling and sympathetic cooling are discussed. The unit of the cooling rates is Hz. At first sight, this may seem like a weird way to express a rate, but we try to reach regimes where a nanoparticle has discrete energy states. A cooling rate of 1 Hz means a decrease in energy of $\hbar\Omega_0$ per second. $\Omega_0$ is the frequency of oscillator.

Optical cooling of nanoparticles differs a lot from optical cooling of atoms or simple molecules. The mechanisms to cool atoms are all very well understood and developed. Most methods of laser cooling rely on a coupling between laser light and well-defined internal states. Atoms have a structure with only a few resonant frequencies, which makes it easy to cool them. Doppler laser cooling works in the following way. An atom is traveling towards a laser beam and absorbs a photon from the laser. By momentum transfer of this photon, the particle is slowed down. Because it is necessary to have more absorptions from head-on photons than from photons from behind, the laser is tuned a little below the resonance absorption in the case of atoms. From the atom's perspective, a head-on photon is Doppler shifted towards its resonant frequency. A photon travelling in the opposite direction is shifted away from resonance. Therefore head-on photons are more strongly absorbed and the overall speed of an atom decreases. However, nanoparticles lack suitable closed optical transitions that could be used to exchange momentum between light and matter. Nanospheres are too big to have such an simple internal structure and only a few discrete resonance frequencies. So the well developed techniques to cool atoms do not work for nanospheres [28].

With cooling we mean cooling the center of mass motion of the nanosphere and thus the external temperature. The internal temperature can get really high. This causes apparent contradictions. For example, while cooling a nanosphere to its quantum ground state ($\mu K$ regime), at the same time we have to make sure that we do not heat the nanosphere so much that it will melt. Sometimes physics is just plain bizarre.

## 5.1 Quantum ground state

The goal of cooling nanoparticles is to bring them in their quantum ground state, with other words, its lowest energy point. The energy of the system in quantum ground state is known as the zero-point energy. The essential quantum features of a harmonic oscillator are quantised energy levels spaced evenly by $\hbar\Omega$, where $\Omega$ is the frequency of the oscillator (see figure 12). In an optical field, the quantised excitations are photons, in a mechanical oscillator, these excitations are called phonons. The mean occupancy of each level is[23]:

$$\bar{n} = \frac{1}{e^{\frac{\hbar\Omega}{k_B T}} - 1} \tag{31}$$

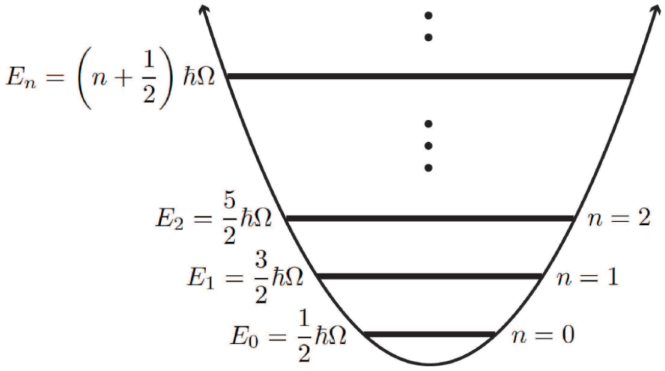


Figure 12: Quantized energy levels of an harmonic oscilator

If $(k_B T >> \hbar\Omega)$ this simplifies to:

$$\bar{n} = \frac{k_B T}{\hbar\Omega} \tag{32}$$

To resolve quantum ground state, $\bar{n} < 1$ is required. For a typical micro- or nano-mechanical oscillator with resonance frequency in the range $\Omega/2\pi = 1 - 1000$MHz at room temperature, $\bar{n}$ turns out to be $10^4 - 10^7$ an thus can be viewed as purely classical. When oscillators become smaller, their resonancy frequency typically increases. Typical frequencies for vibrations of atoms are around $10^{13}$Hz. For that reason, the vibrations of atoms and molecules must be viewed quanutm mechanically. Optical field are considered to be in their ground state as well. Visible light has a frequency of $\Omega/2\pi \sim 6 * 10^{14}$ Hz, a simple calculation shows that at room temperature, $\bar{n}$ is of the order of $10^{-35}$ [23].

We want to derive an expression for the steady state phonon number that depends on the cooling rate, in order to calculate which parameters we need for ground state cooling. The energy of the trapped sphere changes constantly due to interactions with its environment. We can write the time evolution of its average energy as [4]:

$$\frac{d}{dt}\bar{E}(t) = -\gamma[\bar{E}(t) - E_\infty] \tag{33}$$

In this formula $E_\infty$ is the average energy in the steady state, and this is reached by the rate $\gamma$. We have seen that we can write the energy in terms of discrete phonon quanta, the total energy of the nanosphere is $\bar{E} = n\hbar\Omega_0$. Substituting this in equation (33), we get

$$\dot{n} = -\gamma n + \Gamma \tag{34}$$

Here, the heating rate $\Gamma$ is introduced, it is defined as:

$$\Gamma = n_\infty \gamma = \frac{E_\infty}{\hbar\Omega_0}\gamma \tag{35}$$

The heating rate is the rate at which phonons are reintroduced in the mechanical system. The final phonon occupation number in the steady state is given by the ratio between the heating

and cooling rate

$$n_\infty = \frac{\Gamma}{\gamma} \tag{36}$$

So if we know what contributes to the heating and cooling of the nanosphere, it is possible to deduce the minimum phonon number that we can reach, the limit of cooling. Now a few general cooling and heating mechanisms are discussed. These mechanisms do not depend on the kind of cooling that is applied.

The first source of cooling is radiation damping $\gamma_{rad}$ due to the Doppler effect. It arises from the back-action of the scattered field on the motion of the particle $\gamma_{rad} \equiv P_{scatt}/mc^2$. A more important cooling source are collisions with a background gas. Thermalization with the environment damps the particle's motion with a rate $\gamma_{th}$. From kinetic energy theory the following equation for the cooling rate is found [29]:

$$\gamma_{th} = \frac{6\pi\eta R}{m}\frac{0.619}{0.619 + Kn}(1 + c_\kappa) \tag{37}$$

where $\eta$ is the viscosity coefficient of air, R the radius of the microsphere, $c_\kappa = 0.31Kn/(0.785 + 1.152Kn + Kn^2)$ an $Kn = s/R$ is the Knudsen number. s is the mean free path of the air molecules $s \propto 1/P_{gas}$. At low pressures, below 10 mbar, the damping factor is proportional to the pressure and the rate becomes [4]:

$$\gamma_{th} = \frac{15.8R^2 P_{gas}}{m v_{gas}} \tag{38}$$

In this formula, m is the mass of the gas molecules and $v_{gas} = \sqrt{3k_B T/m_{gas}}$ the root mean square velocity. In figure 13 the thermal damping rate for nanospheres with different radii is

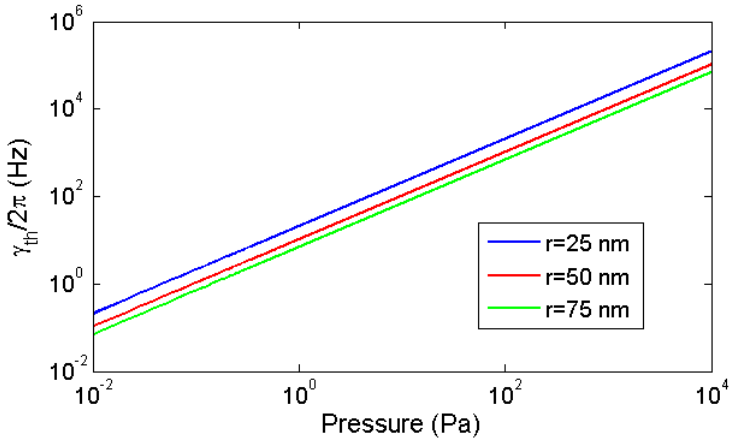


Figure 13: The thermal cooling rate depending on the pressure for different size of nanoparticles.

given. It can be seen that the thermal damping rate is very low (<10 Hz) at low pressures. $\gamma_{th}$ not only causes damping due to air molecules, but heating as well. This heating rate is

given by:

$$\Gamma_{th} = \frac{\gamma_{th} k_B T}{\hbar \Omega_0} \tag{39}$$

The damping and heating terms are in equilibrium for a mechanical energy of $k_B T$, therefore that is the average mechanical energy of of the sphere at thermal equilibrium.

Another contribution to the heating of the sphere is photon scattering, this is also called recoil heating. This is given by two equivalent equations [27][17]:

$$\Gamma_{sc} = \frac{2}{5} \frac{\omega_r}{\Omega_0} R_{sc} \tag{40}$$

and [4]

$$\Gamma_{sc} = \frac{1}{5} \frac{P_{scatt} \omega_0}{mc^2 \Omega_0} \tag{41}$$

The scattering rate is $R_{sc} = \frac{24\pi^3 I V^2}{\lambda^4 \hbar \omega} (\frac{n^2-1}{n^2+2})^2$ and the recoil frequency is given by $\omega_r = \frac{\hbar k^2}{2m}$. Furthermore, $P_{scatt} = \sigma_{scatt} I_0$ and the cross section $\sigma_{scatt} = |\alpha|^2 k^4 / 6\pi\varepsilon_0^2$ where $\alpha$ is the polarizability of the nanosphere. In 2016 the recoil heating rate was measured directly for the first time [4]. It was found that in experiments were the sphere is trapped by a single laser beam, the recoil rate of nanosize silica sphere in the order of 10kHz. There is a limit for the lowest temperature, because of the scattering light that lead to recoil heating.

## 5.2 Feedback cooling

The idea of feedback cooling is to observe the moving of the nanosphere and, using a fast feedback loop, engineer a force in the opposite direction of the movement. An optical position sensor irradiates the nanosphere with light and detects the scattered photons. The feedback lasers change the force they apply. The performance is mainly limited by the accuracy of which the position can be measured [5].

There are two different ways to implement feedback cooling. It can be applied by using the radiation pressure force of three different lasers or by using just the gradient force of a single laser beam [30]. In a scheme with a single laser, the feedback loop slows the particle's motion by increasing the trap stiffness (of the single gradient force) when the nanoparticle moves away from the trap center and reducing the stiffness when the particle falls back to the trap. As explained in the first part of this thesis, the gradient force works in three directions, so 3D cooling can be applied with a single laser beam. When three lasers are used, every laser cools just in one direction. Photodetectors measure the position of the particle and a feedback loop changes the power and position of the cooling lasers [30].

If we only consider thermal cooling and heating effects, then $n_{ss} = \Gamma_{th}/(\gamma_{th}+\gamma_{fb})$. Using equations (32) and (35) we can rewrite this, so that we get an expression for the final temperature

of the center of mass of the nanosphere:

$$T_{c.m.} = T_0 \frac{\gamma_{th}}{\gamma_{th} + \gamma_{fb}} \tag{42}$$

$\gamma_{fb}$ is the additional (feedback) cooling applied [30]. The cooling term $\gamma_{fb}$ only contributes to the damping of the sphere, and is not contained in the heating term. So feedback cooling is also called cold damping. It is favorable to have a good vacuum. Because the lower $\gamma_{th}$ gets, the higher the influence of the applied cooling is. When there is no cooling, a trapped microsphere will exhibit Brownian motion, a random motion resulting from collisions with air molecules. The amplitude of the (Brownian) motion of a trapped microsphere at thermal equilibrium is $x_{rms} = \sqrt{k_B T_0/(M\omega^2)}$. $T_0$ is the environmental temperature, $M$ the mass of the particle and $\omega$ the angular trapping frequency. The detection system must be at least accurate enough to measure this. So the resolution of the detection system has to be really high [5].

### 5.2.1 Measurement uncertainty

The position of the particle has to be measured to operate the feedback cooling loop. Therefore, measurement uncertainty of $x$, $y$ and $z$ introduced by shot noise limits the lowest reachable temperature [30]. A beam with power P is a stream of photons with a mean flux of:

$$\bar{N} = \frac{\lambda}{2\pi\hbar c} P \tag{43}$$

However, there is no guarantee that $\bar{N}$ photons arrive each second. There are some fluctuations, sometimes more or fewer photons will arrive. The standard deviation in a set of counts is $\Delta N = \sqrt{N}$. It follows that the fractional precision for measuring N is $\frac{\Delta N}{N} = \frac{1}{\sqrt{N}}$. So the more photons there are, the more precise the flux can be measured Also, the measurement accuracy has a fundamental quantum limit, which follows from $\Delta x \Delta p \geq \hbar/2$, where $\Delta p = \Delta N \hbar k$. $\Delta N = \sqrt{N}$ is the uncertainty in photon number, and $N_{scat} = \Delta t/(\hbar k c)$ the photon number of the scattered light. Combining this, we obtain $\Delta x \geq \sqrt{\hbar c \lambda/(8\pi P \Delta t)}$. If the bandwith $B = 1/\Delta t$ is known one can calculate the minimal temperature that can be achieved by using a specific set up by using $x = \sqrt{k_B T/(m\omega^2)}$. In ref [30] it is showed that the fundamental limit due to measurement uncertainty is a few microkelvin. This is enough for ground state cooling, so feedback cooling should allow to cool a laser-trapped particle to its quantum ground state.

Increasing the signal power at the detector by using a higher laser power will reduce the measurement uncertainty $\Delta x$. However, a strong scattering leads to recoil heating [30]. So when implementing feedback cooling, one needs to find a good balance.
Ground state cooling with use of feedback cooling demands an optimal detection and feedback scheme. In practice, the biggest difficulty of this system is that the detection system and feedback circuits have electronic noise and it will be really hard to cool a sphere into quantum ground state. The total detection efficiency is given by the parameter $\eta_c$ and comprises of the

photon collection efficiency, optical losses, splitting into separate detection paths. For zero point cooling the total detection efficiency has to be more than 2/5, at the moment the best efficiency is $\eta_c = 0.0005$ [4].
In 2011, Toncang Li and Mark Raizen succeeded in cooling a microsphere from room temperature to 1.5 mK using a feedback cooling scheme with three seperate lasers. The group of Jan Gieseler managed in 2016 to cool a nanosphere with a radius of 50 nm to 45 $\mu$K, which corresponds to a mean phonon number of 63 [4].

All in all the cooling of an trapped microsphere in vacuum towards the quantum ground-state of the particle can be achieved by feedback cooling. The cooling limit for feedback cooling is determined by the measurement uncertainty [29] and the fundamental cooling limit by the recoil heating of a particle.

### 5.3 Cavity cooling

Another cooling method that could be used to cool the mechanical motion of an trapped dielectric particle is cavity cooling. Theoretically, this cooling method could cool a nanosphere to its quantum ground-state from room temperature as well.

Cavity cooling works in the following way. A nanoparticle that travels through a standing light wave feels an attractive force towards the 'bright spots', the regions with highest intensity. This particles alters the distance between the mirrors, due to its index of refraction the light needs a longer time to travel through it. So it shifts the cavity resonance. Even a small length change will lead to a significant phase shift of the cavity field. Dependent on the position of the particle, it allows more or less light to enter the cavity. The overall cavity intensity will be highest whenever the particle leaves a bright region en lowest when the particles approaches it. So the nanoparticle always feels a stronger slowing force than an accelerating one. In conclusion, the particles gains less energy whet it runs downhill than it loses while going uphill [28].

In the case of a single cavity mode, the nanosphere is trapped at an intensity maximum. For small displacements, only coupling terms, between the C.O.M. motion of the nanosphere and the photons in the cavity, that are quadratic in x are relevant. However, linear coupling provides larger coupling rates, but it requires the particle to be positioned outside the intensity maximum. This can be achieved in multiple ways. For example by an optical tweezer external to the cavity [31], an Paul trap [32], by using gravity in a vertical cavity [33] or by using a second cavity mod [34]. This last option is also called 'self-trapping'. By using two cavity modes, there are two standing waves with different frequencies in the cavity. One is used for trapping and generally has a much higher intensity than the other that is used for the cooling. [34] These self-trapping oscillators are fundamentally different from other optomechanically cooled devices in the way that the mechanical frequency $\omega_m$ is not an intrinsic feature of the resonator but determined by the optical field. It is a function of one or both of the detuning

frequencies, $\Delta_1$ or $\Delta_2$ of the modes [35].

### 5.3.1 Cooling rates and cooling limits

The limit of cavity cooling is reached when the heating resulting from the absorbed light power exceeds the cooling power.
When a photon hits the nanoparticle, there are two possible outcomes. The nanosphere emits a photon with lower energy than the initial photon and gains energy, this is called Stokes scattering, or the nanosphere loses energy and the emitted photon has higher energy than the absorbed one. The latter is called anti-Stokes scattering. The laser cooling rate is defined as the difference of the rate of the Stokes anti–Stokes scattering.

$$\Gamma \equiv R_- - R_+ \tag{44}$$

Where $R_-$ is the anti–Stokes (cooling) and $R_+$ is the Stokes (heating) scattering rate.

$$R_\pm = \frac{\kappa \Omega_m^2}{\frac{\kappa^2}{4} + (\delta_2 \mp \omega_m)^2} \tag{45}$$

In this formula $\Omega_m$ is the effective opto-mechanical driving amplitude:

$$\Omega_m \equiv 2gk\sqrt{\frac{\hbar}{2m\omega_m}}|\alpha_2| \tag{46}$$

and the mechanical frequency $\omega_m$ is given by formula (26). This is the case when field mode 1 is purely responsible for trapping and mode 2 for cooling. g is the optomechanical coupling strength, k the wavenumber, $\kappa$ the linewidth, $\delta_2$ the detuning and $\omega_m$ the mechanical frequency.
In the limit $\omega_m >> \kappa$ and taking the maximum cooling rate $\Gamma \approx \kappa$ the phonon number is[17]:

$$\langle n_f \rangle \approx \frac{\kappa^2}{16\omega_m^2} + \phi\frac{\omega_m}{\kappa} \tag{47}$$

where $\phi$ is a dimensionless parameter $\phi \equiv \gamma_{sc}/\omega_m = \frac{4\mathfrak{l}^2}{5}\frac{\epsilon-1}{\epsilon+2}\frac{V}{\lambda^3}$ The first term corresponds to the cooling, the second therm to the photon recoil heating.
In figure 14 the mean phonon number for different sizes of nanoparticles is plotted. It turns out that the phonon number is much less than one for all finesses between 1000 and 10000. In almost all experiments microcavities are used, with length of only a centimeter. With a microcavity it is more difficult to reach a mean phonon number that is less than one, and higher finesse is needed. I did not find a reason why this could not be done with a larger cavity. An intensity of $10W/\mu m^2$ can be reached without exceeding the melting point of the sphere [17]. In the set up we consider, the intensity is about $0.16W/\mu m^2$, so far less.

In the unresolved sideband regime ($\kappa >> \omega_m$), the quantum limit is $n_{min} = \kappa/(4\omega_m)$ for $\delta_2 = \kappa/2$. The minimum phonon number can't reach 1, so ground state cooling is not possible.

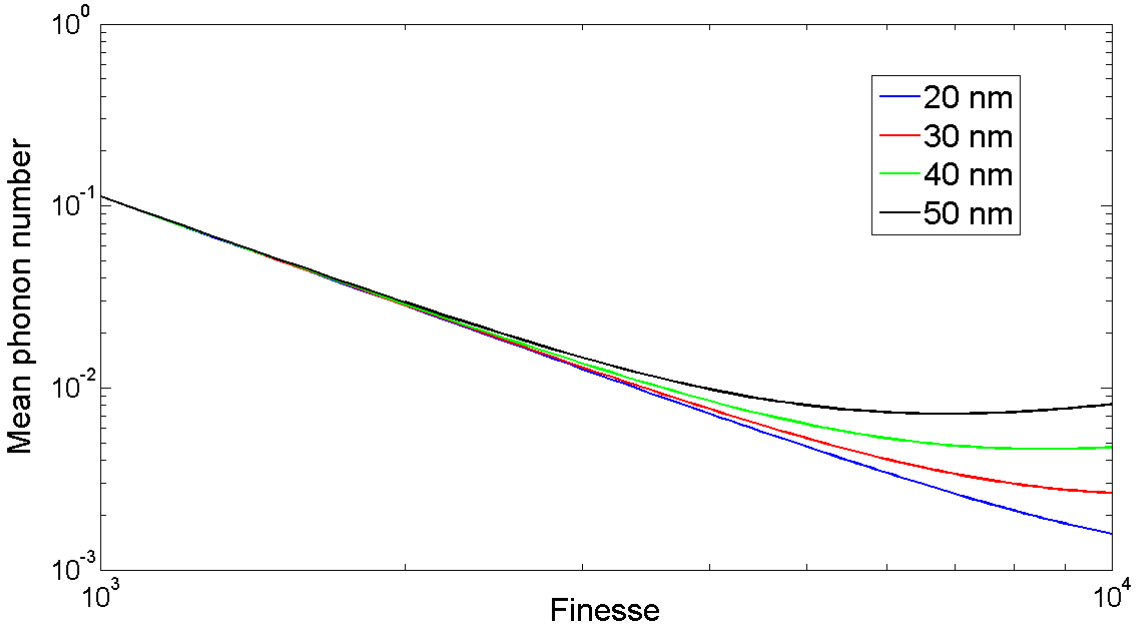


Figure 14: Phonon number, the length of the cavity l=18 cm and the power P=10kW.

But if $\omega_m >> \kappa$ (the resolved sideband limit), the quantum limit is $n_{min} = \kappa^2/(16\omega_m^2)$ and the ground state can be achieved for $\delta_2 = -\omega_m$ [36].
Concluding, we can say that reaching the zero point energy limit by cavity cooling is very well possible. In fig 14 it is showed that really low phonon numbers can be reached. Cavity cooling requires a good sideband limit, where the mechanical frequency of the oscillator is much higher than the linewidth of the cavity. This can be achieved by increasing the length of the cavity, increasing the finesse and/or increasing the intensity. In 2015, the group of Peter Barker [37] used cavity cooling to cool nanospheres to a minimum temperature of 1$\sim$3mK, starting from room temperature. They obtained final phonon occupancies of $n_p = 100 - 1000$.

### 5.4 Sympathetic cooling

As discussed in the introduction of this chapter, the cooling of atoms is a technique that is very well developed. Sympathetic cooling makes use of cold atoms or ions. In this technique, light forms a bridge between the C.O.M. of the atomic ensemble and the nanosphere. The atoms and nanosphere do not have to be close to each other, but may be separated from each other by a macroscopic distance [38]. Sympathetic cooling works in the following way. A sphere is captured inside a cavity. As the sphere moves, it changes the phase of light reflecting from the cavity. The atoms sit in a potential well that depends on the antinode positions of the standing wave. These are effected by the motion of the sphere, and therefore a force is imparted on the atoms. As the atoms move from their trapping minima, photon momenta restore them to their equilibrium position. The intensity of one of the components of the standing wave is increased or decrease, which imparts a force on the nanoparticle inside the cavity [24]. A schematic drawing is given in figure 15

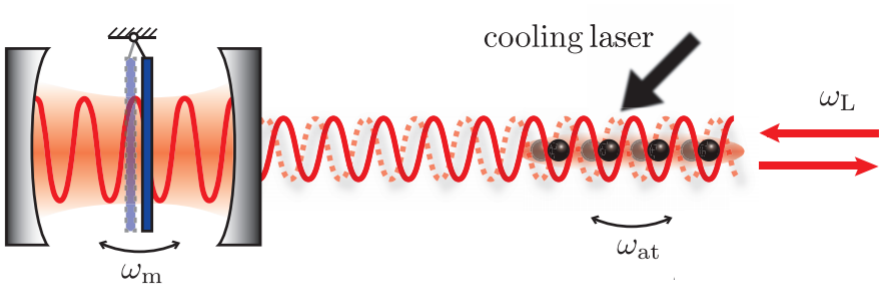


Figure 15: A set-up for sympathetic cooling. In our case the membrane would be a nanosphere. Picture from ref [39]

Until now, sympathetic cooling using atoms has been used to cool microscopic particles up to the size of proteins. In these experiments, the coolant and target thermalize through short range collisional or electrostatic interactions. However, with this method, a large mass difference reduces the cooling performance. So this method is not effective for nanoparticles. Another method is to use laser light to realize a long-distance interaction. The target and the cooling atoms are coupled by radiation pressure forces. By placing the target/nanosphere/membrane in an optical cavity, the forces are enhanced. This results in a large coupling, even though the atoms and nanosphere reside in separate vacuum chambers.

The standing wave in which the atoms sit, is generated in the following way. The optical cavity is driven by a strong laser beam. The reflectivity of the back mirror is much higher than that of the input mirror. Therefore, the light leaves through the input port and interferes with the incoming beam outside the cavity, creating an standing wave. This acts as an lattice potential for a cloud of cold atoms [24].

This sympathetic cooling scheme is analogous to the optomechanical cavity cooling. The role of the cavity is now played by the atoms. The big differences are however that it is tunable, in other words the cooling can be turned on and off and the scheme doesn't require the cavity linewidth to be much less than the mechanical frequency (resolved sideband limit).

To calculate the coupling between the atoms and the sphere, we first need the atom-field coupling rate. The atom-field coupling rate is given by:

$$G_{at} = \frac{\omega_{at}}{2\alpha k_L l_{at}}\sqrt{\pi N} \tag{48}$$

where $\omega_{at}$ is the atom trapping frequency, $l_{at} = \sqrt{\hbar/m\omega_m}$ is the harmonic oscillator length (zero point motion) for the atoms, $N$ is the atom number and $k_L = \omega/c$ is the cooling laser wave number [39] .

The sphere-light coupling rate is:

$$G_m = \frac{3V}{2V_c}\frac{\epsilon - 1}{\epsilon + 2}\omega k_L l_m \frac{\alpha}{\kappa_1\sqrt{\pi}} \tag{49}$$

where $\kappa_1 = 2\pi\kappa$. The effective optomechanical coupling connecting the mechanics of the sphere to the mechanical motion of the atoms is given by:

$$G_{tot} = 2G_{at}G_m \tag{50}$$

In the adiabatic limit, when the atom-cooling rate ($\Gamma_{at}$) is larger than the coupling rate G, the sympathetic cooling rate is:

$$\Gamma_{sym} = \Gamma_{at}\frac{g^2}{\Delta_m^2 + \Gamma_{at}^2/4} \tag{51}$$

In this formula, $\Delta_m$ is the difference of the mechanical trapping frequency of the sphere and the atoms, which is assumed to be zero. The steady state phonon number of the sphere is given by:

$$n_{ss} = \frac{\gamma_g n_m + \gamma_m^{diff}/2 + \gamma_{sc}}{\gamma_g + \Gamma_{sym}} + (\frac{\Gamma_{at}}{4\omega_{at}})^2 + \frac{\gamma_{at}^{diff}}{2\Gamma_{at}} \tag{52}$$

In this formula, $\gamma_{at}^{diff}$ is the heating due to Rayleigh scattering by the atoms, $\gamma_{sc}$ the Rayleigh scattering by the sphere, $\gamma_m^{diff}$ the radiation pressure noise of the sphere and $\gamma_g$ the damping of the sphere by collisions with the background gas. In reference [27] this is evaluated with real experimental parameters and they show that for an atom cooling rate of ∼10 kHZ, quantum ground state cooling is achievable.

### 5.5 Comparison

Cavity cooling, feedback cooling and sympathetic cooling all can in principle reach ground-state cooling. Cavity cooling requires a resolved-sideband regime and feedback cooling a sufficiently good measurement imprecision. Feedback cooling and sympathetic cooling add the possibility of turning off the cooling, to observe the strong-coupling dynamics [27]. Sympathetic cooling is quite analogous to cavity cooling. The biggest differences are that with sympathetic cooling it is easier to switch the cooling on and off and sympathetic cooling does not require the cavity linewidth to be much less than the mechanical frequency. So it relaxes the experimental requirements. On the other hand, the set-up of an experiment with sympathetic cooling is a bit harder, because the atoms and a cooling mechanism for them are needed as well. Over the last few years, feedback cooling improved a lot and the maximal cooling until now has been reached by using feedback cooling. However, noise in the feedback loop limits the cooling and it will be hard to improve the efficiency so much that ground state is reached [4].
In experiments with a good cavity sideband limit, cavity cooling is favorable over feedback cooling. Because ground state cooling by a feedback loop may be hard to achieve because of

the measurement noise and cavity cooling can not easily be turned off, sympathetic cooling seems to be the best option for cooling. There is no method that has big clear advantages over the other. Possibly, combinations of cooling methods will give the best results. For example, feedback cooling can be used to precool the nanoparticle, which makes stable sympathetic cooling easier. The advantages of each system could be used in an optimal way to cool the nanoparticles to ground state. We have seen that with the different cooling methods, it is possible to cool the nanosphere to ground state, and it seems like only a matter of time before the first group will manage to achieve that.
Something that is worth looking in to is the use of a Paul trap for more trapping stability. The use of a Paul trap could reduce the recoil heating of a nanosphere, since hte light intensity can be a lot lower.

# 6 Conclusion

In order to shed light on the transition of quantum to classical behaviour, cooled levitated nanospheres are a good candidate to perform research on. They do not have any physical contact with the environment, which causes a strong decoupling. By cooling them to their quantum ground state, we should be able to observe quantum effects and test the limits of macroscopic quantum behaviour.

Trapping nanoparticles have given rise to a whole new research field. Although the optical forces of light are not noticeable in daily life, for small particles the optical force can be bigger than the gravitational force on the particle. The optical force is can create a potential that is strong enough to trap nanospheres. The scattering force of light could provide a problem by forming a stable trap. Trapping particles in an optical cavity is a good way to decrease the effect of the scattering force and increase the power of the laser. A cavity confines light between two mirrors and thus a standing wave is formed. The result is a lattice of potential wells where the particle can be trapped.

To cool nanospheres to their quantum ground state there are different methods. Feedback cooling, cavity cooling or sympathetic cooling are three very promising cooling techniques. Theoretically, all these cooling mechanisms are capable of ground state cooling. These cooling methods have their own advantages and disadvantages, for different parameter regimes. All in all, we have a slight preference for sympathetic cooling, as it is a relatively unexplored cooling method for nanospheres that is very promising. It can be turned of and does not require good cavity limit. Furthermore, combining different cooling methods may give the best results in the future.

## Afterword

I found writing this thesis to be very interesting and instructive. It required an understanding of the concepts from a lot of different courses that I have taken in the bachelor. Knowledge of optics, quantummechanics, electromagetism and thermodynamics was needed.

I found it to be difficult to get a clear overview of the field and define the subjects I wanted to research. I learned a lot during this process about searching for information. Especially, finding accessible basic knowledge about the principles was hard. In papers not everything is always explained in detail and a lot of the subjects were too advanced to just google it. Looking back, there are some parts where I would have liked some more in depth content and others that could have been skipped. However, in the beginning I chose to cover a lot of subjects because every time I came across something new, I wanted to know more about it.

I really enjoyed writing this thesis. The moments that I all of a sudden got an understanding of what I was trying to or when I found a paper with exactly the information I needed were very rewarding. And, although I was not really part of the research group, I liked to attend all the meeting and get some insight into doing research. It was nice to be in a different environment than normally, when taking classes.

Finally, I would like to thank Steven Hoekstra and Artem Zapara for their help. Their feedback was really useful and Artem always pointed out the most recent papers to me.

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