Investigating the effect of external magnetic fields and polarisation of intense light fields on Ba$^+$ ions

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Abstract

The Ba$^+$ ion is well suited for investigating effects of atomic parity violation (APV). Furthermore it can be exploited to build an atomic clock in the optical frequency range. Optical spectroscopy of the $6s^2S_{\frac{1}{2}} - 6p^2P_{\frac{3}{2}}$, $5d^2D_{\frac{3}{2}} - 6p^2P_{\frac{1}{2}}$ and $6s^2S_{\frac{1}{2}} - 5d^2D_{\frac{3}{2}}$ transition is employed in the work towards measurement of APV on a single trapped Ba$^+$ ion. In preparation to this, experimental parameters affecting the $5d^2D_{\frac{3}{2}} - 6p^2P_{\frac{1}{2}}$ transition are investigated. The magnitude and direction of the magnetic field $\vec{B}$ as a function of current $I$ through sets of Helmholtz coils at the position of the ion was estimated by measuring the field at two locations close to a vacuum chamber. A non-linear dependence of $\vec{B}_z$ on $I$ was observed. This effect is not yet fully explained. Laser light polarisation was controlled using a half-waveplate. This waveplate is inserted in a motorised rotation optical mount. The polarisation direction of this waveplate was found to be in the vertical direction when the rotation mount was put at an angle of 18(1$^\circ$. The polarisation angles of blue and red laser light w.r.t. the vertical direction are determined to be $\theta_{\text{Blue}} = 51.10(19)^\circ$ and $\theta_{\text{Red}} = 56.41(80)^\circ$. The degree of circularity of the blue and red laser light is found to be $\phi_{\text{Blue}} = 1.158(38)^\circ$ and $\phi_{\text{Red}} = 43.8(4.2)^\circ$. An uncertainty in these values may be present as a consequence of the light passing through a dichroic beamsplitter. The transmittance and reflectance through this beamsplitter might depend on the polarisation of the light. This would result in a systematic error. Spectroscopy measurements have been conducted for two polarisation angles. One in the direction of a PMT to observe atomic transitions and one in the vertical direction. Two more measurements were performed for a different direction of the magnetic field, which is equivalent to rotating the polarisation. Matlab code was used to fit the recorded spectrum to a lineshape. The values obtained for polarisation and circularity are yet insufficiently precise to draw an unambiguous conclusion on the polarisation and circularity that the ion ‘sees’. Deviations of the model from previous measurements may be caused by a differing choice of coordinates. The polarisation was determined relative to the lab vertical axis, defined as the $z$-axis. The $z$-axis for the ion is defined by the quantisation axis, which is in the direction of the magnetic field. This direction does not in general correspond to the lab vertical axis. Suggestions for improved measurements are given. More measurements can be made to arrive at a distribution of polarisations, which is expected to peak at the value closest to the values the ion ‘sees’. Some improvements to the model may include accounting for motion of the ion or improving knowledge of the magnetic field or light field intensity.
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Chapter 1

Introduction

Single ions can be confined for long times of order days in particle traps. This enables precision measurements to investigate the effects of atomic parity violation (APV) at low energies. Single ions also have a great potential for very precise atomic clocks.

The Weinberg angle (weak mixing angle) $\theta_W$ describes the relative strength of electromagnetic and weak interactions and can be measured in precision experiments with single trapped ions. This is an important parameter for the electroweak interaction in the Standard Model, as it describes the relative strengths of the electromagnetic (EM) and weak interaction [1]. It can be related to the coupling constants of the electromagnetic interaction ($e$) and the weak interaction ($g_W$), the mass ratio of the $W^\pm$ and $Z^0$ bosons and the fine structure constant $\alpha$ [2]. The effective Weinberg angle depends on the momentum transfer in the interaction due to radiative corrections [2], see figure 1.1. Single trapped ions, in particular $\text{Ba}^+$ and $\text{Ra}^+$ are very well suited to determine the Weinberg angle at low energies. A deviation of $\theta_W$ has been predicted, e.g. if dark $Z$ bosons (or other unknown particles) would exist, in this regime. This could point towards physics beyond the Standard Model.

Figure 1.1: The squared sine of the effective Weinberg angle $\sin^2 \theta_W$ plotted versus momentum transfer in the interactions, $Q$. The solid line is the prediction by the SM. The dashed lines represent possible deviation from the predicted value at low energies. Source: [3].
Another goal of studying the Ba\(^+\) ion is to build an optical clock. Atomic clocks have been studied with ever increasing accuracy since 1945, when Isaac Isidor Rabi first proposed to build a clock based on atomic transitions in cesium [4]. These clocks were based on microwave-frequency transitions, which are on the order of \(10^{10}\) Hz. The Cs clock has been the standard for time ever since 1967 [5]. The precision of an atomic clock scales with the frequency and the inverse of the line width of the transition [1]. For this reason, atomic clocks in the optical regime have been considered for a long time. The invention of the femtosecond frequency comb made it possible to lock lasers phase coherently to frequencies in the microwave range. This is where the Cs clock operates. This could in principle enable for an optical atomic clock to be built [4].

The role of the Ba\(^+\) ion is as follows. The transition from the \(6s^2S_{\frac{1}{2}}\) to the \(5d^2D_{\frac{3}{2}}\) state has a long lifetime and with that, a low line width. The frequency of this transition is in the infrared, which is higher than the microwave frequency, where the Cs clock operates. Because of this, a clock based on Ba\(^+\) can potentially reach a higher precision than that of today’s Cs based clocks [4]. Figure 1.2 displays the development of atomic clocks and shows how clocks using femtosecond frequency combs have surpassed their microwave predecessors in the past decade in terms of precision.

![Figure 1.2: A graph of the precision of atomic clocks over the past decades. Adopted from a poster made by E.A. Dijck.](image)

These experiments are performed by trapping a Ba\(^+\) ion in a radio frequency (RF) Paul trap. The Paul trap is positioned inside a vacuum chamber. The ions are brought into the chamber using a Ba oven. This consists of a small needle, containing BaCO\(_3\). By running a current through the needle, Ba atoms are released. When they enter the Paul trap the Ba atoms are ionised by laser light. The Ba\(^+\) ion is then cooled using light fields. A laser is scanned over a range of frequencies and count rates as a function of frequency are collected. The features of these spectra hold information about the ion, but also about the external parameters.
Chapter 2

Currently Ongoing Research in the Ba$^+$ Ion Project

The determination of $\sin^2(\theta_W)$ is a precision measurement. This means very precise and accurate control over and knowledge of the experimental parameters is required. As of now, this knowledge is not yet sufficient for determining $\sin^2(\theta_W)$. To investigate these parameters, preparatory experiments are performed.

2.1 Description of the Ba$^+$ Ion

Barium has 56 protons in its nucleus and is therefore the 56th element in the periodic table. The Ba$^+$ ion has an electronic configuration that is similar to that of Cs: [Xe]6s$^1$. All electrons that make up the Xe configuration are tightly bound in a closed shell. Therefore, the energy levels of the Ba$^+$ ion are assumed to be completely determined by the one valence electron in the 6s$^1$ shell. This makes the Ba$^+$ ion a relatively simple system to study. The isotope $^{138}$Ba is used because it has no nuclear spin and therefore no hyperfine-splitting due to the coupling of the nuclear magnetic moment with the electron [6].

Earlier experiments, involving a collimated Cs beam, have also determined APV [7]. Unlike the Cs atom, Ba$^+$ is charged and can therefore be trapped using electric and magnetic fields. Trapping an ion at a well-defined location allows for better control over and monitoring of relevant parameters, compared to using an atomic beam. Examples of these parameters are the magnitude of external AC or DC electric and magnetic fields, laser frequency at the location of the ion and the quality of the vacuum. This is an advantage of using Ba$^+$ instead of atomic Cs.

Furthermore, the APV matrix element scales with $Z^3$, here $Z$ is the atomic number [8]. This means that the APV effect is larger for larger $Z$. The weak interaction effect as a function of $Z$ has been given in figure 2.1. The only candidate that has a higher $Z$ than Ba and also meets all of the above criteria is the Ra$^+$ ion. The Ra$^+$ ion is not used, however, because it is radioactive, which compromises the possibility to trap the ion for a long period of time. Eventually an experiment with Ra$^+$ would still be preferable. The stable Ba$^+$ ion is used to gain control over the experimental conditions in preparation to an experiment with Ra$^+$. 
Figure 2.1: The APV matrix element as a function of atomic number $Z$. The red line includes a relativistic correction, making Ra$^{+}$ even more preferable over Ba$^{+}$ [9].

The energy levels of the Ba$^{+}$ ion are schematically displayed in figure 2.2. The $6s^2S_{1/2}$ and $5d^2D_{3/2}$ are mixed. This mixing and its effects are being discussed in [3]. In the currently ongoing research the $6p^2P_{3/2} - 5d^2D_{3/2}$ transition is investigated. Relevant parameters in this research are the magnetic field, the detuning of the light field and the polarisation of the light field. These parameters are discussed in section 2.2, 2.3 and 2.4 respectively.

Figure 2.2: The lowest energy levels of Ba$^{+}$. 
2.2 Light Shift

When a two-level system (for instance, the subsystem consisting of the 6p$^2$P$^{1}_{\frac{1}{2}}$ and 5d$^2$D$^{3}_{\frac{3}{2}}$ levels) is introduced to an oscillatory electric field, e.g. a light field, the AC-Stark effect occurs. If the light field is on resonance with the transition frequency between these states, they are coupled. This means that the population oscillates strongly between the two states. This oscillation occurs at the Rabi frequency, $\Omega$, given by \[\Omega_{ij} = \frac{e}{\hbar} \langle i | \vec{E} \cdot \vec{r} | j \rangle \] \hspace{1cm} (2.1)

Where $\vec{E} \cdot \vec{r}$ is the electric potential at the location of the ion. However, if the light field is off resonance (detuned) with the transition frequency, no population transfer will take place. Instead, the energy levels will either move further apart or closer together. This is displayed in figure 2.3. The amount by which the energy shifts in the case where $|\delta| \gg \Omega$ is given by \[\Delta E_{1,2} = \pm \frac{\hbar \Omega^2}{4\delta} \] \hspace{1cm} (2.2)

The shift of the energy levels due to the interaction with a light field is also referred to as light shift. In this experiment $\delta \sim 10^7$ Hz and the frequencies of the light fields are in the optical frequency range, on the order of $10^{14}$ Hz.

The amount by which the energy levels oscillate is given by $\Delta E_{1,2}$, which is equal to $\pm \frac{\hbar \Omega^2}{4\delta}$.

Figure 2.3: A simple schematic of light shift. Left, the laser has a lower frequency than the transitions. The energy levels move apart. This is called red detuning. Right, the laser has a higher frequency than the transitions. The energy levels move together. This is called blue detuning. Adopted from [3].

In the case of this experiment not only two level systems are considered, but also the three level system consisting of all energy levels displayed in figure 2.2. The ion interacts with two light fields, one corresponding to the 493 nm transition and one corresponding to the 650 nm transition. Both light fields have detuning $\delta_g$ and $\delta_r$, respectively. If one of the light fields is at resonance with its respective transition the population of the 6p$^2$P$^{1}_{\frac{1}{2}}$ state is maximal and a maximum of 493 nm and 650 nm photons is observed. If $\delta_g = \delta_r$, the population of the 6p$^2$P$^{1}_{\frac{1}{2}}$ state is minimal. Direct transitions from the 6s$^2$S$^{1}_{\frac{1}{2}}$ and 5d$^2$D$^{3}_{\frac{3}{2}}$ state occur. These are called two-photon Raman transitions. Because the 6p$^2$P$^{1}_{\frac{1}{2}}$ state is not populated, no photons of 493 nm and 650 nm are emitted. This will be placed in the context of spectroscopy in section 2.5. A more theoretical framework is given in [3].
2.3 The Magnetic Field

If the Ba$^+$ ion is exposed to a non-oscillatory magnetic field $\vec{B}$, the Zeeman effect occurs. This causes splitting of energy levels according to the mechanism described below.

The energy levels of Ba$^+$ have a fine structure due to spin-orbit coupling. The $6p^2P_{1/2}$ level has $j = \frac{1}{2}$ and therefore $m_j = \pm \frac{1}{2}$ and thus has two degenerate sub levels. Similarly, the $5d^2D_{3/2}$ level contains four degenerate sub levels.

When the ion is exposed to an external magnetic field, the energies of these levels start to differ and they are no longer degenerate. Transitions can then occur between the sub levels of the two states if the selection rule $\Delta m_j = 0$, $\pm 1$ is obeyed for dipole transitions [6]. For two photon transitions, $\Delta m_j$ may equal 2. The internal field of the Ba$^+$ ion is far greater than the external field. Thus, the Zeeman effect can be treated in the weak field limit, where

$$\Delta E = g_J m_J \mu_B B_{\text{ext}}$$ (2.3)

Where $\Delta E$ is the amount by which the energy levels deviate from their degenerate states due to the external field $B_{\text{ext}}$, $g_J$ is the Landé g-factor, $\mu_B$ is the Bohr magneton. The magnetic field in this experiment is controlled through a set of Helmholtz-coils in three directions. By running a current through the coils, the field can be manipulated in all three directions. There are also external, uncontrolled magnetic fields present. Possible sources and their effects are listed in section 3.1.

2.4 Polarisation of Light Fields

When the energy levels of the Ba$^+$ ion are split through the Zeeman effect, two types of transitions can occur. Transitions where $\Delta m_j = 0$ are called $\pi$-transitions and correspond to linearly polarized light. Photons of $\pi$ polarisation are emitted or absorbed in the radial direction w.r.t. the quantisation axis of the ion, which is in the direction of $\vec{B}$ [11]. Transitions where $\Delta m_j = \pm 1$ are called $\sigma^\pm$-transitions. They correspond to either clockwise or anti-clockwise circularly polarized light. Photons of $\sigma^\pm$ polarisation are emitted and absorbed in the direction of the quantisation axis of the ion [11]. A schematic representation of this phenomenon is displayed in figure 2.4.

![Schematic](image)

**Figure 2.4:** A simplified depiction of the allowed dipole transitions from the $6p^2P_{1/2}$ state, which has two Zeeman sub levels (upper) to the $5d^2D_{3/2}$ state, which has four Zeeman sub levels (lower). The relative transition probabilities are given next to their respective transitions [13].

When treating this in the context of two-photon Raman transitions between the $6s^2S_{1/2}$ to $5d^2D_{3/2}$ state, $m_j = \pm \frac{1}{2} \rightarrow \mp \frac{1}{2}$ transitions may occur as well.
2.5 Spectroscopy

After the Ba$^+$ ion has been trapped the ion is exposed to three light fields. One light field drives transitions between the 6$s^2$S$_\frac{1}{2}$ and 6$p^2$P$_\frac{3}{2}$ states. This field has a wavelength of approximately 493 nm and is therefore also called the blue laser light. The second light field has a wavelength of approximately 650 nm and is detuned with respect to the 5$d^2$D$_\frac{3}{2}$ to 6$p^2$P$_\frac{3}{2}$ transition. This light field is causing light shift and is therefore also called the light shift laser light, or LS laser light. A third light field also has a wavelength of approximately 650 nm. This is called the red laser light and is scanned over a range of frequencies, including the frequency of the 5$d^2$D$_\frac{3}{2}$ - 6$p^2$P$_\frac{3}{2}$ transition.

Photons are collected using a photomultiplier tube (PMT) and an electron multiplying CCD camera (EMCCD). At the Raman frequency nearly no photons are collected due to the mechanism described in section 2.2. In a frequency spectrum it manifests as a dip. This is called a Raman dip and it holds information on the frequency of a transition.

The Zeeman effect can be observed from the spectra as well. Suppose, for example that the energy levels of the 6$p^2$P$_\frac{3}{2}$ and 5$d^2$D$_\frac{3}{2}$ states split into two and four sub levels respectively. The Raman dip corresponding to the 5$d^2$D$_\frac{3}{2}$ - 6$p^2$P$_\frac{3}{2}$ transition will then split into several other Raman dips. Their separation corresponds to the separation of the energy levels given by equation (2.3). Whether and where these dips occur depends highly on the polarisation of the light fields, due to the mechanism described in section 2.4.

An example of spectrum is given in figure 2.5. The horizontal axis displays the range of frequencies over which the red laser light was scanned. The vertical axis are photon count rates by the PMT. The dip is where Raman transitions occur.

![Figure 2.5: An example of a spectrum obtained from optical spectroscopy. The dip is caused by direct transitions from the 5$d^2$D$_\frac{3}{2}$ to the 6$s^2$S$_\frac{1}{2}$ state. The spectrum indicated in red is shifted to the left w.r.t. the spectrum indicated in blue. This is caused by light shift; the red spectrum is with the effect of light shift and the blue spectrum is without this effect. The experiment is described in section 3.4. Source: [12].](image-url)
Chapter 3

Measurements

The magnetic field at the location of the ion is an important parameter in this experiment. It defines the quantisation axis of the ion and induces the Zeeman effect. This effect is exploited to measure APV. The magnetic field is controlled using Helmholtz coils. In section 3.1, the experiment to determine magnetic field as a function of the current through the coils is described. The polarisation of the incident light influences which transitions between energy levels are likely to occur. The polarisation direction of the light field is controlled using a half-waveplate. In section 3.2 the set of measurements performed to determine the direction of the half-waveplate is described. The polarisation can be controlled, but also needs to be known. Measurements on the polarisation direction and circularity of the blue and red laser is described in section 3.3. After determining the polarisation of the incident light it is important to perform measurements on the ion to investigate its effects. In section 3.4, two spectra with similar experimental conditions, except for the direction of the magnetic field and polarisation are displayed and discussed.

3.1 Magnetic Field as a Function of Current

The magnetic field arises from several sources, which can be categorised into two groups. DC fields are magnetic fields which vary slowly, or not at all over time. Examples include the field caused by the earth or the field caused by the magnetic table on which the setup rests. AC fields are oscillating fields. Potential causes of AC fields are alternating currents close to the setup. This could compromise the homogeneity of the field inside. However, there have been studies showing that the magnetic field of square coils, more than one ‘radius’ apart can be expected to be homogeneous. Up until now the magnetic field $\vec{B}$ as a function of the current through the coils $I$ was not well known. This experiment is designed to calibrate $\vec{B}$ in all directions as a function of $I$.

3.1.1 The Experiment

To control $\vec{B}$ at the location of the ion, the vacuum chamber is placed at the centre of Helmholtz coils. The setup is schematically displayed in figure 3.1. The coils are square-shaped. Each set of coloured lines at the sides of the square represents a coil. The brown square is the coil facing the vertical direction. With approximate Helmholtz configuration is meant that the distance between opposing coils is not equal to their radius. This could compromise the homogeneity of the field inside. However, there have been studies showing that the magnetic field of square coils, more than one ‘radius’ apart can be expec-
Figure 3.1: A sketched top view of the setup. The dashed line (purple) represents the ionisation laser beam. The dotted line (blue) represents the blue laser beam. The twodashed line (red) represents the red laser beam. Probes to measure $\vec{B}$ were placed at location 1 and 2.

...ted to still be homogeneous or even exceed the homogeneity of circular coils [14][15][16]. With these considerations in mind, the magnetic field as a function of the current can be determined.

For measuring $\vec{B}$, a P.W. Bell Model 8010 Gauss/Teslameter, with a transverse probe was used. The direction in which the probe measured $\vec{B}$ was not known at first. This was determined using a different field meter from AlphaLab Inc. model GM2. This meter is capable of measuring the field in different directions. Knowing that earth’s magnetic field points more or less vertical into the ground at the location of this experiment, the direction in which the GM2 measured could be determined.

Using this information the measured values could be compared to the Model 8010 meter. This done, each component of $\vec{B}$ was determined at location 1 and 2 in figure 3.1 at the approximate height of the ion. It was not feasible to measure the field at position 3 due to obstructions. The ion pump, close to position 4 created a field, making measurements at this location unrepresentative. For this reason, measurements at location 4 were omitted as well.

First, the field was brought to zero in all directions by adjusting the current through the coils until the meter read zero field. The current was then varied from -5 A to +5 A, in steps of 1 A though one set of coils at a time. Negative currents create a magnetic field in the opposite direction. For each current through one set of coils, its corresponding component of $\vec{B}$ was determined. So, if a current was run through the coils which induce a field in the x-direction, the x-component of $\vec{B}$ was measured. From electromagnetic theory it is known that $\vec{B}$ of a finite wire of length $L$ is given by

$$B = \frac{\mu_0 I}{2\pi a} \left[ \frac{x}{\sqrt{a^2 + x^2}} + \frac{L - x}{\sqrt{a^2 + (L - x)^2}} \right], \tag{3.1}$$

where $a$ is the distance to the wire, $x$ is the position along the wire and $I$ is the current through the wire. The square Helmholtz coils can be approximated as a superposition of several line currents, obeying equation (3.1). Therefore, a linear relationship between $\vec{B}$ and $I$ is expected.
3.1.2 Results

The measurements have been performed and the data and fitted functions are displayed in figure 3.2. As expected, $\vec{B}(I)$ is linear. A difference in slope is observed. This may be caused by the distance from the probe to the coils under investigation not being equal. From equation (3.1) it can be seen that if $a$ gets smaller, $\frac{\partial \vec{B}}{\partial I}$ gets larger.

The intercept is equal to the background field. The background field is greatest in the negative z-direction (pointing downwards). The total magnitude of the background field calculated from the average field is 0.4 G, which is on the same order as earth’s magnetic field. For these two reasons, it is concluded that the background field is likely mostly due to earth’s magnetic field.

The field at the location of the ion was calculated by taking the average of the field at both locations. This assumption is made by using that there are no (ferro- or dia)magnetic components between location 1 and 2 and the ion. It is also assumed that the field is close to homogeneous between the two locations. Because of that, a linear interpolation is expected to be valid. The functions displayed in the three lower graphs are expected to represent $\vec{B}$ at the location of the ion as a function of $I$.

The uncertainty in $B_z$ is larger than the uncertainty in $B_x$ and $B_y$. To investigate where this comes from, the differences between the fitted function and the measured values have been plotted in figure 3.3. The deviation from the fitted function and the measured values is more or less randomly distributed for $B_x$ and $B_y$ at both locations. For $B_z$ this distribution does not appear to be random. There is extra dependence on $I$, with a minimum at $I = 0$. This dependence is not included in the model. This result is discussed in section 4.1.
Figure 3.2: The magnetic field $B$ in Gauss as a function of the current $I$ in Ampère for each direction. At location 1 and 2 (upper three graphs) and an estimate for the field at the location of the ion (lower three graphs).
Figure 3.3: The differences with the fitted functions and the measured magnetic field. The upper three graphs are for location 1 and the lower three graphs are for location 2.
3.2 Polarisation Direction of Waveplate

3.2.1 The Experiment

For controlling the direction of polarisation of the incident light, a half-waveplate is used. A half-waveplate is a birefringent crystal. A birefringent crystal does not have a uniform index of refraction; it depends on the direction of the electric field vector $\vec{E}$ of the light. The directions in which the index of refraction is maximal or minimal are called the slow and fast axis respectively. The fast axis is equal to the optic axis of the crystal, whereas the slow axis is perpendicular to it in this case. The indices of refraction are called $n_e$ for the fast axis and $n_o$ for the slow axis. If linearly polarised light enters the waveplate, the component parallel to the fast axis will travel faster than the component parallel to the slow axis. This will result in a phase shift between the components. When both components are added in superposition after leaving the waveplate, this may in general result in a change of circularity of the polarisation, or a change of direction in this specific case. The phase shift $\Delta \varphi$ is given by [17]

$$\Delta \varphi = \frac{2\pi}{\lambda_0} d(|n_o - n_e|) \ ,$$

where, $d$ is the thickness of the waveplate and $\lambda_0$ is the wavelength of the light prior to entering the waveplate. In the case of the half-waveplate, $d$ is such that $\Delta \varphi = \pi$. This thickness must then be [17]

$$d(|n_o - n_e|) = (2m + 1)\frac{\lambda_0}{2} \ ,$$

where, $m \in \mathbb{N}$ as any odd integer number of equally thick layers results in the same effective rotation ($\Delta \varphi = \pi, 3\pi, 5\pi$ etc.). If $\Delta \varphi = m\pi$, the resulting light will have its polarisation rotated by $2\theta$. A schematic description of this process is displayed in figure 3.4.

![Figure 3.4: An illustration of the mechanism of a half-waveplate. The wave component parallel to the optic axis travels through the material faster than the component perpendicular to the optic axis. This results in an effective rotation of linearly or elliptically polarised light. Perfectly circularly polarised light is not affected.](image)

The waveplate was mounted in a Thorlabs motorized Rotation Stage/Mount of type PRM1Z8, from now on referred to as stage 1. Stage 1 was positioned such that the laser would have to pass through the waveplate before entering the vacuum chamber. A polarisation filter, from now on referred to as an analyser, was mounted in an equal rotation stage, from now on referred to as stage 2. The setup is schematically displayed in figure 3.5. Both stages can be programmed to make a rotation of any angle, up to a precision of 25 arcseconds. They can also be programmed to rotate to a predefined angle step by step, where they will remain stationary for a predefined period of time before proceeding to the next angle.
Chapter 3. Measurements

Figure 3.5: The setup used for determining the polarisation direction of the half waveplate. The dashed line (purple) represents the ionisation laser beam. The dotted line (blue) represents the blue laser beam. The twodashed line (red) represents the red laser beam. The TMP indicates the location of the turbo pump.

The stages were programmed such that stage 2 would make one full rotation, after which stage 1 would make a step of 10°. Stage 2 would proceed with making one full rotation again, and so on, until stage 1 made one full rotation. The angle of both stages are from now on referred to as θ₁,₂, for stage 1 and 2 respectively. These angles are not to be confused with the polarisation angles of the optical elements which are positioned in the stages.

Photon count rates were collected using a photodiode (PD). A minimum count rate is expected when the polarisation direction waveplate and the analyser are perpendicular. A maximum is expected if they are parallel.

The data was collected using Root, a program developed by CERN for large scale data analysis. For each θ₁, count rate as a function of θ₂ was plotted and fitted to a function of the form

\[ \Gamma_{PD} = p_0 \sin^2 (p_1 \theta_2 + p_2) + p_3, \]  

where, \( \Gamma_{PD} \) is the count rate of the PD and \( p_{0,1,2,3} \) are fitting parameters. The code used to fit the data is given in Appendix A. The parameters are given the labels:

- \( p_0 \) — Amplitude [s⁻¹]
- \( p_1 \) — Frequency (unitless)
- \( p_2 \) — Phase (unitless)
- \( p_3 \) — Offset [s⁻¹]
3.2.2 Results

Plotting and fitting the data yielded sinusoidal graphs for each step of $\theta_1$. Figure 3.6 is an example of one of these plots. For every fit, the fitting parameters were collected and exported to an excel data sheet. These fitting parameters could then again be related to $\theta_1$. A table of these parameters is given in Appendix B.

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<td>$p_3$</td>
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Figure 3.6: An example of PD count rates as a function of $\theta_2$. The blue dots are the data and the red line is the fit. This graph is for $\theta_1 = 80^\circ$.

The frequency of these plots is always close to unity, because rotating stage 2 by an angle corresponds to rotating the analyser by the same angle. This is different for the waveplate, which rotates the polarisation by twice the angle that the stage is rotated in.

The phase is an important parameter, because it holds information on the relative angle between the waveplate and the analyser. The phase as a function of $\theta_1$ was plotted in figure 3.7. From figure 3.7 a linear relationship between the phase and $\theta_1$ is observed. The phase $p_2 \propto 2\theta_1$, as expected from the information above.

From this information, the polarisation direction of the waveplate can be calculated. The phase is zero if $\theta_1 = -44(1)^\circ$. With the phase equaling zero, the function $\Gamma_{PD}(\theta_2)$ reduces to:

$$\Gamma_{PD} = p_0 \sin^2 (p_1 \theta_2) + p_3$$  \hspace{1cm} (3.5)

This function has a maximum for $\theta_2 = 90^\circ + k \cdot 180^\circ$, with $k \in \mathbb{Z}$.

It can be concluded that there is a maximum transition through the analyser when $|\theta_1 - \theta_2| = 134(1)^\circ$. 

Figure 3.7: The phase of the signal against $\theta_1$

Now, using an analyser of known polarisation direction, the direction along which the polarisation is vertical is determined. It was found that the polarisation of the LS laser light is in the vertical direction for $\theta_1 = 18(1)^\circ \pm n \cdot 90^\circ$. This was verified by qualitatively measuring PD count rates. The count rate did drop if $\theta_1$ deviated from $18^\circ$. Thus, this value represents a local maximum for the PD count rate. It can therefore be concluded that the waveplate polarises the light vertically when stage 1 is at $18(1)^\circ$. 
CHAPTER 3. MEASUREMENTS

3.3 Polarisation of the Blue and Red Light Fields

In this experiment the polarisation direction and the degree of circularity (explained in subsection 3.3.2) is determined.

3.3.1 The Experiment

For this experiment, the same setup as described in subsection 3.2.1 is used. In this case, however stage 1, including the waveplate, was removed from the setup. The light was collected using several PDs as displayed in figure 3.8. These PDs were positioned the location indicated by ‘Photodiode’ in figure 3.5.

![Figure 3.8: The setup of photodiodes used for determining the polarisation direction of the red and blue lasers.](image)

The incident light is reflected twice by a non-polarising beam splitter from Thorlabs of type BSX10. The transmitted light passes through a dichroic beamsplitter from Thorlabs of type DMLP567R. This beamsplitter has a cutoff wavelength somewhere between 493 nm and 649 nm. This causes the blue light to be reflected to the blue PD, and the red light to be transmitted to the red PD. A portion of the light is reflected by the second BSX10 beamsplitter and is collected by a dump. This dump does not measure photon count rates, but the total power of the incident light.

It was found that the transmittance and reflectance through the dichroic beamsplitter depend on the polarisation of the light. It was not known what this dependence was and therefore, the blue and red photodiode were not used. The dump was used to collect the data, as the reflectance of the BSX10 beamsplitters did not depend on the polarisation.

The data was collected and analysed using ROOT. The data was plotted and fitted to a function of the form equal to (3.4). The parameters carry the same labels as given in table ??.

3.3.2 Results

Plots of the measured power versus the analyser angle and their fitted graphs are displayed in figures 3.9 and 3.10. Note that the x-axis of the plots display the actual analyser angle and not $\theta_2$. This correction was made using the results from the experiment described in section 3.2. The analyser angle will from now on be called $\theta_A$. The x-axis of the graphs displays $\theta_A$ w.r.t. the vertical. The function parameters are given in the figures. Nearly no power is measured at the minima. This implies linear polar-
isation. The red laser is elliptical. This can be seen from the fact that there is periodicity, but the measured power does not drop all the way to zero. If there were a constant transmittance, the light would be circularly polarised.

Figure 3.9: PD count rate versus $\theta_A$ (blue dots) for the blue laser. The fitted function is displayed in red. The fitting parameters are given in the list next to the graph.

Figure 3.10: PD count rate versus $\theta_A$ (blue dots) for the red laser. The fitted function is displayed in red. The fitting parameters are given in the list next to the graph.
The polarisation of the light relative to the vertical can be calculated by equating $\sin^2 (p_1 \theta_A + p_2) = 1$. This is equivalent to equating $p_1 \theta_A + p_2 = 90^\circ$. Solving for $\theta_A$ gives:

$$\theta_A = \frac{90^\circ - p_2}{p_1} \quad (3.6)$$

Plugging in the fitting parameters for both the blue and red laser yields:

$$\theta_{\text{Blue}} = 0.5110(19)^\circ \quad (3.7)$$
$$\theta_{\text{Red}} = 56.41(80)^\circ \quad (3.8)$$

The electric field component of polarised light travelling in the $z$-direction can be expressed as:

$$\vec{E} = E_0 [\sin (kz - \omega t) \hat{x} + A \cdot \cos (kz - \omega t) \hat{y}] \quad (3.9)$$

Note that the directions defined in this equation do not necessarily correspond to the axes given in figure 3.5. This field can be normalised by equating $|\vec{E}|^2 = E_0$, but this is not relevant for this calculation. What is relevant is the parameter $A$. This is a parameter that describes the circularity of the light, defined as:

$$A = \sin(\phi) = \frac{P_{\text{min}}}{P_{\text{max}}} \quad (3.10)$$

Here, $\phi$ is the degree of circularity and is used in this set of experiments to describe the circularity of the polarised light. The way the degree of circularity is defined is arbitrary, but this is the way it is defined in the model that was used to describe the frequency spectra. If $\phi = 0$, the $y$-component in equation (3.9) vanishes and the light is linear in the $x$-direction. If $\phi = 90^\circ$, both components in equation (3.9) have equal amplitudes, but a phase difference of $\frac{\pi}{2}$. The light is then circularly polarised. Any value of $\phi$ in between 0$^\circ$ and 90$^\circ$ yields elliptically polarised light. The values $P_{\text{min}}$ and $P_{\text{max}}$ are the minimum and maximum value as determined from the fitted functions in figures 3.9 and 3.10. This yields

$$\phi_{\text{Blue}} = 1.158(38)^\circ \quad (3.11)$$
$$\phi_{\text{Red}} = 43.8(4.2)^\circ \quad (3.12)$$

It is concluded that the blue laser is linearly polarised in the vertical direction to good approximation. Both $\theta_{\text{Blue}} = 0.5110(19)^\circ$ and $\phi_{\text{Blue}} = 1.158(38)^\circ$. The red laser is elliptically polarised to a degree of $\phi_{\text{Red}} = 43.8(4.2)^\circ$ with the major axis at an angle of $\theta_{\text{Red}} = 56.41(80)^\circ$ from the lab vertical axis.

The reduced $\chi^2$ value given in figures 3.9 and 3.10 is very large. A more elaborate discussion on this is given in section 4.2.
3.4 Effect of Polarisation on the Spectrum

The polarisation of the incident light fields has been established. It is now possible to control this parameter and measure its effects on the Ba\(^+\) ion by performing spectroscopy.

3.4.1 The Experiment

The setup of this experiment is as depicted in figure 3.5. Frequency spectra have been taken for various detunings and laser powers. Firstly, eight different detunings of the LS laser were taken, four above and four below the resonance frequency. For each detuning, a spectrum was taken for four different laser powers. For each power and detuning, one spectrum was taken with and one without the LS laser on.

These measurements were taken over the course of two days, where one day \(\vec{B}\) was in the x-direction and the other day \(\vec{B}\) was in the z-direction. To examine the effects of the polarisation of the light field, four sets of spectra (each set containing a spectrum with and one without LS laser on) were compared to each other.

3.4.2 Results

Two sets of spectra were taken with equal parameters but with different polarisation of the LS laser light (horizontal and vertical). Two spectra were taken on different days with similar parameters, with exception to \(\vec{B}\), which was changed by 90°. It is expected that this is equivalent to rotating the direction of polarisation by 90°, because the direction of polarisation is measured relative to the direction of \(\vec{B}\) (see section 2.4). The spectra are displayed in figure 3.11 and 3.12.

The spectra are fitted using a matlab script. Running this script returns the parameters given in table 3.1.

<table>
<thead>
<tr>
<th>Spectrum</th>
<th>(\theta_{\text{Blue}}) (degree)</th>
<th>(\phi_{\text{Blue}}) (degree)</th>
<th>(\theta_{\text{Red}}) (degree)</th>
<th>(\phi_{\text{Red}}) (degree)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spectrum (a)</td>
<td>9.3(109)</td>
<td>5.5(17)</td>
<td>38.9(5.8)</td>
<td>1.4(27)</td>
</tr>
<tr>
<td>Spectrum (b)</td>
<td>−0.5(61)</td>
<td>10.9(8.5)</td>
<td>−47.0(4.0)</td>
<td>3.8(24)</td>
</tr>
<tr>
<td>Spectrum (c)</td>
<td>5.7(12)</td>
<td>13.1(4.6)</td>
<td>48.5(3.1)</td>
<td>11.9(8.5)</td>
</tr>
<tr>
<td>Spectrum (d)</td>
<td>6.3(4.8)</td>
<td>13.3(3.4)</td>
<td>48.5(2.0)</td>
<td>9.0(6.8)</td>
</tr>
<tr>
<td>Spectrum (e)</td>
<td>−3.2(7.4)</td>
<td>12.4(3.8)</td>
<td>89.0(72)</td>
<td>43.4(4.6)</td>
</tr>
<tr>
<td>Spectrum (f)</td>
<td>31.3(3.7)</td>
<td>2.8(8.4)</td>
<td>−32.1(4.7)</td>
<td>20.4(5.2)</td>
</tr>
<tr>
<td>Spectrum (g)</td>
<td>−19.0(3.8)</td>
<td>12.0(5.1)</td>
<td>−72.1(9.8)</td>
<td>37.6(9.3)</td>
</tr>
<tr>
<td>Spectrum (h)</td>
<td>40.4(2.8)</td>
<td>9.2(4.5)</td>
<td>−71.5(5.2)</td>
<td>27.0(8.9)</td>
</tr>
</tbody>
</table>

For comparison lineshapes have been fitted to the same data, but now with the parameters for polarisation fixed to \(\theta_{\text{Blue}} = \phi_{\text{Blue}} = 1°\), \(\theta_{\text{Red}} = 55°\) and \(\phi_{\text{Red}} = 41°\). These results are plotted in figures 3.13 and 3.14.

The spectra with fixed polarisation and circularity (labeled 1 till 8) and the spectra with polarisation and circularity as a fitting parameter (labeled a till h) were compared to each other. The goodness of the fit is better when having polarisation and circularity as fitting parameters, with exception to spectra (d) and (4).
Figure 3.11: Four frequency spectra obtained by performing spectroscopy. The two spectra in each row belong to one set of measurements; one with and one without the LS laser turned on. Spectra (a) and (b) are with horizontal polarisation. Spectra (c) and (d) are with vertical polarisation.

The values the fit gives for polarisation and circularity deviate from the values determined in section 3.3. The errors in $\theta_{\text{Blue}}$ and $\phi_{\text{Red}}$ for both spectrum (a) and (b) are large. Possible causes are discussed in section 4.3. No conclusions on the polarisation and circularity can be drawn from the results determined from these spectra. To arrive at more conclusive results it is advisable to take more measurements on the spectrum for different polarisation directions. Arguments for this suggestion are listed in section 4.3.
Figure 3.12: Spectra (e) and (f) are with B in the x-direction and spectra (g) and (h) are with B in the z-direction.
Figure 3.13: Four frequency spectra obtained by performing spectroscopy. The two spectra in each row belong to one set of measurements; one with and one without the ion being exposed to LS laser light. Spectra (1) and (2) are with horizontal polarisation. Spectra (3) and (4) are with vertical polarisation. In this case the fitting parameters for polarisation and circularity were fixed.
Figure 3.14: Spectra (5) and (6) are with $\vec{B}$ in the $x$-direction and spectra (7) and (8) are with $\vec{B}$ in the $z$-direction. In this case the fitting parameters for polarisation and circularity were fixed.
Chapter 4

Discussions

In this chapter the results obtained and the conclusions drawn in the experiments described in sections 3.1, 3.2, 3.3 and 3.4 are discussed. This includes suggestions and proposals for further experimentation. The experiments are discussed in the same order as they were described in this thesis.

4.1 The Magnetic Field as Function of Current

The magnetic field as a function of the current through the Helmholtz coils was determined at two positions close to the vacuum chamber. An estimate was made for \( \vec{B}(I) \) at the location of the Ba\(^+\) ion by taking the average of the two functions. It is presently unknown how well this linear interpolation approaches the actual value for \( \vec{B}(I) \) at the position of the ion. For this it is advisable to perform spectroscopy and deduce the value of \( \vec{B}(I) \) that the ion ‘sees’. These experiments have been performed by P. van Dorp, but are not listed in this work.

Deviations from the actual value can have various causes. Firstly, the estimate for the height of the ion was crude and could be off by about 1 cm. Calculations of the gradient of \( \vec{B} \) indicate that deviations of this order still cause for negligible variation in \( \vec{B} \). Secondly, the field was measured only at two locations. Stray fields from, for example the ion pump, could cause the field to be different from the average of the field at the two locations. Lastly, uncertainty was caused by fluctuations in the measurements of the field itself of order \( 3 \cdot 10^{-3} G \).

The magnetic field in the z-direction was observed to not be entirely linear. There is an extra dependence on \( I \) observed from figure 3.3. This dependence is presently unknown and requires further investigation. It is suggested to investigate the influence of (ferro- or dia)magnetic components around the vacuum chamber.

4.2 The Polarisation of the Waveplate and Lasers

The polarisation of the waveplate and of the light has been determined. Also the degree of circularity of both the blue and red laser has been determined.

In table B.1 the frequency of the first two \( \theta_2 \) are higher than the rest. The phase corresponding to these two frequencies are lower than one would expect from the linear relationship arrived at in section 3.2. This may be caused by the fact that the lasers were switched on right before the measurement started.
and needed time to warm up. After they did, the light field was stable enough to yield a linear relationship for the phase versus $\theta_2$. The first two values for the phase were omitted in the graph arrived at in figure 3.7.

The dichroic beamsplitter which the light passed through to reach the photodiode might have been sensitive to polarisation. This would cause an offset in the angle at which the waveplate transmitted vertically polarised light. It is yet to be determined what this offset is. If this offset is known, it can be corrected for by subtracting it from the currently determined values. This offset may also be the cause of the deviation in the values for $\theta_{\text{Blue}}$ and $\theta_{\text{Red}}$ as determined from the spectrum in section 3.4 from those determined in this experiment.

The data was fitted to a profile plot using ROOT. This method takes separate data points and combines them into a predefined number of data points by taking a weighted average. This has the advantage of being able to average out small fluctuations over short periods of time. The disadvantage is that information is lost on these small fluctuations. For the purpose of this experiment this information is not needed and therefore a low number of data points is chosen.

The quality of the fit in figure 3.9 is high and corresponds to a confidence level of approximately 95%. Thus, the model described by equation (3.4) is expected to describe the polarisation of the blue light field well. The quality of the fit in figure 3.10 is too low. This means that the model given by (3.4) does not describe the measured power as a function of analyser angle sufficiently, or the error in the power measurements was underestimated. A possible cause of error may be that the lasers were not warmed up properly prior to taking these measurements. This may have caused fluctuations in the intensity of the light field. These fluctuations can be expected to be long compared to the time of measurement, as the lasers usually need a long time to warm up. This may have caused outliers in the data and thus a lower quality of fit. It is suggested to perform these measurements again with fully warmed up lasers to investigate possible deviation from this result.

4.3 The Effects of Polarisation on the Spectrum

The obtained values for $\theta_{\text{Blue}}$ have a spread between $-19^\circ$ and $+40^\circ$ approximately. The obtained values for $\theta_{\text{Red}}$ have a spread between $-72^\circ$ and $89^\circ$. The values determined for (c) and (d) correspond well with each other, as well as the values for (g) and (h). Some discrepancies remain difficult to explain. For example, the only parameter that was changed between spectrum (g) and (h) was blocking LS laser light. This does explain a difference in transition frequency, but not in polarisation. A similar argument can be given for spectrum (e) and (f). Thus, a change in experimental parameters can not explain the discrepancy between the different values to present knowledge. Closer analysis on the angle of absorption and emission of photons relative to the orientation of the ion may provide an explanation.

The errors in the fitted values have been given can have two causes. Firstly, the program that was used to fit the data to a function did not recognise the periodicity of the polarisation or degree of circularity. Therefore, it did not find the values closest to zero. More importantly, the program would keep making steps on the order of $360^\circ$ in trying out different fitting parameters. This could cause the program to return a disproportionate error.

Secondly, it is possible that the model, or the Ba$^+$ ion itself is not sensitive to small deviations in polarisation or circularity. This makes for a large spread in possible fitted values, which is also what is observed.

Deviation may also have been caused by the definition of the vertical axis. In the measurements of the
polarisation, the z-axis was chosen to be straight up in the lab frame and all polarisations were measured w.r.t. this axis. The polarisation should actually be measured w.r.t. the z-axis of the Ba\(^+\) ion, which is the quantisation axis. This axis is determined by the magnetic field. The direction of \(\vec{B}\) was not precisely known at the time of the experiment. A deviation in the angle of \(\vec{B}\) w.r.t. the lab vertical may explain a difference in polarisation angle by a different choice of coordinates.

Two suggestions for future experimentation can be made. Firstly, to arrive at a more precise value for the polarisation and circularity, more spectra can be taken for various polarisation directions. It is expected that the values obtained for polarisation and circularity will have a distribution, which peaks around the value closest to the value that the ion ‘sees’. Secondly, more parameters could be included in the model. For example: a better estimate for \(\vec{B}\) at the position of the ion, include the velocity of the ion or a better estimate for the light field intensity at the position of the ion.
Acknowledgements

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Furthermore, I would like to thank K.A. van der Klogt, S.V.J.M. Buijsman, M.P. Leeuw, L. Wimmenhoeve and M. Kwarten. They all supported me greatly in the time when I was in doubt of my capabilities of studying physics. Their continued support during my degree was of immeasurable value. To me, this work is not just a report written in a few months. It is the embodiment of all the knowledge and experience I have acquired over the past five years. For that, I am proud and thankful.
Bibliography

[1] O.O Versolato, Laser spectroscopy of trapped Ra$^+$ ions: towards a single-ion optical clock, University of Groningen, Groningen (2011)


Appendix A

Code Polarisation Waveplate

The code below was used in the experiment described in section 3.2 for arriving at the phase as a function of $\theta_1$. Line 7 imports the raw data file. Line 9 to 11 set the fitting function and the initial values for the parameters. The for loop starting at line 13 specifies the value for $\theta_1$ and fits the predefined fit as a function of $\theta_2$. It also plots graphs of PD count rate versus $\theta_2$. This gives a figure like 3.6 for each $\theta_1$. Line 3 prints the fitting parameters for each fit to a file, named ‘results.csv’. With thanks to E.A. Dijck.

```c
void polarisation() {

FILE* outfile = fopen("data2552016/results.csv","w");
delete gROOT->FindObject("fit");
new TFile("data2552016/binary-2016_05_25-13_15_36.root");
TF1 *fit = new TF1("fit","[0]*sin(([1]*x+[2])*(pi/180))**2+[3]",0,360);
fit->SetParNames("Amplitude","Frequency","Phase","Offset");
fit->SetParameters(0.15,2,0,0.42);
for (double angle = 0; angle < 360; angle += 10)
{
TString selection = TString::Format("abs(PolStage1Pos-%f)<0.1", angle);
TTree* tree=(TTree*)gROOT->FindObject("tree");
tree->Fit("fit","PD650nmLS:PolStage2Pos>>PDvStage2(1000,0,360)",selection,"prof");
printf(outfile,"%f,%f,%f,%f\n", angle, fit->GetParameter(0), fit->GetParameter(1),
fit->GetParameter(2), fit->GetParameter(3));
}
fclose(outfile);
}
polarisation.C
```
Appendix B

Fitting Parameters Polarisation Waveplate

These are the parameters acquired from running the code given in A. The errors in $\theta_2$ are determined by the precision of the rotation stage. This precision is 25 arc seconds, which corresponds to 0.007 of a degree.

Table B.1: The fitted parameters for determining the polarisation of the waveplate.

<table>
<thead>
<tr>
<th>$\theta_2$</th>
<th>Amplitude [s$^{-1}$]</th>
<th>Frequency</th>
<th>Phase</th>
<th>Offset [s$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.000(7)</td>
<td>0.21108(9)</td>
<td>1.4988(2)</td>
<td>77.19(9)</td>
<td>0.3660(7)</td>
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<td>79.14(9)</td>
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<td>128.37(3)</td>
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<td>148.31(3)</td>
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<td>0.9968(2)</td>
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<td>0.30098(3)</td>
<td>0.9988(2)</td>
<td>188.69(3)</td>
<td>0.2879(5)</td>
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<td>0.30508(5)</td>
<td>1.0017(1)</td>
<td>208.87(3)</td>
<td>0.2916(4)</td>
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<td>0.9990(1)</td>
<td>229.87(3)</td>
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<td>80.000(7)</td>
<td>0.31642(6)</td>
<td>0.9982(1)</td>
<td>250.06(2)</td>
<td>0.2864(4)</td>
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<td>0.2842(4)</td>
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<td>380.89(3)</td>
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<td>0.9999(1)</td>
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<td>500.07(3)</td>
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</table>
Appendix C

Error Propagation

Methods for calculating the propagation of uncertainties in certain values are given here.

The values for the fitting parameters in the upper three graphs in figure 3.2 contained uncertainties. These uncertainties propagated to the averages given by the lower three graphs. The uncertainty is given by the error propagation equation for addition and subtraction:

$$\Delta B_i,\text{Average} = \sqrt{(\Delta B_i,1)^2 + (\Delta B_i,2)^2}.$$  \hspace{1cm} (C.1)

Here, $B_i,\text{Average}$ is the average magnetic field, $B_i,1$ and $B_i,2$ are the magnetic fields at location 1 and 2 respectively. The parameter $i$ is either $x$, $y$ or $z$.

The values for the fitting parameters in figures 3.9 and 3.10 contained uncertainties as well. These uncertainties propagated in $\theta_{\text{Blue,Red}}$ and $\phi_{\text{Blue,Red}}$.

Firstly, the uncertainty in $\theta_{\text{Blue,Red}}$ is given by the error propagation equation for multiplication and division:

$$\Delta \theta = \theta \sqrt{\left(\frac{\Delta p_1}{p_1}\right)^2 + \left(\frac{\Delta p_1}{p_2}\right)^2}.$$  \hspace{1cm} (C.2)

Here, $\theta$, $p_1$ and $p_2$ are the polarisation angle, frequency and phase respectively. The $\Delta \theta$, $\Delta p_1$ and $\Delta p_1$ are the uncertainties in the polarisation angle, frequency and phase respectively. This expression was also used in calculating $A$, given by equation (3.10).

The uncertainty in $\phi_{\text{Blue,Red}}$ was calculated by first calculating $A$ and $\Delta A$ using equation (C.2) and then applying

$$\Delta \phi = \arcsin(A + \Delta A) - \arcsin(A).$$  \hspace{1cm} (C.3)

Here, $\Delta \phi$ and $\Delta A$ are the uncertainties in $\phi$ and $A$ respectively.