UNIVERSITY OF GRONINGEN

Distributed Optimal Control of Smart Electricity Grids: Home Battery and Electric Vehicle Implementation

by

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“When wireless is perfectly applied the whole earth will be converted into a huge brain, which in fact it is, all things being particles of a real and rhythmic whole”

Nikola Tesla

UNIVERSITY OF GRONINGEN

Abstract

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In this thesis, we consider the balancing problem in a hierarchical market-based structure for smart energy grids that is based on the Universal Smart Energy Framework. Changes in the electricity markets create a mismatch between the day-ahead-planning and the actual supply and demand. We present a multilevel distributed optimal control algorithm, in which the devices of prosumers that can provide flexibility are optimally dispatched based on local information to minimize this mismatch. Home-batteries and Electric Vehicles (EV's) are added to an existing model. Adequate charging of EV's is controlled by a distributed constraint. The distributed EV constraint uses "The System of Updating Matrices" for information sharing, a new theory proposed in this thesis. The System of Updating Matrices can be used to fine-tune, per agent, per time step, the weight/influence of the neighboring agents in a network.

Acknowledgements

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Contents

Abbreviations

ADS	**Active Demand & Supply**
BRP	**Balance Responsible Party**
μCHP	**Micro Combined Heat and Power**
DAP	**Day-Ahead Planning**
DMPC	**Distributed Model Predictive Control**
DSO	**Distribution System Operator**
EV	**Electric Vehicle**
LQR	**Linear Quadratic Regulator**
MCM	**Market-based Coordination Mechanism**
MPC	**Model Predictive Control**
UFLEX	**Flexibility managed according to USEF's rules and guidelines**
USEF	**Universal Smart Energy Framework**

Symbols

K	total simulation time.
k	general time step.
K_{pred}	length of prediction (receding) horizon.
τ	time step within prediction horizon.
y	information step.
$y_i[k]$	information step of EV user i used at time step k.
$\bar{y}$	information step used by neighbors in prediction horizon.
κ_i	charging time ratio of EV user i
L	generalized heat buffer level.
S	state-of-charge level battery.
$S_i^{desired}$	desired state-of-charge of EV user i.
ξ_i	boolean variable indicating of the EV (i) is in the charging dock.
P	power produced or consumed by a device when 100% ON.
ρ	number of charging levels.
C	conversion factor from power to generalized buffer level.
η	ratio of power consumed and delivered (efficiency).
q	heat demand of prosumer.
δ	device charging level (boolean for HP and μCHP).
F^+, F^-	ramp-up/ramp-down flexibility.
t^{on}, t^{off}	number of time-steps a device has been ON/OFF.
f_i, g_i	flexible and fixed load of prosumer i.
x_i	imbalance of prosumer i.
$goal_i$	goal function, the DAP share of prosumer i.
λ_i, μ	Lagrangian multipliers
γ	sub-gradient iteration step sizes.

ϵ	sub-gradient iteration stopping criterion.
v_i	expected influence of the neighbors
L_{max}	distribution network capacity limit.
N	total number of prosumers.
n	number of prosumers per aggregator.
z	aggregator index.
A	information sharing matrix for prosumers.
E_i	information sharing matrix for EV users i.
σ_i	the number of neighbors of EV user i.
e_i	state that keeps track of error between $S_i^{desired}$ and actual S_i.
$\breve{e}_i$	the state e_i with dummy variable.
β	dummy variable for $\breve{e}_i$.
v_i^{ev}	expected charging influence of the (EV) neighbors of EV user i.
$e_{y_i[k],i}$	information state of EV user i corresponding to current information step $y_i[k]$.

Chapter 1

Introduction

This thesis deals with the design of a distributed control algorithm for a network of electricity prosumers, including Electric Vehicles (EV's) and home battery implementation. A distributed Model Predictive control is used for the simulation study with realistic data. The actors in the system, called prosumers, can base their physical behaviour on the information provided by other prosumers in the system. The information shared by Electric Vehicle user, is shared in a novel way. This novel way is based on changing the weights of the information sharing matrices during time steps in the time horizon. This Chapter will introduce the changes in the electricity market, Smart Grid technology, and gives an overview on past research focused on controlling smart grids. This Chapter ends with the outline of the thesis and the main contributions.

1.1 Changes in Electricity Usage

Environmental concerns and changes in power usage lead to more complexity in the energy sector. The Europe Union under-pined the importance of acting as fast as possible by signing The Paris Agreement. The Paris Agreement central aim is to strengthen the global response to the threat of climate change by keeping a global temperature rise this century well below 2 degrees Celsius above pre-industrial levels and to pursue efforts to limit the temperature increase even further to 1.5 degrees Celsius. It is argued that to reach these ambitious goals, appropriate financial flows, a new technology framework and an enhanced capacity building framework should be put in place [3].

A smarter energy network is needed to balance fluctuating energy supply due to the new energy sources, like solar panels and windmills, which depend on weather conditions. These renewable energy sources could provide cleaner energy, but it becomes more complex to predict the amount of generated energy per time unit. A current trend is that energy consumers (households and companies) start to participate as producers in the energy market. This trend will endure in the future due to decreasing purchasing costs of household energy generating devices.

Energy storage systems, such as home batteries, are becoming increasingly important in power system operations. As the penetration of uncertain and intermittent renewable resources increase, storage systems play a vital role to guarantee the robustness, resiliency, and efficiency of energy systems [17]. Much of these capacities are expected to be achieved by distributed home batteries owned by individuals [19].

The need to accommodate fluctuating generation while avoiding network overloads creates an optimization problem: what is the optimal way to supply the required power demand, while compensating at the same time for short-term deviations between the forecasted and the actual supply and demand of power in the system [11].

Electrical devices become smarter and could be turned off and on autonomously based on software and customer preferences to reduce electricity costs and reduce peak loads. In addition to the development of these energy technologies and changes in demand and supply, our society is digitalizing and giving rise to an Internet of Things. This enabling the almost completely free exchange of information between any device and actor in a system. These technologies will become crucial in optimizing and stabilizing the energy system [5].

Traditionally only power generation is seen as a flexible variable that can be subject to control. But a new concept: demand response, can offer a solution to the the problems associated with the electrification of our society. A framework (USEF) where these two concepts come together is introduced in Chapter 2.

Controllable domestic generators could offer solutions to the uncontrollable renewable energy sources. One example of controllable domestic generator is the micro Combined Heat and Power (μ-CHP) system. The μ-CHP can produce heat and power in a household. By producing power locally, the energy losses in the transportation lines are avoided, and by using the output heat there is no waste of heat. They typically convert energy stored in gas into heat and electricity. The semi-counterpart of the μ-CHP is the Heat-pump who converts electricity into heat. Ongoing research is done with these two Active Demand & Supply (ADS) devices coupled to the USEF framework. In the article of Nguyen at al. [11] a Distributed Optimal Control model is described where the heat pump and the μ-CHP are used to balance a smart electricity grid with congestion management in the USEF framework. This model will be described in Chapter 4, and will be extended throughout this thesis.

Likewise, the switch from fossil fuel vehicles to Electric Vehicles (EV's) will create an increase of the demand for electrical energy significantly. In literature it is argued that uncontrolled charging of electric vehicles, i.e. recharging irrespective of market prices or grid situations, will increase peak load to dangerous levels. Vehicle recharging should thus be carried out carefully in order to minimize negative impacts [16]. Vehicle-to-Grid (V2G) technology allows electric vehicles to not only consume electricity from the grid, but it could also supply energy to the grid. V2G can improve the power system's resiliency and reliability and make customers money.

A smart grid is expected to be the future power network. These grids offer a number of significant advantages. First the Smart Grid allows for communication between actors, which enables demand response for the end users. Secondly, domestic power generation is a key component, which makes the end-user both a producer and a consumer, or a prosumer, of electric power. Thirdly, by producing and consuming power locally, Smart Grids also minimize transportation losses. The last feature offers both economic and environmental gains[9].

Another advantage of the Smart Grid is found in that local matching can lower the fluctuations in power flow over the transformer station in the power system. Smart Grids can therefore ease the control efforts to achieve a power balance in the overall power system. The article of Tekiner-Mogulkoc et al [18] gives a overview of the possibilities the Smart Grid technology could offer. The authors indicate that it is difficult to coordinate the decisions of large number of end-user to benefit from this new power systems. They introduce the concept of load shifting in time but do not tell how this should be done.

In a power system, end-users can have a large variety of electric power demand and production devices, such as washing machines, freezers, home batteries, and μ-CHP systems, which can be controlled even if they are subject to operational constraints and user preferences. Controlling these local electricity generators (or discharging batteries) is harder to control than controlling the "traditional" systems. Large power plants have been used to keep the voltage and frequency of the electricity grid to desired levels. The problem of adequately controlling all the local devices within this smaller household/low voltage network, is refereed to in this thesis as: the optimal control problem (of the smart grid).

Kezunovic et al. [8] research indicated that a centralized solution scheme for the optimal control problem is too time consuming, because of the computational complexity. This observation motivated Larsen to consider network model with a distributed information structure, which will allow the implementation of scalable optimization methods, as well as to include power demand and supply [9], exploiting Model Predictive Control (MPC). The MPC framework of Larsen includes the end-users' future power demand

and technical constraints from the devices. The information sharing network is based on local (power imbalance) information about the system. Each local actor sends this information to its neighbors only. Each actor will use the information coming from its neighbours to decide to turn ON or OFF its electrical devices.

The work of Nguyen et al. [11] is well alligned with the work of Larsen[9]. In the paper, the authors formulate a multilevel distributed optimal control problem, in which the appliances of prosumers (heat pump and μ-CHP) that can provide flexibility are optimally dispatched based on local information. This model takes into account the capacity limitations of the distribution network. The results are based on a distributed Model Predictive Control.

The distributed formulation is obtained via dual decomposition and Lagrangian relaxation [4], [13]. The capacity limitations of the distribution network are based on the work of Biegel[1]. In the work of Biegel, a control method based on dual decomposition to achieve congestion management is proposed.

The work of Giselsson and Rantzer [6] elaborates on the topic of distributed model predictive control, where they focused on feasibility, stability and performance. They developed a stopping condition that allows the iterations to stop when the desired performance, stability and feasibility can be guaranteed. This will accelerate the solving speed of the optimal control problem for each time-step.

1.2 Contributions and Outline of the Thesis

The main contributions of the thesis are:

- Implementation of two new ADS devices in the existing model proposed in [11]: home batteries and electric vehicles. Both devices have promising load balancing characteristics.
- An extension to the current model, that makes it possible to implement ADS devices that could be used for supplying and demanding electricity, instead of only demanding (heat pump) and only supplying (μ-CHP).
- An extension to the current model, that makes it possible to implement power production/consumption levels. In the existing model, is there only the possibility to put the device 100% ON or completely OFF.
- A new strategy to share information among the EV's. A system of information sharing matrices is introduced, to increase flexibility at the beginning of charging

while ensuring that the EV is adequately charged at the end of the charging period. The increased flexibility results in a closer match between the actual load and the day-ahead-planning.

- A distributed method with neighboring information is used for modelling the EV constraint. This is done by introducing a new (information) state variable, which calculates the difference between the desired buffer level and the actual buffer level of the prosumer and its neighbors. This distributed approach ensures fast calculations when the model is scaled up to more EV's.

- An overall functioning model where DSO constraint violation is prevented even with the increased electricity demand partly caused by EV users. This proves that the Distributed Optimal Control in combination with Model Predictive Control is suitable for the implementation of Electric Vehicles and home-batteries into the existing model with μCHP's and heat pumps.

The rest of the thesis is organized as follows:

Chapter 2 elaborates on the Universal Smart Energy Framework (USEF). USEF is a collaboration that focuses on implementing an integrated smart energy framework, in a cost-effective and efficient manner. It focuses on making a standard system where all the "smart" devices could work with. It unlocks the value of flexibility and makes flexibility a tradable product. The paper of Nyugen introduced this framework, and because this thesis will build on the paper [11], the USEF framework is explained.

Chapter 3 presents a method that is used to quantify the flexibility. Thereafter, the optimal control preliminaries are explained and combined with reviewing the theory used to solve our control problems in the smart grid in a completely distributed way. At the end of the Chapter a new theory is introduced: System of updating matrices. Which will be the basis for information sharing between the EV's

Chapter 4 is based on the work of Nyugen et al [11]. Three levels, device, aggregator and prosumer, are mathematically introduced. Also the optimal control formulas are given and explained. This paper is the basis of the research developed in this thesis.

Chapter 5 will introduce the first addition, the home battery. This device is working completely different because itself is a buffer. Also a large difference compared to the heat pump and μCHP, is that the home-battery can demand and supply energy to the grid instead of doing solely demanding or solely supplying. The introduction of multiple charging levels is a large difference in controlling this device as well. These two changes have large influence on the controlling of the smart grid. The Chapter first introduces the problem, then gives an overview of a simplified battery model, and finally a mathematical solution to the problem is proposed.

Chapter 6 will introduce the second additional device, the electric vehicle (EV). The battery of the EV is assumed to be the same as of the battery introduced in Chapter 5 (however with different device parameters). One of the differences is that this device will not be coupled to the grid at any time. Also an electric vehicle charging constrain is introduced in the problem statement of this chapter. Requirements and mathematical substantiation are given throughout the chapter. A mathematical solutions is proposed.

Chapter 7 presents the algorithm that is used to update the variables of the optimal control problem. The algorithm combined with the models presented throughout the thesis should result in a optimally controlled smart grid. Three different optimal control problems are summarized and explained in this Chapter.

Chapter 8 presents the simulation study and the results of these studies. The algorithm and solutions are tested with realistic heat and power demand patterns, also some data will be altered to demonstrate the performance of the proposed algorithm in more extreme cases.

In the end, **Chapter 9** concludes the thesis.

Chapter 2

USEF

This Chapter will elaborate on the USEF framework, this framework is used in the article of Nguyen, and will be used as theoretical foundation of this Thesis. In this Chapter a stakeholder analysis is given, that is used to substantiate the requirements set in Chapters 5 & 6.

2.1 Introduction to USEF

Universal Smart Energy Framework (USEF) has been established to drive the fastest most cost-effective route to an integrated smart energy future. The organization would like to deliver one common standard upon which all smart energy products and services could be build. USEF fits on top of most energy market models, extending existing processes to offer the integration of both new and existing energy markets. USEF is developed, maintained, and audited by the USEF Foundation, a non-profit partnership of seven organizations active in all areas of the smart energy industry: ABB, Alliander, DNV GL, Essent, IBM, ICT automation and Stedin.

This section will be based on the “USEF: The Framework Explained”[5], that is available free of charge on the USEF website.

2.2 Flexibility, the Cornerstone of USEF

Electricity consuming devices like, heat pumps, domestic appliances, electric vehicles and HVAC-systems can offer flexibility by changing their load profile. This is known as Demand Response. An electric vehicle for example could start charging earlier, because a wind front is coming up sooner than expected. The load profile would move forward in time to compensate for the earlier generated supply of electricity out of the wind front. Alternatively, the opposite is also possible, the charging process could be slowed down in case of grid congestion during peak times. The group of applications that could ramp-up or ramp-down their load profile are called Active Demand and Supply (ADS). ADS also encompasses local storage units that can both deliver extra load while charging and extra generation capacity while discharging.

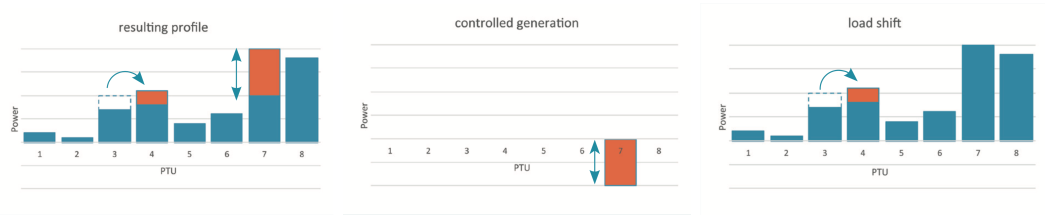


FIGURE 2.1: Flexibility example of one prosumer and one ADS.

The picture above shows a simplified load profile of a prosumer. Horizontally is the time, vertically the consumed power. Flexibility can be obtained by programming ADS's (e.g. a heat pump) to another point in time (left graph). Alternatively, another type of ADS (e.g. a local generation unit like a CHP) is able to reduce or increase the power (middle graph)The resulting profile, being the sum of the two ADS is shown in the right graph. Note that the flexibility is always coupled to the underlying energy supply.

2.3 Stakeholders

Suppliers, balance responsible parties and producers

Nowadays, the main role of energy suppliers is to ensure that they can provide energy to their end users whenever they need it. The relationship between suppliers and their end users is expected to completely change in the near future. Households will become prosumers and start generating power using solar panels. One example of this: households will store energy in battery systems in their homes and electric vehicles, and information systems will make the market transparent to them.

USEF argues that flexibility that is provided by the end users' ADS, could develop in a blessing to the energy market. It provides a way to compensate for the unreliability of renewable energy sources. Doing so effectively requires accurate prediction models, making data the most important asset in the energy supply business of tomorrow. Suppliers with the most accurate prediction models and the algorithms to efficiently dispatch the right assets will gain the most competitive position in the market and make the most efficient use of the available flexibility to optimize their portfolios.

In the future, suppliers need to optimize their generation assets. This means they can use flexibility to reduce peak loads based on the end user's demand profile and hence prevent the dispatch of less efficient generation units. Suppliers can use their customers' flexibility to adapt their consumption profiles to the availability of renewable energy sources such as the wind and sun. This will help balancing the energy system of the future.

Distribution network operators and distribution system operators
Grid operators are responsible for ensuring that the energy in the system can flow freely between suppliers and consumers. This means they must guarantee that sufficient network capacity is always available. Today's distribution grids are designed and controlled to guarantee safety and reliability, providing one of the most stable infrastructures in our society. It is not unacceptable for innovative grid services to impact the performance of this networks. Hence new services and functionality will most likely only be adopted if they improve network stability even further.

One of the most important issues to resolve is the increase in of electrical demand due to electric vehicles and electric heat pumps. Without incentives based on the actual local distribution grid load, the collective simultaneous response from end users' smart devices can increase the peak load on the system even further instead of relieving it. Though it is possible to increase grid capacity through grid reinforcements, this is a time-consuming and costly solution. Hence it would be wise if the flexibility provided by demand-response assets can also be used by grid operators to reduce the loads on their networks and prevent expensive grid capacity investments. As a result, the role of the Distribution Network Operator (DNO) will change into that of a Distribution System Operator (DSO), who will need to actively manage the available capacity in its network and provide market services to do so.

Transmission system operators *The TSOs are out of the system boundaries, because they do not have a direct connection to the outcomes of this research, but it is nonetheless important to explain their role.*
Transmission system operators (TSOs) must ensure that sufficient network transmission capacity is available for energy to flow freely between producers and end users, while maintaining system balance. As the share of renewables increases, TSOs will play an even more active role in the power system than they do today. The intermittent character of these renewable energy sources, such as wind and solar power, requires a continuous optimization process to dispatch power plants throughout Europe in the most economical way. As a result, load flow patterns across the transmission grid are continuously changing. Under these volatile conditions, TSOs must ensure that transmission capacity and system balance are properly managed at every instant.

Prosumers

End users are more and more willing to actively contribute to a sustainable energy future and to invest in renewable energy production by installing PV solar panels or taking joint ownership of wind turbines. In addition, these citizens are working to reduce their energy bills by insulating their homes and making use of energy-efficient technologies such as heat pumps and economical freezers. To verify that these measures result in the expected savings, end users want insight into their energy bills, consumption, production and properly the flexibility offered. Prosumers would like to exercise their right to connect to the system, use their assets whenever they want, and they would like to change their preference settings at each time.

As these citizens become more energy aware and take ownership of power production assets, they transform from passive consumers into prosumers who want to actively participate in the energy market. Prosumers have fundamentally different needs than the classical end user; for example, they would like to charge their electric vehicles using the renewable energy generated by the solar panels on the roofs of their homes. To them it is irrelevant whether the car is parked at home or elsewhere; the network should be able to transport that energy to the vehicle.

Aggregators and energy service companies

In the redesigned electricity market as the EU demands, new roles are introduced. The most important new role is played by the aggregators. Aggregators accumulate the flexibility they obtain from the demand-response resources owned by a set of industrial, commercial, and residential end users. This pool of flexibility is then turned into products to serve the needs of the various stakeholders, as described before. More about the role of aggregators and information about interactions with other agents in the market is provided in section 2.4.

Differences in stakes

There are multiple stakeholders with an interest in the flexibility resulting from Active Demand and Supply. Each of them has its own purpose and characteristics in terms of timing, volume, location, accuracy, and so forth. Flexibility can be derived from various types of ADS from industrial down to residential. The challenge is to optimally divide the available flexibility over the different flexibility services at each point in time. What each actor describes as optimal may differ considerably among stakeholders.

A DSO for example would ideally like to see a flat load profile on its networks, so that the available network capacity is utilized maximally and grid capacity investments are minimized. A wind farm operator, however, would like to see its clients demand follow its volatile generation profile as closely as possible. Finally, a prosumer may just want to use energy whenever he desires, and not be limited in his energy consuming behavior. It is clear that all these wishes cannot be fulfilled simultaneously. Somehow, these stakeholders must share the flexibility resulting from ADS.

The only way to overcome this issue is to capture the importance of these wishes in a neutral parameter that expresses the value it has for each stakeholder. By monetizing this importance stakeholder wishes can be evaluated on an equal basis. This enables each stakeholder to compare the cost of the desired flexibility with that of the alternatives. This in turn enables stakeholders to negotiate the price for the desired flexibility to the maximum that is defined by the costs of their alternatives. Stakeholders that are required to participate in a particular solution will automatically be compensated by those stakeholders that benefit most. This ensures that all stakeholders benefit and the optimal solution is created.

2.4 Market Organization

Demand response through load shifting, storage and management of locally generated energy provide new ways to unlock flexibility in the energy system. Within USEF this flexibility can be generically accessed for multiple purposes and used to serve a variety of stakeholders. This is called the USEF flexibility value chain, and is visualized in figure 2.2. Flexibility managed according to USEF's rules and guidelines is bundled together under the name of **UFLEX**.

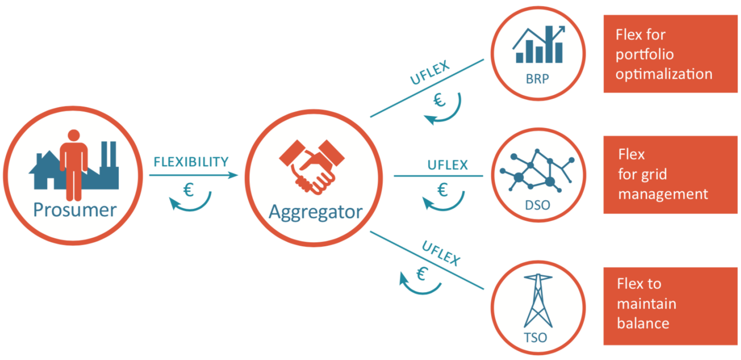


FIGURE 2.2: Summarizing the Aggregator flexibility services for the Prosumer, BRP, DSO and TSO

The Aggregrator is placed centrally within the USEF flexbility value chain. The Aggregator should be responsible for acquiring flexibility from Prosumers, aggregating it into a portfolio, creating services that draw on the accumulated flexibility, and offering these flexibility services to different markets, serving different market players. In return, the Aggregator receives the value it creates with UFLEX on these markets and shares it with the Prosumer as an incentive to shift its load. Therefore with the services of the Aggregator, Prosumers gain access to the energy markets.

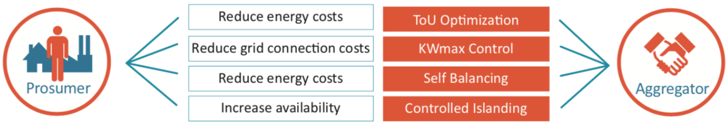


FIGURE 2.3: Summarizing the Aggregator flexibility services for the prosumer

A prosumer can use its own flexibility for in-home optimization, before flexibility is offered to other customers in the energy market. The prosumer can decrease there energy costs by Time-of-Use (ToU) optimization, which is based on load shifting from high-price intervals to low price intervals. They can use flexibility for peak shaving, and self-balancing (more about these options is described in [5]). The main challenge for the prosumer will be matching flexibility earnings, ToU and there preference settings to result in a optimal price portfolio for the prosumer, see figure 2.3. Important to note is that there also will be prosumers where the main goal is not money, but they want to know for sure that they always use green energy from there own sources for example.

FIGURE 2.4: Summarizing the Aggregator flexibility services for the BRP

The Balance Responsible Party (BRP) naturally aims to reduce its sourcing cost (purchase of electricity) as closely as possible to avoid imbalance charges. Demand-Side flexibility from Prosumers within the BRP's client base can be used to optimize its portfolio. There are four potential energy-based services for the BRP, visualized in figure 2.4.

Day-ahead portfolio optimization aims to shift loads from a high-price time interval to a low-price time interval on a day-ahead basis or longer. It enables the BRP to reduce its overall electricity purchase costs.

Intraday portfolio optimization closely resembles day-ahead optimization, but the time frame is constrained after closing of the day- ahead market. Depending on national regulations, the electricity program can be changed until a few hours before the actual time period it refers to. This enables intraday trading, and load flexibility can be used to create value on this market, equivalent to the day- ahead and long-term markets.

The BRP can also use flexbility for self-balancing. Self-balancing is the reduction of imbalance by the BRP within its portfolio to avoid imbalance charges. From a TSO's point of view, further value can be created through passive balancing. In passive balancing, the TSO renumerates a BRP that support the reduction of the system imbalance by deviating the balance position of its own portfolio in the right direction. If this contributes to reducing the total imbalance, the BRP may receive remuneration for its passive contribution. The BRP does not actively bid on the imbalance market using its load flexibility, but uses it within its own portfolio. There are risks involved in this strategy, related to the predictability of the total imbalance. Generally, an online signal for the total imbalance is required, provided by the TSO or other means. Generation optimization, which is extensively described in the framework, refers to optimizing the behavior of central production units as they prepare for their next hourly planned production volume.

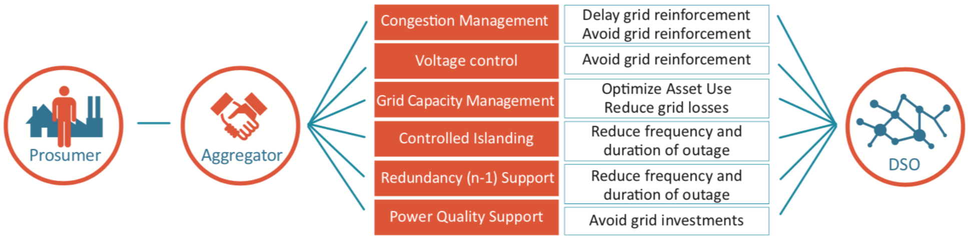


FIGURE 2.5: Summarizing the Aggregator flexibility services for the DSO

The Distribution System Operator (DSO) has six different Aggregator flexibility services, summarized in the figure 2.5. These flexibility services provide value by helping the DSO increase its performance and efficiency in managing the distribution grid.

In this thesis the Congestion management is the most important. Congestion management refers to avoiding the thermal overload of system components by reducing peak loads. In contrast with grid capacity management, this is a situation where failure due to overloading may occur. It is a short-term problem for the DSO that requires a relatively swift response. The conventional solution is grid reinforcement. Load flexibility may defer or even avoid the necessity of grid investments.

Voltage problems typically occur when solar PV systems generate significant amounts of electricity. This will "push up" the voltage level in the grid. Using load flexibility by increasing the load or decreasing generation is an option to avoid exceeding the voltage limits. This mechanism can reduce the need for grid investments (such as automatic tap changers) or prevent generation curtailment. A home battery or EV battery can overcome these problems. Grid capacity management aims to use load flexibility primarily to optimize operational performance and asset dispatch by reducing peak loads, extending component lifetimes, distributing loads evenly, and so forth. An added benefit may be the reduction of grid losses. The other services are not important for these thesis.

2.5 Market Design

To optimize the value of flexibility across all roles in the system, USEF introduces a new market-based coordination mechanism (MCM) along with new processes. The MCM provides all stakeholders with equal access to a smart energy system. The USEF MCM is well aligned with current processes and fits on top of existing markets.

USEF recognizes four different operating regimes. In the Green and Yellow regimes, the MCM assures optimal use of the flexibility available for BRPs (Green and Yellow) and DSOs (Yellow). The Orange regime is introduced as a fallback in case insufficient flexibility is available for the DSO to avoid an outage—the DSO can temporary overrule the market to avoid an outage by limiting connections. For certain parts of the grid, demand will, at peak times, exceed the available capacity. Using USEF, the DSO will identify and publish the locations in the grid where overload might occur: the Congestion Points.

MCM phases The MCM has five phases, summarized in figure 2.6. The aim of USEF's Plan and Validate phases is to make optimal use of grid capacity and to maximize all stakeholders' freedom of dispatch and transaction before the actual delivery of energy takes place. The time scales in these phases range all the way from years and months down to just hours before the Operate phase starts. This broad window facilitates trading on different energy markets (such as the forward market, day-ahead spot market, and intraday spot market) and the ability to accommodate changes in the required grid capacity. A current common practice in energy markets is to close one hour before delivery in the intraday process. The contract and settle phases of the MCM are not

important for the outline of this thesis. The other 3 phases will be shortly discussed. The **plan phase** aims to find an ecnomically optimal program to meet the energy demands of all the Aggregator and BRP portfolios for a certain period. The Plan phase starts when the Aggregator collects forecasts for the Prosumers it represents. Having received the forecasts, the Aggregator optimizes its own portfolio and plans how to maximize the value of the flexibility options in its portfolio, resulting in an A-plan. The Aggregator optimizes its portfolio based on its clients' needs. After this optimization, the Aggregator sends its initial A-plan to the BRP.

Likewise, the BRP optimizes its portfolio of Aggregators, Producers, and Suppliers to attain an economically optimal program. During this process it will negotiate with its Aggregators to exploit the available flexibility in the market and optimize its value. If the BRP identifies market changes that may affect its portfolio, it may reoptimize its portfolio. After the Aggregators' A-plans have been aligned with the BRP portfolio, the

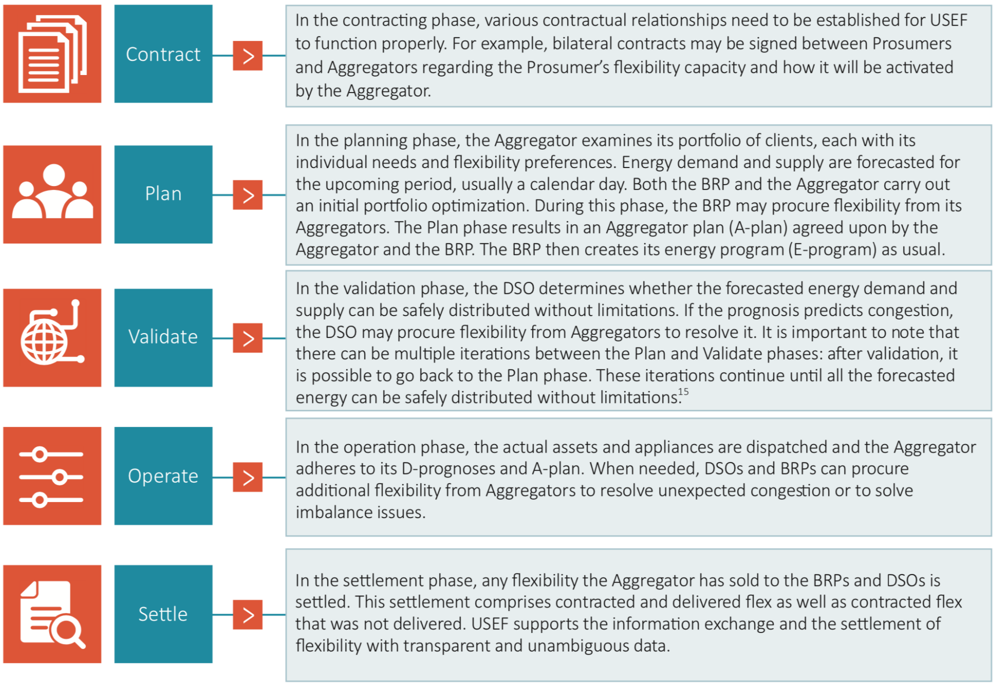


FIGURE 2.6: The five phases of the USEF market coordination mechanism

BRP creates its E-program, which forms the basis for the imbalance settlement process between the BRP and the TSO.

Also as part of the Plan phase, the DSO determines where congestion may take place. Note that, in contrast to the interaction between Aggregators and BRPs which takes place on a daily basis, DSOs declare Congestion Points at a low frequency, likely on the order of a few times a year. When a DSO declares a Congestion Point, the Aggregators active at this Congestion Point can decide to become active on the associated local market by offering UFLEX to the DSO.

The **Validate phase** consists of two intricately linked processes, executed in parallel by different market roles: Validate-D and Validate-E. The Validate-E process, performed by the TSO, is an existing process already in use in many countries. USEF does not alter the Validate-E process. At the start of the Validate-D process, each Aggregator creates D-prognoses for all Congestion Points where it is active, using its A-plan as a basis. The DSO accumulates the D-prognoses from all its Aggregators. This enables the DSO to perform a grid safety analysis. This analysis determines whether the planned energy can be distributed. If it cannot, USEF moves to the Yellow regime and the DSO procures flexibility on the market to resolve the congestion issues. If the available flexibility is not sufficient to resolve the expected congestion, USEF moves to the Orange regime. The DSO's procurement of flexibility may impact an Aggregator's A-plan.18 For this reason, the Validate phase is iterative with the Plan phase; that is, an Aggregator may repeatedly adjust its A-plan to the extent allowed by time and its bilateral agreement with the BRP.

In the **Operate phase**, the actual delivery of energy and flexibility takes place by means of operational interactions. Aggregators deliver the flexibility they have sold to BRPs and DSOs for portfolio optimization and grid capacity management respectively. This flexibility is provided by Prosumers' ADS, which is controlled by the Aggregator.

As long as no deviations from the validated A-plans, D-prognoses, and E-programs occur, the energy system remains in balance with no congestion issues. However, it is unlikely that all A-plans, D-prognoses, and E-programs will be executed exactly according to plan. Deviations can arise from all sorts of sources, ranging from changing weather conditions to a football match running overtime. Deviations can lead to the following events:

- Imbalances in energy supply and demand on the total system level (affecting the BRP)
- Changes in the agreed-upon A-plan (affecting the Aggregator)
- Local congestion in the distribution system (affecting the DSO)

During the Operate phase, additional flexibility can be used to compensate for these deviations.

The Aggregator's main goal is to adhere to its agreed-upon A-plan and its D-prognoses. To achieve this, the Aggregator schedules the operation of ADS assets in a way that reflects the flexibility sold during the Plan and Validate phases. These settings can be adjusted before the Operate phase starts. Second, the Aggregator measures the net demand of its cluster, using smart meter data, to detect deviations from its A-plan or D-prognoses. In the likely event that deviations occur, the Aggregator will have to re-optimize its portfolio. Perhaps deviations can be solved within the portfolio itself; if not, the Aggregator will have to change the operation set- points of the ADS.

The BRP's main interest is to minimize its imbalance costs. If market circumstances change as a result of the TSO maintaining the system balance, or if the BRP detects that it is causing imbalance by deviating from its E-program, the BRP can procure additional flexibility from Aggregators.

For examples and more information about the framework, visit the website of USEF or/and read the framework document [5].

Chapter 3

Preliminaries

This Chapter will give an overview on how to quantify flexibility and the basic mathematical theory used to solve Distributed Optimal Control problems. The flexibility part will extended to on the home-battery and thereby also to the electric vehicle, see Chapter 5. The Distributed Optimal Control theory will be the mathematical basis for Chapters 4, 5 and 6.

3.1 Quantification of Flexibility

As explained in Chapter 2, flexibility is regarded as the ability to shift the production or consumption of an Active Demand & Supply application in time, without changing the total energy production or consumption. By using the flexibility of devices, (i.e., turning ON/OFF devices based on the load measured in the network), demand side management can be performed. The method of quantifying the flexibility that a thermal appliance can offer at a given time, is based on [11]. In Chapter 5 the same approach will be applied to the home battery.

Two scenarios can be distinguished:

- **Ramp-up Flexibility**: Increasing the electricity consumption of the household by turning OFF the μ-CHP or turning ON the heat pump. In the case of the battery and EV: consuming more electricity or supplying less electricity compared to the previous time step.
- **Ramp-down Flexibility**: Decreasing the electricity consumption of the household by turning ON the μ-CHP or turning OFF the heat pump. In the case of the

battery and EV: consuming less electricity or supplying more electricity compared to the previous time step.

In this section is the flexibility quantified for the existing model of Nguyen [11]. For an overview of the preceding model see Chapter 4.

3.1.1 Heat buffer

The produced heat from the μ-CHP and heat pump is stored in a buffer where the water level is assumed to be constant. Only the heat content is changing.

$$L[t] = L_0 + \frac{\eta}{C}|P|t - \frac{\sum_\tau q[t]}{C}, \qquad 0 \leq L[\tau] \leq 1 \tag{3.1}$$

Where $L[t]$ is the generalized buffer level after t time-steps, L_0 is the initial buffer level, η is a ratio between electric and thermal power, C is a conversion factor from thermal power to the buffer level, and P is electrical power produced by the μCHP or consumed by the heat pump while filling the buffer. The household has a heat demand $q[t]$, by which the buffer is drained.
We can derive, from (3.1), the remaining available electrical capacity of the heat buffer.

$$P = \frac{C(1 - L_0) + \sum_\tau q[\tau]}{\eta\tau} \tag{3.2}$$

3.1.2 μCHP and Heat pump

The Boolean variable δ indicates whether the appliance is running at $\tau = 0$.

$$\delta = \begin{cases} 1, \text{ if the appliance is operating} \\ 0, \text{ if the appliance is not operating.} \end{cases} \tag{3.3}$$

In order for the μCHP to have ramp-up flexibility the appliance must be running and has to be turned OFF. Vice versa, for ramp-down flexibility. When the buffer is full, the μCHP can not longer operate and when the buffer is drained, the appliance can no longer remain idle, because it has to fulfill the heat demand of the prosumer. These two considerations provide the upper and lower limit of the available flexibility. Taking into account the power limits, the ramp-up and ramp-down flexibilities of the μCHP are given by:

$$F_C^+[\tau] = min\left\{\frac{\delta C_C L_0 - \sum_\tau q[\tau]}{\eta_C \tau}, -P_C\right\} \tag{3.4}$$

$$F_C^-[\tau] = max\bigg\{ -\frac{(1-\delta)C_C(1-L_0) + \sum_\tau q[\tau]}{\eta_C \tau},\ P_C \bigg\} \tag{3.5}$$

The same reasoning holds for the heat pump. The flexbilities of the heat pump are:

$$F_H^+[\tau] = min\bigg\{ \frac{(1-\delta)C_H(1-L_0) + \sum_\tau q[\tau]}{\eta_H \tau},\ P_H \bigg\} \tag{3.6}$$

$$F_H^-[\tau] = max\bigg\{ \frac{-\delta C_H L_0 + \sum_\tau q[\tau]}{\eta_H \tau},\ -P_H \bigg\} \tag{3.7}$$

3.2 Preliminaries on Optimal Control

This section reviews the general techniques that will be used to solve the optimal control problems of the preliminary model (Chapter 4) and the EV constraint (Chapter 6). The concept of Model Predictive Control (MPC) is introduced and this concept is coupled to distributed optimal control techniques. This last technique is based on dual-decomposition and sub-gradient iterations, and is also referred to as a price mechanism. A combination of the methods mentioned above is Distributed Model Predictive Control (DMPC), where a centralized optimization problem is solved in distributed way using dual decomposition. This Chapter is based on [4],[9] and [11].

3.2.1 Primal and Dual problem Solving Methods

For a given variable $x \in \mathbb{R}^n$ we associate a cost $V : \mathbb{R}^n \to \mathbb{R}$, inequality constraints $f_i(x) \leq 0$ and equality constraint $h_j(x) = 0$ where $i = 1, ..., m$, and $j = 1, ..., p$. The problem is called the primal problem, and is given by:

$$\begin{aligned} &\text{minimize } V(x) \\ &\text{subject to } f_i(x) \leq 0, \quad i = 1, ..., m, \\ &\qquad\qquad h_j(x) = 0, \quad j = 1, ..., p. \end{aligned} \tag{3.8}$$

Solving the primal problem with the hard (normal) constraints can be computational expensive. Therefore a powerful tool for solving problems without the need to explicitly solve the hard constraints is used. The method of Lagrange multipliers, which is a relaxation method. This method used the Lagrange dual function

$$L(x, v, \lambda) = V(x) + \sum_{i=1}^{m} v_i f_i(x) + \sum_{j=1}^{p} \lambda_j h_j(x) \tag{3.9}$$

Where the Lagrangian multiplier vectors $v \in \mathbb{R}^m_+$ and $\lambda \in \mathbb{R}^p$ are associated with the inequality and equality constraints in (3.8). The constraints are relaxed, but violating the constraint results in an additional cost that is linear in the amount of violation. Taking the minimum of $L(x, v, \lambda)$, for fixed v and λ over x provides a lower bound for the optimal value of V. Finding the best lower bound is called the dual problem and is given by

$$L^* = \sup_{v \geq 0, \lambda} \inf_{x} L(x, v, \lambda) \tag{3.10}$$

Where "*" indicates the "best" optimal solution. In (3.10) the Lagrangian multiplier v_i has to be positive because of the inequality in equation (3.8) is corresponding to this multiplier. When $f_i(x)$ is negative, and the constraint is satisfied, it contributes to lower the value of the Lagrange dual function. While the constraint is violated $f_i > 0$, the term acts as a penalization. The other multiplier λ can become negative and positive to punish deviations from $h_i(x) = 0$.

3.2.2 Sub-gradient Iterations and Stopping Criterion

The type of optimization problems from section 3.2.1 can be solved with the by applying a sub-gradient iteration method. This method is often used in combination with decomposition methods to solve problems of large scale systems in a distributed manner. The optimization problem formulated throughout the thesis and summarized in Chapter 7 is not convex, therefore a stopping criterion is implemented.

Suppose we want to solve

$$\min_{x \in \mathbb{R}} \quad V(x), \tag{3.11}$$

where $V : \mathbb{R} \to \mathbb{R}$ is a convex function. To obtain an iterative solution we search for a sequence $\{x(r)\}_{r=0}^{r=\inf}$ that converges to the optimal vector x^* in some sense. The idea is that we choose a direction with some step-size coefficient $\gamma(k)$, such that $V(x[r+1]) < V(x[r])$.

The sub-gradient method can be applied on the dual of (3.8). Given a point $x(r)$, we can find the optimal v and λ for this point by the means of the method. This can be done because the Lagrangian dual function (3.9) is concave in v and λ, even when the problem of (3.8) is not convex [4]. We define

$$L_{Dual,r}(v[r], \lambda[r]) = inf_{x[r]} L(x[r], v[r], \lambda[r]), \tag{3.12}$$

for iteration r. The iterative updates of the Lagrangian multipliers are expressed by

$$v[r+1] = \max(0, v[r] + \gamma[k]f(x[r])), \tag{3.13}$$

$$\lambda[r+1] = \lambda[r] + \gamma[k]h(x[r])), \tag{3.14}$$

where $f(x[r])$ and $h(x[r])$ are sub-gradients of $L_{Dual,r}(v[r], \lambda[r])$ at the current point $x[r]$ of iterations, $\gamma[r] > 0$ is a step size. Notice that when the objective function is differentiable the search direction of the sub-gradient iteration is the same as they of gradient descent. Therefore, to find the local minimum of the the differential function, we take steps proportional to the negative of the gradient of the function in the current point $x[r]$. In this thesis only the difficult constraints of the original problem are included in the Lagrangian, the remaining are treated explicitly in the problem.

Suppose we want to solve a Distributed Model Predictive Control (DMPC), see section 3.3.2. Rather than finding the optimal solution in each time step in the DMPC controller, the most important task is to find a control action that gives desirable closed-loop properties such as stability, feasibility, and desired performance. Such properties can sometimes be ensured well before convergence to the optimal solution. To benefit from this observation, Giselson and Rantzer [6] proposes a stopping condition that allows the iterations to stop when the desired performance, stability and feasibility can be guaranteed. For the analysis for performance and stability, we refer to the article of [6]. The main idea is to keep solving the dual of (3.8), with Lagrangian multipliers (3.13) and (3.14) until the difference between the Lagrangian multipliers from current iteration r and the $r-1$ is less than a stopping value ϵ. This is done for all time steps $k \in \{1, ..., K\}$, and prediction horizon step τ. A brief example algorithm is given:

Algorithm 1: Stopping Condition

for k=*1,...,*K **do**
    initialize begin values
    while $|\hat{\lambda}^r[\tau] - \hat{\lambda}^{r-1}[\tau]| > \epsilon$ ***or***
    $|\hat{v}^r[\tau] - \hat{v}^{r-1}[\tau]| > \epsilon$ **do**
        Solve the dual of (3.8)
        Update Lagrangian Multiplier (3.13)
        Update Lagrangian Multiplier (3.14)
    end
end

3.2.3 Optimal Control in a Network of Decision Makers

A global optimal control problem over a network consisting of n agents (prosumers) is considered.

$$x[k+1] = \mathbf{A}x[k] + \mathbf{B}u[k] - w[k] \tag{3.15}$$

Where $x[k] \in \mathbb{R}^n$ is the state, $u[k] \in \mathbb{R}^n$ is the input and $w[k] \in \mathbb{R}^n$ is the disturbance vector (in this thesis a goal function), at time step k.

The network of agents, is represented by a weighted, directed graph. $G = (\phi_n, \varepsilon_n)$, with ϕ_n =$\{1, 2, ..., n\}$ being the set of agents and $\varepsilon_n \subseteq \phi_n \times \phi_n$ being the set of edges. The agents are connected in order to exchange information, $(i, j) \in \varepsilon$ means that agent i receives information from agent j. The weight on the edge characterizes the importance of the information. Matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$ is the weighted adjacency matrix corresponding to the strongly connected G with the properties as follows:

1) $A_{ij} \geq 0$.

2) $A_{ij} = 0$, if no information is sent from agent j to i.

3) $\sum_i A_{ij} = 1$

Input matrix $\mathbf{B}$ is a $n \times n$ identity matrix. This means that each agent can only control one input signal directly. The objective is to find a sequence of control inputs that minimizes a given quadratic performance index $V(x[k], u[k])$ over K time steps. It is assumed that this performance index is separable for the agents. In game theoretic terms, this is considered a network of decision makers that are dynamically coupled, but have access to different information about the under lying uncertainties. Optimization of such a problem is called *team-optimization*. The aim is to reformulate the problem so that it could be splits in several sub-problems. By introducing prices into this approach, the problem is reformulated from a team-minimization problem to a *non-cooperative* game with additional players [14]. First, the Linear Quadratic Regulator (LQR) is introduced, thereafter coupled to the reformulated problem.

3.2.4 Linear Quadratic Regulator Problem in a Network

A discrete time network with n agents is considered, where at time-step k agent i has a state $x_i \in \mathbb{R}$., and a control input $u_i \in \mathbb{R}$. The agents are dynamically coupled through state equations.

$$x_i[k+1] = \sum_{j=1}^{n} A_{ij}x_j[k] + B_{ii}u_i[k] - w_i[k], \quad i = 1, ..., n, \tag{3.16}$$

where A_{ij} weights the connections in the network. For the EV constraint, the weight will change over k (see Chapter 6 and section 3.4). The input weights on B_{ii} still equals one. The following vectors are defined to write the system in compact form:

$$x[k] = [x_i[k]...x_n[k]]^\top$$
$$u[k] = [u_1[k]...u_n[k]]^\top$$
$$w[k] = [w_1[k]...w_n[k]]^\top$$

Then, the compact form of the distributed system is given by state equation (3.15), where information sharing matrix $A \in \mathbb{R}^{n\times n}$ is stable. And consists of the components A_{ij}. The performance of the system is measured by a quadratic objective function:

$$V(x[k], u[k]) = \sum_{k=1}^{K} x^\top[k]Qx[k] + u^\top[k]Ru[k] \tag{3.17}$$

Where $Q \in \mathbb{R}^{n\times n}$,$R \in \mathbb{R}^{n\times n}$ are positive definite weight matrices. The first term penalizes deviation of $x[k]$ from zero, while the second term penalizes the use of the control input $u[k]$. We assume that Q,R are diagonal, so that the objective function is separable. The goal of the infinite horizon LQR problem is to find the control input that minimizes the infinite horizon cost. The optimal value of this problem is therefore given by

$$\begin{aligned} \min_{u} \quad & \limsup_{k\to\infty} V(x[k], u[k]) \\ & \textbf{subject to } (3.15) \end{aligned} \tag{3.18}$$

3.2.5 Dynamic Dual-Decomposition

The right hand side of the state equation (3.16) depends on neighbor states through the terms $\sum_{j\neq i} A_{ij}x_j$. Therefore it is a coupled system, because the update of one agent also depends on information from neighbors. To decouple this state equation, each agent introduces a local variable, $v_i[k] \in \mathbb{R}$ representing the expected influence from other agents on its state equation. Thus equation (3.16) is now given by the fully decoupled state equations

$$x_i[k+1] = A_{ii}x_i[k] + v_i[k] + B_{ii}u_i[k] - w_i[k], \quad i = 1, ..., n \tag{3.19}$$

With additional equality constraint

$$v_i[k] = \sum_{j\neq i} A_{ij}x_j, \quad i = 1, ..., n \tag{3.20}$$

By defining $A_D \in \mathbb{R}^{n\times n}$ as matrix that has the same diagonal as matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, $A_i i = A_{D,ii}$ and it is zero elsewhere, $A_{D,ij} = 0$ if $j \neq i$. By also defining $A_0 = A - A_D$, equations (3.19) and (3.20) could be written as

$$x[k+1] = A_D x[k] + Bu[k] + v[k] - w[k] \tag{3.21}$$

$$v[k] = A_0 x[k] \tag{3.22}$$

As in [9], to decompose the control problem (3.18), Lagrangian decomposition is applied to the constraint (3.22). The Lagrangian multiplier vector $\lambda(k) \in \mathbb{R}^n$ is introduced to write the problem as an maximization minimization problem.

$$\max_{\lambda} \min_{u,v} \limsup_{k\to\infty} V_i(x[k], u[k], v[k], \lambda[k], K),$$
$$\textbf{subject to } (3.21) \tag{3.23}$$

As explained in section 3.2.1, the aim is to obtain a set of decoupled minimization problems. By rearranging the terms the local problem is now expressed by:

$$\max_{\lambda} \sum_{i=1}^{n} \min_{u_i,v_i} \limsup_{K\to\infty} V_i(x_i[k], u_i[k], v_i[k], \lambda_i[k], K)$$
$$\textbf{subject to } (3.21) \tag{3.24}$$

Where we define

$$V_i(x_i[k], u_i[k], v_i[k], \lambda_i[k], K) = \sum_{k=1}^{K} \overbrace{x_i[k] Q_{ii} x_i[k] + u_i[k] R_{ii} u_i[k]}^{\text{local}} + \lambda_i^r[k] v_i[k]$$
$$- \sum_{j\neq i} \lambda_j^r[k] A_{ji} x_i[k] \tag{3.25}$$

The problem is now completely decoupled, given the shadow prices $\lambda[k]_i^r$ of actor i, for itteration step r, see section 3.2.2. Thus the problem is reformulated to a non-cooperative game with additional players. The new players are market players associated with state variables shared by agents through the $\mathbf{A}$ matrix. They adjust the prices to take advantage of violation of constraints (3.22).

The prices are not calculated by the actor, therefore the model needs central coordination to obtain the prices. In the next section 3.3 sub-gradient iterations are used in order to make the method completely distributed.

3.3 Model Predictive Control

In the application of EV charging and controlling the four devices, the devices in the network are subject to constraints. This implies for example that control inputs u can take only values in a specific set. Therefore the infinite horizon optimal control problem cannot be solved by an algebraic Riccati equation [9]. In this complex system it is not possible to find a closed loop expression $u[k] = f(x[k])$ that solves the infinite horizon control problem. Therefore the optimal control problem is solved by exploiting Model Predictive Control (MPC) setting. In a MPC setting the optimal control problem is solved for each time-step k over a *finite horizon* K_{pred}. It is important to note that the solutions provided by MPC are sub-optimal compared to solving the infinite horizon problem, but it should be designed to ensure that the constraints can be met for all agents i for any time step k.

First the centralized MPC approach is introduced, after that the problem is MPC problem is decomposed into substantially smaller sub-problems.

3.3.1 Centralized Model Predictive Control

In the centralized MPC problem, the goal is to find the optimal control sequence $\{\hat{u}(\tau)\}_{\tau=k}^{k+K_{pred}}$ that minimizes the objective function over the horizon $\tau = k, ..., k+K_{pred}$, given prediction models, initial conditions and constraints.

$$\begin{aligned} V(x[k], u[k]) &= x^\top[k]Qx[k] + u^\top[k]Ru[k] \\ &= \sum_{i=1}^{n} V_i(x_i[k], u_i[k]) \end{aligned} \tag{3.26}$$

With

$$V_i(x_i[k], u_i[k]) = x_i[k]Q_{ii}x_i[k] + u_i[k]R_{ii}u_i[k] \tag{3.27}$$

Then, the problem to solve becomes

$$\begin{aligned} \min_{\hat{u}} \quad & \sum_{\tau=k}^{k+K_{pred}} V(\hat{x}[\tau], \hat{u}[\tau])] \\ \textbf{subject to} \quad & \hat{x}[\tau+1] = A\hat{x}[\tau] + B\hat{u}[\tau] - \hat{w}[\tau], \qquad \tau = k, ..., k+K_{pred} - 1 \\ & \hat{x}[k] = x[k] \\ & \hat{x}[\tau^*] \in X, \hat{u}[\tau^*] \in U, \qquad \tau^* = k, ..., k+K_{pred} \end{aligned} \tag{3.28}$$

Where $X, U \subseteq \mathbb{R}^n$ are convex. A, B are defined in section 3.2.3. and $V[k]$ is defined in (3.26). The minimization is performed at each time step k taking new measurements of

$x[k]$ into account. Once a finite optimal control sequence $\{\hat{u}(\tau)\}_{\tau=k}^{k+K_{pred}}$ is obtained, only the first input is implemented to the system. The horizon is then shifted one sample $\tau = k+1, ..., K_{pred}+k+1$, and (3.28) is solved for this finite horizon.

3.3.2 Distributed Model Predictive Control

As in section 3.2.5, the idea is to replace the original problem by a set of smaller sub-problems. This is more convenient for large scale energy networks. Following the same reasoning as sections 3.2.5 and 3.3.1, the dual problem becomes

$$\begin{aligned} \max_{\hat{\lambda}_i} \sum_{i=1}^{n} \min_{\hat{u}_i, \hat{v}_i} \quad & V_i(\hat{x}_i[\tau], \hat{u}_i[\tau], \hat{v}_i[\tau], \hat{\lambda}_i[k]) \\ \text{subject to} \quad & \hat{x}_i[\tau+1] = A_{ii}\hat{x}_i[\tau] + \hat{v}_i[\tau] + B_{ii}\hat{u}_i[\tau] - \hat{w}[\tau] \quad \tau = k, ..., k+K_{pred}-1 \\ & \hat{x}_i[k] = x_i[k] \\ & \hat{x}_i[\tau^*] \in X_i, \hat{u}_i[\tau^*] \in U_i, \qquad \tau^* = k, ..., k+K_{pred} \end{aligned} \tag{3.29}$$

With additional constraint:

$$\hat{v}_i[\tau] = \sum_{j \neq i} A_{ij}\hat{x}_j[\tau] \tag{3.30}$$

And value function:

$$V_i(\hat{x}_i[\tau], \hat{u}_i[\tau], \hat{v}_i[\tau], \hat{\lambda}_i[k]) = \sum_{\tau=k}^{k+K_{pred}} \hat{x}_i[\tau]Q\hat{x}_i[\tau] + \hat{u}_i[\tau]R\hat{u}_i[\tau] + \hat{\lambda}_i[\tau]\hat{v}_i[\tau] - \sum_{j \neq i} \hat{\lambda}_j[\tau]A_{ji}\hat{x}_i[\tau] \tag{3.31}$$

However, problem (3.29) is still using centralized information that is needed to find the prices. It is observed that only price information $\hat{\lambda}_j[\tau]$ from connected agents are needed to solve the decoupled minimization problems [9]. This observation is used to make the final step towards a fully distributed algorithm.

The final step is to include sub-gradient iterations. This can be done even when the problem is not convex [4]. By including sub-gradient iterations, the problem of (3.29) can be approximated. For all $\tau = k, ..., k+K_{pred}$ the sub-gradient iterations of the prices are updated according to

$$\hat{\lambda}_{i,r+1}[\tau] = \hat{\lambda}_{i,r}[\tau] + \gamma_{i,r}\left(\hat{v}_{i,r}[\tau] - \sum_{j \neq i} A_{ij}\hat{x}_{j,r}[\tau]\right) \tag{3.32}$$

Where r labels the sub-gradient iteration (number), and $\gamma_{i,r}$ is the gradient step size. Using this technique, ensures that the price updates are also distributed, and thereby only depending on local information form the neighboring agents, and the gradient-steps

$\gamma_{i,r}$ are chosen such that we converge to the optimum. To conclude, the distributed MPC algorithm is obtained. The original information structure is preserved and λ can be used as price reference.

3.4 System of Updating Matrices

This section reviews a new technique that is used to update information sharing matrices. The matrices that will be constructed with this techniques are used to satisfy the requirements of the EV charging in Chapter 6. The System of Updating matrices can be used for problems, where each time step requires less or more influence from the neighbors. With the updated matrices it is possible to fine-tune the influence of neighbors while ensuring that there is no calculation error, caused by differences in weights.

Remark 3.1 (Physical/Information state). *We consider a network of actors that send information about their physical system to their neighbors. The information from the neighbors influences the physical behavior of the actor. The System of Updating Matrices, is constructed to* ***change*** *the amount of influence that neighbors have on the physical system of the actor at each time step k. Changing the information network will not change the current and past state of the physical system, it can only influence the behaviour, and thereby the state of the physical system in the future.*

Example 3.1. *The state-of-charge of a battery is the physical system. The battery can charge or discharge (the input). The battery communicates with other batteries, and only wants to charge the battery when other batteries have a higher state-of-charge (behaviour). Each battery sends information about its state-of-charge to its neighbors (information).*

3.4.1 Problem with Unbalanced Information Sharing

We are interested in the dynamics and behaviour of all actors together rather than individual actors. When all the actors share information in a similar way as section 3.2.3, and we assume that all actors have similar weights for their neighbors ($A_{ji} = A_{ij}$), involves the actor's state into a accurate representation of the system, see table 3.1. Each information state x_i is calculated by $x_i[k+1] = A_{ii}x_i[k] + \sum_{i \neq j} A_{ij}x_j[k]$, where k is the time step, x_i is the information state of actor i and u_i is the input of actor i. Each actor converges to approximately the same state after a couple of time steps, see table 3.1. Now we consider a network where still the requirements on the $\mathbf{A}$ matrix of section 3.2.3 are preserved:

Agents	**Initial state**	**k=1**	**k=2**	**k=2**	**k=4**	**k=5**
$x_1[k]$	1.00	1.75	1.94	1.98	1.99	2.00
$x_2[k]$	2.00	2.00	2.00	2.00	2.00	2.00
$x_3[k]$	3.00	2.25	2.06	2.02	2.01	2.00
SUM	6	6	6	6	6	6

TABLE 3.1: Information sharing with 0.5 self-weight and 2 neighbors with 0.25 weight each.

Matrix $\mathbf{A} \in R^{n \times n}$ is the weighted adjacency matrix corresponding to the strongly connected G with the most important properties:

1) $A_{ij} \geq 0$.

2) $A_{ij} = 0$, if no information is sent from agent j to i.

3) $\sum_i A_{ij} = 1$

In the scenario of table 3.2, the first actor is less affected by their neighbors, compared to the scenario of table 3.1.

Agents	**Initial state**	**k=1**	**k=2**	**k=2**	**k=4**	**k=5**
$x_1[k]$	1.00	1.75	1.90	1.95	1.97	1.97
$x_2[k]$	2.00	2.00	2.00	1.99	1.98	1.98
$x_3[k]$	3.00	2.25	2.06	2.00	1.98	1.98
SUM	6	6	5.96	5.94	5.94	5.94

TABLE 3.2: Information sharing with agents 2 and 3 that have 0.5 self-weight and 2 neighbors with 0.25 weight each. Agent 1 has 0.6 self-weight and the weight of the neighbors of 0.2

In the second scenario, a deviation occurs between the sum of the states compared to the initial states, after time step 1. This deviation becomes bigger at each time step. This is a problem, because we want that the information about the state of charge is equal to the real state of charge of the network. Therefore, a fourth property is proposed to prevent calculating errors:

4) $\sum_j A_{ij} = 1$

With the fourth properly, the matrix $\mathbf{A}$ becomes doubly stochastic. The extra property has as downside that it not possible to obtain the desired weights for each time step for each actor. For example consider one agent that would base its behaviour 50% on its own state, and 50% on the neighbors, while the other two would only base their behaviour 100% on their own state. Therefore is it not possible to satisfy all the four properties mentioned above while using the individual "desired" weights in one matrix.

In this thesis we want to update the information sharing matrix at different time steps for different actors. The weight/influence of neighboring actors cannot be individually assigned in one matrix, due to the doubly stochastic properties. Considering the same example as before:

Example 3.2. *one actor i is completely selfish $A_{ii} = 1$ at time step k. The doubly stochastic properties ensure that there is no information shared between the neighbor j and actor i ($A_{ij} = 0, \quad \forall i \neq j$). When the neighbor j desires to base its behaviour on neighboring states for this time step, it is not possible because it will not receive information of its neighboring actor i.*

Also in less extreme cases ($A_{ii} < 1$) will the actors be constraint in how much information they share with neighbors. Therefore, we have designed a System of Updating Matrices to ensure that actors are unconstrained in choosing how much influence their neighbors have on their behaviour.

3.4.2 Weight Update System

In the System of Updating Matrices we want to introduce p parallel information sharing networks. At each time step k, actor i is unconstrained to "choose" which information sharing network (matrix) is used. Each information sharing network has different weights for the information states of neighbors and its own state.

We define $p+1$ information steps:

$$y \in \{0, 1, 2, ..., p\} \tag{3.33}$$

We define a switching function that is used to calculate which information step y is used for actor i at time step k,

$$y_i[k] : \mathbb{R} \rightarrow \{1, 2, ..., p\}. \tag{3.34}$$

Remark 3.2 (Difference between steps)*. The amount of information steps y is usually smaller than the amount of modeling time step k. Each agent can choose (or calculate) which information step is used during solving the optimal control problem, for each time step k. At a point in time during simulations, will k be the same for each agent, while y can be different. Actor i can use a different information step y at time step k compared to other actors in the network. And it is also possible that each actor can use different information steps for different time steps.*

Each information step corresponds to an information state, but also corresponds to one of the information matrices. We introduce a total of $p+1$ information matrices that meet the requirements of section 3.4.1:

$$A_y \in \{A_0, A_1, A_2, ..., A_p\}, \tag{3.35}$$

with properties:

$$\begin{aligned} 0 \leq A_{y,ij} &\leq 1, \\ A_{y,ij} &= 0, \text{ if no information is sent from agent } j \text{ to } i, \\ \sum_j A_{y,ij} &= 1, \\ \sum_i A_{y,ij} &= 1, \\ A_{y,ij} &= A_{y,ji}, \end{aligned} \tag{3.36}$$

We define matrix A_0, as the initial matrix. $A_{0,ii}$ is the initial self-weight of actor i. $A_{0,ij}$ is the initial weight of the connection between actor i and neighbor j, (for $A_{0,ij} \geq 0$ and $i \neq j$). The cascading decrease or increase of the weights for the different matrices A_y is given by the following expressions:

$$A_{y,ii} = A_{0,ii} + l_y(\cdot), \tag{3.37}$$

$$A_{y,ij} = A_{0,ij} - \frac{l_y(\cdot)}{\sigma_i}, \tag{3.38}$$

where σ_i is the number of neighbors of agent i. and

$$l_y(\cdot) \in \{l_1(\cdot), l_2(\cdot), ..., l_p(\cdot) \ \mid -1 \leq l_y(\cdot) \leq 1\}, \tag{3.39}$$

$l_y(\cdot)$ can be a linear or non linear function as long as it satisfies the properties in equation (3.36).

In the System of Updating Matrices, we have information p state variables:

$$x_{y,i} \in \{x_{1,i}, x_{2,i}, ..., x_{p,i}\}. \tag{3.40}$$

The next states are calculated according to:

$$x_{y,i}[k+1] = A_{y,ii}x_{y,i}[k] + \sum_{i \neq j} A_{y,ij}x_{y,j}[k] + u_i[k], \quad \forall y \tag{3.41}$$

The states are calculated for all y at each time step k. This is necessary because actors should be able to switch from one information step to another without occurrence of an error. An error similar to the error in section 3.4.1, occurs when actors calculate their $x_{y_i[k],i}[k+1]$ by using $x_{y_i[k-1],i}[k]$ where $y_i[k-1] \neq y_i[k]$, see equation (3.34, for explanation about $y_i[k]$.

The behaviour of actor i is based on the used information step y calculated by the information step function $y_i[k]$:

$$x_{y_i[k],i}[k+1] = A_{y_i[k],ii} x_{y_i[k],i}[k] + \sum_{i \neq j} A_{y_i[k],ij} x_{y_i[k],j}[k] + u_i[k], \tag{3.42}$$

The input u_i calculated in (3.42) is used to update all the information states in equation (3.41).

After the optimal control problem is solved for a time step k, the actor sends all the states to their neighbors. This calculation of all the states is not computational expensive as long as the number of updating steps (p) is kept within reasonable numbers. Because all information states except $y_i[k]$ are calculated after the solver finds the input u_i.

A graphical representation of the information sharing between 4 agents with the System of Updating Matrices is given in figure 3.1. Each agents has two neighbors ($\sigma_i = 2$).

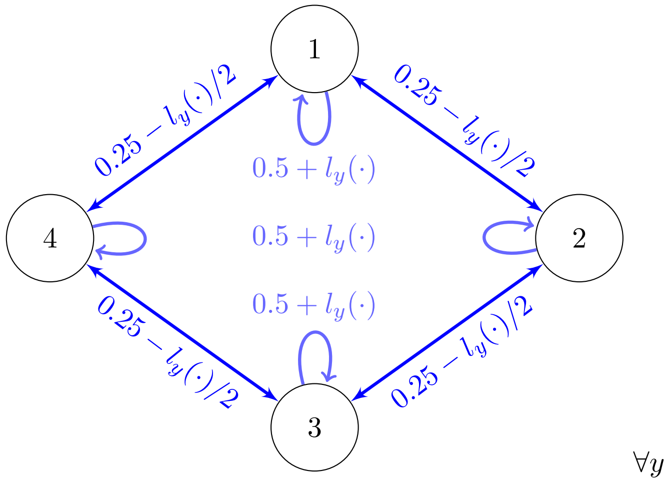


FIGURE 3.1: 4 agents who share information between each other according to the System of Updating Matrices. $A_{0,ii} = 0.5$ and $A_{0,ij}$=0.25

Remark 3.3 (Information vs Physical System). *The System of Updating matrices create p parallel information systems, with information state $x_{y,i}[k]$, that are all calculated with the same input u_i but with different initial states and different weight matrices. The physical system of actor i is not initialized differently each time step. The physical system is not directly depended on the information sharing matrices. There exist one physical system per agent i, that bases his behaviour on the information he gets from the neighbors with the weights and state corresponding to the used information state $\tilde{y}_i$.*

3.4.3 DMPC and the System of Updating Matrices

The model of Distributed Model Predictive Control from section 3.3.2, is extended with the use of the System of Updating Matrices. A couple of extra calculations could be necessary to keep the results and decision making behaviour of the actors accurate. This is foremost needed for DMPC models with long prediction horizons in combination with a significant differences in updated matrices used by neighboring actors.

The relatively small error is caused by differences between matrices used by different actors. The problem is that neighbors will also have information exchange during solving the optimal control problem with the sub-gradient method in a DMPC. This communication is meant to exchange information about what the neighboring agent is expected to do. This information provided by the neighbor j is based on its own $y_j[k]$, therefore is the information provided by the neighbor less reliable for actor i when $y_i[k] \neq y_j[k]$.

When differences in information sharing matrices is minimal, the error in predicted influence, can be neglected. This is reasonable because the outcome of the MPC for the time-step will be corrected based on equation (3.41).

To prevent the error, extra calculations are based on the set of updated matrices $A[\bar{y}]$, where $\bar{y}$ represent the information steps used by the neighbors. The used update steps $\bar{y}$ should be sent towards the neighbors, at the beginning of each time step. The extended optimal control problem becomes:

$$\begin{aligned}
&\sum_{i=1}^{n} \min \quad V_i(\hat{x}_{y_i[k],i}[\tau]), \\
&\textbf{subject to } \hat{x}_{y_i[\tau],i}[\tau+1] = A_{y_i[\tau],ii}\hat{x}_{y_i[\tau],i}[\tau] + \hat{v}_i[\tau] + \quad B_{ii}\hat{u}_i[\tau] - \hat{w}[\tau] \qquad , \\
&\qquad \hat{x}_{y_i[k],i}[k] = x_{y_i[k],i}[k] \qquad \tau = k, ..., k+K_{pred}-1 \\
&\qquad \hat{x}_i[\tau^*] \in X_i, \hat{u}_i[\tau^*] \in U_i, \qquad \tau^* = k, ..., k+K_{pred}
\end{aligned} \tag{3.43}$$

With additional constraint, that is not relaxed by Lagrangian relaxation:

$$\hat{v}_i[\tau] = \sum_{j \neq i} A_{y_i[k],ij}\hat{x}_{y_i[\tau],j}[\tau] \tag{3.44}$$

And the value function becomes:

$$V_i(\hat{x}_{y_i[k],i}[\tau]) = \sum_{k=1}^{k+K_{pred}} \hat{x}_{y_i[k],i}[\tau] Q \hat{x}_{y_i[k],i}[\tau] \tag{3.45}$$

To calculate the expected influence while using the updating matrices without an error. Each actor should calculate during iteration r, their information states with equation (3.41) $\forall \bar{y}$. These states are sent each iteration step to the corresponding neighbor, so that the expected influence of the neighbors could be calculated without an error with equation (3.44). The information sharing is done similar to figure 3.1, but now at each iteration r instead of only at time step k.

Remark 3.4 (Computational expensive). *This extra calculations can become computational expensive, but it will not influence the behaviour of the actor within a iteration directly. The actor will base his behaviour based on control problem (3.43). Therefore it will not require extra solving time for the solver. These information states could be calculated after a solver is used.*

Chapter 4

Preceding System Description

In this section, a summary is given of the most important findings and model components of the model presented in [11]. The three modeling levels are described and the detailed formulas are given.

4.1 Device Modeling

We first describe the models of the μCHP and heat pumps used in the paper. The devices have flexible loads ($f_i[k]$), and are modeled as mixed logical dynamical systems. Running on fossil fuel, the μCHP is generating both electricity and heat simultaneously. The amount of electricity produced is assumed to be a constant value P_C.

$$f_i[k] = \begin{cases} 0, \text{ if } \delta_i[k] = 0 \\ P_C, \text{ if } \delta_i[k] = 1 \end{cases} \tag{4.1}$$

Where $\delta_i[k]$ is the generalization of (3.3) for prosumer (and appliance) i at time-step k. The ratio between the produced electricity and heat is η_C. When the heat pump is operating, it converts electricity into heat with conversion ratio η_H. The electricity consumption during the generation process is assumed to be a constant P_H.

$$f_i[k] = \begin{cases} 0, \text{ if } \delta_i[k] = 0 \\ P_H, \text{ if } \delta_i[k] = 1 \end{cases} \tag{4.2}$$

In both cased, the generated heat is stored in a heat buffer. The buffer storage level decreases if there is a heat demand in the household, this amount is denoted by $q[k]$.

Hence, the storage level can be expressed as:

$$L_i[k+1] = L_i[k] + \frac{\eta}{C}|f_i[k]| - \frac{q[k+1]}{C} \tag{4.3}$$

For efficiency reasons, an appliance is required to keep operating for at least T^{on}_{min} time-steps once it has been turned ON. Correspondingly, it is required to stay switched OFF for at least T^{on}_{min} time-steps before it can run again. It is important to notice that by the introduction of the Boolean variables the problem becomes a mixed integer programming problem, and we lose convexity, which means that the dual gap between the primal and the dual problem is no longer zero.

4.2 Aggregator Level

According to the USEF market-based control mechanism, a day-ahead planning (DAP[k]) is made in the plan and validate phases. The day-ahead planning is devided among the aggregrators, based on the available flexibility they have. At every time-step, the aggregators receive flexibility information from their connected prosumers, and accumulate them in order to make the weighting factor:

$$F_z^+[k] = \sum_{i=1}^{n} F_i^+[k], F_z^-[k] = \sum_{i=1}^{n} F_i^-[k] \tag{4.4}$$

where $i = 1, .., n$ are the prosumers connected to aggregator z. As in the paper of Nyugen, it is assumed that each aggregator has the same amount of prosumers, also it is assumed that each prosumer has only one device. The more flexibility an aggregator has the bigger share of the day-ahead planning it will receive, so that it contributes more to the balancing objective.

$$\text{DAP}_z[k] = \frac{F_z[k]}{\sum_z F_z[k]}\text{DAP}[k] \quad \text{for all } z. \tag{4.5}$$

The day-ahead planning of each aggregator can be in turn spread among the prosumers based on their available flexibility or evenly. In the paper Nguyen uses evenly spreading:

$$\text{goal}_i[k] = \frac{\text{DAP}_z[k]}{n} \quad \text{for all } i. \tag{4.6}$$

4.3 Prosumer Level

Here, we describe the optimal control problem within one Aggregator. We think of the prosumers as agents, and use the following notation.

1) $f_i[k]$: Flexible (controllable) load of prosumer i.
2) $g_i[k]$: Fixed (uncontrollable) load of prosumer i.

The electric load is the sum of supply (production) and demand (consumption), with the convention of using negative sign for supply and positive for demand. The prediction error between the forecasted and actual electricity load is expressed by:

$$\tilde{x}_i[k] = f_i[k] + g_i[k] - goal_i[k] \tag{4.7}$$

or

$$\tilde{x}_i[k+1] = \tilde{x}_i[k] + u_i[k] + w_i[k] - \Delta goal_i[k] \tag{4.8}$$

where

$$u_i[k] = f_i[k+1] - f_i[k] \tag{4.9}$$

$$w_i[k] = g_i[k+1] - g_i[k] \tag{4.10}$$

$$\Delta goal_i[k] = goal_i[k+1] - goal_i[k] \tag{4.11}$$

By introducing the information sharing matrix $\mathbf{A}$ we provide coupling between the prosumers.

$$x_i[k+1] = A_{ii}x_i[k] + \sum_{j \neq i} A_{ij}x_j[k] + [k] + u_i[k] + w_i[k] - \Delta goal_i[k] \tag{4.12}$$

The $\mathbf{A}$ matrix should ensure that the error information is always equal within an aggregator.

$$\sum_{i=1}^{n} x_i[k] = \sum_{i=1}^{n} \tilde{x}_i[k] \tag{4.13}$$

The differences is that $\tilde{x}_i[k]$ denotes the real, physical imbalance of prosumer i, whereas $x_i[k]$ is the imbalance information that includes the weighted sum of the neighboring imbalances as well.

4.4 Distributed Optimal Control Problem

The objective is to minimize the deviation between the predicted and the actual load; therefore, we formulate the problem as

$$\min_{u_i} \sum_{k=0}^{K-1} \sum_{i=1}^{n} (x_i[k])^2 \tag{4.14}$$

Where the expected influence of the connected neighbors is defined as

$$v_i[k] = \sum_{j \neq i} A_{ij} x_j[k], \tag{4.15}$$

this changes the state into

$$x_i[k+1] = A_{ii} x_i[k] + v_i[k] + u_i[k] + w_i[k] - \Delta goal_i[k] \tag{4.16}$$

In the paper is described how two constraints are decentralized and put into the model with Lagrangian relaxation. As described in 3.2.

$$\begin{aligned} \max_{\lambda_i, \mu_i} \min_{u_i, v_i} \sum_{k=0}^{K-1} x_i^2[k] + \overbrace{\lambda_i[k] v_i[k] - x_i[k] \left[\sum_{j \neq i} A_{ij} \lambda_j[k] \right]}^{\textbf{Prosumer decoupling}} \\ + \underbrace{\mu[k] f_i[k] + \mu[k] g_i[k]}_{\textbf{DSO constraint}} \end{aligned} \tag{4.17}$$

The λ and μ are the Lagrangian multipliers of the neighbor coupling constraint and the DSO constraint and are updated with sub-gradient iterations:

$$\lambda_i^{r+1}[k] = \lambda_i^r[k] + \gamma_i^r \left(v_i^r[k] - \sum_{j \neq i} A_{ij} x_j^r[k] \right) \tag{4.18}$$

$$\mu_i^{r+1}[k] = \mu_i^r[k] + \gamma_i^r \left(\sum_i (f_i[k] + g_i[k]) - L_{max} \right) \tag{4.19}$$

where r is the iteration counter and γ is the appropriately chosen step sizes. Note that $\lambda_i[k]$ are distributed, prosumer specific Lagrange multipliers, whereas μ is a centralized, grid operator-specific Lagrange multiplier. The neighbouring coupling constraint works exactly as is explained in section 3.3. The Lagrangian multiplier variable μ can not become negative, this is done to punish violations of the maximal grid capacity L_{max} while not rewarding for using less fixed g_i and flexible f_i while the DSO constraint is not violated.

Remark 4.1 (DSO constraint). The constraint set by the Distribution System Operator is really important in the model. This constraint prevents power outrages and at the same time it prevents that USEF should move to the orange regime. Violations of the day-ahead-planning, the other objective of Bao, are less important than the DSO constraint, see also Chapter 7.

The shadow prices can be interpreted as monetary rewards that the prosumers receive as incentives to modify their loads. The device specific constraints and variables could be find in the model of Nyugen et al [11]. The algorithm that is used to solve the optimal control problem can be found in the article. This algorithm will function as basis for the algorithm described in Chapter 7.

Chapter 5

Home Battery

An overview is given of the stages of implementing the simplified battery model of Ratnam and Weller [15]. In a way that this battery could be an extension to the model Distributed Optimal Control of Nguyen et al.[11]. The simplified battery model will also function as the foundation for modelling an EV battery.

5.1 Problem setting

There is a large increase in the use of renewable energy resources, such as solar panels and windmills. This increase is caused by decrease in purchasing costs and a population that is more aware of the environment. These energy resources are bringing uncertainty to the energy market, due to dependency of these devices on for example the weather conditions. The power supply of these devices cannot be as adequately controlled as for example coal-fired power plants. As the penetration of uncertain and intermittent renewable resources increase, storage systems are critical to the robustness, resiliency, and efficiency of energy systems [17]. Much of these capacities are expected to be achieved by distributed home batteries owned by individuals [19].

There is another reason for the increased interest in home battery systems. People with home owned renewable energy sources want to use there own "green" energy. This energy should temporarily be stored when a prosumer consumes less energy than they generate over a time period.

Therefore, we want to implement a home battery in the hierarchical, three-level structure (BRP, aggregators, and prosumers), in which the day-ahead planning is spread over the aggregators, and subsequently the prosumers connected to them, based on the available flexibility of the prosumers. It is assumed that the electricity portfolio is already forecasted and agreed on by the aggregators, BRP and DSO. This electricity portfolio is referred to as the day-ahead planning. the goal is to fullfill the demand (heat or electricity) while keeping the electricity load as close to the day-ahead planning as possible.

The Distributed Optimal Control of Smart Electricity Grids model [11], does not have the possibility to use a home battery. The model currently only uses devices that can only consume or supply electricity instead of using both. Also, it is not possible to implement charging levels: the μMPC and the heat pump can only be turned off or turned on (100%). This is not realistic for a battery, because you want to store/subtract as much energy as is needed. USEF wants to create one common standard for Active Demand and Supply devices [5], therefore the flexibility calculations and the desired charging levels and states should be modelled/calculated in a similar way as in the preceding system (Chapter 4).

In the preceding model, there is no costs on the use of a certain amount of power. This is not necessary for the μCHP and the heat pump because only full charging and discharging could occur and because the devices could only charge or discharge. This prevents excess switching of the devices. For the home battery excessive switching could happen, for example: one battery is going to charge as much possible, while the neighboring battery (or other device) is discharging, to cumulatively reach the desired value of the DAP. The same desired value could be obtained for example by only charging the battery on 50%. Excess switching has two downsides:

- The devices are subjected to losses while charging and discharging (efficiency of around 90%). This causes higher electricity bills, which is not beneficial for the prosumer and harms the environment by wasting electricity. Also more energy is needed to balance the system which could pressurize the distribution network capacities (DSO constraint).
- The excess switching causes that the solving time of the Distributed Model Predictive Control becomes more difficult (longer). There are much not optimal possibilities that could also solve the imbalance.

A method should prevent this type of inefficient charging patterns. The same method should be used for the electric vehicle in Chapter 6. The requirements are translated into 4 design criteria for the home battery:

- Charging levels should be assigned by the DMPC.
- Discharging and charging should be possible and controllable by the DMPC.

- The home battery should have similarities in modeling and calculating with the preceding system.
- Inefficient switching should be avoided.

5.2 Simplified Battery Model

The closest fit between the design requirements and a battery model in literature is found in the article of Ratnam and Weller [15]. In this article, a deliberately simplified battery model is employed to asses the distributor benefits of coordinating residential battery schedules. The battery model may be extended for more specific battery technologies as presented in [10]. In the article Distributed-Receding Horizon Optimization (D-RHO), which has much similarities with the DMPC model presented in the paper, uses charge and discharge rates of residential battery storage are coordinated to reduce peak loads and reserve power flow in the upstream distribution grid.

In this thesis the same assumptions are made as in the paper [15], excluding the battery constraints, it is assumed there are no additional residential constraints for any proposed battery schedule. To capture the limited "charging/discharging power" of the battery, battery profile constraints $B^{-}\mathbb{1} \leq x \leq B^{+}\mathbb{1}$, where $B^{-} \in \mathbb{R}_{\leq 0}$ and $B^{+} \in \mathbb{R}_{\geq 0}$ are used. Given the battery profile x, the state of charge of the battery (in kW) at time $k\Delta$ is denoted by $S[k]$, where

$$S[k] = S[0] - \sum_{j=1}^{k} x_1[j]\Delta \qquad \text{for all } k \in [1, ..., \tau], \tag{5.1}$$

and $S[0]$ denotes the initial state of charge of the battery. The battery capacity is represented (in kW) by $C \in \mathbb{R}_{\geq 0}$, the state of charge profile by $S = [S(0), ..., S[s]] \in \mathbb{1}^{\tau+1}$, and we represent the state of charge profile constraint by $0 \leq S \leq C[1\mathbb{1}]^{\top}$.

5.3 Flexibility

The approach of quantifying the flexibility of the home battery is based on section 3.1.

5.3.1 Power Buffer

$$S[\tau] = S_0 + \frac{\delta_b \, P \, \eta_b \, \tau}{C_b}, \qquad 0 \leq S[\tau] \leq 1 \tag{5.2}$$

$S[\tau]$ is the generalized state-of-charge level after τ time-steps, S_0 is the initial state-of-charge level, η is a ratio between energy used and stored (efficiency), C is a conversion factor from electrical power to the state-of-charge level, and P is the power produced or consumed while filling the battery.
From (5.2) we can derive the remaining available electrical capacity of the power buffer.

$$P = \frac{C(1 - S_0)}{\eta \, \tau} \tag{5.3}$$

5.3.2 Decision Variable and Quantification of Flexibility

The decision variable of the battery model is different from the Boolean of the Heat pump and the μCHP. The simplification that is proposed in [11], that the applications can only be switched "full" on or that it is switched off is not realistic when we consider the battery. The battery should (dis)charge a desired amount which is constrained by a battery profile boundary. The decision variable cannot be set to continuous, because this will cause problems with the solving possibility and solving time. Therefore we set it as integer and we divide the integer value by the total of charging levels (ρ). Hereby, we force the normalized integer variable (δ_b) to remain between -1 and 1.

$$\delta_b = \frac{\delta_{integer}}{\rho} = \begin{cases} \text{-1} \leq \delta_b < 0, \text{ if the battery is discharging.} \\ 0, \text{ if the battery is not operating.} \\ 0 < \delta_b \leq 1, \text{ if the battery is charging.} \end{cases} \tag{5.4}$$

with:

$$-\rho \leq \delta_{integer} \leq \rho, \qquad \delta_{integer} \in \mathbb{Z} \tag{5.5}$$

The flexibility is then given by

$$F_b^+[\tau] = min\left\{\frac{C_b(1 - S_0)}{\eta_b \tau}, \; P_b(1 - \delta_b)\right\} \tag{5.6}$$

$$F_b^-[\tau] = min\left\{\frac{C_b S_0}{\eta_b \tau}, \; P_b(1 + \delta_b)\right\} \tag{5.7}$$

It is important to notice that in the case of ramp-down, a battery can still be charged as long as it charges less than the last time step. This is due to the possibly to step-wise charge or step-wise discharge, within charging limits.

5.3.3 Device Modeling

The decision variable for the battery has 3 or more states instead of 2 as can be seen in equation (5.4) and it also can become negative. We assume that the battery can charge and discharge with the same power levels. Therefore the controllable load in time step k is calculated with:

$$f_i^b[k] = \delta_b * P_b \tag{5.8}$$

In the battery case, the power is stored in an electricity buffer. The buffer storage level decreases if the battery is discharging. Hence, the storage level is expressed as:

$$S[k+1] = S[k] + \frac{\eta_b}{C_b} f_i^b[k] \tag{5.9}$$

For efficiency reasons does the model of Nguyen use timers for the μCHP and the heat pump. The simplified battery model does not need timers for efficiency reasons. We apply no further changes at the aggregator level, see section 4.2.

5.4 Distributed Optimal Control Problem

Based on the requirements set in the beginning of the chapter, inefficient switching should be avoided. The Optimal Control literature provides a rather simple solution for this problem. In the objective function of the preceding system, deviations from the Day-Ahead-Planning and the deviation from expected influence of the neighbors are penalized. We propose that a penalty is applied for using controllable load (f_i^b). This penalty should be significantly less than the penalty on deviations from the Day-Ahead-Planning, because it should be rewarding to decrease the deviation. This small penalty will ensure that two prosumers will come to an efficient result as graphically represented in figure 5.1.

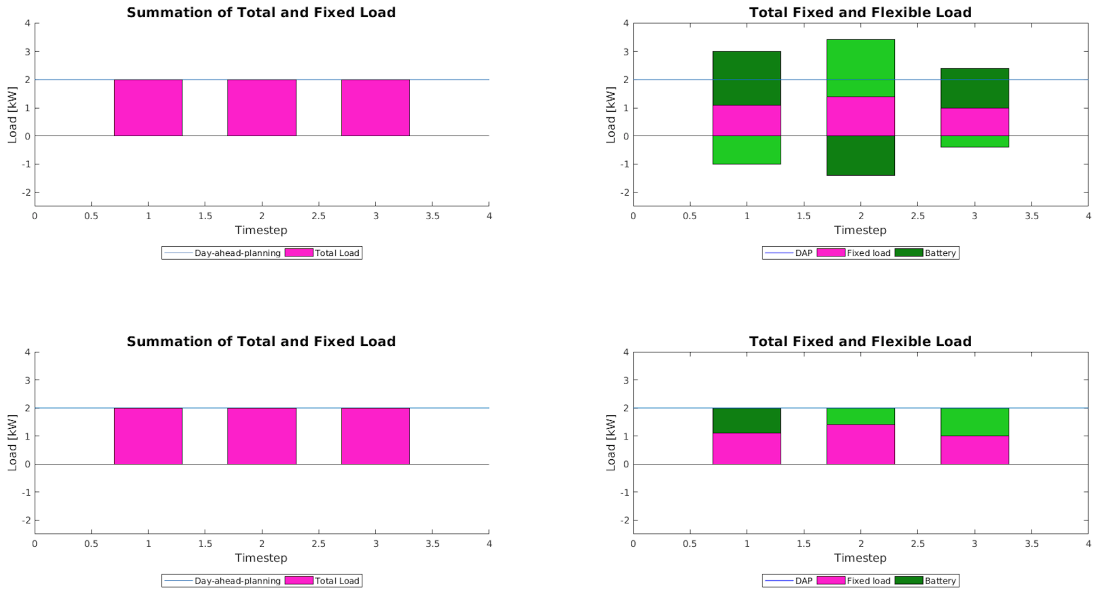


FIGURE 5.1: Two ways of reducing the error between the day-ahead-planning and total load per time step. The upper graphs are representing an inefficient way. The lower graphs indicate the desired (efficient) way to minimize the error.

The penalty should apply for both negative and positive values. Therefore, the appropriate extension to the model should be a quadratic objective that punishes for using f_i with a relatively small weight factor R. The prosumer level, sub-gradient iterations and calculation of the states are the same as the preceding system (see sections 4.3 and 4.4). The objective for the home battery becomes:

$$\max_{\lambda_i,\mu_i} \min_{u_i,v_i} \sum_{k=0}^{K-1} x_i^2[k] + Rf_i^2[k] + \overbrace{\lambda_i[k]v_i[k] - x_i[k]\left[\sum_{j\neq i} A_{ij}\lambda_j[k]\right]}^{\textbf{Prosumer decoupling}}$$
$$+ \underbrace{\mu[k]f_i[k] + \mu[k]g_i[k]}_{\textbf{DSO constraint}} \tag{5.10}$$

subjected to (4.16)

and the device specific constraints

The objective of the home battery with the device specific constraint satisfies the requirements set in section 5.1. The charging levels are implemented, inefficiency is prevented and the home battery could be controlled in a similar way as the model the devices described in Chapter 4. In section 7.2.2 the most important constraints and equations of the home battery are summarized.

Chapter 6

Electric Vehicle

This Chapter will elaborate on a specific constraint for Electric Vehicle (EV) charging. The device modelling of the battery is the same as the home battery in Chapter 5. First the problem is explained and concluded with 5 design criteria, thereafter are the design criteria translated into a mathematical model. In the last section of this Chapter the EV-constraint is coupled to the overall distributed optimal control problem.

6.1 Problem Setting

The price-performance ratio of electric vehicle battery systems has improved to the extent that the market introduction of electric vehicles has become economically viable. This trend will continue over the coming decade, driven by the commercial development of new battery technologies and new companies entering the EV market. EV batteries provide a unique opportunity for the electricity system to store energy, which was until recently only possible using large-scale pumped storage hydroelectric units. This storage enables the local reduction of peak loads on the system and thereby violations of the DSO constraint, and hence a reduction in costs by limiting capacity usage. When sufficient storage capacity is installed, self-balancing and the trading of stored electricity can also be exploited. These storage systems provide another essential form of flexibility to the market[5].

As explained in section 2.3, it is important to guarantee that flexibility will be sufficiently available and that peak loads can be reduced whenever required to maintain network stability. This is the main stake of the DSO. As indicated in sections 1.1, the electrification of the society has the downside of putting allot of pressure on the electricity grid.

Controlling EV charging is necessary, mainly because the EV charging period of a prosumer will overlap with the EV charging periods of other prosumers. For example: 17:00 coming home and leaving at 08:00 each working day. This will create high peaks in the energy network and can result in violations of the DSO constraint and high electricity costs for EV charging.

A smart constraint that incorporates step wise charging over the whole charging period is necessary, mainly because the possible charging time is longer than the prediction horizon of the DMPC. When no EV constraint is implemented, this will create high peaks in power usage when the "end of charging time" is within the prediction horizon for the first time.

It is also important to ensure that all the EV's are sufficiently charged at the end of the charging period. This will be the main stake for the EV user. We assume that the behavior of the EV user is known (and reliable) and set as customer preferences. The start and end of the charging period and the desired state-of-charge level, are known in advantage. Which is done in many Vehicle-to-grid (V2G) solutions.

In the USEF framework, prosumers aggregrators, BRP and DSO benefit from the flexibility (UFLEX) provided by Active Demand and Supply (ADS) devices. Therefore the controlled charging should "produce" (see Chapter 2) as much flexibility as possible. This should create price advantages for charging EV's while maintaining the energy consumption and production as close as possible to the Day-Ahead-Planning (DAP) while maintaining system balance.

We want to implement the EV in the hierarchical, three-level structure (BRP, aggregators, and prosumers), in which the day-ahead planning is spread over the aggregators, and subsequently the prosumers connected to them, based on the available flexibility of the prosumers[11]. In the same way compared to the home battery it is assumed that the electricity portfolio (DAP) is already forecasted and agreed on by the aggregators, BRP and DSO. The goal is to fulfill the demand (heat or electricity) while keeping the electricity load as close to the day-ahead planning as possible while charging the EV to the desired level.

This problem should work together with the controlling of the overall system, where the home battery, μCHP and heat pump are controlled in a fully distributed way. It is important to note that we are not directly interested in each EV on its own, we are interested in the behaviour of the network of EV's. Which is in line with the preceding system [11]. The centralized control of such problems is not realistic as is described in [8]. Therefore a distributed control is more suitable for the model when it is scaled up to more EV's users (and other prosumers).

It is also very important to look at how the EV constraint is "weighted" compared to maintaining and balance between the DAP and actual loads. **Weighting** is referred to as which constraint is more important at which time of the process, a balance should be found between ensuring that the EV's are charged at the end of the charging period, while maintaining the balance. The same reasoning could be applied in comparison with the DSO constraint. We will refer to this dilemma as: **control weighting**.

Six design criteria for the EV constraint are formulated:

- EV charging should be spread over the whole charging period.
- All the individual EV's should be sufficiently charged at the end of the charging period.
- UFLEX should be produced as much as possible.
- Controlled charging should fit in the current system of fully distributed control.
- The control weighting should be considered during the design of the EV constraint

6.2 Charging Period and Step-wise Charging

First, we will define the charging period of the electric vehicle. A variable should indicate if the EV is present in the charging station, in other words it should indicate that the time step k is between the coupling moment k_i^{begin} and the decoupling moment k_i^{end}. We assume that the (de)coupling moments are always on the beginning of a time step. Because the charging period, $k_i^{end} - k_i^{begin}$, could be longer than the horizon of the DMPC, step wise charging of the EV is necessary to overcome peak loads, as explained in section 6.1. For step wise charging it is important to keep track of how much time we already have used to charge and how much time we still have to charge the EV battery to the desired state-of-charge level ($k_i^{current}$), and compare this the the charging period k_i^{period}, in figure 6.1 some examples are given. To make a comparison, the ratio κ_i for each prosumer i is defined:

$$\kappa_i = \begin{cases} \frac{k_i^{current}}{k_i^{period}} = \frac{k - k_i^{begin}}{k_i^{end} - k_i^{begin}}, & \text{if } k_i^{begin} \leq k \leq k_i^{end} \\ 0, & \text{else} \end{cases} \tag{6.1}$$

Note: the ratio will be zero when the EV is not coupled to the electricity grid

The EV's have different charging periods and desired charging levels. The EV can not contribute to balancing the system when it is not plugged in. But because all the charging patterns are assumed to be know, the EV (dis)charging can be taken into account in future time steps, when the coupling moment is within the time horizon.

First a constraint is considered with only local information to introduce the desired charging behaviour. We want to make sure that the EV of prosumer i is adequately charged, based on the ratio κ_i and the desired State-of-Charge $S_i^{desired}$. This local version of the constraint is given as:

$$S_i^{desired}\kappa_i \leq \frac{\eta_b}{C_b} f_i[k] + S_i[k] \tag{6.2}$$

Where $f_i[k]$ is the flexible (controllable) load of prosumer i at time step k, and $S[k]$ is the generalized state-of-charge level after of time step k, η is a ratio between energy used and stored (efficiency), C is a conversion factor from electrical power to the state-of-charge level, and κ_i is also dependent on k, see formula (6.1).

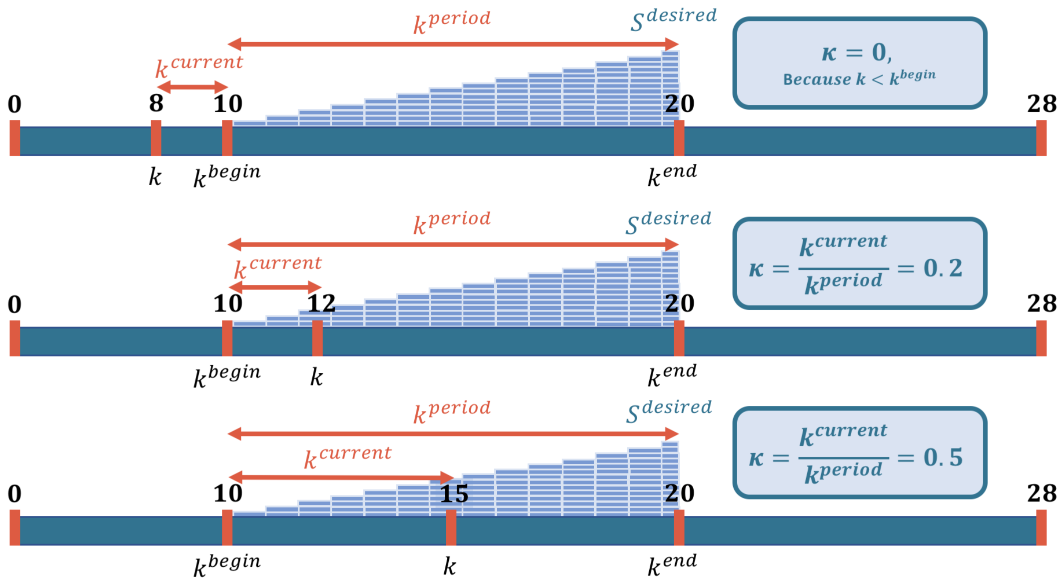


FIGURE 6.1: Visual representation of the calculation of the period ratio variable and a representation of the step-wise charging until the desired charging level

The step wise charging to $S^{desired}$ is graphically represented in figure 6.1. In the upper graph at time step k, $S^{desired}\kappa = 0$, therefore the constraint will not be "active". In the middle graph the actual state of charge plus the generalized controllable load, should be more than $0.2 * S^{desired}$.

Remark 6.1 (Charging profiles). In figure 6.1, a linear growth is visualized. Non-linear growth is also possible, which results in different charging patterns with the same $S_i^{desired}$ at time step k^{end}. Non-linear growth can also be independent of κ_i, and for example be a demand scheme determined by the aggregator.

There is a constraint needed to prevent the possibility of charging when the electric vehicle is not in the charging station. The Boolean variable ($\xi_i[k]$) indicates if the EV of prosumer i is away ($\xi_i[k] = 0$) or present ($\xi_i[k] = 1$) at time step k. The corresponding constraint is:

$$(1 - \xi_i[k])f_i[k] = 0 \tag{6.3}$$

The variable ξ will also be used in constraint (7.21) in section 7.3.

6.3 Flexibility: producing UFLEX

The calculations of flexibility for the EV will be the same as for the home battery, see section 5.3. When the EV is not connected to the grid, the flexibility will be zero. In this section an explanation will be given on how to take into account the design goal of flexibility.

The main disadvantage of equation (6.2) is that it is too **strict**: it will unnecessarily lower the production of UFLEX (flexibility). The constraint will solve the problem of high peak loads but it does not reflect what we are interested in directly. We are interested in the charging behaviour of all the EV's together. It does not matter if one EV is not adequately charged at a certain point in time, as long as this is compensated by another EV in the Network. This is similar to the problem of load balancing between prosumers with the different devices; not each prosumer has to exactly match the DAP as long as they do it as a group of prosumers.

Therefore the state-of-charge levels from all the EV's in the network should be more than the total of desired state of charge levels. The problem becomes:

$$\sum_{i=1}^{n} S_i^{desired}[k]\kappa_i \leq \sum_{i=1}^{n} \left(\frac{\eta_b}{C_b} f_i[k] + S_i[k] \right) \tag{6.4}$$

This problem is completely centralized and needs information of all the EV users in the network. One of the design criteria is to solve the problem in a distributed way (with only information of their neighbors). The distributed solution will be given in the next sections. *Notice that charging can also be independent of k_i (Remark 6.1).*

6.4 EV Information Sharing Network

The distributed constraint requires information on the SOC (desired and generalized) levels of the EV's of the neighbors. This is done by introducing a second information sharing network. With EV weighted adjacency matrices $E_y \in \{E_0, E_1, E_2, ..., E_p | \mathbb{R}^{n \times n}\}$ that only connect the EV-users in the network. The properties of matrices E_y are the same as in section 3.4.2 equation. (3.36). p is the total number of information steps y.

Matrix $\mathbf{A}$ is used for balancing the DAP with the actual total load, and remains constant for each time step k. EV charging takes a different approach towards the information sharing over a network of prosumers. A different approach is needed to meet of requirements of the EV charging, introduced in section 6.1.

To increase flexibility it is "allowed" to compensate charging levels from one EV user by another EV user, as long as the sum of these levels is equal to the sum of desired levels at that moment. To ensure that each individual EV is adequately charged at the decoupling moment, the differences in weights of matrices E_y are exploited, as explained in section 3.4. In the beginning, is the EV's charging behavior mostly based on the behaviour of its neighbouring EV's. When time passes the weight of the neighbours decreases while its own weight increases in the matrices E_y. Therefore, will the EV user base its charging behaviour more and more on its own state-of-charge (SOC).

As is done in section 3.4, the E_y matrix will update according to function $l_y(\cdot)$, that should meet the requirements equation (3.39) and not interfere with the properties of the E_y ((3.36)). Where y is the information step. The weight change according to

$$E_{y,ii}[y] = E_{0,ii} + l_y(\cdot), \tag{6.5}$$

$$E_{y,ij}[y] = E_{0,ij} - \frac{l_y(\cdot)}{\sigma_i}, \tag{6.6}$$

where $E_{0,ii}$ and $E_{0,ij}$ are the initial weights at $y = 1$, and σ_i is the number of neighbors of agent i.

Example 6.1. *In this thesis for one of the simulation models a linear gradient of the function $l_y(\cdot)$ is used, that depends on κ_i, with initial weights $E_{0,ii} = 0.5, E_{0,ij} = 0.25$. And a total of $p = 10$ information steps. At information step 10, the actor should become completely selfish ($E_{ii}[\eta] = 1, E_{ij}[\eta] = 0$). The information updating equation becomes:*

$$l_y(y) = 0.05 * y \tag{6.7}$$

Before solving the optimal control problem, all the updating information matrices could be calculated for all values y. Where the desired information step $y_i[k]$ for prosumer i at time step k is expressed by:

$$y_i[k] = \|p\kappa_i[k]\| = \|10\kappa_i[k]\|, \tag{6.8}$$

Remark 6.2 (Rounding). The mathematical symbol $\|..\|$ means rounding to the nearest integer.

A sample matrix corresponding to this example, is given for a network where each prosumer has an EV and two connected neighbors:

$$E_y = \begin{Bmatrix} 0.5{+}0.05y & 0.25{-}\frac{0.05y}{2} & 0 & 0 & \dots & 0 & 0.25{-}\frac{0.05y}{2} \\ 0.25{-}\frac{0.05y}{2} & 0.5{+}0.05y & 0.25{-}\frac{0.05y}{2} & 0 & \dots & 0 & 0 \\ 0 & 0.25{-}\frac{0.05y}{2} & 0.5{+}0.05y & 0.25{-}\frac{0.05y}{2} & \ddots & 0 & 0 \\ 0 & 0 & 0.25{-}\frac{0.05y}{2} & 0.5{+}0.05y & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0.25{-}\frac{0.05y}{2} & 0 & 0 & 0 & \dots & 0.25{-}\frac{0.05y}{2} & 0.5{+}0.05y \end{Bmatrix} \tag{6.9}$$

For examples on how $l_y(\cdot)$ changes the matrix $\mathbf{E}$ see appendix A.

A graphical representation is given of a network with prosumers that have EV's and prosumers without EV's in figure 7.2. In this figure can be seen that the information sharing matrix $\mathbf{E}$ does not change the information sharing of $\mathbf{A}$. More on this interaction is explained in section 7.2.3.

6.5 The linear Quadratic Regulator Problem in the EV-network

We will introduce a second state variable (e_i) which keeps track of the error between the desired SOC and the actual SOC for each prosumer i with an EV as device. The second information state variable e_i, is useful because we can put this state (indirectly) into a quadratic objective matrix. This is necessary because not violating the DSO constraint (Remark 4.1) is more important than adequately charging the EV. When instead of a state variable in the objective matrix, a method is chosen whereby the EV constraint is solved as a "hard" constraint, it could be that the DSO is violated because many EV's need to charge at the same time. This will force that USEF to move towards the orange and red regimes, see Chapter 2, which is very unfavorable.

To use this state variable in the fully distributed MPC, the same approach will be applied as in section 3.2 A discrete time network with n EV users is considered. Where at time-step k agent i has a state $e_i \in R$, and a control input $f_i \in R$.

Remark 6.3 (Constant information sharing matrix). *In this section, it is chosen to keep the information sharing matrix $\boldsymbol{E}$ constant, to explain the modeling of the EV constraint* *in a basic and understandable manner. In section 6.7 the updating information matrix will be implemented based on section 3.4.*

The new state is defined as:

$$\begin{aligned} e_i[k+1] =& E_{ii}e_i[k] + \sum_{j\neq i} E_{ij}e_j[k] + \Delta S_i[k] - \Delta S_i^{goal}[k] \\ =& E_{ii}e_i[k] + \sum_{j\neq i} E_{ij}e_j[k] + \eta f_i[k] - \Delta S_i^{goal}[k] \end{aligned} \tag{6.10}$$

where

$$\begin{aligned} \Delta S_i &= S_i[k+1] - S_i[k] = \eta * f_i[k] \\ \Delta S_i^{goal} &= S_i^{goal}[k+1] - S_i^{goal}[k+1] \end{aligned}$$

η is a ratio between energy used and energy stored (efficiency), and f_i is the controllable load of prosumer i, E_{ii} and E_{ij} are the weights of the neighboring connections in the network. This is the traditional way of doing the calculations based on a constant matrix. The following vectors are defined to write the system in compact form:

$$\begin{aligned} e[k] &= [e_1[k]...e_n[k]]^\top \\ f[k] &= [f_1[k]...f_n[k]]^\top \end{aligned}$$

Then, the compact form of the distributed information system is given by state equation

$$e[k+1] = Ee[k] + \eta f[k] - \Delta S^{goal}[k] \tag{6.11}$$

The performance of the system is measured by a quadratic objective function, where we introduce a objective variable $\breve{e}$:

$$V(\breve{e}[k]) = \sum_{k=1}^{K} \breve{e}[k]Q\breve{e}[k] \tag{6.12}$$

$$\breve{e}[k] = e[k] + \beta \tag{6.13}$$

$\breve{e}[k]$ in combination with β ensure that only a shortage of charge in the EV battery is punished, while overcharging the battery will not be rewarded or punished. $\beta[k] = [\beta_1[k]...\beta_n[k]]^\top$, with $\beta_i \in \{\beta_1, \beta_2, ..., \beta_n | \mathbb{R}_{\leq 0}\}$, is called a *slack variable* in optimization literature. A slack variable is added to an inequality constraint to transform it into an equality. Where $Q \in \mathbb{R}^{n\times n}$ is positive definite weight matrices. This function penalizes deviation of $\breve{e}[k]$ from zero. By introducing $\beta \in \mathbb{R}_{\leq 0}$ we ensure that the in the overall problem, where also the DSO constraint and the matching of the DAP are considered, will not be brought out of balance.

Remark 6.4 (Slack variable usefulness). *When the slack variable is not introduced, the solver will try to always match the amount of actual state-of-charge to the desired one. This is most likely to create an unnecessary conflict with the objective of matching the day-ahead-planning with the actual load.*

We assume here that Q is diagonal, so that the objective function is separable. Also as standard assumption we use that Q and E are observable and that E is controllable [9]. The goal of the LQR problem, if the problem was infinite horizon, is to find the control input that minimizes the infinite horizon cost based on (6.12). The optimal value of this problem is therefore given by

$$\begin{aligned} &\min \quad \limsup_{k\to\infty} V(\breve{e}[k]) \\ &\textbf{subjected to } (6.11) \textbf{ and } (6.13) \end{aligned} \tag{6.14}$$

The β will behave as follows in the minimization of (6.14):

$$\beta = \begin{cases} 0 & \text{if } e[k] \le 0 \\ -e[k], & \text{if } e[k] > 0 \end{cases} \tag{6.15}$$

6.6 Dynamic Decomposition

The right hand side of the state equation (6.11) depends on neighbor states through the terms $\sum_{j\neq i} E_{ij}e_j$. Therefore it is a coupled system, because the update of one agent also depends on information from neighbors. To decouple this state equation, each agent introduces a local variable, $v_i^{ev}[k] \in \mathbb{R}$ representing the expected influence from other agents on its state equation. Thus (6.11) is now given by the fully decoupled state equations

$$e_i[k+1] = E_{ii}e_i[k] + v_i^{ev}[k] + \mu f_i[k] - \Delta S_i^{goal}[k], \quad i = 1, ..., n \tag{6.16}$$

With additional equality constraint

$$v_i^{ev}[k] = \sum_{j\neq i} E_{ij}e_j[k], \quad i = 1, ..., n \tag{6.17}$$

By defining $E_D \in \mathbb{R}^{n\times n}$ as matrix that has the same diagonal as matrix $E \in \mathbb{R}^{n\times n}$, $E_{ii} = E_{D,ii}$ and it is zero elsewhere, $E_{D,ij} = 0$ if $j \neq i$. By also defining $E_0 = E - E_D$, equations (6.16) and (6.17) could be written as

$$e[k+1] = E_D e[k] + v^{ev}[k] + \mu f_i[k] - \Delta S^{goal}[k] \tag{6.18}$$

$$v^{ev}[k] = E_0 e[k] \tag{6.19}$$

Now the problem is further altered to fit in the Model Predictive Control. Following the same approach of sections 3.2, 3.3, but without the Lagrangian multiplier. The problem is written as a minimization problem:

$$\begin{aligned} \min \quad & \sum_{k=1}^{k+K_{pred}} \left(\hat{\hat{e}}_i[\tau] Q \hat{\hat{e}}_i[\tau] \right) && (6.20) \\ \text{subject to} \quad & \hat{e}_i[\tau+1] = E_{ii}\hat{e}_i[\tau] + \hat{v}_i^{ev}[\tau] + \mu \hat{f}_i[\tau] - \Delta S_i^{goal}[\tau] \\ & \hat{\hat{e}}_i[\tau] = \hat{e}_i[\tau] + \beta_i[\tau] \end{aligned}$$

The final step is to adapt the system in a way that it uses the sub-gradient iterations of the Lagrangian multipliers μ and λ to also find the best solution for all the EV's combined. Instead of sending the sub-gradient values to the neighboring EV's, will the EV-user sent the state variables e_i for the time horizon for all $\tau = k, ..., k+K_{pred}$, to the neighbors. The received state variables are used to make the expected influence of the neighbors and will be filled in for $v_i^{ev}[\tau]$ in the next iteration r, until the sub-gradient multipliers μ and λ_i are below the stopping ϵ, see section 3.2.2. The expected influence is given by:

$$\begin{gathered} \hat{v}_{i,r}^{ev}[\tau] = \sum_{j \neq i} E_{ij}\hat{e}_{j,r}[\tau], \\ \tau = k, ..., k+K_{pred}-1, \end{gathered} \tag{6.21}$$

Remark 6.5 (Driving up the Prices). *The combination of this technique with the pricing mechanism of λ is expected to work really well. The distributed EV constraint is expected to increase the prices by giving them less liberty, because violation of the EV will result in a penalty. The sub-gradient method will ensure that the (sub)optimal balance will be found between matching the day-ahead planning and charging the EV's.*

To conclude, the distributed MPC for the EV-constraint is obtained. In the next section, the new information structure will be implemented.

6.7 Updating Information System

As explained in section 6.4, a system of updating matrices is used for the EV constraint. The same approach as in the section 3.4 is applied. The EV's information state about the state-of-charge with the updating matrices is expressed by

$$e_{y_i[k],i}[k+1] = E_{y_i[k],ii}e_{y_i[k],i}[k] + \sum_{j \neq i} E_{y_i[k],ij}e_{y_i[k],j}[k] + \eta f_i[k] - \Delta S_i^{goal}[k], \tag{6.22}$$

where $e_{y_i[k],i}[k]$ is state of actor i for time step k and used information step $y_i[k]$. Where $y_i[k]$ is calculated based on formulas similar to (6.7) and (6.8). $E_{y_i[k],ii}[k]$ and $E_{y_i[k],ij}$ are the weights that also correspond to the used information step y calculated by switching function $y_i[k]$. When an EV user switches from an information step to the next information step, the actors have to start using a different state $e_{y_i[k],i}$ that corresponds to the new information step y. This is only possible, as explained in section 3.4, when all the prosumers calculate the state $e_{y,i}[k+1]$ for all information steps y, after solving the optimal control problem at each time step k. *For prosumers without EV's is* $e_{y,i}[k] = 0 \forall y, k$.

This will result in a set of $M \in \mathbb{R}^{n \times K \times p}$, where n is the number of prosumers, K the total number of time steps, and p the total number of information steps. In the set M are the information states $e_{y,i}[k] \forall y, k, i$ stored. Based on the following calculations:

$$\begin{aligned}
e_{1,i}[k+1] =& E_{1,ii}\; e_{1,i}[k] + \sum_{i \neq j} E_{1,ij}\; e_{1,j}[k] + \eta f_i[k] \quad && \forall i \\
&\vdots \\
e_{y,i}[k+1] =& E_{y,ii}\; e_{y,i}[k] + \sum_{i \neq j} E_{y,ij}\; e_{y,j}[k] + \eta f_i[k] \quad && \forall i \\
&\vdots \\
e_{p,i}[k+1] =& E_{p,ii}\; e_{p,i}[k] + \sum_{i \neq j} E_{p,ij}\; e_{p,j}[k] + \eta f_i[k] \quad && \forall i
\end{aligned} \tag{6.23}$$

This is done to ensure that the EV users can work without the occurrence of an error, as explained in section 3.4.2. One EV-user can be completely "selfish" without influencing the information sharing of the neighbors. After solving each time step k, the prosumers sends all the information states to their E_y matrix neighbors. This is not computational expensive as long as the number of information steps (p) is kept within reasonable numbers.

The extended optimal control problem from equation (6.20) becomes:

$$
\begin{aligned}
\sum_{i=1}^{n} \min \quad & \sum_{k=1}^{k+K_{pred}} \left(\hat{\breve{e}}[\tau] Q \hat{\breve{e}}[\tau]\right), & \tau = k, ..., k + K_{pred} - 1 \\
\text{subject to} \quad & \hat{e}_{y_i[\tau],i}[\tau+1] = E_{y_i[\tau],ii}\hat{e}_{y_i[\tau],i}[\tau] + \hat{v}_i^{ev}[\tau] + \eta \hat{f}_i[\tau] - \Delta S_i^{goal}[\tau], & \\
& \hat{e}_{y_i[k],i}[k] = e_{y_i[k],i}[k] & \\
& \hat{\breve{e}}_i[\tau] = \hat{e}_{y_i[\tau],i}[\tau] + \beta_i[\tau] & \\
& \hat{e}_i[\tau] \in X_i, \hat{f}_i[\tau] \in U_i, & \tau = k, ..., k + K_{pred}
\end{aligned}
\tag{6.24}
$$

With additional constraint:

$$
\hat{v}_i^{ev}[\tau] = \sum_{j \neq i} E_{y_i[\tau],ij}\ \hat{e}_{y_i[\tau],j}[\tau] \tag{6.25}
$$

To calculate the expected influence with the updating matrices, a choice can be made between calculation time and precision, as is explained in section 3.4. We define a variable that represent the update steps that the neighbors are using for calculating their state within the prediction horizon, $\bar{y}$.

Equation (6.21) is extended, and can be used for the updating matrices:

$$
\begin{gathered}
\hat{v}_{i,r}^{ev}[\tau] = \sum_{j \neq i} E_{y_i[\tau],ij}\ \hat{e}_{y_i[\tau],j,r}[\tau], \qquad \forall \bar{y} \\
\tau = k, ..., k + K_{pred} - 1,
\end{gathered}
\tag{6.26}
$$

where the values $\bar{y}$ represent the updated steps that the neighbors are using for calculating their state within the prediction horizon and r is the iteration step, see section 3.4.3. This calculations requires that each state should be calculated at each iterations for all $\bar{y}$ with equations (6.23). This can become computational expensive, but it will not influence the behaviour of the actor within an iteration directly because the actor will base its behaviour based on control problem (6.24). Therefore, it will not require extra solving time for the solver. These values could be calculated after a solver is used. After calculating these predicted values of $e_{\bar{y},i,r}[\tau+1]$. Each EV user i sends these information states towards the neighboring EV's at each iterations until convergence is obtained. Convergence is based on the Lagrangian multipliers, see section 3.2.2.

To conclude, a decentralized EV constraint is obtained that satisfies all the requirements set in section 6.1. In Chapter 7 is the constraint implemented in the overall control problem.

Chapter 7

Algorithm & Combined Optimal Control

This chapter will give an overview of the 3 different control problems. Sets of constraints are written down in a way that they could be put into a linear quadratic solver. The concept of weight balancing is discussed and hierarchy is given on the importance of the constraints. The Chapter concludes with the algorithm that solves the Combined Optimal Control problem. The Combined Optimal Control problem should result in an (sub)optimal control of the smart grid with 4 devices without DSO constraint violations.

There are 3 different constraint sets:

- For the μCHP and heat pump, described by Nyugen et al. in [11]
- A set for the home battery, described in Chapter 5.
- A set for the EV, where the constraint is described in Chapter 6, the model will be extended with device specific constraint from Chapter 5

7.1 Hierarchy

Before the constraints are summarized, is it important to understand the hierarchy between the objectives of the overall problem. Based on the stakeholder analysis in section 2.3, and objective hierarchy is made, which is graphically visualized in figure 7.1. The most important objective for the Distribution System Operator (DSO), but also for all the other actors in the system, is to prevent violations of the maximal electricity network capacity, also referred to as the DSO constraint. The highest weight should be set on this constraint, therefore the multiplication of the Lagrangian multiplier μ should be always larger than the other objectives when the constraint is violated. The second most important constraint is the EV constraint. The EV-user will only work with USEF product UFLEX when adequately charging of the EV is guaranteed. The System of Updating matrices is somewhat relaxing this constraint to encourage the use of UFLEX, but in the end charging should be more important to the EV-user than matching the day-ahead-planning. *Note that the EV objective does not occur in the problem sets of the home-battery, μCHP and heat pump.* After the DSO and the EV

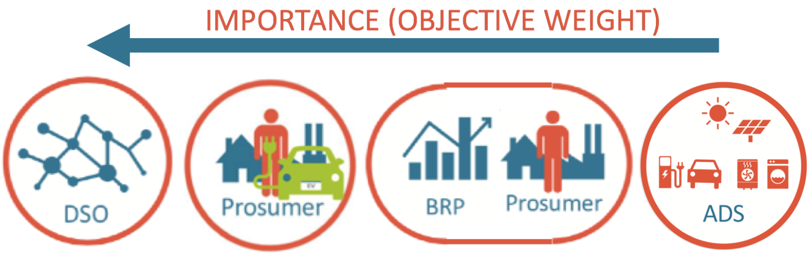


FIGURE 7.1: Increasing importance for the objectives from right to left

objective comes the day-ahead-planning objective. This objective is beneficial for the prosumer, balance responsible party (BRP) and the aggregator. The day-ahead-planning should be matched by the actual and fixed load, this can be done by the Active demand & supply devices (ADS) of the prosumer, with help and coordination from the aggregator and paid for by the BRP. For more information see Chapter 2. This objective is a combination of the Lagrangian multiplier λ and the state (error) x.

The least important constraint is efficient controlling the of ADS devices. While this is important, the weight should be much smaller than matching the day-ahead-planning (and the rest), because it should be always rewarding to match the DAP with applying some flexible load. It is important to note that we assume that it is always more monetarily rewarding to match the day-ahead-planning compared to the costs of energy loss to efficiency.

7.2 Optimal Control Problems Summary

This subsection summarizes and clusters the constraints to make it easier to see which equations are used in the algorithm. This is only an summary of the constraints that are used inside the solver. The calculations outside the solver are provided in section 7.3

7.2.1 micro-CHP and Heat Pump Problem

Chapter 4 and the article [11] provide a detailed explanation of all variables and constraints.

Optimal Control Problem: μCHP and Heat Pump

$$\max_{\lambda_i,\mu_i} \min_{u_i,v_i} \sum_{k=0}^{K-1} \overbrace{x_i[k]Qx_i[k]}^{\textbf{DAP}} + \overbrace{\lambda_i[k]v_i[k] - x_i[k]\Bigg[\sum_{j\neq i} A_{ij}\lambda_j[k]\Bigg]}^{\textbf{Prosumer decoupling}}$$

$$+\underbrace{\mu[k]f_i[k] + \mu[k]g_i[k]}_{\textbf{DSO constraint}} \tag{7.1}$$

Subjected to:

$$x_i[k+1] = A_{ii}x_i[k] + v_i[k] + u_i[k] + w_i[k] - \Delta goal_i[k] \tag{7.2}$$

$$u_i[k] = f_i[k+1] - f_i[k] \tag{7.3}$$

$$w_i[k] = g_i[k+1] - g_i[k] \tag{7.4}$$

$$\Delta goal_i[k] = goal_i[k+1] - goal_i[k] \tag{7.5}$$

$$f_i[k] = P_{device}\delta \tag{7.6}$$

$$L_i[k+1] = L_i[k] + \frac{\eta}{C}|f_i[k]| - \frac{q[k+1]}{C} \tag{7.7}$$

And a set of constraints that work as counters for the OFF (t_{off}) and ON (t_{on}) time for the devices.
Most important lower and upper bounds:

$$L_{min} \leq L[k] \leq L_{max}, \quad -P \leq f_i[k] \leq P, \quad T_{min,on} \leq t_{on}, \quad T_{min,off} \leq t_{off}$$

And

$$\delta_b \in [0,1], \quad t_{on} \in \mathbb{Z}, \quad t_{off} \in \mathbb{Z}$$

7.2.2 Home Battery Problem

Chapter 5 provides a detailed explanation of all variables and constraints.

Optimal Control Problem: Home Battery

$$\max_{\lambda_i,\mu_i}\min_{u_i,v_i}\sum_{k=0}^{K-1}\overbrace{x_i[k]Qx_i[k]}^{\textbf{DAP}}+\overbrace{f_i[k]Rf_i[k]}^{\textbf{Efficiency}}+\overbrace{\lambda_i[k]v_i[k]-x_i[k]\left[\sum_{j\neq i}A_{ij}\lambda_j[k]\right]}^{\textbf{Prosumer decoupling}}$$
$$+\underbrace{\mu[k]f_i[k]+\mu[k]g_i[k]}_{\textbf{DSO constraint}} \tag{7.8}$$

Subjected to:

$$x_i[k+1]=A_{ii}x_i[k]+v_i[k]+u_i[k]+w_i[k]-\Delta goal_i[k] \tag{7.9}$$

$$u_i[k]=f_i[k+1]-f_i[k] \tag{7.10}$$

$$w_i[k]=g_i[k+1]-g_i[k] \tag{7.11}$$

$$\Delta goal_i[k]=goal_i[k+1]-goal_i[k] \tag{7.12}$$

$$\rho f_i[k]=P_b\delta_b[k] \tag{7.13}$$

$$S_i[k+1]=S_i[k]+\frac{\eta_b}{C_b}f_i[k] \tag{7.14}$$

Most important lower and upper bounds:

$$S_{min}\leq S[k]\leq S_{max},\qquad -\rho\leq\delta_b\leq\rho,\qquad -P_b\leq f_i[k]\leq P_b$$

And

$$\delta_b\in\mathbb{Z}$$

7.2.3 Electric Vehicle Problem

The EV constraint is constructed in a way that it could be combined with the preceding system model of Nguyen[11]. The combined optimal control problem consist of two states, two Lagrangian multipliers and two information sharing structures .

Optimal Control Problem: Electric Vehicle

$$\max_{\lambda_i,\mu} \min_{u_i,v_i,v_i^{ev}} \sum_{k=0}^{K-1} \overbrace{x_i[k]Qx_i[k]}^{\textbf{DAP}} + \overbrace{f_i[k]Rf_i[k]}^{\textbf{Efficiency}} + \overbrace{\lambda_i[k]v_i[k] - x_i[k]\left[\sum_{j\neq i} A_{ij}\lambda_j[k]\right]}^{\textbf{Prosumer decoupling}}$$
$$+ \underbrace{\check{e}_i[k]W\check{e}[k]}_{\textbf{EV constraint}} + \underbrace{(\mu[k]f_i[k] + \mu[k]g_i[k])}_{\textbf{DSO constraint}} \tag{7.15}$$

subjected to

$$x_i[k+1] = A_{ii}x_i[k] + v_i[k] + u_i[k] + w_i[k] - \Delta goal_i[k] \tag{7.16}$$

$$u_i[k] = f_i[k+1] - f_i[k] \tag{7.17}$$

$$w_i[k] = g_i[k+1] - g_i[k] \tag{7.18}$$

$$\Delta goal_i[k] = goal_i[k+1] - goal_i[k] \tag{7.19}$$

$$e_{y_i[k],i}[k+1] = E_{y_i[k],ii}e_{y_i[k],i}[k] + v_i^{ev}[k] + \eta f_i[k] - \Delta S_i^{goal}[k] \tag{7.20}$$

$$\check{e}_i[k] = \xi_i[k]e_{y_i[k],i}[k] + \xi_i[k]\beta_i[k] \tag{7.21}$$

$$(1 - \xi_i[k])\delta_{ev} = 0 \tag{7.22}$$

$$\Delta S_i^{goal}[k] = S_i^{goal}[k+1] - S_i^{goal}[k] \tag{7.23}$$

$$\rho f_i[k] = P_{ev}\delta_{ev}[k] \tag{7.24}$$

$$S_i[k+1] = S_i[k] + \frac{\eta_{ev}}{C_{ev}} f_i[k] \tag{7.25}$$

Most important lower and upper bounds:

$$S_{min} \leq S[k] \leq S_{max}, \quad -\rho \leq \delta_{ev} \leq \rho, \quad -P_{ev} \leq f_i[k] \leq P_{ev}$$

And

$$\delta_b \in \mathbb{Z}$$

The interaction of the preceding system with the neighbors will not change, still imbalance information will flow to the neighboring prosumers. The change in information flow, caused by adding a second network of information sharing matrices E_y, results in additional contact between EV-users only. The EV users work together to charge all the EV's to the desired level, while communicating their imbalance information with the neighbors. The information flow is graphically represented in figure 7.2. This alignment within the preceding system is a conscious choice. When balance information of EV-users is also only shared with neighboring EV's instead of other type of users, then it will be impossible to match the load profiles to the day-ahead-planning (DAP) when there are imbalances, when the EV's are unplugged. This is mainly because EV users will have the similar charging periods, as explained in section 6.1. When the EV's are not plugged in, the neighbors could help reducing the error x_i with their "fixed-location" devices.

Example 7.1. *In this example, there are 8 prosumers, 4 of the prosumers have an EV, and the other prosumers are equipped with an home-battery, heat pump or μCHP. The imbalance (and price) is shared via the blue lines, this represents the information sharing according to matrix $\boldsymbol{A}$ of Chapter 4. The orange lines represent the EV-user communication, which is shared according to matrix $\boldsymbol{E}_i[k]$. The matrix $\boldsymbol{E}$ that corresponds to the network of the system illustrated in 7.2, is given by equation(7.26).*

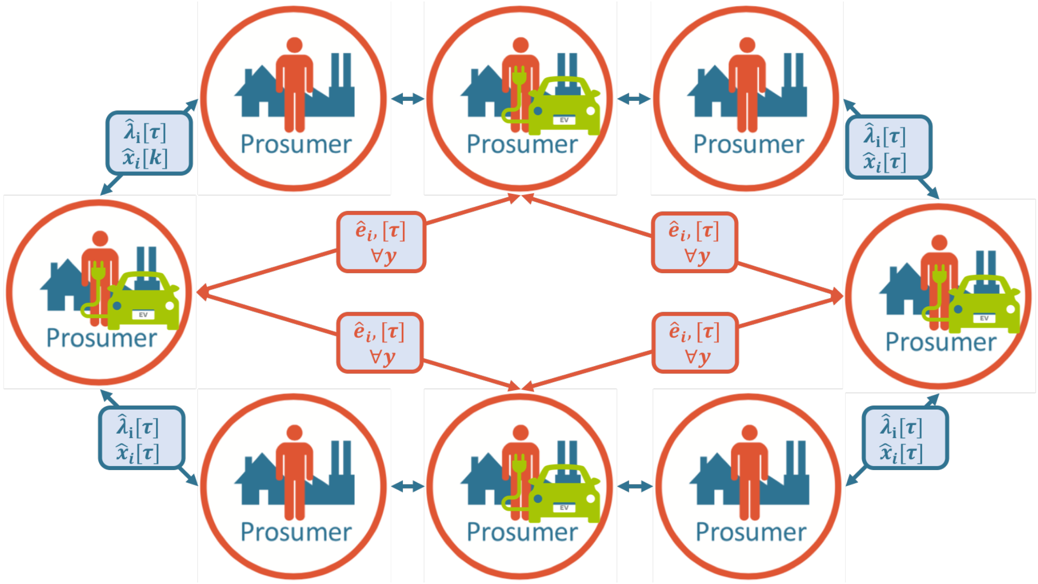


FIGURE 7.2: The information flows between prosumers with EV and without EV's

$$E_y = \left\{ \begin{array}{cccccccc}
0.5+l_y(\cdot) & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0 & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0.5+l_y(\cdot) & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0.5+l_y(\cdot) & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0 & 0 & 0.25\text{-}\frac{l_y(\cdot)}{2} & 0 & 0.5+l_y(\cdot) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right\} \tag{7.26}$$

7.2.4 Lagrangian Multipliers

The Lagrangian sub-gradient updates are:

$$\lambda_i^{r+1}[k] = \lambda_i^r[k] + \gamma_i^r \left(v_i^r[k] - \sum_{j \neq i} A_{ij} x_j^r[k] \right) \tag{7.27}$$

$$\mu_i^{r+1}[k] = \mu_i^r[k] + \gamma_i^r \left(\sum_i (f_i[k] + g_i[k]) - L_{max} \right) \tag{7.28}$$

The properties of the Lagrangian multipliers are explained in section 3.2.2 and Chapter 4. The Lagrangian multipliers are calculated after the individual Optimal Control Problems are solved.

7.3 Algorithm

The algorithmic description of the distributed MPC scheme is shown in Algorithm 2, with $\tau = k, ..., k + K_{pred}$ being the prediction time-step within the receding horizon K_{pred}. The hat notation indicates the predicted values. For clarity we only described the algorithm for one aggregator network. All the aggregators perform the same optimization loop. In this algorithm, the DSO monitors for congestion points at every iteration step, after all aggregators have performed the iteration the following cases can happen.

- If the DSO constraint is violated ($\hat{\mu} > 0$, all aggregators have to perform the optimization loop again. The DSO, acting as the central coordinator for congestion managements, sends out the shadow price $\mu[k]$ to all prosumers to encourage less load consumption.
- If there are no congestion points detected by the DSO, but the λ has not converged yet, then all aggregators have to perform the optimization loop.
- If some aggregators have converged λ_i, then only the aggregators that are not converged have to go in the optimization loop again.

Algorithm 2: Solving the Combined Optimal Control Problem

for $k \leftarrow 0$ ***to*** $K-1$ **do**
  each prosumer i measures $x_i[k], w_i[k], e_i[k]$;
  $\hat{x}_i|_{\tau=k} = x_i[k], \hat{w}_i|_{\tau=k} = w_i[k], \hat{e}_i|_{\tau=k} = e_i[k]$;
  initialize $\hat{\lambda}_i[\tau], \hat{\mu}[\tau]$ and ϵ
  if i ***is*** *EV user* **then**
    each prosumer updates $y_i[\tau]$
    each prosumer updates $E_{y_i[\tau]}$;
    each prosumer updates $e_{y_i[k],i}[\tau]$
    each prosumer sends $\bar{y}[\tau]$ to EV neighbors
  end
  while $|\hat{\lambda}_i^r[\tau] - \hat{\lambda}_i^{r-1}[\tau]| > \epsilon(\forall i)$ ***or***
  $|\hat{\mu}^r[\tau] - \hat{\mu}^{r-1}[\tau]| > \epsilon$ **do**
    for $i \leftarrow 1$ ***to*** n **do**
      each prosumer calculates $\hat{v}_i^{pred} = \sum_{j \neq i} A_{ij} \hat{\lambda}_i^r[\tau]$
      if i ***is*** *EV user* **then**
        each prosumer calculates $\hat{v}_i^{ev} = \sum_{j \neq i} E_{i,ij}[k] e_{y_i[\tau],i,r}[\tau]$
        solve (7.15)
      else if i ***is*** *battery user* **then**
        solve (7.8)
      else
        solve (7.1)
      end
    end
    each prosumer i sends $\{\hat{x}_i[\tau]\}_{\tau=k}^{k+K_{pred}}$ to neighbors;
    each prosumer i sends $\{\hat{e}_{y_i[\tau],i}[\tau]\}_{\tau=k}^{k+K_{pred}}, \forall \bar{y}$ to neighbors;
    for $i \leftarrow 1$ ***to*** n **do**
      subgradient update (7.27)
    end
    each prosumer i sends $\{\hat{\lambda}_i[\tau]\}_{\tau=k}^{k+K_{pred}}$ to neighbors;
    subgradient update (7.28)
    each prosumer i receives $\{\hat{\mu}[\tau]\}_{\tau=k}^{k+K_{pred}}$ from DSO;
  end
  each prosumer i implements $u_i[k] = \hat{u}_i[\tau|_{\tau=k}]$;
  each EV user calculates $e_i[k](\forall y)$ according to (6.23)
end

Remark 7.1 (Aggregators). *We assume that the number of agents for all aggregators is the same. There is no coupling between prosumers or EV users that belong to different aggregators. Using dual decomposition, a multilevel distributed optimization formulation is obtained for the prosumers, that can only communicate locally with the neighbors during the optimization loop it self, therefore is this part of the system fully distributed. The aggregators only send out the goal functions at the beginning of each optimization loop, they do not act during the process.*

Chapter 8

Simulations

This Chapter will elaborate on how we implement our controller in 6 scenarios. First the modelling parameters are given for the simulations with an explanation about the data used. Also the concept of how the demand is determined is explained. Section 8.2 provides the results for the normal experiment and some extremer experiments that are used to show the robustness of the controller. The results are discussed and compared with each other.

8.1 Implementation

The controller is implemented for different situations. We use MATLAB 7.5 with the Gurobi solver 8.0.1 [7], to solve the mixed-integer quadratic programs. A circular topology is chosen for both the EV and prosumer clusters, within an aggregator. The weights of matrix $\mathbf{A}$ are 0.5 as self-weight and two times a weight of 0.25 from the neighbors. The initial weights of the $\mathbf{E}$ matrices are the same as the constant weights of matrix $\mathbf{A}$. For the common simulations we choose the circular sequence: **...-μCHP-HP-Battery-EV-μCHP-...**. A example is given in figure 8.1 for the weights and connections in a graph with 12 prosumers.

The matrices E_y are constructed according to equation (6.9) for all information steps y before the calculations start, to prevent unnecessary calculations during the optimization loops. We work with realistic load profiles acquired from the patern genarators from the Energy research Center of the Netherlands (ECN) [12]. The device parameters of the μCHP and HP are based on devices installed in PowerMatching City, a smart grid demonstration project in the Netherlands [2]. Which is the same data as Nguyen et al. used in the article [11]. The battery is based on the Tesla Powerwall with a

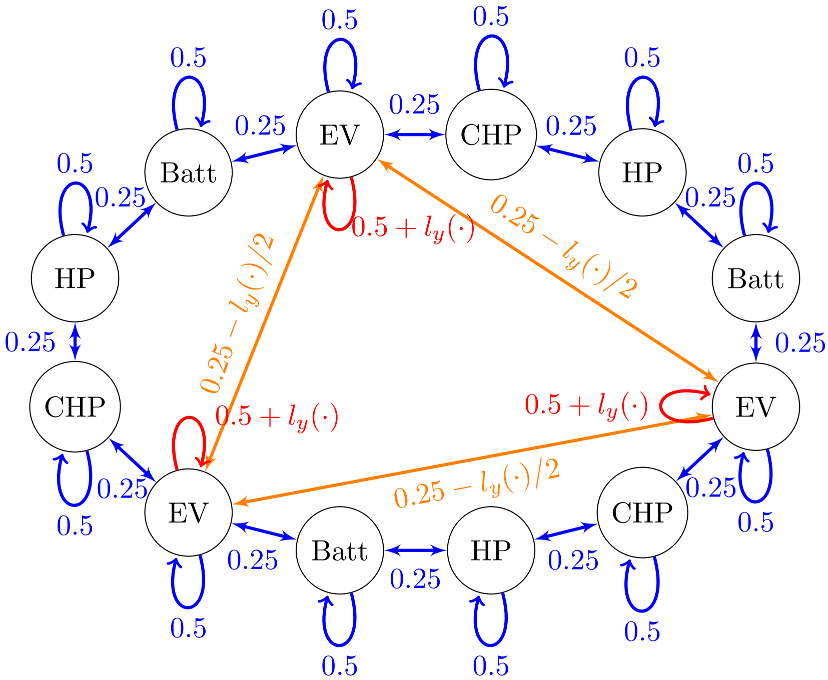


FIGURE 8.1: Example of common simulation with 12 prosumers in the circular sequence

capacity of 13.5kWh, however, the charging and discharging power is lowered for the common experiments to 3kW instead of the actual 5kW. The electric vehicle parameters are based on the Nissan leaf with a battery of 21.3 kWh usable battery capacity, and lowered charging power of 2.7 kW instead of the actual 3.6 kW.

Remark 8.1 (Lowered parameters). *The parameters are lowered to see more effect of the fixed load changes on the behaviour of the model, this is chosen because the mathematics are more important in this thesis than the actual parameters. The same is done for the DSO constraint: 1.1 kW per prosumer in the article of Nyugen et al. [11]. In Appendix B is the result of a simulation with actual parameters.*

The Lagrangian multipliers and the e^{pred} are initialized with zeros. The sub-gradient convergence criterion ϵ for both multipliers, is chosen to be 0.02. In the simulations 1 time step corresponds to 15 minutes in real time, and the prediction horizon for the DMPC is taken to be six time-steps. The heat buffer levels are arbitrarily initialized. The initial state-of-charge levels are 1000kW for the Electric vehicle and 8kWh for the home batteries. The home batteries are half full at the initialization, this is a reasonable assumption because after 1 hour, EV's start to enter the control problem, this is commonly in the evening, when all the home batteries are charged by energy from for example solar panels and wind turbines. The total simulation time will be 12 hours ($K = 48$). The step sizes of the sub-gradient iterations are chosen to be $\gamma = 1/((r)^{0.5}$.

The demand patterns are very important for the simulations. The EV's arrive between time step 4 and time step 12, and they leave between time step 34 and 47 for the common scenarios. The desired state-of-charge steps for the EV's could be linear ascending, however also for some scenarios non-linear. The total desired state of charge is between 12.5kWh and 19 kWh. These levels are reasonable because we initialize the battery at a very low state-of-charge level, this corresponds of to a difference of more than 12.5kWh, and therefore reasonable considering the lowered charging power. The demand for the EV user is the summation of the fixed load plus the desired state of charge step ΔS_i^{goal}. For the home battery, the demand is based on the fixed load plus 0.5kW if the EV's are away, to simulate charging by for example solar panels, and minus 0.5kW in the common EV charging time, to simulate the rise in price of electricity, which encourages selling electricity to neighbors for example, of charging your own EV with energy stored in your home battery. Also, to simulate the discharging of batteries at the moment when they are needed the most, in a way that the batteries could be used for charging the next (sunny) day. The demand for the μCHP and the HP is the summation of fixed loads and normalized heat demand. The day-ahead-planning is the summation of all the "demands" of the prosumers per time step with a moving average filter that has a window of 90 minutes. The DAP is distributed over the aggregators according to equation (4.5) and thereafter distributed over the prosumers by equation (4.6). The day-ahead-planning may never become larger than the DSO constraint. We apply shaving of the DAP when this occurs, because USEF requires that the DAP has to be congestion free.

The hierarchical weights are determined based on the requirements of section 7.1. Q is associated with state variable x_i, R is associated with the efficiency (section 5.4), W is associated with the information state variable $\breve{e}_i$ and M is associated with the DSO constraint (Lagrangian multiplier μ). The weights used in the simulations are presented in Table 8.1

	Q	**R**	**W**	**M**
μCHP	1	-	-	1
Heat Pump	1	-	-	1
Home Battery	1	0.05	-	1
Electric Vehicle	1	0.05	10	15

TABLE 8.1: Objective weights for the Optimal Control Problems

Remark 8.2 (Sensitive Objective Weights). *The objective weights are based on the device parameters but also on for example common values of μ,λ_i and several other values. The most important are given in Table 8.2. These values are obtained from multiple test simulations with varying device parameters.*

	x_i Lower	Upper	$\breve{e}_i$ Lower	Upper	f_i Lower	Upper	μ Lower	Upper
μCHP	-2	2	-	-	-	-	0	5
Heat Pump	-2	2	-	-	-	-	0	5
Home Battery	-2	2	-	-	-3	3	0	5
Electric Vehicle	-1	3	-1	0	-3.5	3.5	0	5

TABLE 8.2: Common approximate values for the associated variables of the objective weights

8.2 Results

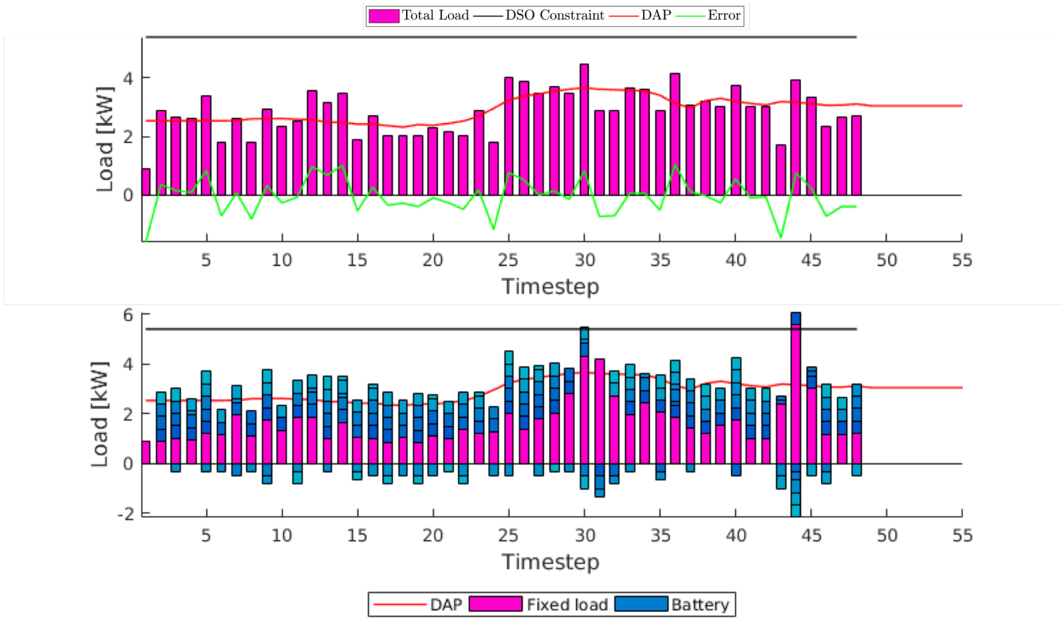


FIGURE 8.2: Scenario 1, 6 prosumers with congestion management and only home battery's. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail. The system aims to follow the forecasted day-ahead-planning. The green line is the mismatch. The network capacity limit (DSO constraint) is not violated.

Scenario 1: In this scenario, the network consists of 6 households, each with the home battery as appliance, and they are connected in a circular order under one aggregator. The DSO constraint is working and the DAP is based only on the fixed loads. There are three discharge and charging levels ρ, which results in the option to charge or discharge for 33%, 66% or 100%. The efficiency constraint on f_i is disabled in this graph, to show the inefficient solutions, for example at time step 9.

The load levels cause that the forecasted day-ahead-planning is more closely matched than the simulations done by Nguyen et al. The congestion point at time step 44 is not only completely resolved by the batteries, but also really close to matching the day-ahead-planning. This is a significant difference with the controlling only with μCHP and heat pumps. The option to use appliances that can both charge and discharge is profitable for the overall control problem and it helps bringing more stability to the system. Less prosumers are needed to balance the system because the prosumers with home batteries can accomplish the role of energy supplier and energy consumer. Arguments stated in literature saying that home batteries and EV could potentially be the back bone of the energy stabilization in the future is endorsed by the results in figure 8.2.

It is also important to note that the characteristics of the DMPC controller became visible in the the results of scenario 1. Instead of looking only at the current time step the MPC looks at the next 6 time steps, where the error of the last time step is remembered. In time step 43 for example there is a quite large negative error, which is compensated by the positive error at time step 44 and 45. Here it can be seen that in time step 43 already anticipates on the high fixed level load level of time step 44, which is assumed to be known. The other way around happens at time step 30 that compensates for time steps 31 and 32. This has to do with forecasting of the MPC but also with the "remembering" of the error of the last time steps.

Scenario 2: 28 prosumers with 4 different ADS devices and 1 aggregator
In this scenario, the network consists of 28 households, each with one of the 4 devices: HP, μCHP, home battery or EV. The prosumers are connected in a circular order under one aggregator. The DSO constraint is enabled and the DAP is based on the common settings, see section 8.1. There are three discharge and charging levels ρ, which results in the option to charge or discharge for 33%, 66% or 100%. The EV's information sharing matrices $\mathbf{E}$ are updated with a **linear gradient** of $l[y]$.

Until time step 7 no EV is in the coupled to the charger. Therefore the load matching of the DAP is done with a combination of home batteries, μCHP's and head pumps. For this reason the home batteries are mostly charging until time step 10. At time step 10 almost all the EV's start charging, as can be seen in figure 8.4. This requires that the home batteries start to discharge and work with the μCHP's to lower the consumption of total load (fixed plus flexible).

In time step 30 a high fixed load causes the EV to start to charge a bit less, which can be seen in figure 8.4b, while the total SOC stays above the total desired state of charge (figure 8.4a). This in combination with the all the batteries discharging and turning ON the μCHP's, it reduces the peak to a small mismatch.

From time step 44, the EV starts to decouple from the system. When the EV's decouple, the home batteries start to recharge again to match the DAP. This is expected to do so until the EV's are coupled to the system again.

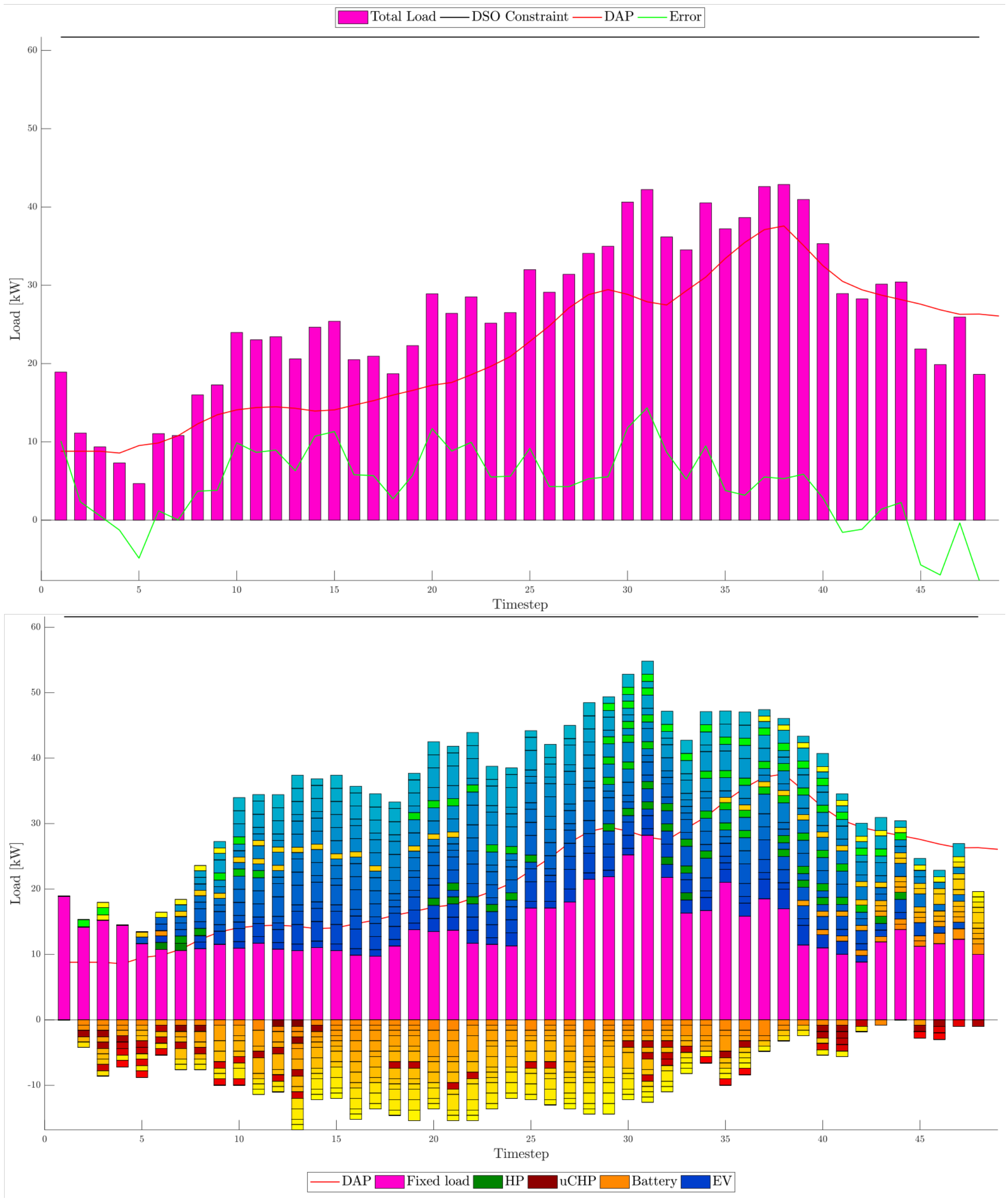


FIGURE 8.3: Scenario 2, 28 prosumers with congestion management, all 4 the ADS devices and 1 aggregator. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail. The green line is the mismatch. The network capacity limit (DSO constraint) is not violated. The red line is the DAP.

During all the time steps DSO violation is prevented and the EV's are adequately charged during the whole time horizon. The combination of charging patterns an the linear gradient of $l[y]$ have not a significant consequence on the individual charging behavior of the EV's. A small negative individual SOC difference compared to the desired state of charge is only occurring between time step 10 and 18 for prosumer 4, (EV 1, top left graph in 8.4 and a small individual difference for EV user 6 (bottom right). The

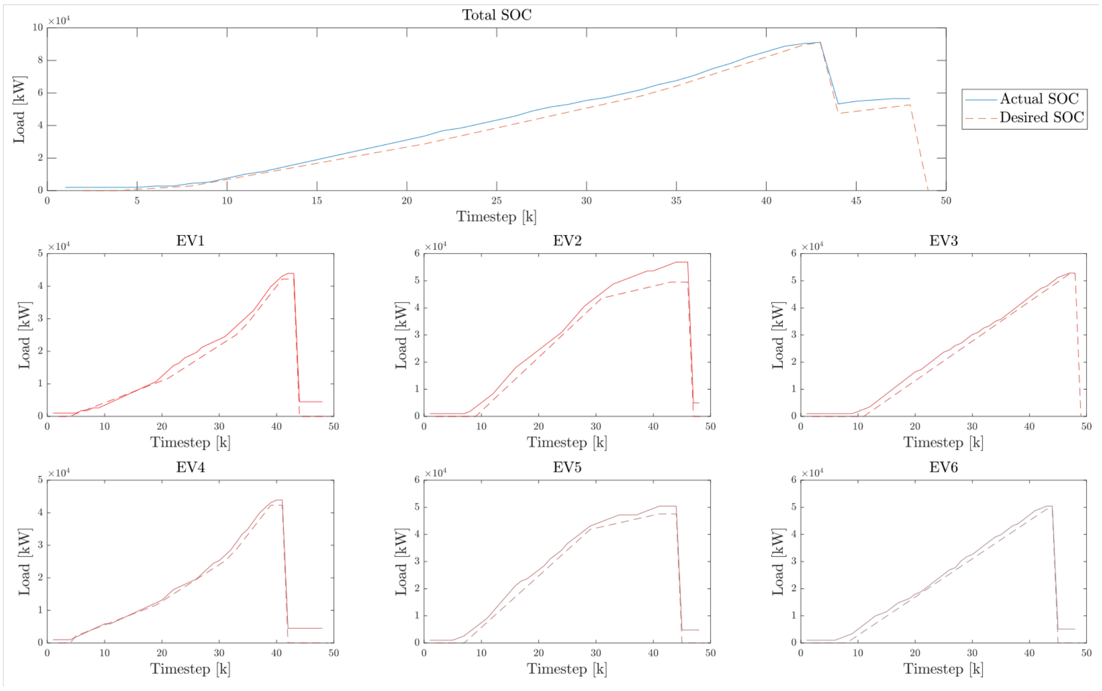


FIGURE 8.4: Scenario 2, the actual SOC of the EV's users compared to the desired SOC per time step. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC.

updating of information sharing matrices **E** to become selfish, results in no (individual) violations of the desired SOC in the end of the charging period.

Inefficient solutions that occurred in scenario 1, are almost completely resolved, see section 5.4, for explanation on inefficient solutions.

The total error, over all the time steps together is 76.34 kW, where compensation of negative errors values with positive errors in the next time step is not done.

Scenario 3: 2x 28 prosumers with 4 different ADS devices and 2 aggregators

In this scenario, the network of an aggregator consists of 28 households, each with one of the 4. The prosumers are connected in a circular order under **two** aggregator. Which makes a total of 56 prosumers. The DSO constraint is enabled and the DAP is based on the common settings, see section 8.1. There are three discharge and charging levels ρ, which results in the option to charge or discharge for 33%, 66% or 100%. The EV's information sharing matrices **E** are updated with a **linear gradient** of $l[y]$. There are larger graphs of figure 8.5 in the appendix.

The charging patterns per aggregator have small differences compared to scenario 2. The different charging patterns indicate that it is possible to work with all sorts of demand patterns as long as the Δ_i^{goal} is smaller than the battery profile constraints, section 5.2.

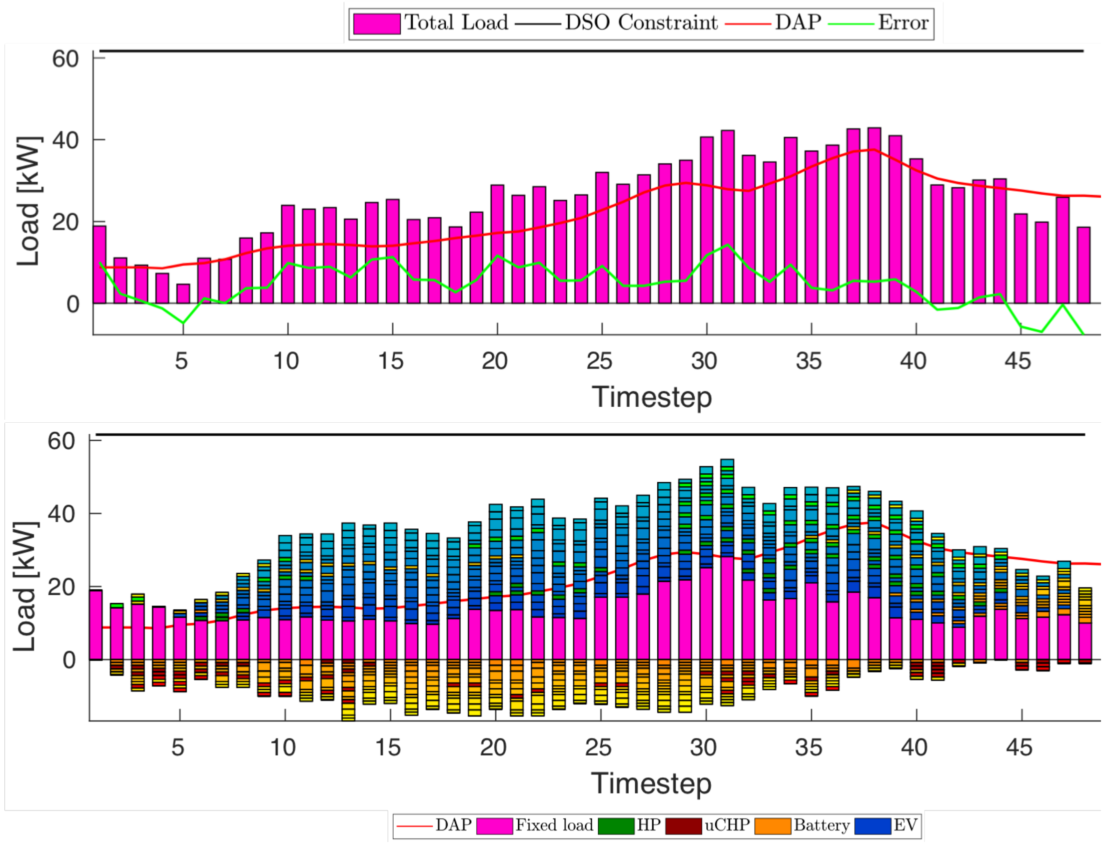


FIGURE 8.5: Scenario 3, 28 prosumers with congestion management, all 4 the ADS devices and 2 aggregators. Top: Total load within the both prosumer networks. Bottom: flexible and fixed loads in detail. The network capacity limit (DSO constraint) is not violated.

Our controller seems not the have any problems satisfying different charging behaviors while matching the DAP within acceptable errors. For both aggregators, there are no EV constraint violations at any time step.

Both aggregators are covered by one DSO. They should ensure that there are no violations of the network capacity limit, without getting imbalance information of the other aggregator network. In this scenario now violations occur, but there is an larger "positive" error in matching the DAP than in scenario 2. The separation of information causes a larger error, which is expected when you consider that less information about the overal systems imbalance is provided. However, the error is still acceptable. It should be noted that the increase in fixed and flexible load also contributes to the rise in error between the DAP and the actual load. The total error of both aggregators together is 255.88 kW.

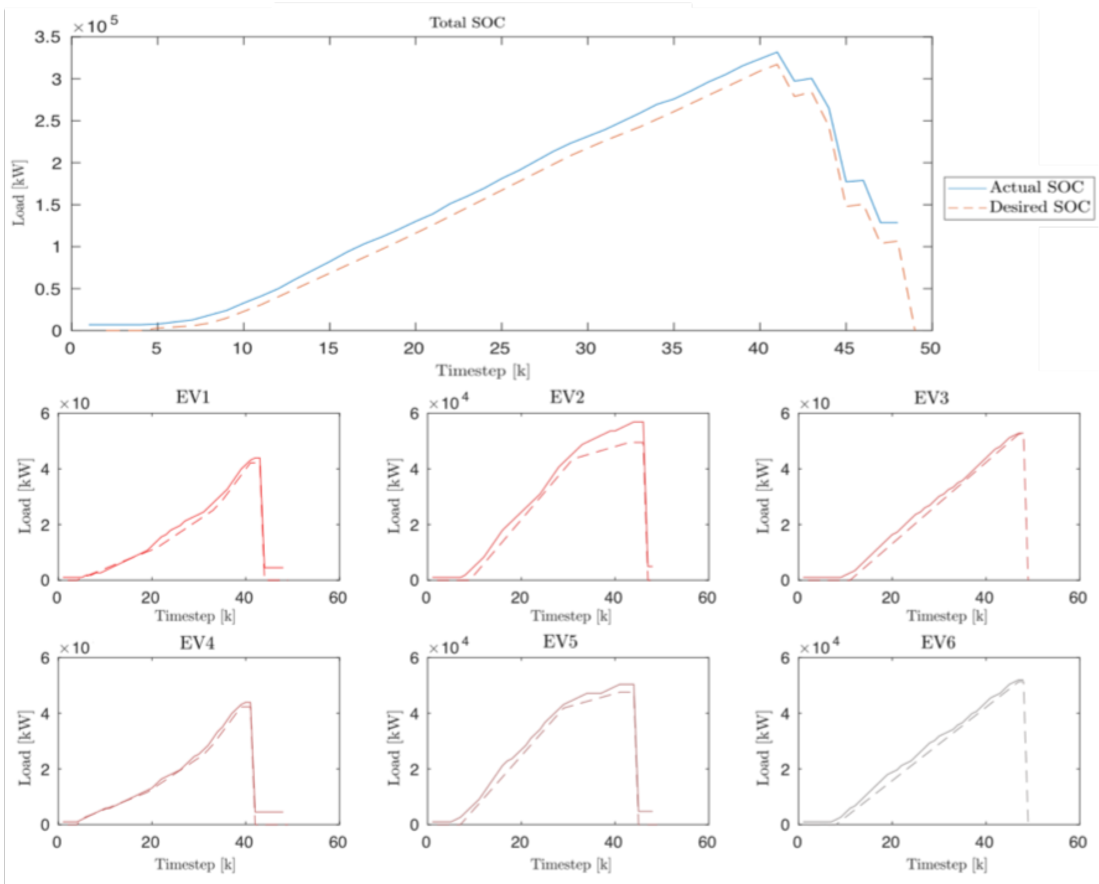


FIGURE 8.6: Scenario 3, the actual SOC of the EV's users compared to the desired SOC per time step of aggregator 1. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC.

Scenario 4: Only selfish EV users $E_{ii} = 1 \forall k$. In this scenario, the network consists of 28 households, each with one of the 4 devices. The prosumers are connected in a circular order under one aggregator. The DSO constraint is enabled and the DAP is based on the common settings. There are three discharge and charging levels ρ. Almost the same setting are applied as scenario 3.

The main difference is, that the EV users are completely selfish at all time steps. Their behaviour and current state is based solely on their own state throughout the whole simulation. As can be seen in figure 8.8, there are no violations of the total SOC but also no violations of the individual EV users. This causes less use of the flexibility, because there is less "freedom". The constraint is to **strict** as explained in section 6.3. This results in a higher total error, over all the time steps together: 80.58 kW. The total error is larger than the error in scenario 2. However for a fair comparison with exactly the same setting see scenario 5.

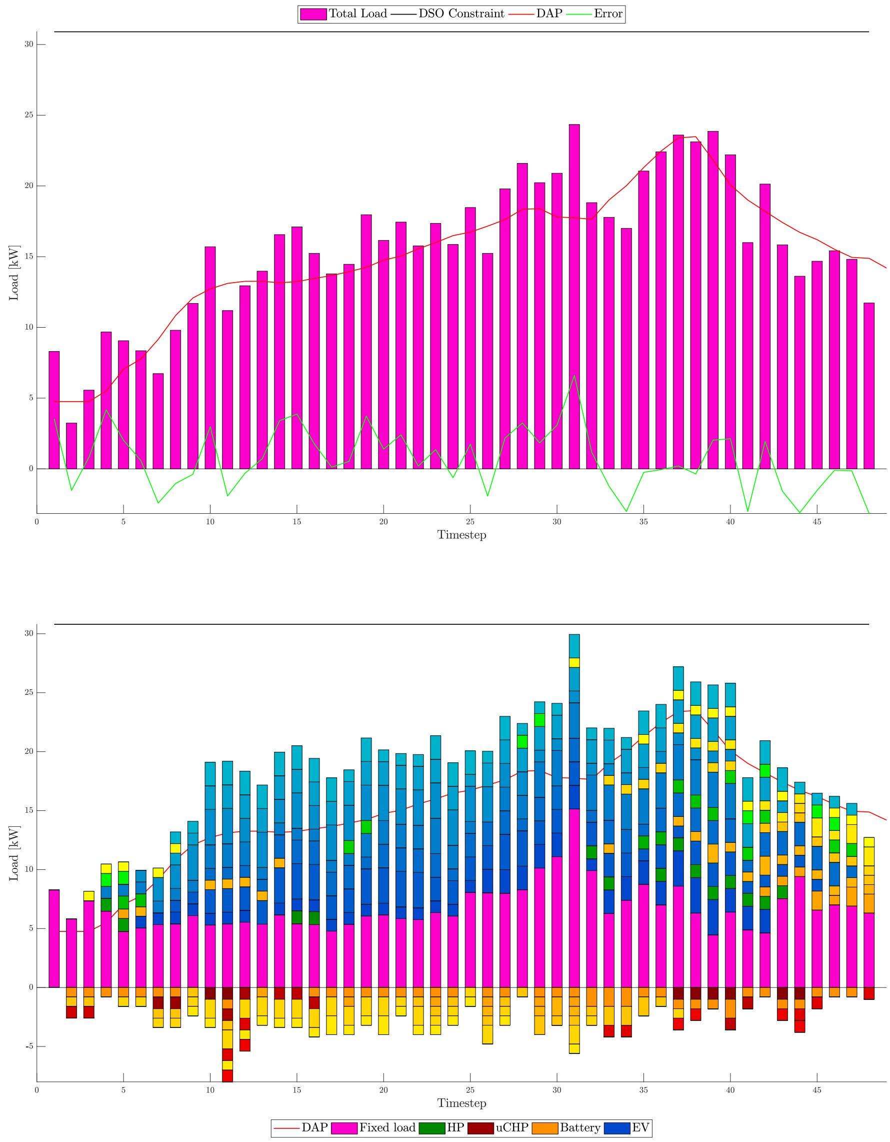


FIGURE 8.7: Scenario 4, 28 prosumers with congestion management and completely selfish EV users ($E_{ii} = 1 \forall k$). Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail.

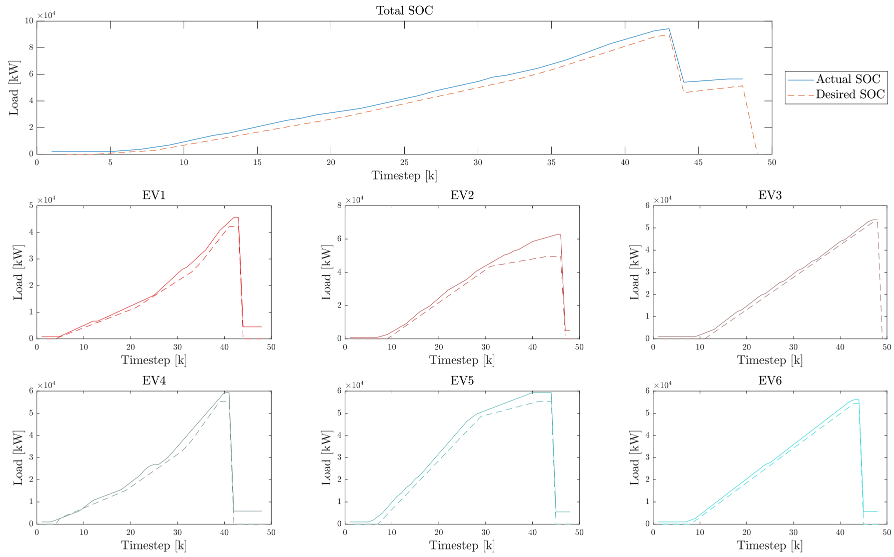


FIGURE 8.8: Scenario 4, the actual SOC of the EV's users compared to the desired SOC per time step. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC. **All the individual EV's have always more SOC than is desired**

Scenario 5: non linear information step function $l[y]$. In this scenario, the settings are exactly the same as scenario 4. However there is an important difference compared to scenario 2 scenario 4. The EV's information sharing matrices **E** are updated with a **non linear gradient** of $l_y(\cdot)$:

y	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
$l_y(\cdot)$	0.05	0.05	0.05	0.10	0.20	0.30	0.40	0.50	0.50	0.50

TABLE 8.3: Non linear gradient of information step function $l[y]$ for scenario 4.

This course of the function results in a less selfish EV user for most of the time, however it does allow the EV user to be completely selfish for the last 3 steps. This has an advantageous result on the error compared to scenario 4. The total error, over all the time steps together is 71.34 kW. This is a decrease of almost 11.5% compared to scenario 4. In figure 8.10, it is visible that the allowed violations of individual state of charge differences is used.

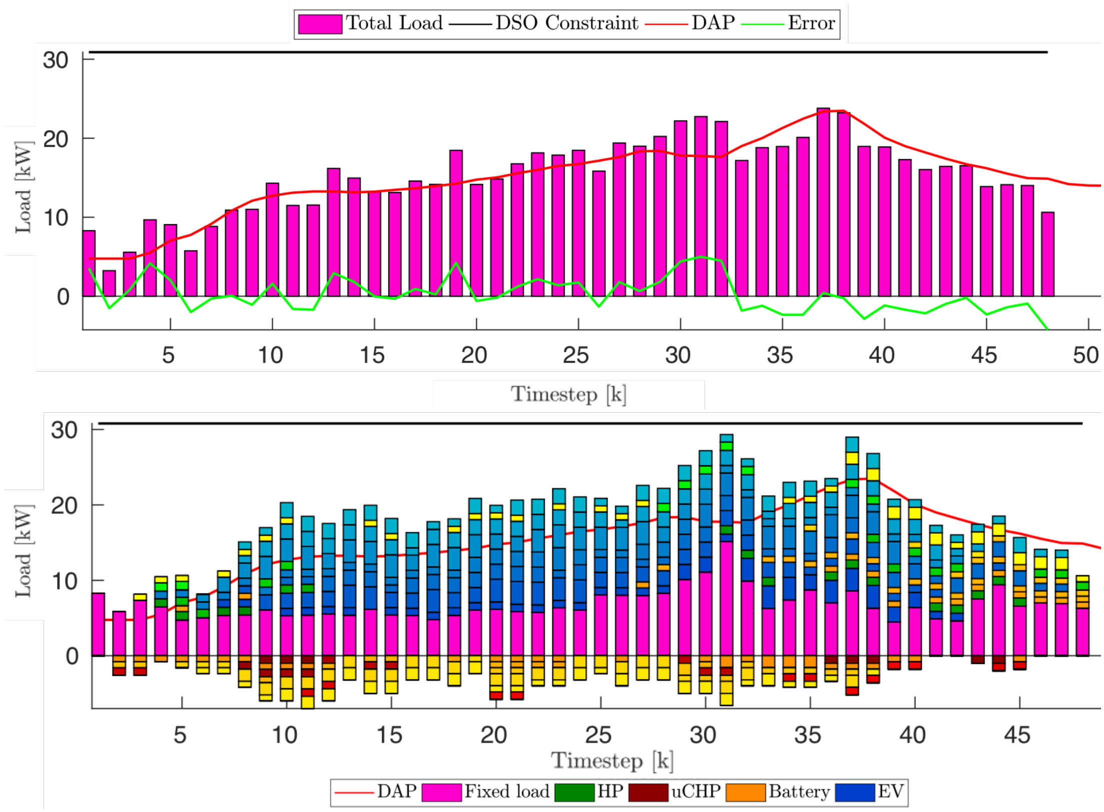


FIGURE 8.9: Scenario 5, 28 prosumers with congestion management and non linear step function $l[y]$. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail.

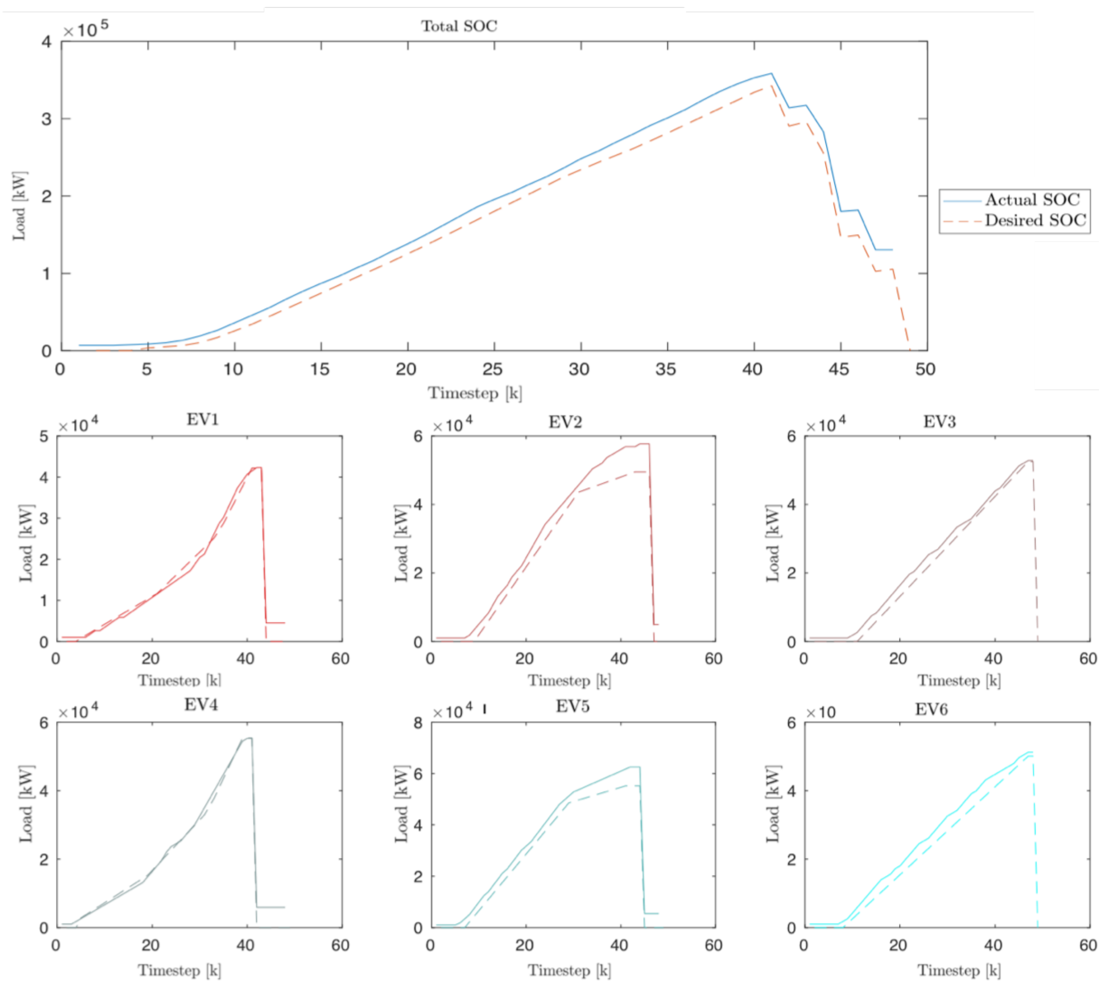


FIGURE 8.10: Scenario 5, the actual SOC of the EV's users compared to the desired SOC per time step. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC. **The individual EV's could have less SOC than is individually desired in the beginning of the charging time**

Scenario 6: extreme scenario with no home batteries and increased $S_i^{desired}$, to show the difference in hierarchical weights.

This scenario has as only purpose to show the effect in hierarchical weights as in Table 3.1. In both scenarios we have 12 prosumers with circular sequence: **...-μCHP-EV-HP-EV-μCHP-...**. Thus we have a total of 6 EV's, 3 heat pumps and 3 μCHP's. In scenario 6a the DSO constraint is turned "OFF". While in scenario 6b the DSO constraint is enabled. $l_y[k]$ increases linear like in scenario 2. The desired state of charge is enlarged to force violations of the DSO constraint in scenario 6a. In figure 8.11 the "shaving" day-ahead-planning can be seen, as explained in section 8.1.

Figure 8.11 shows 16 DSO violations for scenario 6a. This is expected because the most important objective in this scenario is the EV charging. There is no EV charging violations as can be seen in figure 8.12. There occur individual EV violations before the end of the charging time k^{end}, which is allowed and caused by matching the DAP, as explained before.

Figure 8.13 shows 0 DSO violations for scenario 6b. All the violations of scenario 6a are resolved by decreased charging behaviour of the EV users. This results in violations of the EV charging constraint as can be seen in figure 8.14. Each individual EV is not adequately charged at the end of the charging period. These results indicate that the weight balancing works as intended. The DSO constraint will not be violated even for violating the EV charging constraint.

The number of iterations before convergence is obtained is 8 times more for scenario 6b compared to scenario 6a per time step. Solving with the DSO constraint results in switching behaviour between high loads and too low loads. The high loads happen when the Lagrangian multiplier $\mu = 0$, and therefore there is no punishment for using flexible load f_i by the DSO constraint. These high loads results in a violation of the DSO constraint, therefore in the next time step, using positive flexible loads is punished, which results in low loads. Which enlarges the Lagrangian multiplier λ_i for the next iteration. It takes 8 times more iterations to level out this switching behaviour. This presumably indicates that the weights of Table 3.1 are not the optimal weights yet.

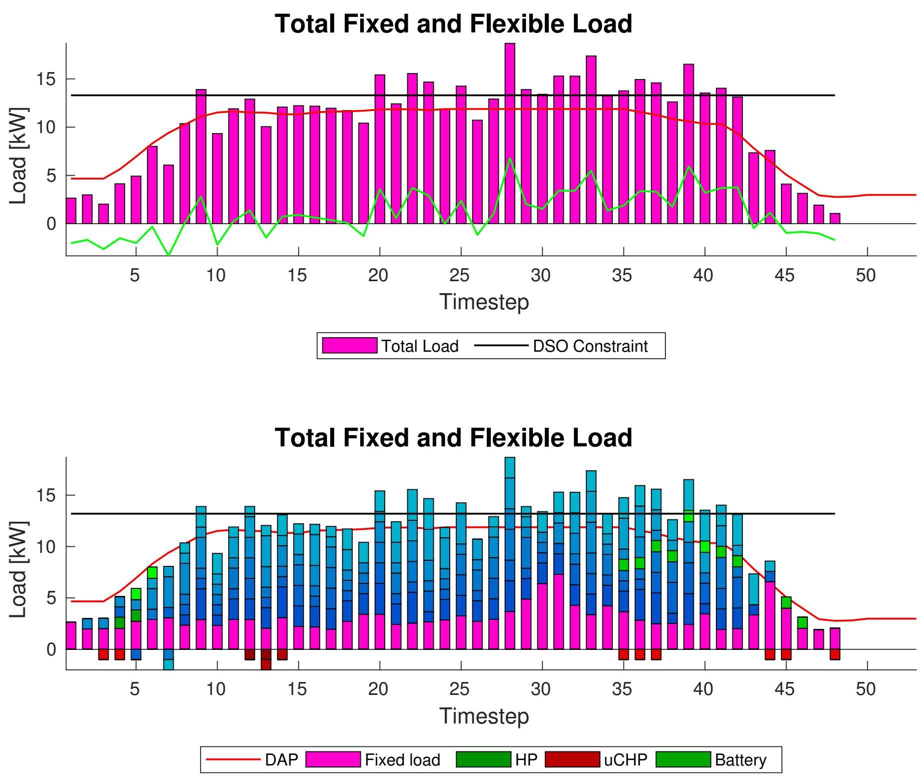


FIGURE 8.11: Scenario 6a, 28 prosumers **without** congestion management without home batteries and increased loads. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail. **There are DSO constraint violations**

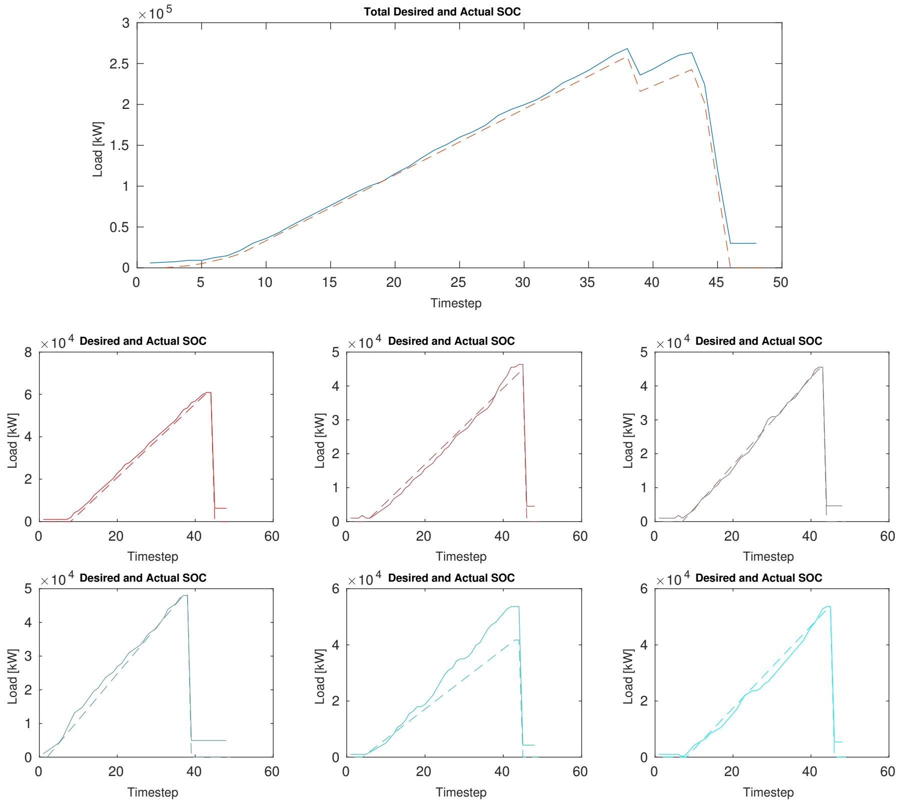
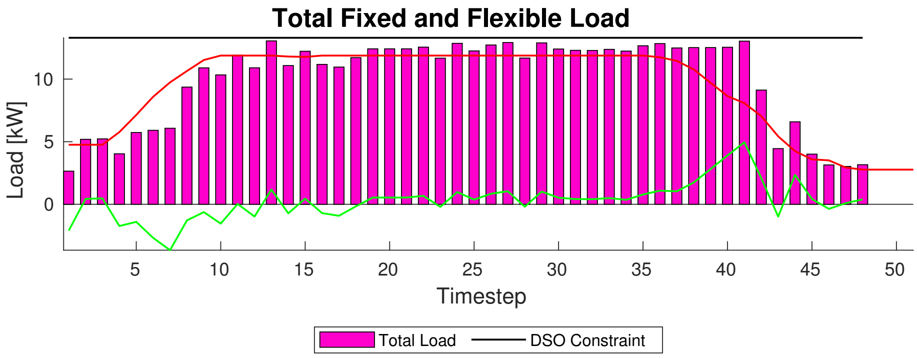


FIGURE 8.12: Scenario 6a, the actual SOC of the EV's users compared to the desired SOC per time step. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC. **There are no EV constraint violations**



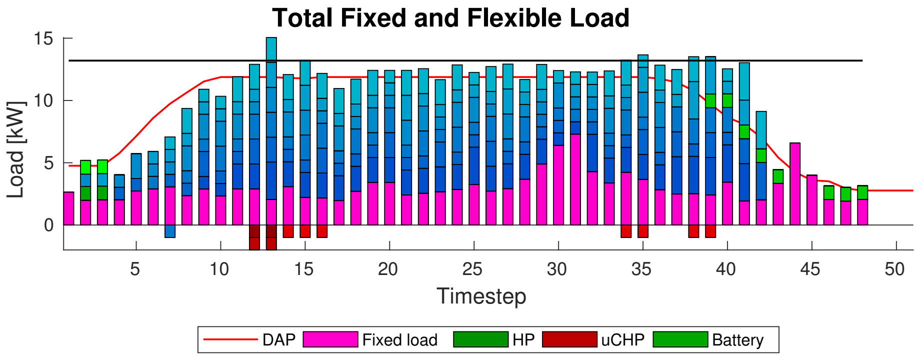


FIGURE 8.13: Scenario 6b, 28 prosumers **with** congestion management, without home batteries and increased loads. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail. **There are no DSO constraint violations**

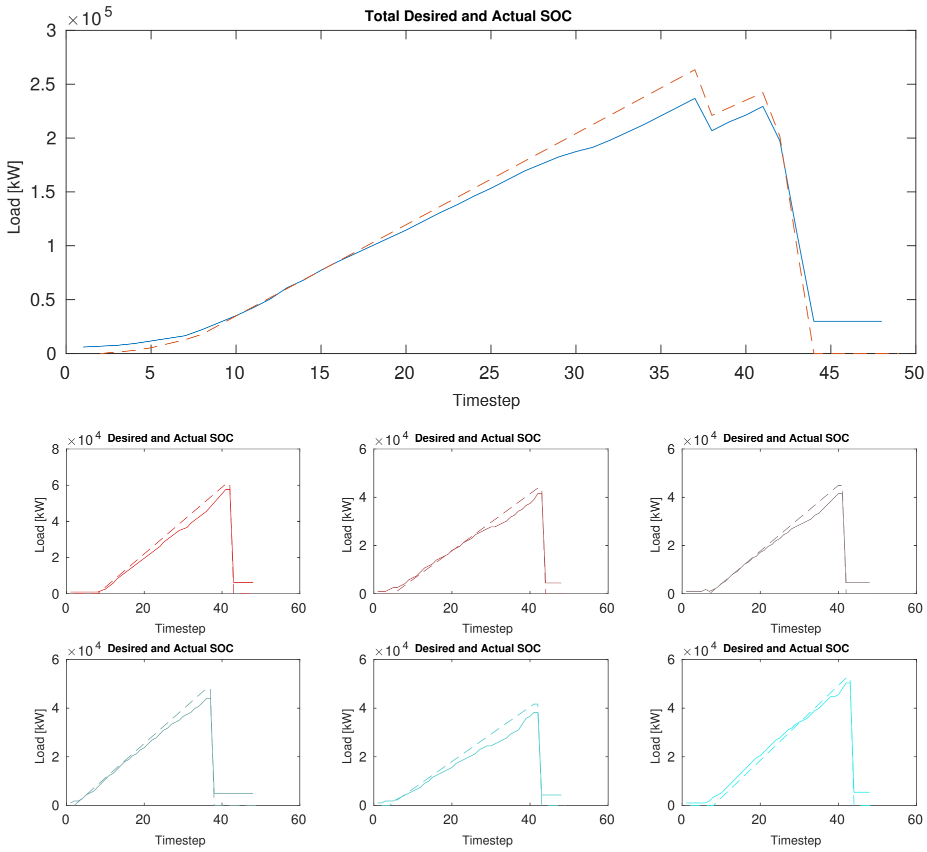


FIGURE 8.14: Scenario 6b, the actual SOC of the EV's users compared to the desired SOC per time step. Top: Summation of SOC of all EV users. bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC. **There are EV constraint violations**

Chapter 9

Conclusion & Further Work

This thesis has investigated distributed control in a multiple electricity prosumers network, where the aim is to reach a global goals in the network only based on distributed decisions and information. Global goals such as: not violating the DSO constraint, adequate charging electric vehicles and matching the day-ahead-planning. In this chapter, we discuss the results and main contributions of this thesis, and we finish with recommendations for further work

9.1 Conclusions

This project was undertaken to implement the home battery and the EV into the infrastructure for the smart grid that allows the agents in the grid to apply distributed control so that the network as a total reaches a global goal for electricity production and consumption. We also extended the distributed control algorithms of Nguyen et al. [11], by introducing among others a System of Updating Matrices. We evaluated this extended control algorithm based on dual decomposition and sub-gradient iterations using MPC, by the means of case studies with supply-side and demand-side appliances.

In Chapter 5 we have designed a model of a home battery based on a simplified battery proposed in the article of Ratman et al. [15]. We introduced charging levels into the DMPC algorithm. We also introduced the possibility to control and quantify (USEF) flexibility for devices that could consume and supply energy to the power grid. We designed a modeling objective function that penalizes inefficient switching of the batteries, for calculation efficiency and power efficiency. And the home battery could be controlled by a DMPC algorithm similar to the one proposed in the article of Nguyen et al. [11]. The battery model will be the basis of the physical model of the EV. The virtual information network is the same as for the preceding system introduced in Chapter 4.

This network enables distributed coordination of electric power in the network. In the model, the agents do not have full information about the bower imbalance in the complete network, but they communicate imbalance information only to their neighbors. The distributed control of home batteries will be used to match the actual load to the Day-Ahead-Planning (DAP).

In Chapter 6 we have designed a specific constraint for Electrical Vehicle (EV) charging. The designed constraint ensures that the EV charging is spread over the whole charging period; all the individual EV's are sufficiently charged at the end of charging period as long as it not violates the DSO constraint; sufficient production and use of UFLEX; EV charging is controlled by the distributed control Algorithm.
We have designed a new information sharing model for EV users, called the System of Updating Matrices. We have distinguished between a virtual information network where the agents exchange information, and the physical charging of the EV's. The EV users exchange information with its neighbors in the information network. The information they share is about the mismatch of actual state of charge levels and the desired state-of-charge levels. This is a useful approach because acotrs can compensate with state-of-charge surpluses the state-of-charge shortages of their neighbors. This enlarges the use of and production of UFLEX, which results in higher financial rewards for the individual prosumers and contribute to load balancing. The System of Updating Matrices introduced in Chapter 3, is a new theory that ensures that actors can "choose" how much they rely on the information of neighbors, at each time step. There are several parallel information states that are calculated with more or less of their neighbors. The requirements on the System of updating Matrices are given in section 3.4.
The System of Updating Matrices is introduced in the EV constraint to ensure that more flexibility is produced in the beginning of charging while ensuring that all individual EV's are adequately charged in the end of the charging period.
The EV charging constraint, is put into the objective matrix to ensure than in the overall problem, the DSO constraint will not be violated. A slack variable is introduced to only penalize a shortage in state-of-charge while not rewarding overcharging the EV battery. Rewarding overcharging an EV's battery would disrupts the DAP balancing objective of the overall system.

In Chapter 7 we successfully coupled the EV charging constraint to the overall model. An explanation about weight balancing in the objective matrix based on constraint hierarchy is provided. The chapter ends with the introduction of the algorithm that solves the combined optimal control problem in an MPC with the System of Updating Matrices.

In Chapter 8, 6 scenario-studies show the effects of the constraints, models, System of Updating Matrices, and the DMPC algorithm. Scenario 1 shows the significant positive effect of the implementation of home-batteries in combination with the charge/discharge levels on the matching with the day-ahead-planning. A significantly smaller normalized error is observed in comparison with the results of Nguyen et al. [11]. Scenario 2 up to and including scenario 5, show that the proposed algorithm is properly working. The error with the DAP are within acceptable margins and there never occur any DSO violations. The absence of the possibility of DSO violations indicates that the EV constraint works as intended. The introduction of the System of Updating Matrices is proven to have a positive effect on the load balancing of 11% for this particular simulation. Scenario 6 is an extreme scenario to show the effects of the hierarchical weights. When the DSO constraint is used, there are no DSO violations but only EV charging violations. This indicates that the weight balancing works as intended, however the significant increase in iterations when DSO violations threaten to happen indicate that the hierarchical objective weights could probably improved.

The findings in this report are subjected to at least three limitations. First, we did not perform experiments in the lab with real devices. Second a large scale test of EV charging is not conducted of around 1000 EV's due to problems with the (constraint) student license of Gurobi and MATLAB, the constraint is expected to work as intended but we cannot show this by simulation. Third we assume that all the EV charging times and desired levels are exactly known, there is no system in place that can deal with late or early decoupling of an EV for example.

9.2 Further work

Simulation studies to optimize the weight balancing to decrease the amount of iterations when DSO constraint is violated. The balance between the weight of the efficiency of the batteries and matching the DAP seem the have no problems but this can change for different system parameters. A research to the mathematical dependency of the objective weights could be useful.

To couple charging and discharging of the EV and home battery to electricity prices and the prices payed for flexibility by for example the BRP and DSO. for example: changing the EV charging patterns based on price fluctuations and researching how different functions $l_y(\cdot)$ provide the best monetary benefits. The EV constraint and the algorithm are designed to manage all kinds of charging patterns. A simulation study to the optimal pattern could be beneficial for the application in the real world. Also finding solutions for less predictable/reliable EV charging patterns will increase the robustness of the overall system in the real world.

Start working with feasible starting values for the Lagrangian multiplier λ, but also seek for possibilities to directly couple this starting values to the starting values for the predicted SOC levels of the neighbors $\hat{e}_i^{pred}$. This will speed up the calculations especially when DSO constraint violation is possible for time step k. In this controller during the iterations, is the DSO constraint often unnecessarily violated. This occurs because the predicted influence of the neighbors is initialized as a zero. This causes that all the prosumers start charging at maximum levels to try to obtain the objectives. This causes an violation of the constraint and a $\mu > 0$. The μ causes an extreme decrease in fixed charge f_i, which significant increases in the Lagrangian multiplier λ_i. Which causes extreme reactions of the prosumers. It takes allot of unnecessary time until this effect is leveled out. This effect has much similarities with the Bullwhip effect in economics or in control theory a controller proportional controller with bad setting that causes large overshoots and large settling time.

In this thesis two different types of information sharing networks are used. For the simulation, only a circular sequence is used. It could be interesting to optimize the settings of the matrices **A** and **B**. One could research for example the effect of adding more neighbors, non-circular sequences. It is expected that for a real world scenario, optimization of all the information sharing settings is necessary to obtain desired results for all the stakeholders.

It is interesting to research how the DMPC algorithm performs in a real environment/lab setting. For example in the PowerMatching City in the Netherlands.

Other aspects that are not touched upon in this thesis are the social, psychological, privacy, and regulation aspects of a future grid and EV charging behaviour. We leave this to the experts of the respective fields.

Appendix A

Examples of EV Sharing Matrices

if all prosumers have an EV:
For $k = k^{begin}$

$$\mathbf{E} = \left\{ \begin{array}{cccccc} 0.5 & 0.25 & 0 & \dots & 0 & 0.25 \\ 0.25 & 0.5 & 0.25 & \dots & 0 & 0 \\ 0 & 0.25 & 0.5 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.25 & 0 & 0 & \dots & 0.25 & 0.5 \end{array} \right\}$$

For $k = \frac{k^{begin} + k_i^{end}}{2}$

$$\mathbf{E} = \left\{ \begin{array}{cccccc} 0.75 & 0.125 & 0 & \dots & 0 & 0.125 \\ 0.125 & 0.75 & 0.125 & \dots & 0 & 0 \\ 0 & 0.125 & 0.75 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0.125 & 0 & 0 & \dots & 0.125 & 0.7 \end{array} \right\}$$

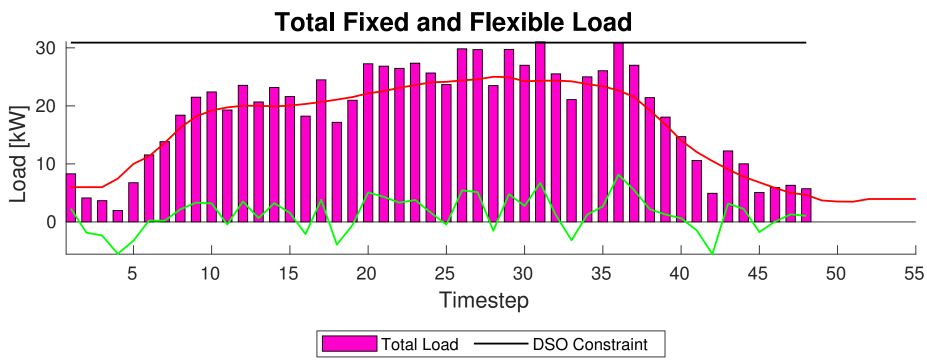
For $k = k^{end}$

$$\mathbf{E} = \left\{ \begin{array}{cccccc} 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & 1 \end{array} \right\}$$

Appendix B

Simulation with Actual Values

This simulation is done with actual values described in section 8.1. The DSO constraint is still the lowered value (1.1 kW per prosumer) to indicate the robustness of the system. Still no DSO and EV violations occur.



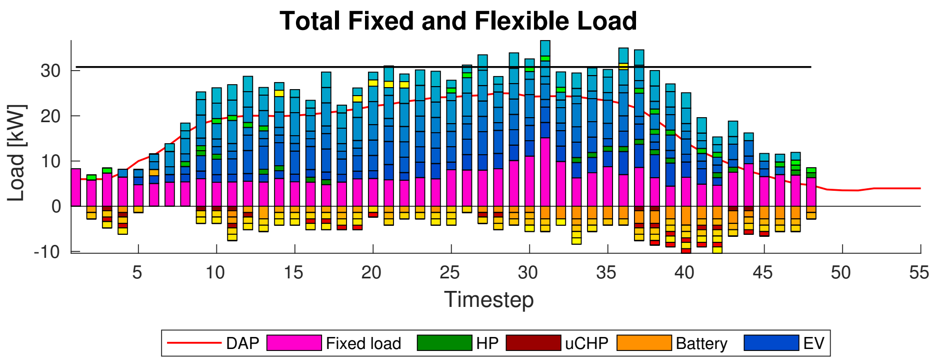


FIGURE B.1: 28 prosumers with congestion management, all 4 the ADS devices and 1 aggregator. Top: Total load within the prosumer network. Bottom: flexible and fixed loads in detail. The network capacity limit (DSO constraint) is not violated.

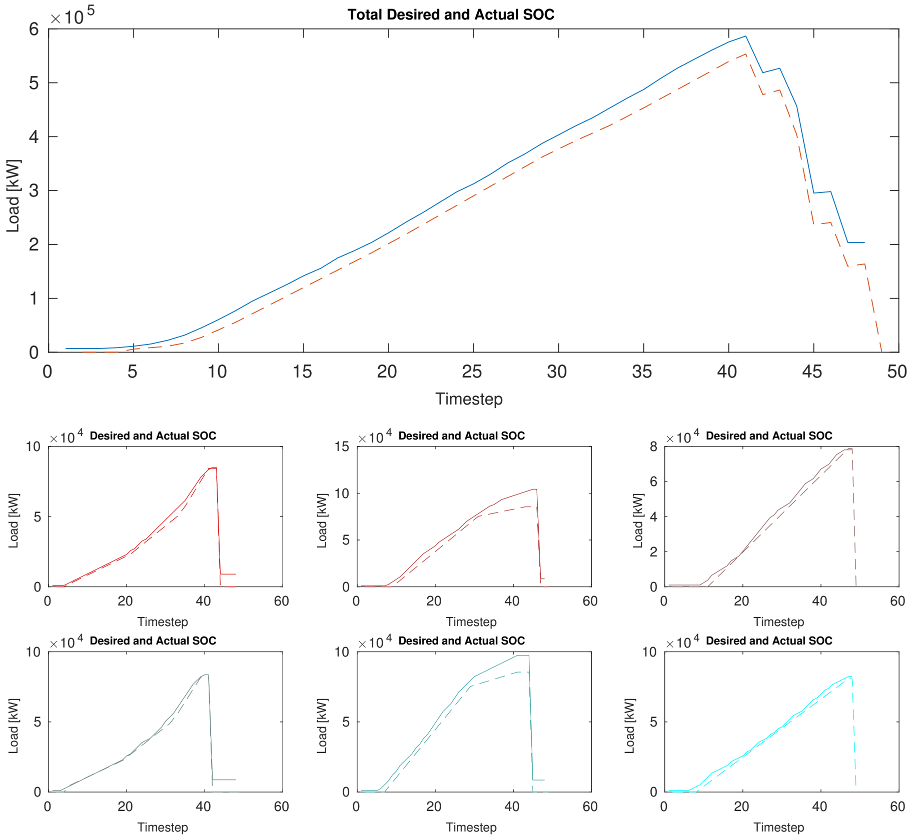


FIGURE B.2: The actual SOC of the EV's users compared to the desired SOC per time step of aggregator 1. Top: Summation of SOC of all EV users. Bottom: 6 individual SOC levels with differences in charging patterns, charging period and desired SOC.

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