EEG Correlates of Learning in the Alpha-Arithmetic Task

Bachelor's Project Thesis

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Abstract: Learning is characterized by three phases as defined by Fitts and Posner (1967): cognitive, associative and autonomous. Previous studies on arithmetic tasks have observed these phases as well as associated cognitive stages such as encoding, solving and responding within each phase. In the current study, we were interested in finding evidence for the learning phases and corresponding stages in the EEG correlates of the alpha-arithmetic task by Zbrodoff (1995). The experiment consisted of 3 blocks of 192 trials where each trial was an alpha-arithmetic equation such as A+2=C and required the participants to respond with 'yes' or 'no'. We hypothesized that the EEG correlates of the task would show a discrete confirmation of three learning phases and for each phase some defined cognitive stages. Stimulus and response locked event related potentials (ERP) showed us some differences in brain activity between the 3 blocks. Additionally, a combination of Hidden Semi-Markov Models and multivariate pattern analysis (HSMM-MVPA) fitted to each block-phase confirmed the presence of different cognitive stages for each phase. Although there is evidence for learning, there are limited indicators for using counting as an initial problem solving strategy, as opposed to previous findings.

Keywords: learning stages; arithmetic; EEG; HSMM

1 Introduction

Learning is perceived as a resulting outcome of practice. Consider the concept of learning in the general context of arithmetic. To facilitate the learning of a new unknown problem such as A+2=C, the learner will start the process by repetitive solving. The problem will eventually be learned with enough repetitions and subsequent retention. In addition, with enough encounters of A+2 and its gradual learning, the response to the problem is expected to get faster over time. Previous research described such a speed-up as a function of practice hence referring to the power law of practice (Newell and Rosenbloom, 1981). However, later research explained this speedup in individual learning due to change in problem-solving strategy from computation to retrieval (Delaney, Reder, Staszewski, and Ritter, 1998).

Prior to this, Fitts and Posner (1967) had already proposed three phases of skill acquisition involved in the learning process. These phases, namely cognitive, associative and autonomous, captured the different transitions of problem-solving strategy involved in learning. The three phases and the learning associated with them is consistent with the two aforementioned reasoning for speedup, that is, practice and change in problem solving strategies. The results of this study continue to be widely explored in different domains including arithmetic tasks.

Anderson and Tenison (2015) identified the aforementioned three stages in the pyramid problem task. In this particular study, skill acquisition for the pyramid problem task was modelled. A pyramid problem task consists of problems such as $b\$n$ where b is referred to the base and n the height. The problem is solved by adding down n numbers from the *base*. For instance, $4\$3 = 4+3+2 = 9$. The goal of the study was to determine the nature of the underlying learning phases and confirm if they were cognitively distinct. They fit Hidden Markov Models (HMM) of learning states one to five to the data. The three-learning-state Hidden Markov

Model (HMM) was the best fit for the data, and Anderson and Tenison (2015) achieved their goal of identifying three learning phases for the pyramid problem. They defined a cognitive phase as the initial phase that includes counting and computation to solve the problem. In this phase, the skill is represented as declarative knowledge and it makes demands on the attentional resources. The associative phase marked the transition from computation to remembering previously solved problems and hence was the phase where the learner relied on retrieval. Here, the previous declarative knowledge is converted to concrete production rules which conditionally perform the required actions. The third phase was described as the autonomous phase. This was the final phase of learning where, instead of employing counting or retrieval, the responses were automatic. Here, the production rules are said to become automatic as a result of practice. This phase is also identified for its little to no cognitive and memory involvement. With each transition between the phases, a speedup in the response to the problem was recorded. Furthermore, they observed differences in cognitive activity and motor activity with each transition. The earlier two phases had a higher cognitive activity while the later phase showed a higher motor activity. This description of phases by Anderson and Tenison (2015) forms the general expectation for learning in an arithmetic task.

On the same task of pyramid problems, Anderson and Fincham (2014) focused on the cognitive stages involved in individual trials. Building on this, Anderson, Fincham, and Tenison (2016b) investigated cognitive stages involved in the three learning phases of the pyramid problem task using fMRI. Anderson et al. (2016b) found evidence for three cognitive stages: encoding, solving and responding. All three cognitive stages were evident in the first learning phase. Once the transition to the second learning phase was made, the participants appeared to skip the solving stage. In the last learning phase, response was reflexive and the duration of the encoding stage diminished. The transition from learning phase 1 to 2 was apparent by the increasing irrelevance of the complexity imposed by the height of the pyramid problem. The second transition, however, did not have such indicators and the change was established from the differences in the cognitive and motor activity between phase 2 and 3. The fMRI data revealed a decreasing activation over phases for LIPFC region of the brain which controls the retrieval of knowledge (Jing, Liu, Lu, Qin, Yao, Zhong, and Zhou, 2011).

The current study was centered on making similar findings on the learning of the arithmetic task by Zbrodoff (1995). The aforementioned study by Anderson et al. (2016b) used fMRI to detect these phases and associated stages. Although it has a high spatial resolution, this technique has a low temporal resolution which means it can only observe stages stretching over multiple seconds. To overcome this, the current study used EEG and took advantage of its high (millisecond) temporal resolution to refine previous findings.

Zbrodoff (1995) verified the process of learning and improvement via practice in the alpha-arithmetic task. The task consists of equations such as A+2= C where the correct answer is determined by counting from the alphabet on the left-hand side. The study investigated the problem-size effect by presenting participants with alpha-arithmetic equations and instructing them to respond with yes or no by calculating the given equation. The experiment was spread over three blocks and integrated the problem-size effect by including three different addends, 2,3 and 4, in the equations. It was observed that the performance (namely, accuracy and response time) got worse with an increase in addend and this effect declined over the blocks as a result of practice and learning. The decreasing trend was associated with a decreasing problem-size effect. Moreover, the trend was proposed to be the result of transition from counting to retrieval of previously learned trials in order to solve the problems. This evidence of transition and the task's similarity with earlier studies pose a high possibility that distinct learning phases and cognitive stages are present in the alpha-arithmetic task.

In the current study, the setup of Zbrodoff (1995) was replicated. The behavioral and simultaneously the EEG data for the alpha-arithmetic task were recorded. The objective of the study was to find evidence for different cognitive phases of learning and concurrently the cognitive stages in each of the phases in the EEG correlates of the alpha-arithmetic task. We hypothesized that the EEG correlates would show a discrete confirmation of three distinct learning phases and some defined cognitive stages in these learning phases of the

alpha-arithmetic task. We also expected to observe changes in the presence and duration of the cognitive stages with each learning phase. The behavioral data was expected to be consistent with the results obtained by Zbrodoff (1995).

2 Methods

2.1 Participants

The experiment was conducted on 29 participants from the University of Groningen and Hanze University of Groningen. The participants performed the alpha-arithmetic task for 45 minutes (1.5 hours including the EEG setup) and received a monetary compensation of 12 euros. All were right-handed, had normal or corrected-to-normal vision and no neurological disorders. The native language of the participants consisted of Latin alphabets (e.g. English, Dutch, German etc.). One participant was excluded due to having a different written script in their native language. Three participants were excluded for incomplete EEG data, one for excessive ocular movements in EEG data and one for inconsistencies between behavioral and EEG data. The analysis was performed on the data of the remaining 23 participants (12 females, Mean age = 23.5). Our research received ethical approval from the research Ethics Committee (CETO) of the Faculty of Arts, University of Groningen.

2.2 Procedure

The current study replicated the setup of Experiment 4 by Zbrodoff (1995). The alpha-arithmetic task consisted of six correct problems with letters from A-F paired with addends 2-4. All addends had the same frequency of occurrence, which in this case added up to two occurrences of each addend. Similarly, there were six incorrect problems where the right-hand side of the equation was one letter ahead of the correct answer. For clarification consider the example A + 2 = C. This is the correct problem and the incorrect of the same is A + 2 = D. Such 12 problems were shown twice each resulting in 24 problems. Eight repetitions of a randomized set of these 24 problems formed 1 block. So, there were 192 trials in one block and hence a total of 576 trials over the three blocks. Overall, each problem was repeated 48 times, which should be sufficient to learn the problems(cf. the pyramid problem task with 36 repetitions; Anderson et al. (2016b)). The experiment paused for a break after every 96 trials i.e. halfway through and at the end of each block and was resumed by the experimenter once the participant was ready. In the break, the participants were shown their average response time (in milliseconds) and average accuracy (in percentage) over the previous 96 trials. To ensure that there were no problems that were always easier than the others, the combination of letter and addend was randomized for each participant.

The experiment started with a set of instructions. This included an example of an alpha-arithmetic equation. The participants were instructed to only use their right hand for responding and to not use their fingers to count on when solving the problems. The participants were required to respond to the presented equation with a corresponding key on the keyboard. Half of the participants had to respond with key 'b' for yes and 'n' for no and the other half had to use the opposite convention.

The experiment started with a short practice round with 6 problems which did not overlap with the problems in the actual experimental blocks. These consisted of letters K-M paired with addends 2-4. The practice round had 3 correct and 3 incorrect problems. Prior to the presentation of the problem, a fixation dot was displayed for a randomized duration (over trials) between 500-1500 milliseconds in order to ensure that the participants do not respond to the problems as a result of expectancy. Following the fixation dot, the problem was presented on the center of the screen with a response timeout of 10,000 milliseconds. In the practice round, the problems had a longer timeout duration of 30,000 milliseconds. A feedback- 'Correct!', 'Incorrect' or 'Late' was displayed after each problem depending on the response made. Figure 2.1 shows the time course of a single trial.

2.3 Behavioral analysis

The behavioral data of 23 participants was used for analysis. The data for the last trial was lost for 4 participants and hence the relevant information including the block, addend and accuracy were filled in using the corresponding EEG data. The RTs which deviated twice the standard deviation

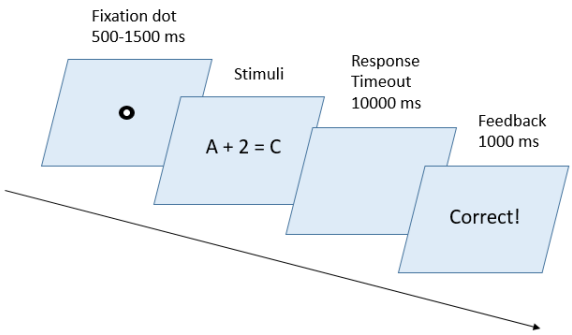

Figure 2.1: Time course of a single trial

from the mean of the condition (addend-block combinations) were considered outliers and were removed. Additionally, the trials with incorrect responses were removed as outliers for both, the response time and EEG, analyses.

2.4 EEG

EEG was recorded from 32 positions with a sampling rate of 512 Hz (Biosemi). The data was post hoc referenced to the average of mastoids. Four additional electrodes were attached to track the horizontal and vertical eye movements. The EEG signals were filtered by a high-pass filter 1.0 Hz and a low-pass filter 40 HZ followed. The data was downsampled to 256 Hz and preprocessed using EEGLAB (Delorme and Makeig, 2004) in MATLAB. The preprocessing consisted of manual artifact rejection after which an average of 96% of data remained. For three participants, noisy channels were removed since the noise covered more than 50% of the data. This was followed by sectioning data into independent components using Independent Component Analysis (ICA) to recognize and reconstruct ocular components. On average 1.5 components per participant were removed by ICA. The previously removed channels were then interpolated. The preprocessed EEG data was matched with the behavioral data to account for all correct trials, events and missing values. This data was then used for plotting Event Related Potentials (ERP) and resampled to 100 Hz for fitting Hidden Markov Models.

2.5 Event-related potentials

In order to analyse the EEG data and detect the relevant effects, it needs to be examined close to fixed point where it is the most informative. This purpose is served by ERPs which are averages of brain activity with respect to certain significant events. To prepare the data for ERPs, it was epoched with respect to the stimulus and the response, and baselined using a pre-stimulus baseline. The baseline was taken as the time frame between -400 to 0 milliseconds relative to the stimulus, the stimulus-locked between -200 to 1200 ms relative to the stimulus onset and the response-locked between -1200 to 200 ms relative to the response onset. The stimulus and response-locked activity was plotted for each of the three blocks with collapsed addends. The ERPs were plotted for all the 32 channels over the scalp for which the EEG was recorded.

2.6 Hidden semi Markov models

We were interested in finding the underlying stages and their duration in the learning of the alpha-arithmetic tasks. Previous studies have successfully used a combination of Hidden semi Markov models (HSMM) and Mutivariate Pattern Analysis (HSMM-MVPA) to uncover the underlying processing stages (Anderson, Borst, Walsh, and Zhang, 2016a). The HSMM-MVPA method can be used to analyse EEG signal of the trials and identify the cognitive stages within each of them. The 'Hidden' in HSMM represents the hidden nature of the cognitive stages and 'semi' indicates that the duration of the found stages can be variable.

First, the EEG activity is expressed in terms of *bumps* and *flats*. Bumps are peaks in the activity signifying the start of a new cognitive stage and flats are flat periods of activity denoting an ongoing stage. Bumps, which are otherwise dormant in the signal, are modeled as half-sine peaks resulting from summation with each other and ongoing sinusoidal noise. Flats are the periods where the mean of the ongoing sinusoidal noise is considered to be 0. A stage, thus, comprises a bump followed by a flat. The first cognitive stage starts with the onset of the trial and does not consist of a bump. Similarly, the last stage ends when the response is made and hence it does not necessarily mean that a new stage has started. As a result of these in-

terpretations, there is always one flat more than bumps. The HSMM then performs an estimation of the location and duration of bumps and flats to determine the underlying stages.

3 Results

The behavioral results were expected to show a speed-up in response as well as a problem-size effect as identified by Zbrodoff (1995).The response time was expected to decreases with each block and for each addend. Figure 3.1 shows the resulting plot of response time (RTs). The x-axis represents the three blocks and the y-axis indicates the average RT in milliseconds. The three different lines are the changes in RTs per addend. The error bars in the plot indicate the standard errors. From the plot, we observe a clear decrease in the RTs over the blocks which means that there was a speedup in response. In terms of addends, we were expecting a problem-size effect as found by Zbrodoff(1995). The RTs were expected to increase with an increase in addends as a result of increased difficulty in computation. Figure 3.1 shows a RT difference between addend 2 and 3, however, there was no such difference between addend 3 and 4.

Similarly, the average accuracy scores per addend per block can be seen in Figure 3.2. The accuracy does appear to get better over the blocks but the overlapping error bars suggest that the differences are not significant.

To measure the significance of the results, a two-way ANOVA was performed with dependent variable as RT and independent variables block and addend as factors. The effect was measured for each of the independent variables and for the interaction between the two. There was a significant differences in the RTs per block, $F(2,44)=84.08$, $p<0.001$. The addend too had significant effect on the RT, $F(2,44)=8.751$, $p<0.001$. There was an interaction between addend and block, $F(2,88)=13.99$, $p<0.001$.

Since ANOVA gives an comprehensive result, pairwise t-tests were performed to analyse the underlying effects. A significant difference was found between the RTs of block 0 and 1, $p<0.001$, but not between block 1 and 2, $p=0.096$. The effects of addends were calculated per block. There was a significant difference between addend 2 and 3 for block

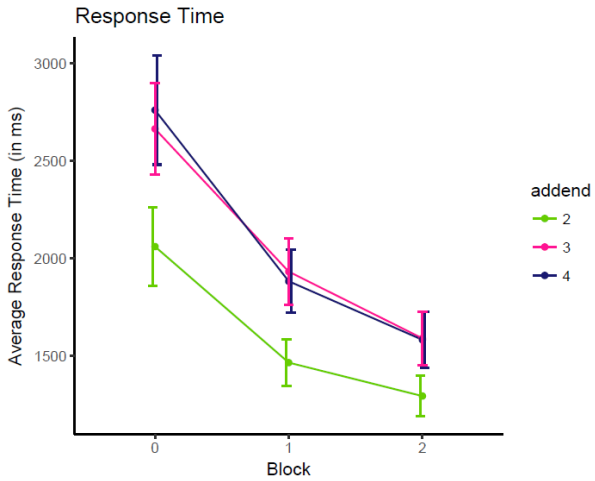

Figure 3.1: Plot showing the average response time (in ms) per addend over the blocks. The blocks 0, 1 and 2 correspond to the first, second and the third block respectively. The three different lines represent the three addends 2, 3 and 4. The error bars show the standard errors.

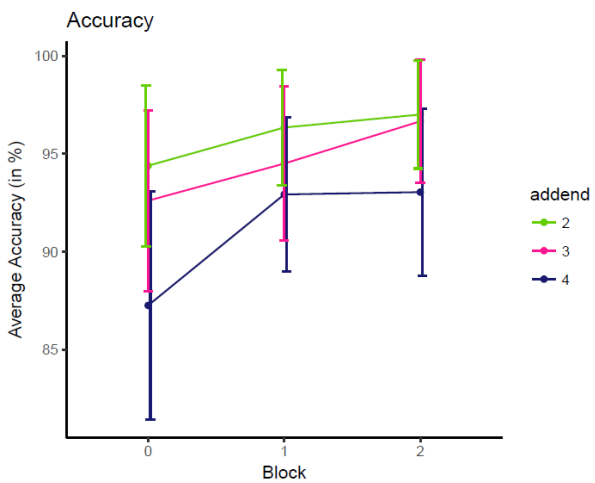

Figure 3.2: Plot showing the average accuracy scores (in %) per addend over the blocks. The blocks 0, 1 and 2 correspond to the first, second and the third block respectively. The three different lines represent the three addends 2, 3 and 4.The error bars show the standard errors.

0, $p<0.001$, and for block 1, $p<0.001$, but not for block 2, $p=0.015$. There was no significant differences between addend 3 and 4 for block 0, $p=0.318$, block 1, $p=0.828$, or block 2, $p=0.898$.

3.1 ERP

The ERPs were plotted for all the 32 channels over the scalp for which the EEG was recorded. For display purposes the plots of nine channels (F3,FZ,F4,C3,CZ,C4,P3,PZ and P4) are presented. These channels represent the overall activity across the scalp. Initially, ERPs were plotted per addend per block. However, no notable differences were found. As a consequence of this, the ERPs per block with collapsed addends were plotted instead.

The stimulus-locked ERPs can be seen in the Figure 3.3. The plots show the time with reference to the stimulus onset on the x-axis and on the y-axis is the brain activity in millivolts. The three lines in the plot represent the activity for the three blocks. In the stimulus-locked ERPs, the most salient differences are found in the parietal channels P3,PZ and P4 as shown (g), (h) and (i) of Figure 3.3. A difference in the activity over the blocks is observed at about 300 ms after the onset of the stimulus. The decrease in the activity over the blocks is a supporter for learning by signifying a reducing mental effort for problem solving.

Similarly, Figure 3.4 shows the response-locked ERPs. The x-axis represents the time in milliseconds with reference to the response and the y-axis shows the corresponding brain activity. Again, the three lines correspond to the three blocks. The defining differences are observed in the frontal channels- F3,FZ and F4 as seen in (a), (b) and (c) of Figure 3.4. The plots show a decreasing activity over the blocks at around 300 ms prior to the response. Here too, the decreasing activity indicates a reducing mental effort.

3.2 HSMM

HSMM were constructed for each block with collapsed addends and for amount of stages ranging from one to ten. A leave-one-out cross validation (LOOCV) was performed where the HSMM was fitted to the trial-by-trial data of 22 participants and a log likelihood was obtained on the remaining participant. The models of stages n and n + 1 were compared to check for best-fit. The model with n + 1 bumps was said to outperform the one with n bumps if the log likelihood increased for 17 out of 23 participants. This threshold of participants is as indicated by a sign test. From this, the highest number of stage where this increase is log likelihood was significant was chosen as the best-fit. The best-fit HSMM for the blocks were 6-bump , 5-bump and 5-bump respectively which maps to 7, 6 and 6 cognitive stages. Figure 3.5 shows the resulting cognitive stages, their duration and corresponding topology found from the best-fit HSMM for each block-phase (block and/or learning phase). The x-axis represents the time series of trials of each block. The results were examined per block with the purpose of intricately understanding each level. Block 0, or the first learning phase, obtained a best-fit of 7 cognitive stages and hence has one additional stage compared to the other blocks. The fifth cognitive stage in this block is notably long, covering about 3/4th of the entire trial. Block 1, which is the second learning phase, shows a long stage prior to the response. The same stage is noticed again in Block 2 and its duration decreases which is an indicator of learning. The results of the HSMM are also comparable with that of the ERPs. Similar to the ERPs, the HSMM show a difference in the three blocks about 300 ms after the onset of the stimulus.

4 Discussion

The current study replicated the setup of the alpha-arithmetic task by Zbrodoff (1995). The behavioral and the EEG data was recorded for the task. The aim of the study was to find evidence for the different phases of learning and the corresponding cognitive stages in each of the phases in the EEG correlates of the task. Following the results of previous studies on similar arithmetic tasks, we hypothesized that the EEG correlates would exhibit the three learning phases- cognitive, associative and autonomous and, along with that, well-defined cognitive stages within each of these phases. We expected to see a shift in problem solving strategy from counting to retrieval. Both the behavioral and the EEG data were analysed. The EEG data was applied to ERP analysis and Hidden semi Markov models.

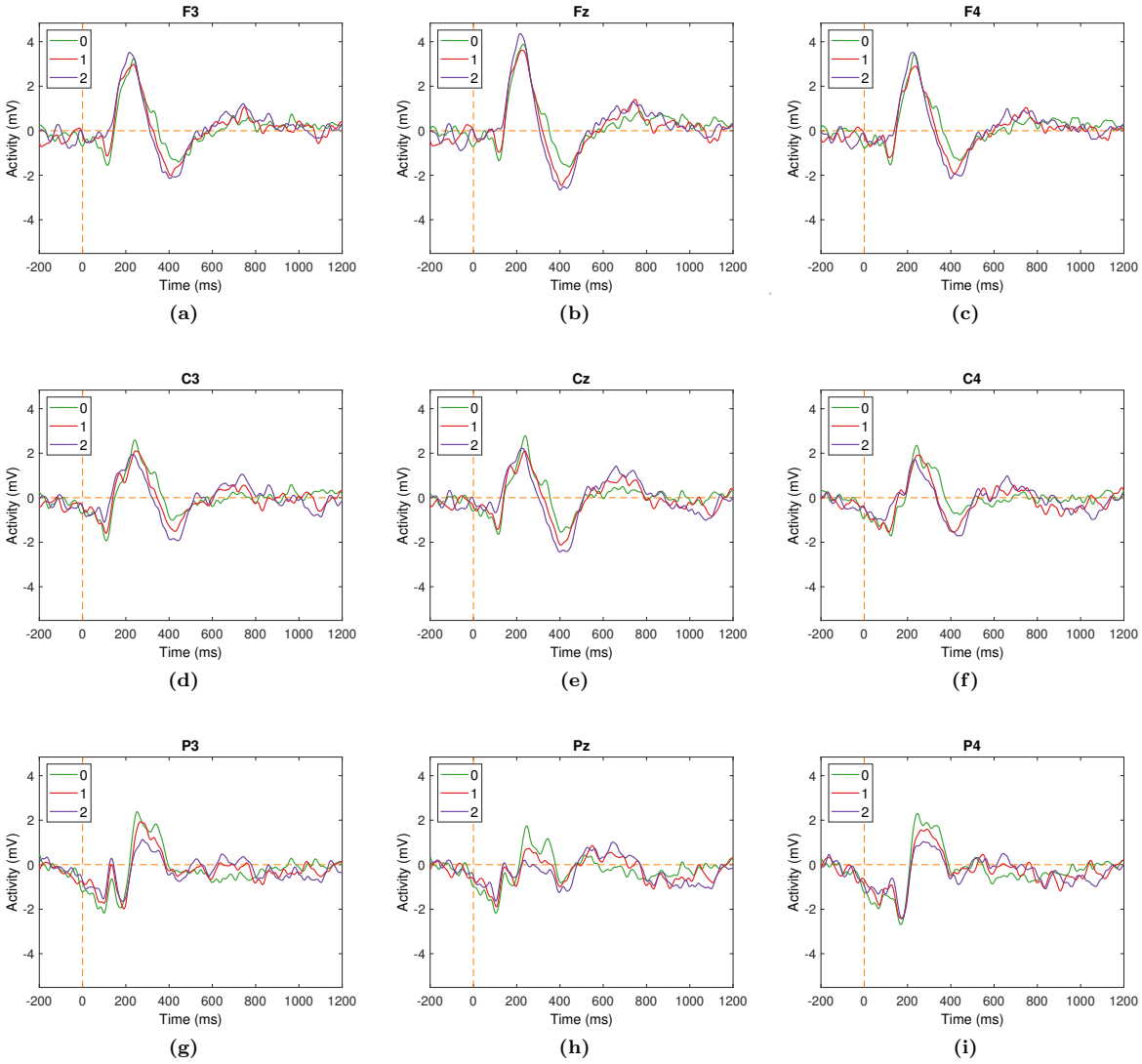

Figure 3.3: Stimlus-locked ERPs; Each plot shows the EEG activity in one of the channels for the three blocks- 0,1,2 corresponding respectively to blocks 1,2,3 of the alpha-arithmetic task

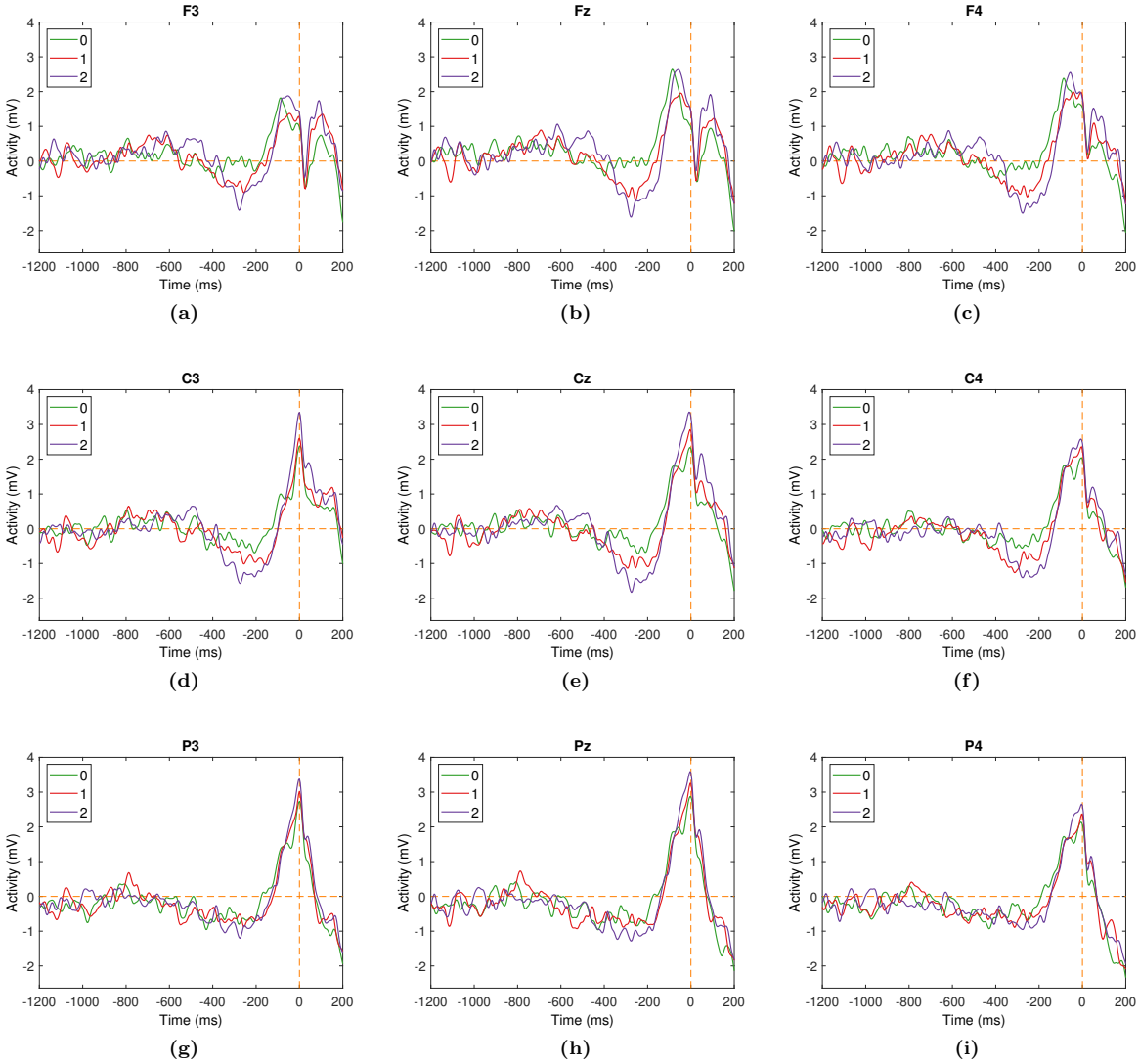

Figure 3.4: Response-locked ERPs; Each plot shows the EEG activity in one of the channels for the three blocks- 0,1,2 corresponding respectively to blocks 1,2,3 of the alpha-arithmetic task

The behavioral analysis of the response time per block and per addend (Figure 3.1) showed a decrease in RT over the blocks and this speedup in response over the blocks indicates the presence of learning. As for the addends, there was a significant difference between the RTs of addend 2 and addend 3 for two out of 3 blocks. However, the RTs of addend 3 and 4 did not illustrate any differences and were overlapping throughout. There can be multiple interpretations of this result. First, there is a clear difference between addend 2 and 3. Since there are no differences between problems with addend 2 and 3, other than the addend themselves, it is definite that the participants are using counting to solve the problem at least for addend 2. Now, since there is no difference between addend 3 and 4, it is possible that the participants are still using counting and that the problems with addend 3 and 4 are of equal difficulty. Or it could be that they are not using counting but rather a different

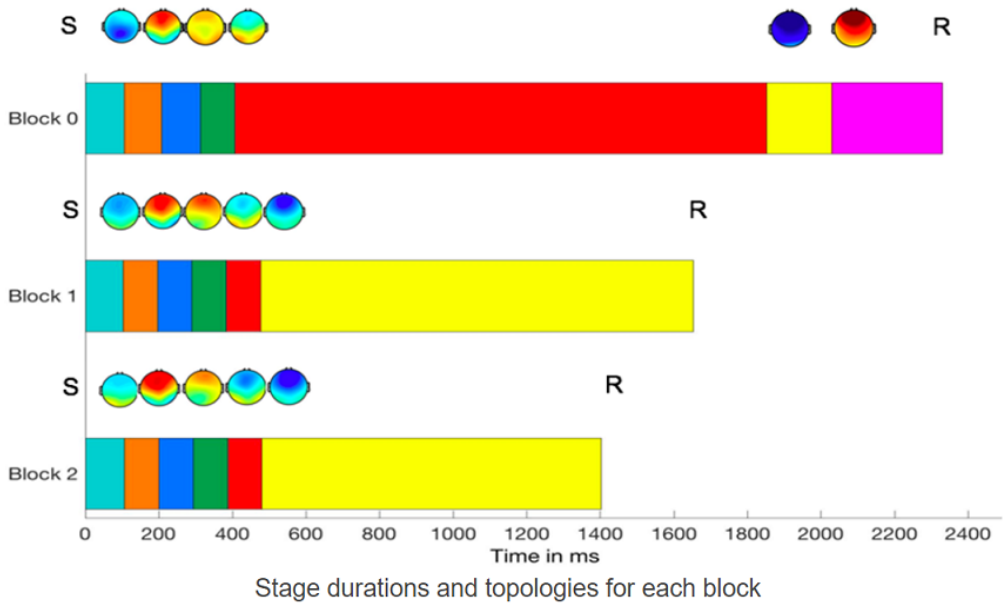

Figure 3.5: Plot showing the duration and topology of the different cognitive stages per phase-block as found by the Hidden semi Markov models. The S represents the stimulus onset and the R represents the response. Note that the colors of the stages are irrelevant and two stages of the same color do not necessarily represent the same mechanism.

strategy. The problem with the first interpretation is that the problems were randomized for each participant. This means that it is highly unlikely for the problems with addend 3 and 4 to always be of the same difficulty for all participants. So, the more probable explanation is that the participants use a different approach for solving the problems. One such approach could be solving by recognition. Since the problems which were presented to participants were equations such as A+3=D and the expected response was 'yes' or 'no', there is a high chance of them not even requiring to count to respond. The participants could just be getting better at recognition. One way to improve this setup to favor counting would be to replace the problems with incomplete equations such as 'A + 2 = ' and instruct the participants to provide the correct answer. By doing so, counting is enforced.

The close resemblance of problems with addend 3 and 4 opens up the discussion that they are more difficult than addend 2 but possibly still not difficult enough to require counting. This can be checked by introducing higher addends such as 5 and 6. A difference between addend 3 and 4 with the higher addends would reject recognition as a strategy and would mean that the problems with the these two addends in our experiment are just too similar. On the other hand, if there are no differences with the higher addend, some interesting findings about problem solving strategies would come forth.

Briefly, the behavioral analysis of RTs confirms the presence of learning and leads us to conclude that the participants are partly using an alternative to counting to solve the problems. As for the accuracy, there was no significant difference between the blocks or the addends. This could be a result of our participants being good at the task as some of them were already obtaining 90+ accuracy scores in the first block of trials. This leaves very little room for improvement and hence explains why the differences were not significant. We can also speculate that this is a result of the experiment design more than the participants themselves. During the breaks, the participants were shown their average accuracy and RT scores. This was expected to motivate them to perform better and hence learn better. However, it seems as though this comes at the cost

of employing strategies which are not expected. The participants could be more focused on improving their accuracy scores and RTs and hence adopting faster or more convenient strategies than counting. This also provides an explanation for why there is partial problem-size effect. Since addend 2 is easier to count, the participants are not compromising on speed or accuracy by using counting with the problems with addend 2. However, for the higher addends, 3 and 4, they appear to be substituting counting with a strategy which will not negatively affect their performance. To account for such actions in the future, the instructions and feedback can be adapted to avoid behavioral disparities. One other issue to consider is that the current study used equations with letters A-F. These letters are all at the beginning of the alphabet and are likely to be remembered better than the ones in the middle. If true, this would also explain the high accuracy scores and the overlap between addend 3 and 4. This can be controlled by replacing A-F with letters K-P, for instance.

The EEG data was first analysed using ERPs from which there are apparent differences in the EEG activity of the three blocks of trials. These differences can be seen in the stimulus-locked parietal channels and the response-locked frontal channels. The differences between the blocks confirms the existence of distinct learning phases involved in the process of learning. As previously mentioned, the ERPs were first plotted per addend per block but no variations were observed between the addends. In contrast to the behavioral results, the ERPs display no evidence for using counting to solve the problems. The overlapping EEG activity between the three addends suggests that the participants could solve problems with addend 4 just as easily as they could solve problems with addend 2.

To find the cognitive stages within each of the learning phases, HSMM were fitted to each-block phase. The results of the models (Figure 3.5) illustrated one additional stage in block 0 than the other blocks and additionally a distinct long phase in this initial block. Previous studies, including the aforementioned by Anderson, Fincham and Tenison (2016), associated such stages with computation. Since the current task also runs on the same principles as Anderson et al. (2016b) and due to the possibility of a problem-size effect, this stage is potentially one for counting. Assuming this to be true, the two stages following the counting could be related to storing the results of the counting and subsequently responding. The long (possibly) counting stage is much shorter in the other two block-phases indicating a reduced use of counting to solve the problem. Alternatively, the long duration of this stage indicates that it could be retrieval rather than counting.

Block 1 shows a long stage right before the response is made and the duration and the related topology of this stage suggests strongly that it is associated with retrieval. The duration of the same stage is reduced in the next block which means that retrieval time decreased and hence learning is affirmed. Overall, the HSMM provide supporting evidence for learning. There is a possibility of using counting as an initial problem solving strategy but there is not enough evidence here to confirm it.

To review, the current study demonstrated learning in the EEG correlates of the alpha-arithmetic task. In terms of the research question of the current study, we confirm our hypothesis of distinct learning phases and corresponding cognitive stages. However, there were varying results about the problem solving strategy which leaves with some unexpected but nevertheless interesting matters to focus on. Further research on the subject with the suggested modifications will help put together the big picture of our findings.

5 Acknowledgements

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