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Mirror symmetry in Calabi-Yau compactifications of type II supergravities

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Abstract

In this thesis we examine the manifestation of mirror symmetry in Calabi-Yau compactifications of type II supergravities, i.e. the low energy limits of type II string theories. We will conclude that mirror symmetry is observed in standard Calabi-Yau compactifications. We then turn on background fluxes and observe that mirror symmetry is observed for fluxes in the RR sector, but not for fluxes in the NS sector. We then look for a suitable, non Calabi-Yau, manifold which could serve as a mirror to electric NS flux compactifications and eventually end up with Half-flat manifolds.

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Chapter 1

Introduction

The holy grail in theoretical physics is to find a theory that unifies general relativity and the standard model. A very promising candidate is *string theory*. String theory no longer takes particles to be pointlike, but rather views them as tiny strings. Different vibrational modes of the string would then correspond to different particles as we know them. There are several different string theories, two of which we will be interested in: type IIA and IIB, which have much in common. One of the important things is that they exhibit *supersymmetry*, which basically means that there is a symmetry present which relates bosons to fermions, i.e. integer spins to half-integer spins.

However there is a problem with all string theories: in order for superstring theory to be Lorentz invariant, one must conclude that space-time is ten dimensional. However our universe seems to be mere four dimensional, thus in order for superstring theory to be a good candidate for a unified theory one must get rid of those six extra dimensions. Now this can be done by twisting and curling up the six extra dimensions into a compact space of small dimensions. If these dimensions are small enough we cannot probe them (at least with the current energy scales accesible to us via experiments). The extra dimensions are thus invisible to us and as a result we view the world as effectively four dimensional. This process is called *compactification*.

Now if we would like our effective theory to preserve a minimal amount of supersymmetry, we are naturally lead to taking our curled up compact space to be a *Calabi-Yau manifold*. These manifold come with many great properties which enables us to make very general statements concerning our effective theory. Calabi-Yau compactifications have very nice properties, but also suffer some deficiencies. One of which is the fact that a lot of massless scalar fields, called *moduli*, turn up in our effective theory, which we should have observed, but we haven't. One can solve this by introducing *background fluxes* to our internal manifold, which will then generate masses for the moduli.

A very interesting concept related to the compactification of IIA/B string theories on Calabi-Yau manifolds is that of *mirror symmetry*. This is a highly non-trivial symmetry which states that if one compactifies IIA on a generic Calabi-Yau manifold Y, one can alway find another manifold $\tilde{Y}$ such that if we compactify IIB on this manifold, we end up with the same effective four dimensional theory. This manifold is called the mirror of Y.

String theories are very complex theories and therefore we will focus solely on the low energy limit. This consists of the massless sector of the string theories and these are called type IIA and IIB supergravities. Just like the respective string theories they live in ten dimensions and are supersymmetric, which is one of the key ingredients. For we are going to demand that after compactifying

The statement of mirror symmetry is done at the level of the full string theory. However in this thesis we will be examining the manifestation of mirror symmetry in the supergravity limit. Our goal is to see if mirror symmetry also holds in this limit, i.e. if we compactify IIA supergravity on Y and IIB on $\tilde{Y}$, the corresponding effective theories are the same. Also we will investigate whether mirror symmetry is also observed when we turn on background fluxes. To tackle this we will make subdivide the fluxes in two types: RR and NS fluxes (each subdivided in electric and magnetic fluxes). We will see that the RR fluxes behave well, but the NS fluxes will prove to be problematic.

Chapter 2

Preliminaries

This chapter gives an introduction to all the concepts needed to perform the reduction of IIA/B on Calabi-Yau manifolds and to be able to assess the possibility of mirror symmetry. We will start off by giving a short review of the IIA/B supergravities and their fieldcontent and actions [3]. Thereafter we will make the idea of compactification more precise, and examine where and how Calabi-Yau manifolds come into play. To be able to discuss the implications of mirror symmetry we will then turn our attention to some properties of Calabi-Yau manifolds, namely their cohomology groups and moduli spaces. Lastly we will review the origin of mirror symmetry and its implications.

2.1 IIA/B supergravities

The field theories which we are eventually going to compactify are, as said before, the IIA and IIB supergravities. They are the low energy limits of IIA/B superstring theories and have a massless field content. Just as these string theories they are supersymmetric, which is a very important aspect of the theories. Let's make a couple of remarks concerning supersymmetry. Supersymmetries are symmetries relating bosons with fermions, i.e. integer spins with half-integer spins. These symmetries are generated by supersymmetry parameters, which are spinors. The amount of supersymmetry in a theory is characterized by the amount of supersymmetry parameters N; the higher N, the more supersymmetric the theory is. We will not go into detail but simply note that the IIA/B supergravities have $N = 2$ supersymmetry in 10 dimensions, i.e. two supersymmetry parameters. There is ofcourse a lot to say about these theories, and there is a rich amount of literature on the subject, but we will stick to a very short description of their field content and the corresponding actions.

2.1.1 Field content

Both theories have different fields present which fall into three sectors: the RR-, the NSNS- and the NSR-sector, which stem from their origin in string theory. The NS sector is the same for both theories and includes the metric g, the dilaton ϕ and a 2-form B_2. For IIA there are only odd forms with even field strengths are present in the RR sector, while the situation is reversed for IIB. Both theories contain multiple fermions in the NSR sector, however let us note that in this thesis, we will solely examine the bosonic part. This can be done since

Table 2.1: Field content of IIA/B supergravities

	IIA	IIB
NS	$\hat{g}_{\mu\nu}$, $\hat{\phi}$	$\hat{H}_3 = d\hat{B}_2$
R	$\hat{A}_1, \hat{C}_3$	$\hat{l}, \hat{C}_2, \hat{A}_4$
NSR	$\Psi^{1,2}$, $\lambda^{1,2}$	$\Psi^{1,2}$, $\lambda^{1,2}$

we are working with supersymmetric theories, so the corresponding fermionic sector can be determined via supersymmetry transformations. An overview of the field content can be found in Table 2.1.

2.1.2 Actions

The dynamics of the fields are ofcourse governed by actions:

IIA

The action consists of three parts: the NSNS-, the RR- and a topological sector.

$$\begin{aligned} S &= S_{NSNS} + S_{RR} + S_{Top} && (2.1)\\ &= \int e^{-2\hat{\phi}}(-\frac{1}{2}\hat{R} * 1 + 2d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4}\hat{H}_3 \wedge *\hat{H}_3)\\ &\quad - \frac{1}{2}\int (\hat{F}_2 \wedge *\hat{F}_2 + \hat{F}_4 \wedge *\hat{F}_4)\\ &\quad - \frac{1}{2}\int \hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \frac{1}{3}(\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1 && (2.2)\end{aligned}$$

Here $\hat{F}_2 = d\hat{A}_1$, $\hat{H}_3 = d\hat{B}_2$ and $\hat{F}_4 = d\hat{C}_3 - d\hat{A}_1 \wedge \hat{B}_2$ (the expressions follow from gauge invariance, but we will not discuss that).

Let us already get ahead of the facts and also introduce the so called *massive* version of IIA. This version will be important when we are going to add RR fluxes to our theories. The transition from the massless to massive is given by making the following modifications to the field strengths:

$\hat{F}_2 = d\hat{A}_1 + m\hat{B}_2$ and $\hat{F}_4 = d\hat{C}_3 - d\hat{A}_1 \wedge \hat{B}_2$

And adding a term $-m^2 * 1$ to the action. Three new terms also need to be added to the topological part of the action:

$$\delta S = -\frac{1}{2}\int -\frac{m}{3}(\hat{B}_2)^3 \wedge d\hat{C}_3 + \frac{m}{4}(\hat{B}_2)^4 \wedge d\hat{A}_1 + \frac{m^2}{20}(\hat{B}_2)^5 \qquad (2.3)$$

IIB

As noted, the IIB theory is very similar to IIA. However, one aspect of IIB needs some extra attention. Namely: the five-form fieldstrength is self dual, i.e. $\hat{F}_5 = *\hat{F}_5$. This basically means that we cannot write down a covariant action which encorporates the self-duality condition. So we have to impose the self-duality condition by hand at the level of the equations of motion; if we would try to do so at the level of the action, our kinetic term for F_5 would vanish. The action we will use is the following:

$$\begin{aligned} S &= S_{NSNS} + S_{RR} + S_{Top} \qquad (2.4) \\ &= \int e^{-2\hat{\phi}}(-\frac{1}{2}\hat{R} * 1 + 2d\hat{\phi} \wedge *d\hat{\phi} - \frac{1}{4}\hat{H}_3 \wedge *\hat{H}_3) \\ &\quad - \frac{1}{2}\int (\hat{l} \wedge *\hat{l} + \hat{F}_3 \wedge *\hat{F}_3 + \frac{1}{2}\hat{F}_5 \wedge *\hat{F}_5) \\ &\quad - \frac{1}{2}\int \hat{A}_4 \wedge d\hat{B}_2 \wedge d\hat{C}_2 \qquad (2.5) \end{aligned}$$

With: $\hat{F}_3 = d\hat{C}_2 - \hat{l}\hat{H}_3$ and $\hat{F}_5 = d\hat{A}_4 - d\hat{B}_2 \wedge \hat{C}_2$.

The problem of the self-duality also makes the compactification of IIB a bit more involved than that of IIA, but we will get back to that later.

2.2 Compactification

Having gone through the properties of the IIA/B supergravities and having examined their field content it is time to make the idea of compactification a bit more precise ([1]). The starting point is that we assume ten dimensional (Lorentzian) space-time $\mathcal{M}_{10}$ to be a product of a four dimensional (Lorentzian) space-time $\mathcal{M}_4$ with infinite dimensions, and a compact six dimensional (Riemannian) space $\mathcal{Y}_6$ with small dimensions, i.e:

$$\mathcal{M}_{10} = \mathcal{M}_4 \times \mathcal{Y}_6 \qquad (2.6)$$

Making this assumption has several implications, the first of which has to do with the *structure group* of our space. The structure group of a manifold is, loosely spoken, the group which captures how the different patches of the tangent bundle are glued together. For a generic non-orientable Riemanian n-dimensional manifold this is $O(n)$, if the manifold is orientable it reduces to $SO(n)$. In our case $\mathcal{M}_{10}$ is Lorentzian and orientable which means that our structure group is $SO(1,9)$. Under the ansatz we made above, the structure group will decompose accordingly:

$$SO(1,9) \to SO(1,3) \times SO(6) \qquad (2.7)$$

Where $SO(1,3)$ is the structure group of our four dimensional space-time and $SO(6)$ that of our internal manifold. Now in general, fields present in a theory live in representations of the structure group of the background space. So in the uncompactified theory our ten dimensional fields (denoted by hats) live in representations of $SO(1,9)$. However, in the process of compactification the structure group decomposes and hence the fields will decompose in representations of the structure groups of our four dimensional space-time and our internal manifold. How, will depend on the fields at hand ofcourse. For example, we get the following decompositions for a 1-form and a 2-form:

$$\hat{A}_1(x,y) = A_1(x)a_0(y) + A_0(x)a_1(y) \qquad (2.8)$$

$$\hat{B}_2(x,y) = B_2(x)b_0(y) + B_1(x)b_1(y) + B_0(x)b_2(y) \qquad (2.9)$$

Here x denotes dependence on four dimensional space-time coordinates, y denotes internal dependence, A_n is an n-form in 4 dimensions and a_n an n-form on our internal manifold. Thus we see that a ten dimensional field will lead to several different fields in four dimensions.

A very general property of such compactifications is that the four dimensional fields acquire masses which are inversely proportional to the dimensions of the internal manifold. Since we take our internal manifold to be very small, the acquired masses are very high. However, we are working in the low energy limit and are thus not interested in these massive modes, but rather focus on the massless fields. The way to procceed then, is to try and find a suitable basis for the internal components which capture the relevant low-energy physics we are interested in. In general this is not an easy task, but we will see that in the Calabi-Yau case this can be done very neatly. Finally we would perform this decomposition and explicitly perform an integration over the internal manifold to end up with an effective four-dimensional action which describes the low-energy physics. But before going into the details, let's first review why Calabi-Yau manifolds are so often taken as internal manifolds.

2.2.1 Road to Calabi-Yau manifolds

It is believed that the Standard Model is a low-energy limit of some four-dimensional supersymmetric field theory. At high energies physics should thus be supersymmetric and eventually were supersymmetry gets broken and the Standard Model emerges. As we have not observed supersymmetry, supersymmetry should be broken at some scale above the current energy scales available to us via experiments, say E_b. The supergravities we are about the compactify are, as we have noted before, $N = 2$ supersymmetric in 10 dimensions. In general, supersymmetry does not survive the procedure of compactification. However the compactification scale, E_c, is thought to be much higher than E_b. Thus to reconcile this and end up with an effective four dimensional theory which is supersymmetric between E_c and E_b, we will thus demand that some minimal degree of supersymmetry should survive the compactification process. In order to make contact with the Standard Model, eventually we would like to break this supersymmetry but we will not discuss this in this thesis. We will see that this preservation of supersymmetry under compactifying our theory is very restrictive regarding our internal manifold ([7]). To see this, let us first note that in order to be able to talk about supersymmetry in four dimensions, we would like to be able to globally define supersymmetry parameters in four dimensions. To examine this, let's decompose a ten dimensional supersymmetry parameter, which is a spinor $\hat{\epsilon}$, in terms of four dimensional space-time and internal components as induced by our compactifications ansatz:

$$\hat{\epsilon} = \sum_I \xi_I \otimes \eta^I \tag{2.10}$$

Here ξ_I are space-time spinors and η^I are internal spinors. In order for such a decomposition to exist, we do need to be able to globally define these nowhere vanishing internal spinors η^I. Now this is very restrictive: not all manifolds admit globally defined spinors. To see this, for example, note that one cannot even globally define a nowhere vanishing vector field on the 2-sphere.

Remembering that the IIA/B supergravities are $N = 2$ supersymmetric, we note that there are two supersymmetry parameters, $\hat{\epsilon}^{1,2}$, present in these theories. Thus by the above analysis see that every globally defined internal spinor η_I, leads to two superparameters,

$\xi_I^{1,2}$, in four dimensions. Since our goal is to preserve the *minimal* degree of supersymmetry in four dimensions, we require that we can define precisely *one* internal spinor η. This would then lead to two supersymmetry parameters, i.e. $N = 2$ supersymmetry, in four dimensions. This has severe consequences regarding the structure group ($SO(6)$) of our manifold, and implies the following reduction of it:

$$SO(6) \rightarrow SU(3) \tag{2.11}$$

It can be shown that under this reduction, the relevant spinorial representation of $SO(6)$ present in our theory will decompose in, among other reps, a singlet under $SU(3)$. This singlet is precisely a globally defined invariant spinor, see for example [1].

Supersymmetric vacuum

Thus we have now found the minimal requirement to be able to talk about supersymmetry in four dimensions, namely the reduction of the structure group to $SU(3)$. However, we are now going to demand some properties regarding the vacuum around which we are going to expand our fields. The first is that our four dimensional space-time should be maximally Poincaré symmetric. This basically means that all fields that transform non-trivially under the Lorentz group, which are the fermions, must have vanishing vacuum expectation values. Also the bosonic fields have to transform as singlets which means that their expectation values should be constant. On our internal manifold nonconstant vevs for the bosonic fields are allowed as they do not reduce the symmetry of our four dimensional space-time.
Next we we demand our background to be supersymmetric. This basically means that the variations of the fermions with respect to the supersymmetry parameters should vanish. This wil prove to be very restrictive. To see this let us consider the ten dimensional supersymmetry variations of the gravitinos present in our theories:

$$\delta\psi_M = \nabla_M \epsilon + (\hat{F}_p)_M \epsilon \tag{2.12}$$

(M denotes indices on $\mathcal{M}_{10}$, μ indices on $\mathcal{M}_4$ and m indices on $\mathcal{Y}_6$.) Now we have a couple of options. The simplest of which is to say that all our bosonic fields vanish (except for the metric ofcourse). This means that also our field strengths $\hat{F}_p$ vanish, both in space-time and on the internal space. This corresponds to the fluxless case. Allowing the bosonic fields to acquire non-zero internal vevs corresponds to introducing fluxes, but we will get back to that later. Let's first consider the fluxless case such that all our bosonic vevs vanish. The supersymmetry condition then reads:

$$\nabla_M \epsilon = 0 \tag{2.13}$$

One can split this equation in space-time and internal parts by taking the spinor ϵ to be the usual direct product:

$$\hat{\epsilon} = \xi \otimes \eta \tag{2.14}$$

Inserting this into the variations formula once can conclude that the four dimensional space-time should be Minkowski (see ref [1]). Regarding the internal components we are led to:

$$\nabla_m \eta = 0 \tag{2.15}$$

This is a very restrictive statement: our spinor η is covariantly constant. This implies that the holonomy group of our manifold should be reduced to $SU(3)$. Remember that the holonomy group is the group of possible transformations acquired by parallel transporting. Thus considering everything we conclude that our (compact) internal manifold should have $SU(3)$ structure and $SU(3)$ holonomy. This precisely means that our internal manifold is a *Calabi-Yau* 3-fold.

2.3 Calabi-Yau manifolds

In the previous section we found that if we want our vaccuum to preserve $N = 2$ supersymmetry we end up with the constraint that that a covariantly constant spinor should exist on our manifold. This implied that our manifold has $SU(3)$-structure and it's holonomy group should also be $SU(3)$. This is one of many equivalent definitions of a Calabi-Yau manifold. Calabi-Yau manifolds come with many nice properties, some of which we are gonna adress here. For a less brief overview see for example [2]. First of all, Calabi-Yau manifolds are complex manifolds, which basically means that locally they look like C^n, just as real manifolds locally look like R^n. On complex manifold we can define complex k-forms, which decompose as:

$$\omega^k = \sum_{k=p+q} \alpha^{p,0} \wedge \beta^{0,q} \tag{2.16}$$

Where a (r, s)-form is a form which has r holomorphic indices and s anti-holomorphic indices. Now for some more special properties of Calabi-Yau manifolds. The fact that we can globally define a spinor η on our manifold enables us to also globally define a non-vanishing $(3, 0)$-form Ω and a $(1, 1)$-form J. If in addition our spinor is covariantly constant we conclude that the forms are closed, i.e. $d\Omega = dJ = 0$. With these forms we can define a complex structure on our manifold, making it a complex manifold. Also the existence of the closed $(1, 1)$-form J, implies that the manifold is a Kähler manifold, with Kähler form J. The Kähler form and the metric are related via: $g_{\alpha\bar{\beta}} = -iJ_{\alpha\bar{\beta}}$. Also it is known that Calabi-Yau manifolds are Ricci-flat, which means that Ricci scalar R vanishes.
Another very important property is the one-to-one correspondence between harmonic forms and the cohomology classes. This is very interesting because we will see that harmonic forms will play an important role in Calabi-Yau compactifications and a lot is known about the cohomology groups of a Calabi-Yau manifold. Let us remember that the (de Rham)-cohomology groups, H^k_d, are defined as:

$$H^k_d = \frac{\{\omega | d\omega = 0\}}{\{\alpha | \alpha = d\beta\}} \tag{2.17}$$

Where ω and α are (k)-forms and β is a $(k-1)$-form. For Calabi-Yau manifolds the groups split as:

$$H^k_d = \bigoplus_{k=p+q} H^{p,q}_{\bar{\partial}} \tag{2.18}$$

Here the groups $H^{p,q}_{\bar{\delta}}$ are called the Dolbeaut-cohomology groups, which are the groups of the equivalence classes defined by:

$$H^{p,q}_{\bar{\partial}} = \frac{\{\omega | \bar{\partial}\omega = 0\}}{\{\alpha | \alpha = \bar{\partial}\beta\}} \tag{2.19}$$

Here ω and α are (p,q)-forms and β is a $(p,q-1)$-form and $\bar{\partial}$ is the differential operator that maps (p,q)-forms to $(p,q+1)$-forms such that $\bar{\partial}^2 = 0$. The dimensions of the de Rham cohomology groups are called the betti number b^k, and by the above relation we get:

$$b^k = \sum_{k=p+q} h^{p,q} \tag{2.20}$$

Where $h^{p,q}$ are the dimensions of the corresponding Dolbeaut-cohomology groups and are called the hodge-numbers.

For Calabi-Yau manifolds we can relate many of the Dolbeaut-cohomology groups to eachother via various isomorphisms. The first is via complex conjugation which induces an isomorphism between $H^{p,q}$ and $H^{q,p}$, and hence we get that $h^{p,q} = h^{q,p}$. Then there is an isomorphism induced by the hodge operator $*$, which maps a (p,q)-form to a $(n-q,n-p)$-form in a bijective manner, where n is the complex dimension of the manifold. Thus we see that $h^{p,q} = h^{n-q,n-p}$. The cohomology groups of a Calabi Yau manifold come with another relation, namely $H^{0,q} \simeq H^{n,q}$. This isomorphism is induced by the uniqueness of the $(3,0)$ form Ω. Now if we assume our Calabi Yau to be connected, which we will, one can show that there is a unique $(0,0)$-form. The last thing we have to note is that there are no 1-forms on a Ricci flat manifold, which our Calabi-Yau manifold is. If we take all these various relations together we see that for a Calabi-Yau three-fold there are only two independent hodge numbers, namely $h^{1,1}$ and $h^{1,2}$ and we end up with the following so called Hodge diamond which is simply a convenient way to order the different hodge numbers.

$$\begin{array}{ccccccc} & & & h^{3,3} & & & \\ & & h^{3,2} & & h^{2,3} & & \\ & h^{3,1} & & h^{2,2} & & h^{1,3} & \\ h^{3,0} & & h^{2,1} & & h^{1,2} & & h^{0,3} \\ & h^{2,0} & & h^{1,1} & & h^{0,2} & \\ & & h^{1,0} & & h^{0,1} & & \\ & & & h^{0,0} & & & \end{array} = \begin{array}{ccccccc} & & & 1 & & & \\ & & 0 & & 0 & & \\ & 0 & & h^{1,1} & & 0 & \\ 1 & & h^{1,2} & & h^{1,2} & & 1 \\ & 0 & & h^{1,1} & & 0 & \\ & & 0 & & 0 & & \\ & & & 1 & & & \end{array} \tag{2.21}$$

We thus see that many of the cohomology groups vanish which greatly simplifies our compactification. We will know introduce a basis for the different harmonic forms which we will use throughout this thesis:

	Basis	
H^0	1	1
H^2	ω_i	$i = 1,..,h^{1,1}$
H^3	α_A, β^A	$A = 0,..,h^{1,2}$
H^4	$\tilde{\omega}^i$	$i = 1,..,h^{1,1}$
H^6	$*1$	1

This basis can be taken to be real and satisfy all kinds of relations which we will sum up:

$$\int_{CY} 1 \wedge *1 = \mathcal{K} \qquad \int_{CY} \omega_i \wedge \tilde{\omega}^j = \delta_i^j$$
$$\int_{CY} \omega_i \wedge *\omega_j = 4\mathcal{K} g_{ij} \qquad \int_{CY} \omega_i \wedge \omega_j \wedge \omega_k = \mathcal{K}_{ijk}$$
$$\int_{CY} \alpha_A \wedge \alpha_B = \int_{CY} \beta^A \wedge \beta^B = 0 \qquad \int_{CY} \tilde{\omega}^i \wedge *\tilde{\omega}^j = \frac{1}{4\mathcal{K}} g^{ij}$$
$$\int_{CY} \alpha_A \wedge *\alpha_B = C_{AB} \qquad \int_{CY} \alpha_A \wedge \beta^B = \delta_A^B$$
$$\int_{CY} \beta^A \wedge *\beta^B = B^{AB} \qquad \int_{CY} \alpha_A \wedge *\beta^B = -A_A^B$$

Where $\mathcal{K}$ is the volume of the Calabi-Yau manifold. We will not go into the details but simply note (and refer to [5]) that due to the special properties of Calabi-Yau manifolds, such a basis exists.

2.4 Truncation and moduli space

We already noted that in general the fields in four dimensions will acquire masses proportional to the dimensions of the internal manifold. We will now see that we can find an appropriate subset of forms in which we are going to expand, such that only the low energ physics we are interested in remains and the massive excitations are discarded. Our starting point is a ten dimensional massless field, as present in the supergravities. (See [9].) The equation of motion for such a field, $\hat{\phi}(x, y)$ is:

$$\Delta\hat{\phi}(x, y) = 0 \tag{2.22}$$

Now assuming the product form of our space-time, the laplacian in ten dimensions splits into a four dimensional and a six dimensional part. Since our manifold is compact, we know that there are finitely many eigenvalues of the internal laplacian on the Calabi-Yau and that they are non-negative. Also the eigenforms are mutually orthogonal and span the whole space of forms. Thus any form can be written as a sum of eigenforms of the laplacian:

$$\hat{\phi}(x, y) = \sum_i \phi_i(x)\lambda^i(y) \tag{2.23}$$

Where $\lambda^i(y)$ are the eigenforms on our Calabi-Yau with eigenvalues m_i^2, and $\phi_i(x)$ are the corresonding four dimensional fields. Now taking into account the split of the laplacian, we can rewrite the equation of motion for our ten dimensional field:

$$\Delta_{10}\hat{\phi}(x, y) = (\Delta_4 + \Delta_6) \sum_i \phi_i(x)\lambda^i(y) \tag{2.24}$$
$$= \sum_i (\Delta_4 + m_i^2)\phi_i(x)\lambda^i(y) = 0 \tag{2.25}$$

I.e:

$$\sum_i (\Delta_4 + m_i^2)\phi_i(x) = 0 \tag{2.26}$$

Thus we see that the four dimensional fields acquire masses given by the eigenvalues of the internal laplacian. It is known that these eigenvalues are inversely proportional to the scale of our internal manifold. Since we take it to be very small, the massess will be very large, certainly compared to the low energy limits we are interested in. So the only relevant contributions are those given fields are in fact the zero modes of the internal laplacian, i.e. the harmonic forms. What we will then do is only expand our forms in harmonic forms and effectively discard all the massive fields. This process is called truncation, or truncating the spectrum. In order for the set of harmonic forms to give rise to a consistent truncation they have to be closed under the various operations we encounter in the action. These are taking the hodge dual and taking the differential. Thus our condition basically tells us that the hodge dual of a harmonic form must again be a harmonic form; the same is required regarding the derivative of a harmonic form. It can be easily checked that this is indeed the case.

2.4.1 Moduli space

We must first say something about what is called the *moduli space* of a Calabi-Yau manifold. Remember that our metric is a dynamical field and is thus allowed to vary. However to be consistent with our compactification ansatz that we compactify on a Calabi-Yau manifold, not all deformations are allowed. We only consider variations that preserve the 'Calabi-Yau'-ness of our metric, i.e. we allow deformations δg such that $g + \delta g$ is still a Calabi-Yau manifold. The implications of this can be examined by writing out the Ricci-flatness condition for our new metric, i.e:

$$\mathcal{R}(g + \delta g) = 0 \tag{2.27}$$

One can write this condition out, and by the special properties of Calabi-Yau manifolds the allowed variations split in mixed and pure types, i.e ([2]):

$$\delta g_{\alpha\bar{\beta}} = -iv^i(\omega_i)_{\alpha\bar{\beta}} \tag{2.28}$$

$$\delta g_{\alpha\beta} = \frac{1}{|\Omega|^2}\bar{z}^a(\eta_a)_{\alpha\bar{\beta}\bar{\gamma}}\Omega_{\beta}^{\bar{\beta}\bar{\gamma}} \tag{2.29}$$

Here ω_i are the harmonic 2-forms and η_a are harmonic complex 3-forms related to α_a and β^a. The v^i are real scalars which can depend on four dimensional space-time and are called the Kähler moduli, which can be seen by remembering that $g_{\alpha\bar{\beta}} = iJ_{\alpha\bar{\beta}}$. The z^a are complex scalars in four dimensional space-time and are called the complex structure moduli and represent deformations of the complex structure. The allowed variations can thus be expanded in harmonic forms and give rise to scalar fields in four dimensions, which nicely fits with our truncation of the other forms. Another way to look at it is by noting that our Calabi-Yau metric g is completely determined by Ω and J, so deformations of g will correspond to deformations of Ω and J. Since these forms are closed we can expand them in terms of our harmonic basis, i.e:

$$\Omega = z^A \alpha_A - \mathcal{F}_A \beta^A \tag{2.30}$$

$$J = v^i \omega_i \tag{2.31}$$

One can choose the basis of harmonic forms, such that $z^A = (1, z^a)$ coincide with the complex structure moduli as above and v^i with the Kähler moduli. (F_A is some prepotential which does not interest us at the moment, see appendix A.) Now we can indeed see that in general z^A and v^i can vary such that Ω and J remain closed, since:

$$d\Omega = z^A d\alpha_A - \mathcal{F}_A d\beta^A = 0 \tag{2.32}$$

$$dJ = v^i d\omega_i = 0 \tag{2.33}$$

(Note that this is the case because we have taking the d-operator on our internal manifold which only acts on the harmonic forms).

This truncation to harmonic forms is thus consistent as the metric and the other fields are treated on the same footing. This is a very special and also very nice property of Calabi-Yau compactifications. If we would compactify on a generic $SU(3)$ manifold the forms Ω and J are no longer closed and we cannot expand them in harmonic forms, for if we would do so we would lose most of the structure of our manifold, and in effect treat it as if it were Calabi-Yau. Thus, in the generic case we must find some other subset of forms on which we are going to expand such that it is all consistent. This means that we are also going to expand Ω and J in the same set of forms. Since these defining forms are not closed, we can already conclude that at least some of the expansions forms are also not closed. That in contrast to the harmonic forms we take in Calabi-Yau compactifications. We will get back to this in later chapters.

The complex structure and Kähler moduli actually come with a lot of structure. One can view the moduli as coordinates on what is called the *moduli space* of a Calabi-Yau manifold, which we denote $\mathcal{M}$. This space is very interesting in itself and is actually a very special manifold in it's own right. First of all, due to the split of the moduli in moduli of the complex structure and of the Kähler structure we see that the moduli space is a direct product of the complex structure moduli space $\mathcal{M}_C$ and that of the Kähler moduli $\mathcal{M}_K$:

$$\mathcal{M} = \mathcal{M}_K \times \mathcal{M}_C \tag{2.34}$$

On these moduli spaces we can define all kinds of objects which all very much depend on the cohomology groups of the given Calabi-Yau manifold. For example, the metric on $\mathcal{M}_K$ is completely determined by the harmonic 2-forms ω_i. For a more thorough treatment, we refer to appendix A.

Getting a bit ahead of ourselves we first note that if we want to be able to discuss mirror symmetry, we have to make the following observation: both IIA and IIB contain an antisymmetric tensor B_2 which, as we will see in the next section, gives rise to $h^{1,1}$ scalar fields b^i. What we will see is that these can be neatly combined with the Kähler moduli to form a basis for the so called complexified Kähler cone $\mathcal{M}_{CK}$, with coördinates $t^i = b^i + iv^i$. This provides an extension of the geometrical moduli space as discussed above, and the moduli space we will consider is thus:

$$\mathcal{M} = \mathcal{M}_{CK} \times \mathcal{M}_C \tag{2.35}$$

As we will now see, this is the relevant moduli space regarding mirror symmetry.

2.5 Mirror Symmetry

Now that we have covered all the aspects involved in Calabi-Yau compactifications we are finally able to treat mirror symmetry. Let us first say a little about the origin of the mirror conjecture following [2].

String theory has different interpretations, one of which is the point of view in which one takes string theory to be a conformal field theory on what is called the worldsheet (the worldsheet is the surface a string when moving through space-time). This conformal field theory contains moduli, which are deformations of some operators, and which are essentially non-geometric. Now here comes the crux: in this conformal field theory one encounters a $U(1)$ charge, whose sign is conventional. However depending on the sign we can interprete the field theory as either IIA compactified on a Calabi-Yau manifold Y, or as IIB compactified on some other manifold $\tilde{Y}$. In this interpretation, one sees that the non-geometrical moduli can actually be viewed as moduli of Calabi-Yau manifolds. Some of those moduli would then correspond to complex structure moduli and some to Kähler moduli. But whether they can be viewed as (complexified) Kähler or complex structure moduli also depends on the sign the $U(1)$ charge in the theory. This basically means that the complex structure and (complexified) Kähler moduli space are interchanged for the manifolds Y and $\tilde{Y}$. From the CFT picture this is a trivial statement, it's just the matter of a conventional sign. However, from the geometrical point of view, it is highly non-trivial as the Kähler and complex structure moduli spaces are very different objects. This observation lead to the mirror conjecture: for any Calabi Yau manifold Y, there exists a *mirror manifold* $\tilde{Y}$, such that if one compactifies IIA on Y, and IIB on $\tilde{Y}$, that the two resulting 4-dimensional theories coïncide.

For mirror pairs, the roles of the (complexified) Kähler moduli space and the Complex structure moduli space thus interchanged, i.e:

$$\mathcal{M}_C(Y) = \mathcal{M}_{CK}(\tilde{Y}) \tag{2.36}$$

$$\mathcal{M}_K(CY) = \mathcal{M}_C(\tilde{Y}) \tag{2.37}$$

This of course implies that also the different structures encaptured in the moduli space, such as their metrics and their dimensions. The fact that their dimensions are interchanged means that for mirror pairs, the hodge number $h^{1,1}$ and $h^{1,2}$ are interchanged. This corresponds to mirroring the hodge diamond in a diagonal, hence the term mirror symmetry. Let's note that these statements are made in the full non-perturbative string theories. The beauty of mirror symmetry is that some aspects which are non-perturbative on one side, are perturbative on the other. So things that are difficult to calculate on one side, can be easily calculated on the other side. But this also means that the relations between the moduli spaces hold only if worldsheet instanton corrections are taken into account. Thus this statement about the moduli spaces is really one of the non-classical string corrected moduli spaces.

However throughout this thesis we are working in the supergravity limit and this basically means that we only consider the classical moduli spaces. We should therefor also take a suitable limit of our Calabi-Yaus Y and $\tilde{Y}$ consistent with this approximation. We are not going into the details of this, but simply note that such suitable limits exist. Concretely, this means that on one side we have to work in the large volume limit and on the other side in the large complex structure limit and in effect we are working with the classical moduli spaces. In these limits the relations will between the string corrected moduli spaces will also hold for the classical moduli spaces. For a more detailed review of the structure of the moduli spaces and the implications of mirror symmetry we refer to appendix A.

Chapter 3

Calabi Yau compactification without fluxes

Having covered all the preliminaries concerning the supergravities, Calabi-Yau compactifications and mirror symmetry, we are ready to perform an explicit reduction over a generic Calabi-Yau manifold Y. Recall that we will only consider the bosonic sector of the theories and claim that the corresponding fermionic sector follows by supersymmetry. We will first decompose all the fields present in the theories, and try te see whether we can already say something about mirror symmetry at that level. Then we will perform the integration over our Calabi-Yau manifold resulting in the effective four dimensional actions. Lastly we will examine mirror symmetry at the level of the actions and try to find an explicit mapping between the fields in IIA and IIB. This section is largely based on [5] and [7].

3.1 Field content and decompositions

Now for the actual decomposition of our ten dimensional fields in terms of the harmonic basis given by:

	Basis	
H^0	1	1
H^2	ω_i	$i = 1, .., h^{1,1}$
H^3	α_A, β^A	A = 0,..,h^{1,2}
H^4	$\tilde{\omega}^i$	$i = 1, .., h^{1,1}$
H^6	$*1$	1

With all the properties discussed in chapter 2 and appendix A. Also remember that,

$$i, j, .. = 1, .., h^{1,1} \tag{3.1}$$

$$I, J, .. = 0, .., h^{1,1} \tag{3.2}$$

$$a, b, .. = 1, .., h^{1,2} \tag{3.3}$$

$$A, B, .. = 0, .., h^{1,2} \tag{3.4}$$

and that hats denote fields in ten dimensions.

3.1.1 IIA

Before expanding let's quickly recall the (bosonic) field content of type IIA supergravity: the RR-sector of IIA consists of a one-form A_1 and a three form C_3, and the NSNS-sector includes the dilaton ϕ, a 2-form B_2 and the metric g. Now let's decompose them in terms of our harmonic basis:

$$\hat{A}_1 = A_1^0 \tag{3.5}$$

$$\hat{B}_2 = B_2 + b^i\omega_i \tag{3.6}$$

$$\hat{C}_3 = C_3 + A_1^i \wedge \omega_i + \xi^A \wedge \alpha_A + \tilde{\xi}_A \wedge \beta^A \tag{3.7}$$

Thus we end up with several new fields $A_1^0, B_2, b^i, C_3, A_1^i, \xi^A, \tilde{\xi}_A$ and ofcourse the geometrical moduli z^a, v^i and the dilaton ϕ. These will neatly arrange themselves in supersymmetric multiplets. We will not go into the details, but simply note that these multiplets can be constructed using supersymmetry considerations. We see that apart from a lot of scalars and vectors, we have a three-form C_3 and a two-form B_2. However, the field content is most often written in terms of scalar fields only. This can be done by noting that we can dualize B_2 to a scalars by making use of Poincaré dualities, see appendix C. In fact, C_3 dualizes to a constant which we will set to zero since it, as we will see, corresponds to a flux. We will get back to this later.

3.1.2 IIB

The RR-section of IIB consists of a scalar l, a 2-form C_2 and a 4-form A_4 with self-dual field strength, and the NSNS-sector contains the same fields as IIA. Just as in the IIA case we can easily decompose the different field (strenghts) on our harmonic basis:

$$\hat{l} = l \tag{3.8}$$

$$\hat{B}_2 = B_2 + b^i\omega_i \tag{3.9}$$

$$\hat{C}_2 = C_2 + c^i\omega_i \tag{3.10}$$

$$\hat{A}_4 = A_4 + D_2^i \wedge \omega_i + \rho_i \wedge \tilde{\omega}^i + V^A \wedge \alpha_A - U_A \wedge \beta^A \tag{3.11}$$

Thus in this case we end up with: $B_2, C_2, A_4, c^i, b^i, l, D_2^i, \rho_i, V^A, U_A$ in addition to the geometrical fluxes and the dilaton. Again we want to describe the content only in scalar and vector degrees of freedom, which can be done by dualizing C_2 and B_2 to the scalars h_1 and h_2 respectively. Also we note that A_4 can be ommited since the only terms in which it enters the action are via it's fieldstrength, and since there are no 5-forms in fourdimensional space it vanishes. Also the self-duality condition implies that D_2^i and ρ_i are in fact related and we choose to express everything in terms of the ρ_i. The same holds for V^A and U_A and we will choose to use V^A. The arrangement of the different fields in multiplets (both IIA and IIB case) is given in table 3.1.

3.1.3 Mirror symmetry

Let's see if we can already say something about the possibility of mirror symmetry at this level. Let's first note that via the dualization of the 2-forms the tensor multiplets

Table 3.1: Multiplets in four dimensions

	IIA		IIB	
Gravity multiplet	1	(g,A_1^0)	1	(g,V_1^0)
Vector multiplets	$h^{1,1}$	(A_1^i, v^i, b^i)	$h^{1,2}$	(V_1^a, z^a)
Hypermultiplets	$h^{1,2}$	($z^a, \xi^a, \bar{\xi}_a$)	$h^{1,1}$	(v^i, b^i, ρ_a)
Tensor multiplet	1	($B_2, \phi, \xi^0, \bar{\xi}_0$)	1	(B_2, C_2, ϕ, l)

dualize to an additional hypermultiplet containing only scalar degrees of freedom in both the IIA and IIB case. Now from our quick analysis of the different fields that arise in the compactifications, and by referring to table 3.1, we can directly see that IIA compactified on Y and IIB compactified on its mirror $\tilde{Y}$ indeed yield the correct amount of different multiplets such that mirror symmetry should be possible. This since for mirror pairs the hodge numbers $h^{1,1}$ and $h^{1,2}$ are interchanged.
However, the fact that both field contents are compatible does not mean that their dynamics also coïncide. In order to compare those dynamics we have to compactify the 10 dimensional actions and reduce them to effective 4-dimensional actions by integrating the dependence on our internal manifold out. Let us first compactify the part of the actions that IIA and IIB have in common consisting of the Ricci scalar and the dilaton.

3.2 Compactifying the Ricci scalar and the dilaton

Let's first look at how the part of the action with the Ricci-scalar and the dilaton reduce. We considere these together since they prove to be related. One must first note that when looking at the 10-dimensional action, the sign of the kinetic dilaton term is wrong. This is due to the fact that we are in the so called 'string frame'. However by a Weyl rescaling of the metric, we can move from this frame to the 'Einstein frame' in which the correct sign appears. This is possible because the action is in fact Weyl invariant. This is achieved by the following rescaling, with $\Omega = e^{-\frac{\hat{\phi}}{4}}$ (see appendix D on Weyl transformations). This results in the followingaction:

$$S = \int -\frac{1}{2}\hat{R} * 1 - \frac{1}{4}d\hat{\phi} \wedge *d\hat{\phi} \tag{3.12}$$

Now for the actual compactification: we will not go through the steps but simply state the result. For a thorough treatment see for example [5].

$$S = \int -\frac{1}{2}\mathcal{K}\mathcal{R} * 1 - \mathcal{K}\frac{1}{4}d\phi \wedge *d\phi - \frac{1}{2}P_{ij}dv^i \wedge *dv^j - \frac{1}{2}Q_{ab}dz^a \wedge *d\bar{z}^b \tag{3.13}$$

Here P_{ij} and Q_{ab} are related to the metrics on the Kähler and complex structure moduli respectively. Now we will perform another Weyl rescaling in order to get the correct normalization of our Ricci scalar. We use $\Omega = \mathcal{K}^{\frac{1}{2}}$, resulting in:

$$
\begin{aligned}
S = \int & -\frac{1}{2}\mathcal{R} * 1 - \frac{3}{4} d\ln\mathcal{K} \wedge * d\ln\mathcal{K} - \frac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} \\
& - \frac{1}{2\mathcal{K}} P_{ij} dv^i \wedge * dv^j - \frac{1}{2\mathcal{K}} Q_{ab} dz^a \wedge * d\bar{z}^b
\end{aligned}
\tag{3.14}
$$

However, now we have this nasty term $\frac{3}{4}d\ln\mathcal{K} \wedge * d\ln\mathcal{K}$ which we want to get rid off. This can be achieved by redefining the Kähler moduli via:

$$
v^i = e^{-\frac{1}{2}\hat{\phi}} \tilde{v}^i \tag{3.15}
$$

Due to this term, many of the others will in fact also transform. Again we will not go into the details, but refer to appendix D. Let us just note that it amounts to different factors of $e^{-\frac{1}{2}\hat{\phi}}$ depending on the terms at hand. After defining the four dimensional dilaton as $e^{-\phi} = \mathcal{K} e^{-\hat{\phi}}$ and dropping the tildes for our Kähler moduli, our final result is:

$$
\int -\frac{1}{2} R * 1 - d\phi \wedge * d\phi - g_{ij} dv^i \wedge * dv^j - q_{ab} dz^a \wedge * d\bar{z}^b \tag{3.16}
$$

3.2.1 IIA

In compactifying the Ricci scalar we had to do various rescalings and redefinitions. Obviously we also need to perform these same operations on the rest of the action.

Weyl rescaling

Applying the same Weyl rescaling with $\Omega = e^{-\frac{\hat{\phi}}{4}}$ results in:

$$
\begin{aligned}
S = & -\frac{1}{4} \int e^{\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3 - \frac{1}{2} \int e^{\frac{3}{2}\hat{\phi}} \hat{F}_2 \wedge * \hat{F}_2 - \frac{1}{2} \int e^{\frac{1}{2}\hat{\phi}} \hat{F}_4 \wedge * \hat{F}_4 \\
& - \frac{1}{2} \int \hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1 + \frac{1}{3} (\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1
\end{aligned}
\tag{3.17}
$$

The way the various terms get modified depends on the amount of metric factors entering via the hodge operator. As we can see the topological terms do not change since there is no metric dependence in these terms.

Decompositions

Now we will perform the actual integration of our terms over our Calabi-Yau manifold. For this we must first get expressions for the fieldstrengths in terms of the harmonic expansion. This can be done very easily by simply inserting the decomposition of the fields in the field strength definitions. By making use of the fact that $d(A \wedge B) = dA \wedge B + (-1)^d A \wedge dB$, and the fact that we are expanding in harmonic forms so the second term vanishes, we end up with:

$$\hat{F}_2 = dA_1 \tag{3.18}$$
$$\hat{H}_3 = dB_2 + db^i \wedge \omega_i \tag{3.19}$$
$$\hat{F}_4 = dC_3 + dA_1^i \wedge \omega_i + d\xi^A \wedge \alpha_A + d\tilde{\xi}_A \wedge \beta^A \tag{3.20}$$

Now for the actual integration we will give the results. We have included the full calculations as appendix B. Let's start off with the kinetic term of our NS 2-form:

$$-\frac{1}{4}e^{-\hat{\phi}}\int_{CY} \hat{H}_3 \wedge *\hat{H}_3 = -\frac{\mathcal{K}}{4}e^{-\hat{\phi}}(dB_2 \wedge *dB_2) - \mathcal{K}e^{-\hat{\phi}}g_{ij}db^i \wedge *db^j \tag{3.21}$$

Now for the RR-sector:

$$-\frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\int_{CY} \hat{F}_2 \wedge *\hat{F}_2 = -\frac{\mathcal{K}}{2}e^{\frac{3}{2}\hat{\phi}}dA_1 \wedge *dA_1 \tag{3.22}$$
$$\begin{aligned} -\frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\int_{CY} \hat{F}_4 \wedge *\hat{F}_4 = &-\frac{\mathcal{K}}{2}e^{\frac{1}{2}\hat{\phi}}(dC_3 - B_2 \wedge dA_1^0) \wedge *(dC_3 - B_2 \wedge dA_1^0) \\ &- 2\mathcal{K}e^{\frac{1}{2}\hat{\phi}}g_{ij}(dA_1^i - b^i dA_1^0) \wedge *(dA_1^j - b^i dA_1^0) \\ &+ \frac{1}{2}e^{\frac{1}{2}\hat{\phi}}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C) \wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D) \end{aligned} \tag{3.23}$$

Here the matrix $\mathcal{M}$ is defined in appendix A. Finally we turn our attention to the topological terms:

$$\int_{CY} \hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 = \mathcal{K}_{ijk}b^i dA_1^j \wedge dA_1^k + B_2 \wedge d(\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \tag{3.24}$$
$$\int_{CY} (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge dA_1 = \mathcal{K}_{ijk}b^i b^j dA_1^k \wedge dA_1^0 \tag{3.25}$$
$$\int_{CY} (\hat{B}_2)^3 \wedge d\hat{A}_1 \wedge d\hat{A}_1 = \mathcal{K}_{ijk}b^i b^j b^k dA_1^0 \wedge dA_1^0 \tag{3.26}$$

Weyl rescaling

Now we will perform the Weyl rescaling needed to get the right normalization for our Ricci scalar ($\Omega = \mathcal{K}^{\frac{1}{2}}$):

$$-\frac{1}{4}e^{-\hat{\phi}}\int_{CY}\hat{H}_3\wedge *\hat{H}_3 = -\frac{\mathcal{K}^2}{4}e^{-\hat{\phi}}(dB_2\wedge *dB_2) - e^{-\hat{\phi}}g_{ij}db^i\wedge *db^j \tag{3.27}$$

$$-\frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\int_{CY}\hat{F}_2\wedge *\hat{F}_2 = -\frac{\mathcal{K}}{2}e^{\frac{3}{2}\hat{\phi}}dA_1\wedge *dA_1 \tag{3.28}$$

$$-\frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\int_{CY}\hat{F}_4\wedge *\hat{F}_4 = -\frac{\mathcal{K}^3}{2}e^{\frac{1}{2}\hat{\phi}}(dC_3 - B_2\wedge dA_1^0)\wedge *(dC_3 - B_2\wedge dA_1^0) \tag{3.29}$$

$$\begin{aligned}&- 2\mathcal{K}e^{\frac{1}{2}\hat{\phi}}g_{ij}(dA_1^i - b^i dA_1^0)\wedge *(dA_1^j - b^i dA_1^0)\\ &+ \frac{1}{2\mathcal{K}}e^{\frac{1}{2}\hat{\phi}}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C)\wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D)\end{aligned} \tag{3.30}$$

The topological part remains unaffected since it is metric independent.

Rotation of Kähler moduli

Now we need to redefine our Kähler moduli and at the same time define our four dimensional dilaton as $\phi = \hat{\phi} - \frac{1}{2}\ln\mathcal{K}$. By applying these transformations we get:

$$-\frac{1}{4}e^{-\hat{\phi}}\int_{CY}\hat{H}_3\wedge *\hat{H}_3 = -\frac{1}{4}e^{-4\phi}(dB_2\wedge *dB_2) - g_{ij}db^i\wedge *db^j \tag{3.31}$$

$$-\frac{1}{2}e^{\frac{3}{2}\hat{\phi}}\int_{CY}\hat{F}_2\wedge *\hat{F}_2 = -\frac{\mathcal{K}}{2}dA_1\wedge *dA_1 \tag{3.32}$$

$$\begin{aligned}-\frac{1}{2}e^{\frac{1}{2}\hat{\phi}}\int_{CY}\hat{F}_4\wedge *\hat{F}_4 = &-\frac{\mathcal{K}}{2}e^{\frac{1}{2}\phi}(dC_3 - B_2\wedge dA_1^0)\wedge *(dC_3 - B_2\wedge dA_1^0)\\ &- 2\mathcal{K}g_{ij}(dA_1^i - b^i dA_1^0)\wedge *(dA_1^j - b^i dA_1^0)\\ &+ \frac{1}{2\mathcal{K}}e^{2\phi}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C)\wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D)\end{aligned} \tag{3.33}$$

Again, the topological part remains unaffected since it does not depend on the metric, nor the moduli.

Dualization

The terms that we have calculated involve C_3 and B_2. However, as noted before, we would like to dualize them to scalar degrees of freedom. The general recipe for this is adding an appropriate lagrange multiplier involving the dual scalar to the action, and then eliminating the original fields. For a more thorough treatment see appendix C.

From the appendix we see that term with C_3 is actually dual to a constant and does not describe any degrees of freedom. The dual constant which we dub e_0 for reasons to be clear in the next sections, can be viewed as an RR-flux. Since we are considering the fluxless case, we will take $e_0 = 0$ and the dual action involving e_0 can be discarded.

Now for our 2-form B_2 we collect all the terms involving it and add the lagrange multiplier $\frac{1}{2}H_3 \wedge da$ to the action, where a is the scalar dual of B_2:

$$S_{B_2} = \int -\frac{1}{4}e^{-4\phi}H_3 \wedge *H_3 + \frac{1}{2}H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) + \frac{1}{2}H_3 \wedge da \tag{3.34}$$

By referring to appendix C we see that our new action in terms of a is:

$$S_a = \int -\frac{1}{4}e^{4\phi}(da + \tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \wedge *(da + \tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \tag{3.35}$$

Collecting terms

We have now made all the necessary steps to put the effective action in its final form. Let's group all the terms involving fields in the vectormultiplets:

$$\begin{aligned}
S_{vm} = \int & -g_{ij}dv^i \wedge *dv^j - g_{ij}db^i \wedge *db^j + \mathcal{K}dA_1 \wedge *dA_1 && (3.36)\\
& + 4\mathcal{K}g_{ij}(dA_1^i - b^i dA_1^0) \wedge *(dA_1^j - b^i dA_1^0)\mathcal{K}_{ijk} + b^i dA_1^j \wedge dA_1^k && (3.37)\\
& + \mathcal{K}_{ijk}b^i b^j dA_1^k \wedge dA_1^0 + \mathcal{K}_{ijk}b^i b^j b^k dA_1^0 \wedge dA_1^0 && (3.38)\\
= & - g_{ij}dt^i \wedge *d\bar{t}^j + Re\mathcal{N}_{IJ}F^I \wedge F^J + Im\mathcal{N}_{IJ}F^I \wedge *F^J && (3.39)\\
& && (3.40)
\end{aligned}$$

Where we have defined $\mathcal{N}$ in appendix A and introduced $F^I = (A^0, A^i)$. Also we have grouped the Kähler moduli with the moduli coming from the NS 2-form to form the moduli t^i of the complexified Kähler moduli space. One can easily check that it indeed can be written as such.

Now lets group all the terms involving fields in the hypermultiplets:

$$S_{hm} = \int -d\phi \wedge *d\phi - q_{ab} dz^a \wedge *d\bar{z}^b \tag{3.41}$$

$$- \frac{1}{4} e^{4\phi}(da + \tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \wedge *(da + \tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \tag{3.42}$$

$$+ \frac{1}{2} e^{2\phi}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC} d\xi^C) \wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD} d\xi^D) \tag{3.43}$$

$$= \int -h_{uv} q^u \wedge *dq^v \tag{3.44}$$

$$\tag{3.45}$$

Where q^u is the collection of all hyperscalars and h_{uv} is the metric on the hypermultiplet sector. And now collecting everything, we can compactly write our final result:

$$S_{IIA} = \int -\frac{1}{2} R * 1 - g_{ij} dt^i \wedge *d\bar{t}^j - h_{uv} q^u \wedge *dq^v \tag{3.46}$$

$$+ Re\mathcal{N}_{IJ} F^I \wedge F^J + Im\mathcal{N}_{IJ} F^I \wedge *F^J \tag{3.47}$$

$$\tag{3.48}$$

3.2.2 IIB

The compactification of IIB, goes in much the same way as that of IIA. Apart from the field content, the only real difference is that we have to pay extra attention to the fact that our 5-form fieldstrength is self-dual. The rest of the steps are identical to those made in the compactification of IIA. We will therefore not repeat all of the different rescalings and redefinitions, but simply note that the combined effect of all these steps are simply factors of $e^{-\frac{1}{2}\phi}$ and $\mathcal{K}$, depending on the terms at hand.

Decompositions

The fieldstrength decompositions are given by:

$$\hat{H}_3 = dB_2 + db^i \omega_i \tag{3.49}$$

$$\hat{F}_2 = (dC_3 - l dB_2) + (dc^i - l db^i)\omega_i \tag{3.50}$$

$$\begin{aligned} \hat{F}_5 &= (dD_2^i \wedge \omega_i + F^A \alpha_A - G_A \beta^A + d\rho_i \wedge \tilde{\omega}^i) \\ &\quad + (B_2 + b^i \wedge \omega_i) \wedge (dC_2 + dc^i \wedge \omega_i) \\ &= (dD_2^i + b^i dC_2 + B_2 \wedge dc^i) \wedge \omega_i + F^A \alpha_A - G_A \beta^A \\ &\quad + (d\rho_i + K_{ijk} b^j \wedge dc^k) \wedge \tilde{\omega}^i \end{aligned} \tag{3.51}$$

The last equality follows since $\omega_i \wedge \omega_j = \mathcal{K}_{ijk} \tilde{\omega}^k$. We have also introduced $F^A = dV^A$ and $G_A = dU_A$

Now for the actual integration (we will ignore the different dilaton dependend factors and reinsert them at a later stage whilst at the same time correcting for the different rescalings etc.):

$$-\frac{1}{4}\int_{CY}\hat{H}_3\wedge *\hat{H}_3 = -\frac{\mathcal{K}}{4}(dB_2\wedge *dB_2) - \mathcal{K}g_{ij}db^i\wedge *db^j \tag{3.52}$$

$$-\frac{1}{2}\int_{CY}d\hat{l}\wedge *d\hat{l} = -\frac{\mathcal{K}}{2}dl\wedge *dl \tag{3.53}$$

$$-\frac{1}{2}\int_{CY}\hat{F}_3\wedge *\hat{F}_3 = -\frac{\mathcal{K}}{2}(dC_2 - ldB_2)\wedge *(dC_2 - ldB_2) - 2\mathcal{K}g_{ij}(dc^i - ldb^i)\wedge *(dc^j - ldb^j) \tag{3.54}$$

$$\begin{aligned}-\frac{1}{4}\int_{CY}\hat{F}_5\wedge *\hat{F}_5 = &-\mathcal{K}g_{ij}(dD_2^i + b^i dC_2 + B_2\wedge dc^i)\wedge *(dD_2^j + b^j dC_2 + B_2\wedge dc^j)\\ &-\frac{1}{4\mathcal{K}}g^{ij}(d\rho_i + \mathcal{K}_{ikl}b^k\wedge dc^l)\wedge *(d\rho_j + \mathcal{K}_{jmn}b^m\wedge dc^n)\\ &+\frac{1}{4}(Im\mathcal{M}^{-1})^{AB}(G_A - \mathcal{M}_{AC}F^C)\wedge *(G_B - \bar{\mathcal{M}}_{BD}F^D)\end{aligned} \tag{3.55}$$

$$-\frac{1}{2}\int_{CY}\hat{A}_4\wedge d\hat{B}_2\wedge d\hat{C}_2 = -\frac{1}{2}\mathcal{K}_{ijk}D_2^i\wedge db^j\wedge dc^k - \frac{1}{2}\rho_i(dB_2\wedge dc^i + db^i\wedge dC_2) \tag{3.56}$$

Implications of self-duality

The fact that $F_5 = *F_5$ has the implication that our fields D_2^i and ρ_i, and V^A and U_A are in fact related. This can be easily seen by explicitly calculating $*\hat{F}_5$ and equating it with $\hat{F}_5$. Working that out, we get the following relations:

$$dD_2^i + b^i dC_2 + B_2\wedge dc^i = \frac{1}{4\mathcal{K}}g^{ij} * (d\rho_j + K_{jkl}b^k\wedge dc^l) \tag{3.57}$$

$$G_A = Re\mathcal{M}_{AB}F^B + Im\mathcal{M}_{AB} * F^B \tag{3.58}$$

What we would like is to eliminate half of those fields in favor of the other half, namely D_2^i in favor of ρ_i and G_A in favor of F^A. However, we cannot use these equations directly to elimate the fields by inserting them into the action. If we would do so the kinetic terms would vanish. So we must find another way to impose these four-dimensional relations and find out what our acion in terms of chosen fields only is. This can be done by adding an appropriate total derivative term to the action. On then considers all the fields to be independent and calculates the equations of motion. The duality conditions will then directly follow from the equations of motion and now we *can* insert these back into the action and we effectively eliminating half of the fields. The following total derivative is suitable:

$$\frac{1}{2}dD_2^i\wedge d\rho_i + \frac{1}{2}F^A\wedge G_A \tag{3.59}$$

By now reinserting the duality conditions into the action we end up with:

$$S_{F^A} = \int \frac{1}{2}Re\mathcal{M}_{AB}F^A\wedge F^B + \frac{1}{2}Im\mathcal{M}_{AB}F^A\wedge *F^B \tag{3.60}$$

$$\begin{aligned}S_{\rho_i} = \int &-\frac{1}{8\mathcal{K}}g^{ij}(d\rho_i - \mathcal{K}_{ikl}c^k db^l)\wedge *(d\rho_j - \mathcal{K}_{jmn}c^m db^n)\\ &- d\rho_i\wedge(c^i dB_2 + db^i\wedge C_2) - \frac{1}{2}\mathcal{K}_{ijk}c^i c^j dB_2\wedge db^k\end{aligned} \tag{3.61}$$

Dualization

What remains to be done is dualizing the remaining two-forms C_2 and B_2, starting off with the first. We must add the lagrang multiplier $dC_2 \wedge dh_1$ to the part of the action containing C_2 (where we have already performed the rescalings), and we end up with:

$$S_{C_2} = \int \frac{1}{2} e^{-2\phi} \mathcal{K}(dC_2 - ldB_2) \wedge *(dC_2 - ldB_2) - b^i dC_2 \wedge d\rho_i + dC_2 \wedge dh_1 \tag{3.62}$$

Referring to appendix C we see that our dual action is:

$$S_{h_1} = \int \frac{e^{2\phi}}{2\mathcal{K}} (dh_1 - b^i d\rho_i) \wedge *(dh_1 - b^j d\rho_j) + ldB_2 \wedge (dh_1 - b^i d\rho_i) \tag{3.63}$$

Now collecting all the terms with B_2:

$$S_{H_3} = \int -\frac{1}{4} e^{-4\phi} H_3 \wedge *H_3 + H_3 \wedge (\frac{1}{2} \mathcal{K}_{ijk} c^i c^j \wedge db^k + ldh_1 + (c^i - lb^i) d\rho_i) \tag{3.64}$$

Dualizing to h_2 gives:

$$S_{h_2} = \int -e^{4\phi} D\tilde{h} \wedge *D\tilde{h} \tag{3.65}$$

Where $D\tilde{h} = \frac{1}{2} dh_2 + ldh_1 + (c^i - lb^i) d\rho_i - \frac{1}{2} \mathcal{K}_{ijk} c^i c^j db^k$

Collecting all the terms

Collecting all the terms and keeping in mind the various rescalings, we get:

$$\begin{aligned} S_{IIB} = \int (&-\frac{1}{2} R * 1 - q_{ab} dz^a \wedge *d\bar{z}^b - g_{ij} dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi \\ &- \frac{e^{2\phi}}{8\mathcal{K}} g^{ij} (d\rho_i - \mathcal{K}_{ikl} c^k db^l) \wedge *(d\rho_j - \mathcal{K}_{jmn} c^m db^n) \\ &- 2\mathcal{K} e^{2\phi} g_{ij} (dc^i - ldb^i) \wedge *(dc^j - ldb^j) - \frac{1}{2} \mathcal{K} e^{2\phi} dl \wedge *dl \\ &- \frac{1}{2} \mathcal{K} e^{2\phi} (dh_1 - b^i d\rho_i) \wedge *(dh_1 - b^j d\rho_j) - e^{4\phi} D\tilde{h} \wedge *D\tilde{h} \\ &+ \frac{1}{2} Im\mathcal{M}_{AB} F^A \wedge *F^B + \frac{1}{2} Re\mathcal{M}_{AB} F^A \wedge F^B \end{aligned} \tag{3.66}$$

3.2.3 Mirror Symmetry

Now that we have calculated the effective actions for both IIA and IIB, we can examine them and check whether they are mirror symmetric. I.e. does the following hold:

$$S_{IIB}(Y) = S_{IIA}(\tilde{Y}) \tag{3.67}$$

$$S_{IIA}(Y) = S_{IIB}(\tilde{Y}) \tag{3.68}$$

When comparing the actions for IIA and IIB it is difficult to see whether they are mirror symmetric or not, since they seem to have rather different forms. However, let's see what the action of IIA compactified on the mirror $\tilde{Y}$ looks like. This can be very easily obtained by noting that the only things that change are the dimensions of the cohomology groups. (Note that we will denote the corresponding moduli, metrics and coupling matrices of the mirror $\tilde{Y}$ with tildes):

$$\begin{aligned} S_{IIA}(\tilde{Y}) = \int & -\frac{1}{2}R*1 - \tilde{g}_{ab}d\tilde{t}^a \wedge *d\bar{\tilde{t}}^b + Re\tilde{\mathcal{N}}_{AB}F^A \wedge F^B + Im\tilde{\mathcal{N}}_{AB}F^A \wedge *F^B \\ & - d\phi \wedge *d\phi - \tilde{q}_{ij}d\tilde{z}^i \wedge *d\bar{\tilde{z}}^j \\ & - \frac{1}{4}e^{4\phi}(da + \tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_A) \wedge *(da + \tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I) \\ & + \frac{1}{2}e^{2\phi}(Im\tilde{\mathcal{M}}^{-1})^{IJ}(d\tilde{\xi}_I + \tilde{\mathcal{M}}_{IK}d\xi^K) \wedge *(d\tilde{\xi}_J + \bar{\tilde{\mathcal{M}}}_{JL}d\xi^L) \end{aligned} \tag{3.69}$$

We would now like to relate this to the action of IIB on Y, which can be done by remembering the various relations between the moduli, metrics and coupling matrices for mirror pairs, as described in appendix A:

$$h^{1,1} \leftrightarrow h^{1,2} \tag{3.70}$$
$$t \leftrightarrow z \tag{3.71}$$
$$g \leftrightarrow q \tag{3.72}$$
$$\mathcal{M} \leftrightarrow \mathcal{N} \tag{3.73}$$

Making use of these relations we then get:

$$\begin{aligned} S_{IIA}(\tilde{Y}) = \int & \frac{1}{2}R*1 - q_{ab}dz^a \wedge *d\bar{z}^b + Re\mathcal{M}_{AB}F^A \wedge F^B + Im\mathcal{M}_{AB}F^A \wedge *F^B \\ & - d\phi \wedge *d\phi - g_{ij}dt^i \wedge *d\bar{t}^j \\ & - \frac{1}{4}e^{4\phi}(da + \tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I) \wedge *(da + \tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I) \\ & + \frac{1}{2}e^{2\phi}(Im\mathcal{N}^{-1})^{IJ}(d\tilde{\xi}_I + \mathcal{N}_{IK}d\xi^K) \wedge *(d\tilde{\xi}_J + \bar{\mathcal{N}}_{JL}d\xi^L) \end{aligned} \tag{3.74}$$

Thus if mirror symmetry holds, we should be able to put the action of IIB in the form above by performing some field redefinitions. We will now proceed to do so. We claim that indeed the IIB action can be rewritten as such, and subsequently we will use the explicit expression for $\mathcal{N}$ to write everything out and then systematically compare the various terms by their dependence on ϕ, g_{ij} and g^{ij}. We start off with identifying the dilatons due to their somewhat special role. We can ofcourse also identify the moduli and therefor also b^i since it enters in t^i. In addition we can swiftly see that we can make the straightforward identifications between the vectorfields:

$$A^a = V^a \tag{3.75}$$

To find the other identifications, let's start by rewriting the last line of 3.74 as:

$$\frac{e^{2\phi}}{2}(Im\mathcal{N}^{-1})^{IJ}(d\tilde{\xi}_I + Re\mathcal{N}_{IK}d\xi^K)\wedge *(d\tilde{\xi}_J + Re\mathcal{N}_{JL}d\xi^L)$$
$$+\frac{e^{2\phi}}{2}Im\mathcal{N}_{IJ}d\xi^I\wedge *d\xi^J \tag{3.76}$$

We know use theexpression for $\mathcal{N}$ to rewrite the second term:

$$\frac{e^{2\phi}}{2}Im\mathcal{N}_{IJ}d\xi^I\wedge *d\xi^J = -2e^{2\phi}\mathcal{K}g_{ij}(d\xi^i - b^i\xi^0)\wedge *(d\xi^j - b^j\xi^0)$$
$$-\frac{e^{2\phi}}{2}\mathcal{K}d\xi^0\wedge *d\xi^0 \tag{3.77}$$

Comparing with 3.66 we make the following identifications:

$$d\xi^0 = \pm dl \tag{3.78}$$
$$-4\mathcal{K}g_{ij}(d\xi^i - b^i d\xi^0) = \pm(dc^i - ldb^j) \tag{3.79}$$

From which we deduce:

$$\xi^0 = l \tag{3.80}$$
$$\xi^i = lb^i - c^i \tag{3.81}$$

Now looking at the terms coming from $(Im\mathcal{N}^{-1})^{ij}$ which are proportional to g^{ij}:

$$-\frac{e^{2\phi}}{8\mathcal{K}}g^{ij}(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K)\wedge *(d\tilde{\xi}_j + Re\mathcal{N}_{jL}d\xi^L) \tag{3.82}$$

And equating with the terms proportional to g^{ij} in the IIB action:

$$d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K = \pm(d\rho_i - \mathcal{K}_{ikl}c^k db^l) \tag{3.83}$$

Which is solved by:

$$\tilde{\xi}_i = \rho_i + \mathcal{K}_{ikl}(\frac{1}{2}b^k b^l l - c^k b^l) \tag{3.84}$$

The remainder of 3.76 is given by:

$$-\frac{e^{2\phi}}{2\mathcal{K}}(d\tilde{\xi}_0 + Re\mathcal{N}_{0K}d\xi^K + b^i(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K)\wedge *(d\tilde{\xi}_0 + Re\mathcal{N}_{0L}d\xi^L + b^j(d\tilde{\xi}_j + Re\mathcal{N}_{jL}d\xi^L) \tag{3.85}$$

Comparing with 3.66 we get:

$$d\tilde{\xi}_0 + Re\mathcal{N}_{0K}d\xi^K + b^i(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K) = \pm(dh_1 - b^i d\rho_i) \tag{3.86}$$

Resulting in:

$$\tilde{\xi}_0 = -h_1 + \frac{1}{2}K_{ikl}(b^i b^k c^l - \frac{1}{3}b^i b^k b^l l) \tag{3.87}$$

Considering the terms proportional to $e^{4\phi}$:

$$-\frac{1}{4}e^{4\phi}(da+\tilde{\xi}_I d\xi^I-\xi^I d\tilde{\xi}_I)\wedge *(da+\tilde{\xi}_I d\xi^I-\xi^I d\tilde{\xi}_I) \tag{3.88}$$

And comparing gives:

$$da+(\tilde{\xi}_I d\xi^I-\xi^I d\tilde{\xi}_I)=\pm 2(\frac{1}{2}dh_2+ldh_1+(c^i-lb^i)d\rho_i-\frac{1}{2}\mathcal{K}_{ijk}c^ic^jdb^k) \tag{3.89}$$

Which gives us our last identification:

$$a=h_2+lh_1+\rho_i(c^i-lb^i) \tag{3.90}$$

We have now found an explicit map which puts the IIB action in the appropriate form and therefor we can now directly observe that mirror symmetry holds.

Chapter 4

Calabi Yau compactification with fluxes

4.1 Introducing fluxes

In the previous chapter we compactified our IIA/B supergravities and concluded that mirror symmetry is manifest. This is of course a nice result. However, these ordinary Calabi-Yau compactifications suffer some deficiencies which do not make them attractive from a phenomological point of view. One of which is the appearence of the many *massless* scalar fields, i.e. the moduli. This poses a problem: if such theories should describe the physics of our universe, we would have already observed these scalar fields. However, as of now, the only scalar field we have observed is the higgs field and it has a mass of around 125 GeV. So if we could be able to somehow generate masses for our scalar fields of the order of the higgs field, we could resolve this problem. One way to do this is by introducing fluxes on our internal manifold. So what are fluxes? Basically they are sources that we add to our internal manifold such that if we integrate our bosonic fields along non-trivial cycles of our manifold they acquire non-zero values. This means that our bosonic fields acquire non-zero vacuum expectation values. This in contrast with the standard Calabi-Yau compactifications for which we assumed the bosonic vevs to vanish. It can be shown that not all fluxes are allowed as there are some identities the field strengths need to satisfy, such as the Bianci identity, see [1]. One then sees that the allowed flux parameters have to be constant and we should get expressions of the following form:

$$\int_{\Sigma_i} F_n = e_i \tag{4.1}$$

Where the e_i are the constant flux parameters, F_n is some n-form fieldstrength and Σ_i are non-trivial cycles. Now we know the homology groups which capture the amount and structure of non-trivial cycles are dual to the cohomology groups which capture the amount and structure of harmonic forms. This basically means that for every non-trivial cycle there is exactly one harmonic form which is dual to it, which means that integrated over the cycle will result in a nonzero value. Let Σ_i be the collection of all non-trivial cycles and η_i be the corresponding set of dual harmonic forms. Then explicitly we get:

$$\int_{\Sigma_i} \eta_j = \delta^i_j \tag{4.2}$$

Thus our vev's for our bosonic fields can be expanded in a harmonic basis, which ofcourse fit's perfectly with the standard compactification procedure in which we expand all the fields in harmonic forms. We can thus say that the amount of harmonic forms directly determines the number of fluxparameters we can introduce for our various bosonic fields. So for instance if we would like to introduce fluxes for a four-form fieldstrength, we must do so in a basis of harmonic four-forms and the amount fluxes we can introduce is $h^{1,1}$.

As we noted, the IIA/B theories contain bosonic fields in two sectors: the RR and the NS sector. From the string theory perspective these are really very different fields with very different origins. This basically means that the fluxes we turn on via fieldstrenghts of fields in the RR sector should be distinguished from those turned on via fieldstrengths of NS fields. There are a couple of other things we can say about fluxes.

Besides the RR/NS distinction, there is a another subdivision we can make: we have electric fluxes, e_i, and their magnetic duals m^i. The details are not important for our further discussions in this thesis and we will not cover them, but let's just note that it is analagous to electric and magnetic charges in classical electromagnetism. Summarizing, we end up with RR fluxes and NS fluxes, which each are subdivided into electric and magnetic variants.

The main goal of this chapter is to calculate the effective four dimensional actions when fluxes are turned on and to examine whether supersymmetry is still manifest. Since the RR fluxes are fundamentally different from NS fluxes, we can already note that RR fluxes on one side can never be mapped to NS fluxes on the other and vice-versa. We will therefor tackle the problem for RR and NS fluxes separately, i.e we want to check if

$$S_{IIA}(Y, e_{RR}, m_{RR}) = S_{IIB}(\tilde{Y}, e_{RR}, m_{RR}) \tag{4.3}$$

$$S_{IIB}(Y, e_{RR}, m_{RR}) = S_{IIA}(\tilde{Y}, e_{RR}, m_{RR}) \tag{4.4}$$

holds and also:

$$S_{IIA}(Y, e_{NS}, m_{NS}) = S_{IIB}(\tilde{Y}, e_{NS}, m_{NS}) \tag{4.5}$$

$$S_{IIB}(Y, e_{NS}, m_{NS}) = S_{IIA}(\tilde{Y}, e_{NS}, m_{NS}) \tag{4.6}$$

We will start off with the RR flux case. Before we do so however, let's note that there are some subtleties we ought to keep in mind. First of all, when we introduce fluxes on our Calabi-Yau manifold these fluxes will in general backreact on the geometry, and in the process alter it. This means that our original Calabi-Yau manifold has changed in the process and our ansatz about our manifold being Calabi-Yau no longer holds. However we wil work in the limit that the fluxes are small, such that the backreaction of the geometry is also small and can in fact be neglected. That is: we still compactify on a Calabi-Yau manifold and benefit from all it's nice properties although, strictly, this is no longer the case. Another thing we have to adress is that the fluxes are actually quantized due to a Dirac quantization condition (just like the familiar Dirac condition in classical electromagnetism if magnetic monopoles were to exist). However, in our large volume limit we can effectively treat them as small continuous parameters.

4.2 RR flux compactification

Now let's turn on fluxes in the RR sector. For IIA we have two bosonic field strengths F_2 and F_4 which are both even forms. We can thus introduce $h^{1,1}$ fluxes in H^2 via F_2 and $h^{1,1}$ fluxes in H^4 via F_4, leading to a total of $2h^{1,1}$ flux parameters.

The IIB theory contains three fieldstrengths, F_1, F_3, F_5, which are all odd. Now let's first note that we cannot add fluxes in H^1 and H^5 via F_1 and F_5 respectively, since these cohomology groups vanish on Calabi-Yau manifolds. We are thus left with F_3 as the only possible fieldstrength via which we can introduce RR fluxes. Since there are $2h^{1,2} + 2$ harmonic 3-forms, that is the total amount of flux parameters we can introduce in the IIB case.

Let's see if we can already say something about mirror symmetry at this level. Recall that mirror symmetry interchanges $h^{1,1}$ and $h^{1,2}$. So if we would compactify IIB on the mirror we would get a total of $2h^{1,1}+2$ parameters, while on the IIA side we have $2h^{1,1}$. We thus seem to be missing two RR fluxes on the IIA side. Now remember that the RR fluxes in the IIA case lie in H^2 and H^4, i.e. in the even comohomogy groups. However, there are ofcourse also H^0 and H^6 as even cohomology groups, both of which have dimension 1. Thus if we could somehow turn on fluxes in these groups we would get 2 extra flux parameters fixing the apparent asymmetry. Now since we have no field strengths via which we can introduce them, we must find another way. The first flux can be turned on by noting that the field C_3 which appears in our effective theory does not describe degrees of freedom and is in fact poincaré dual to a constant which, as we already noted several times before, can be viewed as an RR flux. In the standard compactification we put this constant to zero and C_3 doesn't contribute to the action. However, now we will take it to be non-zero and in this way it will fullfill the role of a flux in H^0 ([3]).
The other missing flux can be introduced by not starting off with the standard massless IIA theory, but with the massive IIA theory as described in section 2.1. In this theory our NS 2-form is in fact massive, with a mass parameter which we are gonna dub m^0. This mass parameter precisely fulfills the role of the missing H^6 flux.

Thus mirror symmetry might actually hold since we can introduce the right amount of flux parameters in both cases. To examine the actual dynamics we proceed by calculating our effective four dimensional actions, starting with IIA. We follow [3].

4.2.1 IIA

As just noted we need to turn to the massive version of IIA as described in section 2.1. Adding the fluxes amounts to the following changes:

$$dA_1 \to dA_1 + dA_1^{flux} = dA_1 - m^i\omega_i \tag{4.7}$$

$$dC_3 \to dC_3 + dC_3^{flux} = dC_3 + e_i\tilde{\omega}^i \tag{4.8}$$

Which result in the following modified expansions of the field strengths:

$$\hat{F}_2 = (dA_1 + mB_2)\wedge 1 - (m^i - mb^i)\wedge\omega_i \tag{4.9}$$

$$\begin{aligned}\hat{F}_4 = {} & dC_3 - B_2\wedge dA^0 - \frac{m}{2}(B_2)^2 \\ & + (dA^i - b^i dA^0 + m^i B_2 - m^0 b^i B_2)\wedge\omega_i \\ & + (d\xi^A\wedge\alpha_A + d\tilde{\xi}_A\wedge\beta^A) \\ & + (b^i m^j - \frac{1}{2}mb^i b^j)K_{ijk}\tilde{\omega}^k + e_i\tilde{\omega}^i \end{aligned}\tag{4.10}$$

$$\hat{H}_3 = dB_2 + db^i\wedge\omega_i \tag{4.11}$$

Now for the actual integrations:

$$\begin{aligned}-\frac{1}{2}\int_{CY}\hat{F}_2\wedge *\hat{F}_2 = {} & -\frac{\mathcal{K}}{2}dA_1\wedge *dA_1 - m\mathcal{K}B_2\wedge *dA^0 \\ & -\frac{1}{2}m^2\mathcal{K}B_2\wedge *B_2 - 2\mathcal{K}(m^i - mb^i)(m^j - mb^j)g_{ij}\end{aligned}\tag{4.12}$$

$$\begin{aligned}-\frac{1}{2}\int_{CY}\hat{F}_4\wedge *\hat{F}_4 = {} & -\frac{1}{2}\mathcal{K}(dC_3 - B_2\wedge dA^0 - \frac{m}{2}(B_2)^2)\wedge *(dC_3 - B_2\wedge dA^0 - \frac{m}{2}(B_2)^2) \\ & - 2\mathcal{K}g_{ij}(dA^i - b^i dA^0 + m^i B_2 - m^0 b^i B_2)\wedge *(dA^j - b^j dA^0 + m^j B_2 - m^0 b^j B_2) \\ & - \frac{1}{8\mathcal{K}}g^{ij}(e_i + b^k m^l\mathcal{K}_{ikl} - \frac{1}{2}mb^k b^l\mathcal{K}_{ikl})\wedge *(e_j + b^m m^n\mathcal{K}_{jmn} - \frac{1}{2}mb^m b^n\mathcal{K}_{jmn}) \\ & + \frac{1}{2}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C)\wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D)\end{aligned}\tag{4.13}$$

The kinetic term for B_2 does not change. For the different tolopogical terms we find:

$$\begin{aligned}-\frac{1}{2}\int_{CY}\hat{B}_2\wedge d\hat{C}_3\wedge d\hat{C}_3 = & -\frac{1}{2}\mathcal{K}_{ijk}b^i dA_1^j\wedge dA_1^k + -\frac{1}{2}dB_2\wedge(\xi^A d\tilde{\xi}_A - \tilde{\xi}_A d\xi^A) \\ & - e_i B_2\wedge dA^i + 2b^i e_i dC_3\end{aligned}\tag{4.14}$$

$$\begin{aligned}\frac{1}{2}\int_{CY}(\hat{B}_2)^2\wedge d\hat{C}_3\wedge d\hat{A}_1 = & \frac{1}{2}\mathcal{K}_{ijk}(b^i b^j dA_1^k\wedge dA_1^0 - b^i b^j m^k dC_3 - 2b^i m^j B_2\wedge dA_1^0) \\ & + b^i e_i B_2\wedge dA^0 - \frac{1}{2}e_i m^i B_2\wedge B_2\end{aligned}\tag{4.15}$$

$$-\frac{1}{6}\int_{CY}(\hat{B}_2)^3\wedge d\hat{A}_1\wedge d\hat{A}_1 = -\frac{1}{6}\mathcal{K}_{ijk}(b^i b^j b^k dA_1^0\wedge dA_1^0 - 6b^i b^j m^k B_2\wedge dA^0 + 3b^i m^j m^k B_2\wedge B_2) \tag{4.16}$$

$$\frac{m}{6}\int_{CY}(\hat{B}_2)^3\wedge d\hat{C}_3 = \frac{m}{6}\mathcal{K}_{ijk}b^i b^j b^k dC_3 + \frac{m}{2}\mathcal{K}_{ijk}b^i b^j B_2\wedge dA^k + \frac{m}{2}b^i e_i B_2\wedge B_2 \tag{4.17}$$

$$-\frac{m}{8}\int_{CY}(\hat{B}_2)^4\wedge d\hat{A}_1 = -\frac{m}{2}\mathcal{K}_{ijk}b^i b^j b^k B_2\wedge dA^0 + \frac{3m}{4}\mathcal{K}_{ijk}b^i b^j m^k B_2\wedge B_2 \tag{4.18}$$

$$-\frac{m^2}{40}\int_{CY}(\hat{B}_2)^5 = -\frac{m^2}{4}\mathcal{K}_{ijk}b^i b^j b^k B_2\wedge B_2 \tag{4.19}$$

Also we must not forget the extra term $-m^2 * 1$:

$$\int_{CY} -m^2 * 1 = -\mathcal{K}m^2 \tag{4.20}$$

Dualization

Next we dualize C_3 to a constant, e_0. Collecting all the terms with C_3 we get:

$$S_{C_3} - \int (\frac{\mathcal{K}}{2}(dC_3 - J_4) \wedge *(dC_3 - J_4) + \frac{h}{2} dC_3 \tag{4.21}$$

With $J_4 = B_2 \wedge dA^0 - \frac{m}{2}(B_2)^2$ and $\frac{h}{2} = b^i e_i + \frac{1}{2} b^i b^j m^k K_{ijk} - \frac{m}{6} b^i b^j b^k K_{ijk}$.

Now adding the term $\frac{e_0}{2} dC_3$ to the action and referring to appendix C we get:

$$S_{e_0} = -\int \frac{1}{2\mathcal{K}}(h + e_0)^2 * 1 + \frac{1}{2}(h + e_0) J_4 \tag{4.22}$$

$$\tag{4.23}$$

In the fluxless case we also dualized B_2 to a scalar. However, this time, it is no longer massless and can thus no longer be dualized to a scalar. It *can* be dualized to a massive vector but we will not do so. We will content ourselves with keeping B_2 and, as we will see, this is good enough to examine mirror symmetry.

Collecting all the terms

Now let's collect all the new terms, starting with all terms contributing to a potential:

$$\begin{aligned} V &= 2\mathcal{K}(m^i - mb^i)(m^j - mb^j) g_{ij} - m^2 \\ &\quad + \frac{1}{8\mathcal{K}} g^{ij} (e_i + b^k m^l \mathcal{K}_{ikl} - \frac{1}{2} m b^k b^l \mathcal{K}_{ikl})(e_j + b^m m^n \mathcal{K}_{jmn} - \frac{1}{2} m b^m b^n \mathcal{K}_{jmn}) \\ &\quad - \frac{1}{2K}(e_0 + e_i b^i \frac{1}{2} b^i b^j m^k \mathcal{K}_{ijk} - \frac{m}{6} b^i b^j b^k \mathcal{K}_{ijk})^2 \qquad (4.24) \\ &= -\frac{1}{2}(e_I - \bar{\mathcal{N}}_{IK} m^K)(Im\mathcal{N})^{-1IJ}(e_J - \mathcal{N}_{JL} m^L) \qquad (4.25) \end{aligned}$$

One can easily verify that one can indeed write the potential term as such. It can now be directly seen that the RR fluxes introduce potentials for some of the scalars in the vectormultiplets.

Besides a potential we find several new terms involving B_2. Firstly we get a mass term:

$$\begin{aligned} \mathcal{S}_M &= \int (-\frac{1}{2} m^2 \mathcal{K} - 2\mathcal{K} g_{ij} (m^i - mb^i)(m^j - mb^j)) B_2 \wedge * B_2 \\ &= \quad -\int \frac{1}{2} M^2 B_2 \wedge * B_2 \qquad (4.26) \end{aligned}$$

With $M^2 = -m^I Im\mathcal{N}_{IJ} m^J$. Which can be easily checked by remembering that $-4\mathcal{K} g_{ij} = (-\mathcal{K}_{ij} - \frac{1}{4}\frac{\mathcal{K}_i\mathcal{K}_j}{\mathcal{K}})b^i b^j$.

We also get a topological mass term:

$$\begin{aligned}\mathcal{S}_{M_T} = &-\int \frac{1}{2}(me_0 + e_i m^i - mK_{ijk}b^i b^j m^k + mb^i e_i + -\mathcal{K}_{ijk}b^i b^j b^k \\ &- \mathcal{K}_{ijk}b^i b^j m^k + \frac{m^2}{3}K_{ijk}b^i b^j b^k)B_2 \wedge B_2 \\ = &-\int \frac{1}{2}M_T^2 B_2 \wedge B_2 \qquad (4.27) \\ & \qquad (4.28)\end{aligned}$$

With $M_T^2 = -m^I Re\mathcal{N}_{IJ} m^J + m^I e_I$.

And finally we get an interaction term:

$$\begin{aligned}\mathcal{S}_{J_2} &= -\int B_2 \wedge (e_I F^I - m^I(Re\mathcal{N}_{IJ}F^J + Im\mathcal{N}_{IJ} * F^J) \\ &= -\int B_2 \wedge J_2 \qquad (4.29)\end{aligned}$$

Now putting everything together and performing the right rescalings etc, we arrive at our final expression:

$$\begin{aligned}S_{IIA} = &\int(-\frac{1}{2}R * 1 - g_{ij}dt^i \wedge *d\bar{t}^j - q_{ab}dz^a \wedge *d\bar{z}^b - d\phi \wedge *d\phi \qquad (4.30) \\ &- \frac{1}{4}e^{-4\phi}H_3 \wedge *H_3 + \frac{1}{2}H_3 \wedge (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) \qquad (4.31) \\ &+ \frac{e^{-2\phi}}{2}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C) \wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D) \qquad (4.32) \\ &+ \frac{1}{2}Re\mathcal{N}_{IJ}F^I \wedge F^J + \frac{1}{2}Im\mathcal{N}_{IJ}F^I \wedge *F^J \qquad (4.33) \\ &- B_2 \wedge J_2 - \frac{1}{2}B_2 \wedge *B_2 - \frac{1}{2}M_T^2 B_2 \wedge B_2 \\ &- \frac{1}{2}e^{4\phi}(e_I - \bar{\mathcal{N}}_{IK}m^K)(Im\mathcal{N})^{-1IJ}(e_J - \mathcal{N}_{JL}m^L) \qquad (4.34)\end{aligned}$$

4.2.2 IIB

As said before, in the IIB case we introduce RR fluxes via F_3, which can be achieved by the following modification:

$$dC_2 \to dC_2 + dC_2^{flux} = dC_2 + m^A \alpha_A - e_A \beta^A \qquad (4.35)$$

Leading to the following decompositions of our fieldstrenghts:

$$\hat{F}_3 = dC_2 - lH_3 + (dc^i - ldb^i)\omega_i + m^A\alpha_A - e_A\beta^A \tag{4.36}$$

$$\begin{aligned}\hat{F}_5 &= (dD_2^i + b^i dC_2 + B_2 \wedge dc^i) \wedge \omega_i + F^A\alpha_A - G_A\beta^A \\ &\quad + (d\rho_i + \mathcal{K}_{ijk}b^j dc^k) \wedge \tilde{\omega}^i + m^A B_2 \wedge \alpha^A - e_A B_2 \wedge \beta^A \end{aligned}\tag{4.37}$$

$$\begin{aligned} &= (dD_2^i + b^i dC_2 + B_2 \wedge dc^i) \wedge \omega_i + \tilde{F}^A\alpha_A - \tilde{G}_A\beta^A \\ &\quad + (d\rho_i + \mathcal{K}_{ijk}b^j dc^k) \wedge \tilde{\omega}^i \end{aligned}\tag{4.38}$$

$$\hat{H}_3 = dB_2 + db^i \wedge \omega_i \tag{4.39}$$

Here we have introduced $\tilde{F}^A = F^A + m^A B_2$ and $\tilde{G}_A = G_A + e_A B_2$ for convenience. Performing the actual integration:

$$\begin{aligned}-\frac{1}{2}\int_{CY} \hat{F}_3 \wedge *\hat{F}_3 = &-\frac{\mathcal{K}}{2}(dC_2 - ldB_2) \wedge *(dC_2 - ldB_2) \\ &- 2\mathcal{K}g_{ij}(dc^i - ldb^i) \wedge *(dc^i - ldb^i) \\ &- \frac{1}{2}(e_A - \bar{\mathcal{M}}_{AC}m^C)(Im\mathcal{M})^{-1AB}(e_B - \mathcal{M}_{BD}m^D)\end{aligned}\tag{4.40}$$

$$\begin{aligned}-\frac{1}{4}\int_{CY} \hat{F}_5 \wedge *\hat{F}_5 = &-\mathcal{K}g_{ij}(dD_2^i + b^i dC_2 + B_2 \wedge dc^i) \wedge *(dD_2^j + b^j dC_2 + B_2 \wedge dc^j) \\ &- \frac{1}{4\mathcal{K}}g^{ij}(d\rho_i + \mathcal{K}_{ikl}b^k \wedge dc^l) \wedge *(d\rho_j + \mathcal{K}_{jmn}b^m \wedge dc^n) \\ &+ \frac{1}{4}(Im\mathcal{M}^{-1})^{AB}(\tilde{G}_A - \mathcal{M}_{AC}\tilde{F}^C) \wedge *(\tilde{G}_B - \bar{\mathcal{M}}_{BD}\tilde{F}^D)\end{aligned}\tag{4.41}$$

(Please note the tildes.)

$$\begin{aligned}-\frac{1}{2}\int \hat{A}_4 \wedge d\hat{B}_2 \wedge d\hat{C}_2 = &-\frac{1}{2}\mathcal{K}_{ijk}D_2^i \wedge db^j \wedge dc^k \\ &- \frac{1}{2}\rho_i(dB_2 \wedge dc^i + db^i \wedge dC_2) - \frac{1}{2}(F^A e_A - G_A m^A) \wedge B_2\end{aligned}\tag{4.42}$$

Self-duality condition

To eliminate half of the fields we procceed in the same way as in the fluxless case. The scalar section is unmodified by the RR fluxess so the result will be the same. However, as we have seen, the vector terms are modified. The self-duality condition for our four dimensional vectors turns out to be:

$$\tilde{G}_A = Re\mathcal{M}_{AB}\tilde{F}^B + Im\mathcal{M}_{AB} * \tilde{F}^B \tag{4.43}$$

(Please note the tildes.)

Just as in the fluxless case we add the appropriate total derivative $\frac{1}{2}F^A \wedge G_A$ (note the absence of tildes) to our action and after elimination of half of the fields by inserting the self-duality conditions in the action, we get:

$$S_{F^A} = \int \frac{1}{2} Re\mathcal{M}_{AB} \tilde{F}^A \wedge \tilde{F}^B + \frac{1}{2} Im\mathcal{M}_{AB} \tilde{F}^A \wedge *\tilde{F}^B - \frac{1}{2} B_2 \wedge (F^A + \tilde{F}^A) e_A \tag{4.44}$$

Note that in terms of F^A instead of $\tilde{F}^A (= F^A + m^A B_2)$ we get:

$$S_{F^A} = \int \frac{1}{2} Re\mathcal{M}_{AB} F^A \wedge F^B + \frac{1}{2} Im\mathcal{M}_{AB} F^A \wedge *F^B - B_2 \wedge I_2 - \frac{1}{2} N^2 B_2 \wedge *B_2 - \frac{1}{2} N_T^2 B_2 \wedge B_2 \tag{4.45}$$

With:

$$I_2 = e_A F^A - m^A (Re\mathcal{M}_{AB} F^B + Im\mathcal{M}_{AB} * F^B) \tag{4.46}$$

$$N_T^2 = -m^A Re\mathcal{M}_{AB} m^B + m^A e_A \tag{4.47}$$

$$N^2 = -m^A Im\mathcal{M}_{AB} m^B \tag{4.48}$$

Which already very much reminds us of the corresponding terms in the IIA case.

Dualization

B_2 is massive, just as in IIA, and we will not dualize it for the same reasons. However C_2 remains massless and we will dualize it to the scalar h_1 just as in the fluxless case. Actually the sector containing C_2 is not altered at all by adding RR fluxes so we get the same result as in the fluxless case (apart from the rescalings):

$$S_{h_1} = \int \frac{1}{2\mathcal{K}} (dh_1 - b^i d\rho_i) \wedge *(dh_1 - b^j d\rho_j) + l dB_2 \wedge (dh_1 - b^i d\rho_i) \tag{4.49}$$

Collecting all the terms

Thus collecting everything we end up with and taking into account the various rescalings we end up with:

$$
\begin{aligned}
S_{IIB} = \int (&-\frac{1}{2}R*1 - q_{ab}dz^a \wedge *d\bar{z}^b - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi \\
&- \frac{e^{2\phi}}{8\mathcal{K}} g^{ij}(d\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(d\rho_j - \mathcal{K}_{jmn}c^m db^n) \\
&- 2\mathcal{K}e^{2\phi}g_{ij}(dc^i - ldb^i) \wedge *(dc^j - ldb^j) - \frac{1}{2}\mathcal{K}e^{2\phi}dl \wedge *dl \\
&- \frac{1}{2}\mathcal{K}e^{2\phi}(dh_1 - b^i d\rho_i) \wedge *(dh_1 - b^j d\rho_j) \\
&+ ldB_2 \wedge (dh_1 - b^i d\rho_i) - \frac{1}{4}e^{-4\phi}dB_2 \wedge *dB_2 \\
&- \frac{1}{2}\mathcal{K}_{ijk}c^i c^j dB_2 \wedge db^k + c^i dB_2 \wedge d\rho_i \\
&+ \frac{1}{2}Re\mathcal{M}_{AB}F^A \wedge F^B + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B - B_2 \wedge J_2 \\
&- \frac{1}{2}M^2 B_2 \wedge *B_2 - \frac{1}{2}M_T^2 B_2 \wedge B_2 \\
&- \frac{1}{2}e^{4\phi}(e_A - \bar{\mathcal{M}}_{AC}m^C)(Im\mathcal{M})^{-1AB}(e_B - \mathcal{M}_{BD}m^D)
\end{aligned} \tag{4.50}
$$

4.2.3 Mirror Symmetry

We are now ready to examine whether mirror symmetry still holds in the RR flux case. Let's first give the action of IIA compactified on the mirror manifold (again using tildes):

$$
\begin{aligned}
S_{IIA}(\tilde{Y}) = \int (&-\frac{1}{2}R*1 - \tilde{g}_{ab}d\tilde{t}^a \wedge *d\bar{\tilde{t}}^b - \tilde{q}_{ij}d\tilde{z}^i \wedge *d\bar{\tilde{z}}^j - d\phi \wedge *d\phi \\
&- \frac{1}{4}e^{-4\phi}H_3 \wedge *H_3 + \frac{1}{2}H_3 \wedge (\tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I) \\
&+ \frac{e^{-2\phi}}{2}(Im\tilde{\mathcal{M}}^{-1})^{IJ}(d\tilde{\xi}_I + \tilde{\mathcal{M}}_{IK}d\xi^K) \wedge *(d\tilde{\xi}_J + \bar{\tilde{\mathcal{M}}}_{JL}d\xi^L) \\
&+ \frac{1}{2}Re\tilde{\mathcal{N}}_{AB}F^A \wedge F^B + \frac{1}{2}Im\tilde{\mathcal{N}}_{AB}F^A \wedge *F^B \\
&- B_2 \wedge \tilde{J}_2 - \frac{1}{2}\tilde{M}B_2 \wedge *B_2 - \frac{1}{2}\tilde{M}_T^2 B_2 \wedge B_2 \\
&- \frac{1}{2}e^{4\phi}(\tilde{e}_A - \bar{\tilde{\mathcal{N}}}_{AC}\tilde{m}^C)(Im\tilde{\mathcal{N}})^{-1AB}(\tilde{e}_B - \tilde{\mathcal{N}}_{BD}\tilde{m}^D)
\end{aligned} \tag{4.51}
$$

Here we have introduced:

$$\tilde{J}_2 = \tilde{e}_A F^A - \tilde{m}^A(Re\tilde{\mathcal{N}}_{AB}F^B + Im\tilde{\mathcal{N}}_{AB} * F^B) \tag{4.52}$$

$$\tilde{M}_T^2 = -\tilde{m}^A Re\tilde{\mathcal{N}}_{AB}\tilde{m}^B + \tilde{m}^A\tilde{e}_A \tag{4.53}$$

$$\tilde{M}^2 = -\tilde{m}^A Im\tilde{\mathcal{N}}_{AB}\tilde{m}^B \tag{4.54}$$

We can now easily put the IIB action in a similar form by making use of the mirror mapping as calculated in the previous chapter (with the exception of the mapping for a ofcourse). We see that, up to a total derivative $d(H_3 \wedge \xi^i)$, our IIB action reduces to:

$$
\begin{aligned}
S_{IIA}(Y) = \int (& -\frac{1}{2}R*1 - g_{ij}dt^i \wedge *d\bar{t}^i - q_{ab}dz^a \wedge *d\bar{z}^b - d\phi \wedge *d\phi \\
& - \frac{1}{4}e^{-4\phi}H_3 \wedge *H_3 + \frac{1}{2}H_3 \wedge (\tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I) \\
& + \frac{e^{-2\phi}}{2}(Im\mathcal{N}^{-1})^{IJ}(d\tilde{\xi}_I + \mathcal{N}_{IK}d\xi^K) \wedge *(d\tilde{\xi}_J + \bar{\mathcal{N}}_{JL}d\xi^L) \\
& + \frac{1}{2}Re\mathcal{N}_{AB}F^A \wedge F^B + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B \\
& - B_2 \wedge I_2 - \frac{1}{2}NB_2 \wedge *B_2 - \frac{1}{2}N_T^2 B_2 \wedge B_2 \\
& - \frac{1}{2}e^{4\phi}(e_A - \bar{\mathcal{M}}_{AC}m^C)(Im\mathcal{M})^{-1AB}(e_B - \mathcal{M}_{BD}m^D)
\end{aligned} \tag{4.55}
$$

If we now make the familiar identifications between the metrics, coupling matrices and moduli for mirror pairs we note that, if in addition we make the following identifications regarding the flux parameters:

$$\tilde{e}_I = e_I \tag{4.56}$$

$$\tilde{m}^I = m^I \tag{4.57}$$

we get the following relations between the B_2 masses and interactions:

$$\tilde{M} = N \tag{4.58}$$

$$\tilde{M}_T = N_T \tag{4.59}$$

$$\tilde{J}_2 = I_2 \tag{4.60}$$

Applying all these identifications we see that indeed:

$$S_{IIB}(Y, e, m) = S_{IIA}(\tilde{Y}, \tilde{e}, \tilde{m}) \tag{4.61}$$

Thus we conclude that mirror symmetry is preserved in the RR flux case if one directly identifies the fluxes on the IIA side with those on the IIB side.

4.3 NS flux compactification

Having seen that mirror symmetry holds in the RR flux case, let's examine the NS flux case. Considering NS fluxes we see that they can only be turned on for H_3 since this is the only NSNS fieldstrength present in both IIA and IIB. This poses an immediate problem regarding mirror symmetry. The NS fluxes lie in the cohomology group H^3 for both IIA and IIB and are thus counted by $h^{1,2}$ in either case. Thus compactifying IIA on Y results in fluxes counted by $h^{1,2}$, but IIB compactified on it's mirror $\tilde{Y}$ results in fluxes counted by $h^{1,1}$. This means we can't match the amount of NS fluxes in this case and we can immediatly conclude that mirror symmetry no longer holds.

One might then think as this of the end. However, there is a possible solution to this problem by noting that the metric g and the dilaton ϕ are also NS fields, and we may be able to generate fluxes via these fields. What we will see is that we can indeed generate fluxes with the metric, but in effect have to part from Calabi-Yau compactifications and have to consider somewhat more general manifolds (namely $SU(3)$ manifolds) as our internal space. But before we will get into this procedure, let us first calculate the effective actions in the NS flux case as we will need them later on anyway. This section is based on [3] and [4].

4.3.1 IIA

Before we can proceed with the compactification of IIA we must first note someting about the topological term. The one we have used in the previous cases depends on B_2. However if we want to add fluxes for it's fieldstrength H_3, we need our action to solely depend on H_3 and not on B_2. This can be achieved by the following redefinition:

$$\hat{C}_3 \to \hat{C}_3 + \hat{A}_1 \wedge \hat{B}_2 \tag{4.62}$$

Under which our topological term will then be modified into:

$$S_{top} = \int \hat{H}_3 \wedge \hat{C}_2 \wedge d\hat{C}_3 \tag{4.63}$$

In order to recover our standard formulation as we got in the previous sections, we have to keep in mind that in the end, that is after compactification, we need to redefine our vectorfields A_1^i according to:

$$A^i \to A^i - b^i A^0 \tag{4.64}$$

Having said the above, let's turn on the fluxes in the NS sector, which amounts to the following modification of our NS fieldstrength:

$$H_3 \to H_3 + H_3^{flux} = H_3 + m^A \alpha_A + e_A \beta^A \tag{4.65}$$

Which results in the following decompositions:

$$\hat{F}_2 = dA_1^0 \tag{4.66}$$

$$\hat{H}_3 = dB_2 + db^i \wedge \omega_i + m^A \alpha_A + e_A \beta^A \tag{4.67}$$

$$\begin{aligned}\hat{F}_4 = {} & dC_3 - A^0 \wedge dB_2 + (dA^i - A^0 \wedge db^i) \wedge \omega_i) \\ & + D\xi^A \wedge \alpha_A + D\tilde{\xi}_A \wedge \beta^A \end{aligned} \tag{4.68}$$

Where we have introduced: $D\xi^A = d\xi^A - m^A A^0$ and $D\tilde{\xi}_A = d\tilde{\xi}_A - e_A A^0$. We thus see that some of the hyperscalars get charged. The actualy integration gives:

$$-\frac{1}{4}\int H_3 \wedge *H_3 = -\frac{1}{4}\mathcal{K} dB_2 \wedge *dB_2 - 2\mathcal{K} g_{ij} db^i \wedge *db^j \tag{4.69}$$

$$- \frac{1}{4}(e_A + \mathcal{M}_{AC} m^C)(Im\mathcal{M}^{-1})^{AB}(e_B + \bar{\mathcal{M}}_{BD} m^D) \tag{4.70}$$

$$-\frac{1}{2}\int F_2 \wedge *F_2 = -\frac{\mathcal{K}}{2} dA^0 \wedge *dA^0 \tag{4.71}$$

$$-\frac{1}{2}\int F_4 \wedge *F_4 = -\frac{\mathcal{K}}{2}(dC_3 - A^0 \wedge dB_2) \wedge *(dC_3 - A^0 \wedge dB_2) \tag{4.72}$$

$$- 2\mathcal{K} g_{ij}(dA^i - A^0 db^i) \wedge *(dA^i - A^0 db^i) \tag{4.73}$$

$$+ \frac{1}{2}(Im\mathcal{M}^{-1})^{AB}(D\tilde{\xi}_A + \mathcal{M}_{AC} D\xi^C) \wedge *(D\tilde{\xi}_B + \bar{\mathcal{M}}_{BD} D\xi^D) \tag{4.74}$$

Besides the fact that ordinary derivatives are replaced by covariant derivative in the hyperscalar sector, we see that the kinetic term for B_2 induces a potential involving the hyperscalars. Turning to our topological term:

$$\frac{1}{2}\int H_3 \wedge C_3 \wedge dC_3 = -\frac{1}{2}dB_2 \wedge (\xi^A d\tilde{\xi}_A - \tilde{\xi}_A d\xi^A) + \frac{1}{2}\mathcal{K}_{ijk}db^i \wedge A^j \wedge dA^k \tag{4.75}$$

$$+ (m^A \tilde{\xi}_A - e_A \xi^A)dC_3 \tag{4.76}$$

Dualization

Now we dualize C_3 to a constant λ which we will put to zero since it corresponds to a RR-flux. However due to the new terms introduced by the NS fluxes, we see that the dual action does not vanish even if we put $\lambda = 0$. Thus we do have to perform the actual dualizing, starting with collecting all the terms involving C_3:

$$\mathcal{S}_{C_3} = -\int \frac{\mathcal{K}}{2}(dC_3 - A^0 \wedge H_3) \wedge *(dC_3 - A^0 \wedge H_3) + (m^A \tilde{\xi}_A - e_A \xi^A)dC_3 \tag{4.77}$$

Now referring to appendix C we get, after putting $\lambda = 0$:

$$\mathcal{S}_\lambda = -\int \frac{1}{2\mathcal{K}}(m^A \tilde{\xi}_A - e_A \xi^A + e)^2 * 1 + (m^A \tilde{\xi}_A - e_A \xi^A)A^0 \wedge dB_2 \tag{4.78}$$

Next we dualize B_2 to the scalar a. Collecting the terms involving B_2:

$$\mathcal{S}_{B_2} = \int -\frac{\mathcal{K}}{4}H_3 \wedge *H_3 - \frac{1}{2}H_3 \wedge (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A + (m^A \tilde{\xi}_A - e_A \xi^A)A^0) \tag{4.79}$$

And subsequently dualizing:

$$\mathcal{S}_a = \int -\frac{1}{4\mathcal{K}}(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \wedge *(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \tag{4.80}$$

Where we have introduced $Da = da - (m^A \tilde{\xi}_A - e_A \xi^A)A^0$. The dual scalar thus also becomes charged (which is not entirely surprising since it is a hyperscalar).

Collecting the terms

After rescaling and redefining the vectors we end up with:

$$\begin{aligned}
S_{IIA} = \int & (-\frac{1}{2}\mathcal{R} * 1 - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi - g_{ab}dz^a \wedge *d\bar{z}^b \\
& - \frac{e^{4\phi}}{4}(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \wedge *(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \\
& + \frac{e^{2\phi}}{2}(Im\mathcal{M}^{-1})^{AB}(D\tilde{\xi}_A + \mathcal{M}_{AC}D\xi^C) \wedge *(D\tilde{\xi}_B + \mathcal{M}_{BD}D\xi^D) \\
& + \frac{1}{2}Im\mathcal{N}_{IJ}F^I \wedge *F^J + \frac{1}{2}Re\mathcal{N}_{IJ}F^I \wedge F^J \\
& + \frac{1}{4\mathcal{K}}e^{2\phi}(e_A + \mathcal{M}_{AC}m^C)(Im\mathcal{M}^{-1})^{AB}(e_B + \bar{\mathcal{M}}_{BD}m^D) \\
& - \frac{1}{2\mathcal{K}}e^{4\phi}(m^A\tilde{\xi}_A - e_A\xi^A)^2
\end{aligned} \tag{4.81}$$

This result is very similar to the fluxless case apart from the covariant derivatives and the potential for some of the hyperscalars.

4.3.2 IIB

Unlike in the IIA case, we don't have to redefine any of the fields since the action already depends on H_3 only. We can introduce the NS fluxes in a very similar way:

$$H_3 \to H_3 + H_3^{flux} = H_3 + m^A\alpha_A - e_A\beta^A \tag{4.82}$$

This leads to the following decompositions of the fieldstrengths:

$$\hat{H}_3 = dB_2 + db^i \wedge \omega_i + m^A\alpha_A - e_A\beta^A \tag{4.83}$$

$$d\hat{l} = dl \tag{4.84}$$

$$\hat{F}_3 = C_2 - ldB_2 + (c^i - ldb^i) \wedge \omega_i - l(m^A\alpha_A - e_A\beta^A) \tag{4.85}$$

$$\begin{aligned}
\hat{F}_5 = & (dD_2^i - db^i \wedge C_2 - c^i dB_2) \wedge \omega_i + \tilde{F}^A \wedge \alpha_A + \tilde{G}_A \wedge \beta^A \\
& d\rho_i \wedge \tilde{\omega}^i - c^i db^j \wedge \omega_i \wedge \omega_j
\end{aligned} \tag{4.86}$$

Where we have introduced $\tilde{F}^A = F^A - m^A C_2$ and $\tilde{G}_A = G_A - e_A C_2$.
Performing the integration gives:

$$
\begin{aligned}
-\frac{1}{4}\int_{CY} H_3 \wedge *H_3 &= -\frac{1}{4}\mathcal{K} dB_2 \wedge *dB_2 - \mathcal{K} g_{ij} db^i \wedge *db^j \\
&+ \frac{1}{4}(e - \mathcal{M}m) Im\mathcal{M}^{-1}(e - \mathcal{M}m) * 1 \qquad (4.87)\\
-\frac{1}{2}\int dl \wedge *dl &= -\frac{1}{2}\mathcal{K} dl \wedge *dl \qquad (4.88)\\
-\frac{1}{2}\int F_3 \wedge *F_3 &= -\frac{1}{2}\mathcal{K}(dC_2 - ldB_2) \wedge *(dC_2 - ldB_2) \\
&- 2\mathcal{K} g_{ij}(dc^i - ldb^i) \wedge *(dc^i - ldb^i) \\
&+ \frac{1}{2} l^2 (e_A - \mathcal{M}_{AC} m^C)(Im\mathcal{M}^{-1})^{AB}(e_B - \bar{\mathcal{M}}_{BD} m^D) * 1 \qquad (4.89)
\end{aligned}
$$

Self duality

The terms involving D_2^i and ρ_i are unmodified by the introduction of the fluxes, so we can directly use the results of the fluxless case to eliminate D_2^i.

To eliminate G_A in favor of F^A we again add the term $\frac{1}{2}F^A \wedge G_A$ to the action. This term will again provide the right duality condition as an equation of motion:

$$
\tilde{G}_A = Re\mathcal{M}_{AB}\tilde{F}^B + Im\mathcal{M}_{AB} * \tilde{F}^B \qquad (4.90)
$$

Eliminating in the usual fashion we end up with:

$$
\begin{aligned}
S_{F^A} &= \frac{1}{2} Re\mathcal{M}\tilde{F}^A \wedge \tilde{F}^B + \frac{1}{2} Im\mathcal{M}\tilde{F}^A \wedge *\tilde{F}^B \\
&\frac{1}{2} e_A C_2 \wedge (F^A + \tilde{F}^A) \qquad (4.91)
\end{aligned}
$$

It is interesting to note that since $\tilde{F}^A = F^A - m^A C_2$, several mass terms are introduced for C_2 which are all proportional to the magnetic flux parameters m^A. This is already a very different result in comparison to the IIA case.

Dualization

Since C_2 is now massive, we can no longer dualize it to a scalar. B_2 can still be dualized to a scalar, but we will postpone it for a moment and first collect all the terms and discuss some of the differences between IIA and IIB.

Collecting the terms

$$
\begin{aligned}
S_{IIB} = \int (&-\frac{1}{2}R*1 - q_{ab}dz^a \wedge *d\bar{z}^b - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi \\
&- \frac{1}{4}e^{-4\phi}dB_2 \wedge dB_2 - \frac{1}{2}e^{-2\phi}\mathcal{K}(dC_2 - ldB_2) \wedge *(dC_2 - ldB_2) \\
&- \frac{e^{2\phi}}{8\mathcal{K}}g^{ij}(d\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(d\rho_j - \mathcal{K}_{jmn}c^m db^n) \\
&- 2\mathcal{K}e^{2\phi}g_{ij}(dc^i - ldb^i) \wedge *(dc^j - ldb^j) - \frac{1}{2}\mathcal{K}e^{2\phi}dl \wedge *dl \\
&+ (db^i \wedge C_2 + c^i dB_2) \wedge (d\rho_i - \mathcal{K}_{ijk}c^j db^k) + \frac{1}{2}\mathcal{K}_{ijk}c^i c^j dB_2 \wedge db^k \\
&+ \frac{1}{2}Re\mathcal{M}\tilde{F}^A \wedge \tilde{F}^B + \frac{1}{2}Im\mathcal{M}\tilde{F}^A \wedge *\tilde{F}^B \\
&+ \frac{1}{2}e_A C_2 \wedge (F^A + \tilde{F}^A) \\
&+ \frac{1}{2}e^{4\phi}(l^2 + \frac{e^{-2\phi}}{2\mathcal{K}})(e_A - \mathcal{M}_{AC}m^C)(Im\mathcal{M}^{-1})^{AB}(e_B - \bar{\mathcal{M}}_{BD}m^D) * 1
\end{aligned}
\tag{4.92}
$$

Comparing the two actions, we observe very different dynamics as we already anticipated. We see that in the IIA case some of the hyperscalars get charged under A^0 and a potential for the hyperscalars is introduced. In the IIB case also some of the hyperscalars get charged but in a very different manner, namely under V^i. Also and a potential for some of the hyperscalars is induced, which differs from that in the IIA case. In addition the two-form C_2 becomes massive proportional to the magnetic fluxes.

As said before, the question is now whether we can somehow generate fluxes via the metric that are mirror symmetric to the NS fluxes. It has been shown that for both electric and magnetic NS fluxes this is rather complicated, and unfortunately beyond the scope of this thesis. The case of either only electric NS fluxes or magnetic NS fluxes, is much simpler. We will choose to only turn on electric NS fluxes noting that we could just has easily have chosen the magnetic fluxes.

4.3.3 Vanishing magnetic flux

We will now give the actions of IIA and IIB without magnetic fluxes:

IIA

We can quickly see that the IIA action turns into:

$$
\begin{aligned}
S_{IIA} = \int & (-\frac{1}{2}\mathcal{R} * 1 - g_{ij} dt^i \wedge * d\bar{t}^j - d\phi \wedge * d\phi - g_{ab} dz^a \wedge * d\bar{z}^b \\
& - \frac{e^{4\phi}}{4}(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \wedge *(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \\
& + \frac{e^{2\phi}}{2}(Im\mathcal{M}^{-1})^{AB}(D\tilde{\xi}_A + \mathcal{M}_{AC} D\xi^C) \wedge *(D\tilde{\xi}_B + \mathcal{M}_{BD} D\xi^D) \\
& + \frac{1}{2} Im\mathcal{N}_{IJ} F^I \wedge * F^J + \frac{1}{2} Re\mathcal{N}_{IJ} F^I \wedge F^J \\
& + \frac{1}{4\mathcal{K}} e^{2\phi}(Im\mathcal{M}^{-1})^{AB} e_B - \frac{1}{2\mathcal{K}} e^{4\phi}(e_A \xi^A)^2
\end{aligned} \tag{4.93}
$$

Where in this case the covariant derivatives are given by: $Da = da + e_A \xi^A A^0$, $D\xi^A = d\xi^A$ and $D\tilde{\xi}_A = d\tilde{\xi}_A + e_A A^0$.

IIB

Let's see what we can do in the IIB case. First of all we see that C_2 becomes massless again and we can dualize it to a scalar in the usual fashion. Collecting all the terms involving C_2:

$$
\begin{aligned}
S_{C_2} = \int & -\frac{1}{2}\mathcal{K}(dC_2 - l dB_2) \wedge *(dC_2 - l dB_2) \\
& - b^i dC_2 \wedge d\rho_i - dC_2 \wedge e_A V^A
\end{aligned} \tag{4.94}
$$

Dualizing to h_1 gives:

$$
\begin{aligned}
S_{h_1} = \int & -\frac{1}{2\mathcal{K}}(dh_1 - b^i d\rho_i - e_A V^A) \wedge *(dh_1 - b^j d\rho_j - e_B V^B) \\
& + l dB_2 \wedge (dh_1 - b^i d\rho_i - e_A V^A)
\end{aligned} \tag{4.95}
$$

Turning our attention to B_2:

$$
S_{B_2} = \int -\frac{1}{4}\mathcal{K} dB_3 \wedge * dB_3 + dB_3 \wedge (\frac{1}{2}\mathcal{K}_{ijk} c^i c^j \wedge db^k + l dh_1 - l e_A V^A + (c^i - l b^i) d\rho_i) \tag{4.96}
$$

And dualizing to h_2:

$$
S_{h_2} = \int -\frac{1}{\mathcal{K}} D\tilde{h} \wedge * D\tilde{h} \tag{4.97}
$$

Where $D\tilde{h} = \frac{1}{2} dh_2 + l dh_1 + (c^i - l b^i) d\rho_i - l e_A V^A - \frac{1}{2}\mathcal{K}_{ijk} c^i c^j db^k$.

Putting everything together we find (after the correct rescalings):

$$
\begin{aligned}
S_{IIB} = \int & -\frac{1}{2}R*1 - q_{ab}dz^a \wedge *d\bar{z}^b - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi \\
& - \frac{e^{2\phi}}{8\mathcal{K}}g^{ij}(d\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(d\rho_j - \mathcal{K}_{jmn}c^m db^n) \\
& - 2\mathcal{K}e^{2\phi}g_{ij}(dc^i - l db^i) \wedge *(dc^j - l db^j) - \frac{1}{2}\mathcal{K}e^{2\phi}dl \wedge *dl \\
& - \frac{1}{2}\mathcal{K}e^{2\phi}(dh_1 - b^i d\rho_i - e_A V^A) \wedge *(dh_1 - b^j d\rho_j - e_A V^A) \\
& - e^{4\phi}D\tilde{h} \wedge *D\tilde{h} \\
& + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B + \frac{1}{2}Re\mathcal{M}_{AB}F^A \wedge F^B \\
& + \frac{1}{2}e^{4\phi}(l^2 + \frac{e^{-2\phi}}{2\mathcal{K}})e_A(Im\mathcal{M}^{-1})^{AB}e_B
\end{aligned} \tag{4.98}
$$

We will put in a the standard form by adding everything up and performing the usual mirror mapping. The result is:

$$
\begin{aligned}
S_{IIB} = \int & -\frac{1}{2}R*1 - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi - g_{ab}dz^a \wedge *d\bar{z}^b \\
& - \frac{e^{4\phi}}{4}(Da - (\xi^I D\tilde{\xi}_I - \tilde{\xi}_I D\xi^I)) \wedge *(Da - (\xi^I D\tilde{\xi}_I - \tilde{\xi}_I D\xi^I)) \\
& + \frac{e^{2\phi}}{2}(Im\mathcal{N}^{-1})^{IJ}(D\tilde{\xi}_I + \mathcal{N}_{IK}D\xi^K) \wedge *(D\tilde{\xi}_J + \mathcal{N}_{JL}D\xi^L) \\
& + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B + \frac{1}{2}Re\mathcal{M}_{AB}F^A \wedge F^B \\
& + \frac{1}{2}e^{4\phi}(l^2 + \frac{e^{-2\phi}}{2\mathcal{K}})e_A(Im\mathcal{M}^{-1})^{AB}e_B
\end{aligned} \tag{4.99}
$$

With $Da = da - \xi^0 e_A V^A$, $D\tilde{\xi}_0 = d\tilde{\xi}_0 + e_A V^A$, $D\tilde{\xi}_i = d\tilde{\xi}_i$ and $D\xi^I = d\xi^I$.

Chapter 5

Finding a NS-flux mirror candidate

In the previous section we saw that if we turn on RR fluxes we still have a nice mirror symmetric theory. However when we turned on NS fluxes we observed, already by counting the amount of flux parameters, that mirror symmetry is broken. Thus if we compactify, say IIA, on a Calabi-Yau with NS fluxes, there is no corresponding mirror Calabi-Yau such that IIB compactified on this mirror with NS fluxes turned on, results in the same effective theory. This was all due to the fact that both in IIA and IIB the only NS field strength for which we can turn on fluxes is our NS 2-form fieldstrength H_3. However, we also noted that the NS sector also consists of the metric g and the dilaton ϕ. It might be possible to somehow generate fluxes with these fields. The dilaton isn't going to help us a lot as it could only generate a flux in H^1 which vanishes on a Calabi-Yau manifold. So let's say we want to generate fluxes with the metric. In the Calabi-Yau case we had no fluxes coming from the metric and the only terms coming from the metric were the kinetic terms from the geometrical moduli. The reason for this is that Calabi-Yau manifolds are torsionless. Remember that this was equivalent to saying that our spinor should be covariantly constant. If we want to be able to generate fluxes with the metric, we need to allow for manifolds which do have torsion and in effect generate new terms which could serve as fluxes. Thus we naturally part from Calabi-Yau compactifications and need to consider a more general setup.

Thus in this chapter we are looking for a manifold $\tilde{Z}$, which is necessarily not Calabi-Yau, that can serve as a mirror to NS flux compactifications. I.e:

$$S(IIA, Y, NS) = S(IIB, \tilde{Z}, NS) \tag{5.1}$$

and vice-versa. However, that is not all: we must also try to find out basis we must expand our internal forms in. In the Calabi-Yau case we could neatly expand in harmonic forms, however in a more general setup this will no longer be possible.

The line of reasoning we will give here directly follows that given in [5] and [4].

Table 5.1: Difference between Calabi-Yau manifolds and a generic $SU(3)$-structure manifold

Calabi-Yau	Generic $SU(3)$
$\nabla\eta = 0$	$\nabla\eta = T\eta$
$R = 0$	$R \neq 0$
$d\Omega = 0, dJ = 0$	$d\Omega \neq 0, dJ \neq 0$

5.1 The road to Half-flat manifolds

We can already quickly narrow down the possibilities by noting that when we turned on fluxes, we assumed that they are very small. This basically meant that we still expanded in harmonic forms and that our field content did not change. Now if we want $\tilde{Z}$ to be a suitable candidate, ofcourse, it should produce the same field content. Also the kinetic terms should be the same and all the fields should be arranged in hypermultiplets determined by $N = 2$ supersymmetry. Now, we have noted that a necessary condition for the fields to arrange themselves in these multiplets is that our manifold should have $SU(3)$ structure. Thus we can already say that we are going to stay within the collection of manifolds with $SU(3)$ structure (of which Calabi-Yau manifolds are a special case). Let's give a short review of some of the properties of manifolds with $SU(3)$-structure.
One can define an $SU(3)$ manifold by demanding a globally non-vanishing spinor to exist. From this spinor we can define two forms, just as in the Calabi-Yau case, Ω and J. However in this general setup these forms are no longer closed, as was the case with Calabi-Yau manifolds. Also the spinor is no longer covariantly constant (i.e. torsionless), but we see that $\nabla_m \eta = \frac{1}{4}\kappa_{mnp}\Gamma^{np}\eta$, where κ_{mnp} is the the contorsion, which is related to the torsion, T, via: $T_{mnp} = \frac{1}{2}(\kappa_{mnp} - \kappa_{nmp})$.

Also we know for general manifolds with $SU(3)$ structure that the Riemann tensor decomposes in a Calabi-Yau part and a torsion part:

$$\mathcal{R}_{SU(3)} = \mathcal{R}_{CY} + \mathcal{R}_T \tag{5.2}$$

Here $\mathcal{R}_{CY}$ has all the nice symmetry properties of a Calabi-Yau and $\mathcal{R}_T$ is a part solely determined by the torsion of the manifold. The corresponding Ricci scalar R no longer vanishes. A summary is given in table 5.1.

Since we want to generate fluxes with the intrinsic torsion, T, of the manifold, it is usefull to examine it in a bit more detail. The torsion can be decomposed in terms of irreducible $SU(3)$ representations, $\mathcal{W}_i$, which are called the torsion classes. In other words:

$$T \in \mathcal{W}_1 \oplus \mathcal{W}_2 \oplus \mathcal{W}_3 \oplus \mathcal{W}_4 \oplus \mathcal{W}_5 \tag{5.3}$$

Where the corresponding parts of T are labeled T_i. These torsion classes each have different interpretations as various combinations of J, Ω and their derivatives. See for an overview table 5.2.

5.1.1 Calabi-Yau part

Let's first focus on the Calabi-Yau part of our manifold. Since the fluxes are assumed to be small, we would also like the geometrical fluxes to be small. Since the fluxes are generated by the torsion, this basically means that our torsion should be very small. Now if we assume

Table 5.2: Torsion classes

Component	Interpretation	$SU(3)$-representation
$\mathcal{W}_1$	$J \wedge d\Omega$ or $\Omega \wedge dJ$	$1 \oplus 1$
$\mathcal{W}_2$	$d\Omega^{2,2}$	$8 \oplus 8$
$\mathcal{W}_3$	$dJ^{2,1} + dJ^{1,2}$	$6 \oplus \bar{6}$
$\mathcal{W}_4$	$J \wedge dJ$	$3 \oplus \bar{3}$
$\mathcal{W}_5$	$d\Omega^{3,1}$	$3 \oplus 3$

our manifold to be a perturbation of a Calabi-Yau, it is natural to consider manifolds in which the 'correction' term $\mathcal{R}_T$ which measures the deviation from a Calabi-Yau, to be very small. We will thus view our possible mirror candidate Z as a perturbation of a Calabi-Yau manifold.
Now we let ourselves be guided by mirror symmetry, and note that if $\tilde{Z}$ is to produce the same field content and kinetic terms as our mirror manifold $\tilde{Y}$, then the Calabi-Yau part should be the same. Thus we view Z as a perturbation of $\tilde{Y}$, and thus demand $\mathcal{R}_{CY}(Z) = \mathcal{R}(\tilde{Y})$. These Calabi-Yau parts also have the same moduli space end we end up with: $\mathcal{M}(\tilde{Z}) = \mathcal{M}(\tilde{Y})$. This will ensure that indeed the produced kinetic terms for the moduli terms will be the same.

5.1.2 Torsion part

So much for the Calabi-Yau part of our manifold, and let's focus on the torsion part. Remembering that in NS flux compactifications we could add fluxes in the (odd) cohomology group H^3 and resulting in $h^{1,2}+1$ flux parameters we somehow want to be able to generate even fluxes with the metric on $\tilde{Z}$.

We know that for two Calabi-Yau manifolds to be mirrors their odd and even cohomologies get interchanged. That is, the complexified Kähler moduli space of Y equals the complex structure moduli space of $\tilde{Y}$ and vice-versa. Introducing fluxes in H_3 basically means that the real part of our complexified Kahler form $B_2 + iJ$ is no longer closed. Then by letting mirror symmetry guide us, we see that on our mirror manifold we would like the real part of Ω to be no longer closed, while still requiring the imaginary part to be closed. From table 5.2 we see that the components of the torsion which a related to $d\Omega$ are T_1, T_2 and T_5. We can extract information about the real part of $d\Omega$ denoted by $d\Omega^+$ and the imaginairy part denoted by $d\Omega^-$ by taking appropriate combinations of these torsion classes. To properly do so we first note that the torsion classes $\mathcal{W}_1$ and $\mathcal{W}_2$ can be decomposed further as:

$$\mathcal{W}_1 = \mathcal{W}_1^+ \oplus \mathcal{W}_1^- \tag{5.4}$$

$$\mathcal{W}_2 = \mathcal{W}_2^+ \oplus \mathcal{W}_2^- \tag{5.5}$$

By now taking correct combinations of these we can relate the wanted components to these classes:

$$T^+_{1\oplus 2} \quad \text{corresponds to} \quad (d\Omega^+)^{2,2} \tag{5.6}$$

$$T^-_{1\oplus 2} \quad \text{corresponds to} \quad (d\Omega^-)^{2,2} \tag{5.7}$$

$$T_5 \quad \text{corresponds to} \quad (d\Omega^+)^{3,1} \quad \text{and} \quad (d\Omega^-)^{3,1} \tag{5.8}$$

Thus letting the imaginairy parts vanish we conclude that:

$$T^{-}_{1\oplus 2} = 0 \tag{5.9}$$

and also by noting that $(d\Omega^{+})^{3,1}$ and $(d\Omega^{-})^{3,1}$ are related we conclude that:

$$T_5 = 0 \tag{5.10}$$

Another way to look at it is that we would somehow like to try and matchup the different torsion classes with the cohomology groups of the Calabi-Yau manifold. We will do this to ensure that now new terms arise via the torsion that are not present in Calabi-Yau compactifications. For example we do not want terms produced by torsion parts that represent 5-forms, as the corresponding cohomology group vanishes on our Calabi-Yau. Again we refer to table 5.2 and we see that we should put:

$$T_4 = T_5 = 0 \tag{5.11}$$

as these transform as $(3,2)$- and $(3,1)$-forms respectively, and the corresponding cohomology groups vanish on Calabi-Yau manifolds. Combining the two arguments we conclude that

$$T^{-}_{1\oplus 2} = T_4 = T_5 = 0 \tag{5.12}$$

might be appropriate conditions. Written out in terms of Ω and J these conditions translate into:

$$d\Omega^{-} = 0 \tag{5.13}$$

$$d(J \wedge J) = 0 \tag{5.14}$$

An $SU(3)$-manifold whose Ω and J obey the above relations is called a *Half-flat* manifold. Half-flat manifolds might thus be suitable mirror candidates.

5.2 Finding an expansion basis

Now that we have found a possible mirror candidate, we need to find a suitable expansions basis. In order to not produce terms not present in Calabi-Yau compactifications we will not allow any 1- or 5-forms as there are no such forms present in that case. (This ofcourse fits nicely with the fact that we have taken the various torsion classes representing vanishing cohomology groups on the Calabi-Yau manifold to zero.) We thus only allow a certain subset of 0-, 2-, 3-, 4- and 6-forms and we will suggestively call:

$$(1, \omega_i, \alpha_A, \beta^A, \tilde{\omega}^i, *1) \tag{5.15}$$

As we already discussed in chapter 2, in order for such a basis to be consistent it has to be closed under the differential operator d and the hodge operater $*$. Also, as noted in chapter 2, we not merely want to expand our fields in this basis, but we also want the expansion basis to capture the half-flat structure of the manifold. We therefore demand that we can expand our invariant forms in this basis, as these forms determine the structure of the manifold. I.e:

$$\Omega = z^A \alpha_A - \mathcal{G}_A \beta^A \tag{5.16}$$

$$J = v^i \omega_i \tag{5.17}$$

This is of course completely analogous to the Calabi-Yau case, but now the expansion forms can no longer be closed since J and Ω fail to be closed. However, as we have noted before, we want to produce the same kinetic terms for the moduli (and the other fields for that matter). We are therefore going to take a basis which in the limit of zero fluxes will coïncide with the harmonic forms on $\tilde{Y}$. Thus we are going to view our expansion basis as a perturbation of those harmonic forms. Also we will demand that the relations which hold for those harmonic forms, also hold for our expansion basis. That is all the various integrals of combinations of the forms as given in appendices A and B.

One can show ([4],[8]) that, if such a basis were to exist, the most general form consistent with the above relations, is given by:

$$d\omega_i = m_i^A \alpha_A + e_{iA}\beta^A \tag{5.18}$$
$$d\tilde{\omega}^i = 0 \tag{5.19}$$
$$d\alpha_A = e_{iA}\tilde{\omega}^i \tag{5.20}$$
$$d\beta^A = -m_i^A \tilde{\omega}^i \tag{5.21}$$

Also we can put some restraints on the possible fluxparameters by making use of $d^2 = 0$, but we will not be interested in that.

What we would now like to do is put some further restraints on the basis, to make it compatible with the structure of a Half-flat manifold. To do this lets calculate $d\Omega$ and dJ, for their properties characterize Half-flat manifolds:

$$d\Omega = z^A d\alpha_A - \mathcal{G}_A d\beta^A \tag{5.22}$$
$$= (z^A e_{iA} + \mathcal{G}_A m_i^A)\tilde{\omega}^i \tag{5.23}$$
$$dJ = v^i d\omega_i \tag{5.24}$$
$$= v^i (m_i^A \alpha_A + e_{iA}\beta^A) \tag{5.25}$$

Now for this basis to be compactible with a Half-flat manifold, we must demand $d\Omega^- = 0$. Now the complex moduli z^a are as well as the pre-potential $\mathcal{G}$, not suprisingly, complex so in general $d\Omega$ has a non-vanishing imaginairy component. However we can easily solve this by only considering the term with z^0, which is taking to be 1 (see appendix A). This corresponds to taking $e_{ia} = m_i^A = 0$ and the only non-vanishing fluxes are e_{i0} (from now on dubbed e_i). This has the following implications regarding our expansions forms:

$$d\alpha_0 = e_i \tilde{\omega}^i \tag{5.26}$$
$$d\alpha_a = d\beta^A = 0 \tag{5.27}$$
$$d\omega_i = e_i \beta^i \tag{5.28}$$
$$d\tilde{\omega}^i = 0 \tag{5.29}$$

And to check that these are indeed compatible with a half-flat manifold:

$$d\Omega^- = 0 \tag{5.30}$$
$$d\Omega^+ = e_i \tilde{\omega}^i \tag{5.31}$$
$$dJ = v^i e_i \beta^0 \tag{5.32}$$
$$d(J \wedge J) = 0 \tag{5.33}$$

Thus indeed this defines the structure of a Half-flat manifold, and we have found the correct conditions for a possible expansion basis to be suitable. However, the question is whether we can find such a basis. We have stated all kinds of conditions on the basis which are very restrictive, and it is not at all clear that such a basis indeed exists. It all kind of depends on whether we can continuously deform our original Calabi-Yau manifold to a Half-flat manifold such that we can indeed view it as a perturbation and in the limit of small fluxes the expansion set nicely coïncides with the original harmonic basis. For the rest of the thesis we will just assume that such a basis does exist and calculate the actions using this expansions basis.

Chapter 6

Half flat compactification

Having seen in the previous chapter that Half-flat manifolds might be suitable candidates and having found a possible appropriate expansion basis, we now turn to the actual computation of the effective theory resulting from such compactifications with the goal to show that indeed half-flat manifolds are mirrors to Calabi-Yau compactifications with electric NS fluxes. We will do this for both IIA and IIB to ensure that it holds for both cases. First we take a look at the common sector that consists of the metric and the kinetic term for the dilaton.

6.1 Decomposition of Ricci scalar

As in the Calabi-Yau case we will first look at the Ricci scalar part. We noted that for a general $SU(3)$ manifold we had: $\mathcal{R} = \mathcal{R}_{CY} + \mathcal{R}_T$. $\mathcal{R}_{CY}$ will decompose in the same way as in the Calabi-Yau case resulting in the usual kinetic terms for the moduli. R_T will result in new terms depending on the moduli. These can be obtained via a rather lengthy and tedious calculations involving a carefull analysis of the torsion classes, which we will not repeat here. We simply state the result as calculated in [4]. After applying the correct Weyl rotations etc. we end up with:

$$\int_{CY} -\frac{1}{2}\hat{R} * 1 - d\hat{\phi} \wedge *d\hat{\phi} = -\frac{1}{2} R * 1 - q_{ab} dz^a \wedge *d\bar{z}^b - g_{ij} dt^i \wedge *d\bar{t}^j \tag{6.1}$$

$$- d\phi \wedge *d\phi - \frac{-e^{2\phi}}{16\mathcal{K}} e_i e_j g^{ij} [(Im\mathcal{M})^{-1}]^{00} \tag{6.2}$$

We see that a potential emerges.

6.2 IIA

Due to the fact that our new expansion basis also includes forms which are not closed we get additional terms when expanding our fields:

$$
\begin{aligned}
d\hat{A}_1 &= dA_1 && (6.3)\\
d\hat{B}_2 &= dB_2 + db^i \wedge \omega_i + e_i b^i \beta^0 && (6.4)\\
d\hat{C}_3 &= dC_3 + d(A \wedge \omega_i) + d(\xi^A \wedge \alpha_A) + d(\tilde{\xi}_A \wedge \beta^A)\\
&= dC_3 + dA^i \wedge \omega_i - e_i dA^i \wedge \beta^0 + d\xi^A \wedge \alpha_A + \xi^0 e_i \tilde{\omega}^i + d\tilde{\xi}_A \wedge \beta^A\\
&= dC_3 + dA^i \wedge \omega_i + (d\tilde{\xi}_0 - e_i dA^i) \wedge \beta^0 + d\xi^A \wedge \alpha_A + \xi^0 e_i \tilde{\omega}^i + d\tilde{\xi}_a \wedge \beta^a && (6.5)
\end{aligned}
$$

As noted before we can only generate $h^{1,1}$ fluxes via the metric, which means we are still one short. However, this additional flux parameter, e_0, can be added via H_3. To be sure it pairs up nicely with the other fluxes we take it to be in the β^0 direction, i.e:

$$H_3 \to H_3 + e_0 \beta^0 \tag{6.6}$$

Thus we get the following decompositions of the fieldstrengths:

$$
\begin{aligned}
F_2 &= dA_1 && (6.7)\\
H_3 &= dB_2 + db^i \wedge \omega_i + (e_i b^i + e_0)\beta^0 && (6.8)\\
F_4 &= (dC_3 - A^0 \wedge dB_2) + (dA^i - A^0 \wedge db^i) \wedge \omega_i\\
&\quad + D\xi^A \wedge \alpha_A + D\tilde{\xi}_A \beta^A + \xi^0 e_i \tilde{\omega}^i && (6.9)
\end{aligned}
$$

With $D\tilde{\xi}_0 - e_i(A^i + b^i A^0) - e_0 A^0$, $D\xi^A = d\xi^A$ and $D\tilde{\xi}_a = d\tilde{\xi}_a$. So we see that some of the hyperscalars get gauged.

Let us also note that since we are introducing a flux via H_3 we must use the alternate form for the topological term, and have to keep in mind the redefinition of the vectorfields we have to do in order to end up with an action in the standard form as in the previous sections. Now let's proceed with the actual integration:

$$
\begin{aligned}
-\frac{1}{4}\int \hat{H}_3 \wedge *\hat{H}_3 &= -\frac{1}{4}\mathcal{K} dB_2 \wedge *dB_2 - \mathcal{K} g_{ij} db^i \wedge *db^j \\
&- (e_i b^i)^2 (Im\mathcal{M}^{-1})^{00} \\
-\frac{1}{2}\int F_4 \wedge *F_4 &= -\frac{\mathcal{K}}{2}(dC_3 - A^0 \wedge dB_2) \wedge *(dC_3 - A^0 \wedge dB_2) \qquad (6.10) \\
&- 2\mathcal{K} g_{ij}(dA^i - A^0 db^i) \wedge *(dA^i - A^0 db^i) \qquad (6.11) \\
&+ \frac{1}{2}(Im\mathcal{M}^{-1})^{AB}(D\tilde{\xi}_A + \mathcal{M}_{AC} D\xi^C) \wedge *(D\tilde{\xi}_B + \bar{\mathcal{M}}_{BD} D\xi^D) \\
&- \frac{(\xi^0)^2}{8\mathcal{K}} g^{ij} e_i e_j \\
\frac{1}{2}\int H_3 \wedge C_3 \wedge dC_3 &= \frac{1}{2}\mathcal{K}_{ijk} db^i \wedge A^j \wedge dA^k + \xi^0 e_i dB_2 \wedge A^i + \xi^0 e_i db^i \wedge dC_3 \\
&+ dB_2 \wedge (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A d\xi^A) + \xi^0 e_i db^i \wedge dC_3 \\
&- (e_i b^i + e_0)\xi^0 dC_3 - (e_i b^i + e_0) C_3 \wedge d\xi^0 \qquad (6.12)
\end{aligned}
$$

Dualization

Just as in the ordinary case we would like to dualize C_3. Again our 3-form is dual to a constant, which can be viewed as an RR flux and we will put to zero. However due to new interactions of C_3 with the fluxparameters the dual action does not vanish. Collecting all the terms involving C_3:

$$
\begin{aligned}
\mathcal{S}_{C_3} &= \int -\frac{\mathcal{K}}{2}(dC_3 - A^0 \wedge dB_2) \wedge *(dC_3 - A^0 \wedge dB_2) - (e_i b^i + e_0)\xi^0 dC_3 \\
&\quad - (e_i b^i + e_0) C_3 \wedge d\xi^0 \\
&= \int -\frac{\mathcal{K}}{2}(dC_3 - A^0 \wedge dB_2) \wedge *(dC_3 - A^0 \wedge dB_2) - \xi^0 (e_i b^i + e_0) dC_3 \qquad (6.13)
\end{aligned}
$$

Where in the second equality we have added a total derivative $d(C_3 \wedge \xi^0(e_i b^i + e_0))$. Performing the dualization we get:

$$
S = \int -\frac{(\xi^0)^2}{2\mathcal{K}}(e_i b^i + e_0)^2 - \xi^0(e_i b^i + e_0) A^0 \wedge dB_2 \qquad (6.14)
$$

Now we turn to the dualization of B_2. Collecting all the terms:

$$
S_{B_2} = \int -\frac{1}{4\mathcal{K}} dB_2 \wedge *dB_2 - \frac{1}{2} dB_2 \wedge (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A d\xi^A - \xi^0(e_i b^i + e_0) A^0 + \xi^0 e_i \wedge A^i) \qquad (6.15)
$$

And after dualizing we end up with:

$$
S_a = \int -\frac{1}{2\mathcal{K}}(Da - \xi^A D\tilde{\xi}_A + \tilde{\xi}_A d\xi^A) \wedge *(Da - \xi^A D\tilde{\xi}_A + \tilde{\xi}_A d\xi^A) \qquad (6.16)
$$

Where $Da = da - \xi^0(e_i b^i + e_0) A^0 + \xi^0 e_i \wedge A^i$.

6.2.1 Mirror symmetry

Collecting all the terms and performing the right rescalings and redefinitions we get:

$$
\begin{aligned}
S_{IIA} = \int &(-\frac{1}{2}\mathcal{R} * 1 - g_{ij} dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi - g_{ab} dz^a \wedge *d\bar{z}^b \\
&- \frac{e^{4\phi}}{4}(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \wedge *(Da - (\xi^A D\tilde{\xi}_A - \tilde{\xi}_A D\xi^A)) \\
&+ \frac{e^{2\phi}}{2}(Im\mathcal{M}^{-1})^{AB}(D\tilde{\xi}_A + \mathcal{M}_{AC} D\xi^C) \wedge *(D\tilde{\xi}_B + \mathcal{M}_{BD} D\xi^D) \\
&+ \frac{1}{2} Im\mathcal{N}_{IJ} F^I \wedge *F^J + \frac{1}{2} Re\mathcal{N}_{IJ} F^I \wedge F^J \\
&- \frac{1}{4\mathcal{K}} e^{2\phi}(Im\mathcal{M}^{-1})^{00} e_I (Im\mathcal{N}^{-1})^{IJ} e_J + \frac{1}{2\mathcal{K}} e^{4\phi} e_I (Im\mathcal{N}^{-1})^{IJ} e_J
\end{aligned} \tag{6.17}
$$

With $Da = da - \xi^0 e_I A^I$, $D\tilde{\xi}_0 - e_I A^I$, $D\xi^A = d\xi^A$ and $D\tilde{\xi}_a = d\tilde{\xi}_a$. Here we have

Recalling all the relations between compactifying on Y and on it's mirror $\tilde{Y}$ we see that, apart from the potential term, the actions are indeed the same for IIA on $\tilde{Y}_{hf}$ and IIB on Y if one identifies the NS fluxes directly (i.e. $\tilde{e}_A = e_A$). To see that the potential terms will also coïncide requires us to remember that $(Im\tilde{\mathcal{M}}^{-1})^{00} = (Im\mathcal{N}^{-1})^{00} = -\frac{1}{\mathcal{K}}$.

Thus we conclude that indeed:

$$
S_{IIA}(\tilde{Y}_{hf}, e_0) = S_{IIB}(Y, e_I) \tag{6.18}
$$

6.3 IIB

The field decompositions in the IIB case are given by

$$
d\hat{l} = dl \tag{6.19}
$$

$$
d\hat{B}_2 = dB_2 + db^i \wedge \omega_i + e_i b^i \beta^0 \tag{6.20}
$$

$$
d\hat{C}_2 = dC_2 + dc^i \wedge \omega_i + e_i c^i \beta^0 \tag{6.21}
$$

$$
\begin{aligned}
d\hat{A}_4 = &dD_2^i \wedge \omega_i + e_i D_2^i \wedge \beta^0 + F^A \wedge \alpha_A - G_A \wedge \beta^A \\
&+ (d\rho_i - e_i V^0) \wedge \tilde{\omega}^i
\end{aligned} \tag{6.22}
$$

We introduce the additional NS flux via H_3:

$$
H_3 \to H_3 + e_0 \beta^0 \tag{6.23}
$$

And we can decompose the fieldstrengths:

$$
\hat{H}_3 = dB_2 + db^i \wedge \omega_i + (e_i b^i + e_0)\beta^0 \tag{6.24}
$$

$$
\hat{F}_3 = (dC_2 - ldB_2) + (dc^i - ldb^i)\omega_i + e_i(c^i - lb^i)\beta^0 - le_0\beta^0 \tag{6.25}
$$

$$
\begin{aligned}
\hat{F}_5 = &(dD_2^i - db^i \wedge C_2 - c^i dB_2) \wedge \omega_i + (D\rho_i - \mathcal{K}_{ijk} c^j db^k) \wedge \tilde{\omega}^i \\
&+ F^A \alpha_A - \tilde{G}_A \beta^A
\end{aligned} \tag{6.26}
$$

Where, for brevity, we defined: $D\rho_i = d\rho_i - e_i V^0$, $\tilde{G}_0 = G_0 - e_i(D_2^i - b^i C_2) + e_0 C_2$ and $\tilde{G}_A = G_a$.

Performing the integrations:

$$
\begin{aligned}
-\frac{1}{4}\int H_3 \wedge *H_3 &= -\frac{1}{4}\mathcal{K} dB_2 \wedge *dB_2 - \mathcal{K} g_{ij} db^i \wedge *db^j \\
&\quad + \frac{1}{4}(e_i b^i + e_0)^2 (Im\mathcal{M}^{-1})^{00} \qquad (6.27)\\
-\frac{1}{2}\int F_3 \wedge *F_3 &= -\frac{\mathcal{K}}{2}(dC_2 - l dB_2) \wedge *(dC_2 - l dB_2) \\
&\quad - 2\mathcal{K} g_{ij}(dc^i - l db^i) \wedge *(dc^j - l db^j) \\
&\quad + \frac{1}{2}(e_i(c^i - l db^i) - l e_0)^2 (Im\mathcal{M}^{-1})^{00} \qquad (6.28)\\
-\frac{1}{4}\int \hat{F}_4 \wedge *\hat{F}_4 &= \frac{1}{4}(Im\mathcal{M}^{-1})^{AB}(\tilde{G}_A - \mathcal{M}_{AC}F^C) \wedge *(\tilde{G}_B - \bar{\mathcal{M}}_{BD}F^D) \\
&\quad - \mathcal{K} g_{ij}(dD_2^i - db^i \wedge C_2 - c^i dB_2) \wedge *(dD_2^j - db^j \wedge C_2 - c^j dB_2) \\
&\quad \frac{1}{16\mathcal{K}} g^{ij}(D\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(D\rho_j - \mathcal{K}_{imn}c^m db^n) \qquad (6.29)\\
-\frac{1}{2}\int \hat{A}_4 \wedge \hat{H}_3 \wedge d\hat{C}_2 &= -\frac{1}{2}\mathcal{K}_{ijk} D_2^i \wedge db^j \wedge dc^k - \frac{1}{2}\rho_i(dB_2 \wedge dc^i + db^i \wedge dC_2) \\
&\quad + \frac{1}{2} e_i V^0 \wedge (c^i dB_2 - b^i dC_2) - \frac{1}{2} e_0 V^0 \wedge dC_2 \qquad (6.30)
\end{aligned}
$$

Selfduality

The self-duality condition in terms of our four dimensional fields can be calculated to be:

$$
dD_2^i - db^i \wedge C_2 - c^i dB_2 = \frac{1}{4\mathcal{K}} g^{ij} * (D\rho_i - \mathcal{K}_{ijk} c^j db^k) \qquad (6.31)
$$

$$
\tilde{G}_A = Re\mathcal{M}_{AC}F^C + Im\mathcal{M}_{AC} * F^C \qquad (6.32)
$$

Again we add the term $\frac{1}{2} dD_2^i \wedge d\rho_i + \frac{1}{2} F^A \wedge G_A$ to the action so that we can eliminate half of the fields via the equations of motions. The resulting action is given by:

$$
\begin{aligned}
S_{\rho_i, F^A} &= \frac{1}{2} Re\mathcal{M}_{AB} F^A \wedge F^B + \frac{1}{2} Im\mathcal{M}_{AB} F^A \wedge *F^B \\
&\quad - \frac{1}{8\mathcal{K}} g^{ij}(D\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(D\rho_j - \mathcal{K}_{jmn}c^m db^n) \\
&\quad - D\rho_i \wedge (c^i dB_2 + db^i \wedge C_2) - \frac{1}{2}\mathcal{K}_{ijk} c^i c^j dB_2 \wedge db^k \qquad (6.33)
\end{aligned}
$$

Dualization

We will now dualize C_2 and B_2 to the scalars h_1 and h_2. First collecting all the terms with C_2:

$$
\begin{aligned}
S_{C_2} &= \int \frac{1}{2}\mathcal{K}(dC_2 - l dB_2) \wedge *(dC_2 - l dB_2) \\
&\quad + \frac{1}{2} dC_2 \wedge (-b^i D\rho_i + e_0) V^0) \qquad (6.34)
\end{aligned}
$$

After dualization we get:

$$S_{h_1} = \int -\frac{1}{2\mathcal{K}}(h_1 - b^i D\rho_i + e_0)V^0) \wedge *(h_1 - b^i D\rho_i + e_0)V^0)$$
$$+ ldB_2 \wedge (h_1 - b^i D\rho_i + e_0)V^0) \tag{6.35}$$

Collecting the terms involving B_2:

$$S_{B_2} = \int -\frac{1}{4}\mathcal{K}dB_2 \wedge *dB_2 + dB_3 \wedge (\frac{1}{2}\mathcal{K}_{ijk}c^i c^j db^k + ldh_1 + (c^i - lb^i)D\rho_i + le_0 V^0) \tag{6.36}$$

And we find for the dual action:

$$S_{h_2} = \int -\frac{1}{\mathcal{K}} D\tilde{h} \wedge *D\tilde{h} \tag{6.37}$$

With $D\tilde{h} = \frac{1}{2}dh_2 + ldh_1 + (c^i - lb^i)D\rho_i + le_0 V^0 - \frac{1}{2}\mathcal{K}_{ijk}c^i c^j db^k$.

Collecting the terms

Collecting all the terms, we end up with:

$$\begin{aligned} S_{IIB} = \int & -\frac{1}{2}R * 1 - q_{ab}dz^a \wedge *d\bar{z}^b - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi \\ & - \frac{e^{2\phi}}{8\mathcal{K}} g^{ij}(D\rho_i - \mathcal{K}_{ikl}c^k db^l) \wedge *(D\rho_j - \mathcal{K}_{jmn}c^m db^n) \\ & - 2\mathcal{K}e^{2\phi} g_{ij}(dc^i - ldb^i) \wedge *(dc^j - ldb^j) - \frac{1}{2}\mathcal{K}e^{2\phi} dl \wedge *dl \\ & - \frac{1}{2}\mathcal{K}e^{2\phi}(dh_1 - b^i D\rho_i - e_0 V^0) \wedge *(dh_1 - b^j D\rho_j - e_0 V^0) \\ & - e^{4\phi} D\tilde{h} \wedge *D\tilde{h} \\ & + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B + \frac{1}{2}Re\mathcal{M}_{AB}F^A \wedge F^B \\ & + \frac{1}{4}e^{2\phi}(Im\mathcal{M}^{-1})^{00} e_I (Im\mathcal{N}^{-1})^{IJ} e_J - \frac{1}{2}e^{4\phi}(Im\mathcal{M}^{-1})^{00}(e_I \xi^I)^2 \end{aligned} \tag{6.38}$$

6.3.1 Mirror Symmetry

After the familiar mirror mapping we see that our action turns into:

$$\begin{aligned} S_{IIB} = \int & -\frac{1}{2}R * 1 - g_{ij}dt^i \wedge *d\bar{t}^j - d\phi \wedge *d\phi - g_{ab}dz^a \wedge *d\bar{z}^b \\ & - \frac{e^{4\phi}}{4}(Da - (\xi^I D\tilde{\xi}_I - \tilde{\xi}_I D\xi^I)) \wedge *(Da - (\xi^I D\tilde{\xi}_I - \tilde{\xi}_I D\xi^I)) \\ & + \frac{e^{2\phi}}{2}(Im\mathcal{N}^{-1})^{IJ}(D\tilde{\xi}_I + \mathcal{N}_{IK}D\xi^K) \wedge *(D\tilde{\xi}_J + \mathcal{N}_{JL}D\xi^L) \\ & + \frac{1}{2}Im\mathcal{M}_{AB}F^A \wedge *F^B + \frac{1}{2}Re\mathcal{M}_{AB}F^A \wedge F^B \\ & + \frac{1}{4}e^{2\phi}(Im\mathcal{M}^{-1})^{00} e_I (Im\mathcal{N}^{-1})^{IJ} e_J - \frac{1}{2}e^{4\phi}(Im\mathcal{M}^{-1})^{00}(e_I \xi^I)^2 \end{aligned} \tag{6.39}$$

With: $Da = da + e_I \xi^I V^0$, $D\tilde{\xi}_I = d\tilde{\xi}_I - e_I V^0$ and $D\xi^I = d\xi^I$.

One can now quickly observe that if one compactifies IIB on the 'mirror' half-flat manifold that indeed we recover the IIA action on Y, by the same reasoning as done before. I.e:

$$S_{IIB}(\tilde{Y}_{hf}, e_0) = S_{IIA}(Y, e_I) \tag{6.40}$$

We have thus shown that half-flat manifolds are the correct mirror candidates when one turns on NS fluxes.

Chapter 7

Conclusions

In this thesis we examined the manifestation of mirror symmetry in Calabi-Yau compactifications of type IIA and IIB supergravities with and without fluxes. We started by giving a short review of the type IIA/B supergravities, compactication in general and Calabi-Yau manifolds and the idea of mirror symmetry and the mirror conjecture in chapter 2. Here we noted that by demanding that a minimal amount of supersymmetry should be preserved when compactifying one concludes that the compactification manifold has to have $SU(3)$-structure. By also demanding the vacuum to be supersymmetric we can further reduce this to torsionless $SU(3)$ manifolds, i.e. Calabi-Yau manifolds. We then noted that a consistent basis of internal forms in which to expand the ten dimensional field content such that the relevant low energy physics is captured, is the collection of harmonic forms. Also we examined the moduli space $\mathcal{M}$ of a Calabi-Yau manifold, it's splitting in a Kähler ($\mathcal{M}_k$) and a complex structure ($\mathcal{M}_c$) part and it's significance regarding the emergence of geometrical moduli. We concluded the chapter with the mirror conjecture, which states that for any Calabi-Yau manifold Y there is another Calabi-Yau manifold $\tilde{Y}$, such that IIA string theory compactified on Y results in the same effective action as IIB compactified on $\tilde{Y}$ and vice-versa. In order examine the implications of the mirror conjecture in the low energy limit of the supergravities we had to work in an appropriate limit, namely the large volume and the large complex structure limit. We concluded that for Y and $\tilde{Y}$ to be mirror partners their Kähler and complex structure moduli spaces need to be interchanged and in effect also their odd and even cohomology groups.

In chapter 3 we first noted that in the fluxless case the four dimensional field content was compatible with mirror symmetry and we subsequently calculated the effective actions. The IIA action was seen to be that of a standard four dimensional $N = 2$ supergravity theory. The IIB action however, was not immediatly seen to be of such a form. However, by redefining the fields we were able to put it in the standard form and by making use of the relations between mirror manifolds we quickly established mirror symmetry and effectively found an explicit map between the fields in the IIA and IIB case.

We then turned our attention to the more interesting case of flux compactications in chapter 4. We first turned on fluxes in the RR sector and by explicitely calculating the effective actions we swiftly saw that the theories are still mirror symmetric. Due to the RR fluxes a potential for some of the scalars is introduced and the NS two-form was seen to become massive. When we turned on NS fluxes we quickly noted, by simply counting the amount of fluxes, that the theories are not mirror symmetric, as in both cases the only NS fieldstrength able to generate fluxes is H_3 and remembering that the hodge numbers get interchanged under mirror symmetry. We still continued to calculate the effective actions for further discussions and found very different dynamics in the IIA and IIB case. At the IIA side one sees that some of the hyperscalars get gauged with respect to the graviphoton (the vector in the gravity multiplet) and also a potential is introduced for some of the hyperscalars. In the IIB theory the hyperscalars also get gauged, but this time with respect to a lineair combination of the vectors in the vectormultiplets, a potential gets introduced and C_2 becomes massive.

We then examined to the possibility of generating additional NS fluxes via the metric to restore the lack of mirror symmetry. We saw that this means that we have to compactify on manifolds with torsion, and thus have to part from Calabi-Yau compactifications. By referring to the literature we focused on the electric NS flux case only. Our goal in chapter 4 was thus to find a non Calabi-Yau manifold $\tilde{Z}$ that would be the mirror of a Calabi-Yau compactification with electric NS fluxes. We could swiftly say that in order to stay within the standard $N = 2$ supersymmetry framework $\tilde{Z}$ has to be an $SU(3)$ structure manifold. The fact that we treat the fluxes perturbatively and by making use of mirror symmetry, we could deduce that the candidate must be a perturbation of $\tilde{Y}$ with a small torsion component. By analyzing the different torsion components and by matching them up with the corresponding cohomology groups of Y, we were lead to the conclusion that so called half-flat manifolds might be suitable candidates. These have the property that 'half' of Ω fails to be closed. We were then faced with the problem that a standard truncation to the massless, i.e. the harmonic, forms was no longer possible as these do not capture the structure of the half-flat manifold. However we were able to find restrictions on such a basis and found that these forms are necessarily not all closed and should satisfy the relations of the harmonic basis of Y (metrics, intersection numbers, etc.). In chapter 5 we calculated the effective actions of IIA and IIB compactified on a half-flat manifold and by comparing with electric NS flux compactifications we concluded that indeed, half-flat compactifications are mirrors to electric NS flux compactifications.

However, let us make a couple of remarks. The considerations in chapter 4 were far from mathematically solid. They were merely physical considerations as to what conditions appropriate manifolds must satisfy to generate the right physics. However the author has yet to come across a thorough treatment of the existence of half-flat manifolds as perturbations of Calabi-Yau manifolds, let alone the existence of an expansion basis meeting the requirements. It might be possible that in order to treat such questions one must turn to generalized geometry. It is shown (see also [8]) that one can neatly describe $SU(3)$ structure manifolds in terms of invariant spinors, Φ_J and Φ_Ω, which are formal sums of different forms. One can then extend the idea of moduli spaces from the Calabi-Yau case to the set of $SU(3)$ manifolds. The moduli space would then correspond to the space of deformations of the two defining spinors. Perhaps one can show that indeed by considering such deformations one can continuously deform a Calabi-Yau to a half-flat manifold. This naturally leads to the question whether some more general mirror conjecture can be made involving fluxes and $SU(3)$ manifolds. A first exploration of such considerations is also given in [8]. A more general mirror conjecture would then be that for any $SU(3)$-structure manifold Y there is a mirror $SU(3)$ manifold $\tilde{Y}$ such that if we compactify IIA (with fluxes) on Y and

IIB (also with fluxes) on $\tilde{Y}$ the effective actions will be the same. The existence of a half-flat manifold mirror to electric NS flux Calabi-Yau compactification could then be included in this statement. All these considerations are made in the supergravity limit however and one must ofcourse go back to the original notion of mirror symmetry in terms of the full string theories to see whether a more generalized statement is meaningfull.

Another interesting question not treated in this thesis is of course how everything works out if we also allow non-zero magnetic NS fluxes. One can find in the literature (see [8]) that in order to produce the correct terms proportional to magnetic NS fluxes one has to consider manifolds with $SU(3) \times SU(3)$ structure as possible mirror candidates. These manifolds admit *two* globally invariant spinors $\eta_{1,2}$, and with these one can construct two sets of invariant forms $J_{1,2}$ and $\Omega_{1,2}$. In order to be able to describe such compactifications one has to use generalized geometry. Sadly this was beyond the scope of this thesis, but the interested reader is referred to [9].

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Appendix A

Moduli spaces

Let's be a bit more precise about the structure of the Calabi-Yau moduli spaces, and introduce some quantities that we will frequently encounter in during the compactification process. We will be working in the appropriate limits, i.e. the large volume limit and the large complex structure limits. Finally we will say a little bit about mirror symmetry. Let's start off with the complexified Kähler moduli space. See [5].

A.1 Complexified Kähler moduli space

The complexified Kähler moduli space is parametrized by the coëfficients of the harmonic expansion of the combine two-form:

$$B_2 + iJ = t^i\omega_i \tag{A.1}$$

Now, it is known that the moduli space itself are very special manifolds and come with many nice properties. We will not go into the details but simply give a very short summary of the different properties, relevant to the compactification.

The Kähler moduli space is in fact in itself a special Kähler manifold, and therefore has a metric, g_{ij}, which can be expressed in terms of a potential K_K, which is turn is determined by a pre-potential $\mathcal{F}$:

$$g_{ij} = \partial_i\partial_j K_k \tag{A.2}$$

$$e^{-K_k} = i(\bar{T}^I\mathcal{F}_I - T^I\bar{\mathcal{F}}_I) \tag{A.3}$$

Where $\mathcal{F}_I = \partial_I\mathcal{F}$ and $T^I = (1, t^i)$.

Let us first introduce some convenient objects we will encounter many times:

$$\mathcal{K}_{ijk} = \int \omega_i \wedge \omega_j \wedge \omega_k \tag{A.4}$$

$$\mathcal{K}_{ij} = \mathcal{K}_{ijk}v^k \tag{A.5}$$

$$\mathcal{K}_i = \mathcal{K}_{ijk}v^j v^k \tag{A.6}$$

$$\mathcal{K} = \mathcal{K}_{ijk}v^i v^j v^k \tag{A.7}$$

In terms of these objects we can find explicit expressions for the metric g_{ij} and it's inverse g^{ij}:

$$g_{ij} = \frac{1}{4\mathcal{K}} \int \omega_i \wedge *\omega_j = -\frac{1}{4}(\frac{\mathcal{K}_{ij}}{\mathcal{K}} - \frac{K_i \mathcal{K}_j}{4\mathcal{K}^2}) \tag{A.8}$$

$$g^{ij} = 4\mathcal{K} \int \tilde{\omega}^i \wedge *\tilde{\omega}^j = -4\mathcal{K}(\mathcal{K}^{ij} - \frac{v^i v^j}{\mathcal{K}}) \tag{A.9}$$

Where $\mathcal{K}^{ij}$ is defined by $\mathcal{K}^{ij}\mathcal{K}_{ik} = \delta^j_k$.

The complexified Kähler moduli space completely determines the different couplings in the vector multiplet sector via the following gauge coupling matrix $\mathcal{N}$ defined by:

$$Re\mathcal{N}_{00} = -\frac{1}{3}\mathcal{K}_{ijk} b^i b^j b^k \tag{A.10}$$

$$Re\mathcal{N}_{i0} = \frac{1}{2}\mathcal{K}_{ijk} b^j b^k \tag{A.11}$$

$$Re\mathcal{N}_{ij} = -\mathcal{K}_{ijk} b^k \tag{A.12}$$

$$Im\mathcal{N}_{00} = -\mathcal{K} + (\mathcal{K}_{ij} - \frac{1}{4}\frac{\mathcal{K}_i \mathcal{K}_j}{\mathcal{K}}) b^i b^i \tag{A.13}$$

$$Im\mathcal{N}_{i0} = -(\mathcal{K}_{ij} - \frac{1}{4}\frac{\mathcal{K}_i \mathcal{K}_j}{\mathcal{K}}) b^i \tag{A.14}$$

$$Im\mathcal{N}_{ij} = (\mathcal{K}_{ij} - \frac{1}{4}\frac{\mathcal{K}_i \mathcal{K}_j}{\mathcal{K}}) \tag{A.15}$$

This matrix can be determined via the Kähler potential and the vector moduli in the following way:

$$\mathcal{N}_{IJ} = \bar{\mathcal{F}}_{IJ} + 2i \frac{(Im\mathcal{F})_{IK} T^K (Im\mathcal{F})_{JL} T^L}{T^K (Im\mathcal{F})_{KL} T^L} \tag{A.16}$$

We will also encounter the inverse of the imaginary part of $\mathcal{N}$:

$$(Im\mathcal{N}^{-1})^{00} = 1 \tag{A.17}$$

$$(Im\mathcal{N}^{-1})^{i0} = b^i \tag{A.18}$$

$$(Im\mathcal{N}^{-1})^{ij} = \frac{g^{ij}}{4} + b^i b^j \tag{A.19}$$

A.2 Complex structure moduli space

Just as the complexified Kähler moduli space, the complex structure moduli space comes with metric q_{ab} determined by a Kähler potential K_c, which in turn is determinede by a pre-potential $\mathcal{G}$:

$$q_{ab} = \partial_a \partial_b K_c \tag{A.20}$$

$$e^{-K_c} = i(\bar{z}^A \mathcal{G}_A - z^A \bar{G}_A) \tag{A.21}$$

Where $\mathcal{G}_A = \mathcal{G}$

As the Kähler moduli space determines the couplings in the vector multiplet sector, so does the complex structure moduli space in the hypermultiplet sector. This via gauge coupling matrix $\mathcal{M}$ defined by:

$$A = \int (Re\mathcal{M})(Im\mathcal{M})^{-1} \tag{A.22}$$

$$B = -(Im\mathcal{M}) - (Re\mathcal{M})(Im\mathcal{M})^{-1}(Re\mathcal{M}) \tag{A.23}$$

$$C = (Im\mathcal{M})^{-1} \tag{A.24}$$

Where A, B and C are the matrices defined by the different integrals over the basis (α_A, β^A) given in chapter 2.

The matrix can be determined via the complex structure pre-potential $\mathcal{G}$ and the complex structure moduli via:

$$\mathcal{M}_{AB} = \bar{\mathcal{G}}_{AB} + 2i\frac{(Im\mathcal{G})_{AC}Z^C(Im\mathcal{G})_{BD}Z^D}{Z^C(Im\mathcal{G})_{CD}Z^D} \tag{A.25}$$

Where $Z^A = (1, z^a)$

A.3 Mirror symmetry

Consider a Calabi-Yau manifold Y and it's mirror $\tilde{Y}$. We noted that this meant the following:

$$\mathcal{M}_k = \tilde{\mathcal{M}_c} \tag{A.26}$$

$$\mathcal{M}_c = \tilde{\mathcal{M}_k} \tag{A.27}$$

Which in turns implies

$$h^{1,1} = \tilde{h}^{1,2} \tag{A.28}$$

$$h^{1,2} = \tilde{h}^{1,1} \tag{A.29}$$

So if there are moduli in the complexified Kähler moduli space for Y, there , and since the moduli spaces we note:

$$t^i = \tilde{z}^i \tag{A.30}$$

$$z^a = \tilde{t}^a \tag{A.31}$$

Also by identifying the moduli spaces we can make similar identifications for the pre-potentials:

$$\mathcal{F} = \tilde{\mathcal{G}} \tag{A.32}$$

$$\mathcal{G} = \tilde{\mathcal{F}} \tag{A.33}$$

And since the Kähler and complex structure metrics and the coupling matrices are determined by the prepotentials and moduli in exactly the same way only with the roles of the pre-potentials reversed as well as the roles of the moduli, we see:

$$g_{ij} = \tilde{q}_{ij} \tag{A.34}$$

$$q_{ab} = \tilde{g}_{ab} \tag{A.35}$$

$$\mathcal{M} = \tilde{\mathcal{N}} \tag{A.36}$$

$$\mathcal{N} = \tilde{\mathcal{M}} \tag{A.37}$$

Appendix B

IIA

In this appendix we will show the full calculations of the integration of our different terms over our Calabi-Yau manifold in the IIA case. This to show how different etc. The full calculations for the IIB case and the flux compactifications and also the half-flat compactifications are not given since they proceed in a completely analagous fashion.

We first note that we will always end up with some integrals involving combinations of harmonic forms over our Calabi-Yau manifold. The key point in these calculations is that most of these terms will vanish and do not contribute to the effective action. In order to be able to integrate over our CY we must integrate over a 6-form, so for example integrals over combinations as $\omega_i \wedge \alpha_A$ do not contribute since they are 5-forms. The only possibly contributing combinations we expect to find are the ones given in chapter 2.

Remembering this, we can easily perform the integrations:

$$\int_{CY} H_3 \wedge *H_3 = \int_{CY} (dB_2 \wedge 1 + db^i \wedge \omega_i) \wedge *(dB_2 \wedge 1 + db^j \wedge \omega_j) \tag{B.1}$$

$$= \int_{CY} (dB_2 \wedge *dB_2) \wedge (1 \wedge *1) + (db^i \wedge *db^j) \wedge (\omega_i \wedge *\omega_j) \tag{B.2}$$

$$= \mathcal{K}(dB_2 \wedge *dB_2) + 4\mathcal{K} g_{ij} (db^i \wedge *db^j) \tag{B.3}$$

$$\begin{aligned} \int_{CY} F_2 \wedge *F_2 &= \int_{CY} (dA_1 \wedge 1) \wedge *(dA_1 \wedge 1) \\ &= \int_{CY} (dA_1 \wedge *dA_1) \wedge (1 \wedge *1) \\ &= (dA_1 \wedge *dA_1) \int_{CY} (1 \wedge *1) \\ &= \mathcal{K}(dA_1 \wedge *dA_1) \end{aligned} \tag{B.4}$$

$$
\begin{aligned}
\int_{CY}\tilde{F}_4\wedge *\tilde{F}_4 &= (dC_3 - B_2\wedge dA_1^0)\wedge *(dC_3 - B_2\wedge dA_1^0)\int_{CY}(1\wedge *1)\\
&+ (dA_1^i - b^i dA_1^0)\wedge *(dA_1^j - b^i dA_1^0)\int_{CY}(\omega_i\wedge *\omega_j)\\
&+ (d\xi^A\wedge *d\tilde{\xi}^B)\int_{CY}(\alpha_A\wedge *\beta^B) + (d\xi^A\wedge *d\xi^B)\int_{CY}(\alpha_A\wedge *\alpha_B)\\
&+ (d\tilde{\xi}_A\wedge *d\xi^B)\int_{CY}(\beta^A\wedge *\alpha_B) + (d\tilde{\xi}_A\wedge *d\tilde{\xi}_B)\int_{CY}(\beta^A\wedge *\beta^B)\\
&= \mathcal{K}(dC_3 - B_2\wedge dA_1^0)\wedge *(dC_3 - B_2\wedge dA_1^0)\\
&+ 4\mathcal{K}g_{ij}(dA_1^i - b^i dA_1^0)\wedge *(dA_1^j - b^i dA_1^0) - A_A^B(d\xi^A\wedge *d\tilde{\xi}^B)\\
&+ B_{AB}(d\xi^A\wedge *d\xi^B) + A_B^A(d\tilde{\xi}_A\wedge *d\xi^B) + C_{AB}(d\tilde{\xi}_A\wedge *d\tilde{\zeta}_B)\\
&= \mathcal{K}(dC_3 - B_2\wedge dA_1^0)\wedge *(dC_3 - B_2\wedge dA_1^0)\\
&+ 4Kg_{ij}(dA_1^i - b^i dA_1^0)\wedge *(dA_1^j - b^i dA_1^0) - A_A^B(d\xi^A\wedge *d\tilde{\xi}^B)\\
&+ \frac{1}{2}(Im\mathcal{M}^{-1})^{AB}(d\tilde{\xi}_A + \mathcal{M}_{AC}d\xi^C)\wedge *(d\tilde{\xi}_B + \bar{\mathcal{M}}_{BD}d\xi^D)
\end{aligned}
$$

$$
\begin{aligned}
\int_{CY}B_2\wedge dC_3\wedge dC_3 &= b^i dA_1^j\wedge dA_1^k\int_{CY}\omega_i\wedge\omega_j\wedge\omega_k + && \text{(B.5)}\\
&= \mathcal{K}_{ijk}b^i dA_1^j\wedge dA_1^k + B_2\wedge d(\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) && \text{(B.6)}\\
&= \mathcal{K}_{ijk}b^i dA_1^j\wedge dA_1^k + dB_2\wedge (\tilde{\xi}_A d\xi^A - \xi^A d\tilde{\xi}_A) && \text{(B.7)}
\end{aligned}
$$

$$
\begin{aligned}
\int_{CY}(B_2)^2\wedge dC_3\wedge dA_1 &= b^i b^j dA_1^k\wedge dA_1^0\int_{CY}\omega_i\wedge\omega_j\wedge\omega_k && \text{(B.8)}\\
&= \mathcal{K}_{ijk}b^i b^j dA_1^k\wedge dA_1^0 && \text{(B.9)}
\end{aligned}
$$

$$
\begin{aligned}
\int_{CY}(B_2)^3\wedge dA_1\wedge dA_1 &= b^i b^j b^k dA_1^0\wedge dA_1^0\int_{CY}\omega_i\wedge\omega_j\wedge\omega_k && \text{(B.10)}\\
&= \mathcal{K}_{ijk}b^i b^j b^k dA_1^0\wedge dA_1^0 && \text{(B.11)}
\end{aligned}
$$

Appendix C

Poincaré duality

For a general field theory one there are multiple ways in which one describe the physical degrees of freedom. For instance, an action involving a p-form in d dimensions can always be described in terms of it's Poincaré dual action involving a $d-p-2$-form. In the special case of $p = d-1$ one can describe it in terms of a constant. Applied to our situation we would get that a massless 2-form is dual to a scalar and a massless 3-form is dual to a constant. Let's examine the dualization process for these two cases ([3]):

C.1 Dualization of massless 2-form

Consider the massless two-form B_2 with field strength H_3. The most general action we can encounter is involving H_3 is:

$$S_{H_3} = -\int \frac{g}{4} H_3 \wedge *H_3 - \frac{1}{2} H_3 \wedge J_1 \tag{C.1}$$

Where g is some function of scalars and J_1 is a one-form depending on vectorfield and scalars. The first step to dualization is adding a Lagrange multiplier of the form $H_3 \wedge da$, where a is the scalar dual to H_3. We will then treat H_3 as an independent variable, that is: H_3 is no longer dB_2. Next we derive the equations of motions for our fields H_3 and a which turn out to be $dH_3 = 0$ (i.e. $H_3 = dB_2$, since we work in Minkowski space) for a, and $*H_3 = \frac{1}{g}(da + J_1)$. Inserting these back into the original action we get:

$$S_a = -\frac{1}{4g}(da + J_1) \wedge *(da + dJ_1) \tag{C.2}$$

Which is our dual action in terms of a.

C.2 Dualization of massless 3-form

Now we consider a massless three-form C_3. The most general action we will encounter is:

$$S_{C_3} = -\int \frac{g}{4}(dC_3 - J_4) \wedge *(dC_3 - J_4) + \frac{h}{2} dC_3 \tag{C.3}$$

Where g and h are arbitrary scalar functions and J_4 is a four-form depending on the 2-forms, 1-forms and scalars present in the theory. Again we add an appropriate lagrange multiplier to the action: $\frac{e_0}{2} dC_3$:

$$S_{C_3} = -\int \frac{g}{4}(dC_3 - J_4) \wedge *(dC_3 - J_4) + \frac{h + e_0}{2} dC_3 \tag{C.4}$$

The equations of motion for dC_3 imply:

$$\frac{g}{2} * (dC_3 - J_4) = -\frac{1}{2}(h + e_0) \tag{C.5}$$

Inserted in the original action, we end up with our action in terms of the dual e_0 constant:

$$S_{e_0} = -\int \frac{1}{4g}(h + e_0)^2 * 1 + \frac{1}{2}(h + e_0) J_4 \tag{C.6}$$

Appendix D

Weyl rescalings

D.1 Weyl rescalings

Under a Weyl rescaling with Ω we get the following transformations:

$$g_{\mu\nu} = \Omega^{-2}\tilde{g}_{\mu\nu} \tag{D.1}$$
$$g^{\mu\nu} = \Omega^{2}\tilde{g}^{\mu\nu} \tag{D.2}$$
$$\sqrt{-g} = \Omega^{d}\sqrt{-\tilde{g}} \tag{D.3}$$

Where d is the dimension of space-time.

D.2 Redefining the moduli

Due toe the redefinition of the moduli via $v^i = e^{-\frac{1}{2}\hat{\phi}}\tilde{v}^i$ we get the following transformations for the different objects present:

$$\mathcal{K}_{ijk} = \tilde{\mathcal{K}}_{ijk} \tag{D.4}$$
$$\mathcal{K}_{ij} = e^{-\frac{1}{2}\hat{\phi}}\tilde{\mathcal{K}}_{ij} \tag{D.5}$$
$$\mathcal{K}_{i} = e^{-\hat{\phi}}\tilde{\mathcal{K}}_{i} \tag{D.6}$$
$$\mathcal{K} = e^{-\frac{3}{2}\hat{\phi}}\tilde{\mathcal{K}} \tag{D.7}$$
$$g_{\alpha\bar{\beta}} = e^{-\frac{1}{2}\hat{\phi}}\tilde{g}_{\alpha\bar{\beta}} \tag{D.8}$$
$$\sqrt{g} = e^{-\frac{3}{2}\hat{\phi}}\sqrt{\tilde{g}} \tag{D.9}$$

Since the definition of the hodge dual contains metrics, the various kinetic terms also acquire factors of $e^{-\frac{1}{2}\hat{\phi}}$ depending on the forms at hand.

Appendix E

Mirror mapping

E.1 Finding the Mirror map

In this appendix we give the full calculations regarding the mirror map.

The equations,

$$d\xi^0 = \pm dl \tag{E.1}$$

$$d\xi^i - b^i d\xi^0 = \pm(dc^i - ldb^i) \tag{E.2}$$

are easily solved. We choose the + variant for the first equation, resulting in:

$$\xi^0 = l \tag{E.3}$$

Inserting this in the second, for which we then have to choose the negative variant:

$$\xi^i = (b^i dl + ldb^i) - c^i = lb^i - c^i \tag{E.4}$$

The next one already requires a bit more work:

$$d\rho_i - \mathcal{K}_{ikl}c^k db^l = \pm(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K) \tag{E.5}$$

Rewriting and taking the plus variant:

$$\begin{aligned} d\tilde{\xi}_i &= d\rho_i - \mathcal{K}_{ikl}c^k db^l - Re\mathcal{N}_{iK}d\xi^K \\ &= d\rho_i - \frac{1}{2}\mathcal{K}_{ikl}b^k b^l dl + \mathcal{K}_{ikl}b^l d(ld^k - c^k) \\ &= d\rho_i + \mathcal{K}_{ikl}(-c^k db^l - b^l dc^k + \frac{1}{2}b^k b^l dl + db^k b^l l) \\ &= d\rho_i + \mathcal{K}_{ikl}(\frac{1}{2}d(b^k b^l l) - d(c^k b^l)) \end{aligned} \tag{E.6}$$

Here we have used the expressions for $Re\mathcal{N}_{iK}$, the expressions for ξ^K and that $K_{ikl}db^k b^l = K_{ikl}b^k db^l$, due to the symmetry properties of $\mathcal{K}_{ijk}$. I.e. we get:

$$\tilde{\xi}_i = \rho_i + \mathcal{K}_{ikl}(\frac{1}{2}b^k b^l l - c^k b^l) \tag{E.7}$$

Next we consider:

$$d\tilde{\xi}_0 + Re\mathcal{N}_{0K}d\xi^K + b^i(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K) = +/-(dh_1 - b^i d\rho_i) \tag{E.8}$$

The plus version won't work, so we take the negative one:

$$\begin{aligned}
d\tilde{\xi}_0 &= -dh_1 + b^i d\rho_i - Re\mathcal{N}_{0K}d\xi^K - b^i(d\tilde{\xi}_i + Re\mathcal{N}_{iK}d\xi^K) \\
&= -dh_1 + b^i d\rho_i + \frac{1}{3}\mathcal{K}_{ikl}b^i b^k b^l d\xi^0 - \frac{1}{2}\mathcal{K}_{ikl}b^i b^l d\xi^k \\
&\quad - b^i(d\tilde{\xi}_i + \frac{1}{2}\mathcal{K}_{ikl}b^k b^l d\xi^0 - \mathcal{K}_{ikl}b^l d\xi^k) \\
&= -dh_1 + b^i d\rho_i + \frac{1}{3}\mathcal{K}_{ikl}b^i b^k b^l dl - \frac{1}{2}\mathcal{K}_{ikl}b^i b^l d(lb^k - c^k) \\
&\quad - b^i(d\rho_i + \frac{1}{2}\mathcal{K}_{ikl}d(b^k b^l l) - \mathcal{K}_{ikl}d(c^k b^l) - \frac{1}{2}\mathcal{K}_{ikl}b^k b^l dl - \mathcal{K}_{ikl}b^l d(lb^k - c^k)) \\
&= -dh_1 + \mathcal{K}_{ikl}(b^i db^l c^k + \frac{1}{2}b^i b^k dc^k) - \mathcal{K}_{ikl}(\frac{1}{2}db^i b^k b^l l + \frac{1}{6}b^i b^k b^l) \\
&= -dh_1 + \frac{1}{2}\mathcal{K}_{ikl}(d(b^i b^k c^l) - \frac{1}{3}d(b^i b^k b^l l))
\end{aligned} \tag{E.9}$$

I.e.

$$\tilde{\xi}_0 = -h_1 + \frac{1}{2}\mathcal{K}_{ikl}(b^i b^k c^l - \frac{1}{3}b^i b^k b^l l) \tag{E.10}$$

Considering the term proportional to $e^{4\phi}$:

$$da + \tilde{\xi}_I d\xi^I - \xi^I d\tilde{\xi}_I = +/-2(\frac{1}{2}dh_2 + ldh_1 + (c^i - lb^i)d\rho_i - \frac{1}{2}K_{ijk}c^i c^j db^k) \tag{E.11}$$

We will look at plus variant:

$$\begin{aligned}
da &= 2(\frac{1}{2}dh_2 + ldh_1 + (c^i - lb^i)d\rho_i - \frac{1}{2}K_{ijk}c^i c^j db^k) - \tilde{\xi}_I d\xi^I + \xi^I d\tilde{\xi}_I \\
&= 2(\frac{1}{2}dh_2 + ldh_1 + (c^i - lb^i)d\rho_i - \frac{1}{2}K_{ijk}c^i c^j db^k) + h_1 dl - \rho_i d(lb^i - c^i) \\
&\quad - ldh_1 + d\rho_i(lb^i - c^i) + \mathcal{K}_{ikl}c^i c^k db^l \\
&= dh_2 + d(lh_1) + d(\rho_i(c^i - lb^i))
\end{aligned} \tag{E.12}$$

I.e.

$$a = 2h_2 + lh_1 + \rho_i(c^i - lb^i) \tag{E.13}$$

Where we have used:

$$
\begin{aligned}
\tilde{\xi}_0 d\xi^0 &= (-h_1 + \frac{1}{2}\mathcal{K}_{ijk}b^i b^k c^l - \frac{1}{6}\mathcal{K}_{ijk}b^i b^k b^l l)dl \\
&= -h_1 dl + \mathcal{K}_{ijk}(\frac{1}{2}b^i b^k c^l - \frac{1}{6}b^i b^k b^l l)dl \qquad \text{(E.14)} \\
\tilde{\xi}_i d\xi^i &= (\rho_i + \frac{1}{2}\mathcal{K}_{ijk}b^k b^l l - \mathcal{K}_{ijk}c^k b^l)d(lb^i - c^i) \\
&= \rho_i d(lb^i - c^i) + \mathcal{K}_{ijk}(\frac{1}{2}b^k b^l b^i l - c^k b^l b^i)dl + \mathcal{K}_{ijk}(\frac{1}{2}b^k b^l l^2 - c^k b^l l)db^i + \mathcal{K}_{ijk}(-\frac{1}{2}b^k b^l l + c^k b^l)dc^i \qquad \text{(E.15)} \\
\xi^0 d\tilde{\xi}_0 &= l(-dh_1 + \mathcal{K}_{ijk}b^i c^k db^l + \frac{1}{2}\mathcal{K}_{ijk}b^i b^k dc^l - \frac{1}{6}\mathcal{K}_{ijk}b^i b^k b^l dl - \frac{1}{2}\mathcal{K}_{ijk}b^i b^k l db^l) \\
&= -l dh_1 + \mathcal{K}_{ijk}(b^i c^k l - \frac{1}{2}b^i b^k l^2)db^l + \frac{1}{2}\mathcal{K}_{ijk}b^i b^k l dc^l - \frac{1}{6}\mathcal{K}_{ijk}b^i b^k b^l l dl \qquad \text{(E.16)} \\
\xi^i d\tilde{\xi}_i &= (lb^i - c^i)(d\rho_i + \mathcal{K}_{ijk}\frac{1}{2}b^k b^l dl + \mathcal{K}_{ijk}b^k l db^l - \mathcal{K}_{ijk}b^l dc^k - \mathcal{K}_{ijk}c^k db^l) \qquad \text{(E.17)} \\
&= (lb^i - c^i)d\rho_i + \mathcal{K}_{ijk}(\frac{1}{2}b^i b^k b^l l - \frac{1}{2}b^k b^l c^i)dl + \mathcal{K}_{ijk}(b^i b^k l^2 - 2b^i c^k l + c^i c^k)db^l + \mathcal{K}_{ijk}(c^i b^l - b^i b^l l)dc^k \qquad \text{(E.18)}
\end{aligned}
$$

And we can see by comparing the terms with dl, dc^i and db^i that our final result is:

$$-\tilde{\xi}_I d\xi^I + \xi^I d\tilde{\xi}_I = h_1 dl - \rho_i d(lb^i - c^i) - l dh_1 + d\rho_i(lb^i - c^i) + \mathcal{K}_{ikl}c^i c^k db^l \qquad \text{(E.19)}$$

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