Enhancement cavity for light shift measurements

by

M.L. Reitsma

A bachelor project thesis

Supervisors:
Prof. dr. K.H.K.J. Jungmann
Dr. L. Willmann

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RIJKSUNIVERSITEIT GRONINGEN

Abstract

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To measure light shifts for a planned parity violation experiment on radium ions, a standing wave of high light intensity is required. An optical enhancement cavity is employed for this purpose, which needs to be stable to fractions of a wavelength. This is accomplished by locking laser light to the cavity using the Pound-Drever-Hall frequency locking technique. This has resulted in a cavity that is stable to approximately $10^{-10}$ m. Improvements to this setup can be done by stabilizing the temperature and shielding from vibrations as well as acquiring more massive mirror mounts.
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Chapter 1

Introduction

This thesis is a report on the enhancement of laser light in an optical cavity to create a high intensity standing wave of light. For this the Pound-Drever-Hall technique is employed to lock the cavity length to the laser frequency. This cavity enhancement is an important step towards planned experiments to measure atomic parity violation on single radium ions (Ra\(^+\)). This research is conducted by the radium ion subgroup of the Fundamental Interactions and Symmetries (previously TRImP) group in the Van Swinderen Institute of FWN at the University of Groningen.

1.1 Radium ion experiment

1.1.1 Atomic parity violation

Some transitions in atoms are forbidden by the electromagnetic interaction. Nevertheless the electroweak mixing of states allows for the otherwise forbidden E1 transition (Figure 1.1). The exchange of Z\(^0\) bosons between electrons and the nucleus happens aside the electromagnetic exchange of photons. These cause a shift in the energy levels that is strong in for example radium ions (Ra\(^+\)) and barium ions (Ba\(^+\)). Shifts in transition energy called light shifts arise if the ion is exposed to a light field. The light shift can be measured to extract the Weinberg angle.
The Weinberg angle, actually $\sin^2 \theta_w$, defines the mixing of the weak and electromagnetic interaction. The measured quantity can be compared to predictions by the Standard Model (Figure 1.2). If the results do not agree with the Standard Model it could indicate the possibility for new physics, for example an new $Z'$ boson. [2]

Figure 1.1: Relevant transitions in $\text{Ra}^+$, the $\text{E1}$ transition is forbidden by parity. Light shift occurs in a strong light field and can be measured to extract the Weinberg angle. Taken from [1].

Figure 1.2: Weinberg angle by energy scale as predicted by the Standard Model. Results from the atomic parity violation experiment might indicate possible new physics. Taken from [2].

For accurate spectroscopy measurements a standing wave with very stable position and frequency is required. This can be achieved by employing an enhancement cavity that is
in resonance with the laser light. This thesis will describe how such a cavity is stabilized to a laser beam.

1.1.2 Stable low-cost atomic clock

The same experimental setup can be employed to create an atomic clock with a level of frequency stability of $10^{-18}$. The very same parity violating transition $(7^2 S_\frac{1}{2} - 6^2 D_\frac{3}{2})$ is used as it has a very narrow frequency. It provides a precise and accurate frequency reference for an atomic clock that uses low cost semiconductor diode lasers.
Chapter 2

Laser light

2.1 Characteristics of laser light

Laser light can be used for many purposes. In the radium ion (Ra\(^+\)) experiment lasers are for example used for cooling of the radium ions as well as for generating oscillating electric fields for measuring light shifts. For the latter purpose, laser light is enhanced in an optical cavity as described in this thesis. An understanding of the characteristics of laser light is required to create a stable working setup.

2.1.1 Frequency and linewidth

To be able to conduct measurements with a laser, the laser frequency and linewidth has to be well known and understood. The linewidth can be defined as the full width at half maximum (FWHM) of the lasers frequency spectrum, which is centered around the main frequency at which the highest intensity is found. A smaller linewidth translates to a narrower frequency distribution which means that a higher fraction of the intensity is carried by the center frequency. Figure 2.1 shows a sketch of a laser frequency spectrum and its FWHM indicated.

2.1.2 Gaussian beam

The intensity distribution across a cross section of the laser beam can be approximated by a Gaussian function. Gaussian beams can be fully coupled into an optical cavity by
Chapter 2. *Light in an optical cavity*

Figure 2.1: Sketch of a laser spectrum with the full width at half maximum (FWHM) indicated.

proper alignment with lenses. The beam is fully characterized by its minimum diameter (the waist) and divergence [3]. The divergence in the beam can be translated to a radius of curvature that is useful when coupling a beam of light into a cavity that consists of spherically curved mirrors. Chapter 3 will elaborate on this subject. The curved wavefront of a Gaussian beam and the beam waist is illustrated in Figure 2.2.

Figure 2.2: Gaussian beam profile with its waist indicated. Curvature of the wave front is shown by dotted lines.
Chapter 3

Optical enhancement cavity

For the radium ion experiment, an optical cavity is used to enhance the intensity of the laser light. It is a set of mirrors that act together as a resonator for standing waves of light. If a cavity is stable and in resonance with the laser light, it is possible to generate light intensities inside the cavity that are much higher than the incoming laser light. The criterion for a stable cavity will be explained in Section 3.2.

3.1 Requirements for enhancement

To be able to enhance the electric field at a specific point at which an ion is located, it is necessary to have the cavity on resonance with the laser light to the precision of a fraction of the wavelength of the light. Therefore we will be satisfied with a stability at the order of 10nm. In Chapter 5 there will be an explanation of how we quantify the stability of the cavity-laser lock (Chapter 4) and how this is translated in an approximate length unit.

3.2 Stability criterion

Using a ray tracing method, the path of the laser beam depending on the mirrors and the cavity length can be calculated. This method is further explained by Verdeyen [4]. For a cavity of length L consisting of mirrors with radii of curvature $R_1$ and $R_2$, this
turns out to be
\[ 0 \leq (1 - \frac{L}{R_1})(1 - \frac{L}{R_2}) \leq 1 \] (3.1)

In Figure 3.1, the stability of a cavity is nicely plotted. The dotted line shows the values for which \( R_1 = R_2 \).

Two mirrors of equal radii of curvature are preferred for this setup to focus the laser beam in the center of the cavity. Equation (3.1) then becomes
\[ (1 - \frac{L}{R})^2 \leq 1 \]

It turns out that stability requires \( 0 \leq L \leq 2R \), meaning that the length of the cavity should be smaller than twice the radius of curvature of the mirrors.

![Figure 3.1: Stability diagram of a cavity with \( g_{12} = (1 - L/R_{12}) \). Taken from [4].](image)

### 3.3 Alignment and mode-matching

When a laser beam enters a two-mirror cavity, it will bounce back and forth between the two mirrors. Besides the stability criterion the alignment has to be correct in order to have a stable cavity. Figure 3.2 shows the ray tracing of stable and unstable alignment.
3.3.1 Gaussian beam alignment

As discussed in the previous chapter a Gaussian beam is fully characterized by its diameter and radius of curvature. The properties of a Gaussian beam are such that it can be fully coupled into the cavity when it is aligned and focused to match the geometry of the cavity. This means that the optics have to be chosen and aligned in a way that the beam waist is located in the center while the radius of curvature of the wavefront matches that of both cavity mirrors. Figure 3.3 illustrates the profile of a well coupled light beam in a cavity.

3.3.2 Transverse optical modes

A different challenge in setting up an enhancement cavity is to properly align the laser beam to the cavity. By measuring the properties of the Gaussian laser beam, one can calculate how the setup has to look like in terms of lenses and geometry. This will not be discussed in this thesis but is thoroughly explained by Quimby [5].

When the beam is aligned so that it is coming into the cavity, it has to be in the right mode. Different modes appear when the laser beam is not aligned exactly on the optical axis of the cavity. A mode is basically a pattern in which the beam is propagating inside the cavity, which can be seen in the light transmitted by the cavity. The modes in the cavity used in this experiment mainly produce $TEM_{nm}$ modes where $n$ and $m$ respectively define the nodes in the horizontal and vertical direction of the pattern. The
Chapter 3. *Optical enhancement cavity*

$TEM_{00}$ mode is the required state in which one dot appears. A few modes are shown in Figure 3.4.

![Figure 3.4: Intensities of different modes of the enhancement cavity. The small dots are due to the camera that was used, the mode shapes should be smooth.](image)

### 3.4 Resonance, reflection and transmission of light

Besides the geometry of the cavity and the alignment of the beam that was discussed in previous sections, there is one more requirement for a functioning enhancement cavity. To create a standing wave, the cavity has to be on-resonance with the frequency of the laser light. The cavity is on-resonance when an integer number of wavelengths of the laser light fits inside of the cavity, which requires the optical distance to be an integer multiple ($m$) of half of the wavelength ($\frac{1}{2}$) of the laser light. This resonance length ($L$) depends on the wavelength and therefore on the light frequency ($\nu$) and the refractive index of the medium ($n$).

$$L = m\frac{\lambda}{2} = m\frac{c}{2\nu n}$$  \hspace{1cm} (3.2)

Matching the length of the cavity to the laser frequency is called frequency locking and is covered in Chapter 4.

Light that is off-resonance will be reflected by the cavity. Only the resonant frequency of light satisfying equation 3.2 can enter the cavity through one of the mirrors and will also be transmitted on the other side after a number of round-trips, depending on the properties of the cavity. Because of this property, the cavity can be used to scan over a range of frequencies to find the laser frequency. Then, by looking at the signal of a photosensitive diode, the shape of the frequency spectrum can be found.
Chapter 4

Locking cavity to laser frequency resonance

For a cavity to be locked to resonance with the laser light, the length of the cavity must be an integer multiple of the wavelength of the laser light. This concept is very simple, but is in practice not straightforward.

4.1 Issues in locking

A number of perturbations can cause a cavity to shift out of resonance with the laser light. These perturbations can alter the optical distance inside of the cavity by moving the mirrors or by changing the refractive index of the air between them. The former is caused by either temperature fluctuations that act slowly, or acoustic vibrations acting on a fast kHz time scale. Air flow can effect the system in similar ways but is easily dealt with by creating a housing for the cavity.

Another cause for instability of the system lies at the functioning of the laser itself. It is important to know that a laser diode itself also consists of a small cavity. This means that temperature changes as well as vibrations have similar effects on the laser diode as on the enhancement cavity.
4.2 Pound-Drever-Hall locking technique

There are several methods to lock a cavity to the light frequency, which use a feedback system to stabilize either the cavity to the laser or vice versa. In the setup discussed in this thesis the cavity will be locked to the laser by employing the Pound-Drever-Hall (PDH) locking technique [6]. It involves adjusting the voltage on a piezo actuator attached to one of the mirrors, which expands and contracts proportionally with applied voltage. One could also lock the laser to the cavity. In that case, a signal is fed back to the laser to adjust its parameters accordingly.

4.2.1 Physical concept

To be able to adjust the piezo actuator in the right direction a signal is needed that represents how far the cavity is off resonance with the laser light. Furthermore it needs to be antisymmetric around the resonance frequency to change the length in the right direction. This error signal should keep the cavity in lock with the laser light when proper gain parameters are set.

The input for creating the error signal is the intensity of the light which is reflected off the cavity. A sketch of the reflection signal is shown in Figure 4.1. For an electronic feedback system to generate the error signal, the laser light is phase modulated. To explain why this is useful, consider the expression for the electric field of the laser light

![Figure 4.1: Intensity of light reflected off the cavity. By changing the cavity length, a change in intensity can be measured.](image-url)
Chapter 4. Locking cavity to laser frequency resonance

with resonance frequency $\omega$.

$$E = E_0 e^{i \omega t} \quad (4.1)$$

Adding a sinusoidally varying phase modulation with small amplitude $\beta$ and frequency $\Omega$ adds a phase to the electric field.

$$E = E_0 e^{i \omega t + i \beta \sin(\Omega t)}$$

$$= E_0 e^{i \omega t} e^{i \beta \sin(\Omega t)}$$

Now using Taylor expansion to express the latter exponent for small $\beta$ and expressing sine in complex exponentials gives

$$E = E_0 e^{i \omega t} (1 + i \beta \sin(\Omega t))$$

$$= E_0 (e^{i \omega t} + \frac{\beta}{2} e^{i(\omega+\Omega)t} - \frac{\beta}{2} e^{i(\omega-\Omega)t}) \quad (4.2)$$

Expression 4.2 clearly shows that frequency sidebands appear if a small amplitude phase modulation is applied to the electric field. Using this and the fact that there is a transfer function for a two-mirror cavity

$$R(\omega) = \frac{E_{refl}}{E_{in}},$$

which can be a complex function, we can express the electric field of the beam reflected off the cavity $E_{refl}$ and get the power in the beam, because that is what is measured by a photosensitive detector. The measured power is given by equation 4.3.

$$P_{refl} = P_0 |R(\omega)|^2 + P_0 \frac{\beta^2}{4} (|R(\omega+\Omega)|^2 + |R(\omega-\Omega)|^2)$$

$$+ P_0 \beta (Re[\chi(\omega)] \cos(\Omega t) + Im[\chi(\omega)] \sin(\Omega t)) \quad \text{terms in } 2\Omega \quad (4.3)$$

Here $\chi$ is defined by

$$\chi(\omega) = R(\omega) R^*(\omega+\Omega) - R^*(\omega-\Omega) R(\omega) \quad (4.4)$$

This shows that phase modulation of the laser light creates an antisymmetric part in the reflected intensity. [7] is recommended for further mathematical derivation of the construction of the error signal.

When the modulation frequency is high enough that it is never in resonance with the
cavity, the sidebands are always directly reflected by the input mirror. If the cavity is on resonance the sidebands are $180^\circ$ out of phase and their signals cancel themselves. If it is off resonance the sidebands are phase shifted relative to each other depending on the direction of the frequency deviation, giving an opposite signal above or below resonance. The measured signal is then mixed with the local oscillator signal at the modulation frequency to extract only the modulated (asymmetric) part of the signal. The phase shift should be chosen in such a way that the error signal looks like the one in Figure 4.2.

![Figure 4.2: Example of reflected intensity (Main in) and Pound-Drever-Hall error signal (PDH out) as taken from [8].](image)

A drawing of the PDH feedback system is shown in Figure 4.3. The laser light is first phase modulated by the electro-optic phase modulator (EOM). Next, resonant light enters the cavity while off resonance light is reflected. The reflected light is measured by a photodiode (PD) and mixed with the local oscillator (LO), which also drives the EOM. The signal is then send to one of the piezo driven mirrors via a PID controller.

![Figure 4.3: Schematic drawing of a PDH controlled setup. Taken from [9].](image)
4.3 Experimental setup

4.3.1 Optics

The optical part of the setup that is used for this experiment is schematically drawn in Figure 4.4. A diode laser is frequency stabilized by an optical grating. To optically isolate the laser from the enhancement cavity setup a Faraday isolator is used. The beam is then sent through an optical fiber to the main setup, where a half-wave plate orients the polarization of the electric field of the laser light to the orientation preferred by the EOM.

A beam splitter directs a low intensity part of the beam to a cavity for diagnostics purposes. This is to check the incoming laser beam for higher transverse modes and frequency drifts.

The main beam continues through several lenses and mirrors which are employed to align and focus the beam into the cavity. Next, the beam passes through a polarizing beam splitter, which is oriented such that the incoming beam is directed opposite the first photodiode (PD 1). It then passes through a quarter-wave plate $\lambda/4$ into the cavity. Light reflected by the cavity goes through the quarter-wave plate again which brings the
polarization of the light in the opposite direction, such that the polarizing beam splitter directs the beam into PD 1. The signal from PD 1 is used to generate the PDH error signal.

Light that passes through the cavity is focused onto another photodiode (PD 2), which is used to check the alignment of the beam into the cavity. There is a camera installed to look at different modes if the cavity is purposely misaligned (Figure 3.4).

The enhancement cavity consists of two equal mirrors with radius of curvature $R=50\text{mm}$ placed in the opposite direction, separated by a distance of $L=48\text{mm}$. Both mirrors are attached to a piezo actuator.

### 4.3.2 Controller

The Toptica DigiLock 110 laser locking controller is employed to generate the proper error signal. The controller creates the modulation signal and functions as a PID controller which generates the error signal. The PID controller is an electronic system that can generate an output by a proportional, integral and derivative gain. The relative gain for each of these can be adjusted independently.
Chapter 5

Results

5.1 Cavity locked to light resonance

With the laser beam properly aligned to the cavity and with a good feedback signal to the mirror piezo actuator, the enhancement cavity was locked to the frequency of the laser light. The stability of this lock is discussed in Section 5.2.

5.2 Stability of the system

A measure for the stability of the lock of the enhancement cavity to the laser light is obtained by examining the PDH error signal as shown in Figure 5.1. A calibration of the time scale of this signal is obtained by exploiting the phase modulation of the laser light. The modulation creates sidebands at ± 25MHz on the light spectrum, which show up as zero crossings in the PDH error signal.

We can assume that the error signal is roughly linear if the cavity is near resonance with the laser. We can do this assumption as we require that the lock signal will be centered around the resonance frequency and an order of magnitude approximation of the lock stability is sufficient. The validity of this assumption is supported by Figure 5.1, which displays the roughly linear part of the signal near resonance.

To measure the stability of the system, the feedback signal is fed back to the piezo actuator which moves one of the mirrors to lock the cavity. The root mean square of the
Chapter 5. Results

Figure 5.1: Pound-Drever-Hall error signal as produced by the feedback controller as the optical cavity is scanned near resonance. The zero crossing in the center represents the cavity exactly on resonance with the laser light.

Figure 5.2: Pound-Drever-Hall error signal as produced by the feedback controller as the optical cavity is locked to the laser frequency.

The feedback signal seen in Figure 5.2 is used as the stability measure. Using the calibration of the oscilloscope the stability in frequency scale is measured to be approximately 600kHz. Taking into account that this measurement was fluctuating considerably, we will round this stability to 1MHz to approximate the stability of the lock. For laser light at 740nm ($\nu \approx 4 \cdot 10^{14}$ Hz) this translates to an absolute stability of about $2.5 \cdot 10^{-9}$.

This can be translated to a length scale by comparing this to the cavity length of 10cm, which translates to a stability in position of $\Delta L = 10^{-10} m$. 

Chapter 6

Conclusions and outlook

A reasonably stable cavity for creating a standing wave for light shift measurements was set up during this research project. The stability of the cavity as well as the consequences for the APV experiment are discussed in the next sections. Recommendations for future improvements are also given.

6.1 Stability

The position of the mirror and therefore the position of the standing wave in the cavity was found to be stable at sub-nanometer scale, derived from an approximate frequency stability of 1 MHz. This approximation of stability might have been optimistic taking into account the assumptions that were made. Nevertheless a stability of an order of magnitude of several nanometers is achieved, which very well meets the requirement of having a cavity that is stable to the order of a fraction of the wavelength of the resonant light, which in this case is visible light of 740 nm.

In a time range of ten minutes the PDH lock was able to keep the lock stable. After a few minutes however it would shift out of lock. The cause of this lies at perturbations that exceeded the range within the electronics could keep the cavity in resonance lock. These issues were mainly caused by the laser drifting out of range too fast such that the piezo actuator could not be moved far and fast enough to keep it in lock.
6.2 Atomic parity violation experiment

The cavity lock is sufficient to generate a stable standing wave of light. The APV experiment however requires that measurements can be conducted for a period of hours or days. This means that there is still some improvement to be done to the enhancement cavity. We should keep in mind that the enhancement cavity will eventually be used in a vacuum, which will significantly decrease perturbations that are caused by changes in the refractive index.

6.3 Recommendations

There are several suggestions to be done regarding the improvement of the enhancement cavity locking system. First of all, the diode laser has the tendency to drift in frequency and to produce higher modes. This effect can be reduced by creating a feedback system that controls the laser frequency similar to the method that was described in this thesis.

The locking period can also be extended by improving the response of the piezo actuator attached to the cavity mirror. By providing a more stable mount for the cavity mirrors it is possible to reduce mechanical resonances within the cavity, thereby increasing the response at frequency which currently drive resonances of the mirrors and their mounts.

In conclusion, a lot of progress has been made to create a stable enhancement cavity for light shift measurements. Improvements are to be made in the future and a setup will come to meet all the requirements needed for the APV experiment.
Bibliography


