



MODELING OF A MICROGRID:  
POWER SHARING IN  
SYNCHRONOUS GENERATORS AND  
INVERTERS

Mathijs Lomme

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Supervisors: Prof. dr. A.J. (Arjan) van der Schaft  
& prof. dr. C. (Claudio) De Persis & P. (Pooya)  
Monshizadeh Naini

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# Contents

<b>1</b>	<b>Introduction</b>	<b>4</b>
<b>2</b>	<b>Apparent, active and reactive power</b>	<b>6</b>
2.1	Calculating the active and reactive power . . . . .	6
2.2	Phase difference voltage and current . . . . .	8
2.3	Active power and reactive power for different transmission lines .	10
2.3.1	Purely inductive transmission line . . . . .	10
2.3.2	A purely resistive transmission line . . . . .	11
2.3.3	Combined resistive/inductive transmission lines . . . . .	12
2.4	Relation between frequency and active power . . . . .	13
<b>3</b>	<b>Synchronous Generator</b>	<b>16</b>
3.1	A single generator . . . . .	18
3.2	A synchronous generator connected to the main grid . . . . .	20
3.3	Two parallel SG . . . . .	21
3.3.1	Different $K_d$ . . . . .	22
3.3.2	Different $J$ . . . . .	24
3.3.3	Different nominal power . . . . .	26
3.3.4	Different nominal frequency . . . . .	29
<b>4</b>	<b>Voltage source inverter</b>	<b>32</b>
4.1	SG and inverters . . . . .	32
4.2	Different mathematical techniques . . . . .	34
4.2.1	Phase-Locked Loop . . . . .	34
4.2.2	dq0-transformation . . . . .	35
4.2.3	PWM . . . . .	37
4.2.4	LC-Filter . . . . .	39
4.3	Structure of inverter . . . . .	40
4.4	A single inverter . . . . .	42
4.5	A single inverter connected to the grid . . . . .	42
4.6	Parallel inverters . . . . .	43
4.6.1	Different droop constants . . . . .	43
4.6.2	Different nominal power . . . . .	44
4.6.3	Different nominal frequency . . . . .	45
<b>5</b>	<b>Synchronous generator and inverter</b>	<b>46</b>
5.1	Relation between damping factor and droop coefficient . . . . .	46
5.2	Transient response . . . . .	48

<b>6</b>	<b>Secondary droop control</b>	<b>50</b>
6.1	Secondary control in a single SG . . . . .	50
6.2	Secondary control for parallel SGs . . . . .	51
<b>7</b>	<b>Reactive power droop control</b>	<b>54</b>
7.1	Single inverter using both active as reactive droop control . . . .	55
7.1.1	Error in load demand . . . . .	55
7.1.2	Both P-f and Q-E droop control . . . . .	57
7.1.3	Secondary droop control on voltage magnitude . . . . .	57
7.2	Parallel inverters using both $P - f$ as $Q - E$ control . . . . .	58
<b>8</b>	<b>Grid-supporting power converter acting as a current source</b>	<b>60</b>
<b>9</b>	<b>Conclusions</b>	<b>63</b>
<b>10</b>	<b>Future work</b>	<b>65</b>
<b>11</b>	<b>Appendix</b>	<b>66</b>
11.0.1	Power invariant dq0-transformation . . . . .	66

# Chapter 1

## Introduction

A **microgrid** is an electrical distribution network, heterogeneously composed of distributed generation, storage and load, and managed autonomously from the larger transmission network. Microgrids are able to connect to a larger electric power system, but are also able to island themselves and operate independently. In a microgrid we are dealing with both **DC-current** and **AC-current**. The DC-current is flowing in one direction and the AC-current is flowing in both directions. This is given by a sinusoidal graph, where the upward movement represent the positive direction and the downward movement represents the negative direction. AC is due to inductors and capacitors. The portion of power flow that, averaged over a complete cycle of the AC waveform, results in the net transfer of energy in one direction is known as **real power/active power**,  $P$ . **Reactive power**,  $Q$ , is linked with energy storage in inductors and capacitors, and can be calculated, but the physical definition is not clear.

When there is a demand of active/reactive power, **generators** and **inverters** are working together to ensure synchronization, voltage stability, power balance, load sharing and economic operation.

The total power, also called the **apparent power**,  $S$ , is defined by:  $S = P + jQ$ . The active power, transmitted across a purely inductive line between point 1 and point 2 has a relation with the frequency:

$$2\pi \int f = \frac{PX}{|U_1||U_2|}$$

where  $X$  is called the reactance and  $|U_1|$ ,  $|U_2|$  are the voltage magnitude measured at point 1, respectively point 2. Likewise, the reactive power has a relation with difference in voltage magnitude:

$$|U_2| - |U_1| = \frac{QX}{|U_2|}$$

This means that by controlling  $P$  and  $Q$  one controls the frequency and the voltage magnitude, [6], [7], [10]. A generator is using the **swing equation** to control the frequency and the inverter is using **droop control** to regulate the frequency and voltage magnitude. The active power droop equation relates

a change in active power with respect to the nominal active power,  $P_0$ , to a change in frequency with respect to the nominal frequency  $f_0$ . The relation is controlled by the active droop coefficient,  $k_p$ . This yields the following active droop equation:

$$f = f_0 - k_p(P - P_0) \quad (1.1)$$

Likewise, the reactive power droop equation relates a change in reactive power with respect to the nominal reactive power,  $Q_0$ , to a change in voltage magnitude with respect to the nominal voltage magnitude  $E_0$ . The relation is controlled by the reactive droop coefficient,  $k_q$ . This yields the following reactive droop equation:

$$E = E_0 - k_q(Q - Q_0) \quad (1.2)$$

When a load is increasing, multiple parallel units with the same droop characteristic can react to the fall in frequency by increasing their active power outputs simultaneously. The increase in active power will counteract the reduction in frequency and the units will settle at active power outputs and frequency at a steady-state point on the droop control characteristic. A same thing holds for the voltage droop control.

There are some levels of hierarchy the generators and inverters are working on, [1], [4].

### **Primary control**

Rapidly balances generation and demand, while sharing the load, synchronizing the AC voltage frequencies and stabilizing their magnitudes. Accomplished by decentralized droop control, where generators are controlled such that their power injections are proportional to their voltage frequencies and magnitudes. This is the type of control we will be focusing on.

### **Secondary control**

Droop controllers induce steady state errors in frequency and voltage magnitudes, which are corrected in a secondary control layer. We will briefly look at this subject.

### **Tertiary control**

Fair load sharing among the sources, or dispatch the generation to minimize optimal costs. We will not be working on this level, we are focusing on the droop control.

We will mainly look at (1.1) and how power is being shared between parallel generator/inverters. Later on we will also look at reactive power, which proved to be a bigger challenge. We will be modeling our theories, for which we can use either pscad, [11], or SIMULINK. We have chosen SIMULINK, since we allready had experience with Matlab. From SIMULINK, we will be using the SimPowerSystems toolbox explained in [13].

## Chapter 2

# Apparent, active and reactive power

In an electrical circuit the power is the rate of flow of energy past a certain point in the circuit. Inductors and capacitors in an AC circuit can reverse the direction of the energy flow. We call the portion of power that, averaged over a complete cycle of the AC waveform, results in net transfer of energy in one direction the **active power**. **Reactive power** has something to do with stored energy in inductors and capacitors. While it can be calculated, a real physical meaning is still not clear. We denote the active power by  $P$ , which has unity Watt and the reactive power by  $Q$ , which has unity volt-ampere reactive (var). The apparent power is defined as  $S = P + jQ$  and had unity voltage-ampere (VA).

In this chapter we start by calculating the active and reactive power, see section 2.1. Then we will show how the phase angle changes for different transmission lines in section 2.2. From this we go to the calculation of the active and reactive power for different transmission lines and show how their dependence on frequency and voltage magnitude changes, see section 2.3. Finally, we will write out a different way that shows the connection between active power and frequency, based on the swing equation, in section 2.4.

First we will express the active and reactive powers in terms of voltage and current.

### 2.1 Calculating the active and reactive power

We start with a single phase electric circuit.

The voltage in an electric circuit can be written as:

$$U(t) = |U| \cos(\omega t) \tag{2.1}$$

in which  $|U|$  is the magnitude and  $\omega$  is the angular frequency. Also for the

current:

$$I(t) = |I| \cos(\omega t - \phi) \quad (2.2)$$

where  $|I|$  is the amplitude of the current and  $\phi$  is the phase difference between current and voltage [5]. For purely resistive circuits  $\phi = 0$  and for purely inductive circuits  $\phi = \frac{\pi}{2}$  (see section 2.2).

In stead of looking at (2.1) and (2.2) we write:

$$U(t) = |U| e^{j\omega t} \quad (2.3)$$

$$I(t) = |I| e^{j(\omega t - \phi)} \quad (2.4)$$

The reason why we can do so, is because of Euler's formula  $\cos(\omega t - \phi) = \frac{1}{2} (e^{j(\omega t - \phi)} + e^{-j(\omega t - \phi)})$ . This means that the voltage and current can be written as the sum of two complex-valued functions. Because of the symmetry we only need to analyze one of the two terms, the results will be identical. To get back to the original voltage or current, note that  $\cos(\omega t - \phi) = \Re\{e^{j(\omega t - \phi)}\}$ . Also note that another way of writing (2.3) and (2.4) is by:

$$U(t) = |U| e^{j0} e^{j\omega t} \quad (2.5)$$

$$I(t) = |I| e^{-j\phi} e^{j\omega t} \quad (2.6)$$

When not observing the time dependence, these equations result to the so-called phasors:

$$U = |U| e^{j0} \quad (2.7)$$

$$I = |I| e^{-j\phi} \quad (2.8)$$

The phasor notation is very common to use, but we will stick to the time domain equations (2.3) and (2.4).

The apparent power  $S$  is defined as  $S = U(t)I(t)^*$  where  $I(t)^*$  is the complex conjugate of the current. When we write the voltage and current as:

$$U(t) = |U| e^{j\omega t} = |U| (\cos(\omega t) + j \sin(\omega t)) \quad (2.9)$$

$$I(t) = |I| e^{j(\omega t - \phi)} = |I| (\cos(\omega t - \phi) + j \sin(\omega t - \phi)) \quad (2.10)$$

and we calculate the apparent power, we find it as:

$$\begin{aligned}
S = U(t)I(t)^* &= |U||I| \left( (\cos(\omega t) + j \sin(\omega t)) (\cos(\omega t - \phi) - j \sin(\omega t - \phi)) \right) \\
&= |U||I| \left( \cos(\omega t) \cos(\omega t - \phi) + \sin(\omega t) \sin(\omega t - \phi) \right. \\
&\quad \left. + j (\sin(\omega t) \cos(\omega t - \phi) - \cos(\omega t) \sin(\omega t - \phi)) \right) \\
&= |U||I| \left( \cos(\omega t) (\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi)) \right. \\
&\quad \left. + \sin(\omega t) (\sin(\omega t) \cos(\phi) - \cos(\omega t) \sin(\phi)) \right. \\
&\quad \left. + j (\sin(\omega t) (\cos(\omega t) \cos(\phi) + \sin(\omega t) \sin(\phi)) \right. \\
&\quad \left. + \cos(\omega t) (\cos(\omega t) \sin(\phi) - \sin(\omega t) \cos(\phi))) \right) \\
&= |U||I| \left( \cos(\phi) (\cos^2(\omega t) + \sin^2(\omega t)) \right. \\
&\quad \left. + j (\sin(\phi) (\cos^2(\omega t) + \sin^2(\omega t))) \right) \\
&= |U||I| \cos(\phi) + j |U||I| \sin(\phi)
\end{aligned} \tag{2.11}$$

Since  $S = P + jQ$ , we obtain, referring to equation (2.11) that  $P = |U||I| \cos(\phi)$  and  $Q = |U||I| \sin(\phi)$ . This means that when  $\phi = 0$ , the purely resistive circuit,  $P = |U||I|$  and  $Q = 0$ .

Next we will explain the term phase difference and how big this value is in a resistor, inductor and capacitor.

## 2.2 Phase difference voltage and current

When there are inductors and capacitors in a AC-circuit, the voltage and current do not peak at the same time. Expressing the fraction of time of this difference in radians yields the phase difference between voltage and current. For this phase difference,  $\phi$ , it holds that:

$$\phi \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right] \tag{2.12}$$

where  $-\frac{\pi}{2}$  is corresponding with  $\frac{\pi}{2}$ .

The impedance is the measure of resistance the circuit presents to the current when a voltage is applied. The impedance of a RLC-circuit is given by  $Z = R + j(X_L - X_C)$ , [7], where  $R$  is the resistance,  $X_L$  is the inductive reactance and  $X_C$  is the capacitive reactance. The impedance is thus a complex resistive value. We denote the voltage by  $U(t) = |U|e^{j\omega t}$  and the current by  $I(t) = |I|e^{j(\omega t - \phi)}$ . In this way, the impedance can be expressed by the complex ratio between voltage and current:

$$Z = \frac{U(t)}{I(t)} \tag{2.13}$$



If there is just DC flow, the phase angle is zero and the impedance corresponds to the resistance  $R$ . In AC flow however, there is a nonzero phase angle and thus the impedance becomes complex. First we will write down the impedance in a resistor, inductor and capacitor.

For the resistor we know that:

$$U(t) = RI(t) \quad (2.14)$$

For which the impedance is of course just  $R$ , which means that the voltage and current are in phase.

For the inductor we have:

$$U(t) = L \frac{dI(t)}{dt} \quad (2.15)$$

$$\text{Impedance} = \frac{L \frac{dI(t)}{dt}}{I(t)} = \frac{j\omega L |I| e^{j(\omega t - \phi)}}{|I| e^{j(\omega t - \phi)}} = j\omega L$$

And for the capacitor:

$$I(t) = C \frac{dV(t)}{dt} \quad (2.16)$$

$$\text{Impedance} = \frac{\int |I| e^{j(\omega t - \phi)} dt}{C \int |I| e^{j(\omega t - \phi)} dt} = \frac{\frac{1}{j\omega} |I| e^{j(\omega t - \phi)}}{C |I| e^{j(\omega t - \phi)}} = \frac{1}{j\omega C}$$

Now, there is a way to rewrite the imaginary unit  $j$ :

$$j = e^{j\frac{\pi}{2}} \quad (2.17)$$

In the same way:

$$\frac{1}{j} = e^{j\frac{-\pi}{2}} \quad (2.18)$$

This means that the impedance over an inductor is  $X_L = \omega L e^{j\frac{\pi}{2}}$  and over a capacitor  $X_C = \frac{1}{\omega C} e^{j\frac{-\pi}{2}}$  which shows that the phase difference over an inductor is  $\frac{\pi}{2}$  and over a capacitor is  $\frac{-\pi}{2}$ .

The phase difference of  $\frac{\pi}{2}$  corresponds to a voltage leading the current, or the current lagging the voltage and for the phase difference of  $\frac{-\pi}{2}$  it is vice versa.

In a RLC-circuit, the total impedance is  $Z = R + j(X_L - X_C) = R + jX$ . Or written in polar form,  $Z = |Z| e^{j\phi} = |Z| \cos(\phi) + j|Z| \sin(\phi)$ . This yields that  $R = |Z| \cos(\phi)$  and  $X = |Z| \sin(\phi)$  and therefore:

$$\phi = \arctan \left( \frac{X}{R} \right) \quad (2.19)$$

(2.19) is called the  $\frac{X}{R}$  ratio and it determines how much the current lags or leads the voltage.

## 2.3 Active power and reactive power for different transmission lines

We consider a network with two nodes as depicted in Figure 2.1.

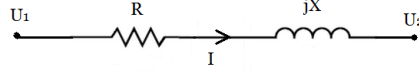


Figure 2.1: Transmission line represented by a resistor and an inductor

The voltage at point 1 is given by  $U_1 = |U_1|e^{j\theta_1}$  and at point 2 by  $U_2 = |U_2|e^{j\theta_2}$ . We are using phasor notation at this point because it is common in the literature and it gives a clearer view of what we want to show at this point. The impedance on the line is the complex value  $Z = R + jX$ , where  $R$  is the resistance and  $X$  the reactance. The current flows from point 1 to point 2 and is calculated by  $I = \frac{U}{Z}$ , where  $U = U_1 - U_2$ . The apparent power at point 2 is calculated by  $S = U_2 I^*$ , where  $I^*$  is the complex conjugate of  $I$ , which is equal to:

$$\begin{aligned} I^* &= \left( \frac{U_1 - U_2}{R + jX} \right)^* \\ &= \frac{U_1^* - U_2^*}{R - jX} \\ &= \frac{|U_1|e^{-j\theta_1} - |U_2|e^{-j\theta_2}}{R - jX} \end{aligned} \quad (2.20)$$

The apparent power at point 2 is then given by:

$$\begin{aligned} S &= U_2 I^* \\ &= |U_2|e^{j\theta_2} \frac{|U_1|e^{-j\theta_1} - |U_2|e^{-j\theta_2}}{R - jX} \\ &= \frac{|U_1||U_2|e^{-j(\theta_1 - \theta_2)} - |U_2|^2}{R - jX} \end{aligned} \quad (2.21)$$

### 2.3.1 Purely inductive transmission line

Very often the special case is considered, where the transmission line has no resistance, i.e.  $R = 0$ . In this case, the apparent power at point 2, (2.21), becomes:

$$\begin{aligned} S &= \frac{|U_1||U_2|e^{-j(\theta_1 - \theta_2)} - |U_2|^2}{-jX} \\ &= j \frac{|U_1||U_2|e^{-j(\theta_1 - \theta_2)}}{X} - j \frac{|U_2|^2}{X} \\ &= \frac{|U_1||U_2|}{X} \sin(\theta_1 - \theta_2) + j \left( \frac{|U_1||U_2|}{X} \cos(\theta_1 - \theta_2) - \frac{|U_2|^2}{X} \right) \end{aligned} \quad (2.22)$$

Since the apparent power is defined as  $S = P + jQ$ , we observe that:

$$\begin{aligned} P &= \frac{|U_1||U_2|}{X} \sin(\theta_1 - \theta_2) \\ Q &= \frac{|U_1||U_2|}{X} \cos(\theta_1 - \theta_2) - \frac{|U_2|^2}{X} \end{aligned} \quad (2.23)$$

The difference  $\theta_1 - \theta_2$  is called the phase angle and is denoted by  $\theta$ . Often this value is  $\approx 0$ , [23], since the voltages are almost in phase. This means that  $\sin(\theta) \approx \theta$  and  $\cos(\theta) \approx 1$ . This results in:

$$\begin{aligned} P &= \theta \frac{|U_1||U_2|}{X} \\ Q &= \frac{|U_1||U_2|}{X} - \frac{|U_2|^2}{X} = \frac{|U_2|}{X} (|U_1| - |U_2|) \end{aligned} \quad (2.24)$$

There is a relation between the angular frequency and the phase angle,  $\omega = \dot{\theta}$ , so it becomes clear that in this specific case the real power  $P$  depends on the frequency, since  $f = \frac{\omega}{2\pi}$ . The reactive power  $Q$  on the other hand, depends on  $|U_1| - |U_2|$ , which is the change in voltage amplitude. So the classic linear droop control equations, which focuses on purely inductive lines, are now clear. We should remember that this is the simple case, where we assumed that there was no resistance on the line. We will now consider the case of a purely resistive line.

### 2.3.2 A purely resistive transmission line

We will look at a purely resistive transmission line, i.e.  $X = 0$  in (2.21). In this case, the apparent power at point 2, (2.21), becomes:

$$\begin{aligned} S &= |U_2| e^{j\theta_2} \frac{|U_1| e^{-j\theta_1} - |U_2| e^{-j\theta_2}}{R} \\ &= \frac{|U_1||U_2| e^{-j(\theta_1 - \theta_2)}}{R} - \frac{|U_2|^2}{R} \\ &= \frac{|U_1||U_2|}{R} \cos(\theta_1 - \theta_2) - \frac{|U_2|^2}{R} - j \frac{|U_1||U_2|}{X} \sin(\theta_1 - \theta_2) \end{aligned} \quad (2.25)$$

This time, the active and reactive power are thus:

$$\begin{aligned} P &= \frac{|U_1||U_2|}{R} \cos(\theta_1 - \theta_2) - \frac{|U_2|^2}{R} \\ Q &= - \frac{|U_1||U_2|}{X} \sin(\theta_1 - \theta_2) \end{aligned} \quad (2.26)$$

Again, we take the  $\theta \approx 0$ , which results in:

$$\begin{aligned} P &= \frac{|U_2|}{R} (|U_1| - |U_2|) \\ Q &= - \frac{|U_1||U_2|}{X} \theta \end{aligned} \quad (2.27)$$

This time, the active power depends on  $|U_1| - |U_2|$ , which is the change in voltage amplitude and the reactive power depends on the frequency.

Finally, we will look at the combined case.

### 2.3.3 Combined resistive/inductive transmission lines

We assume that  $R \neq 0$  and  $X \neq 0$ , so  $Z = R + jX = |Z|e^{j\phi}$ . The apparent power now becomes:

$$\begin{aligned}
S &= U_2 \left( \frac{U_1 - U_2}{Z} \right)^* \\
&= |U_2| e^{j\theta_2} \frac{|U_1| e^{-j\theta_1} - |U_2| e^{-j\theta_2}}{|Z| e^{-j\phi}} \\
&= \frac{|U_1||U_2|}{|Z|} e^{-j(\theta_1 - \theta_2 - \phi)} - \frac{|U_2|^2}{|Z|} e^{j\phi} \\
&= \frac{|U_1||U_2|}{|Z|} \cos(\theta - \phi) - j \frac{|U_1||U_2|}{|Z|} \sin(\theta - \phi) - \frac{|U_2|^2}{|Z|} \cos(\phi) - j \frac{|U_2|^2}{|Z|} \sin(\phi)
\end{aligned} \tag{2.28}$$

This means that, since  $S = P + jQ$ , the active and reactive power are given by:

$$\begin{aligned}
P &= \frac{|U_1||U_2|}{|Z|} \cos(\theta - \phi) - \frac{|U_2|^2}{|Z|} \cos(\phi) \\
&= \frac{|U_2|}{|Z|} \left( |U_1| (\cos(\theta) \cos(\phi) + \sin(\theta) \sin(\phi)) - |U_2| \cos(\phi) \right) \\
&= \frac{|U_2|}{|Z|} \left( (|U_1| - |U_2|) \cos(\phi) + |U_1| \theta \sin(\phi) \right)
\end{aligned} \tag{2.29}$$

$$\begin{aligned}
Q &= -\frac{|U_1||U_2|}{|Z|} \sin(\theta - \phi) - \frac{|U_2|^2}{|Z|} \sin(\phi) \\
&= \frac{|U_2|}{|Z|} \left( -|U_1| (\sin(\theta) \cos(\phi) - \sin(\phi) \cos(\theta)) - |U_2| \sin(\phi) \right) \\
&= \frac{|U_2|}{|Z|} \left( (|U_1| - |U_2|) \sin(\phi) - |U_1| \theta \cos(\phi) \right)
\end{aligned} \tag{2.30}$$

assumed that  $\theta \approx 0$ , so  $\sin(\theta) \approx \theta$  and  $\cos(\theta) \approx 1$ .

Writing  $P$  and  $Q$  in matrix form yields:

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \sin(\phi) & \cos(\phi) \\ -\cos(\phi) & \sin(\phi) \end{pmatrix} \begin{pmatrix} \frac{|U_1||U_2|}{|Z|} \theta \\ \frac{|U_2|}{|Z|} (|U_1| - |U_2|) \end{pmatrix} \tag{2.31}$$

One observes that the right-hand side of the equation above is nothing more than a transformation matrix times the active and reactive power of the simplified case. We will call these simplified  $P$  and  $Q$ ,  $P_1$  respectively  $Q_1$ . Furthermore, since  $Z = R + jX$  and  $Z = |Z|e^{j\phi} = |Z| \cos(\phi) + j|Z| \sin(\phi)$ , we can write  $R = |Z| \cos(\phi)$  and  $X = |Z| \sin(\phi)$ . This way we end up with:

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} \frac{X}{|Z|} & \frac{R}{|Z|} \\ -\frac{R}{|Z|} & \frac{X}{|Z|} \end{pmatrix} \begin{pmatrix} P_1 \\ Q_1 \end{pmatrix} \tag{2.32}$$

This means that the inductance of the transmission line plays a huge role in the droop control methods. We will look, as in done in most literature, at purely

inductive lines. This is because it is the simplest case, and because we are looking at high voltage circuits, which are mostly inductive, see Figure 2.2. This means that we will be observing the behavior of the original droop equations, where  $P$  depends on the frequency and  $Q$  on the voltage amplitude.

TYPICAL LINE IMPEDANCES VALUES			
Type of Line	R ( $\Omega/\text{km}$ )	X ( $\Omega/\text{km}$ )	R/X (p.u.)
Low Voltage Line	0.642	0.083	7.7
Medium Voltage Line	0.161	0.190	0.85
High Voltage Line	0.06	0.191	0.31

Figure 2.2: Impedance values for transmission lines at different voltage

## 2.4 Relation between frequency and active power

We know that  $P \approx \frac{|U_1||U_2|}{X}\theta$ , and that the phase angle is in relation with the frequency. It is however not clear how we can explain the structure of the frequency droop controller, i.e.  $f - f_0 = -k_p(P - P_0)$  or its less known form:  $f - f_0 = -\frac{k_p}{\tau s + 1}(P - P_0)$ .

We now will explain the relations stated above, by reformulating known formulas. We denote by  $\Delta P = P - P_0$  the change in active power, where  $P_0$  is the base value of the active power. In the same way we denote by  $\Delta\tau = \tau - \tau_0$  the change in torque and by  $\Delta\omega = \omega - \omega_0$  the change in angular frequency. The torque can be expressed as  $\tau = \tau_m - \tau_e$ , where  $\tau_m$  is the mechanical torque and  $\tau_e$  the electrical torque. In steady state,  $\tau_{m0} = \tau_{e0}$ , i.e.  $\tau_0 = 0$  [26], [19].

A well-known formula is

$$P = \omega\tau \quad (2.33)$$

Another way of writing 2.33 is the following:

$$P = P_0 + \Delta P = (\omega_0 + \Delta\omega)(\tau_0 + \Delta\tau) \quad (2.34)$$

$$= \omega_0\tau_0 + \Delta\omega\tau_0 + \omega_0\Delta\tau + \Delta\omega\Delta\tau \quad (2.35)$$

$$\approx P_0 + \omega_0\Delta\tau \quad (2.36)$$

The cross term  $\Delta\omega\Delta\tau$  is neglected as it is often  $\approx 0$  and since  $\tau_0 = 0$  we did not write the term  $\Delta\omega\tau_0$ . We now cross of the  $P_0$  terms on both sides to obtain:

$$\Delta P = \omega_0\Delta\tau \quad (2.37)$$

Next, the torque can also be expressed as:

$$\Delta\tau = J \frac{d}{dt} \Delta\omega \quad (2.38)$$

where  $J$  is called the moment of inertia. Using the Laplace transform we obtain:

$$\Delta\tau = J \Delta\omega s \quad (2.39)$$

Rearranging terms yields:

$$\Delta\omega = \frac{1}{Js} \Delta\tau \quad (2.40)$$

Now, the moment of inertia can be written as  $J = \frac{2H}{\omega_0}$ , where  $H$  is called the inertia constant and  $\omega_0$  is just the nominal angular frequency. Substituting this into (2.40) yields:

$$\Delta\omega = \frac{1}{2Hs} \omega_0 \Delta\tau \quad (2.41)$$

Using the fact that  $\omega = 2\pi f$  and substituting 2.37, we obtain:

$$2\pi \Delta f = \frac{1}{2Hs} \Delta P \quad (2.42)$$

which can be rewritten to:

$$f - f_0 = \frac{1}{4\pi Hs} (P - P_0) \quad (2.43)$$

This is of the well known form, where the droop constant seems to be  $k_p = -\frac{1}{4\pi Hs}$ .

Very often, a torque damping component is added to 2.39:

$$\Delta\tau = J \frac{d}{dt} \Delta\omega + \frac{k_d}{\omega_0} \Delta\omega \quad (2.44)$$

Here,  $k_d$  is called the damping torque coefficient. Switching to the Laplace form again, and substitute  $J = \frac{2H}{\omega_0}$  we obtain:

$$\left( \frac{2H}{\omega_0} s + \frac{k_d}{\omega_0} \right) \Delta\omega = \Delta\tau \quad (2.45)$$

Substituting 2.37 and rewrite a little yields:

$$\Delta\omega = \frac{1}{2Hs + k_d} \Delta P \quad (2.46)$$

Writing the angular frequency as  $2\pi f$  again we get the very often used frequency

droop control equation:

$$f - f_0 = \frac{1}{2\pi(2Hs + k_d)}(P - P_0) \quad (2.47)$$

The last equation can be rewritten to:

$$2Hs(\omega - \omega_0) + k_d(\omega - \omega_0) = P - P_0 \quad (2.48)$$

which is the equation the synchronous generator uses to control its frequency and active power output. We will look extensively to the synchronous generator in the next chapter.

## Chapter 3

# Synchronous Generator

In this section we will look at one way to produce power, that is very common in creating a microgrid, namely by using generators. Specifically, we will be using synchronous generators [20]. In SIMULINK there is a block available that is called Simplified Synchronous Machine (SSM). There is an option to either use the block as a motor, or as a generator. The difference is that a synchronous motor demands energy from the grid and a synchronous generator delivers it. A generator turns mechanical power into electrical power, and a motor turns electrical power into mechanical power. Since we are using the generator form of the SSM block, we must have some sort of mechanical power source. In real life, this source can be a steam turbine, solar cells, a water turbine or a diesel engine. In our analysis we keep the mechanical power,  $P_m$ , as a constant value, determined by the nominal power of the grid.

A SSM consists of a rotor and a stator. The rotor is a rotating cylinder which is driven by the mechanical power. Due to this rotation, a magnetic field is generated which moves through the stationary stator where the coils induce a voltage. This generating of voltages can be seen in Faraday's law. The voltage that is generated will have in each of the three phases the same amplitude, same frequency and the phases are  $120^\circ$  displaced from each other, since the coils within the stator are displaced in this matter. The frequency of the electrical power that is produced by the SSM is synchronized with the mechanical rotational speed;  $f_e = \frac{p}{120}n_m$ . Here,  $f_e$  is the electrical frequency in Hertz,  $p$  is the number of poles, and  $n_m$  is the rotor speed of the machine in rpm (revolutions per minute). The number of poles is determined by the circumstances; steam turbines are most efficient when rotating at high speed, i.e. to generate 60 Hz, the rotor is rotating at 3600 rpm, so the number of poles is 2. On the other hand, water turbines are most efficient at low speeds, around 300 rpm, so the number of poles is much higher. In our analysis we will be using  $p = 2$ , since this is most common in real life. The voltage that is generated within the stator is not exactly the same as the voltage leaving the synchronous generator. Reasons for the difference are found by the fact that the coils have a small resistance as well as a self-inductance. The small loss will be seen in the experiments we looked at. The relation between the electrical power generated by the SSM,



$P$  and the angular frequency  $\omega$  is given by the swing equation [19]:

$$J \frac{d}{dt}(\omega - \omega_0) + K_d(\omega - \omega_0) = P_0 - P \quad (3.1)$$

Here,  $J$  is the inertia momentum,  $K_d$  is the damping coefficient,  $\omega_0$  is the reference angular frequency and  $P_0$  is the nominal power reference, corresponding to the mechanical power. Notice the resemblance with (2.48) from the previous chapter. In the SSM block we are using in our experiments, we can specify not only the frequency, line to line voltage and nominal power of the generator, but also the coefficients  $J$  and  $K_d$ . In the upcoming sections, we will look at how these values influence the behavior of the generator. Note that from now on out we will refer to the synchronous generator as SG. In the first section we will observe the behavior of a SG when the loads in the grid are varying. In the second section we will add an ideal voltage source to the model in order to simulate a connected to the grid situation. Next, in section three, we will look at two parallel generators and examine what happens if we alter:

- $K_d$  (subsection 3.3.1)
- $J$  (subsection 3.3.2)
- The nominal power  $P_0$  (subsection 3.3.3)
- The nominal angular frequency  $\omega_0$  (subsection 3.3.4)

We will only look at the active power-frequency relation, since this is a built in equation in the SG. The SG is however also capable of handling the reactive power-voltage magnitude, since it has an internal impedance, [?]. We are just focusing on  $P - f$  control at this point.

### 3.1 A single generator

We begin with the simplest form where we have constant load connected to a SG. The schematics are shown in Figure 3.1. The nominal power of the SG is set at 1000 MW and the phase to phase voltage is 380 kV. The load has an active power demand equal to the nominal power. This means that when one load is connected, the frequency of the generator should be the value of your chosen  $\omega_0$  divided by  $2\pi$ , which in our case is 60 Hz. This is because at this time  $P = P_0$ , i.e. there is nominal power.

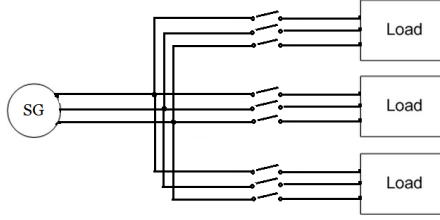


Figure 3.1: Schematic of a SG and three identical loads which can be connected and disconnected by use of the breakers

The same can be seen from (3.1). When the transition time after a power change is over, i.e. the  $\frac{d}{dt}(\omega - \omega_0)$  term is zero, the swing equation results in:

$$\omega = \omega_0 - \frac{1}{K_d}(P - P_0) \quad (3.2)$$

When  $P = P_0$ , the equation results in  $\omega = \omega_0$ , when  $P > P_0$ ,  $\omega < \omega_0$  and when  $P < P_0$ ,  $\omega > \omega_0$ .

The two graphs Figure 3.2(a) and Figure 3.2(b) also make clear what happens to the angular frequency  $\omega$  when the active power increases or decreases. Figure 3.2(a) shows that when a load is added, hence the active power demand increases, the angular frequency at which the generator is working should decrease. In the same way Figure 3.2(b) shows the reverse: when a load is removed, hence the active power demand decreases, the angular frequency of the generator is increased.

We model the situation where we start with no load connected to the generator. At time  $t = 0.5$ ,  $t = 1$  and  $t = 1.5$  the first, second and respectively the third load are connected, all of equal size. At time  $t = 2$ ,  $t = 2.5$  and  $t = 3$ , the loads are disconnected one at a time. Figure 3.3 shows the results.

We can observe that the generator decreases its frequency when the power demand is increased, and increases its frequency when the power demand is decreased. The SG is working like we expected. Observe that at  $t = 0.5$ , there is only one load connected, which has the value of  $P_0$ . One observes that the frequency indeed move close to our chosen  $f_0 = 60$  Hz. The reason why the frequency of the generator is not exactly 60 Hz, is due to the resistance and self-impedance of the coils in the SG as well as the transmission line.

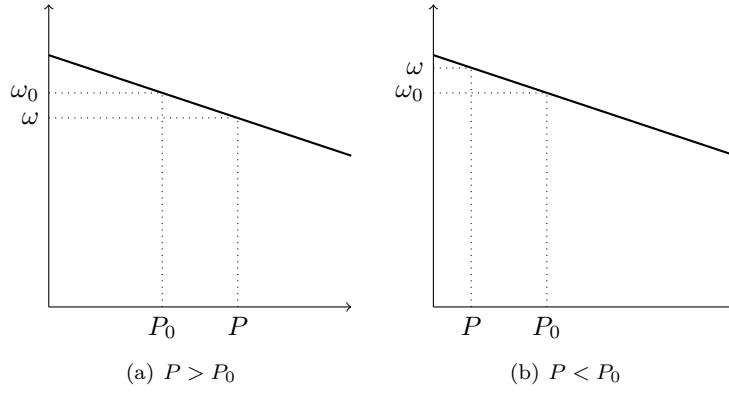


Figure 3.2:  $\omega - P$  droop control

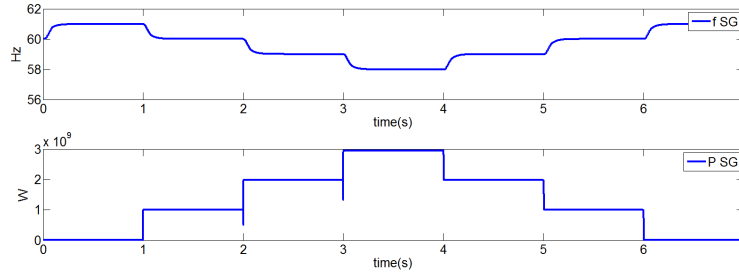


Figure 3.3: Frequency and active power generation of the SG

The SG acted the way the theory explained and now we can expand the network to more difficult situations. In the next section an ideal voltage source will be added to the network in order to simulate how the SG behaves when connected to the network.

## 3.2 A synchronous generator connected to the main grid

In this subsection we will add a ideal voltage source to the network of the previous section in order to create a situation where the SG is connected to the main grid. The loads remain the same, only the time interval is doubled for a more clearer view. As can be seen in Figure 3.4

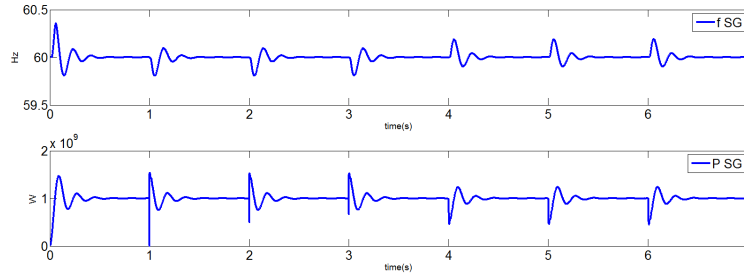


Figure 3.4: The frequency and active power output of the SG when connected to the grid

The first thing that becomes clear is that the SG is not powerful enough to compete with the grid. Since the frequency of the ideal voltage source has been set to 60 Hz, the SG operates at the same frequency. When a load is added, the frequency does decrease, but quickly get's back to the frequency of the main grid. This can be seen at time  $t = 1$ ,  $t = 2$  and  $t = 3$ . As for when a load is removed, the frequency indeed increases, but again falls back to the grid frequency very quickly. This can be seen at time  $t = 4$ ,  $t = 5$  and  $t = 6$ . Also observe that the active power output of the SG converges back to it's nominal power, which has been set to 1000 MW. In real life, there will be multiple generators acting at the same time, so their contribution will be bigger. In the next section we look at a grid with two synchronous generators, without a connection to the main grid.

### 3.3 Two parallel SG

We look at two identical synchronous generators, placed in parallel, working on the three loads which are connected and disconnected at the same time as in the previous section. The schematics are shown in Figure 3.5. Since the SGs are identical, and the loads have not changed, the total nominal power is now 2 GW. this means that when two loads, each of power 1 GW, are connected to the network, the frequency of both generators should be the nominal frequency. In our case this is 60 Hz. We plotted the frequency and output active power of both generators in Figure 3.6.

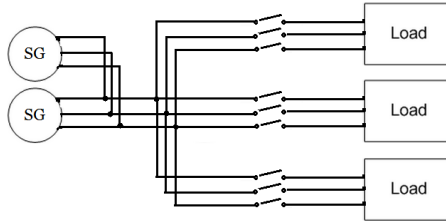


Figure 3.5: Schematic of two parallel SGs with three identical loads

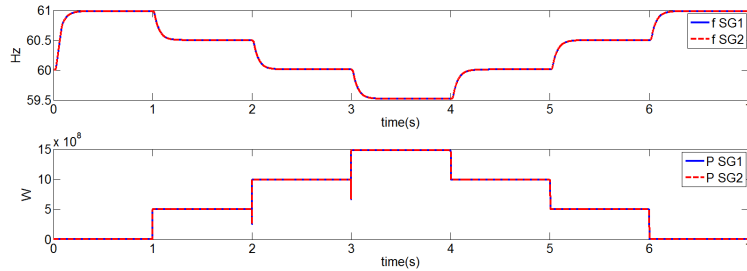


Figure 3.6: Two identical parallel SGs

The two plots of both the two frequencies as well as the active power output are identical, since the SGs have all the same values. As we said, at time  $t = 2$  and  $t = 4$ , there are two loads connected to the grid, which means that after a transition period  $P = P_0$  and so  $\omega = \omega_0$  or  $f = f_0 = 60$  Hz. Again there where some losses within the SGs, so it is not exactly 60 Hz, but we explained the reasons for this in a previous section.

Next we will alter the parameters of one of the two SGs in order to see their influence on the frequency and active power. We will start with  $K_d$ , the damping factor.

### 3.3.1 Different $K_d$

We start by stating the swing equation again.

$$J \frac{d}{dt}(\omega - \omega_0) + K_d(\omega - \omega_0) = P_0 - P$$

We define one of the SGs as SG1 and the other as SG2. Equivalently, all coefficients get the same extension, so  $K_{d1}$  is the damping coefficient of SG1 and  $K_{d2}$  is the damping coefficient of SG2.

For the investigation of the contribution of the damping factor, we set  $K_{d2} = \frac{K_{d1}}{2}$ .

The term  $J \frac{d}{dt}(\omega - \omega_0)$  is only of influence in the transition periods, so we could neglect them and only look outside the transition periods. The swing equation results in:

$$K_d(\omega - \omega_0) = P_0 - P$$

or, equivalently:

$$\omega = \omega_0 - \frac{1}{K_d} (P - P_0) \quad (3.3)$$

For both generators (3.3) holds and if we write them in terms:

$$\omega_1 = \omega_0 - \frac{1}{K_{d1}} (P_1 - P_0) \quad (3.4)$$

and

$$\omega_2 = \omega_0 - \frac{1}{K_{d2}} (P_2 - P_0) = \omega_0 - \frac{2}{K_{d1}} (P_2 - P_0) \quad (3.5)$$

With these two equations we can draw the  $\omega - P$  graph again, now for both SGs, which is depicted in Figure 3.7.

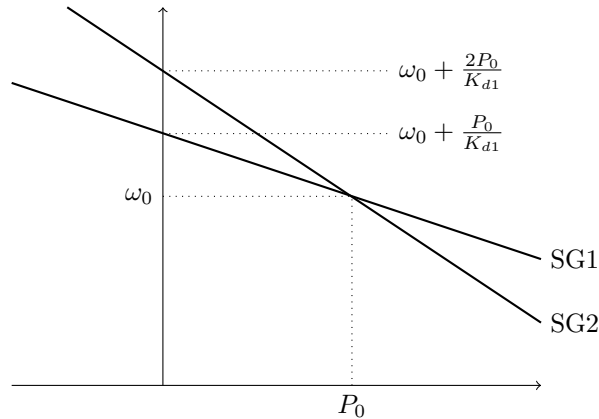


Figure 3.7:  $\omega - P$  droop control for two parallel generators with different  $K_d$

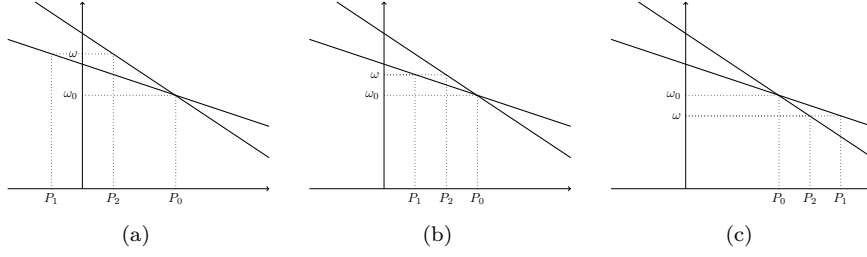


Figure 3.8: (a) shows the situation where there are no loads,  $P_1$  is canceled against  $P_2$ , (b) shows  $P_T = P_0$  and (c) shows  $P_T = 3P_0$

We can establish a relation between the active power outputs  $P_1$  and  $P_2$ . In steady state, so after the transition period, we have that  $\omega_1 = \omega_2$ . This means that (3.4) is equal to (3.5) and we can write:

$$2(P_2 - P_0) = P_1 - P_0 \quad (3.6)$$

which results in:

$$P_1 = 2P_2 - P_0 \quad (3.7)$$

Now we will look at different situations. We start with the case where no load is connected, which means that there is no active power demand. The total delivered power,  $P_T = P_1 + P_2$  is equal to zero. This means that  $P_2 = -P_1$ . Substituting this in (3.7) yields  $P_1 = -\frac{1}{3}P_0$  and thus  $P_2 = \frac{1}{3}P_0$ . This can be seen in Figure 3.8(a).  $P_1$  cancels out  $P_2$  by acting like a synchronous motor, i.e. it is demanding active power of the network.

Next we look at the situation where one load is connected. This means that the sum of the active power delivered by the SGs should be equal to  $P_0$ , i.e.  $P_T = P_0$ . This means that  $P_2 = P_0 - P_1$ . Substituting this in (3.7) results in  $P_1 = \frac{1}{3}P_0$  and  $P_2 = \frac{2}{3}P_0$ . This situation is shown in Figure 3.8(b).

Then, when we add another load, the total power demand will be  $2P_0$ . This means that  $P_2 = 2P_0 - P_1$ . Substituting this in (3.7) yields  $P_1 = P_0$  and  $P_2 = P_0$ . This case was already shown in Figure 3.7.

Finally, when the total demand is  $3P_0$  we have  $P_2 = 3P_0 - P_1$ . Substituting this in (3.7) results in  $P_1 = \frac{5}{3}P_0$  and  $P_2 = \frac{4}{3}P_0$ . This case is shown in Figure 3.8(c).

We again model the situation. Figure 3.9 shows the frequency and output active power of the parallel SGs. Now we can look at different situations. At time  $t = 0$  and  $t = 6$  there is no load connected. We observe that the active power output are as we predicted,  $P_1 = -\frac{1}{3}P_0$  and  $P_2 = \frac{1}{3}P_0$ . At time  $t = 1$  and  $t = 5$ , there is only one load connected. This means that the sum of the active power delivered by the SGs should be equal to  $P_0$ . Again the active power outputs are in line with the theory, namely  $P_1 = \frac{1}{3}P_0$  and  $P_2 = \frac{2}{3}P_0$ . At time  $t = 2$  and  $t = 4$  we observe that  $P_1 = P_2 = P_0$  and  $\omega_1 = \omega_2 = \omega_0$ . Finally, at time  $t = 3$ , we see that the third load is added, which results indeed in  $P_1 = \frac{5}{3}P_0$  and  $P_2 = \frac{4}{3}P_0$ . We can conclude that our model is working correctly.

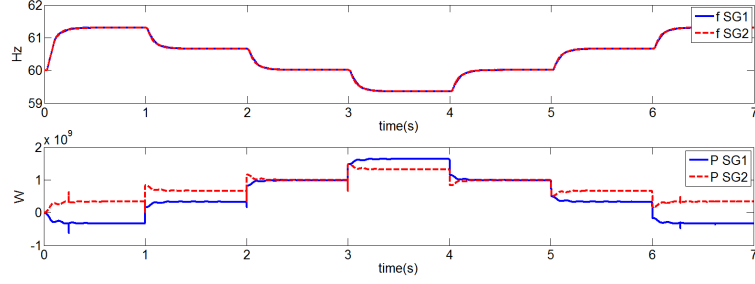


Figure 3.9: Frequency of two parallel SGs with  $K_{d2} = \frac{K_{d1}}{2}$

This concludes the situation where we alter the damping factor  $K_d$ . We started with neglecting the inertia term of the swing equation, in the next section this is the term that will be investigated.

### 3.3.2 Different $J$

In this subsection we will look at different values for the moment of inertia. First we switch back to the network with just one SG and measure the frequency and active power output for a moment of inertia  $J$ . Then we change the moment of inertia to  $\frac{J}{2}$  and plot both cases in the same graph shown in Figure 3.10.

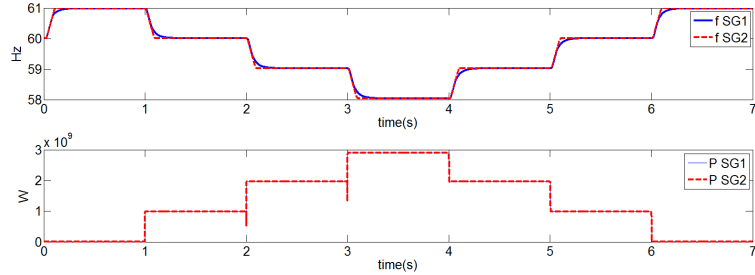


Figure 3.10: Frequency and active power output of a SG with two different moments of inertia

It can be observed that the case with the smaller moment of inertia, so the red graph, is quicker in getting to the steady state. This means that the term  $J \frac{d}{dt} (\omega - \omega_0)$  has become zero. It is not difficult to see why a smaller moment of inertia results in a faster convergence to steady state.

In the first case the moment of inertia is equal to  $J$ . The swing equation can be rewritten to:

$$\frac{d}{dt} \Delta\omega = \frac{\Delta P}{J} + \frac{K_d}{J} \Delta\omega$$

where  $\Delta\omega = \omega - \omega_0$  and  $\Delta P = P - P_0$ .

In the second case the moment of inertia is  $\frac{J}{2}$ . This time:



$$\begin{aligned}
\frac{d}{dt}\Delta\omega &= \frac{\Delta P}{\frac{J}{2}} + \frac{K_d}{\frac{J}{2}}\Delta\omega \\
&= \frac{2\Delta P}{J} + \frac{2K_d}{J}\Delta\omega \\
&= 2\frac{d}{dt}\Delta\omega
\end{aligned}$$

This means that when you decrease  $J$  to half the value in the original case, you double the speed of convergence to steady state. It should also be observed from Figure 3.10 that the active power output of the generator remains the same in the two different cases.

However, this section is about parallel SGs, which means that we should look at the situation of two parallel SGs where the moments of inertia are different from one another. Again, the moments of inertia are chosen as  $J_2 = \frac{J_1}{2}$  and the result is shown in Figure 3.11. Note that we start the simulation when there is already a load of size nominal active power connected. At  $t = 1$  and  $t = 2$  the second and third identical loads are connected and at  $t = 3$  and  $t = 4$  they are disconnected again.

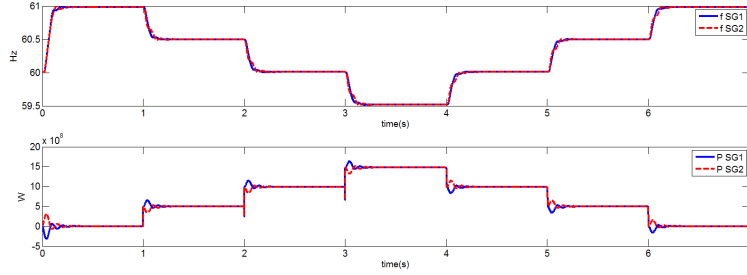


Figure 3.11: Frequency and active power output of two parallel SGs with different moments of inertia

We observe that the first SG with the higher moment of inertia reaches the steady state frequency faster, which was to be expected. Because it reaches this frequency faster, the power it produces will have an overshoot whenever a load is added, and an undershoot whenever a load is deleted. This can be seen in the active power output plot.

In the next subsection we alter the nominal power  $P_0$ .

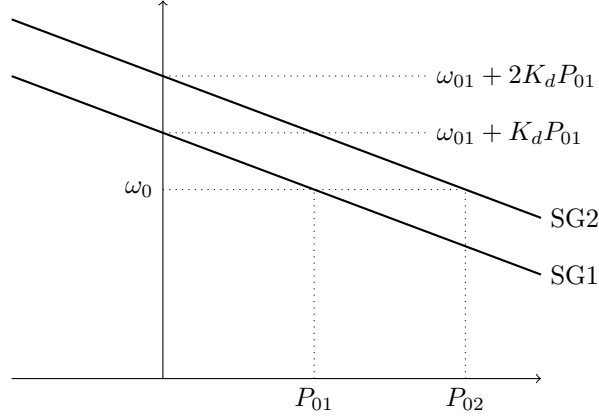


Figure 3.12:  $\omega - P$  droop control

### 3.3.3 Different nominal power

The nominal power  $P_0$  is a parameter that can be chosen in the synchronous machines available in SIMULINK. When the load is equal to the nominal power, the frequency at which the generator is working should be equal to the nominal frequency, in our case chosen as 60 Hz. We will now examine what will happen when the nominal powers of the two parallel generators are not equal. We denote the nominal power of SG 1 and SG 2 by  $P_{01}$  respectively  $P_{02}$ . We choose the nominal power as  $P_{02} = 2P_{01}$ . The total nominal power is now  $3P_{01}$ , which means that when three loads are connected to the two parallel generators, each with active power demand equal to  $P_{01}$ , the frequency of the grid should become 60 Hz.

For both generators we write out (3.3):

$$\omega_1 = \omega_0 - \frac{1}{K_d} (P_1 - P_{01}) \quad (3.8)$$

and

$$\omega_2 = \omega_0 - \frac{1}{K_d} (P_2 - P_{02}) = \omega_0 - \frac{1}{K_d} (P_2 - 2P_{01}) \quad (3.9)$$

We draw the  $\omega - P$  graph again, which is depicted in Figure 3.12.

We can establish a relation between the active power outputs  $P_1$  and  $P_2$ . In steady state, so after the transition period, we have that  $\omega_1 = \omega_2$ . This means that (3.8) is equal to (3.9) and we can write:

$$P_1 - P_{01} = P_2 - P_{01} \quad (3.10)$$

which results in:

$$P_1 = P_2 - P_0 \quad (3.11)$$

where we have chosen  $P_{01} = P_0$

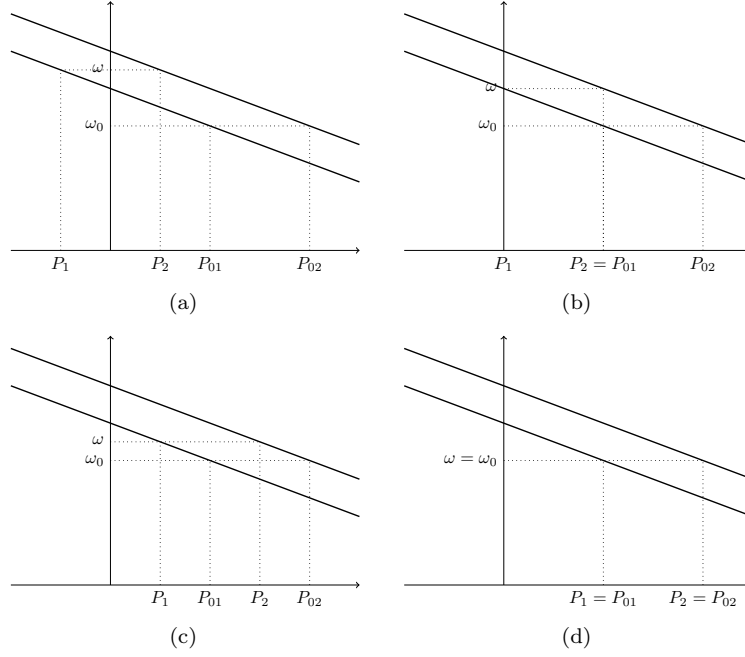


Figure 3.13: (a) shows the situation where there are no loads,  $P_1$  is canceled against  $P_2$ , (b) shows the situation  $P_T = P_0$ , (c) shows  $P_T = 2P_0$  and (d) shows  $P_T = 3P_0$

Now we will look at different situations. We start with the case where no load is connected, which means that there is no active power demand. The total delivered power,  $P_T = P_1 + P_2$  is equal to zero. This means that  $P_2 = -P_1$ . Substituting this in (3.11) yields  $P_1 = -\frac{1}{2}P_0$  and thus  $P_2 = \frac{1}{2}P_0$ . This can be seen in Figure 3.13(a).

Next we look at the situation where one load is connected. This means that the sum of the active power delivered by the SGs should be equal to  $P_0$ , i.e.  $P_T = P_0$ . This means that  $P_2 = P_0 - P_1$ . Substituting this in (3.11) results in  $P_1 = 0$  and  $P_2 = P_0$ . This situation is shown in Figure 3.13(b).

Then, when we add another load, the total power demand will be  $2P_0$ . This means that  $P_2 = 2P_0 - P_1$ . Substituting this in (3.7) yields  $P_1 = \frac{1}{2}P_0$  and  $P_2 = \frac{3}{2}P_0$ . This case is shown in Figure 3.13(c).

In the last case, when the total demand is  $3P_0$  we have  $P_2 = 3P_0 - P_1$ . Substituting this in (3.7) results in  $P_1 = P_0$  and  $P_2 = 2P_0$ . This case is shown in Figure 3.13(d).

Now we will show the result we found in our SIMULINK model. Again the loads are the same as before and so are there operating periods. The results are seen in Figure 3.14

We observe that at  $t = 3$ , when the third load is added to the network, both

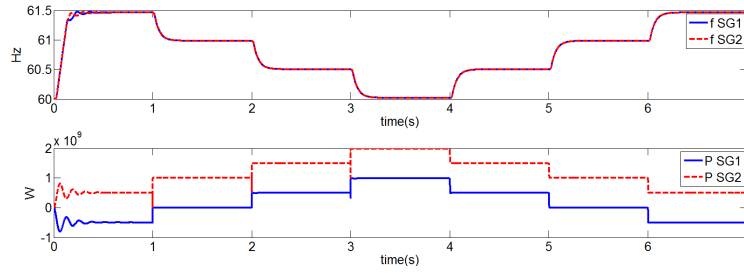


Figure 3.14: Frequency and active power output of two parallel SGs with different nominal power

SGs have reached their nominal power. At this time both frequencies of the generators have reached the nominal frequency of 60 Hz. Again there are some losses due to the internal resistance and self inductance of the generators. Furthermore, SG2 delivers twice as much power in comparison to SG1 whenever there is a load connected to the network.

At time  $t = 0$  and  $t = 6$  there is no load connected. The frequency the two SGs are operating on, will be the average of the two measured on the x-axis of the  $\omega - P$  graph. At this frequency  $P_1$  is negative and  $P_2$  is positive and they cancel each other out to get a total power of zero. The values are as we predicted.

At time  $t = 1$  and  $t = 5$ , there is one load connected with a demand of  $P_0$ . The frequency of both SGs is adjusted to the only frequency at which the sum of  $P_1$  and  $P_2$  is equal to  $P_0$ . This results in the point where  $P_1 = 0$  and  $P_2 = P_0$ , which is as we predicted.

Finally, at time  $t = 2$  and  $t = 4$  the total load is  $2P_0$ . The only frequency at which  $P_1 + P_2 = 2P_0$  is found which results indeed in  $P_1 = \frac{1}{2}P_0$  and  $P_2 = \frac{3}{2}P_0$ .

We again conclude that our model is working correctly.

Next, and as a final case, we will look at what happens when we alter the nominal frequency of one of the two parallel SGs.

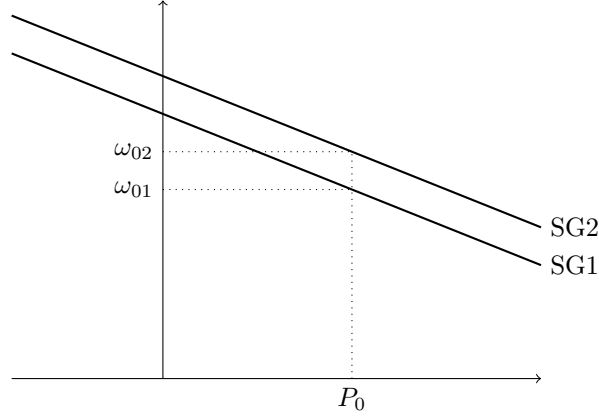


Figure 3.15:  $\omega - P$  droop control

### 3.3.4 Different nominal frequency

The nominal frequency is the frequency at which the generator is working when it is supplying the system with nominal power. We are going to look at what happens when we change the nominal frequency of the second parallel generator,  $\omega_{02}$ , to  $2\pi 60.5$  Hz in stead of  $2\pi 60$  Hz, which is the nominal frequency of the first SG, denoted by  $\omega_{01}$ .

For both generators we write out (3.3):

$$\omega_1 = \omega_{01} - \frac{1}{K_d} (P_1 - P_0) \quad (3.12)$$

and

$$\omega_2 = \omega_{02} - \frac{1}{K_d} (P_2 - P_0) \quad (3.13)$$

We can draw the  $\omega - P$  graph again, which is depicted in Figure 3.15.

The nominal active power of both generators are the same and so is the damping factor  $K_d$ . This means that the slope of both graphs are the same, which causes the difference between active power output of the two generators two have a constant difference. What happens is the following; The total load connected to the network changes. This causes the frequency of the network to take on a new value. Both generators frequencies converge to this value and the corresponding power is given to the network.

We can establish a relation between the active power outputs  $P_1$  and  $P_2$ . In steady state, so after the transition period, we have that  $\omega_1 = \omega_2$ . This means that (3.12) is equal to (3.13) and we can write:

$$P_1 = \frac{\omega_{01} - \omega_{02}}{2K_d} + P_2 \quad (3.14)$$

(3.14) shows that the difference between active power output stays constant, regardless of the load that is demanded.

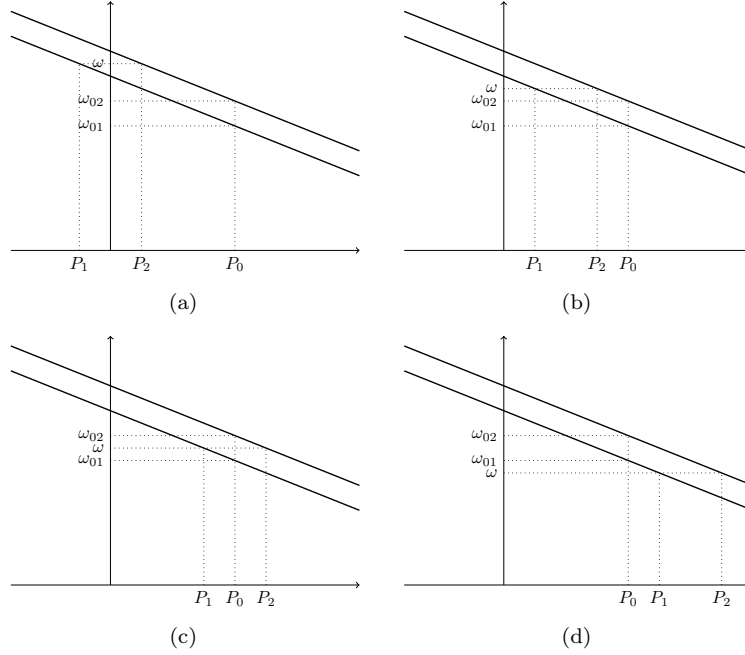


Figure 3.16: (a) shows the situation where there are no loads,  $P_1$  is canceled against  $P_2$ , (b) shows the situation  $P_T = P_0$ , (c) shows  $P_T = 2P_0$  and (d) shows  $P_T = 3P_0$

We will explain, with use of Figure 3.16, which is the  $\omega - P$  graph, what is happening at each different load.

We start with the case where there are no loads connected to the network. It should be noted that at these times the two power outputs cancel each other, which makes sense, since the total load is zero. Figure 3.16(a) shows this situation.

Next, there is just one load connected. The total active power is equal to  $P_0$ , so  $\omega > \omega_0$  and this results in  $P_1 < P_0$  and  $P_2 < P_0$  which can be seen in Figure 3.16(b).

Then two loads are connected, meaning that the nominal power is asked. This leads to a grid frequency of 60.25 Hz, since this is the average of the two nominal frequencies. This results in the active power outputs shown in Figure 3.16(c), where  $P_1 < P_0$  and  $P_2 > P_0$ .

In the last graph, Figure 3.16(d), the case is shown where the total power load is bigger than the nominal power. This results in a grid frequency below 60.25 Hz, resulting in an active power output greater than  $P_0$  for both generators.

We will again use a simulation of our SIMULINK model to check if our theories are correct. These can be seen in Figure 3.17

First, when we look at the frequency plot, we observe that at time  $t = 2$  and

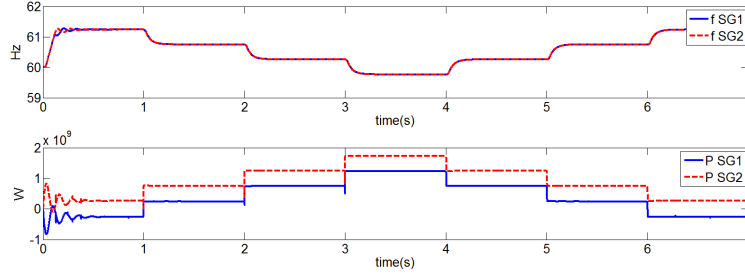


Figure 3.17: Frequency and active power output of two parallel SGs with different nominal frequency

$t = 4$  the frequency is roughly 60.25 Hz. At these time instances, there are two loads connected both of the same value as the nominal power of the SGs, which results in the fact that the generators will work on their nominal frequency. However, since the two generators are identical in every parameter except the nominal frequency, their frequencies will converge to the exact average of the two, which is 60.25 at nominal power.

At time  $t = 0$  and  $t = 6$ , no loads are connected. The frequency is higher than either of the two nominal values and one of the SGs is operating as a motor again. At  $t = 1$  and  $t = 5$ , one load is connected, resulting in still a frequency higher than both nominal values. Finally, at  $t = 3$  three identical loads are connected, resulting in a frequency lower than both nominal values.

Again, the SIMULINK model worked as we expected.

This concludes our investigation of the  $P - f$  relation in parallel Synchronous Generators and with that we conclude the chapter about SG. It is shown by the test results that the models are operating with high accuracy, since our findings were the same as we would have expected by working out the equations. The Synchronous Generator is working quite accurate in delivering the active power demand, not only at standalone mode but also when multiple SGs are working at the same time.

In the next chapter we will look at a second power source, the voltage source inverter. In the chapter after that one we will look at the similarities and differences between the SG and this inverter.

## Chapter 4

# Voltage source inverter

Besides generators, the most common way to produce power to share the required loads, are with the use of inverters. Inverters transfer a given DC-input into a AC-output, with the use of droop control. One of these inverters is the so called voltage source inverter. We look at the case where the original droop equation, (1.1) holds [8], [18], [12], [14].

In this chapter we will outline the idea behind, the construction and the outcomes of using a voltage source inverter as a power source to deliver the active power demand of the loads. From now on we will refer to the voltage source inverter as simply the inverter.

We start by showing how the inverter tries to act as an SG in section 4.1. After this we will explain, in section 4.2, some mathematical techniques that we are using inside our inverter. Then we will explain how the inverter works in section 4.3. In section 4.4 we will observe the behavior of a single inverter connected to multiple loads. Then we look at the grid-connected mode in section 4.5. From there we will look at two parallel inverters, where we will change the droop constants, the nominal frequencies and the nominal power in order to observe the influences of these factors on the active power droop control. This will be done in section 4.6.

So first the relation between a SG and an inverter.

### 4.1 SG and inverters

Synchronous generators follow the swing equation given by (3.1). Inverters do not have an inherent physical relation between frequency and the active power it generates, nor does it have a an inherent physical relation between voltage amplitude and generated reactive power. What we try to achieve with inverters, is copying the behavior of a SG to create such relations for the inverter [22].

We are only looking at the ordinary droop equation at this point, i.e. we are looking at  $P - f$  droop control. This is because we are looking at small, purely inductive lines. This equation means that the frequency is in relation with the active power. We use a simple proportional controller which are:



$$\omega = \omega_0 - k_p (P_m - P_0) \quad (4.1)$$

The measured active,  $P_m$ , is being filtered to get rid of the resonance. This is done by a transfer function  $H(s)$  defined by  $\frac{1}{\tau s + 1}$ . Here,  $\tau$  is called the delay and the bigger it is selected, the less resonance we will obtain in the active power. The downside is, however, that the transition time becomes longer. Actually, the delay factor  $\tau$  can be compared with the inertia constant  $J$  in the synchronous generator. This can be seen as follows.

The transfer function represents a low pass filter, which we will explain in an upcoming section. The filter can be written as:

$$\dot{P}_m = \frac{P - P_m}{\tau} \quad (4.2)$$

If we differentiate (4.1) with respect to time we will get  $\dot{\omega} = -k_p \dot{P}_m$ . Substituting (4.2) yields:

$$\tau \dot{\omega} = -\omega + \omega_0 - k_p (P - P_0) \quad (4.3)$$

Comparing (4.3) to the swing equation (3.1) shows that indeed  $\tau$  corresponds to the inertia term  $J$ . In Figure 4.1 one can see the transition times for different values of  $\tau$ . It holds that for larger  $\tau$  the transition time becomes larger but the resonance becomes smaller. It is actually a trade off between the two.

But we are getting ahead of ourselves. First we have to explain how the inverter operates and before we do that we have to explain some mathematical techniques, which we will do in the next section.

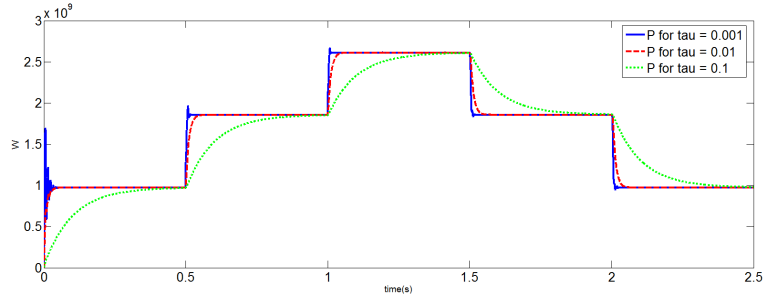


Figure 4.1: Transition time and resonance of the active power with different time delays

## 4.2 Different mathematical techniques

In this section we will explain some techniques we will be using in the construction of our inverter. We will look at:

- Phase-Locked Loop (subsection 4.2.1)
- dq0-transformation (subsection 4.2.2)
- Pulse Width Modulation (subsection 4.2.3)
- LC-filter (subsection 4.2.4)

Figure 4.2 shows the schematics of the inverter and where all techniques listed above are situated.

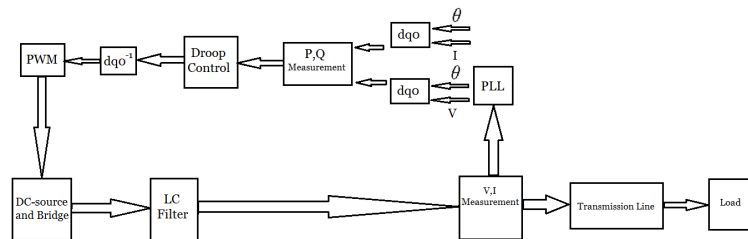


Figure 4.2: Schematic overview of the voltage source inverter

### 4.2.1 Phase-Locked Loop

The goal of a Phase-Locked Loop, PLL, is to determine the frequency and phase of an incoming signal, in our case a three-phase voltage [?]. Figure 4.3 shows the diagram of the PLL we used, which is the standard block from SIMULINK.

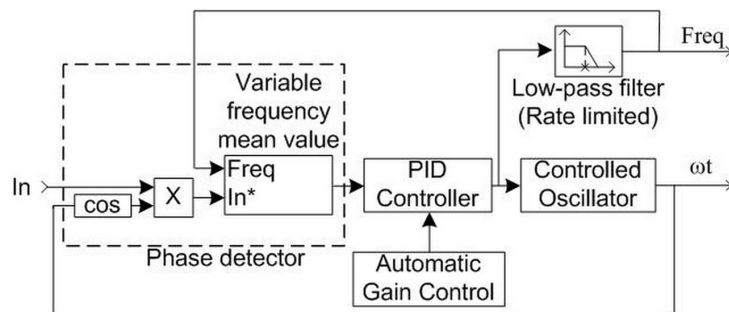


Figure 4.3: Phase-Locked Loop from SIMULINK

The PLL block consists of five main elements: The phase detector, the controlled oscillator, the PID-controller, the automatic gain controller and the low-pass filter.

The first element in the PLL is the phase detector. It takes the input signal and the output of the controlled oscillator and mixes them to produce a voltage proportional to the phase difference between the two signals. Next a PID-controller and an automatic gain controller make sure that the phase difference becomes constant by letting the internal oscillator adjust its frequency. The frequency of the oscillator is drawn to the frequency of the input signal which makes the phase difference will become constant. At this point the loop is locked. The low-pass filter is used to filter out higher order of harmonics in the angular velocity and then we divide by  $2\pi$  to obtain the frequency.

The PLL block outputs besides the frequency also  $\omega t$  which can be used as input for the so called dq0-transformation which we will explain next.

#### 4.2.2 dq0-transformation

The direct-quadrature-zero-transformation rewrites the three-phase input signal from a stationary reference frame into a new rotational reference frame [9]. The angular speed at which this new frame rotates is synchronized with the input, in our case the voltage. First we will explain how the transformation works and then the benefits are explained.

We define the three-phase balanced voltage by:

$$U(t) = \begin{pmatrix} U_a \\ U_b \\ U_c \end{pmatrix} = \begin{pmatrix} |U| \cos(\omega t) \\ |U| \cos\left(\omega t - \frac{2\pi}{3}\right) \\ |U| \cos\left(\omega t + \frac{2\pi}{3}\right) \end{pmatrix} \quad (4.4)$$

Here,  $|U|$  is the magnitude of the voltage. It is clear that  $U(t)$  is balanced, since all the coordinates are  $120^\circ$  apart from one another. The first step in the dq0-transformation is the Clark transformation. The Clark transformation is just a rotation around the a-axis followed by a rotation around the b-axis in order to drive the third coordinate to zero. We rotate first  $-45^\circ$  around the a-axis and then  $35.26^\circ$  around the b-axis. Writing this in transformation matrices we have that the Clark transformation matrix  $T_1$  is given by:

$$T_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & 0 & \frac{-1}{\sqrt{3}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{3}} & 0 & \frac{\sqrt{2}}{\sqrt{3}} \end{pmatrix} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (4.5)$$

When we multiply (4.4) with this transformation matrix we observe that the third coordinate has become zero:

$$T_1 U(t) = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} |U| \cos(\omega t) \\ |U| \cos\left(\omega t - \frac{2\pi}{3}\right) \\ |U| \cos\left(\omega t + \frac{2\pi}{3}\right) \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{3}}{\sqrt{2}} |U| \cos(\omega t) \\ \frac{\sqrt{3}}{\sqrt{2}} |U| \sin(\omega t) \\ 0 \end{pmatrix} \quad (4.6)$$

The three coordinates are now  $\alpha$ ,  $\beta$  and  $z$ , in which  $z$  is zero.

After the Clark transformation, we rotate one more time. This time we rotate around the  $z$ -axis with angular speed  $\theta$  and this transformation  $T_2$  is given by:

$$T_2 = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (4.7)$$

The total dq0-transformation matrix  $T$  is given by  $T_2 T_1$ :

$$\begin{aligned} T &= \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \\ &= \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\theta) & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ -\sin(\theta) & -\sin(\theta - \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \end{aligned} \quad (4.8)$$

This transformation is power invariant, meaning that  $P = u_a i_a + u_b i_b + u_c i_c = u_d i_d + u_q i_q$ . A proof of this statement can be found in the Appendix.

Now we will explain why it is useful to switch to this new reference frame. A variant of  $T$  is called the Park's transformation and the only difference it has with  $T$  is that the constant factor in front of the transformation matrix is  $\frac{2}{3}$  instead of  $\sqrt{\frac{2}{3}}$ . This means that (4.5) has become:

$$T'_1 = \frac{2}{3} \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (4.9)$$

When we multiply (4.4) with  $T'_1$ , we obtain:

$$T'_1 U(t) = \frac{2}{3} \begin{pmatrix} 1 & \frac{-1}{2} & \frac{-1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{-\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} |U| \cos(\omega t) \\ |U| \cos(\omega t - \frac{2\pi}{3}) \\ |U| \cos(\omega t + \frac{2\pi}{3}) \end{pmatrix} = \begin{pmatrix} |U| \cos(\omega t) \\ |U| \sin(\omega t) \\ 0 \end{pmatrix} \quad (4.10)$$

Now, if we let the second transformation rotate with a speed that is synchronized with the angular speed of the voltage, i.e.  $\theta = \omega t$ , we get:

$$\begin{pmatrix} u_d \\ u_q \end{pmatrix} = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} |U| \cos(\omega t) \\ |U| \sin(\omega t) \end{pmatrix} = \begin{pmatrix} |U| \\ 0 \end{pmatrix} \quad (4.11)$$

We neglected the  $z$ -axis since we showed that this became zero. We have obtained that the  $u_q$  term is driven to zero and  $u_d$  is the same as the voltage magnitude. We can do the same for the current, where the current 'lags' the voltage by phase  $\phi$ , i.e.  $I(t) = |I| \cos(\omega t - \phi)$ . Here  $|I|$  is the magnitude of the current. The second transformation is still synchronized with the voltage, so (4.11) becomes the following for the current:

$$\begin{pmatrix} i_d \\ i_q \end{pmatrix} = \begin{pmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{pmatrix} \begin{pmatrix} |I| \cos(\omega t - \phi) \\ |I| \sin(\omega t - \phi) \end{pmatrix} = \begin{pmatrix} |I| \cos(\phi) \\ -|I| \sin(\phi) \end{pmatrix} \quad (4.12)$$

This means that when  $\phi = 0$ , which is the case for a resistive element, the  $i_q$  term is zero and the  $i_d$  term is the magnitude of the current. For inductive or capacitive elements,  $\phi \neq 0$ .

The apparent power was calculated by  $S = UI^*$ , were we could write the voltage and current as:

$$U = u_d + ju_q \quad (4.13)$$

$$I = i_d + ji_q \quad (4.14)$$

Now, the apparent power becomes:

$$\begin{aligned} S &= UI^* \\ &= (u_d + ju_q)(i_d - ji_q) \\ &= u_d i_d + u_q i_q + j(u_q i_d - u_d i_q) \end{aligned} \quad (4.15)$$

This means that the active power and reactive power are given by:

$$P = u_d i_d + u_q i_q \quad (4.16)$$

$$Q = u_q i_d - u_d i_q \quad (4.17)$$

Since we just showed that  $u_q = 0$ , we have that  $P = u_d i_d$  and  $Q = -u_d i_q$ . This means that, in the dq0 reference frame, we can control the active and reactive power by independently  $i_d$  and  $i_q$ . An other benefit is that we can now work with just two quantities, in stead of the original three, which makes calculations easier.

The inverse dq0-transformation, the power invariant form, is given by:

$$T^{-1} = \sqrt{\frac{2}{3}} \begin{pmatrix} \cos(\theta) & -\sin(\theta) & \frac{1}{\sqrt{2}} \\ \cos(\theta - \frac{2\pi}{3}) & -\sin(\theta - \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \\ \cos(\theta + \frac{2\pi}{3}) & -\sin(\theta + \frac{2\pi}{3}) & \frac{1}{\sqrt{2}} \end{pmatrix} \quad (4.18)$$

This matrix is needed to switch back to the abc reference frame.

In the next section we will explain how the Pulse Width Modulation works.

### 4.2.3 PWM

Pulse Width Modulation (PWM) is used to translate an input signal into a pulsing signal. In our case, this input signal is the voltage. The output signal is a pulse which differs between 1 and 0. Here, 1 corresponds to the voltage that is being switched on, and 0 corresponds to the voltage being switched of. The way the PWM accomplishes this task is by comparing the input signal to the

triangle carrier function. Whenever the input signal has a value that is higher than the carrier function, the output is 1 and whenever the input is lower than the carrier function, the output is 0. This technique is showed in Figure 4.4. One can specify the frequency of this carrier function. This frequency has to be much higher than the frequency of the grid so it does not negatively influences the load.

In the PWM, pulse width is adjusted in order to increase or decrease the average output value. As can be seen in Figure 4.5, the bigger the pulse width, the higher the average value will be. In this way the difference in voltage magnitude obtained from the droop control are delivered back to the network.

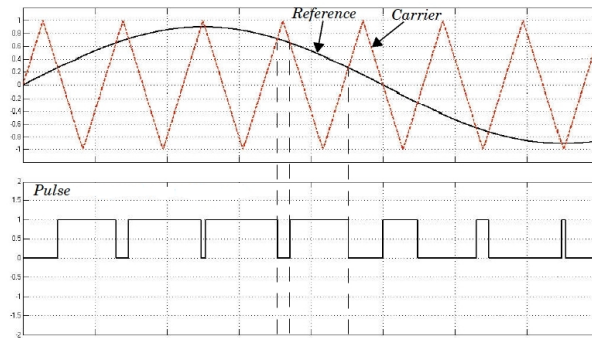


Figure 4.4: Pulse modulation by comparing the reference signal to the carrier signal

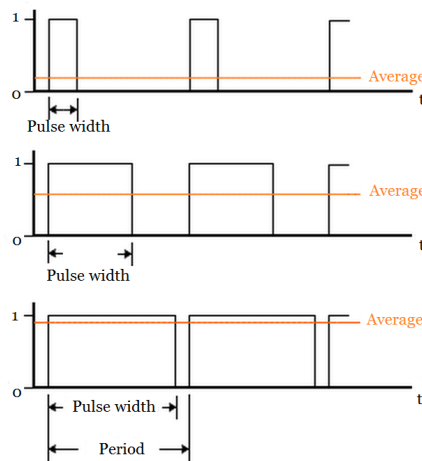


Figure 4.5: PWM: How the pulse width influences the average value

The PWM outputs in total six pulses, two for each phase. One of these two pulses is the signal we described above and the second is the inverse of this signal, i.e. whenever the first signal has value 1, the second has value 0 and vice

verse. These six pulses are given to the universal bridge. Since the output of the bridge is not yet a smooth abc-signal, we use a LC-filter to obtain a harmonic signal. This is a very time consuming part in the construction of the model. We will show how you can use inductors and capacitors to change the signal in the next subsection.

#### 4.2.4 LC-Filter

In a LC-filter, we have a inductor in series and a capacitor in parallel with the grid. Their impedances are denoted by  $Z_1$  and  $Z_2$ , as can be seen in Figure 4.6.

The ratio between voltages,  $\frac{V_{out}}{V_{in}}$ , defines the Laplace transfer function. This transfer function is evaluated in the following way.

The current is calculated by:

$$I = \frac{V_{in}}{Z_1 + Z_2}$$

$V_{out}$  is the current multiplied by the second impedance, i.e.  $V_{out} = IZ_2$ . Substituting the current yields:

$$\begin{aligned} V_{out} &= IZ_2 \\ &= \frac{V_{in}}{Z_1 + Z_2} Z_2 \end{aligned}$$

$$<=>$$

$$\frac{V_{out}}{V_{in}} = \frac{Z_2}{Z_1 + Z_2}$$

Now we implement the impedances for an inductor and a capacitor to obtain:

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{\frac{1}{Cs}}{Ls + \frac{1}{Cs}} \\ &= \frac{1}{LCs^2 + 1} \end{aligned}$$

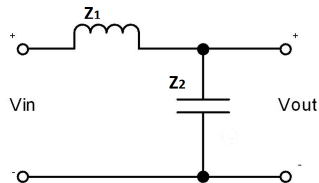


Figure 4.6: LC-filter

Now we have obtained our transfer function which we need to smooth the output of the inverter. It is much more convenient to filter the output with this transfer function in order to find the correct values for  $L$  and  $C$ , than to change the inductor and capacitor in the grid. Once a correct filter is found, it is known what the product of  $LC$  has to be and one can alter the inductor and capacitor accordingly.

This concludes the mathematical techniques we need to build our inverter. In the next section we will explain how it is constructed.

### 4.3 Structure of inverter

As we said in the introduction, an inverter computes a AC-output from a DC-input. We use a simple DC-voltage source, which is connected to a universal bridge. The bridge consists of an IGBT (Insulated-Gate Bipolar Transistor) connected to some diodes. We used the standard block in SIMULINK called Universal bridge. The bridge needs a PWM (Pulse Width Modulation) as input and the output is the three phase connected to the grid.

The pulse is generated with the use of the droop method, which we will explain next.

We start by measuring the voltage and current of the output of the inverter. We call these  $U_i$  respectively  $I_i$ . These measurements are in abc-coordinates, but what we need are dq0 coordinates. We use the dq0-transformation, explained in subsection 4.2.2, to obtain two DC quantities in stead of three AC quantities. The benefit of working in this new framework is that it simplifies the equations.

Now we have obtained the dq0 quantities, we can measure the active power  $P_m$ . The dq0 quantities are denoted by  $(u_d, u_q, u_0)^T$  and  $(i_d, i_q, i_0)^T$  for the voltage, respectively the current. From subsection 4.2.2 we know that

$$P_m = u_d i_d + u_q i_q \quad (4.19)$$

After filtering the measured active power, we apply droop control. At this point, we just look at  $P - f$  droop control:

$$\omega = \omega_0 - k_p(P_m - P_0) \quad (4.20)$$

where  $k_p$  is the frequency droop constant,  $\omega_0$  is the nominal angular frequency (note that  $\omega = 2\pi f$ , where  $f$  is the frequency), and  $P_0$  is the nominal active power.

Since we are not looking at voltage magnitude droop control, we choose our amplitude  $E$  as our nominal voltage, which is a constant. Note that the amplitude  $E$  corresponds to the d-coordinate, and we can choose the q-coordinate and 0-coordinate to be zero. When one is using  $Q - E$  droop control this does not hold, since the q coordinate is not equal to zero. This will be explained



later on when we are investigating reactive power.

We now have two values, the amplitude  $E$  and the angular frequency  $\omega$ . We integrate the angular frequency to obtain  $\omega t$ . Now we do the inverse dq0-transformation, see subsection 4.2.2, to obtain the abc-quantities again.

The obtained abc-signal is given to the PWM Generator as input, which creates a pulse that is sent to the universal bridge. The output of the PWM is not a smooth sinusoidal signal. This means that we need to filter this signal and we use a LC-filter to do so.

After filtering, the signal, a smooth sinusoidal signal, is given back to the network.

This concludes the most simplest case of the inverter, where we used the original droop controllers.

In the next section we will show how the inverter behaves when connected to multiple active loads.

## 4.4 A single inverter

In this subsection we look at the case where there is a single inverter connected to three identical loads, which are only active. We are just looking at active loads and  $P-f$  droop control. We will use a constant voltage magnitude which we give back to the inverse dq0-transformation.

In Figure 4.7 we see the result of the simulation. One can observe that the inverter behaves similar to the synchronous generator. Whenever a load is connected to the system, the frequency of the inverter lower and the active output it produces increases. Note that when the load is equal to the nominal power, which we set at 1000 MW, the frequency of the inverter is equal to the nominal frequency, i.e. 60 Hz. There are some errors, which are caused by the LC-filter and the transmission lines. We are still using a inductive line, as explained before.

Next we will look at a single inverter that is connected to the grid.

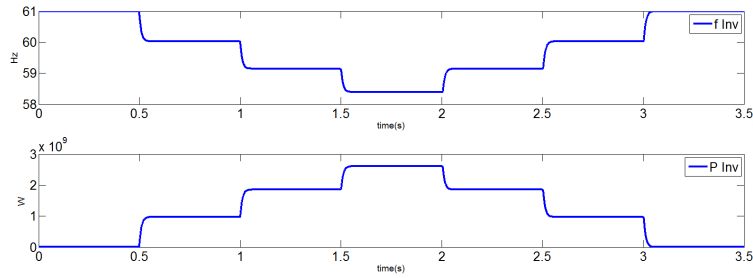


Figure 4.7: Frequency and active power output of the inverter

## 4.5 A single inverter connected to the grid

Figure 4.8 shows the result of a single inverter connected to the grid, which again we simulates with a voltage source. The frequency of the grid is chosen as 60 and we observe that the inverter is working at the same frequency. Whenever a load is added or removed the frequency decreases, respectively increases, but the frequency will quickly go back to the frequency of the grid. This is because the voltage source delivers more power, just as this was the case with the synchronous generator connected to the grid. In real life, multiple parallel inverters are connected to the grid, so they will have a bigger influence on the power demand. Also note that the active power output of the inverter converges quickly back to the nominal active power, which we set at 1000 MW. Next we will look at two parallel inverters, where we alter the values of the droop constant, the nominal frequency and the nominal power.

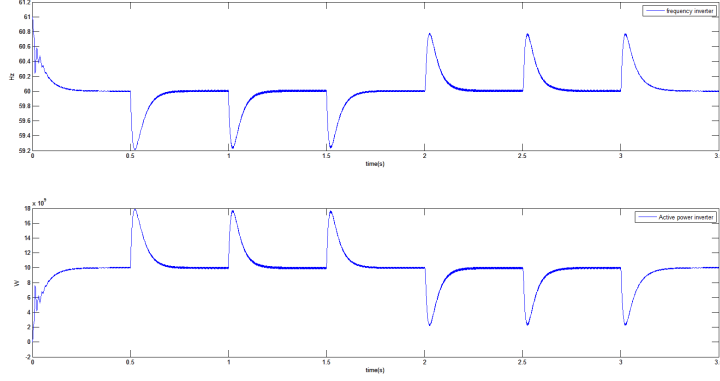


Figure 4.8: Frequency and active power output of an inverter that is connected to the grid

## 4.6 Parallel inverters

In this section we will be checking if the inverters operate in the same manner as SGs when put in parallel [16]. We look at three different cases:

- Different droop constant (subsection 4.6.1)
- Different nominal power (subsection 4.6.2)
- Different nominal frequency (subsection 4.6.3)

### 4.6.1 Different droop constants

The first thing we will look at is the droop constant  $k_p$ , which has the same function as the damping factor  $K_d$  of the synchronous generator. We let the droop constant of the second inverter be half of the droop constant of the first inverter. So we have:  $k_{p2} = \frac{1}{2}k_{p1}$ . Figure 4.9 shows the resulting frequency and active power output.

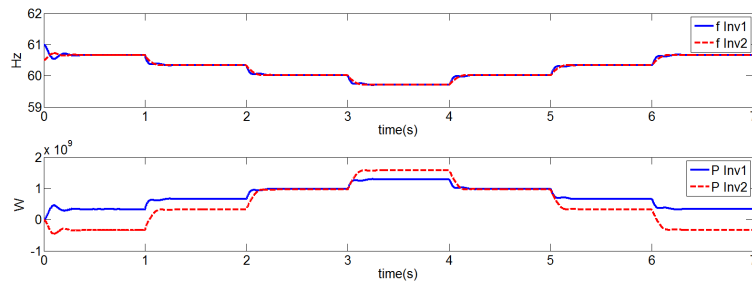


Figure 4.9: Two parallel inverters with  $k_{p2} = \frac{1}{2}k_{p1}$

We observe that the inverters are operating the same as the synchronous generators. When there are no loads connected, the total active power output is

zero, i.e.  $P_1 + P_2 = 0$ . This is the case at time  $t = 0$  and  $t = 3$ . Because the total power needs to be 0, the inverters are searching for a frequency for which  $P_1 + P_2 = 0$ . The only frequency for which this holds is the average of the two starting frequencies, in our case 60.75 Hz. When there is a load demand, the inverters behave in the same way as the synchronous generators. At time  $t = 2$  and  $t = 4$ , the total load is the same as the total nominal active power and one can observe that the frequency of the inverters is close to nominal and the active power outputs are roughly the same, namely  $P_0$ . There are some errors, mainly due to the filtering which is done by the LC-filter. When we compare this graph to Figure 3.9 we conclude that our model is working correctly.

Next we will look at what happens when the two inverters have different nominal power and check if the outcome is the same as in the case of two parallel SGs.

#### 4.6.2 Different nominal power

In this subsection we choose  $P_{02} = 2P_{01}$ . Figure 4.10 shows the results. Again we observe that the inverters behave the same as the generators did by referring to Figure 3.14. The difference in active power output is again constant.

We observe that at  $t = 3$ , when the third load is added to the network, both inverters have reached their nominal power. At this time both frequencies of the generators have reached the nominal frequency of 60 Hz. Again there where some losses due to the LC-filter and the transmission line.

At time  $t = 0$  and  $t = 6$  there is no load connected. At this frequency  $P_1$  is negative and  $P_2$  is positive and they cancel each other out to get a total power of zero.  $P_1 = -\frac{1}{2}P_0$  and  $P_2 = \frac{1}{2}P_0$  as we predicted in the case of the parallel SGs.

At time  $t = 1$  and  $t = 5$ , there is one load connected with a demand of  $P_0$ . The frequency of both inverters is adjusted to the only frequency at which the sum of  $P_1$  and  $P_2$  is equal to  $P_0$ . This results in the point where  $P_1 = 0$  and  $P_2 = P_0$ , which is as we predicted before.

Finally, at time  $t = 2$  and  $t = 4$  the total load is  $2P_0$ . This results in  $P_1 = \frac{1}{2}P_0$  and  $P_2 = \frac{3}{2}P_0$ , just as we obtained in section 3.3.3.

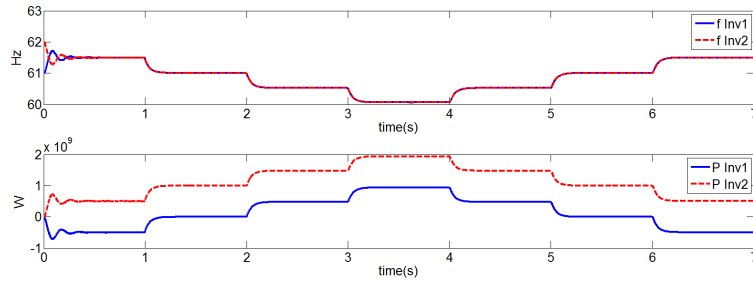


Figure 4.10: Frequency and active power output where  $P_{02} = 2P_{01}$

Finally we will alter the nominal frequency of one inverter and see if the network

behaves the same as it did with two SGs.

### 4.6.3 Different nominal frequency

In this subsection we choose  $\omega_{01} = 60$  Hz and  $\omega_{02} = 60.5$  Hz. Figure 4.11 shows the results. We observe that the inverters behave in the same way as the SGs did, by referring to Figure 3.17. The difference in output active power is always constant as was the case with the parallel generators in section 3.3.4.

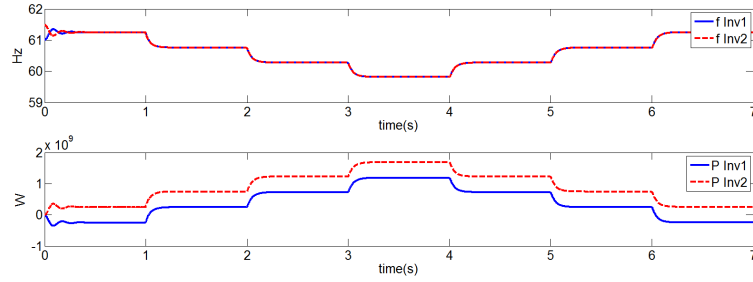


Figure 4.11: Frequency and active power output with  $f_{01} = 60$  Hz and  $f_{02} = 60.5$  Hz

With this, we end this chapter about the voltage source inverter. We have showed that it should operate in the same way as a SG and the results show that this is in fact the case. In the next chapter we will look at the case where we are using a SG and an inverter at the same time.

## Chapter 5

# Synchronous generator and inverter

In this chapter we are examining a network where both a synchronous generator and an inverter are connected in order to deliver the power that is demanded by the loads. The main difference between the two power sources, is the inertia term that is at hand in the generator. This term,  $J$ , is the term that determines the length of the transition time as is explained in section 3.3.2. In the inverter, there is a filter placed after the power measurement which acts as this inertia term. However, it is assumed, as can be verified in [25], that the inverter operates much quicker than the generator. Whenever a load is connected, the inverter delivers most of this load step and the generator raises its power output slowly until finally the inverter and the generator reach a steady state determined by their droop constant respectively damping factor. In the next section we will give the relation between the droop constant and the damping factor.

### 5.1 Relation between damping factor and droop coefficient

In order for the generator and the inverter to share the load equally, we will choose the droop coefficient  $k_p$  accordingly based on the damping factor  $K_d$ . This is done by first observing that the damping factor of the synchronous generator is in unit of torque over angular frequency, i.e.  $\frac{\tau}{\omega} = P$ . So it just has unit of power. It is also known, by the SIMULINK block for the generator, that the value of  $K_d$  is in per unit. This means that the damping factor in SI units is equal to the damping factor in per unit times the nominal power  $P_0$ . This means that the damping factor in SI is  $K_d P_0$ .

Note that the damping factor is in front of the  $\omega - \omega_0$  term in the swing equation:

$$J \frac{d}{dt}(\omega - \omega_0) + K_d(\omega - \omega_0) = P_0 - P$$

The inverter, if we neglect the filter, uses the simple droop controller:

$$\omega - \omega_0 = -k_p (P - P_0)$$

Here, the droop coefficient is in front of the  $P - P_0$  term. The unit of the coefficient is in angular frequency over power, i.e.  $\frac{\omega}{P}$ . The power  $P$  should be equal to the damping factor of the generator, since it was in terms of power, which leads to

$$k_p = \frac{\omega}{K_d P_0}$$

As can be seen in Figure 5.1, it is clear that the generator and inverter share the power equally, which means that this choice of droop coefficient was correct.

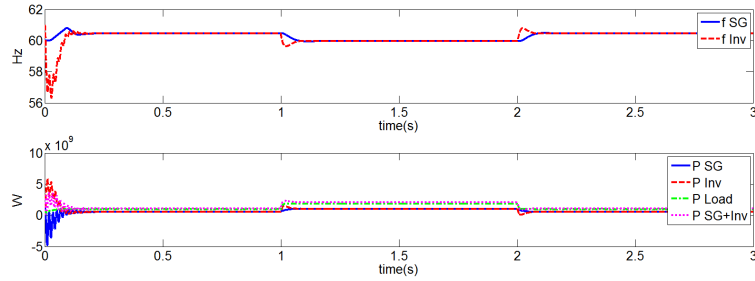


Figure 5.1: Frequency and active power output of a SG and an inverter acting in parallel where their droop characteristics are chosen in such a way that they share the load equally. In the second plot the active power load and the sum of the delivered load is also plotted.

It is also clear that the inverter acts quicker when the load changes. At time  $t = 1$  the load is doubled and one observes that the frequency of the inverter drops almost immediately, whereas the frequency of the SG slowly decreases to the steady state frequency. Note that since the total nominal power is 2 GW, we put the nominal power of the SG equal to the nominal power of the inverter, the steady state frequency after the second load is connected will be our nominal frequency of 60 Hz. There seems to be some start up time before the SG and inverter are operating on a steady state frequency, which is causing the strong deviations in frequency and active power output at the beginning of the simulation.

In the plot of the active powers, there are also the graphs of the active power of the load and the graph of the sum of the active power outputs of the SG and the inverter. It is to be observed that there is a difference between these two graphs. The total delivered active power will always be bigger than the load that is demanded. The difference is caused by the transmission lines and is called the 'active power loss'. When the load is doubled this difference becomes obviously bigger, the loss is bigger for larger load demands.

But why is there this difference in transition time, why does the inverter takes

one the load step. We will give an explanation in the next section.

## 5.2 Transient response

The generator uses the swing equation. This time we use  $P_m$  and  $P_e$ , to indicate that the mechanical power  $P_m$  is the nominal power  $P_0$  and the electrical power  $P_e$  is the generated power  $P$ :

$$J \frac{d}{dt}(\omega - \omega_0) + K_d(\omega - \omega_0) = P_m - P_e$$

The swing equation means that the generator adjusts the active power based on the error in the angular frequency, after which it adjusts the angular frequency in proportion to this measured power[25]. Figure 5.2 shows the dynamics of the generator. The inverter, however, directly adjusts its frequency in proportion to the power it measures. These are two fundamental different ways to adjust frequency, which causes the difference in the beginning of the load change.

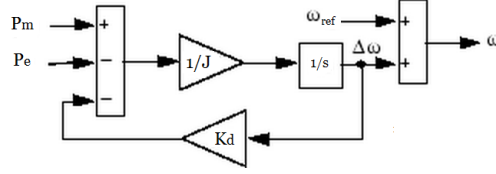


Figure 5.2: The feedback loop that drives the error in the angular frequency to zero

So it seems that the factor  $J$  is influencing the settling time to steady state. Since we divide by  $J$ , see Figure 5.2, a smaller value for  $J$  would correspond to a faster settling time. We run the simulation for three different values of the moment of inertia,  $2J$ ,  $J$  and  $\frac{1}{2}J$ . This result is shown in Figure 5.3. As can be seen, a smaller  $J$  correspond to a faster settling time to steady state.

In the next chapter we will look at secondary control.



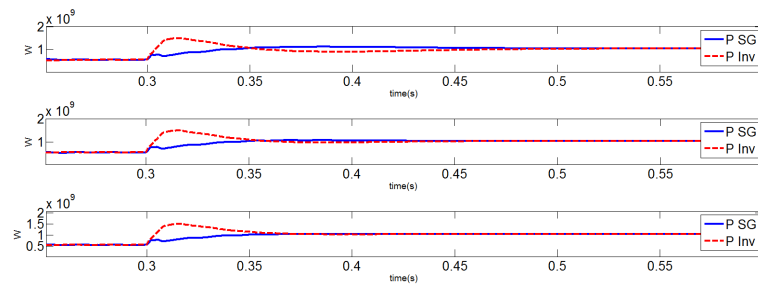


Figure 5.3: Different settling time to steady state. Upper graph is with inertia  $2J$ , the middle is with inertia  $J$  and the lower graph is with inertia  $\frac{1}{2}J$

## Chapter 6

# Secondary droop control

Both synchronous generators and inverters can be used to model the relation between frequency and active power output. The generators use the swing equation and the inverters use the droop control. Both power suppliers have deviations in the frequency they are working on. The purpose of secondary droop control is to eliminate those deviations despite of variable load [2]. We will look at both the single SG, section 6.1, and parallel SGs, section 6.2, and examine how the swing equations change. We could have done the same for the inverter, but since we showed that the inverter works in the same way as the SG, we will not look at these cases.

### 6.1 Secondary control in a single SG

In stead of the swing equation (3.1) we obtain:

$$J \frac{d}{dt}(\omega - \omega_0) + K_d(\omega - \omega_0) = P_0 - P + u \quad (6.1)$$

Here,  $u$  is the new secondary control. As choice of  $u$  we use the proposed integrated angular frequency deviation, so  $u = k_g \int (\omega - \omega_0) dt$ , see [2]. Here,  $k_g$  is a gain constant and has a unit that is the torque per second, i.e.  $\frac{N \cdot m}{s}$ . This extra term is subtracted from our mechanical power input, i.e. we are controlling the input in stead of keeping it constant, in order to deliver the required power. The input is changed in such a way that the measured power minus the nominal power converges to zero and hence  $\omega$  will converge to  $\omega_0$ .

In state space notation it will look like the following:

$$\dot{\theta} = \omega \quad (6.2)$$

$$u = k_g \theta \quad (6.3)$$

We now show the case where there is a single SG connected to a load and

two other identical loads are added to the system, the first at  $t = 3$  and the second at  $t = 6$ . Then at  $t = 9$  the first one is removed and at  $t = 12$  the second one is removed. We use secondary control on the active power input with  $u$  chosen as stated above. We plot both the input active power as well as the active power load together with the frequency at which the SG operates.

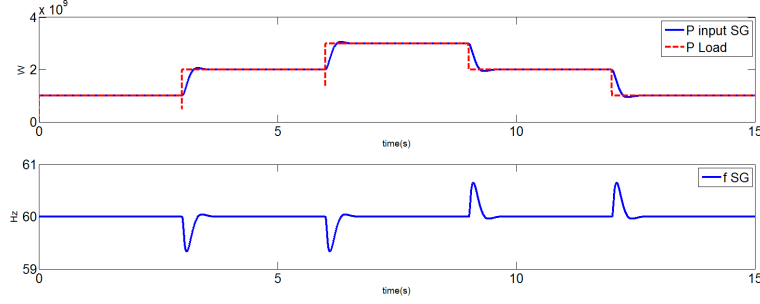


Figure 6.1: Active power input and frequency of a SG

As can be seen, the active power input is changed to be equal to the active power load. The speed at which the input power converges to the power load is determined by  $k_g$ . Furthermore one observes that the frequency of the SG converges to the nominal, 60 Hz due to the change in active power input.

Next we will look at two parallel SGs with different secondary control.

## 6.2 Secondary control for parallel SGs

We look at two parallel SGs, where we have two different values of  $k_g$ . The extra input  $u$  of the first SG,  $u_1$  is denoted by:

$$u_1 = -k_{g1} \int (\omega_1 - \omega_0) dt \quad (6.4)$$

with gain constant  $k_{g1}$ . The extra input  $u$  of the second SG,  $u_2$  is given by:

$$u_2 = -k_{g2} \int (\omega_2 - \omega_0) dt \quad (6.5)$$

with gain constant  $k_{g2}$ .

We look at the steady state solution, i.e.  $\omega_1 = \omega_2 = \omega$ . This is of course not totally correct, we might have long term instability because of this assumption. But if we do not make this assumption, we can not relate the active power outputs to each other. Next we choose  $k_{g2} = 2k_{g1}$ . In this way (6.5) changes to:

$$u_2 = -2k_{g1} \int (\omega - \omega_0) dt \quad (6.6)$$

We look at the steady state of the swing equation, so we neglect the  $J$  term. Furthermore we rearrange the terms to achieve:

$$\omega - \omega_0 = \frac{P_0 + u_1 - P_1}{K_d} \quad (6.7)$$

for the first SG and for the second SG we have:

$$\omega - \omega_0 = \frac{P_0 + 2u_1 - P_2}{K_d} \quad (6.8)$$

In steady state, as we explained in the previous section,  $\omega = \omega_0$  and hence the right hand terms of (6.7) and (6.8) will become zero. Multiplying with  $K_d$  we get:

$$P_0 + u_1 - P_1 = 0 \quad (6.9)$$

and

$$P_0 + 2u_1 - P_2 = 0 \quad (6.10)$$

or, written in a different way:

$$u_1 = P_1 - P_0 \quad (6.11)$$

and

$$u_1 = \frac{P_2 - P_0}{2} \quad (6.12)$$

these two equations gives the following relation between  $P_1$  and  $P_2$ :

$$P_2 = 2P_1 - P_0 \quad (6.13)$$

The total active power demanded by the load,  $P_T$  is delivered by both SGs, so  $P_T = P_1 + P_2$ . Whenever  $P_T < 2P_0$ , where  $2P_0$  is the total nominal power, we will see that  $P_1 > P_2$ . Likewise, when  $P_T > 2P_0$ , we will have that  $P_1 < P_2$ . When  $P_T = 2P_0$ , both SGs will deliver equal power.

We will show this by an experiment. We start with one load connected to the grid with demand  $P_0$ . Then, after 1 second a second identical load is connected, after 2 seconds a third load is added, after 3 seconds the second load is disconnected and after 4 seconds the third load is disconnected. This means that at time  $t = 0$  and  $t = 4$ ,  $P_T = P_0$ . Since  $P_T = P_1 + P_2$  we know that  $P_1 = P_0 - P_2$ . Substituting this in (6.13) gives us:  $P_2 = \frac{1}{3}P_0$ , which means that  $P_1 = \frac{2}{3}P_0$ .

At time  $t = 1$  and  $t = 3$ ,  $P_T = 2P_0$ . This means that  $P_1 = 2P_0 - P_2$ . Substituting in (6.13) yields:  $P_2 = P_0$ , which means that also  $P_1 = P_0$ .

Finally at time  $t = 2$ ,  $P_T = 3P_0$ . This means that  $P_1 = 3P_0 - P_2$ . Substituting in (6.13) yields:  $P_2 = \frac{5}{3}P_0$ , which means that  $P_1 = \frac{4}{3}P_0$ .

Figure 6.2 shows the results of our SIMULINK model. We observe that the behavior is as we expected. There are some errors in the amount of power that is being produced, which is caused by our assumption  $\omega_1 = \omega_2$ . This can be seen in the frequency plot, where SG2 is faster in converging back to the nominal

frequency. If  $\omega_1 = \omega_2$ , these two frequency plots should have been identical. This might be a good subject for future studies; how does the difference  $\omega_2 - \omega_1$  effects the active output sharing and does it cause long term instability? Furthermore, the total power produced is larger than the demanded power which is realistic since there is a loss in the transmission lines.

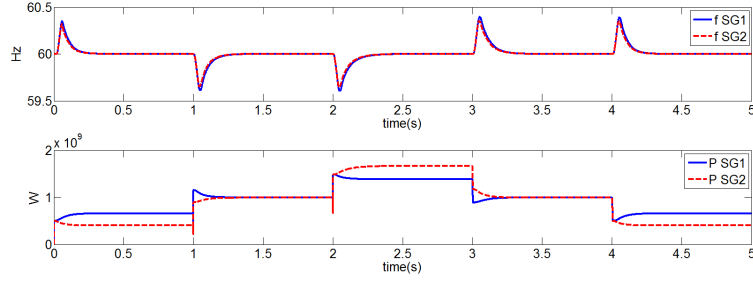


Figure 6.2: Secondary control for two parallel SGs, with  $k_{g2} = 2k_{g1}$

With this we conclude this chapter about secondary control. The reason we looked at this was because it was quite easy to implement in our models and it did not took a lot of time. Next we will look at reactive power droop control.

## Chapter 7

# Reactive power droop control

In this chapter we will look at combined active-reactive power droop control. Since we look at purely inductive transmission lines, this means that the active power is in relation with the frequency,  $P - f$  droop control, and reactive power is in relation with the voltage magnitude,  $Q - E$  droop control, [3].

We have looked at the Synchronous Generator when we were just dealing with active power. The SG does not have a built in equation that deals with reactive power. One could try to fabricate such equations, but we will just observe the case where we are using inverters.

When we were dealing with purely active loads, i.e. purely resistive loads, we used a constant magnitude that was used as input for our PWM. We said that this magnitude corresponded with the d-component of our created signal. Let us call this created signal  $v$ , which is used, together with our calculated  $\omega t$ , to switch back to the abc quantities. These abc quantities are used as the input of our PWM, which was explained in section 4.3. It holds that  $v = (v_d \ v_q \ v_0)^T$ .

The reason why we could let the magnitude be equal to the value  $v_d$  was because we showed that  $u_q = u_0 = 0$ . This meant that the value  $u_d$  was equal to the magnitude of the voltage. This of course was the case since we used a PLL to synchronize with the voltage. We could simply set  $v_q = v_0 = 0$ , since the voltage magnitude was not changed when adding purely resistive loads.

Now we are dealing with combined active-reactive loads. This time the voltage magnitude will deviate from its nominal value. We can not simply choose  $v_q$  to be zero again. We will have to control both the direct signal  $v_d$  as the quadrature signal  $v_q$ . We will use the conventions and model explained in [15], with some small changes. Figure 7.1 shows the structure of our new model.

As can be seen, we do not perform a PLL this time. We use the phase we obtain from the  $P - f$  droop equation to perform the dq0-transformations for the voltage and current. Because of this, the term  $u_q$  will not be zero, as we

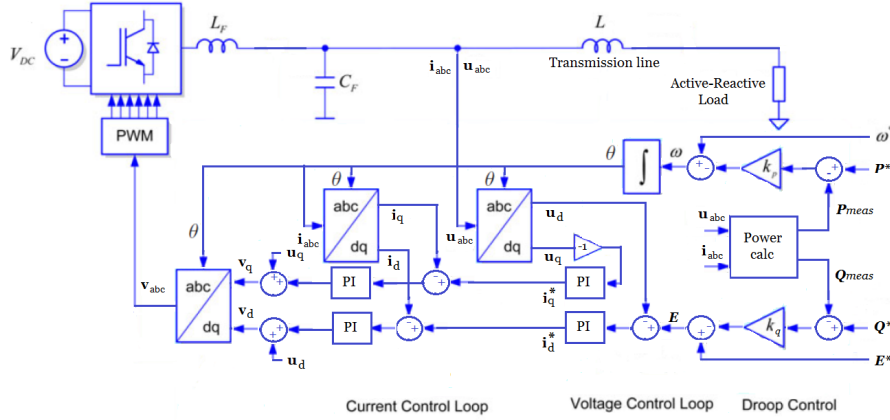


Figure 7.1: The VSI with both droop equations

showed in section 4.2.2. We will however drive this term again to zero, with the use of the PI controller seen in Figure 7.1. Furthermore we drive the error  $E - u_d$  to zero, again by using a PI controller. These two actions are called the Voltage Control Loop. Note that we obtain  $E$  by the  $Q - E$  droop equation.

Next, we drive the errors  $i_d^* - i_d$  and  $i_q^* - i_q$  to zero by two more PI controllers. This part is called the Current Control Loop. The output of both PI controllers are added to  $u_d$ , respectively  $u_q$ , to obtain our signal values  $v_d$ , respectively  $v_q$ . Using the same phase  $\omega t$  (or  $\theta$ ) we used for the dq0-transformations, we switch back to the abc-quantities. This signal is given to the PWM and this concludes the dynamics.

We will first look at a single inverter connected to two identical, both active as reactive, loads. In section 7.2 we we look at parallel inverters.

## 7.1 Single inverter using both active as reactive droop control

In this section we will observe the case where a single inverter is connected to four identical loads. The loads are both active (1 GW) as reactive (50 Mvar). The inverter has the dynamics as explained in the introduction of this chapter.

Since the reactive power demand increases, the voltage magnitude of the droop equation,  $E$ , will decrease. The magnitude of the grids voltage, measured at the output of the inverter, will synchronize with  $E$ , causing the voltage to stabilize. Now here comes the part where there is an error in our SIMULINK model.

### 7.1.1 Error in load demand

The loads we are using in all of our models, are standard load blocks available in SIMULINK. It consists of a RLC in series, where we can specify the nominal

frequency, the active power  $P$ , the inductive reactive power  $Q_L$ , the capacitive reactive power  $Q_C$  and the nominal phase-to-phase voltage (see [13]). When the voltage is equal to the nominal value, the load will have the active and reactive demand as we specified in this block. Since we are changing the magnitude of the voltage, the demand will be different from what we set as our demand. This is why the active and reactive power measurements will not be doubled when we add a second load. They will both increase, but because the voltage is below nominal value, the increase will be less than what we are asking for.

We will demonstrate this with our SIMULINK model. We will look at three different cases, where in each case we look at different  $k_q$  values. Remember that this is the reactive droop equation constant. The higher this value, the bigger the change in voltage magnitude  $E$ . The bigger the change in  $E$ , the smaller the increase in active and reactive power demand at the load when we increase the load. This yields a smaller increase in our active and reactive power measurements.

We show both the voltage magnitude  $E$  and the measured active power  $P$ . We start with one load connected and the other three loads are connected at  $t = 0.3$ ,  $t = 0.6$  and  $t = 0.9$ . Figure 7.2 shows the results.

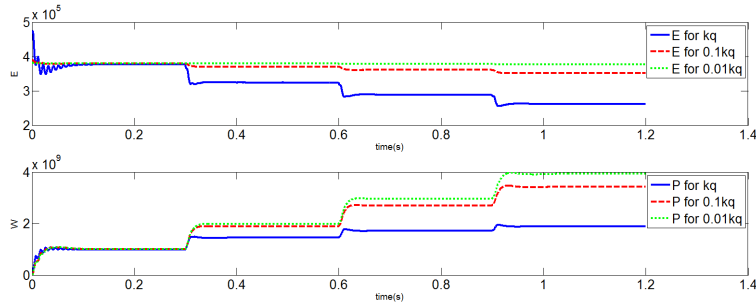


Figure 7.2: Inverters voltage magnitude and active power measurement for (a)  $k_q$ , (b)  $0.1k_q$  and (c)  $0.01k_q$ .

We observe that for small a small value of  $k_q$ , the change is magnitude is indeed very small, but the active power measurement is close to what we want. However, we will look at  $Q - E$  droop control in this chapter, meaning that we want to observe a proportional large change in voltage magnitude. This is why we will use the value of  $k_q$  that we used for the red graph in Figure 7.2. We observe a change is voltage magnitude and the active and reactive power measurements are not to far of what we demand. Note that  $k_q$  is defined as the nominal voltage magnitude divided by the nominal reactive power, i.e.  $k_q = \frac{E_0}{Q_0}$ .

In the next subsection we will look at both droop equations.



### 7.1.2 Both P-f and Q-E droop control

We will show that our model is working as we expected. We add combined loads, which means that the frequency and voltage magnitude should decrease in the grid. We start with one load connected. At time  $t = 0.5$  the second is connected and at time  $t = 1$  the third is connected. the seond load is disconnected at time  $t = 1.5$  and the third at time  $t = 2$ . Figure 7.3 shows the result.

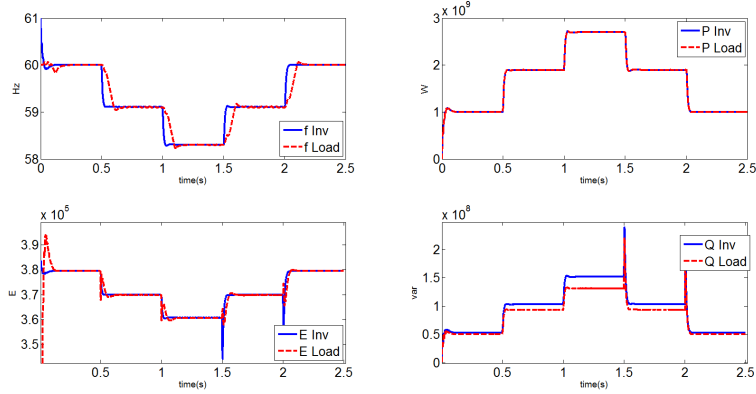


Figure 7.3: Upper left shows the frequency of the inverter and the load. Upper right shows the corresponding active powers. In the lower left graph the voltage magnitude is depicted and the corresponding reactive powers are shown in the lower right.

We observe that indeed the frequency and the voltage magnitude are stabilized and synchronized with the frequency and magnitude obtained from the droop equations. We also observe that the inverter has a higher value for  $Q$  than the demand from the load. This is of course caused by the inductive transmission lines. The next subsection will apply secondary droop control on the voltage magnitude.

### 7.1.3 Secondary droop control on voltage magnitude

In real life situations, secondary droop control on the voltage magnitude is applied in order to get rid off the error in this magnitude, that the droop equations are creating. Just as in section 6.1 we use an extra input, this time for the nominal reactive power  $Q_0$ . This yields the following equation.

$$E = E_0 - k_q(Q - (Q_0 - u)) \quad (7.1)$$

Here,  $u$  is the new secondary control. As choice of  $u$  we use the proposed integrated voltage magnitude deviation, so  $u = k_g \int (E - E_0)dt$ , see [2]. Here,  $k_g$  is a gain constant and has a unit that is the torque per second, i.e.  $\frac{N \cdot m}{s}$ . This extra term is subtracted from our nominal reactive power, i.e. we are controlling the input instead of keeping it constant, in order to deliver the required reactive power. The input is changed in such a way that the measured power minus the nominal power converges to zero and hence  $E$  will converge to  $E_0$ .

We will show this with our SIMULINK model. We expect that, since the voltage magnitude converges to its nominal value, the measurements for  $P$  and  $Q$  will be exactly the load demands we specified. Figure 7.4 shows the result. We are adding a second load at time  $t = 0.5$  and a third load at time  $t = 1$ .

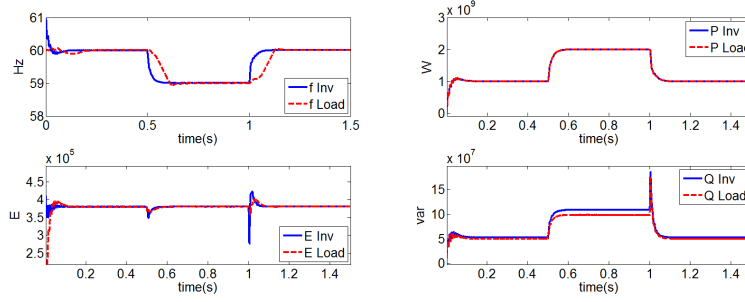


Figure 7.4: Upper left shows the frequency of the inverter and the load. Upper right shows the corresponding active powers. In the lower left graph the voltage magnitude is depicted, where we applied secondary control, and the corresponding reactive powers are shown in the lower right.

We observe that indeed the voltage magnitude converges back to the nominal value. This causes the measurement for  $P$  to be correct, or equal to our expected active power demand.

With this we conclude this section about both droop equations for a single inverter. In the next section we will look at parallel inverters, which are using both droop equations.

## 7.2 Parallel inverters using both $P - f$ as $Q - E$ control

In this section we did an attempt to look at how the reactive power is being shared when we are using parallel inverters. In this, however, we did not succeed. We will explain why our model is not capable of working with parallel inverters.

We start by showing the result of our SIMULINK model. We use two identical inverters, where the dynamics are the same as in the previous section. We have one load connected, which is both active as reactive. Figure 7.5 shows the results.

It is clear from Figure 7.5 that our model is unstable. But what is the reason for this instability? We can explain it by showing the current and voltage at the output of one of the two inverters. These can be seen in Figure 7.6.

As can be seen, the error lies within the current. Apparently, some current is

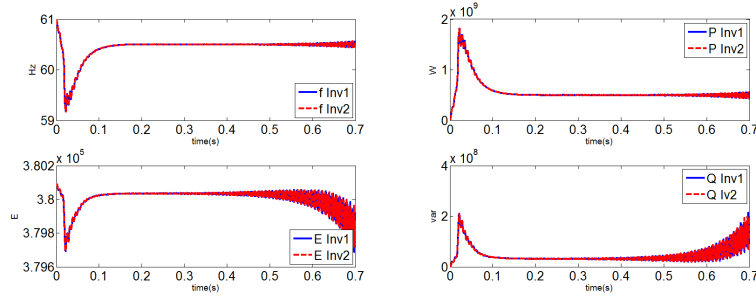


Figure 7.5: Two identical parallel inverters. Upper left shows the frequency, upper right the active power outputs. Lower left shows the voltage magnitude and lower right the reactive power output

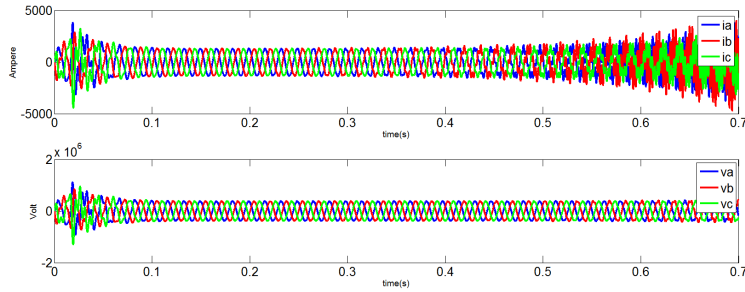


Figure 7.6: The three-phase current and the three-phase voltage of one of the two parallel inverters

flowing from one inverter to the other and vice versa. This phenomenon is called *circulating current* [27], [28]. Because the current is unstable, the voltage output of the inverters becomes unstable, causing error in reactive power, voltage magnitude, active power and frequency.

Some time was spend on trying to fix this problem. Some solutions are mentioned in various papers, primarily by using a Space-Vector Pulse Width Modulation (SV-PWM) in stead of a standard PWM [27] or by adding impedance to cancel out the circulating current [28]. Due to time limitations, we have not found a method that solved the problem. This, however, could be a good subject for future work. When the circulating current problem is solved, one can add multiple inverters and observe the power sharing.

Note that we have this problem because of the transmission lines. If we were to neglect the transmission lines, the reactive power is shared in a similar way as the active power. The error seems to lie within the mismatching in voltage between the two inverters. If this voltage is exactly the same, as is the case where there are no transmission lines, the reactive power sharing is accurate. This problem is being addressed in [?].

This concludes this chapter about reactive power. In the next chapter we will

explain the third kind of power source, the ICS. Because we will be looking at the combined active-reactive loads and because the ICS needs to work in parallel with another grid former (a SG in our case), there will be again circulating current. This means that we did not derive good results but still we would like to explain the model.

## Chapter 8

# Grid-supporting power converter acting as a current source

There are two types of grid-supporting power converters. The first acts as an ideal ac voltage source which is in series with an impedance, the second one is acting as an ideal ac current source in parallel with a shunt impedance. The idea behind these two converters is that they keep the frequency and voltage of the grid close to their rated values, by regulating their output voltage/current. We will be focusing on the second case, the converter acting as a current source, because it is more common in real life, [15].

The first thing to observe is that this new grid-supporting power converter acting as a current source, which we will call ICS from now on, is not able to work in island mode on its own. It needs at least one grid former, i.e. a synchronous generator or a controlled voltage source, to be able to operate. This is because the grid voltage needs to be formed in order for the ICS to work. The main task of the ICS is to participate in regulating the voltage  $E$  and the frequency  $\omega$  of the grid, by controlling the amount of active and reactive power it delivers to the grid. The dynamics of the ICS are illustrated in Figure 8.1, which we used from REF.

As can be seen in this figure, we have an power inverter, our ICS where the output goes through a LC-filter, and a load which is purely resistive. The measurements for our current and voltage are done after the inductor for the current and after the total LC-filter for the voltage.

The voltage is transferred into dq coordinates by the dq0-transformation. The  $v_q$  term is driven to zero by using a PI controller and the output of this controller is added to the reference angular frequency, in our case  $2\pi 60$ . The result, which is the estimated grid frequency  $\omega'$ , is integrated and then we have found our  $\omega t$ , in the figure denoted by  $\theta$  which is called the phase. These calculations, where we have the voltage in abc-coordinates as input and the phase  $\omega t$  as output is just the same as a phase-locked loop, where the goal is to synchronize with the grid voltage.



we get a reference for the q-coordinate of the current  $i_q^*$ . The reactive power in dq-coordinates is determined as:

$$Q = v_q i_d - v_d i_q \quad (8.2)$$

Since, again, we drive the term  $v_q$  to zero,  $Q = -v_d i_q$  and when divided by  $v_d$ , we get an estimate of  $-i_q$ .

We then subtract the measured value of  $i_q$  from our estimate and we do a PI control over the result in order to let this error,  $i_q^* - i_q$ , go to zero.

Now we are at the last step, the construction of  $u_d$  and  $u_q$ , which are the new dq-coordinates for the voltage. The term  $u_d$  is determined by adding the measured d-coordinate  $v_d$  to the outcome of the PI controller that drives the error  $i_d^* - i_d$  to zero, and we add a cross term which we will explain in a moment. The q-coordinate is determined by adding the measured q-coordinate  $v_q$  to the output of the PI controller that drives the error  $i_q^* - i_q$  to zero, and again we add a cross term.

These new dq-coordinates are used, together with the phase  $\omega t = \theta$  to switch back to the three-phase voltage. This is sent into a PWM, from which the output is sent into a bridge which is connected to a DC voltage source. We will not explain again how the PWM operates as we done so already in the chapter about the voltage source inverter.

Next the cross terms need to be explained. As we said, the output of the universal bridge is filtered using a LC-filter. If we call the voltage before the inductor  $E$  and after the inductor  $V$ , then,  $E - V = j\omega LI$ , where  $j\omega LI$  is the voltage over the inductor. Writing in dq factor, remember  $u = u_d + ju_q$ , we get:

$$\begin{aligned} E_d + jE_q - (V_d + jV_q) &= j\omega L(I_d + jI_q) \\ E_d - V_d + j(E_q - V_q) &= -\omega LI_q + j\omega LI_d \end{aligned} \quad (8.3)$$

We observe that there is a negative effect of  $I_q$  on  $E_d - V_d$  and a positive effect of  $I_d$  on  $E_q - V_q$ . This is why the cross terms are in the system. They cancel out the effects that the inductor has on the voltage difference. Indeed, the factor  $L$  in Figure 8.1 is the same as the inductor in the LC-filter,  $L_f$ .

Note that a big difference between the ICS and the IVS, the original inverter, is what they measure. For the IVS we changed the load in the grid, so the active power, and observed how the inverter changed its frequency according to the droop equation. The ICS however, measures a change in frequency in the grid and adjusts its active power it delivers to the grid. Likewise, the ICS detects a change in voltage measurement and adjusts its reactive power output instead of vice versa.

Now we tested our new inverter, but we found the same error as in section 7.2. Since we connected our ICS to a SG and we change the voltage magnitude, there will be circulating current. Because we did not solve the problem of circulating current, we are not able to discuss any results about the ICS here. It is however a good topic for future work, since in real life, the ICS is commonly used as a power source.

## Chapter 9

# Conclusions

In this thesis we investigated and explained the dynamics of power suppliers. We started with explaining how active and reactive power are determined and how they have a connection with frequency and/or voltage magnitude, depending on the transmission lines. We choose to take a time-domain approach, instead of the more common method of phasor notation.

Synchronized generators were used to control and share the active power by changing their frequency according to the swing equation. We observed different cases when working with parallel SGs, where we changed one of the parameters in each case. We explained what should happen to the active power sharing and we modeled our ideas using SIMULINK. Our results were as we expected.

After the SGs, we looked at the voltage source inverter. A detailed explanation was given of how the inverter operates. We explained how the inverter adjusts its active power output by changing its frequency according to the droop equation. We showed the correspondence between the swing and droop method and with the use of our SIMULINK model, we looked at power sharing between parallel inverters. The results were again as we expected, the inverter behaves in a same way as the SG.

From here the next step was obvious. We put a synchronous generator in parallel with our inverter and showed the relation between the damping factor and the droop coefficient. The difference in transient time was explained and with our SIMULINK model we showed that by adjusting the moment of inertia term, we could change the difference in transient periods.

Since it is common in real life, and because it was a small step from our creating SIMULINK models, we looked at secondary control. An extra input term is used to adjust the nominal power, which causes the error in frequency to converge to zero, yielding a nominal frequency. Both single case as the parallel case were explained. In the parallel case, the power sharing was again as we expected.

Next we looked at reactive power. The loads were now both active as reactive, which is more realistic in real life. We used our inverter and showed how the voltage is stabilized by using the second droop equation, which depends



on reactive power. The deviation in voltage magnitude, as we showed, has a large influence on the measurements of active and reactive power. We explained that this was because of the load blocks we used from the SIMULINK library. The more realistic case was also viewed, where we applied secondary control on the nominal reactive power, causing the voltage magnitude to converge to the nominal value.

Finally we wanted to observe what happened when we used parallel inverters, where they are both using reactive power droop control. In this, however, we did not succeed, because of the circulating current. Due to time limitations, we did not find a solution for this problem. The same problem influences our model of the current source inverter, which is why we were not able to get good results for this model either.

In total, this thesis is an overview of how power is being shared in a microgrid. We showed that primary control can be used to rapidly balance generation and demand, while sharing the load, synchronize the AC voltage frequency and stabilize the voltage magnitude.

## Chapter 10

### Future work

Due to time limitations we could not look at each case that we would have wanted to investigate. We started by explaining how the transmission lines influence the behavior of the droop equations. In real life, the transmission lines are more inductive than resistive, but they are not purely inductive as we assumed. A future investigation topic would be to use the combined droop equations and investigate how the power is being shared.

Furthermore, the problem of circulating current needs to be solved. There are existing ways to prevent circulating current, but we were not able to correctly implement these methods within our time period. When the problem is solved, one could observe the reactive power sharing and furthermore one could use the current source inverter we explained.

Since we mainly investigated primary control, one could extend our work by going more deeply into secondary or even tertiary control. A good topic regarding secondary control would be to investigate the transition period for parallel inverters (or SGs).

Finally, we only looked at the stand-alone case. This means that the microgrid was not connected to the main grid. In real life the microgrid is able to work in both connected as disconnected mode. One could investigate the connected case and see how parallel SGs and inverters share the load under these circumstances.

# Chapter 11

## Appendix

### 11.1 Power invariant dq0-transformation

In the power invariant dq0-transformation, it must hold that:

$$P = VI = v_a i_a + v_b i_b + v_c i_c = v_d i_d + v_q i_q + v_0 i_0 \quad (11.1)$$

In the ordinary dq0-transformation, d, q and 0 are determined by:

$$\begin{pmatrix} x_d \\ x_q \\ x_0 \end{pmatrix} = c_1 \begin{pmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \\ c_2 & c_2 & c_2 \end{pmatrix} \begin{pmatrix} x_a \\ x_b \\ x_c \end{pmatrix} \quad (11.2)$$

In our case,  $x$  is either the voltage  $v$  or the current  $i$ . We will now show which values one has to choose for  $c_1$  and  $c_2$  in order to (11.1) to hold.

$$\begin{aligned} v_d i_d + v_q i_q + v_0 i_0 &= c_1^2 \left( \left( v_a \cos(\theta) + v_b \cos\left(\theta - \frac{2\pi}{3}\right) + v_c \cos\left(\theta + \frac{2\pi}{3}\right) \right) \right. \\ &\quad \left( i_a \cos(\theta) + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta + \frac{2\pi}{3}\right) \right) \\ &\quad + \left( -v_a \sin(\theta) - v_b \sin\left(\theta - \frac{2\pi}{3}\right) - v_c \sin\left(\theta + \frac{2\pi}{3}\right) \right) \\ &\quad \left( -i_a \sin(\theta) - i_b \sin\left(\theta - \frac{2\pi}{3}\right) - i_c \sin\left(\theta + \frac{2\pi}{3}\right) \right) \\ &\quad \left. + c_2^2 (v_a + v_b + v_c) (i_a + i_b + i_c) \right) \end{aligned} \quad (11.3)$$

Now, using the double angle formulas, we know that:

$$\cos\left(\theta \pm \frac{2\pi}{3}\right) = \cos(\theta) \cos\left(\frac{2\pi}{3}\right) \mp \sin(\theta) \sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \cos(\theta) \mp \frac{\sqrt{3}}{2} \sin(\theta) \quad (11.4)$$

$$\sin\left(\theta \pm \frac{2\pi}{3}\right) = \sin(\theta) \cos\left(\frac{2\pi}{3}\right) \pm \cos(\theta) \sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} \sin(\theta) \mp \frac{\sqrt{3}}{2} \cos(\theta) \quad (11.5)$$

These observations yield that the cross terms in (11.3) become:

$$\cos(\theta) \cos\left(\theta \pm \frac{2\pi}{3}\right) = -\frac{1}{2} \cos^2(\theta) \mp \frac{\sqrt{3}}{2} \sin(\theta) \cos(\theta) \quad (11.6)$$

$$\cos\left(\theta - \frac{2\pi}{3}\right) \cos\left(\theta + \frac{2\pi}{3}\right) = \frac{1}{4} \cos^2(\theta) - \frac{3}{4} \sin^2(\theta) \quad (11.7)$$

$$\sin(\theta) \sin\left(\theta \pm \frac{2\pi}{3}\right) = -\frac{1}{2} \sin^2(\theta) \pm \frac{\sqrt{3}}{2} \sin(\theta) \cos(\theta) \quad (11.8)$$

$$\sin\left(\theta - \frac{2\pi}{3}\right) \sin\left(\theta + \frac{2\pi}{3}\right) = \frac{1}{4} \sin^2(\theta) - \frac{3}{4} \cos^2(\theta) \quad (11.9)$$

Adding (11.6) and (11.8) yields:

$$\cos(\theta) \cos\left(\theta \pm \frac{2\pi}{3}\right) + \sin(\theta) \sin\left(\theta \pm \frac{2\pi}{3}\right) = \frac{1}{2} (\sin^2(\theta) + \cos^2(\theta)) = -\frac{1}{2} \quad (11.10)$$

Adding (11.7) and (11.9) yields:

$$\cos\left(\theta - \frac{2\pi}{3}\right) \cos\left(\theta + \frac{2\pi}{3}\right) + \sin\left(\theta - \frac{2\pi}{3}\right) \sin\left(\theta + \frac{2\pi}{3}\right) = \left(\frac{1}{4} - \frac{3}{4}\right) (\sin^2(\theta) + \cos^2(\theta)) = -\frac{1}{2} \quad (11.11)$$

Of course it also holds that:

$$\cos^2(\theta + k) + \sin^2(\theta + k) = 1 \quad (11.12)$$

where  $k = 0$ ,  $k = -\frac{2\pi}{3}$  or  $k = \frac{2\pi}{3}$ .

Substituting (11.10), (11.11) and (11.12) into (11.3) yields:

$$v_d i_d + v_q i_q + v_0 i_0 = c_1^2 \left( (1 + c_2^2) (v_a i_a + v_b i_b + v_c i_c) + \left(\frac{1}{2} + c_2^2\right) (v_a (i_b + i_c) + v_b (i_a + i_c) + v_c (i_a + i_b)) \right) \quad (11.13)$$

In order for the transformation to be power invariant, the second term with all the cross terms in (11.13) should be zero. This means that  $-\frac{1}{2} + c_2^2 = 0$ , hence  $c_2 = \pm \frac{1}{\sqrt{2}}$ . Substituting this value in the first term of (11.13) yields:

$$v_d i_d + v_q i_q + v_0 i_0 = \frac{3}{2} c_1^2 (v_a i_a + v_b i_b + v_c i_c) \quad (11.14)$$

In order for (11.14) to be power invariant,  $c_1 = \pm \sqrt{\frac{2}{3}}$ .

Hence, when we choose  $c_1 = \pm \sqrt{\frac{2}{3}}$  and  $c_2 = \pm \frac{1}{\sqrt{2}}$ , the dq0-transformation is power invariant.

## 11.2 SIMULINK models

In this section we give a short summary on how to use the models we used to simulate our findings.

### 11.2.1 Parallel Synchronous Generators

Open the file ParallelSGDroopPfSecondair. In this model we have two SGs connected in parallel to three identical active loads. One can choose to add secondary control by choosing a value of 1 in the Sec Droop Enable blocks. Setting these values to zero results in disabling secondary control.

Double click on the SG blocks to change the values of  $J$ ,  $K_d$  and  $w_0$ . If one would like to change  $P_0$ , you will have to change the  $P_m$  block in front of the SG.

The three plots under each other show the voltage, current, frequency and active power for the first SG, the second SG, respectively the load. The plot on the right shows how the active power is shared between the two SGs and it also shows how the frequency is being synchronized.

### 11.2.2 Parallel Voltage Source Inverters

Open the file ParallelInvertersVSDroopPf. In this model we have two voltage source inverters connected in parallel to three identical active loads.

Double click on the inverter blocks to observe the dynamics. One can change the LC-filter, change the carrier function by double clicking on the PWM lock or change the filtering after the power measurement by altering the transfer function.

One can change the values of  $k_p$ ,  $P_0$  and  $w_0$  in front of the inverter and observe how the active power is being shared in different situations.

The three plots under each other show the voltage, current, frequency and active power for the first inverter, the second inverter, respectively the load. The plot on the right shows how the active power is shared between the two inverters and it also shows how the frequency is being synchronized.

### 11.2.3 SG and Inverter in Parallel

Open the file InverterVSandSGDroopPf. In this model we have a synchronous generator and a voltage source inverter connected in parallel to three identical active loads.

The values of  $K_d$  and  $k_p$  are chosen in such a way that the SG and Inverter share the load equally. One can change the value of  $K_d$  by double clicking on the SG or change the  $k_d$  by altering the value that is in front of the inverter. One can also choose to increase the inertia term  $J$  in the SG, to observe how this value influences the transient time.

The three plots under each other show the voltage, current, frequency and active power for the SG, the inverter, respectively the load. The plot on the right shows how the active power is shared between the SG and the inverter and it also shows how the frequency is being synchronized.

#### 11.2.4 Parallel Inverters with reactive power droop control

Open the file `ParallelVSDroopPfandQE`. In this model we have two voltage source inverters connected in parallel to three identical active/reactive loads.

One can choose all of the parameters in front of both the inverters. These parameters include  $P_0$ ,  $\omega_0$ ,  $k_p$ ,  $Q_0$ ,  $E_0$  and  $k_q$ . To observe the power sharing in different situations, let just one of these parameters be different in the inverters and let all the other parameters be identical.

Note that this model becomes unstable after some time, because of the transmission lines, since the voltages mismatch. One could delete the transmission line blocks to observe the power sharing in the correct way.

The two plots under each other show the voltage, current, frequency, active power, voltage magnitude and reactive power for both inverters. The plot on the right shows how the active power is shared between the inverters and it also shows how the frequency is being synchronized. Furthermore it shows the voltage magnitudes and the sharing of the reactive powers.

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