University of Groningen

Bachelor thesis

Observational status of inflation and the influence of eternal inflation on the inflationary paradigm

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July 7, 2015

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1 Introduction

Inflation is a theory which explains the properties of the early universe and provides a quantum origin for the primordial density perturbations that seeded the large-scale structure observed nowadays. One could argue that inflation can be realised in so many different models, it could fit any observation we make of the current state of the universe. There are however still some model-independent predictions from inflation. In this thesis I will discuss the old Big Bang model, its problems and how inflation solved many of these problems. Furthermore I will discuss which predictions inflation made, how these predictions are proven or disproven throughout the years, and which predictions are still to be confirmed or falsified. Then I will continue to explain the concept of eternal inflation and the implications of an eternally inflating universe for the inflationary paradigm.

2 History of the Big Bang model

First I will give a brief history of how scientists came to the conclusion that there must have been some sort of Big Bang which created our universe. The foundation for any theory describing our universe should be the theory of general relativity (GR), published by Einstein in 1915. This theory describes gravitation not as a gravitational field but as the distortion of space and time themselves. For describing a universe you need a metric, which captures all the geometric and causal structures of spacetime. In the 1920's and 1930's Friedmann, Robertson, Walker and Lemaître all found a metric of an expanding isotropic and homogeneous universe independently [1, 2, 3, 4, 5, 6]. This does not require the Einstein field equations, but just the assumption that the universe is isotropic and homogeneous. When you insert this metric in the field equations and assume the the universe is a perfect fluid, you obtain the so-called Friedmann equations. These equations describe the time-evolution of the expansion of the space in our universe. In 1929 Hubble found the first evidence that galaxies far away from us are traveling away with a velocity which is proportional to the distance [7]. This was the first evidence that our universe is indeed expanding and was later confirmed by other observations. After they found out that the universe was expanding, there were two theories which could explain the expansion and are in accordance with the Friedmann equations; the Big Bang theory and the Steady State theory. The Big Bang theory was the most logical solution to the expansion of the universe, because if you trace the expansion back in time you get, as Lemaître called it, a 'primordial atom'. The idea is that the universe started in a hot, high density state and then expanded and cooled. There were many people which could not digest the idea that there was some beginning of the universe, which introduced according to them religious concepts into physics. Therefore they created another model which could explain the expansion of the universe. The steady state theory wanted to keep the 'wide cosmological principle': the universe should be the same at any time and at any place. To make up for the fact that the universe was expanding, they proposed that new matter continuously was created. Evidence favoured the Big Bang theory more, through the counting of radio sources in our universe. When in 1964 Penzias and Wilson discovered the Cosmic Microwave Background Radiation (CMB) [8], cosmologists generally accepted the Big Bang theory. If you look at any direction in the sky, you can detect a faint glow of microwave radiation which does not come from any stars or galaxies. This CMB was the first light that was created in our universe. This observation supports the Big Bang theory because the Big Bang theory predicts a time of photon decoupling. At first the universe was a very hot and dense plasma where the photons mean free path was very small because they encountered many electrons. After the universe expanded and cooled, the formation of the hydrogen atom was energetically favoured and electrons were captured by the hydrogen. Therefore the photons could decouple of matter and were free to travel through the universe without interacting with matter. These photons are the CMB we measure nowadays. One very interesting property of the CMB is that no matter at which direction in the sky you look, it always has the exact same temperature. It is completeley homogeneous.

3 Big Bang model

3.1 Success of the Big Bang model

For a long time the Big Bang theory was the model which cosmologists thought was a correct description of the evolution of the universe we live in. They thought so because its implications agreed to a great extent with the observations made. Some key observations at that time were:

• The universe is expanding.
• The existence and spectrum of the CMB radiation.
• The abundances of light elements in the Universe (nucleosynthesis).
• The predicted age of the universe is comparable to direct age measurements of the objects within the universe.
• That given irregularities in the CMB there exists a reasonable explanation for the development of structure in the universe, through gravitational collapse.

All these observations considered, the Big Bang model seems a good explanation of the evolution of the universe. However, there are some crucial problems which the Big Bang model still leaves unanswered. But first, let us explain the physics which govern the Big Bang model.

3.2 FRW metric

The central premise of modern cosmology is the "cosmological principle": nobody is at the center of the universe, and the cosmos from any point looks the same as from any other point. This implies that the universe is homogeneous and isotropic. This principle is supported by the observational data from the last decades. The CMB and large scale structure surveys show that the universe is indeed homogeneous and isotropic on sufficiently large scales (>100 Mpc). The average spacetime in an isotropic and homogeneous universe is described by the Friedmann-Robertson-Walker (FRW) metric:

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{3.1}$$

where $a(t)$ is the time-dependent cosmic scale factor, and $k = -1$, $k = 0$ and $k = 1$ indicate an open (positively curved), flat, or closed (negatively curved) universe respectively. Throughout this thesis I will mostly assume $k = 0$, which is in agreement with experiments. A particle at rest at a given set of coordinates (r, θ, ϕ) and which has no external forces acting on it, will have constant (r, θ, ϕ). These coordinates are then called comoving coordinates. You can calcultate the physical distance between particles at $(t, 0)$ and (t, r) by:

$$d(r,t) = \int ds = a(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}}. \tag{3.2}$$

3.3 Friedmann equations

The expansion of the universe is determined by the properties of the material within it. These properties can be specified by the energy density $\rho(t)$ and the pressure $p(t)$. For a perfect fluid these two are related by an equation of state, which gives p as a function of ρ, defined as:

$$\omega = \frac{p}{\rho}. \tag{3.3}$$

Some classical examples of equations of state are:

$$\omega = \frac{1}{3}, \qquad \text{Radiation} \tag{3.4}$$

$$\omega = 0, \qquad \text{Non-relativistic matter.} \tag{3.5}$$

For an entire universe there need not be a simple equation of state. For example there can be different mixtures of material, such as a combination of radiation and non-relativistic matter.

The crucial equations for describing the evolution of the scale factor $a(t)$ are the Friedmann equations:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{pl}^2}\rho \qquad \text{Friedmann equation} \tag{3.6}$$

$$\dot{\rho} + 3H(\rho + p) = 0 \qquad \text{Fluid equation} \tag{3.7}$$

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p) \qquad \text{Acceleration equation} \tag{3.8}$$

where $M_{Pl} = \sqrt{\frac{\hbar c}{8\pi G}}$ is the Planck mass[1] and $H(t) \equiv \frac{\dot{a}}{a}$ is the Hubble parameter. The Hubble parameter is used in Hubble's law, which states that galaxies recede from each other with a velocity that is proportional to the distance.

3.4 Standard cosmological solution

The Friedmann equations can be solved for the two equations of state given earlier (equation (3.4) and (3.5)), which give the classic cosmological solutions:

$$\text{Matter Domination} \qquad \omega = 0: \qquad \rho \propto a^{-3} \qquad a(t) \propto t^{2/3} \tag{3.9}$$

$$\text{Radiation Domination} \qquad \omega = \frac{1}{3}: \qquad \rho \propto a^{-4} \qquad a(t) \propto t^{1/2}. \tag{3.10}$$

In both cases the density falls as t^{-2}. When $k = 0$ we have the freedom to rescale a and it is normally chosen to be equal to unity at the present, so that physical and comoving scales coincide. We can then fix the proportionality constants by setting the density to be ρ_0 at time t_0, where subscript 0 denotes the present value.

A more interesting solution appears for the case of the so-called cosmological constant, because this ensures that there is an quasi-exponential expansion in our universe. This corresponds to an equation of state $p = -\rho$. Equation (3.7) then gives $\dot{\rho} = 0$ and hence $\rho = \rho_0$, leading to:

$$a(t) \propto exp(Ht). \tag{3.11}$$

[1] Throughout this thesis I will assume $\hbar$ and c to be equal to 1, so $M_{pl}^{-2} = 8\pi G$.

3.5 Critical density and the density parameter

Consider adding spatial curvature to the Friedmann equation (3.6):

$$H^2 = \frac{\rho}{3M_{pl}^2} - \frac{k}{a^2}. \tag{3.12}$$

If we divide both sides by the Hubble parameter squared, we can write this as:

$$1 - \Omega = \frac{-k}{(aH)^2}. \tag{3.13}$$

where:

$$\Omega \equiv \frac{\rho}{\rho_{crit}}, \qquad \rho_{crit} \equiv 3M_{pl}^2 H^2. \tag{3.14}$$

The deviation of the density parameter Ω from unity is a measurement for the curvature of the universe. As you can see, to have a flat universe, $\rho = 3M_{pl}^2 H^2$.

3.6 Problems of the Big Bang model

The Big Bang model is a very convincing theory for the evolution of the universe, but there are some crucial problems which it cannot solve. Most of these problems are concerned with the initial conditions of the universe. That is the reason why cosmologists started to look into the theory of inflation, coined by A. Guth [9]. While historically these problems were very important, they are now somewhat marginalized because inflation nowadays is most important due to its relevance in the origin of the cosmic structure.

3.6.1 The horizon problem

Probably the most important observation which the Big Bang model cannot explain is the uniformity of the CMB. In the Big Bang cosmology the CMB consisted of 10^4 causally independent patches at the time of decoupling. This means that points on the sky which have a seperation of 2 degrees should have never been in causal contact, but they are observed to have the same temperature to an extremely high precision. There was no time for these regions to interact, because of the finite horizon size:

$$\int_{t_*}^{t_{dec}} \frac{dt}{a(t)} \ll \int_{t_{dec}}^{t_0} \frac{dt}{a(t)} \tag{3.15}$$

where t_* is the beginning of the universe, t_{dec} is the time of the decoupling of photons and t_0 is the present time. The Big Bang model has no way to explain how these patches can have an equal temperature if they were never in causal contact.

3.6.2 The flatness problem

Let us consider the Friedmann equations written in the form of (3.13). In the standard Big Bang description of the universe $(aH)^{-1}$ is increasing so $\frac{1}{(aH)^2}$ is decreasing. This means that the curvature of the universe grows in time. If we extrapolate our current value of $|1-\Omega|$ to the beginning of the universe, we discover that the value which $|1-\Omega|$ must have had is extremely close to zero. From observations we know that today $|1-\Omega| \lesssim 0.01$ [10]. According to this current value, we have to explain a much more extreme flatness at early times, e.g. $|1-\Omega| \lesssim 10^{-55}$. It is very hard to explain why $|1-\Omega|$ had such a precise value, with a small margin for error. For all we care, its value was 0.1 or 10^5, because the physical laws in the universe will still be the same. The Big Bang model cannot explain why we need to have such precise initial conditions in our universe.

3.6.3 The monopole problem

It is likely that the universe went through different phase transitions. There are for example the electroweak and QCD phase transitions, and possibly others at even higher scales, like the grand unified theory (GUT). The problem is, there are expected to be phase transitions which create 'unwanted relics'. Unwanted relics are particles which are predicted by modern particle theories, but have not yet been observed. These include magnetic monopoles and cosmological defects. Monopoles are heavy pointlike objects, which behave as cold matter $\rho \propto a^{-3}$. The energy density of monopoles decreases slower than the radiation background ($\rho \propto a^{-4}$), so it would become the dominant substance in our universe at some point, in contradiction to our observations. One has to get rid of these monopoles without harming the conventional matter in the universe.

4 Cosmological inflation and the Big Bang problems

The horizon and the flatness problem of the Big Bang model are problems of the initial conditions of the universe. These problems can be solved by finetuning these initial conditions, but it must be done by such an enormous amount that it is very unlikely.

Inflation, the concept that the universe went through a period of quasi-exponential expansion in its earliest stages, solves all problems of the Big Bang model very

convincingly. The isotropy and homogeneity of the CMB and the flatness of the universe emerge dynamically. They are not specific values of the initial conditions anymore. Even the monopole problem is addressed by inflation. It explains why our universe is so big, flat and homogeneous. One of the most remarkable consequences of inflation is that it can provide the seeds for large scale structure formation through quantum fluctuations.
As stated earlier, inflation is a period of accelerated expansion. Another way to define inflation is to say that inflation is a period where the comoving Hubble radius is decreasing:

$$\frac{d}{dt}(aH)^{-1} < 0. \tag{4.1}$$

The decreasing Hubble radius is a more logical definition of inflation, because if we define it this way it will be easier to understand why inflation is a solution to the horizon problem. Of course we can show that a decreasing Hubble radius is the same as accelerated expansion:

$$\frac{d}{dt}(aH)^{-1} = \frac{d}{dt}(\dot{a}^{-1}) = -\frac{\ddot{a}}{\dot{a}^2} < 0. \tag{4.2}$$

because $\dot{a}^2$ is always positive, $\ddot{a} > 0$. So a shrinking Hubble radius implies an accelerated expansion.

4.1 The horizon problem

Everything that exists inside the Hubble radius can be in causal contact. The horizon problem was the problem that the CMB had many independent patches which would have never been in causal contact. A shrinking Hubble radius in the earliest stages of the universe solves this problem. You can imagine that the Hubble radius was very big before the period of inflation. This area is inside the Hubble radius so it can reach thermal equilibrium. During inflation, the Hubble radius decreases, which you can see as a zooming in to a small part of the enormous area which is in thermal equilibrium. Nowadays, when the Hubble radius increases, we observe the CMB to have almost the exact same temperature in every direction, because it was in causal contact at the earliest stages of the universe. In the same way, irregularities on scales much bigger than the observable universe can be caused by causal processes, given that they were created at sufficiently early times that those scales were within causal contact. Of course there should be a minimal amount of e-folds[2] of expansion during inflation to solve the horizon problem. The minimal amount of expansion during inflation should be equal to the amount of expansion

[2] The number of e-folds N is defined as $\int H dt$, such that $a(t) = a_0 e^{\int H dt} = a_0 e^N$.

after inflation. [11]. This translates to an amount of e-folds, which is dependent on the energy scale during inflation. If we take the upper bound of the energy scale 1.9×10^{16} GeV as measured by Planck satellite [12], we get a minimal amount of 67 e-folds of expansion during inflation. Of course this value of inflation is a lower bound, it could be that there was much more inflation, so much more e-folds of expansion.

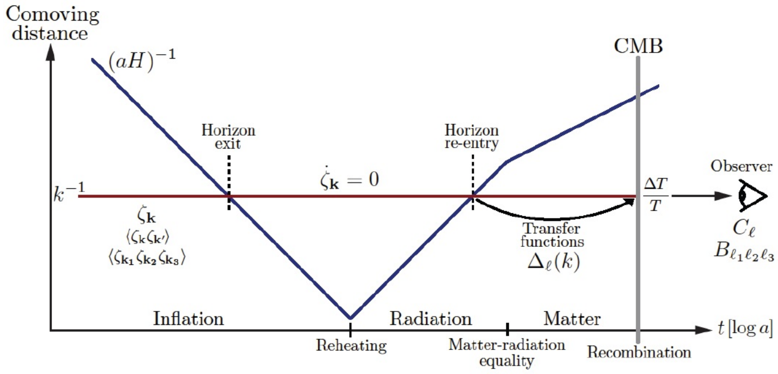

Figure 1: In this figure you can nicely see how inflation is a solution to the horizon problem and how quantum fluctuations could be in the Hubble radius. Figure from [13].

4.2 The flatness problem

The flatness problem was the problem that in our current universe $|1 - \Omega|$ is driven away from 0, while observations show that it is nowadays very close to zero. It is driven away from zero because $(aH)^{-1} > 0$ in our Friedmann universe. $|1 - \Omega| = 0$ is an unstable state. Inflation solves this problem almost by definition. As explained above inflation is defined as a period where the Hubble radius is decreasing, so $(aH)^{-1} < 0$ (see also figure 1). Because the denominator in equation (3.13) is increasing, $|1 - \Omega|$ is driven towards zero. So for inflation, $|1 - \Omega| = 0$ is actually an attractor, as one can see in figure 2. It is true that inflation must happen for a sufficiently long time to reach a state close enough to $|1 - \Omega| = 0$, but that is surely possible. If it is close enough, then it will stay very close to zero, even though in the post-inflationary period it is repelled from this state. To solve the flatness problem, we need some amount of inflation which turns out to be the same as for the horizon problem; 67 e-folds of expansion [11].

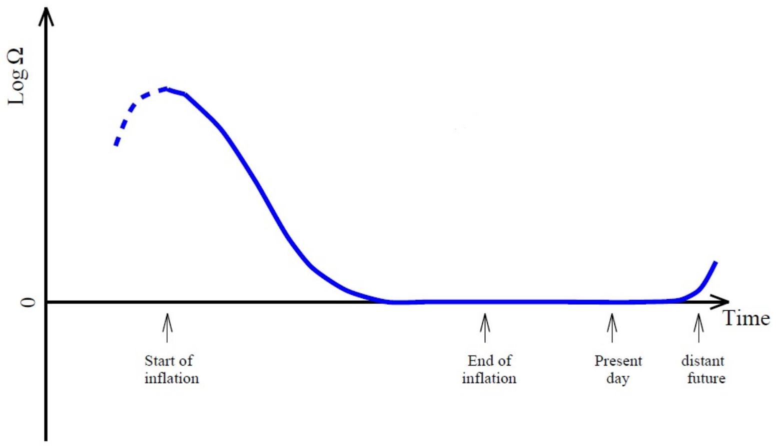

Figure 2: Evolution of the density parameter Ω, during and after inflation. Figure from [14].

4.3 The monopole problem

The monopole problem refers to the problem that we expect phase transitions in the early universe which create monopoles, but we do not observe these nowadays. Inflation also takes care of this problem, because inflation rapidly dilutes the unwanted relics. The energy density during inflation falls slower (a^{-2} or slower) than that of unwanted relics (a^{-3}). So their energy density becomes negligible very quickly.

5 The physics of inflation

In the previous chapter I discussed the idea behind the concept of inflation, and now we turn to the physics which drives inflation. To obtain inflation, we need to have a substance which has $\omega = -1$, so $\rho = -p$, because that produces exponential expansion, as shown in (3.11). Something that can fulfil this property is a scalar field, which in the case of inflation is often called the inflaton.
For a universe filled with a homogeneous scalar field $\phi(t)$, the energy density and pressure are given by:

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{5.1}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{5.2}$$

If we substitute these definitions for the energy density and pressure in the Friedmann equation (equation (3.6)), we get:

$$H^2 = \frac{1}{3M_{Pl}^2}[V(\phi) + \frac{1}{2}\dot{\phi}^2]. \tag{5.3}$$

Because the inflaton is a scalar field, the time evolution of the field $\phi(t)$ is governed by the Klein-Gordon equation:

$$\ddot{\phi} + 3H\dot{\phi} = -V'. \tag{5.4}$$

5.1 Slow roll approximation

The standard approximation used for inflation is the slow-roll approximation. The slow-roll approximation assumes that $T \ll V$, which ensures $\rho = -p$. It turns out that experiments agree with the predictions made by slow-roll. . You can define two conditions which, if they hold, make sure that inflation is happening for a sufficiently long time. As said earlier, inflation is defined as a period where $\ddot{a} > 0$:

$$\frac{\ddot{a}}{a} > 0 \Rightarrow \dot{H} + H^2 > 0 \Rightarrow \epsilon_H \equiv -\frac{\dot{H}}{H^2} < 1.$$

This $\epsilon_H < 1$ ensures us that accelareted expansion is happening. For exponential expansion, we need $\epsilon_H \ll 1$. We also want inflation to happen a sufficiently long time (around 50-60 e-folds) to solve the cosmological problems the Big Bang model had. To make sure this happens, we define:

$$\eta_H = \frac{\dot{\epsilon}_H}{H\epsilon_H}. \tag{5.5}$$

If $|\eta_H| \ll 1$, the change of ϵ_H per Hubble time is small and inflation persists. From equation (5.3) and (5.4), we can derive the continuity equation:

$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{M_{Pl}^2}. \tag{5.6}$$

With these equations we can very easily see why it is called the slow-roll approximation. If we combine equation (5.3), (5.1) and (5.6), we get:

$$\epsilon_H = -\frac{\dot{H}}{H^2} = \frac{\frac{1}{2}\dot{\phi}^2}{3[\frac{1}{2}\dot{\phi}^2 + V]} \ll 1. \tag{5.7}$$

So we can see that for $\epsilon_H \ll 1$, the kinetic term must be much lower than the potential term, hence slow-roll (see figure 3).

Another way to define the slow-roll parameters, is not in terms of the Hubble parameter H, but in terms of the scalar potential V:

$$\epsilon_V = \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 \tag{5.8}$$

$$\eta_V = M_{Pl}^2 \frac{V''}{V}. \tag{5.9}$$

Where a prime is a derivative with respect to the scalar field ϕ. We can calculate the scalar spectral index in terms of the Hubble slow-roll parameters:

$$n_s - 1 = -2\epsilon_H - \eta_H. \tag{5.10}$$

The spectral index says something about the scale dependence of the primordial perturbations. The primordial perturbations are the perturbations in the early universe which we get by treating the inflaton quantum mechanically. These perturbations are the seed for the large-scale structure in our present universe. Because the power spectrum of the primordial perturbations $P(k) \propto k^{n_s - 1}$, $n_s = 1$ means scale-invariant primordial perturbations. According to slow-roll inflation, primordial perturbations should be almost scale invariant; n_s should be very close to one.

5.2 Quantum fluctuations

As we have seen in the previous subsection, the inflaton $\phi(t)$ governs the energy density ρ of the early universe. You can imagine ϕ as a microscopic clock reading off the amount of inflation remaining. Because microscopic quantum mechnanical objects are under influence of the uncertainty principle, the field $\phi(t)$ can have some spatially varying fluctuations, as one can see in figure 3. These variations have the consequence that different regions of space undergo inflation by different amounts. These quantum fluctuations therefore lead to fluctuations in the energy densitiy $\delta\rho$. These variations in energy densities lead to temperature anisotropies $\Delta T(x)$ in the CMB, because photons from denser regions have to overcome a bigger gravitational force and therefore lose energy. In this way, temperature anisotropies arise naturally from a quantum description of inflation.

5.2.1 Scalar, vector and tensor perturbations

The quantum fluctuations of the inflaton can be of three types, namely scalar, vector and tensor perturbations. Scalar fluctations are the simplest form, which are caused by energy density fluctuations and therefore lead to temperature anisotropies. Vector fluctuations are another type of perturbation which are caused by vorticity

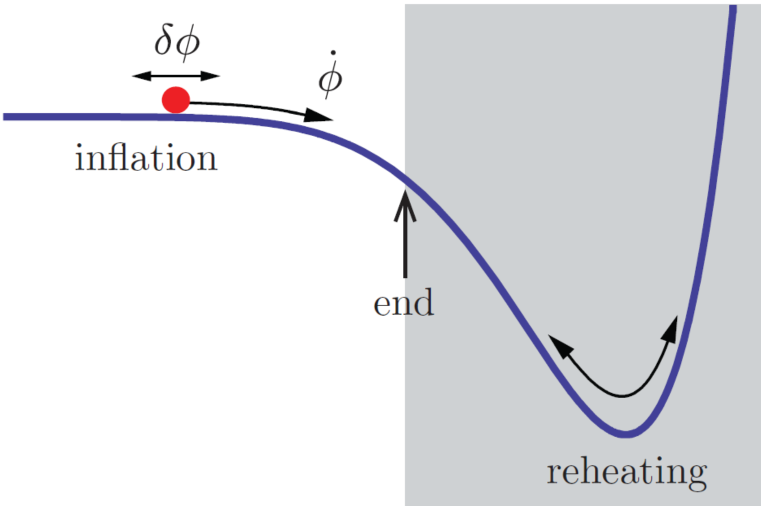

Figure 3: The typical scalar potential for slow-roll inflation. On top of the classical evolution of ϕ you also have the quantum fluctuations $\delta\phi$. Figure from [15].

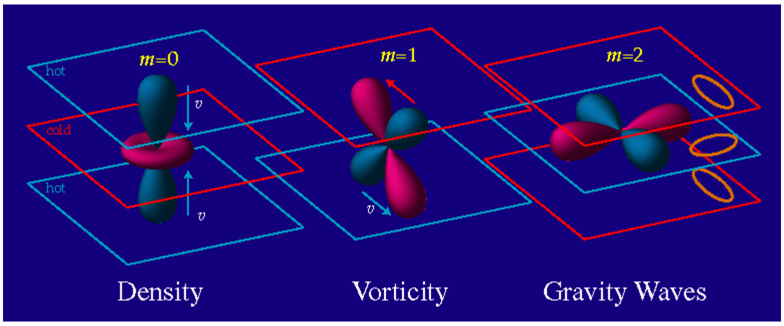

Figure 4: The different perturbations; scalar, vector and tensor. Figure from [16].

in the primordial plasma. Because vorticity is brought to a very low value by inflation (see section (6.1.1)) there will be almost no vector fluctuations left in the CMB. The third and last perturbation you can have is tensor fluctuations, which are caused by gravity waves who stretch and squeeze space in orthogonal directions. The three types of perturbations are represented in figure 4.

Another important property of the CMB which is caused by scalar, vector (negligible) and tensor perturbations is the polarization. The polarization can be split into E-modes and B-modes, similar to an electromagnetic wave which you can split into an electric and magnetic part. The scalar perturbations only create an E-mode polarization. This is because scalar perturbations cause temperature anisotropies. When an electromagnetic wave is incident on a free electron, the scattered wave is polarized perpendicular to the incidence direction. If there were no anisotropies in temperature, no polarization can be made. But because of the anisotropies, photons from different angles have different intensities, which gives you a linear E-mode polarization. Vector perturbations only cause B-mode polarization, but as said earlier these will be negligible. Tensor perturbations give rise to E-mode and B-mode polarization. But because the amount of E-mode polarization from scalar perturbations is much more than that of E-mode polarization from tensor perturbations, we are looking for B-mode polarization in the CMB to detect gravitational waves. We can measure this value by measuring the value r, the tensor-to-scalar ratio, which is defined as:

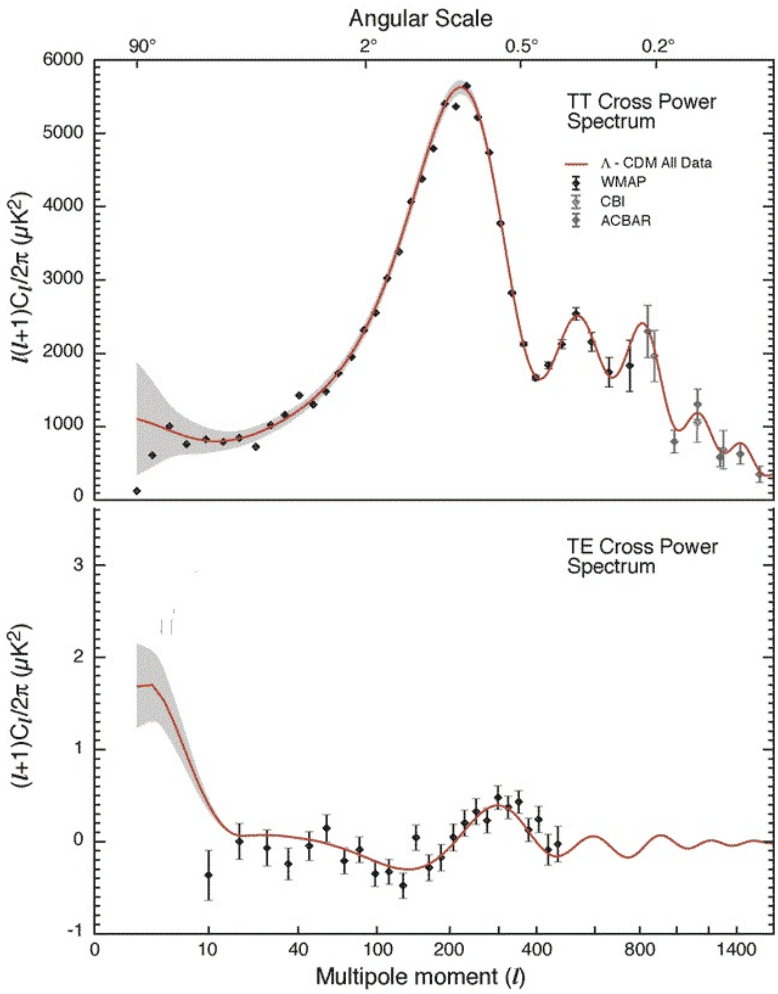

Figure 5: In the top side you can see the temperature power spectrum as seen in the CMB. In the bottom side you can see the correlation spectrum between the temperature anisotropies and the E-mode polarization. Figure from [17].

$$r \equiv \frac{\Delta_t^2}{\Delta_s^2} \tag{5.11}$$

where Δ_t^2 is the dimensionless tensor spectrum and Δ_s^2 is the dimensionless scalar spectrum, both of the primordial perturbations.

In figure 5 one can see in the top side the temperature anisotropies power spectrum and in the bottom side the TE- correlation power spectrum. In the top side one sees the points of the temperature anisotropies power spectrum we measure in the CMB. The peaks and troughs are exactly what inflation predicts. By treating the scalar field quantum mechanically we end up with primordial perturbations, which then lead to the CMB temperature power spectrum we see in figure 5.

6 Predictions and observational tests

Of course inflation is not just a purely theoretical model, it also has its predictions about the current state of the universe. Most of the observations which cannot be explained by the Big Bang model, like the horizon problem, the flatness problem and the monopole problem, supported the theory of inflation. In this chapter I will discuss what inflationary models in general predict, what the observational tests were and still are, and what future observations can tell us about the validity of inflation.

6.1 Predictions of inflation

The main reason for Guth to introduce the inflationary model in 1981 was of course the problems of the initial conditions the Big Bang model had. But inflation would not be a good scientific theory if it did not have some predictions about the state of the present universe which we can observe. These predictions can in some cases be tested to see if inflation can serve as the best theory so far for the origin of the universe.

6.1.1 Rotation of the universe

One of the properties of the universe which was not certain at the time inflation was invented was the so-called rotation of the universe. Inflation had a specific prediction about this property. If the vorticity (rotation) of the early universe was low enough to allow inflation, the vorticity would be driven towards an extremely low value due to inflation [18]. During the inflationary era it is estimated that the factor by which the vorticity decays is of the order 10^{-142} [19]. This means that if they observed at that time a significant rotation of the universe, it would make the inflationary model extremely implausible.

6.1.2 Anisotropies in the CMB

Until the early 90's it seemed like the CMB was completely homogeneous. This was a very big problem for inflation. As was said in the previous chapter, perturbations in the scalar field cause energy density fluctuations in the photon-baryon fluid, so there must be anisotropies. In the slow-roll models I discussed in section 5.1 these anisotropies must have specific properties; they must be consisted with adiabaticity and Guassianity [20]. If these anisotropies were not observed, inflation would lose a great part of its plausibility. This prediction was for the first time tested by the COBE satelite. Another thing inflation predicts about the primordial fluctuations, is that they must be *almost* scale invariant [21]. As said earlier, this is measured by the spectral index n_s. If $n_s = 1$, the perturbations are scale-invariant. Inflation predicts n_s very close to one.

6.1.3 Flatness of the universe

As explained in the previous chapter, $\Omega = 1$ is an attractor in the inflationary model. All of the earlier inflationary models predicted that Ω should be very close to 1, so that the universe would be flat. Later on there were inflationary models which could explain an open universe, but almost all inflationary models predict $\Omega = 1$ [22]. For a long time it did not look like $\Omega = 1$, because until 1998 people had not found out about dark energy [23]. In our universe $\Omega_\Lambda = 0.692 \pm 0.010$[3] [10], so before 1998 it was thought that the matter in our universe only came to an $\Omega = 0.3$ [24]. This was a problem for inflation, because it predicts a flat universe with $\Omega = 1$.

6.1.4 T-mode and E-mode correlation

The CMB does not only have temperature anisotropies, but also differing polarization angles. Adiabatic density fluctuations only produce E-mode polarization. This is because density fluctuations cause the temperature differences in the CMB. At the time of recombination photons interact with free electrons. A unique test for inflation would be the correlation between the T-mode and E-mode spectrum. The T-mode and E-mode spectrum should be out of phase, a peak in the temperature spectrum should mean an trough in the E-mode spectrum.

[3] Ω_Λ is density of dark energy in our universe.

6.1.5 Gravitational waves

Another important prediction inflation makes is the existence of gravitational waves, which are produced by ripples in the spacetime. While scalar perturbations cause density anisotropies and therefore temperature anisotropies, tensor perturbations are the source of gravitational waves. Gravitational waves are an important prediction because it is completely model independent. If there was inflation in the early universe, there must be gravitational waves in the CMB. The downside of the gravitational waves are that they are very hard to detect. One way to detect gravitational waves is through B-mode polarization in the CMB.

6.2 Observational status

6.2.1 Rotation of the universe

One of the first experiments which could make a precise measurement of the rotation of the universe was the COBE satellite. It was only possible to lower the boundary the global rotation of the universe has, it was not possible to completely rule out any rotation of the universe. After one year of observing the sky the limits of the global rotation of the universe were lowered to $\frac{\omega}{H_0} < 10^{-6}$ [18]. This was at a level that begins to probe upper limits of inflationary models. After the 4-year sky map of COBE they could lower the limit of rotation to an even smaller value, namely $\frac{\omega}{H_0} < 6 \times 10^{-8}$ [25]. Further observations did not show a rotation of the universe, so inflation overcame this observational test.

6.2.2 Anisotropies in the CMB

The Cosmic Background Explorer (COBE) satellite was also searching for anisotropies in the CMB, which was a must for inflation because that is what it predicts. After one year of observing the CMB, anisotropies were still not found. After 4 years of observing the sky they found anisotropies in the CBM, with temperature $2.725 \pm 0.020K$ [26]. Of course these anisotropies were not the only test. They could use the data from the COBE satellite also for testing the prediction that the perturbations must be adiabatic, Gaussian and scale-invariant. As it turned out, they could verify that the anisotropies are consistent with Gaussian statistics, and a spectral index of $n_s = 1.2 \pm 0.3$. So COBE was of great importance for cosmologists for showing the anisotropies in the CMB. But predictions of inflation could be tested to a greater precision, which is exactly what the Wilkinson Microwave Anisotropy Probe (WMAP) did. Of course it was already sure that there were anisotropies, but if you measure them with greater precision, you can improve the bounds on non-Gaussianity and adiabaticity of the anistropies. They also measured smaller scales, which allows for a better determination of cosmological parameters. The results confirmed what inflation predicted; the perburbations were still consistent with Gaussianity and adiabatic [27]. From the WMAP data they could also determine the spectral index much more precise: $n_s = 0.971 \pm 0.010$. After WMAP there came another satellite which could measure the anisotropies to an even higher precision, the Planck satellite (see also figure 6). The results from Planck are in good agreement with Gaussian and adiabatic fluctuations in the CMB. The spectral index they measured was $n_s = 0.9608 \pm 0.0054$ [10]. So far everything inflation predicted was measured; the CMB had anisotropies, which are consistent with Gaussianity and adiabatic, and almost scale-invariant, because n_s is slightly less than one.

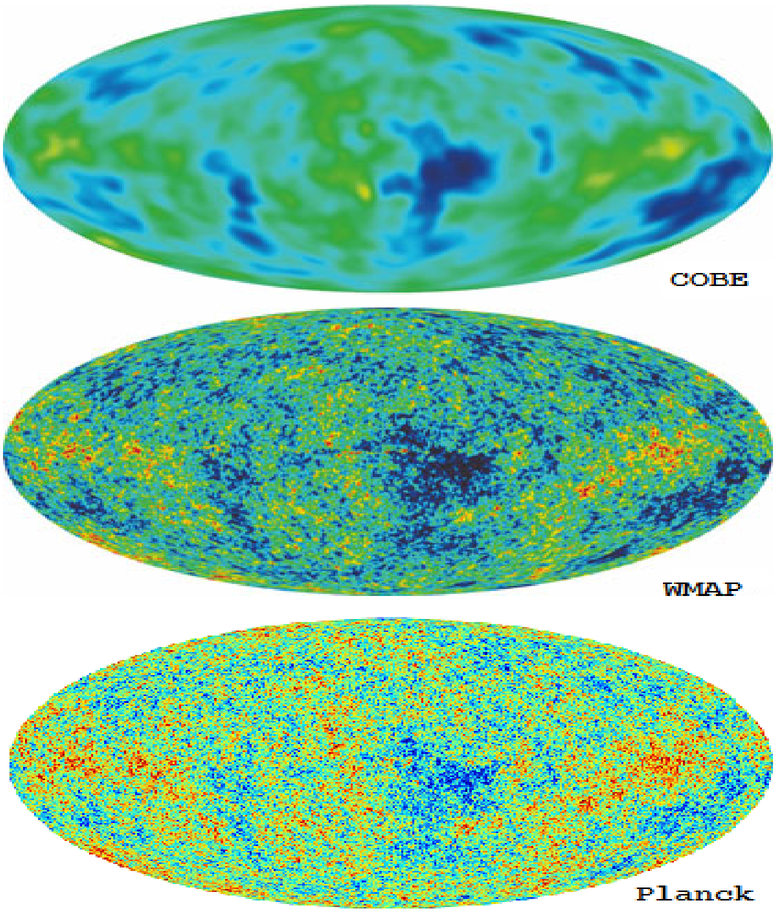

Figure 6: CMB sky as observed by the COBE, WMAP and Planck satellite. One can see by eye that each satellite measured the anisotropies with greater precision.

6.2.3 Flatness of the universe

For a long time it seemed like the density Ω was equal to 0.3 ± 0.1. This was of course a big problem for inflation, which predicted $\Omega = 1$. Then, in 1998, Perlmutter *et al* found through looking at distant supernovae, that the universe will expand forever by means of a positive cosmological constant, called dark energy [23]. With the data from Perlmutter, they found that $\Omega_m = 0.25^{+0.18}_{-0.12}$ and $\Omega_\Lambda = 0.63^{+0.17}_{-0.23}$ [28]. These results strongly favoured a nearly spatially flat universe. This was a big success for inflation. A few years later WMAP measured Ω to be $0.9973^{+0.0039}_{-0.0038}$ [27]. This was even better for inflation, because the density parameter was very close to one. It seems like our universe is extremely flat and the prediction of inflation was right.

6.2.4 T-mode and E-mode correlation

When you look at the TT spectrum (see figure **??**), you see that the peaks and troughs, which are what inflation predicts, only start at $l > 200$ (first peak), which corresponds to angular scales $\theta < 1^\circ$ [15]. This is a scale which is within the causal horizon at the time of recombination. The feature which the TE spectrum brings extra is the negative peak around $100 < l < 200$. This is a scale not within causal contact at the time of recombination. Therefore, one is forced to consider a theory like inflation, where there is a horizon exit and re-entry. This TE spectrum therefore is an observation which makes inflation much more likely than any other theory about the origin of the universe.

6.2.5 Gravitational waves

Gravitational waves is the so-called smoking gun for inflation. These waves are measured through the parameter r, called the tensor-to-scalar ratio. It measures the ratio of tensor perturbations to scalar perturbations, and is given in (5.11) The absence of tensor perturbations is signaled by $r = 0$. WMAP found the tensor-to-scalar ratio to be $r < 0.22$ [27]. Then later on Planck lowered the value to $r < 0.11$ [10]. This means that if there are tensor waves, they are not very abundant in the CMB. Of course in future observations, this value can be measured more precise. It is worth mentioning that in March 2014 the Background Imaging of Cosmic Extragalactic Polarization (BICEP2) published a paper where they claimed to have found the first evidence of the primordial B-mode polarization in the CMB [29]. After inspection of the BICEP2 and Planck data combined the researchers found that they had to withdraw their claim, as it turned out that the B-modes they detected were from dust in the Milky Way [30]. If they ever find a value of $r > 0$, it is very positive for the theory of inflation. If not, then it is not a disaster. It makes the theory of inlation only very slightly less possible.

7 Discussion on the observational evidence

As shown in the previous chapter, inflation had and still has many predictions which could have been falsified. Making such predictions is a condition which must be satisfied to be called a good scientific theory. Inflation could have been falsified by a measurement of the rotation of the universe, but it passed this observational test. Another property of the CMB which we had to measure and eventually was measured by the COBE sattelite was the existence of temperature anisotropies due to scalar perturbations. Even another generic prediction of all models of inflation, that the universe must be spatially flat, turned out to be the case. Later, when they began to measure the polarization of the CMB, it turned out that you definitely need inflation to explain the TE-spectrum. Gravitational waves or tensor perturbations in the CMB spectrum is another prediction of all inflationary models, but they have not been found yet.

So what is the observational status of the paradigm of inflation? You can say that almost all predictions are already confirmed, so what is left? Of course it is true that most of the predictions are already observed and cannot be changed, like the rotation of the universe. But the anisotropies in the CMB could always be measured with a greater precision, which gives you better bounds on Gaussianity and the spectral index, so you can build better models. Also the flatness of the universe can be measured more precise, which can tell you more about the amount of e-folds inflation minimally needs. And the existence of gravitational waves is still a very inflation-specific prediction, which if observed, almost assures us that inflation must have happened.

It seems like nowadays inflation is a well supported scientific theory which can, through changing the potential of the scalar field, predict all possible universes within our current measurement errors. But I think that specific quality, that it can predict anything by changing its potential, is also something which is negative for the inflationary paradigm. As you may remember, inflation was proposed to solve the problem of the initial conditions. Inflation is a theory which can predict almost any universe, but nowadays cosmologists only make models which can give the parameters we measure in our universe. Therefore it seems like the problem of the initial conditions has not changed that much. Why of all the inflationary potentials possible is it just the ones we construct in our models that must have been present in the early universe? It seems like you still need very specific initial conditions, for inflation to produce the results we measure today. While this argument may sound compelling, its not entirely correct. Because inflation does make some generic predictions, which are discussed in chapter 6. Inflation always drives the universe to spatial flatness and always has perturbations, which should be seen in the CMB. The critique which still stands, is the critique that there is no reason to assume a scalar field with negative pressure in the early universe. It seems like putting in the special initial conditions in the Big Bang theory (flatness, homogeneity and anistropies in CMB) have shifted towards putting in the special condition that there must have been some very specific kind of scalar field with negative pressure. What favours inflation is that when the early universe had this special scalar field, the evolution of the whole universe follows naturally, governed by the laws of physics. But still, the problems of inflation are maybe not the observational tests, but the very logical foundation on which it is based. In chapter (8) I will show that if inflation happens it almost always is eternal; if inflation started in a universe it will keep inflating forever. This fact also has many implications for the likelihood that inflation is indeed the correct theory.

8 Eternal inflation

By now it should be clear that inflation must be driven by a scalar field and that this scalar field has quantum fluctuations. Another important feature of inflation which is the direct effect of these quantum fluctuations is that inflation, once it has started, must be eternal. Eternal in the sense that it must go on forever in the future. There are two mechanisms by which inflation can be eternal, for new inflation (small field) and for chaotic inflation (large field).

8.1 New inflation

The first models of new inflation were proposed by Steinhardt [31] and Linde [32], while later Vilenkin [33] showed that eternal inflation is a generic feature of new inflation. With new inflation the scalar field is at a metastable state on the potential, which is called the false vacuum. The false vacuum is the inflationary stage. When the scalar field rolls down from this metastable state, it ends inflation and a universe is created. The decay from the metastable state is an exponentially suppressed process [34]. However, at the same time that it exponentially decays, the false vacuum also exponentially expands. For new inflation to work, the exponential expansion must be much larger than the exponential decay. Therefore, while the inflaton decays at some points, it exponentially expands at other points, which makes the amount of false vacuum always grows in the time that a universe is created. A figure which explains this process nicely is figure 8. First there is only the false vacuum, which eventually decays into a universe, as you can see in the second line of figure 8.

In this second line the two false vacuum regions are just as big as the false vacuum region from the first line, because the false vacuum exponentially grows. In the third and fourth line the false vacuum regions undergo the same process,

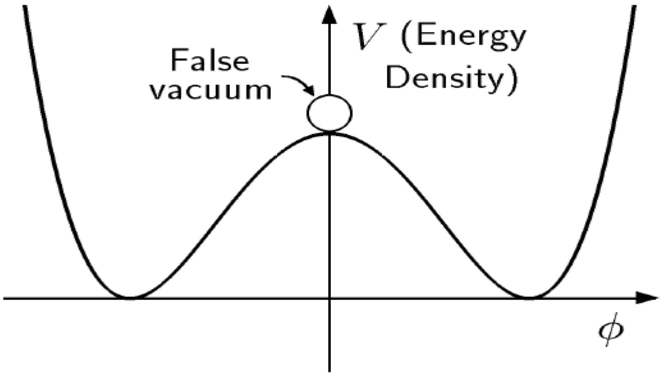

Figure 7: In this figure you can see the scalar potential of new inflation. The inflaton stays on top of a metastable state where it is inflating. Figure from [34].

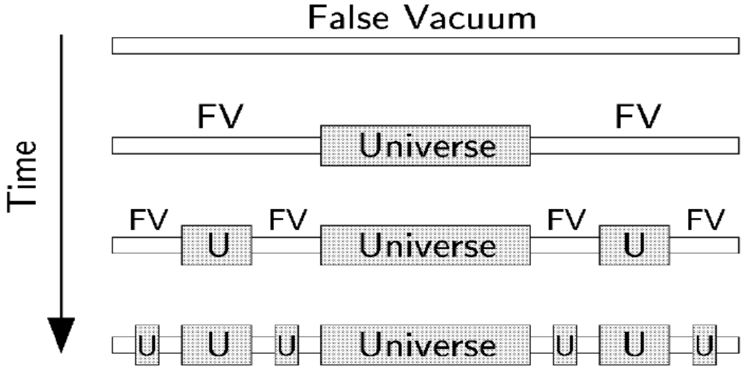

Figure 8: The fractal structure of new inflation. Figure from [34].

creating many different universes. One can imagine that this process will go on forever. Of course this figure is a simplified one-dimensional description of a three dimensional process. The assumption that the decay is systematic is false too, but it makes it easier to think and understand the process. But even when one corrects for these simplifications, we are left with an fractal structure with different universes.

8.2 Chaotic inflation

In 1983 Linde proposed a new model of inflation where inflation would be a natural consequence of chaotic initial conditions in the early universe, hence the name chaotic inflation. Chaotic inflation does not depend on the scalar field lingering on the top of the hill, but on the scalar field rolling down the potential (see figure

9). The easiest way to think about the mechanism of eternal inflation for chaotic inflation is to see what happens during a time interval of duration $\Delta t = H^{-1}$ (one Hubble time). By definition, during one Hubble time interval the space will expand by a factor e. Consider a field ϕ in a Hubble volume of H^{-3}, while we assume the average value is ϕ_0 in this region. During one Hubble time it will expand by a factor e^3, which corresponds roughly to a factor 20. So after one time interval the region will be divided in 20 independent Hubble-sized regions.

When the scalar field ϕ is running down the hill, it follows the classical path in addition to the the quantum fluctuations. These quantum fluctuations can be downhill or uphill on the trajectory. So you can describe the evolution of ϕ during one time interval as:

$$\Delta\phi = \phi_{cl} + \phi_{qu} \tag{8.1}$$

where ϕ_{cl} is the classical trajectory and ϕ_{qu} are the quantum fluctuations during one time interval. One can imagine that if in one of the 20 regions the quantum fluctuation uphill is bigger than the classical evolution downhill, $\phi > \phi_0$, and there will always be more inflating regions after a time interval Δt than there were before. This process will go on forever, so in this way chaotic inflation will also create eternal inflation.

Let's see if the quantum fluctuations can indeed bring the scalar field uphill by a greater amount than the classical trajectory brings it downhill. During a time interval Δt the field will be reduced by $\Delta\phi_{cl} = \frac{2}{\phi}$ [35]. The quantum fluctuations can be estimated to be $|\Delta\phi_{qu}(x)| \approx \frac{H}{2\pi} = \frac{m\phi}{2\pi\sqrt{6}}$. So you can see that if ϕ is smaller than $\phi^* \sim \frac{5}{\sqrt{m}}$, then the motion downhill from the classical path is much greater than the quantum fluctuation. However, if $\phi \gg \phi^*$, you have $\Delta\phi_{qu}(x) \gg \Delta\phi_{cl}$. Because the flucutations have a Gaussian distribution, in half of the 20 domains after time interval Δt the field increases with $|\Delta\phi_{qu}(x)| - \Delta\phi_{cl} \approx |\Delta\phi_{qu}(x)| = \frac{H}{2\pi}$, rather than decreases. This leads to 10 times as much inflating fields after Δt than before. After two time intervals the total volume of the growing scalar field is already 100 times bigger. The universe thus enters an eternal inflationary process of self-reproduction.

8.3 Implications for the inflationary paradigm

At first eternal inflation seems like a very interesting theory, although without any testable predictions. This is because all the universes eternal inflation create, are out of causal contact. For any observation which could prove or disprove eternal inflation, we must observe beyond our own universe. There is however an important implication of eternal inflation. Due to the eternal process, there will be an infinite amount of pocket universes. Maybe they have the same laws of physics

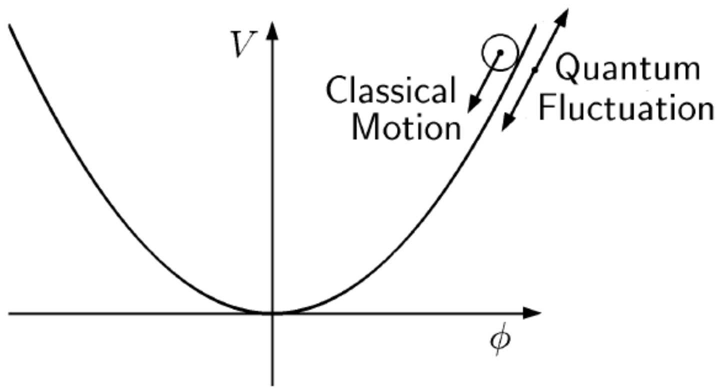

Figure 9: The characteristic potential of chaotic inflation. On top of the classical motion you have the quantum fluctuations. Figure from [34].

like our universe, maybe a little different, or maybe the universe does not look in the slightest amount like our universe. Because eternal inflation is in fact eternal, there will be an infinite amount of different universes created an infinite amount of times. One scientist who is for this reason quite sure that inflation is the a wrong theory, is Paul Steinhardt [36]. According to him, because eternal inflation make all possible universes with all the possible properties, inflation loses its predictive power. Because any physically possible cosmic property will occur in this multiverse, there is no way to falsify inflation. It is quite a philosophical problem. Let us assume that eternal inflation is correct and we live in a pocket universe within a multiverse where every possible universe physically possible exists. With this assumption the problem is that you can argue both ways. One way is to say that because inflation predicts every possible universe and thus also ours, inflation is the right theory. Another way, which Steinhardt takes, is to argue that due to the multiverse, inflation loses its predictive power because it predicts any possible universe. Here we have to ask ourselves what a good scientific theory should look like. Is it enough to explain the observations we make today, or does it have to make predictions which can be tested?

One property which is assumed in this reasoning is that inflation does produce all possible universes with all possible properties. As far as I understand the subject, inflation as we understand it today does make some falsifiable predictions, for example the anisotropies in the CMB and the gravitational waves. Therefore I tend to favour the conclusion that inflation is a good scientific theory with falsifiable predictions.

9 Conclusion

There were a few problems for the Big Bang model and the concept of inflation seemed an answer to all of them. It solved the horizon problem, the flatness problem and the monopole problem. Inflationary theories also made general predictions about our current state of the universe. The global rotation of the universe could disprove inflation, but turned out not to. Inflation predicted that there were anisotropies in the CMB, which were found by COBE and measured to higher precision by WMAP and the Planck satellite. These anisotropies were Gaussian, adiabatic and almost scale-invariant, like inflation predicts. Another prediction inflation made was a flat universe. Until 1998 it looked like this was not the case, but due to the existence of dark energy this prediction from inflation was true too. The TE mode spectrum showed us that we need a concept like inflation to explain the anti-correlation between causally disconnected regions in space. Gravitational waves will be a big support for inflation, but are not yet observed.

Eternal inflation is in both new inflation and in chaotic inflation an unavoidable outcome. New inflation is eternal because while it exponentially decays, the false vacuum also exponentially grows. This way the inflating region keeps growing. Chaotic inflation is eternal if ϕ is big enough for the quantum fluctuations to be bigger than the classical directory. There are people who argue that eternal inflation has an implication for the inflationary paradigm, but in my opinion inflation is still a good scientific theory.

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