



university of
 groningen

faculty of mathematics
and natural sciences

The Study of Calculations of Systematic Uncertainty in BESIII Analyses

Master Thesis

Student: Yang Ding

Supervisor: Dr. M.Kavatsyuk

August 2015

This thesis is submitted by:

Yang Ding

Student number: S2444844

Master Program: Physics

To obtain the degree of Master at the University of Groningen

Research group: Hadronic and Nuclear Physics

Supervisor: Dr. M.Kavatsyuk

KVI-Center for Advanced Radiation Technology

University of Groningen

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Chapter 1 Introduction

An ancient question that human beings want to answer is “what is matter around us made of”. People try to describe the world in terms of some fundamental particles. However, the word “fundamental particles” refer to different ones in different ages. For example, as time goes by, those have ever been regarded as “the most fundamental particles” are elements, molecules, atoms and currently subatomic particles. Particle physics is the study of the fundamental particles and their interactions ^[1]. Most of these subatomic particles can only be produced with high energy accelerators and colliders; therefore particle physics is also called high energy physics.

1.1 The Standard Model

1.1.1 Fundamental Particles and Interactions in the Standard Model

Until now, the best theory that describes subatomic particles and their interactions in theoretical particle physics is the Standard Model ^[2-3]. The Standard Model includes twelve fundamental fermions, four gauge bosons and the Higgs boson.

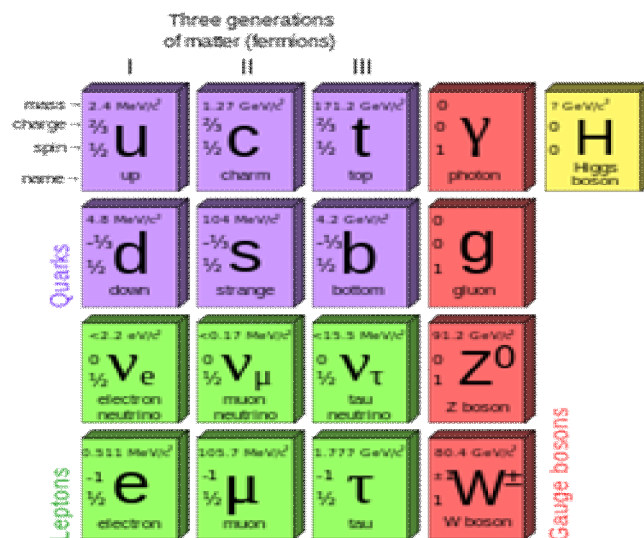


Figure 1.1: Fundamental particles in the Standard Model ^[4].

The twelve fermions divided into three generations contain six quarks (up, down, charm, strange, top, bottom) and six leptons (electron, electron neutrino, muon, muon neutrino, tau, tau neutrino); while each generation contains two quarks and two leptons. The total charge of each generation is equal.

Quarks carry color charge and an isolated quark can not be directly observed due to a phenomenon called color confinement. It means that a quark is always strongly bound to an antiquark (mesons) or another two quarks (baryons) to form composite particles called hadrons which are color-neutral. For example, proton and neutron are the two most common baryons. Quarks are the only fundamental particles that can take part in all the four fundamental interactions in nature.

Leptons do not carry color charge so that they can not experience the strong interaction. The three neutrinos do not carry electric charge therefore they can only interact with other fermions via the weak interaction, which makes them very difficult to detect. The electron, muon and tau carry electric charge therefore they can experience the weak interaction and the electromagnetic interaction.

The four gauge bosons are force carriers. According to the Standard Model, matter particles interact with others by exchanging force mediating particles called gauge bosons. Massless photons are the mediating particles of the electromagnetic interaction; W^+ , W^- and Z bosons are the mediating particles of the weak interaction; while the eight gluons are the mediating particles of the strong interaction between color charged particles.

The massive Higgs boson is a unique member in the Standard Model and used to explain why other particles are massive or massless in terms of the Higgs mechanism.

The information of the four fundamental interactions in nature is listed in

table 1.1. The Standard Model focuses on the weak, electromagnetic and strong interaction, while the gravitation is another topic which is beyond the Standard Model.

Table 1.1: The four fundamental interactions in nature ^[5].

PROPERTIES OF THE INTERACTIONS					
Property \ Interaction	Gravitational	Weak (Electroweak)	Electromagnetic	Strong	
				Fundamental	Residual
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge	See Residual Strong Interaction Note
Particles experiencing:	All	Quarks, Leptons	Electrically charged	Quarks, Gluons	Hadrons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons	Mesons
Strength relative to electromag for two u quarks at:	10^{-41}	0.8	1	25	Not applicable
	10^{-41}	10^{-4}	1	60	to quarks
	10^{-36}	10^{-7}	1	Not applicable to hadrons	20

1.1.2 QCD and the Electroweak Theory

The Standard Model based on the local $SU(3) \times SU(2) \times U(1)$ symmetry is a gauge theory which consists of the quantum chromodynamics and the electroweak theory.

The quantum chromodynamics (QCD) is a non-abelian gauge theory with $SU(3)$ symmetry and a study of the strong interaction. The two unique properties in QCD are color confinement and asymptotic freedom. The former one is a mechanism that keeps quarks and antiquarks inside the hadrons, while the latter one means that the coupling strength increases as the particles with color charge get closer to each other.

The electroweak theory is a non-abelian gauge theory with $SU(2) \times U(1)$ symmetry which uniformly describes the electromagnetic and weak interaction. These two interactions could be unified when the energy is larger than the unification energy, which happened during the early epoch of the universe in terms of the Big Bang theory ^[6].

One goal of physicists is to seek a theory that can unify all these interactions to some extent.

1.2 Experimental Methods in Particle Physics

Experiments in particle physics mainly contain three parts: sources of high energy particles, particle detectors and data analysis tools.

1.2.1 Sources of High Energy Particles

Sources of high energy particles contain natural sources and artificial sources.

The main natural sources are cosmic rays ^[6]. They originate from the cosmos and could produce showers of high energy secondary particles that can penetrate the Earth's atmosphere. The energy of these particles are much larger than those produced in artificial sources, however they are difficult to detect.

Artificial sources are mainly accelerators ^[2-3]. An accelerator is a device that accelerates particles produced artificially to high speeds with electromagnetic fields.

A collider is a kind of accelerator that accelerates a beam of particles to high speed and then let them collide with another beam of high energy particles. The advantage of colliders is that particles produced in the collision process will own larger center-of-mass energy.

Different colliders operate at different energy regions and use different initial state particles. For example, the LHC located at CERN ^[7] is the largest and most powerful collider in the world. It use two beams of protons as incoming particles and its main aim is to seek the Higgs boson and test the supersymmetric theory. The Beijing Electron-Positron Collider II (BEPCII) ^[8], discussed in this thesis, uses beams of electrons and positrons as incoming particles and works at the tau-charm energy

region.



Figure 1.2: The layout of the LHC ^[9].

1.2.2 Particle Detectors

A particle detector ^[2-3] is a device designed to measure information of high energy particles and is a part of a collider. The physical quantities that can be measured are mainly the velocity, the lifetime, the electric charge, the momentum and the energy. For example, in figure 1.2, the detectors installed at the Large Hadron Collider are the CMS ^[10], the ALICE ^[11], the ATLAS ^[12] and the LHCb ^[13]. The detector of the BEPCII is called the Beijing Spectrometer III (BESIII) ^[14].

1.2.3 Data Analysis Tools

Data, produced by detectors, have to be analysed with powerful software systems to process the large amount of experimental data. In early stage, softwares were usually written with the FORTRAN language based on the idea of Procedure-Oriented Programming. Currently, most mainstream softwares used in the particle physics field are written with the C++ language based on the idea of Object-Oriented Programming, such as GEANT4 ^[15] and ROOT ^[16].

Data generation and analysis in this thesis is done using the ROOT

framework. The details of ROOT and other used softwares will be introduced in chapter 3.

1.3 Background and Motivation

The Beijing Electron-Positron Collider II (BEPCII) is a double-ring e^+e^- collider located at Beijing, China. The spectrometer of the BEPCII, namely the Beijing Spectrometer III (BESIII), is composed of a series of sub-detectors. The details of BEPCII and BESIII will be introduced in the next chapter.

The BESIII experiment focuses on studying the physics in the tau-charm energy region.

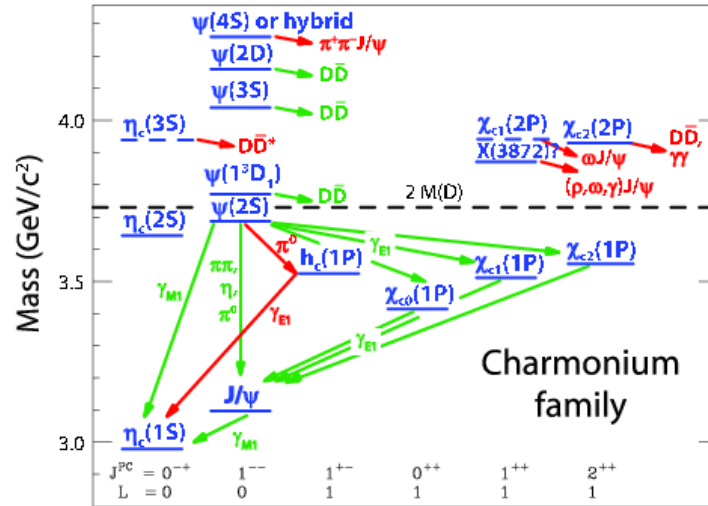


Figure 1.3: The spectrum of charmonium family ^[17].

The collision between electron and positron beams can produce a large amount of charmonium events such as J/ψ and ψ' events. The QCD theory allows the existence of exotic hadrons such as glueballs, hybrid hadrons, exotic baryons and exotic mesons. One important example of exotic hadrons is the $Z_c(3900)$. It was first reported by two independent groups in 2013: the BESIII collaboration ^[18] and the Belle collaboration ^[19]. The components of $Z_c(3900)$ are not clear so far; one possible explanation is that it is composed of four valence quarks. If this is

confirmed, it will be a strong evidence of the QCD theory and the understanding of sub-atomic world will get into a new realm. Therefore the research of $Z_c(3900)$ is very meaningful.

However, the number of $Z_c(3900)$ events is too small compared to the large amount of background events in experiments, which makes it very difficult to detect. In high energy physics experiments, when dealing with the small signal events such as the branching fraction of a forbidden process and this $Z_c(3900)$ example, the precision and upper limits of the measurement are very important.

The outcomes of high energy physics experiments depend on many factors such as the physics model, the resolutions, etc. Therefore the outcome is not an exact value but one with some dispersion. The systematic uncertainty is used to show the dispersion in experimental analyses ^[20]. It is very important for upper-limit measurements because in these measurements, the numbers of signal events are always very small; therefore, even if the systematic uncertainty is not large, it may affect the final result obviously.

In high energy physics experiments, how to deal with the systematic uncertainty is an important problem in the data analysis process. This thesis will make a preliminary exploration of this topic. This thesis is focused on calculations of systematic uncertainty for upper-limit measurements.

In the BESIII analyses, the systematic uncertainty is divided into two parts: uncertainty of signal efficiency and common uncertainties. For a forbidden process, the branching fraction is equal to the number of signal events divided by the number of total events. Equation (1.1) is used to calculate the upper limit value of the corrected branching fraction ^[21]

$$B < \frac{N'_{total}}{(1 - \sigma_{common}^{sys})N_{all}} \quad (1.1)$$

Where N_{total} means the total number of signal events and N'_{total} is the corrected value of N_{total} after taking the systematic uncertainties of the signal efficiency into account, and σ_{common}^{sys} is the common uncertainty. Without taking the systematic uncertainty into account, the value of N_{total} was calculated at a certain confidence level (usually 90%), therefore the confidence level of B was also 90%. However, after considering the systematic uncertainties, the confidence level of the corrected value of B might change.

This thesis aims to test before and after correction with the systematic uncertainty whether the confidence level will change or not.

1.4 The Outline of the Thesis

Chapter 1 is a brief introduction of particle physics; and then introduces the task of this thesis.

Chapter 2 introduces the BEPCII accelerator and BESIII detector. Some main parts of the BESIII detector are discussed in details.

Chapter 3 introduces of statistics and softwares used in this thesis. The Monte Carlo method and interval estimation are discussed in details. Meanwhile, the outline of ROOT, RooFit and RooStats packages is also given.

Chapter 4 is the core part of the thesis. It explains the basic idea of the simulation. And then show the results of simulation and make specific analyses.

Chapter 5 is an overall summary. It reviews the physical background and motivations of this thesis, and gives the conclusion of the research.

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Chapter 2 BEPCII and BESIII

The Beijing Electron-Positron Collider (BEPC) and its detector, the Beijing Spectrometer (BES), started to run in 1989. The first upgrade for the accelerator and the detector (called BEPC and BESII afterwards) was implemented from 1994 to 1996, and the second upgrade for them (called BEPCII and BESIII afterwards) was implemented from 2004 to 2008.

This chapter is a brief introduction of the BEPCII accelerator and the BESIII detector.

2.1 BEPCII

The BEPCII is composed of the injector, the transportation line, the storage ring, the BESIII detector and the Beijing Synchrotron Radiation Facility (BSRF)^[1].

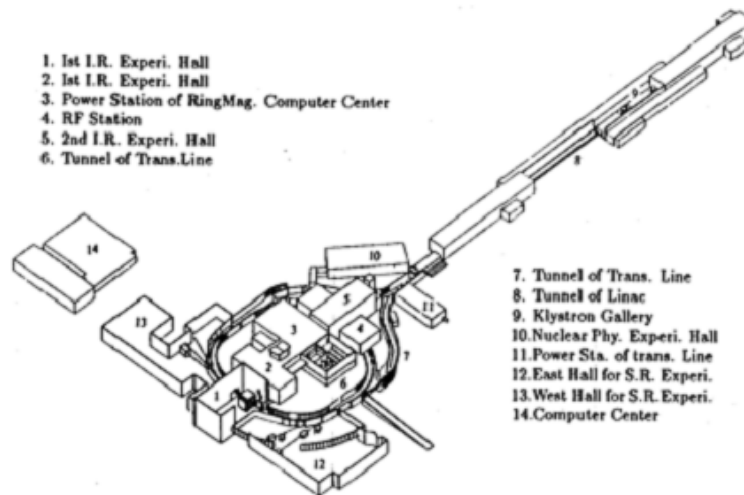


Figure 2.1: The layout of BEPCII^[2].

The injector is a 202 m long linear accelerator which is able to accelerate the electrons and positrons to 1.3 GeV. The transportation line is used to connect the injector and the storage ring. It transports the electron and positron beams coming from the injector to the storage ring. The storage ring is a circle shaped accelerator with a circumference of 240 m. Its task

is to accelerate the electrons and positrons again and then store them. The main parameters of BEPCII are listed in table 2.1.

Table 2.1: The main parameters of BEPCII ^[1].

Beam Energy E	GeV	1.55 (1.89)
Circumference C	m	237.53
Bunch Number k_b		93
Beam Currents per Ring I_{beam}	mA	1116
RF Frequency f_{RF}	MHz	499.8
RF Voltage per ring V_{RF}	MV	1.5
Beta Function at IP β_x^*/β_y^*	cm	100/1.5
Emittance $\varepsilon_x/\varepsilon_y$	m·mr	0.21/0.00315
Bunch Length σ_{z0}/σ_z	cm	1.1/1.5
Bunch Spacing S_b	m	2.4
Impedance $ Z/n _0$	Ω	< 0.7
Tune $\nu_x/\nu_y/\nu_z$		6.58/7.64/.047
Damping Time $\tau_x/\tau_y/\tau_z$	ms	44/44/22
Beam-beam Parameter ξ_x/ξ_y		0.04/0.04
Crossing Angle ϕ_c	mrاد	11×2
Luminosity L	$\text{cm}^{-2}\text{s}^{-1}$	1×10^{33}

2.2 BESIII

BESIII is the detector installed at the BEPCII accelerator. It works at the tau-charm energy region and is used to detect the final state particles produced in e^+e^- annihilation process. Its three main physics goals are: study of electroweak interactions, study of strong interactions and search for new physics ^[3].

The BESIII detector consists of beam pipe, main drift chamber, Time-of-Flight system, electromagnetic calorimeter, muon identifier, superconducting magnet, electronics system, trigger system and data acquisition system.

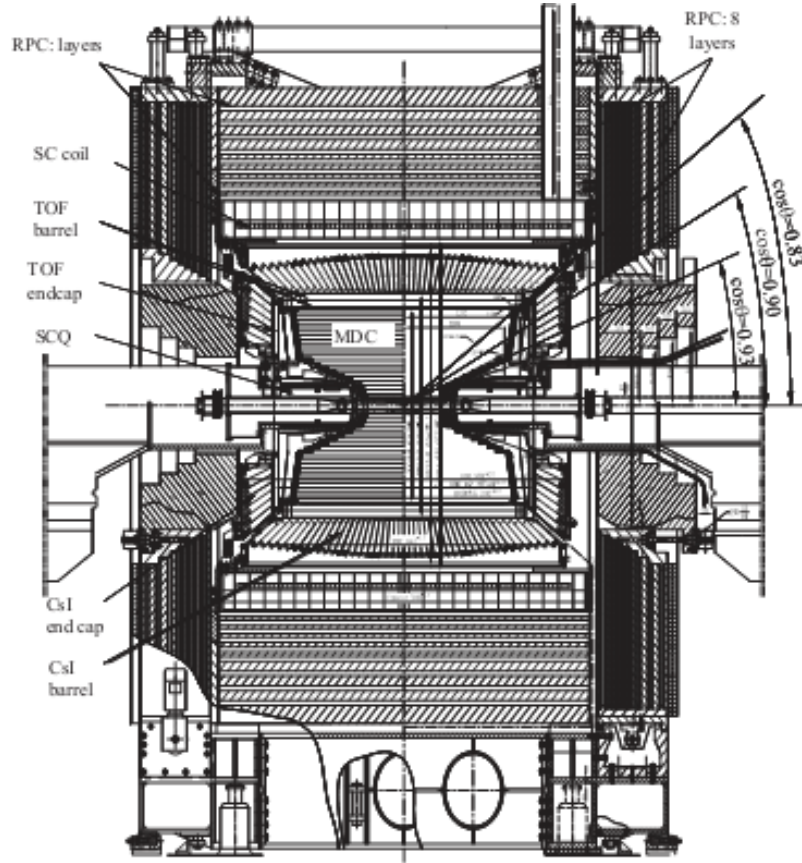


Fig. 1. Schematic drawing of the BESIII detector.

Figure 2.2: The structure picture of BESIII detector^[4]

2.2.1 Beam Pipe

The beam pipe^[5] is the innermost part of the BESIII detector. The total length of the beam pipe is 1000 mm. The central *Be* part is 300 mm long and it is welded to an extension section which is 350 mm long on both sides.

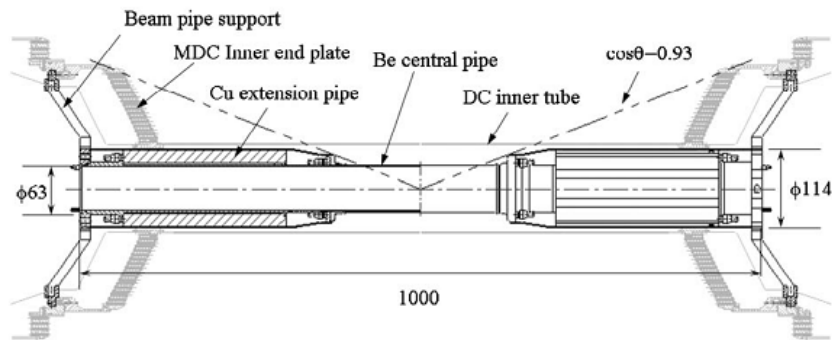


Figure 2.3: The cross-sectional picture of the beam pipe^[4]

2.2.2 Main Drift Chamber

The main drift chamber (MDC) ^[5] is the innermost sub-detector of the BESIII detector. It contains two chambers, an inner one and an outer one. In MDC, by measuring the trajectory of a charged particle in a known magnetic field, the momentum of this particle can be determined. And the particle type can also be identified by measuring the specific energy deposits (dE/dx).

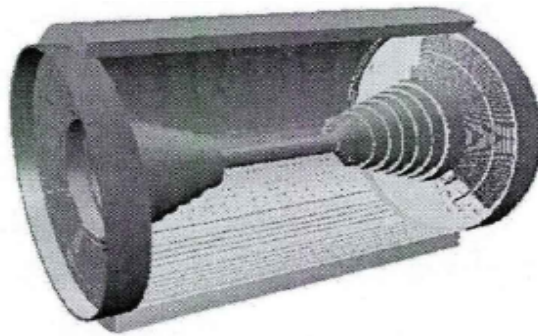


Figure 2.4: The overview of MDC ^[3]

2.2.3 The Time-of-Flight System

The Time-of-Flight (TOF) ^[5] system is made up of plastic scintillator bars and read out by fine-mesh phototubes. The position of TOF is between the main drift chamber and the electromagnetic calorimeter. It is designed to identify the particle type by measuring the flight time of charged particles.

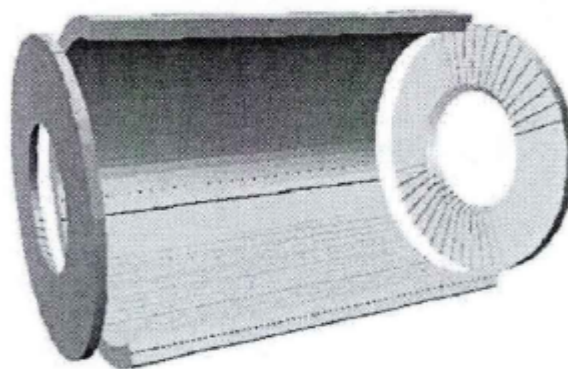


Figure 2.5: The overview of TOF ^[3]

2.2.4 Electromagnetic Calorimeter

The Electro-Magnetic Calorimeter (EMC) ^[5] is used to measure the energies and positions of electrons and photons precisely. It contains one barrel and two endcap sections and has 6272 CsI(Tl) crystals in total.

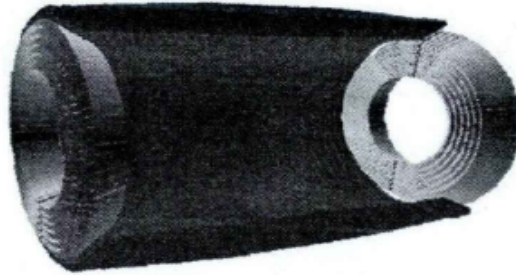


Figure 2.6: The overview of EMC ^[3]

2.2.5 Muon Identifier

The muon identifier ^[5] is the outmost sub-section of the BESIII detector. It is composed of muon counters and hadron absorbers and mainly used to distinguish muons produced in e^+e^- annihilation process from hadrons.

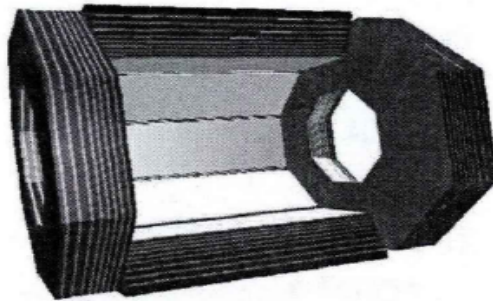


Figure 2.7: The overview of muon identifier ^[3]

2.2.6 Superconducting Magnet

The superconducting magnet ^[5] in the BESIII detector provides a 1T axial magnetic field with good field uniformity over the tracking volume. When a charged particle gets into the superconducting magnet, its trajectory will be deflected. By measuring the radius of the deflection trajectory, the particle detector can measure the charge-to-mass ratio of the charged particle.

2.2.7 Readout Electronics System

The electronics system ^[5] in the BESIII detector consists of 4 parts: the MDC electronics system, the EMC read out electronics, the TOF electronics and the muon counter electronics.

- (1) The MDC electronics system is designed to process the output signals from 6796 sense wires of MDC.
- (2) The function of EMC read out electronics is to measure the charge to determine the energy deposited in the CsI crystals.
- (3) The TOF electronics has 3 main tasks: time measurement, charge measurement and fast timing signals for trigger.
- (4) The muon counter electronics consists of the readout system which can scan data and a test sub-system which is able to test the readout system.

2.2.8 Trigger System

The trigger system ^[5] of the BESIII detector is a fast real-time event selection and control system. It is designed to select signal events and suppress backgrounds to a level which can be sustained by the data acquisition system. It is composed of MDC, TOF, EMC, Track Matching and global trigger subsystems.

The BESIII trigger system is in two levels: a level-1 (L1) hardware trigger and a level-2 software event filter. Some basic trigger information are number of tracks and angle/position of tracks in MDC, timing signal and hit counts in TOF, and the total energy, the energy balance and the cluster counting in EMC. They will be assembled by the global trigger subsystems. When a valid trigger condition is satisfied, which means that the combination of these information can correspond to a signal event, all BESIII sub-detectors are read out.

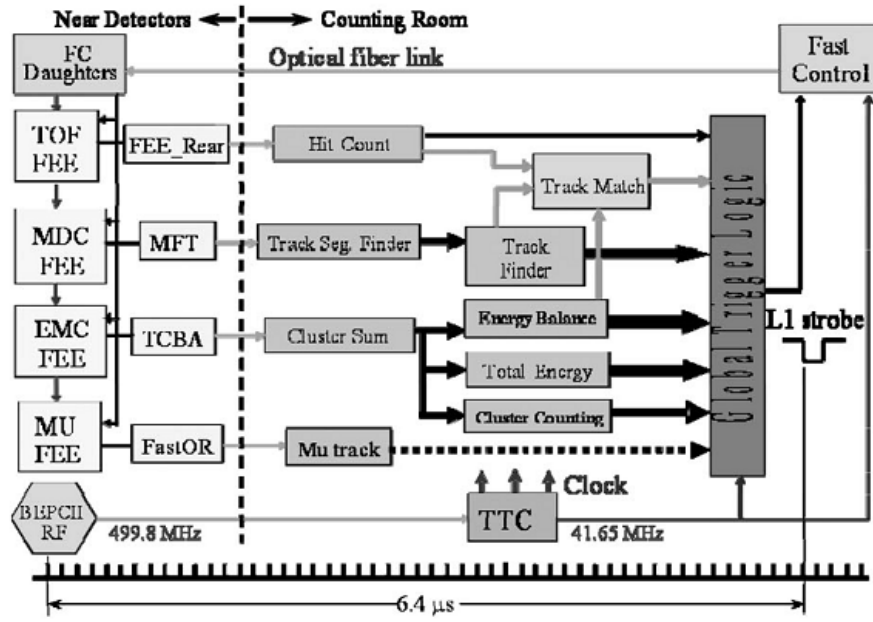


Figure 2.8: The block picture of the trigger system ^[4]

2.2.9 Data Acquisition System

The data acquisition system (DAQ) ^[5] is composed of the readout system, the online control and monitoring system, the calibration system and other support/service systems.

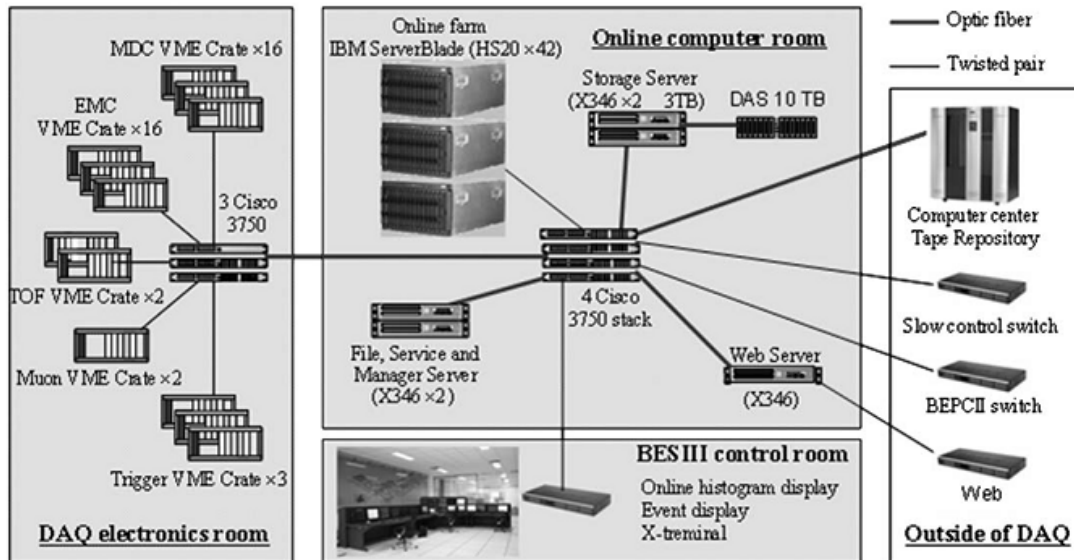


Figure 2.9: The Architecture of the DAQ System ^[4]

The functions of the DAQ system are collecting, transferring, assembling, filtering event data and at last writing accepted events into a persistent

media.

The DAQ system uses multi-level buffering, parallel processing, high-speed VME readout and network transmissions techniques to improve the efficiency of readout of data.

2.3 BESIII Offline Software

The BESIII Offline Software System (BOSS) ^[3,5] uses object-oriented technology by choosing the C++ language. Its two main tasks are processing experimental data and MC data and managing tools and documents. The whole data handling and physics analysis software system contains five parts: a framework, the MC simulation, the data reconstruction package, and the calibration package and analysis tools.

2.3.1 Framework

The BOSS framework is developed based on the Gaudi package ^[6]; it mainly consists of the following parts: algorithm, application manager, transient data store, service and converters.

2.3.2 Simulation

The BESIII Object Oriented Simulation Tool (BOOST) is composed of event generators, detector description, particle tracking and detector response. It is based on the GEANT4 package ^[7].

2.3.3 Reconstruction

The reconstruction package mainly consists of algorithms for track finding and fitting, particle identification, shower- and cluster-finding, and muon track finding.

2.3.4 Calibration

The calibration software contains a calibration framework and calibration algorithms. Data calibration needs to be done both online and offline.

In the BESIII detector, double-muon events are used to do the calibration of the muon identifier, while Bhabha events are used to do the calibration of other sub-detectors.

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Chapter 3 Statistics and Softwares

This chapter introduces the statistical background and computer softwares used in this research.

3.1 Monte Carlo method and Random Number

3.1.1 Monte Carlo method

The Monte Carlo method ^[1-3] (also called the statistical simulation method) is a method which uses the statistical sampling theory to solve mathematical or physical problems approximately.

In 1940s, S. Ulam and J. von Neumann first used this method in the “Manhattan Project” and named it with the name of the famous gambling casino city of Monaco. The Monte Carlo method developed fast with the development of computer technology. It is currently widely used in macroeconomics, computational physics, etc.

The basic principle of the Monte Carlo (MC) method is to construct a model distribution in terms of the problem to make sure the quantities of interest correspond to features of the model distribution, and then do the sampling process to the model to get a sample and calculate the values of the features of the model in terms of the sample. These values can be regarded as the approximate solutions of the quantities of interest.

The MC method based on the probability and statistics theory needs to use random numbers to do the calculation and the key in MC simulation is the quality, i.e. randomness of these random numbers.

The Monte Carlo method is widely used in high energy physics. One example is the GEANT4 package ^[4]. The GEANT4 package is a toolkit for the simulation of propagation of particles through matter and uses the idea of the Monte Carlo method to do the simulation. In the upgrade process from BESII to BESIII, GEANT4 is used to do the simulation

work of sub-detectors.

Another important application of the Monte Carlo method is to calculate the numerical integration. In high energy physics experiments, it can be used to calculate cross sections of reactions.

3.1.2 Pseudorandom Numbers

In the MC simulation process, a set of values of a random variable which has a certain probability distribution will be produced. These values are usually called random numbers ^[3,5]. Random number contains the real random number and the pseudorandom number.

A random number generator is a physical or computational device which is able to generate a sequence of random numbers.

The real random number, which can only be produced by real random number generators, i.e. real physics phenomena, such as dice, coin flipping, the noise of the stereo system and radioactive decays, is not easy to generate. Large amounts of random numbers are needed in MC simulation, which requires a lot of work and time, so that in most cases it is not convenient to use the real random number in MC simulation.

The pseudorandom number can be generated by pseudorandom number generators, i.e. deterministic algorithms. It appears to be random but in fact is not. Because a series of values generated by such a algorithm is usually determined by a special term (called seed) in the algorithm, which means that if the value of seed is fixed and run this algorithm a few times to get a few series of random numbers, the results will be the same. For example, the expression of one common used algorithm (called linear congruential generator) is:

$$X_{n+1} = (aX_n + b) \pmod{m} \quad (3.1)$$

In equation (3.1), a , b and m are large fixed integers, and X_0 is the seed of this algorithm. Given two different initial values of X_0 , this algorithm will

generate two different series of random numbers. However, if these two initial values are the same, the algorithm will generate two same series of random numbers. The initial value of X_0 can be arbitrary numbers, which make sure the outcome is random.

Generating random numbers from a known probability density function is a common procedure in MC simulation and two main methods to implement it are the inversion method and the acceptance-rejection method.

The randomness of the pseudorandom number is determined by a series of statistical tests. The two basic statistical tests are the test of homogeneity (also called frequency test) and test for independence. The former one refers to test whether the random number sequence generated by a generator is uniformly distributed in the $[0,1]$ region; while the latter one test whether the statistical correlation between the random numbers in a random number sequence is significant. The pseudorandom number can be used in MC simulation if it can pass these statistical tests with a given degree of significance.

Most computer languages and softwares have implemented random number generators. For example, in the standard library of the C++ language, the `rand()` function is used to generate random numbers. In ROOT and RooFit, the `TRandom` class is designed to generate random numbers from defined probability distributions and the `SetSeed()` function is designed to determine the initial value of the seed in the algorithm.

3.2 Interval Estimation

In statistics, a common task is to estimate the value of a fixed unknown parameter of a known probability distribution. There are two main methods to do the estimation: the point estimation and the interval

estimation. The former one gives a signal value result of the parameter of interest while the latter one gives an interval result which consists of a lower bound value and an upper bound value of the parameter.

The method of estimation is widely used in high energy physics experiments. For example, when measuring the branching fraction of a forbidden process, the result is usually an upper limit confidence interval accompanied by a certain confidence level, which is calculated with one method of interval estimation. In this thesis, I will focus on researching upper limits confidence intervals.

3.2.1 The definition of confidence interval

A dataset x_1, \dots, x_n which is the realization of random variables X_1, \dots, X_n is given. The parameter of interest is θ and γ is a number between 0 and 1. If two sample statistics $L_n = f(X_1, \dots, X_n)$ and $U_n = g(X_1, \dots, X_n)$ exist and satisfy

$$P(L_n < \theta < U_n) = \gamma \quad (3.2)$$

for each value of θ , then (l_n, u_n) , where $l_n = f(x_1, \dots, x_n)$ and $u_n = g(x_1, \dots, x_n)$, is called a $100\gamma\%$ confidence interval for θ . γ is called the confidence level.

Sometimes the two sample statistics L_n and U_n as defined above do not exist, but one could find L_n and U_n that satisfy

$$P(L_n < \theta < U_n) \geq \gamma \quad (3.3)$$

then the result (l_n, u_n) is called a conservative $100\gamma\%$ confidence interval for θ , which means that the actual confidence level may be higher^[6].

3.2.2 The meaning of confidence interval

In the research of particle physics, two different kinds of confidence intervals are widely used.

The first type is the Bayesian confidence interval^[7] (also called credible

interval). The meaning of this kind of confidence intervals is that if an interval calculated in terms of a sample is $[\mu_1, \mu_2]$ and the confidence level is 90%, then the probability that $[\mu_1, \mu_2]$ covers the true value μ_t is 90%.

The second type is the Newman's confidence interval ^[8]. The meaning of this kind of confidence intervals is that if there are 100 confidence intervals calculated in terms of 100 different samples and the confidence level is 90%, then in principle, 90 of them cover the true value μ_t . It does not describe any signal interval because for each signal one, μ_t is either in it or not.

3.2.3 One-sided confidence intervals

The definition of one-sided confidence intervals could be formulated based on the definition of the central confidence intervals. If a sample statistic L_n exists and satisfies

$$P(L_n < \theta) = \gamma \quad (3.4)$$

for each value of θ , then (l_n, ∞) is called a $100\gamma\%$ lower limits confidence interval. Similarly, if a sample statistic U_n exists and satisfies

$$P(\theta < U_n) = \gamma \quad (3.5)$$

for each value of θ , then $(-\infty, u_n)$ is called a $100\gamma\%$ upper limits confidence interval ^[6].

3.2.4 Confidence Belt

The confidence belt ^[7] which contains one unknown parameter and one observable can be used to construct the confidence intervals.

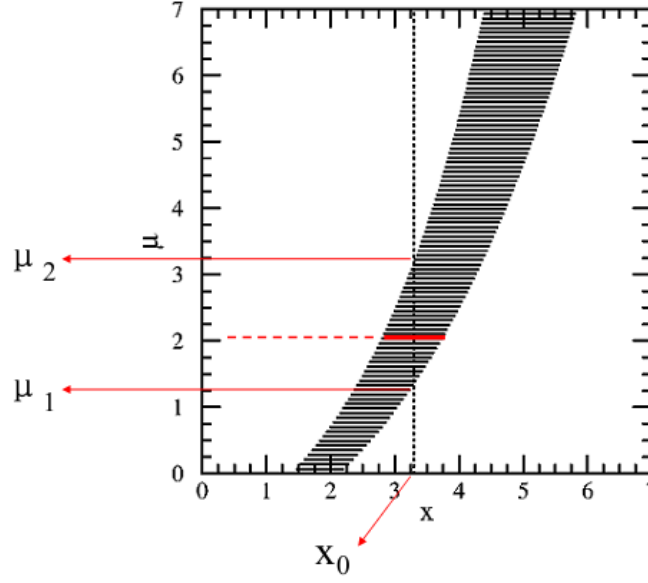


Figure 3.1: An example of the confidence belt ^[9].

Figure 3.1 shows an example of the confidence belt. In this example, x is the observable and μ is the parameter of interest. The method to construct it is the following steps. Firstly, for each value of μ , construct a confidence interval of x which satisfies (α is the confidence level)

$$P\langle x \in [x_1, x_2] | \mu \rangle = \alpha \quad (3.6)$$

And then draw a horizontal acceptance interval $[x_1, x_2]$. For example, when $\mu=2$, the horizon acceptance interval is the solid red line without arrow in this picture. For each value of μ we need to draw a line like this. Secondly, perform an experiment to measure x and obtain its value x_0 , and then draw a line through x_0 , which is the dashed black vertical line in this picture. The resulting confidence interval of μ is $[\mu_1, \mu_2]$ in this picture, which is the union set of all allowed values of μ for those the corresponding acceptance intervals of x are intercepted by the dashed vertical line.

This is the whole process to construct the confidence belt between a random variable and an unknown parameter.

The content in this sub-section is a basic introduction of the traditional Newman's interval estimation theory.

3.3 The Feldman-Cousins's method

The Feldman-Cousins's (FC) method ^[9] is presented by J. Feldman and D.Cousins. It is a new method to construct confidence intervals based on the traditional Newman's method. It uses the ordering principle based on likelihood ratio to construct confidence intervals.

3.3.1 The drawbacks of Newman's confidence intervals

If a confidence interval is constructed with the traditional Newman's method, sometimes it may suffer from two problems: empty set and undercover.

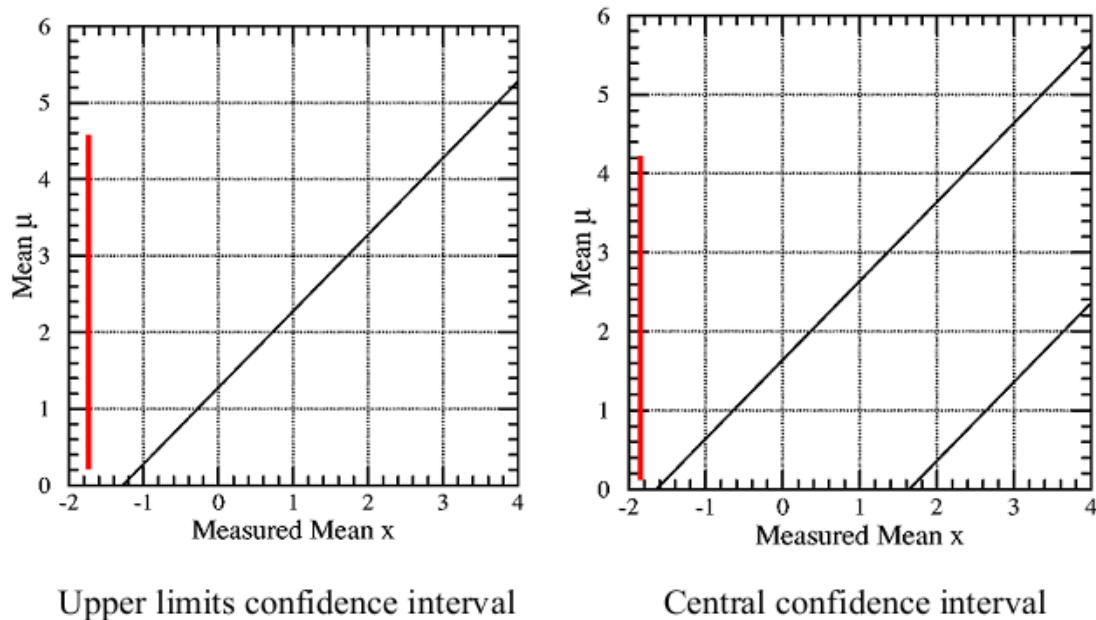


Figure 3.2: The confidence belts for 90% C.L. for the mean of a normal distribution^[9]. Figure 3.2 shows the confidence belts for 90% C.L. for the mean of a standard normal distribution; the left picture is the upper limits confidence interval while the right is the central confidence interval. As shown by the red line, in both of them, when $x=-1.8$, there is no corresponding values of μ . This phenomenon is called empty set.

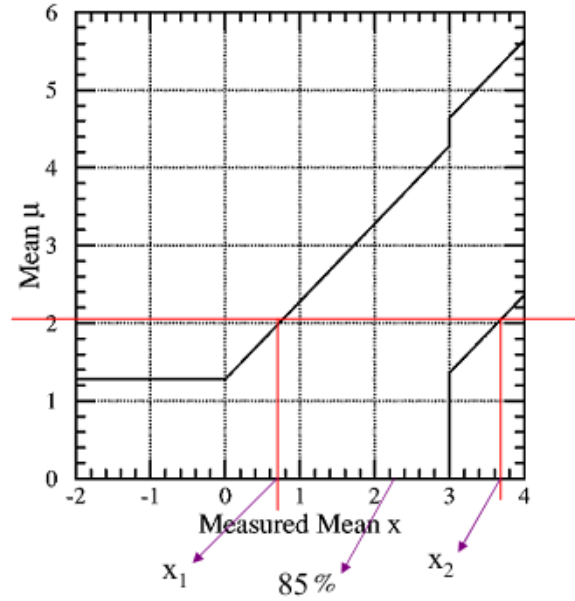


Figure 3.3: The combination of the upper limits and central confidence intervals ^[9]. Figure 3.3 shows the combination of the two pictures in figure 3.2. In this figure, the transition between upper limits and central confidence intervals is not smooth, which leads to a problem called undercover. For example, when $\mu=2$, the confidence level of the horizontal interval $[x_1, x_2]$ is 85% which is smaller than 90%, this phenomenon is called undercover. Because of these two drawbacks, a better method than the Newman's method is needed. A potential choice is the Feldman-Cousins's method. Figure 3.4 shows the confidence interval constructed with the FC method for the same problem as discussed above. In this picture, for each value of x , the confidence interval of μ is not an empty set, and the confidence level is always 90% because the transition between upper limits and central confidence intervals is smooth. The two main disadvantages do not exist in this case.

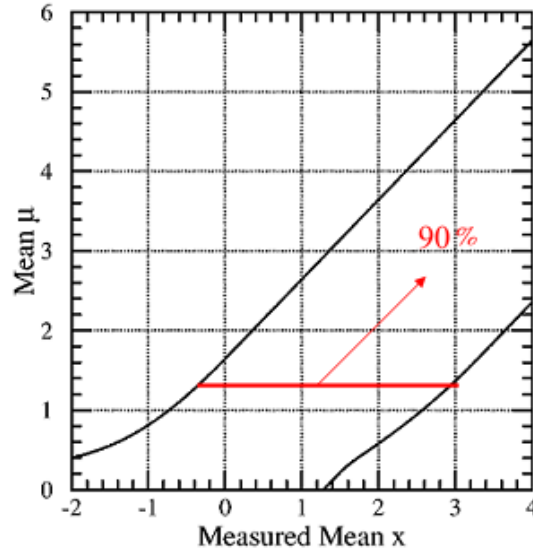


Figure 3.4: The confidence interval constructed with the FC method ^[9].

Based on the comparison between the Newman's method and the FC method, this thesis chooses the latter one to construct confidence intervals in the research.

3.3.2 Construct FC confidence intervals

Here a numerical example is used to illustrate how to use the FC method to construct confidence intervals.

The model is a Poisson process with background. The probability density function is

$$P\langle n|\mu\rangle = \frac{\langle \mu + b \rangle^n \exp[-\langle \mu + b \rangle]}{n!} \quad (3.7)$$

In equation (3.7), n is used to denote the discrete random variable x which stands for the total number of observed events, μ is the parameter of interest which stands for the number of signal events and b is a known parameter ($b=3$ here) which stands for the number of background events. The task is to construct the confidence belt between μ and n . In order to do this we need to calculate the acceptance interval for each value of μ . Here $\mu=0.5$ is used to do the calculation in this example. The values of n are listed in the first column of table 3.1.

Table 3.1: Illustrative calculations in the confidence belt construction ^[9].

b=3, $\mu=0.5$							
n	$P(n \mu)$	μ_{best}	$P(n \mu_{\text{best}})$	R	rank	U.L.	central
0	0.030	0.0	0.050	0.607	6		
1	0.106	0.0	0.149	0.708	5	✓	✓
2	0.185	0.0	0.224	0.826	3	✓	✓
3	0.216	0.0	0.224	0.963	2	✓	✓
4	0.189	1.0	0.195	0.966	1	✓	✓
5	0.132	2.0	0.175	0.753	4	✓	✓
6	0.077	3.0	0.161	0.480	7	✓	✓
7	0.039	4.0	0.149	0.259		✓	✓
8	0.017	5.0	0.140	0.121		✓	
9	0.007	6.0	0.132	0.050		✓	
10	0.002	7.0	0.125	0.018		✓	
11	0.001	8.0	0.119	0.006		✓	

Steps:

- (1) Calculate the probability $P\langle n|\mu \rangle$ with formula (3.7) for each value of n ; the results are given in the second column of table 3.1.
- (2) Calculate the value of μ which can maximize $P\langle n|\mu \rangle$ by using formula (3.8), we call it μ_{best} , the results are listed in the third column of table 3.1.

$$\partial P\langle n|\mu \rangle / \partial n = 0 \Rightarrow \mu_{\text{best}} \quad (3.8)$$

- (3) The μ_{best} should be non-negative and we set it as the bigger one between 0 and $(n-b)$ and then plug μ_{best} into equation (3.7) to calculate $P\langle n|\mu_{\text{best}} \rangle$, the results are listed in the forth column of table 3.1.

- (4) Calculate the likelihood ratio with equation (3.9) for each value of n , the results are listed in the fifth column of table 3.1.

$$R = P\langle n|\mu \rangle / P\langle n|\mu_{\text{best}} \rangle \quad (3.9)$$

- (5) Determine for $\mu=0.5$ which values of n should be added to the acceptance region. Rank these terms in terms of the value of R from large

to small. In this example, when $n=4$, R has the biggest value; we set this term as the first term; $n=3$ is the second term, and so on. And then we add up the $P\langle n|\mu\rangle$ of the first term with the second term, and then the third term until the sum is larger than the confidence level α . For example, if $\alpha=0.9$, we need to add up the first 7 terms. It means that for $u=0.5$, the acceptance region of n is $[0, 6]$. The principle is shown in equation (3.10)

$$P\langle n|\mu\rangle_{(rank=1)} + P\langle n|\mu\rangle_{(rank=2)} + \dots + P\langle n|\mu\rangle_{(rank=k)} \geq \alpha \quad (3.10)$$

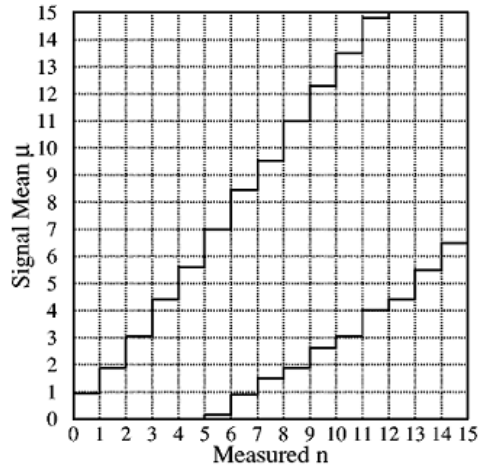


Figure 3.5: The confidence belt of this Poisson problem ^[9].

Repeat this whole process to get the acceptance region of n for each value of μ and at last construct the confidence belt between n and μ . Figure 3.5 shows the confidence belt of this problem ($b=3$, $\alpha=0.9$).

This is an example for discrete random variable. For continuous random variable cases, the basic principle and process are similar.

The content in this sub-section is a basic introduction of Feldman and Cousins's interval estimation theory. In the research of this thesis, the FC method is used to do the interval estimation.

3.4 Softwares

3.4.1 ROOT

ROOT ^[10-11] is an Object-Oriented data analysis framework and designed to scale up to the challenges coming from the LHC. It was created by

R.Brun and F. Rademakers in CERN and its independent C++ interpreter, namely CINT, was created by M. Goto in Japan.

An interesting feature of ROOT is that it was developed not only by developers but also by users, which means that physicists developed ROOT for themselves. This liberal development style makes it specific, appropriate and powerful. Developers and users can exchange ideas via the RootTalk Forum ^[12].

Being a good HEP analysis framework, ROOT can save physicists much work by providing a lot of HEP specific utilities. For example, some more commonly used components are Histograms, Fitting and 2D Graphics; while some less commonly used components are 3D Graphics and Network Communication.

In the ROOT framework, an environment variable called ROOTSYS can hold the path of the top ROOT directory. Figure 3.6 shows the structure of directories of the ROOT framework.

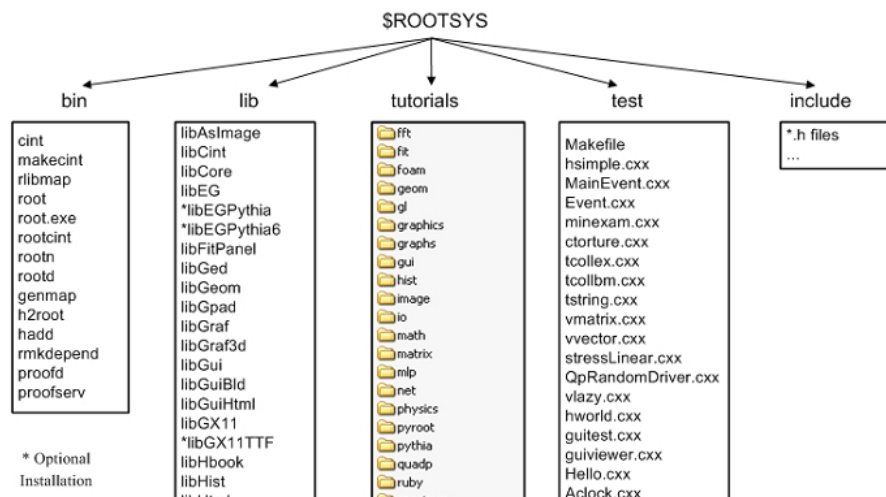


Figure 3.6: ROOT framework directories ^[11].

There are many defined libraries as shown in figure 3.7, that are used to minimize dependencies, such that users only need to load enough code for the task but not all libraries. When programming, related libraries should be linked with to make the classes contained in these libraries

available.

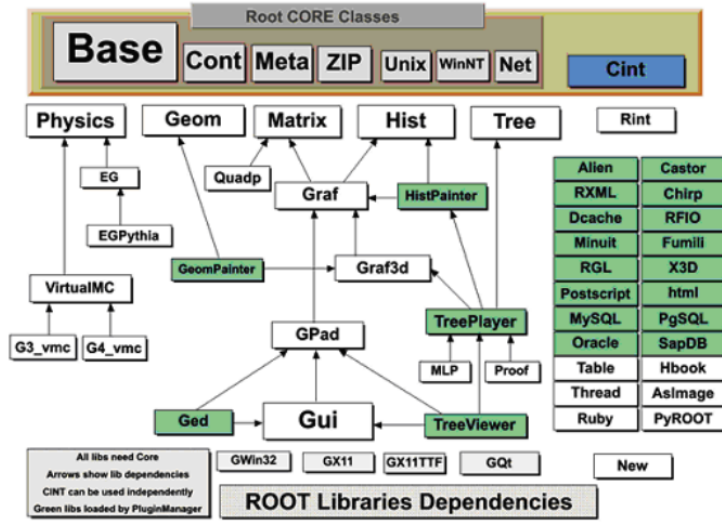


Figure 3.7: ROOT libraries dependencies ^[11].

As shown in the figure 3.8, there are three user interfaces: GUI (windows, buttons and menus), command line (CINT) and script processor (C++ compiler and interpreter).

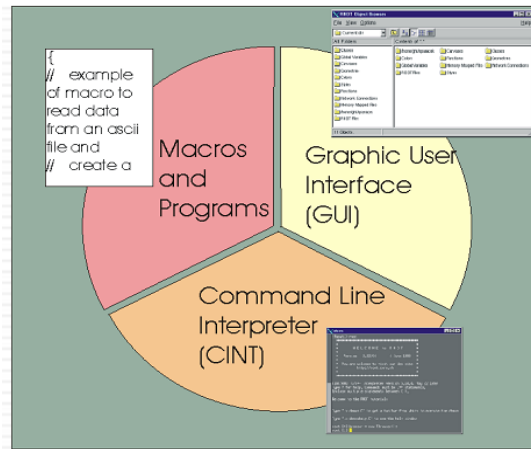


Figure 3.8: ROOT user interfaces ^[11].

More information about ROOT classes and tutorials are available on the ROOT home website ^[13-14].

3.4.2 RooFit

RooFit ^[15-16] is a library of C++ classes. It is designed to provide a toolkit to model the expected probability distribution of events in a HEP analysis. It was originally developed for the BaBar collaboration at the SLAC ^[17].

The RooFit package is integrated and distributed with the ROOT environment and focuses on constructing a probability density function (PDF). The ROOT built-in models are sufficient for simple problems. However, when the model is a complicated PDF, RooFit is more powerful than ROOT. To some extent, RooFit is an extension of ROOT. Figure 3.9 shows the relation between them.

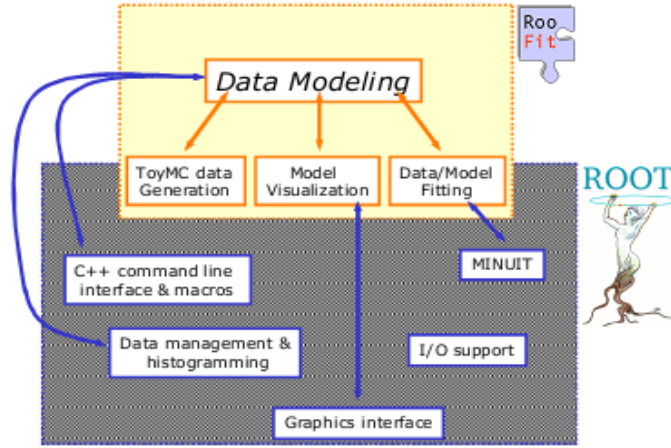


Figure 3.9: The relation between RooFit and ROOT ^[16].

In RooFit, each mathematical object is represented by a C++ object. Table 3.2 shows some examples.

Table 3.2: Correspondence between some math concepts and RooFit classes ^[16].

<i>Math concept</i>	<i>Math symbol</i>	<i>RooFit (base)class</i>
Variable	x	RooRealVar
Function	$f(x)$	RooAbsReal
P.D.F.	$F(x; p)$	RooAbsPdf
Integral	$\int_{x_{min}}^{x_{max}} f(x) dx$	RooRealIntegral
Space point	\vec{x}	RooArgSet
Addition	$fF(x) + (1 - f)G(x)$	RooAddPdf
Convolution	$f(x) \otimes g(x)$	RooFFTConvPdf

Some basic functions of RooFit are constructing one-dimensional or multi-dimensional models in terms of continuous or discrete variables, using a defined PDF to generate data, fitting the experimental data, plotting a PDF or a series of data, convolving a PDF with another one, and so on.

More information about RooFit classes and tutorials are available on the RooFit home website ^[18-19].

3.4.3 RooStats

RooStats ^[20-21] is a new software package, designed to provide advanced statistical tools for the data analysis in high energy physics experiments. It is a joint contribution of ATLAS ^[22] and CMS ^[23] and has been built on top of ROOT and RooFit. The four core developers are K. Cranmer (ATLAS), G. Schott (CMS), L. Moneta (ROOT) and W. Verkerke (RooFit).

The three main goals of RooStats are: standardize interface for commonly used statistical procedures to make sure that they are able to work on an arbitrary RooFit dataset and model; implement most accepted statistical techniques from Frequentist, Bayesian and Likelihood-based approaches; provide utilities for combined analysis.

Most statistical questions can be classified into four types: parameter estimation, interval estimation, hypothesis testing and goodness of fit. RooFit provides tools for the first one while RooStats provides functionality for the second and third one. Currently, the ROOT libraries provide classes for the last one. In RooStats, the interface for confidence interval calculations is *IntervalCalculator* and the interface for hypothesis tests is *HypoTestCalculator*. Each method for constructing confidence intervals is represented by a C++ class as shown in figure 3.10.

Each statistical method needs a PDF as the input model and this task is usually implemented by the RooFit. And then confidence intervals can be constructed by the high-level statistical tools provided by the RooStats. Basically, RooFit is responsible for the infrastructure, namely the model which consists of a PDF, observables and parameters of interest, and so on; while RooStats is responsible for the superstructure, namely

calculations of confidence intervals.

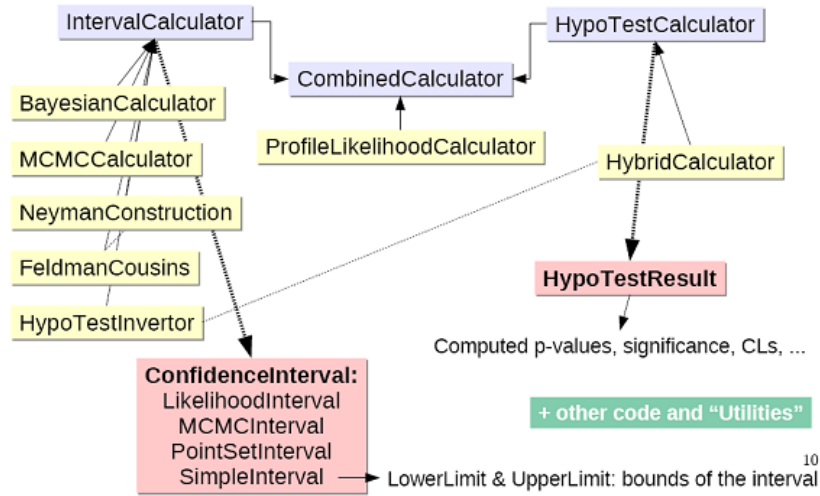


Figure 3.10: The overview of RooStats classes ^[20].

This thesis uses the FeldmanCousins class to do the interval estimation. It is an additional class which enforces a particular configuration of test statistic, distribution creator, limit type, etc.

More information about RooStats classes and tutorials are available on the RooStats home website ^[24-25].

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Chapter 4 Results and Analysis

As described in Chapter 1, the motivation of this thesis is to make a preliminary exploration of the method used to process the systematic uncertainty in the BESIII analyses. This chapter first describes the basic idea of the program aiming to do it, then shows the outcomes, discusses them and gives the conclusion. The details of the program will be introduced in the Appendix.

4.1 Testing Method

4.1.1 Basic Idea of the Simulation

The BESIII experiments detect final state particles produced in e^+e^- annihilation processes. Signal events and background events are usually coexisting in a certain region of the spectrum. When picking signal events out of the total observed events, the systematic uncertainty needs to be taken into account to do the correction. Therefore a reasonable method to deal with the systematic uncertainty is very important. This thesis simulates the process that picks signal events out of the total observed events with and without considering the systematic uncertainty, and focus on the confidence levels before and after corrections.

The basic idea of the simulating method is to use a Gaussian PDF to generate random numbers to stand for the numbers of signal events (n_{sig}) and use an Uniform PDF to generate random numbers to stand for the numbers of background events (n_{bkg}), and then add them up to get the total number of observed events (n_{observed}) at first; the spectrum is the combination of these two PDFs. Secondly, do the core procedure, i.e. construct a confidence interval of the parameter of interest which is the number of signal events in this thesis. Thirdly,

repeat the simulation 5000 times to make sure the sample is large enough. The whole process has been done for cases with and without systematic uncertainty, and then analysis obtained results to get the conclusion at last.

In the program, a reasonable PDF and an appropriate method to construct the confidence interval should be chosen. The observable in this simulation is the total number of observed events (n_{observed}), it should have a Poisson distribution; therefore, the PDF used in the interval estimation is a Poisson PDF. In terms of the two advantages (avoid empty set and undercover) introduced in Chapter 3.3, the Feldman-Cousins's method is used to do the interval estimation in this thesis.

Equation (4.1) is the expression of the Poisson PDF.

$$P\langle n|\mu\rangle = \frac{\langle \mu + N_{\text{bkg}} \rangle^n \exp[-\langle \mu + N_{\text{bkg}} \rangle]}{n!} \quad (4.1)$$

As can be seen in table 4.1, n stands for the total number of observed events (n_{observed}), N_{bkg} stands for the number of background events, and μ stands for the number of signal events. They play different roles in the process of the interval estimation as shown in the table.

Table 4.1: The information of variables used in the Poisson PDF.

name	meaning	role	initial value	region
n	number of total events	observable	$n_{\text{sig}} + n_{\text{bkg}}$	[0, upper value]
N_{bkg}	number of background events	known parameter	a chosen value	fixed
μ	number of signal events	parameter of interest	a chosen value	[0, upper value]

4.1.2 Introduce the Systematic Uncertainty

As described in Chapter 1.3, the equation used to do the correction in BESIII experiments is ^[1]

$$B < \frac{N'_{total}}{(1 - \sigma_{common}^{sys})N_{all}} \quad (4.2)$$

the original value needs to be processed by two steps to get the final result. In this thesis, the first step which appears in the numerator is simulated in the algorithm; this step is used to introduce the systematic uncertainty. While the second step which appears in the denominator is simulated after sampling; this step is used to do the correction.

(1) In order to simulate the first step, the initial value of N_{bkg} is not a fixed value but a series of random numbers generated from a chosen distribution around this value. For example, if this value is 50 and the systematic uncertainty is 10%, the initial value of N_{bkg} is a number between [45, 55]. The true distribution of the systematic uncertainty is unknown; therefore the distributions used to simulate the systematic uncertainty have to be chosen in terms of experience. In this thesis, two most basic and common kinds of distributions in statistics are used to generate random numbers to introduce the systematic uncertainty: the Gaussian distribution and the Uniform distribution. In the analysis part, the symbols **Gaussian Error** and **Uniform Error** are used to mark these two cases.

(2) After getting a confidence interval $[L, U]_{initial}$ with the first step, the second step is simulated artificially with the equation

$$[L, U]_{final} = \frac{[L, U]_{initial}}{1 - 10\%} \quad (4.2)$$

Then the confidence intervals which do not contain the true value of μ (use the term “outflow confidence intervals” later) among the 5000 $[L, U]_{final}$ will be picked out and the actual confidence level will be

determined.

This is the method to introduce the systematic uncertainty in this thesis.

4.1.3 Parameters in the Simulation

Table 4.2: The percentage of upper limit confidence intervals in each simulation.

Systematic Uncertainty=10%	No Systematic Error	Gaussian Error and Correction	Uniform Error and Correction
$N_{\text{bkg}}=30$	90.5%	90.5%	90.6%
$N_{\text{bkg}}=50$	94.1%	92.1%	94.4%
$N_{\text{bkg}}=100$	93.3%	92.6%	93.1%
$N_{\text{bkg}}=500$	94.3%	93.9%	94.3%
$N_{\text{bkg}}=1000$	96.2%	95.7%	98.1%
Systematic Uncertainty=20%	No Systematic Error	Gaussian Error and Correction	Uniform Error and Correction
$N_{\text{bkg}}=50$	94.1%	91.8%	92.0%

This thesis only does research on the upper limit confidence intervals, so the numbers of background events are selected to be much larger than the number of signal events. Simulations are time-consuming. As long as the number of signal is much smaller than the number of background events, conclusions of the analysis are the same. Therefore, the true value of μ is fixed at 2 and the appropriate values of b are $\{30, 50, 100, 500, 1000\}$. When the number of background events is 1000, basically all events are background events, which means that the signal events can be ignored. Therefore, it is meaningless to increase the number of background events

sequentially. Table 4.2 lists the percentage of upper limit confidence intervals in each simulation. All values are larger than 90%, which means that the chosen values of N_{bkg} above are reasonable.

In most BESIII analyses, the systematic uncertainties are from 5% to 20%, only a few ones are beyond this region ^[1-9]. Therefore the systematic uncertainty is selected to be 10% to perform the simulation in this thesis. In order to see what will happen when the systematic uncertainty is very large, one more simulation with the 20% systematic uncertainty has been done. As can be seen later, when N_{bkg} is very large, the confidence levels decrease dramatically after correction even the systematic uncertainty is not very large. Therefore the simulation of the 20% systematic uncertainty case is only done for one of the small N_{bkg} cases. Because of the time-consuming of simulations, in this thesis, only the $N_{\text{bkg}}=50$ case is chosen to do this simulation.

4.2 Results and Analysis

4.2.1 Result and Analysis for 10% Systematic Uncertainty Case

(1) Background=30

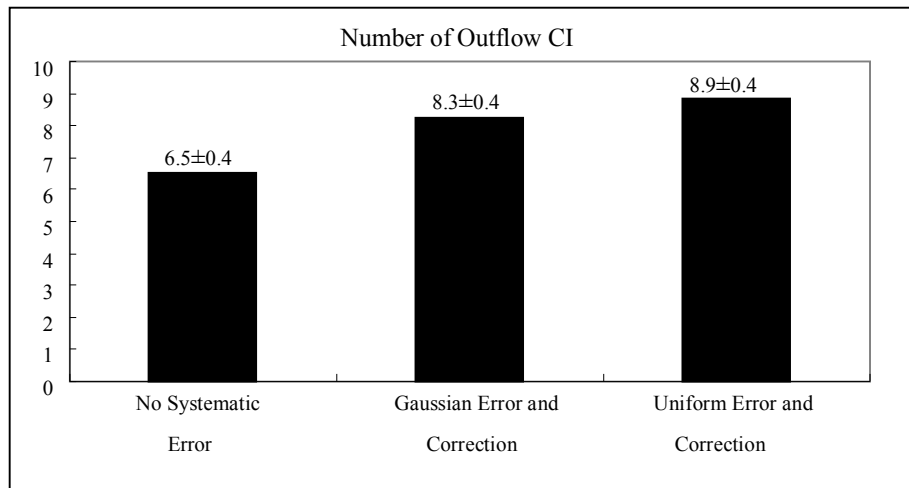


Figure 4.1: The mean of numbers of outflow confidence intervals.

The result is shown in figure 4.1. The horizontal axis stands for the three

types of simulations. For each type of simulation the sample contains 5000 confidence intervals which are divided into 50 groups so that each group consists of 100 confidence intervals. The vertical axis stands for the mean of number of outflow confidence intervals in these 100 confidence intervals. According to the statistics, the larger the vertical value is, the lower the confidence level is. The equation to calculate the confidence level in terms of the vertical value is

$$\text{Confidence Level (C.L.)} = (100 - y) \% \quad (4.2)$$

As shown in the figure 4.1, the mean values of number of outflow confidence intervals of three types of simulations are 6.5, 8.3 and 8.6 respectively. As shown in figure 4.2, the corresponding mean values of confidence levels are 93.5%, 91.7% and 91.1% respectively.

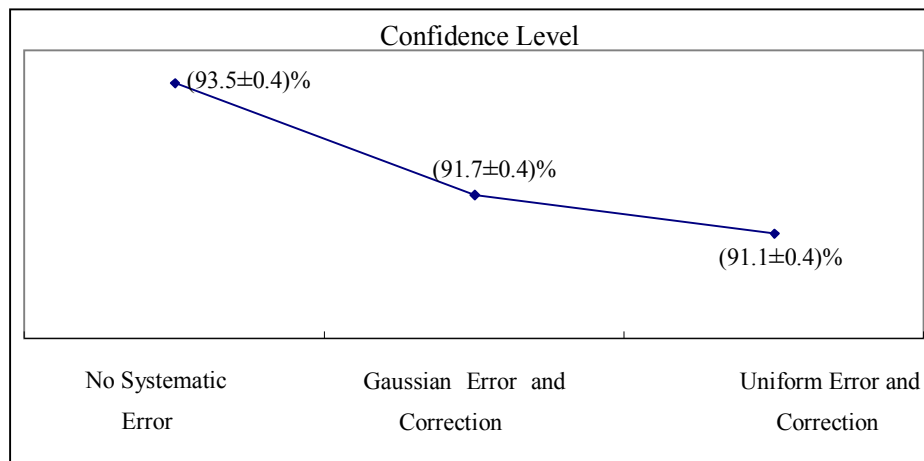


Figure 4.2: The mean of confidence levels.

It means that when $N_{\text{bkg}}=30$, the confidence level decreases slightly after considering the systematic uncertainty. However, the differences are not obvious. And when the systematic uncertainty is introduced via the Uniform distribution, the confidence level is the smallest one.

In principle, the confidence levels without considering the systematic uncertainty should be 90%. However, the simulation result is larger than 90%. This phenomenon may be attributed to the fact that the implementation of the Feldman-Cousins's method does not work very

well. Nevertheless, this confidence level is taken as a reference and relative deviation to this value are studied.

The results which only contain systematic uncertainty but not correction are shown in table 4.3. The confidence levels decrease after each step explained in the sub-chapter 4.1.2. However, the differences between all these five values are very small; therefore it is not clear which step is the dominating reason that leads to the decrease of confidence levels.

Table 4.3: The specific results of $N_{\text{bkg}}=30$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	326	388	413	434	443
Confidence Level	93.48%	92.24%	91.74%	91.32%	91.14%

(2) Background=50

The results of $N_{\text{bkg}}=50$ case are shown in figure 4.3 and 4.4. As well as the first case, the confidence level decreases slightly after considering the systematic uncertainty and its value is the smallest one for the Uniform one. The three mean values of confidence levels are 93.4%, 92.0% and 91.4%, respectively. The differences between them are still small.

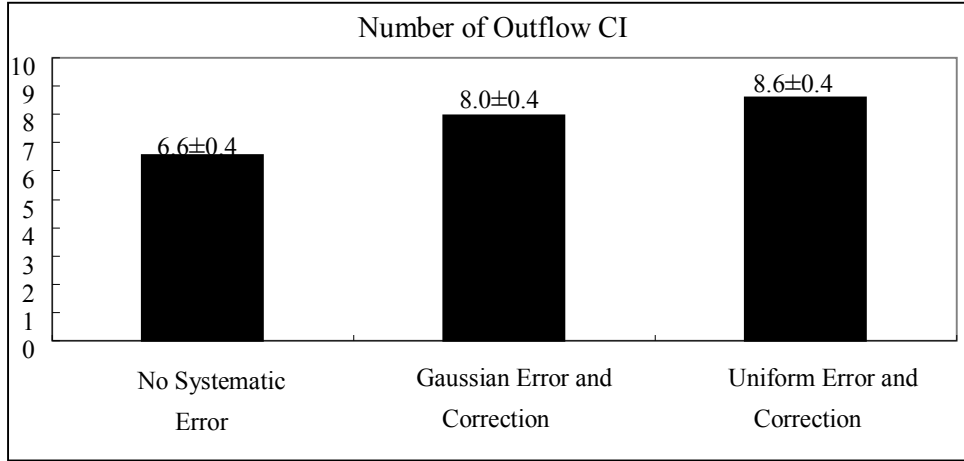


Figure 4.3: The mean of numbers of outflow confidence intervals.

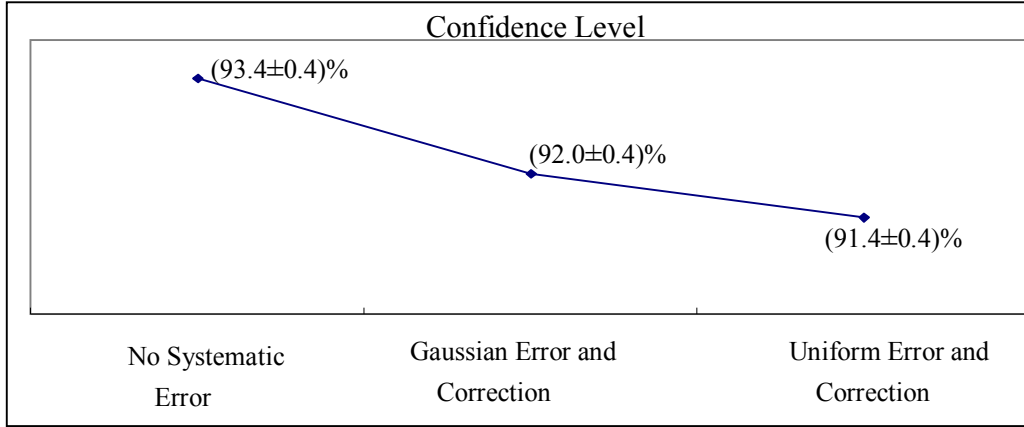


Figure 4.4: The mean of confidence levels.

Table 4.4: The specific results of $N_{\text{bkg}}=50$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	328	375	398	403	429
Confidence Level	93.44%	92.50%	92.04%	91.94%	91.42%

The results which only contain systematic uncertainty but not correction are shown in table 4.4. As with the first case, the confidence levels decrease after each step and it is not clear which step is the dominating one so far.

(3) Background=100

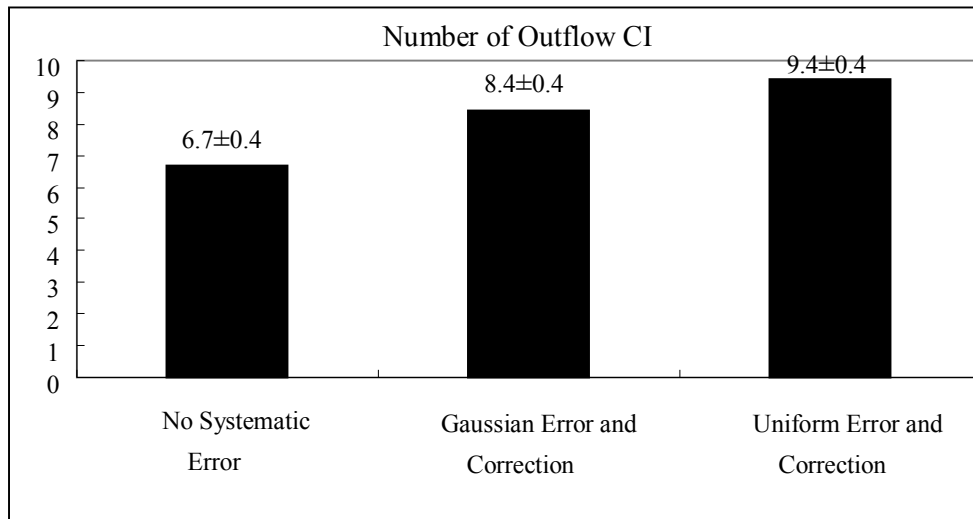


Figure 4.5: The mean of numbers of outflow confidence intervals.

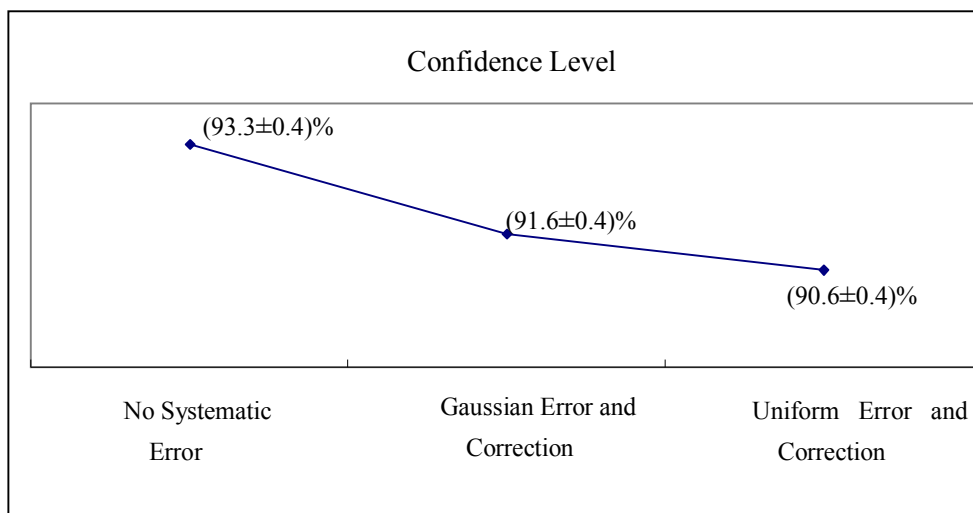


Figure 4.6: The mean of confidence levels.

The results of $N_{\text{bkg}} = 100$ case are shown in figure 4.5 and 4.6. The regularity is the same as above. After taking the systematic uncertainty into account, the confidence level decreases, and the Uniform one is corresponding to the lowest confidence level. The three mean values of

their confidence levels are 93.3%, 91.6% and 90.6%, respectively. The differences between them are a little larger than before. However, all values are still above 90%.

The results which only contain systematic uncertainty but not correction are shown in table 4.5. The confidence levels decrease after each step and the effect of each step is nearly the same.

Table 4.5: The specific results of $N_{\text{bkg}}=100$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	334	399	422	454	471
Confidence Level	93.32%	92.02%	91.56%	90.92%	90.58%

(4) Background=500

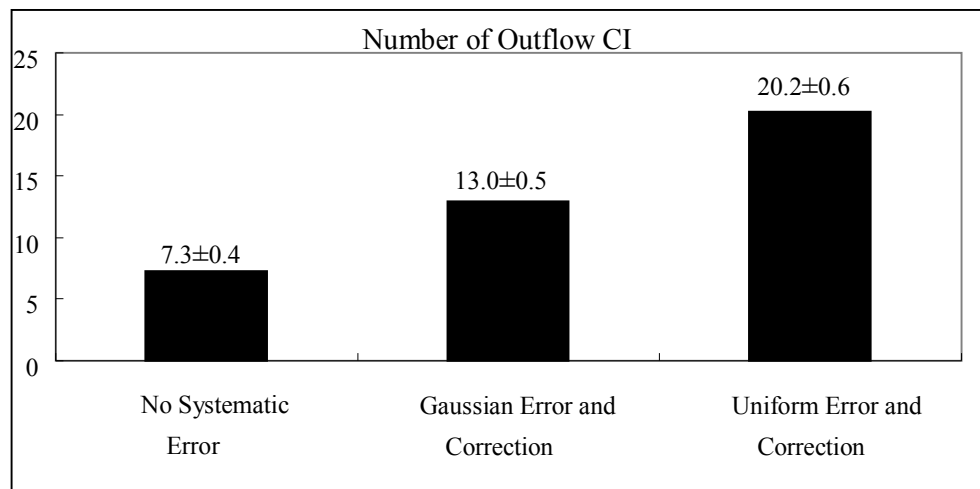


Figure 4.7: The mean of numbers of outflow confidence intervals.

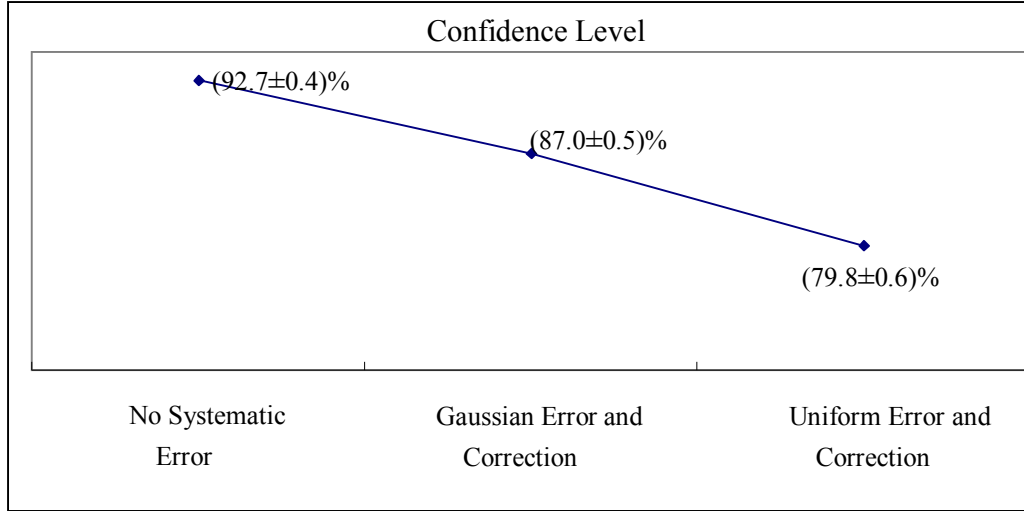


Figure 4.8: The mean of confidence levels.

The results of $N_{\text{bkg}}=500$ case are shown in figure 4.7 and 4.8. Now the number of background events is increased dramatically; meanwhile the confidence level decreases markedly after the correction. The three mean values of confidence levels are 92.7%, 87.0% and 79.8%, respectively. It is even below 80% in the Uniform case. The differences between them are obvious now.

Table 4.6: The specific results of $N_{\text{bkg}}=500$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	365	637	649	1000	1011
Confidence Level	92.70%	87.26%	87.02%	80.00%	79.78%

The results which only contain systematic uncertainty but not correction

are shown in table 4.6. In this case, the correction step just affects the results a little. The dominating reason which leads to the decrease of confidence levels is the introduction of the systematic uncertainty.

(5) Background=1000

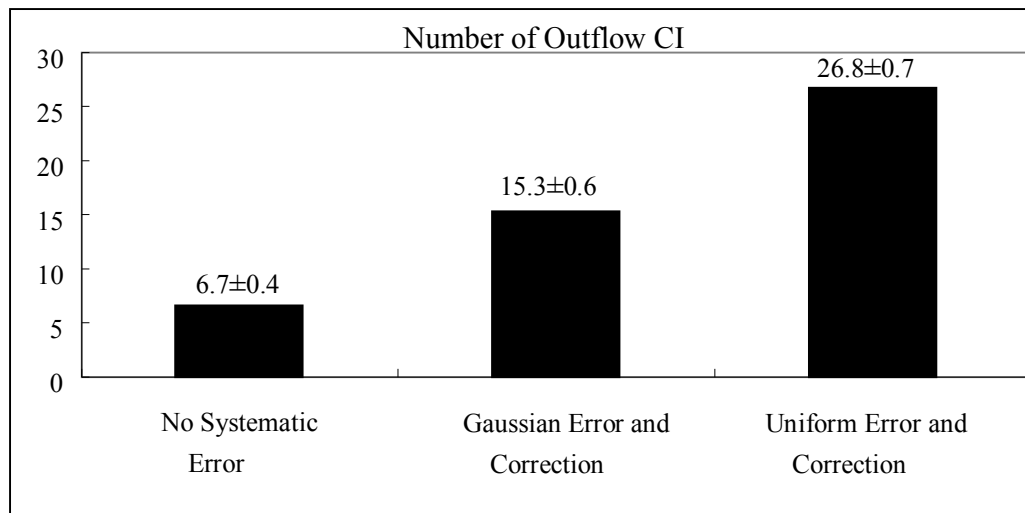


Figure 4.9: The mean of numbers of outflow confidence intervals.

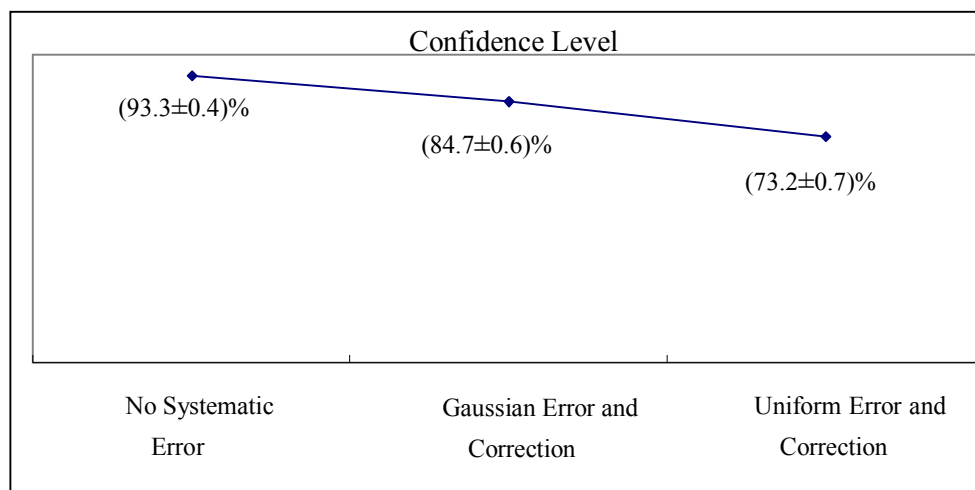


Figure 4.10: The mean of confidence levels.

The results of $N_{\text{bkg}}=1000$ case are shown in figure 4.9 and 4.10. After correction with the systematic uncertainty, the confidence level remains its declining trend as the above cases. The three mean values of confidence levels are 93.3%, 84.7% and 73.2%, respectively; the Uniform

one is still corresponding to the lowest value. The differences between them are more evident now.

Table 4.7: The specific results of $N_{\text{bkg}}=1000$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	333	754	765	1323	1339
Confidence Level	93.34%	84.92%	84.70%	73.54%	73.22%

The results which only contain systematic uncertainty but not correction are shown in table 4.7. As with the $N_{\text{bkg}}=500$ case, the effect of the correction step is not obvious. The first step which is used to introduce the systematic uncertainty becomes the dominating one.

When the number of background events is small, the effects of the first step and the correction step are basically the same. However, when the number of background events is much larger than the number of signal events, the first step which is used to introduce the systematic uncertainty is the dominating step.

For each case, the regularity is similar: the confidence level decreases after two types of correction and the confidence level of the Uniform case is smaller than that of the Gaussian case.

When consider the systematic uncertainty, the extra randomness is introduced by changing the initial value of N_{bkg} ; therefore, the confidence level will decrease after correction.

The concentration of the Gaussian distribution is better than that of the Uniform distribution; therefore, when the systematic uncertainty is introduced with the Gaussian distribution, the randomness of it will be smaller than the Uniform case. That is why the Uniform case is always corresponding to the lowest confidence level.

The overall results are shown in table 4.8 and figure 4.11. The table lists the specific mean values of confidence levels, while the figure is able to show the variation trend clearly.

Table 4.8: The results of the simulation.

	No Systematic Error	Gaussian Error and Correction	Uniform Error and Correction
$N_{\text{bkg}}=30$	93.5%	91.7%	91.1%
$N_{\text{bkg}}=50$	93.4%	92.0%	91.4%
$N_{\text{bkg}}=100$	93.3%	91.6%	90.6%
$N_{\text{bkg}}=500$	92.7%	87.0%	79.8%
$N_{\text{bkg}}=1000$	93.3%	84.7%	73.2%

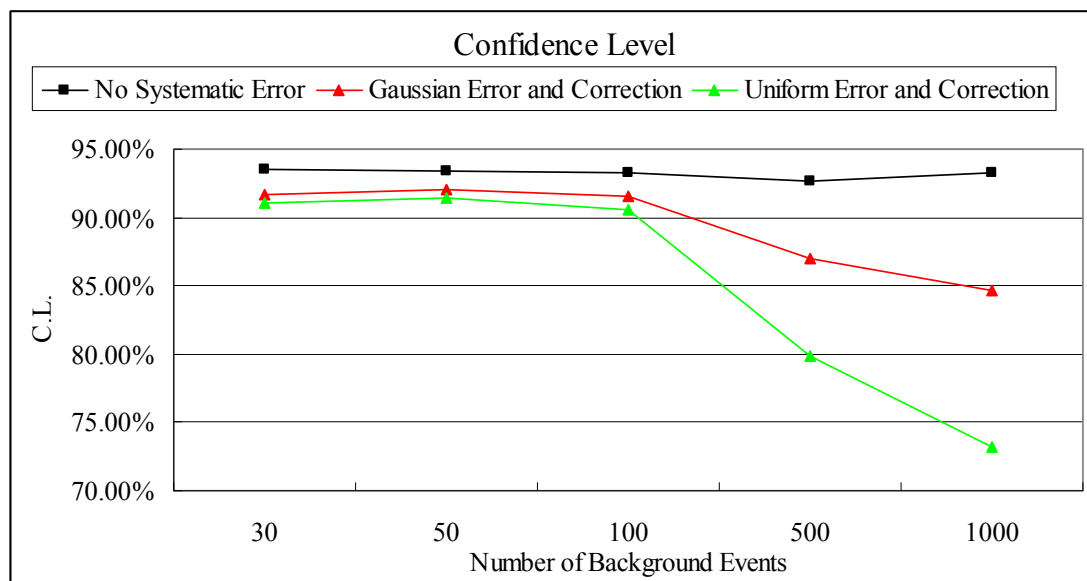


Figure 4.11: The global figure of the simulation result.

In figure 4.11, the horizontal axis stands for the number of background events (value of N_{bkg}), and the vertical axis stands for the values of confidence levels. The data of simulations without systematic uncertainty, with Gaussian-like systematic uncertainty and Uniform-like systematic uncertainty are marked by the black line, the red line and the green line, respectively.

For the first case, when N_{bkg} is increased from 30 to 1000, the black line is nearly a flat straight line with very small fluctuations, and the mean values of confidence levels are always between 90% and 95%.

For the two cases with correction, the trends of the red line and the green line are similar. They keep flat when N_{bkg} is smaller than 100 and decrease dramatically when N_{bkg} is increased by a large margin. A reasonably foreseeable conclusion is that the confidence level will retain its declining trend when N_{bkg} is increased to larger values. It means that when the number of background events is much larger than the number of signal events, the confidence level will decrease if the systematic uncertainty is taken into account.

4.2.2 Result and Analysis for 20% Systematic Uncertainty Case

For the $N_{\text{bkg}} = 50$ case, one more simulation is implemented with 20% systematic uncertainty. The results are shown in figure 4.12 and 4.13.

The confidence level decreases after correction. The three mean values of confidence levels are 93.44%, 89.42% and 86.2%, respectively. Compared with the 10% systematic uncertainty case, this time the differences between them are more obvious. Both of the values with correction are smaller than 90%.

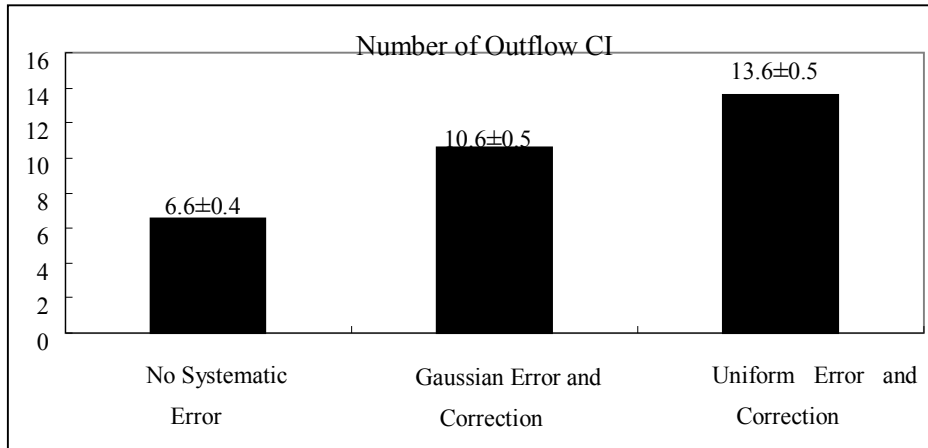


Figure 4.12: The mean of numbers of outflow confidence intervals.

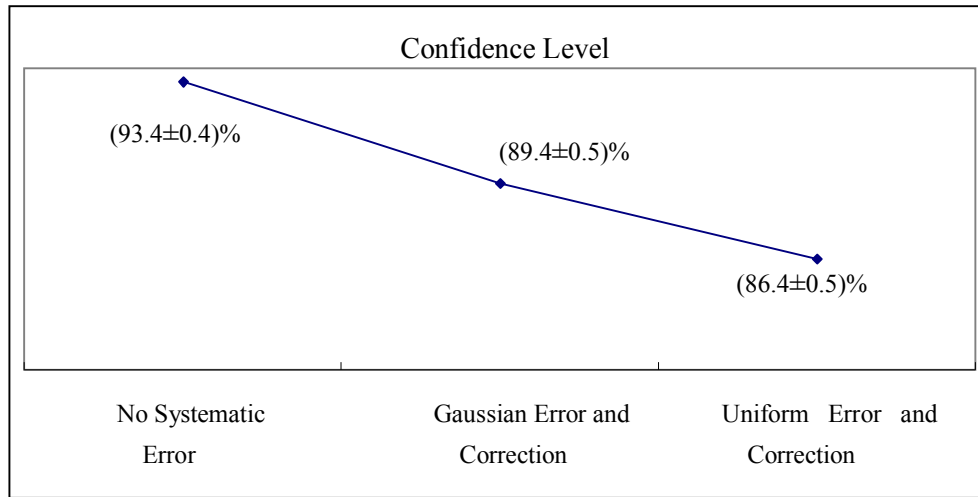


Figure 4.13: The mean of confidence levels.

Table 4.9: The specific results of $N_{\text{bkg}}=50$ case.

	No Systematic Error	Gaussian Error	Gaussian Error and Correction	Uniform Error	Uniform Error and Correction
Total Number of Outflow CI In the 5000 Samples	328	488	529	628	679
Confidence Level	93.44%	90.24%	89.42%	87.44%	86.42%

The results which only contain systematic uncertainty but not correction are shown in table 4.9. As can be seen, the effect of the first step is larger than that of the correction step.

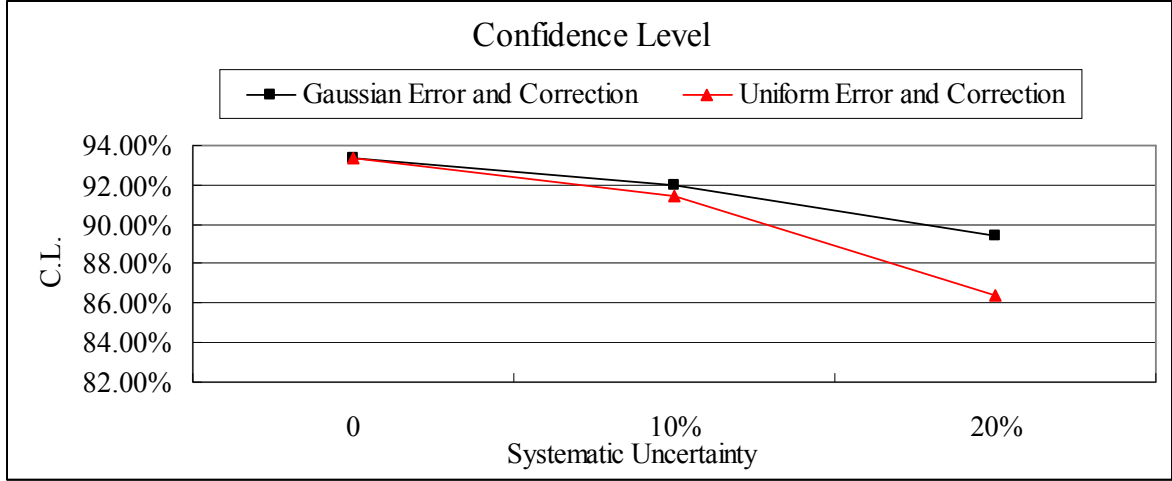


Figure 4.14: The global figure of the simulation result.

The overall results are shown in figure 4.14. The horizontal axis stands for the value of systematic uncertainty, and the vertical axis stands for the values of confidence levels. The data of simulations with Gaussian-like systematic uncertainty and Uniform-like systematic uncertainty are marked by the black line and the red line respectively.

For both cases, the trends are the same: the mean values of confidence levels decrease as the systematic uncertainties increase. A reasonably foreseeable conclusion is that the confidence level will retain its declining trend if the systematic uncertainty keeps increasing.

4.3 Conclusion

In terms of the simulation results and analyses above, the method that is used to process the systematic uncertainty in the BESIII experiment does not work well.

- (1) In principle, the confidence levels without considering the systematic uncertainty should be 90% for each case. However, all values are larger than 90%. This phenomenon may be attributed to the fact that

the implementation of the Feldman-Cousins's method can not reach the standard more or less. Nevertheless, this confidence level is taken as a reference and relative deviation to this value are studied.

- (2) As the number of background events increases, the confidence level will decrease after correction; and when the number of background events is much larger than that of signal events, the confidence level decreases dramatically.
- (3) When the number of background events is small, both the first step and the correction step make sense. However, when the number of background events is much larger than the number of signal events, the first step which is used to introduce the systematic uncertainty is the dominating step.
- (4) As the systematic uncertainty increases, the confidence level will decrease after correction.

For large background cases, the effect of the correction can be ignored. Therefore, a more reasonable technique is needed to process the systematic uncertainty. One possible method is to correct the number of signal events and the number of all observed events with the systematic uncertainty, respectively, and then use the two corrected values to calculate the branching fraction.

References

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Chapter 5 Summary and Outlook

Particle physics is the study of the fundamental particles and their interactions. The best theory in particle physics field is the Standard Model which consists of the electroweak theory and the QCD. Experiments in particle physics are implemented on colliders to look for new particles predicted by the Standard Model.

The work of this thesis is based on research at the Beijing Electron-Positron Collider II (BEPCII). The detector of BEPCII is called the Beijing Spectrometer III (BESIII). The BESIII experiment focuses on physics in the tau-charm energy region. When looking for the upper-limit measurements for small signal events produced in the experiments, a reasonable method to deal with the systematic uncertainty is necessary. This thesis tests the method used in the BESIII analyses and focuses on calculations of systematic uncertainty for upper-limit measurements.

This thesis does research for two cases: the systematic uncertainty=10% case and the systematic uncertainty=20% case. The final conclusions are:

(1) The confidence level should be 90% without introducing the systematic uncertainty. However, the simulation results are always larger than 90%. It means that the implementation of the Feldman-Cousins's method does work very well. Nevertheless, this confidence level is taken as a reference and relative deviation to this value are studied.

(2) When the value of systematic uncertainty is fixed at 10%, the confidence level will decrease after taking the systematic uncertainty into account; and when the number of background events is much larger than the number of signal events, the confidence level decreases obviously.

(3) When the number of background events is small, both the first step and the correction step make sense. However, when the number of

background events is much larger than the number of signal events, the first step which is used to introduce the systematic uncertainty is the dominating step, while the effect of the correction step can be ignored.

(4) When the value of systematic uncertainty increases from 10% to 20%, the confidence level will decrease after taking the systematic uncertainty into account.

For large background cases, the correction does not work well. Therefore, the method used to deal with the systematic uncertainty in the BESIII analyses does not work well; and a more reasonable method is needed in the future.

One possible method is to correct the number of signal events and the number of all observed events with the systematic uncertainty, and then use the two corrected values to calculate the branching fraction.

Appendix A The Program of the Simulation

The Flow Chart of the Algorithm

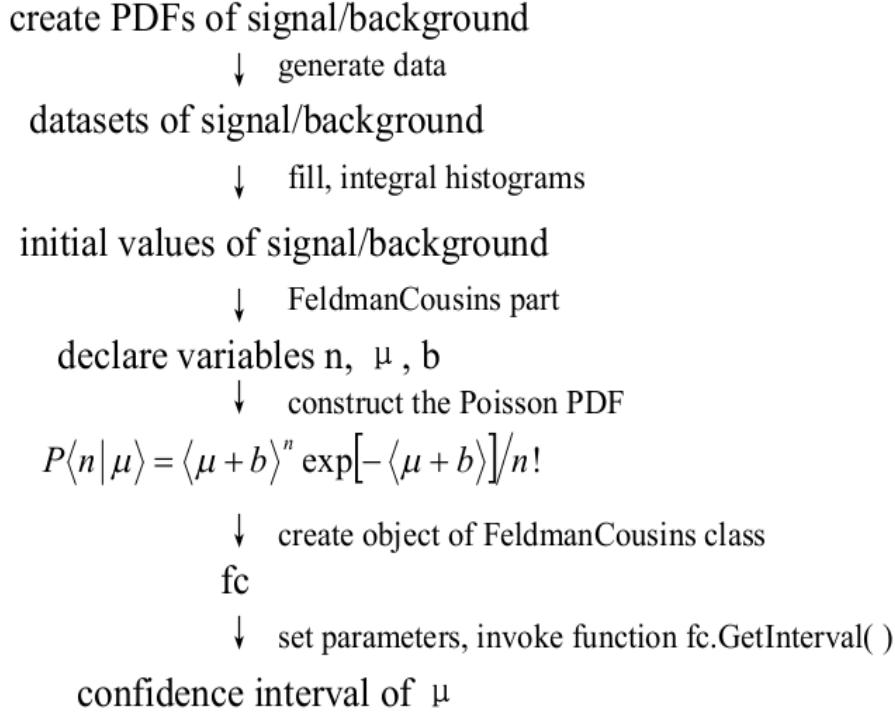


Figure A.1: The flow chart of the algorithm.

Figure A.1 shows the main procedures to construct a confidence interval of the parameter of interest. The specific steps are:

- (1) Create a Gaussian PDF to stand for the signal and a Uniform PDF to stand for the background.
- (2) Use these two PDFs to generate two datasets used to create two histograms, and then do the integral for them in the three sigma (the variance of the Gaussian PDF) region to get the initial values of signal and background ($nsig$ and $nbkg$) used to calculate the initial value of total observed events later.
- (3) Start the FeldmanCousins part. Declare three variables n , μ and b . The total number of observed events has a Poisson distribution; therefore the PDF used in the interval estimation is a Poisson PDF. The information of variables is listed in table A.1.

Table A.1: The information of variables used in the Poisson PDF.

name	meaning	role	initial value	region
n	number of total events	observable	nsig+nbkg	[0, upper value]
b	number of background events	known parameter	a chosen value	fixed
μ	number of signal events	parameter of interest	a chosen value	[0, upper value]

- (4) Create an object belonging to the FeldmanCousins class.
- (5) Implement the necessary setup. The two main parameters are: the confidence level is 0.9 and the step between two testing points is 0.2. At last, invoke the member function GetInterval() to calculate the confidence interval of μ .

This is the whole process to construct one confidence interval. Each time this process is implemented for 100 times with a for loop inserted in the algorithm. The programmed is implemented for 50 times so that the total number of the sample is 5000. The wrong confidence intervals which do not contain the true value of μ will be picked out and the actual confidence level will be determined.

Appendix B The Code of One Simulation

```
#ifndef __CINT__
#include "RooGlobalFunc.h"
#endif
#include "RooRealVar.h"
#include "RooDataSet.h"
#include "RooGaussian.h"
#include "RooPlot.h"
#include "RooFormulaVar.h"
#include "RooAddition.h"
#include "RooProduct.h"
#include "RooPolyVar.h"
#include "RooUniform.h"
#include "Math/DistFunc.h"
#include "RooRandom.h"
#include "RooStats/ConfInterval.h"
#include "RooStats/PointSetInterval.h"
#include "RooStats/ConfidenceBelt.h"
#include "RooStats/FeldmanCousins.h"
#include "RooStats/ModelConfig.h"
#include "RooWorkspace.h"
#include "RooConstVar.h"
#include "RooDataHist.h"
#include "RooPoisson.h"
#include "TCanvas.h"
#include "TAxis.h"
#include "TH1.h"
#include "TMath.h"
#include "TF1.h"
```

```

#include "TTree.h"
#include "TH1F.h"
#include "TMarker.h"
#include "TRandom.h"
#include "TRandom3.h"
#include "TStopwatch.h"

using namespace RooFit ;
using namespace RooStats;
#include<iostream>

TH1* h_signal=0;
TH1* h_background=0;

void original()
{
//-----
Double_t signal;
Double_t background;
Double_t observed;
Double_t ci_lower;
Double_t ci_upper;

TTree* tci=new TTree("tci","tci");
//tci->Branch("background",&background,"background/D");
//tci->Branch("signal",&signal,"signal/D");
tci->Branch("observed",&observed,"observed/D");
tci->Branch("ci_lower",&ci_lower,"ci_lower/D");
tci->Branch("ci_upper",&ci_upper,"ci_upper/D");

```



```

//pdf-----
RooRealVar x("x","x",0,1000);
RooGaussian sig("sig","sig",x,RooConst(500),RooConst(50));
RooUniform y("y","y",RooArgSet(x));
RooGenericPdf bkg("bkg","bkg","y",RooArgSet(y));
RooRealVar bkgfrac("bkgfrac","bkgfrac",0.9,0.,1.);
RooAddPdf model("model","model",RooArgList(bkg,sig),bkgfrac);

TRandom* rg=RooRandom::randomGenerator();
rg->SetSeed();
//for loop-----
for(Int_t i=0;i<100;i++){
TTree* tdata=new TTree("tdata","tdata");
tdata->Branch("observed",&observed,"observed/D");

gRandom->SetSeed();
Int_t nsig=gRandom->Poisson(2);
RooDataSet* data1=sig.generate(x,nsig);
h_signal=data1->createHistogram("x",1000);
signal = h_signal->Integral(350,650);

Int_t nbkg=gRandom->Poisson(100);
RooDataSet* data2=bkg.generate(x,nbkg);
h_background=data2->createHistogram("x",1000);
background = h_background->Integral(350,650);

observed=signal+background;

delete h_signal;

```

```

delete h_background;
tdata->Fill();

//FeldmanCousins part-----
RooRealVar n("n","n",observed,0,100);
RooConstVar b("b","b",30);
RooRealVar mu("mu","mu",2,0,20);
RooAddition mean("mean","mean",RooArgList(mu,b));
RooPoisson pois("pois","pois",n,mean);

RooDataSet data3("data3","data3",RooArgSet(n),Import(*tdata));
data3.Print("v");

RooWorkspace* w = new RooWorkspace();
ModelConfig modelConfig("poissonProblem",w);
modelConfig.SetPdf(pois);
modelConfig.SetParametersOfInterest(RooArgSet(mu));
modelConfig.SetObservables(RooArgSet(n));
w->Print();

//-----
RooStats::FeldmanCousins fc(data3,modelConfig);
fc.SetConfidenceLevel(0.9); // set size of test
fc.UseAdaptiveSampling(true);
fc.FluctuateNumDataEntries(false); // number counting analysis: dataset
always has 1 entry with N events observed
fc.SetNBins(100); // number of points to test per parameter

// use the Feldman-Cousins tool

```

```
PointSetInterval* interval = (PointSetInterval*)fc.GetInterval();
```

```
ci_lower=interval->LowerLimit(mu);
```

```
ci_upper=interval->UpperLimit(mu) ;
```

```
tci->Fill();
```

```
delete tdata;
```

```
}
```

```
tci->Scan();
```

```
}
```

Acknowledgements

Firstly, I would like to thank Dr. M.Kavatsyuk for his help during last year. Without his patient guidance, I can not finish my thesis successfully. Secondly, I would thank everyone in my research group to regard me as a member of the group and help me a lot.

Finally, I would thank my parents. Living and studying in a foreign country is difficult for me. Without you understanding and support, I can not reach my goal.