Integration of argumentative, narrative and probabilistic reasoning in court

A case study comparison of two methods

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Abstract. In order to trace and potentially prevent miscarriages of justice old court cases are analyzed. However there is no standardized method to model these cases yet. In this paper two methods are evaluated, both with a new approach to integrate scenario-based and probability-based ways of reasoning with evidence. One method uses Bayesian network idioms to model scenarios and evidence, and the other uses standard probability theory and propositional logic to represent scenarios for and against which arguments (in the form of evidence) can be made. The advantages of both methods are evaluated by means of a comparative case study of a solved Dutch murder case. The main advantage of the Bayesian method being its scenario-structure and the main advantage of the logico-probabilistic method being its formal approach which, unlike the Bayesian approach, doesn't require more numbers than are available. The logico-probabilistic method is then extended with the scenario-structure of the Bayesian method, creating a third method which uses propositional logic idioms to create scenarios every event of which is treated as a hypothesis for and against which arguments can be made. The model which results from testing the extended method on the case study is as specific as the Bayesian model which doesn't require elicitation of unknown numbers.

Keywords. Evidential reasoning, legal modelling, scenarios, probabilities.

1. Introduction

Every major miscarriage of justice in which an innocent person is wrongly convicted is a state caused tragedy with grave consequences for the victim, including the waste of years of their lives, the ruining of their public image, and in some places in the world, death row. One of the causes of some of the major miscarriages of justice in the last ten years is misrepresentation of forensic evidence or other evidence of a statistic nature in court cases. This misrepresentation of evidence has occurred when a certain piece of forensic or statistical evidence in a trial was wrongly interpreted or when it was unfairly dismissed (Aitken et al., 2010). Examples include the prosecutor's fallacy (Fenton, 2011) and the fallacy of the incredible coincidence (Derksen and Meijsing, 2009). Widely documented cases of the latter include the Lucia de Berk case and the Sally Clark case (Buchanan, 2007) (Derksen and Meijsing, 2009). There are three distinct approaches of evidential reasoning: the argumentative approach, the narrative approach and the probabilistic approach (**?**). The latter approach involves the use of numbers, making it quantitative in nature, whereas the former two approaches don't and can be called qualitative. The prob-

abilistic approach is favored by forensic experts, while judges and jurors primarily use the other two (qualitative) approaches. This leads to a gap in communication between them and forensics experts. The solution to the gap in communication in the field of law is an area of tension. Some are arguing Bayesian methods are inadmissible for presenting evidence, except for DNA evidence and possibly other areas with a firm statistical base, such as seen in the R v T. ruling [1], where a suspect was acquitted because the judge ruled an argument for a shoe print match from Bayesian statistics inadmissible. While others, some of which protested this same ruling (Berger et al., 2011), are supporting broader application of Bayesian statistics (Fenton, 2011).Also still some scientists are questioning whether Bayesian statistics have a place in court at all. (van Koppen, 2011) Instead of arguing for the superiority of one singular approach to evidential reasoning, in this paper a solution is sought which utilizes the strength of both qualitative and quantitative approaches: The advantage of narrative evidential reasoning is the persuasiveness and orderly structure resulting from the coherence that scenarios have (Pennington and Hastie, 1993) (Wagenaar et al., 1993). Argumentative reasoning distinguishes itself with its strong adversarial setting (Verheij, 2014). Lastly probabilistic methods lend the ability to reason about uncertainty in gradations and a nearly.

Several methods have been developed for the purpose of modelling court cases. This study evaluates and compares two recent methods, which aim to integrate quantitative evidential reasoning based on probabilities with qualitative evidential reasoning based around arguments and scenarios in a way that accurately represents the probabilities involved. One of these methods, by Vlek et al. (2014), tries to bridge the gap between Bayesian statistics the qualitative argument and scenario based methods by modelling scenarios with Bayesian networking tools. The other method, developed by Verheij (2014), is based on propositional logic and incorporates core elements from probability theory in order to be able to express uncertainty in gradations. Both methods' advantages and known limitations have been briefly summarized previously (Verheij et al., 2015): The primary limitation of modelling a court case using Bayesian networks is that it requires the specification of probabilities that aren't known. Therefore the Bayesian model will be a subjective interpretation of the case it is modelled on (Vlek et al., 2014). However there are a number of elicitation techniques which can be used to specify unknown probabilities in the probability tables of nodes in a Bayesian belief network (Renooij, 2001). The propositional logic based method doesn't capture the internal structure of scenarios. Arguments and scenarios in the propositional logic model are treated like they have an elementary structure (Verheij et al., 2015), potentially leading to some loss of information about the arguments and scenarios involved.

Research Question

This research aims to investigate for both these methods what their respective advantages are in comparison to each other and what method would be the most advantageous when modelling case studies. In this paper we want to address the following two questions: What are the benefits of both the Bayesian method and the logico-probabilistic method of modelling court cases? And if both methods have distinct benefits in modelling court cases: How do we find, if possible, a compromise to combine these benefits?

[1] R. v. T [2010] All ER (D) 240 (Oct); [2010] EWCA Crim 2439

To answer the first research question a case study analysis will be made. For this case study analysis a solved Dutch murder case which will be used. [2] For privacy reasons the names of those involved have been changed: the victim of this will be referred to as Nelline van der Vaart in this paper and the convicted suspect will be referred to as Patrick F. The case will be modelled both as a Bayesian network and as a formal system of inferences using propositional logic and probability theory and a comparison between these models and the methods they're made with will be made. The relative ease or difficulty of representing different aspects of the case will be looked at and compared for both models. prime suspect Patrick F.'s guilt should follow from both models, because the Nelline van der Vaart case is a relatively straightforward case, in the sense that the evidence points nearly exclusively in his direction. This evaluation will be used to shape a third method which could potentially be the answer to the second research question. However to answer this research question, another analysis with the same case study will be done to make sure the resulting model does actually combine the benefits of the original methods.

2. Methods

2.1. Bayesian networks

A Bayesian network is a graphical representation of a joint probability distribution consisting of nodes representing variables and connecting arrows to represent which nodes are conditionally dependent on each other. The result is a directed acyclic graph. A variable is conditionally dependent on the variables of its node's parent nodes. A node's parents are nodes 'above' the node in the hierarchy of the graph, meaning they are connected to the child node via arrows pointing in the child node's direction. An example can be seen in Figure 1 Nodes in Bayesian networks have states with a probability value between zero and one. The states' combined values sum to one. Nodes with parents have a conditional probability table, detailing their probability values for every combination of states their parents are in. This study uses the Bayesian modelling tool Genie 2.0 (Druzdzel, 2005) to construct the Bayesian model for the case.

2.2. Bayesian method design and procedure

Nodes in the Bayesian network of this research are binary with states 'True' and 'False', with probability value zero representing certain falsehood and one representing certainty. Some of the conditional probabilities in the Bayesian network are based on forensic evidence which often comes accompanied with a clear value of the uncertainty of the lab results. Many other conditional probabilities aren't known and must be elicited using a verbal-literal scale (Renooij, 2001). The verbal-literal scale used in this paper is based on the one used Vlek et al. (2014) and contains a range from 'very likely' (translated to a probability of 0.999) to 'very unlikely' (with a probability of 0.001) as shown in Figure 2.

[2] Official court rulings are available (in Dutch) on http://uitspraken.rechtspraak.nl/, verdict ECLI:NL:RBUTR:2004:AO3150 and appeal ECLI:NL:GHAMS:2005:AT5190

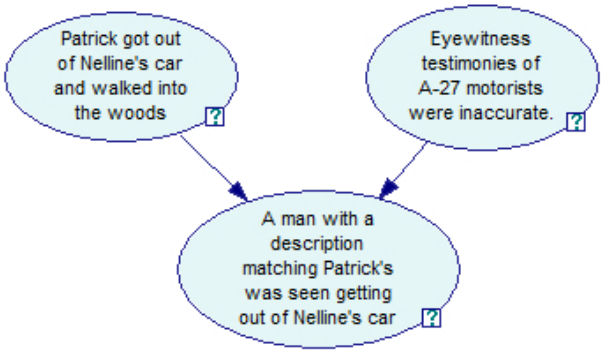

Figure 1. A simple Bayesian belief network with two parent nodes and a child node.

Very likely	0.999
Likely	0.95
Quite Likely	0.9
Uncertain	0.5
Quite unlikely	0.1
Unlikely	0.05
Very unlikely	0.001

Figure 2. The verbal-literal scale used in this research.

In order to represent the coherence of a scenario in a Bayesian network certain Bayesian network structures called idioms are used. Idioms can be thought of as building blocks for building Bayesian representation of a court case. Lagnado, Fenton and Neil devised a number of idioms for modelling certain aspects of a court case, including evidence to a (legal) hypothesis, representing the reliability of evidence, and representing the reliability of alibi's (Lagnado et al., 2013). These idioms will be used in the Bayesian model, but the idioms that define the Bayesian method are the narrative idioms devised by Vlek et al. (2014): The (sub)scenario idiom, the variation idiom and the merged scenarios idiom. A scenario idiom consists of a scenario node with outgoing arrows pointing to a number of nodes representing the events which are part of the scenario. The subscenario idiom has the same general structure as a scenario idiom, but is always used inside a larger scenario. The variation idiom is an alternative to constructing an entirely new scenario to account for variation and will be used for small variations when the different ways an event in a scenario could have taken place have no impact on the conclusion of the scenario itself. Both the (sub)scenario and variation idioms tie into the process of unfolding: Scenarios can be told at various levels of detail. In modelling a legal case some events in a scenario a require a greater level of detail then others. Every time more detail is needed for some part of the scenario, that part will be unfolded. Meaning the event node which requires more detail, is replaced by a subscenario idiom. To determine whether it is necessary to unfold a scenario at a certain node, three questions serve as a guideline. First, is there evidence that can be directly connected to the event node?

If so, the event node is at the right level of detail, and no unfolding is required. If not, the second question is: Is there relevant evidence for details of a subscenario revolving around this event. If so, unfolding is required. If not, the final question one should ask is: Would it be possible to find relevant evidence for the details of a subscenario revolving around this event? If so, unfolding is a required. The variation idiom is used in unfolding if variation is encountered and there is evidence which is relevant for some options, but not for others. The merged scenarios idiom is used to model all possible scenarios of a crime in one Bayesian network. Scenarios are merged with the merged scenarios idiom after they are fully unfolded. The last step in creating a Bayesian network model of the case is including the evidence. For each piece of evidence a node is connected to the node it supports and instantiated (set to 'True').

2.3. Logico-probabilistic method

The logical language used in this method consists of sentences in classical proposition logic. The language has three connecting operators: the conjunction ($\wedge$), disjunction ($\vee$) and negation ($\neg$) operators. It also uses the classical conductive consequence relation ($\models$) to express logical truth and logical incompatibility. A sentence P in the language of this method (represented as $P \in L$), is a logical truth when $\models P$ and two sentences P and Q (P and $Q \in L$) are logically incompatible when $\models \neg(P \wedge Q)$. The probability functions of this method are from the standard probability theory and satisfy the Kolmogorov axioms (Verheij, 2014), also see Hájek (2003): $p(P) \geq 0$, for all $P \in L$, if $\models P$, for any $P \in L$, then $p(P) = 1$ and for any P and $Q \in L$, if $\models \neg(P \wedge Q)$, then $p(P \vee Q) = p(P) + p(Q)$. This method relies on the use of conditional probabilities, which are defined as follows: for every P and $Q \in L$, $p(Q|P) := \frac{p(P \wedge Q)}{p(P)}$.

2.4. Logico-probabilistic design and procedure

Pieces of evidence and possible events are atomic sentences represented as propositional variables which can be combined with logical connectives. The cumulative evidence (E) in favor of a full scenario (H_i) is modelled as a conditional probability of the scenario given the evidence: $p(H_i|E)$. This conditional probability represents the strength of the argument based on E in favor of H_i. Arguments can also be based on a scenario in favor of an expectation, in that case their strength is represented as the conditional probability of a certain expectation given the scenario. If one such expectation is contradicted by new evidence, the scenario stops being plausible. The evidence supports every hypothetical scenario with a varying strength. If multiple scenarios are mutually exclusive, an additional piece of evidence which strengthens the argument in favor of one scenario, can weaken the argument in favor of the other(s). Using the integrated formal method evidential reasoning is treated as a gradual process where each piece of evidence is treated as a step where the argument strength of each scenario is reevaluated.

3. Case study

3.1. The case

On 1 October 2002, between 8:30 and 9:00 a.m., Nelline van der Vaart is on the phone with one of her friends. She is doing laundry while calling hands-free. At 8:53 Nelline's

friend hears Nelline greet someone with a casual ”Goedemorgen”, then she hears her screaming six or seven times, then a loud thumping noise, after that the call is disconnected. That evening the police finds Nelline dead in the kitchen of her roommate and landlord Patrick F. Patrick himself is nowhere to be found. It would later turn out he had spend two days in the woods near Utrecht and after that fled the country and gone to Poland. He is apprehended four months later, when he needed to return to The Netherlands to transfer ownership of his house.

While Patrick was a fugitive in Poland, the forensic investigation linked the bullets used to kill Nelline, to another then unsolved murder. Patrick was subsequently tried for that murder as well. He was initially found guilty of double homicide and given a lifelong prison sentence, but the appeal court aquitted him of the second murder and gave him a 20 years sentence for the murder of Nelline van der Vaart. Hence only the murder of Nelline van der Vaart will be modelled in this case study.

3.2. The main suspect

Patrick himself consistently denied any involvement with the murder. He claimed blanket amnesia at his hearing. He testified having a memory of drinking a beer with a friend on the evening before the murder, and then after that of waking up in what he described as ”a wooded area” on the day after the murder, with nothing in between. He followed a bike trail back into Utrecht, and despite not knowing what had happened at his home he went straight to the station and took a train to his parents’ house in Amersfoort. He claimed to have been framed and to possibly have been kidnapped. He told the court he knew with certainty he didn’t murder Nelline and that he fled to Poland because he was afraid of becoming the scapegoat for a murder he didn’t commit. He claimed the gun attributes found in his house weren’t his and he claimed never to have seen them before. Patrick’s lawyer suggested that his client had fallen victim of some unkown third party with a personal grievance against Nelline, potentially her presumably jealous ex-boyfriend.

3.3. The evidence

There are several other key pieces of evidence in this case next to the testimony of Nelline’s friend and (in case of his innocence) the testimony of Patrick: The autopsy report from Nelline van der Vaart’s body, indicated that she was shot at least five separate times, including one time in her arm, one time in her body and three times in her head from a close range, the brain damage of which caused her to die. Nelline’s body was found in the kitchen of Patrick’s locked apartment, and the police found no signs of forced entry. Bloody footprints were found at the scene of the crime belonging to a specific type of Reebock shoe of which Patrick owned a pair. Patrick’s colleague testified having spoken to Patrick at 8:30, when the latter called in sick from work. The police investigation verified that a call was made at 8:30 from Patrick’s landline to Patrick’s workplace. During the police search of Patricks house, different gun attributes were found in various places, namely: An ammunition clip belonging to an uzi, a piece of sawed off barrel from an uzi and several boxes of live ammunition made from blank rounds. Normal blank rounds and equipment to modify blank rounds into live ammunition were also found. Nellines crashed car was found beside the A27 highway, with blood samples matching hers on the mat and joystick. Patricks odourprint was found in the car, and

a man matching his description was seen getting out of the crashed vehicle as testified by the motorists who witnessed the accident. Lastly following suspicion of Patrick's involvement in Nelline's murder, the police had wiretapped Patrick's relatives, and several of these wiretapped phone conversations indicate that Patrick confessed committing the murder to his parents. Patrick was also diagnosed with a severe personality disorder in the Pieter Baan psychiatric centre, where he was committed for evaluation following his interrogation by the police.

3.4. The first scenario

Considering and comparing multiple scenarios, helps to prevent tunnel vision, which is why two scenario's will be modelled using both methods. In the first scenario the killer is the main suspect, Patrick F. This scenario is based on the police investigation and is roughly the same as the scenario outlined in the court hearings against Patrick, on the basis of which he was convicted.

The fact that Patrick consistently denied killing Nelline, makes it difficult to pin down his exact motive. It's been reported by Patrick's parents and by Nelline's other roommates (but also denied by Patrick) that Patrick and Nelline had previously had a disagreement or falling out about some trivial household matters which could have caused tension between them. Although it is a weak motive for a crime as brutal as murder, this motive will be used in the first scenario. Seeing as there is evidence that Patrick owned a gun and self-made ammunition, it is assumed he used said gun and ammunition to kill Nelline. In order for that to be true, Patrick had to confront Nelline with a gun while Nelline was doing laundry, and then proceed to kill her. There is also evidence that Patrick fled the scene in Nelline's car, which would be a very strange thing to do for an innocent person. The full initial scenario is as follows: Patrick experienced tension between himself and Nelline, because of a prior disagreement or falling out between them, Nelline was at home doing housework, Patrick either entered the house with a gun or had a gun in reach right after entering the house, Patrick then killed Nelline with aforementioned gun and finally Patrick fled the scene in Nelline's car.

3.5. The alternative scenario

There were no suspects arrested or interrogated during the police investigation in the Nelline van der Vaart case other than Patrick, because all the available evidence pointed overwhelmingly to him and only to him. This, combined with the fact that Patrick's own testimony is internally contradictory, makes it difficult to devise a coherent alternative scenario. One of the most evidenced facts about Nelline's murder is the way she was killed, with gunfire. Or more specifically, with bullets made from blanks. Therefore, it is assumed in the alternative scenario, that whoever killed Nelline, shot her in the same manner Patrick did according to the police investigation. Combined with the fact that Patrick consistently denied owning or even knowing about the numerous gun-related items in Patrick's apartment, and the fact that Nelline's body was found in Patrick's kitchen, it is assumed in the alternative scenario, that someone tried to frame Patrick for the murder of Nelline. Patrick's memory loss is incorporated in the alternative scenario because if Patrick wasn't the murderer, the likeliest explanation for his testimony would be that he was speaking the truth. The suggestion by Patrick's defence attorney that

Nelline might have been revenge killed by her jealous ex-boyfriend is incorporated into the alternative scenario as an option to add a tangible motive for the murder. This results in a full alternative scenario where: A mystery killer, who will be referred to as Nelline's ex-boyfriend was possibly bitter about their break up, entered Nelline's house with a gun when she was home alone, killed her and then tried to frame Patrick to make him appear to be a prime suspect. All the while Patrick was suffering memory loss.

4. The Bayesian case model

4.1. The Bayesian model of the first scenario

4.1.1. The initial scenario

The initial scenario is modelled with a standard scenario idiom and can be seen in Figure 3. Several events within the scenario connect to each other: Because Patrick experienced tension between him and Nelline, and because Nelline was at home alone, there was a motive and opportunity for Patrick to hold Nelline at gun point, or maybe even to kill her premeditated. In either case, entering the house with a gun gave Patrick the possibility to kill Nelline, and killing Nelline gave him the impulse to flee the scene in her car.

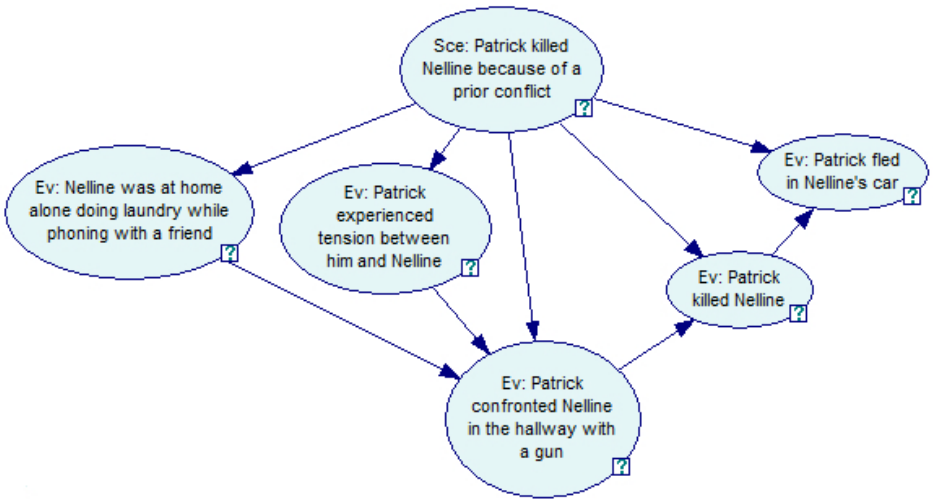

Figure 3. Bayesian model of the initial scenario of Patrick F. murdering Nelline van der Vaart before unfolding. The scenario node (Sce:) represents the scenario itself, the other nodes represent the subscenarios (Sub-sce:) and events (Ev:) which make up the scenario.

4.1.2. Nelline is doing housework and Patrick is experiencing tension

There is direct evidence for the fact that Nelline was at home doing housework, in the form of the testimony of her friend, so using the guideline of the three questions for unfolding events (described in the methods section), this node does not require unfolding. There is no direct evidence that Patrick experienced tension between himself and

Nelline, however there is evidence that there was a prior disagreement or falling out between them, which could have lead to tension. And there is evidence that Patrick wasn't psychologically healthy at the time of the crime, which would make it more plausible he'd repay something as seemingly innocuous as a disagreement about household matters with something as gruesome as murder.

4.1.3. Patrick confronts Nelline with a gun

The event of Patrick confronting Nelline with a gun requires unfolding, because there is no direct evidence for it. However, there are several bits of indirect evidence which can be connected to this event. The evidence that pascal called in sick at 9:30 using his land line places him at the crime scene shortly before the moment of the crime. The testimony of Nellines friend provides details which serve as indirect evidence that Nelline was confronted by Patrick, and there is indirect evidence that Patrick had acquired a loaded gun in the past in the form of the gun related attributes and self-made ammunition found in his house. Therefore the event of Patrick entering the hallway with a gun, requires unfolding into a subscenario with a node about Patrick acquiring a loaded gun in the past. Because there is no direct evidence for this either this node itself then requires unfolding to be able to link the evidence that Patrick made the ammunition himself. The event of Patrick confronting Nelline requires unfolding to fit in the facts that Nelline's friend heard her casually greeting someone, and then screaming; presumably due to Patrick pointing a gun at her. The completely unfolded tree of events relating to Patrick confronting Nelline with a gun is shown in Figure 4: Patrick acquires a loaded gun by first acquiring a gun and gunpowder via some way, buying blank rounds abroad (where they are freely available) and modifying these blanks into live ammo. He also sawed off the barrel of his acquired gun. One the morning of the murder Patrick calls in sick from work, after this he confronts Nelline who greets him, and then screams when he pulls his previously acquired gun on her.

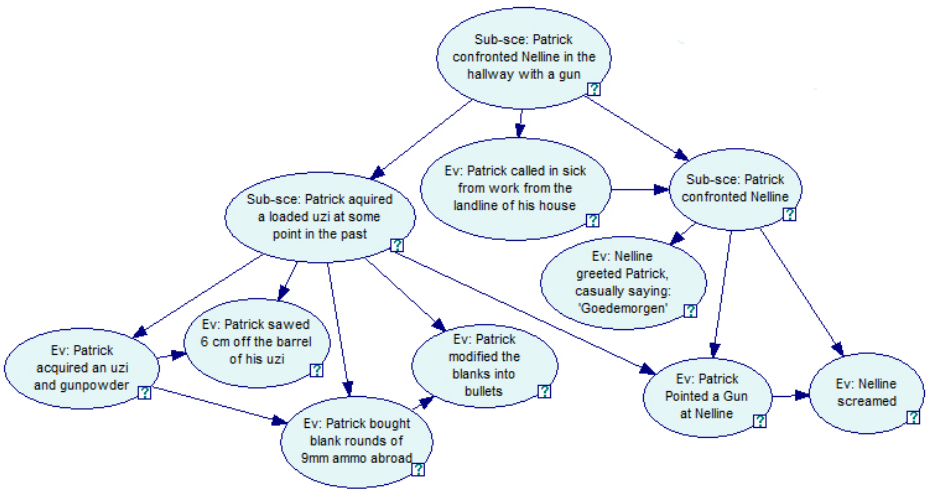

Figure 4. Unfolded subscenario of the event of Patrick confronting Nelline with a gun.

4.1.4. *Patrick kills Nelline*

There is weak direct evidence that Patrick killed Nelline, because of the wiretapped phone conversations between Patrick's close relatives. Therefore unfolding wouldn't strictly be required according to the guideline of the three questions. However there a lot more indirect evidence for the fact that Patrick killed Nelline, like the blood and cartridge cases in the hallway, which indicate that he first shot her in the hallway, and the fact that Nelline's body was found in Patrick's apartment, which indicates that he dragged her there. Without unfolding the event of Patrick killing Nelline these two pieces of evidence would seem to contradict each other instead of forming a coherent scenario. Therefore the choice to unfold event of Patrick killing Nelline is made. The wiretapped phone conversations will in this case be construed as direct evidence only for the fact that Patrick confessed his murder of Nelline to his parents, making it indirect evidence for the murder itself. The full subscenario of Patrick killing Nelline is shown in Figure 5: Patrick shoots Nelline in the hallway, drags her to is room and proceeds to shoot her up close in the head in the head. nelline dies from her injuries.

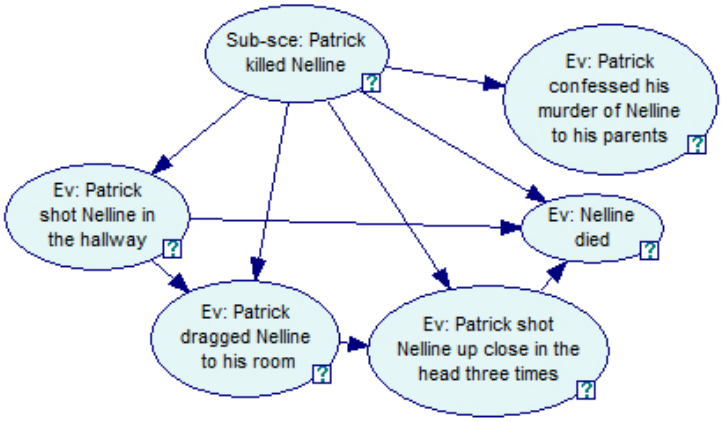

Figure 5. Unfolded subscenario of the Patrick killing Nelline.

4.1.5. *Patrick flees in Nelline's car*

Finally there is no direct evidence and there is indirect evidence for the event that Patrick fled the scene in Nelline's car. Patrick's odourprint in Nelline's car implies that he was in the car at some point, the testimonies of the A-27 motorists imply that someone who looks like Patrick was driving the car when it crashed and Nadia's blood in her car implies that her murderer used her car to flee the scene. The complete subscenario consists of Patrick taking Nelline's car keys, driving away in her car, driving her car of the road and getting out of the carwreck to wander off in the woods.

4.2. *The Bayesian model of the alternative scenario*

In Figure 6 the initial scenario of Nelline's murder by her ex-boyfriend is shown. Nelline's ex-boyfriend was jealous about their break up, so he entered the house with a

gun while Nelline was home alone, after which he killed her and tried to frame Patrick, who was suffering from memory loss, for the murder.

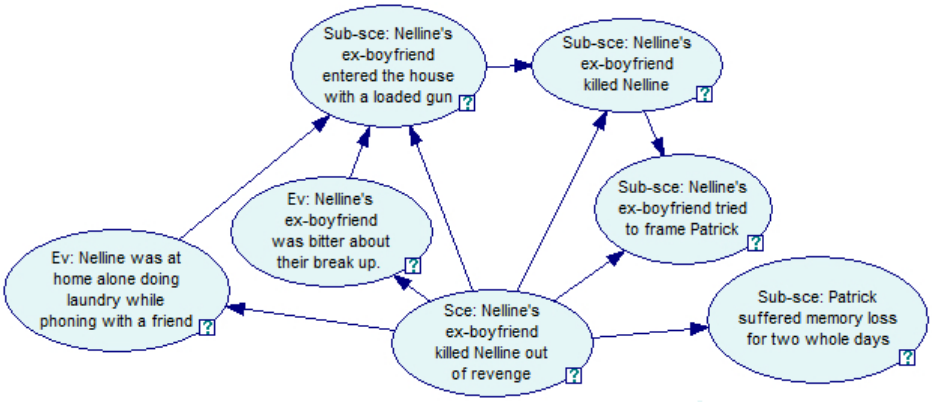

Figure 6. Initial scenario of Nelline's ex-boyfriend killing Nelline.

4.2.1. Nelline is doing housework and her ex-boyfriend is jealous

The motive will be unfolded with a variation idiom to incorporate the defence attorney plea that Nelline was in fact killed by her jealous ex-boyfriend while also leaving open the option that someone had a completely different unknown motive for wanting Nelline dead. There is no direct or indirect evidence for either option, nor is it likely that unfolding this event will lead to any relevant evidence so no further unfolding is required for the motive in the alternative scenario. The evidence for the event of Nelline being at home during housework is the same as in the first scenario (direct), so this node also doesn't need to be unfolded.

4.2.2. Nelline's ex-boyfriend confronts her with a gun

The next event, of Nelline's ex-boyfriend entering the house with a gun does require unfolding, because no direct evidence can be connected to it, but indirect evidence can be connected to it. If Nelline's boyfriend did enter the house with a loaded gun with which he confronted Nelline he had to have acquired a gun and ammunition made from blanks previously. Unlike in the first scenario there is no direct or indirect evidence of this being the case, therefore this node of the subscenario won't be further unfolded. The node that Nelline's ex-boyfriend entered the house, which itself is an event in the subscenario of Nelline's ex-boyfriend entering the house with a gun, needs to be unfolded into a variation idiom. There is evidence against the notion that Nelline's ex-boyfriend broke in, because the police found no signs of forced entry into the house and there is evidence against the notion that Nelline let her ex-boyfriend in, because Nelline's friend didn't hear her opening the front door. Both of these pieces are indirect evidence against the notion that Nelline's ex-boyfriend was able to enter the house at all. Nelline's friend's testimony indirectly provides evidence for the event of Nelline being confronted by her ex-boyfriend, which is unfolded in a similar manner as with the first scenario. In this

subscenario there is direct evidence via Nelline's friend's testimony that Nelline casually greeted her ex-boyfriend who she hasn't seen in a year with: "Goeiemorgen.". This isn't likely regardless whether Nelline's ex-boyfriend confronted her or not, which makes this subscenario partially incoherent. In Figure 7 the complete subscenario of Nelline's ex-boyfriend acquiring a gun loaded with modified blanks, entering Nelline's house by either breaking in or being let in, and confronting Nelline, who greeted him and screamed when he pointed his gun at her.

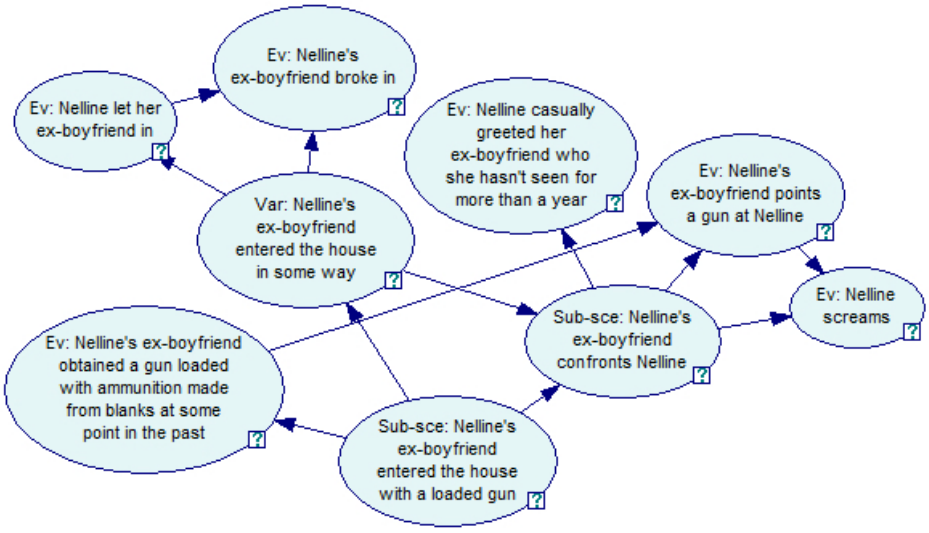

Figure 7. Unfolded subscenario of Nelline's ex-boyfriend entering Nelline's house with a gun.

4.2.3. Nelline gets killed by her ex-boyfriend

There is no direct evidence that Nelline's ex-boyfriend killed Nelline, but same indirect evidence that Patrick killed Nelline could also be used in the alternative scenario against Nelline's ex-boyfriend, provided he killed her in the same exact way that Patrick did in the first scenario. The node about Nelline's ex-boyfriend killing Nelline is therefore unfolded in nearly same way as in the first scenario, except for the fact that event of Nelline's ex-boyfriend moving Nelline (or her body) from the hallway to Patrick's kitchen would be part of the subscenario of Nelline's ex-boyfriend trying to frame Patrick. The unfolded subscenario is similar to the unfolded murder subscenario in the first scenario.

4.2.4. Nelline's ex-boyfriend tries to frame Patrick

The event of Nelline's ex-boyfriend trying to frame requires unfolding as well because there is no direct evidence for it, but there is is indirect evidence. There is evidence for the notion that Nelline's ex-boyfriend planted ammunition and gun attributes in Patrick's house, due to Patrick's testimony. There is also evidence for and against different aspects of the idea that Nelline's ex-boyfriend dragged Nelline from the hallway to Patrick's

kitchen. The fact that Nelline's body was found in Patrick's kitchen behind locked doors, could imply she was dragged there to frame Patrick, but for that to be true he must also have been able to somehow leave Patrick's apartment locked, which wouldn't have been likely, creating another partially incoherent subscenario. Nelline's ex-boyfriend dragged Nelline into Patrick's kitchen and planted gun attributes and ammunition in various places in Patrick's house after which he was able to leave Patrick's house locked.

4.2.5. Patrick's memory loss

The testimony of Patrick serves as direct evidence that he lost his memory for two days, and the fact that fact Patrick never sought or wanted medical attention for his sudden memory loss directly speaks against it, therefore it's possible not to unfold this node. However the choice to unfold this node was made in this case, because Patrick's actions after he came to are unlikely under the assumption that he didn't know what had happened at his house. Unfolding this event as a subscenario shows the inconsistency of it, which has an impact on the credibility of this event and thereby also on the credibility of the scenario as a whole. Patrick drank beer with a friend on the evening before the murder, which might have contributed to him calling in sick the next morning. Either way after this he somehow ended up in the woods. He also lost his memory at some point after or during drinking beer with his friend, which caused him to come to without knowing what had happened at the his house. Lastly he went straight to his parents' house instead of going home.

4.2.6. Merging the scenarios

When both scenarios are fully unfolded all the relevant evidence is connected to the appropriate events, the scenario's are merged using the merged scenarios idiom a few more dependencies can be modelled in the complete scenario: Patrick's reliability as a witness, depends on his guilt and the evidence of Patrick apartment is locked can be explained if he left it locked, and can therefore be tied to the event of him dragging Nelline to his kitchen. Not only are there multiple pieces of evidence which point only to Patrick as the killer and no evidence that only points to a frame up. Moreover the ex-boyfriend scenario has internal inconsistencies and evidence against it. When the scenarios are merged using the merged scenarios idiom and all the relevant evidence is instantiated, the scenario of Patrick murdering Nelline is the most likely scenario according to the network. The network of the fully unfolded and merged scenarios, with all the evidence nodes connected, can be seen in 9 in the first appendix.

4.3. The logico-probabilistic case model

The variables in this model are listed below:

- H_p: The hypothesis that Patrick F. is the killer.
- H_{exb}: The hypothesis that Nelline's ex-boyfriend is the killer.
- E_1: The initial police investigation.
- E_2: The fact that Patrick is nowhere to be found.
- E_3: The further police investigation.
- M: Nelline was murdered.

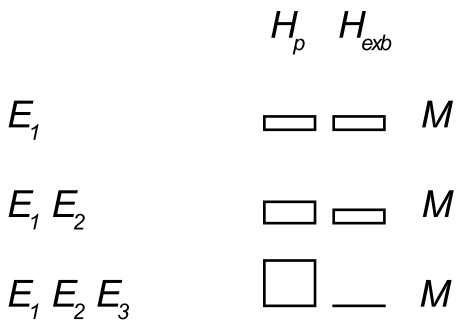

Figure 8. Development of the evidence (E), hypotheses (H) and expectations (in this case M, the expectation that Nelline was murdered). The height of the rectangle represents the strength of the argument for an hypothesis given the evidence, with a flat line denoting 0.

For the logico-probabilistic case model two hypotheses,H_p and H_{exb} , will be considered. These correspond to the first and alternative murder scenario's respectively as previously described (Under H_p Patrick is the murderer, under H_{exb} Nelline's ex-boyfriend is the murderer). The two hypotheses are mutually exclusive:

$$p(H_p \wedge H_{exb}) = 0$$

The first evidence to be considered (termed evidence E_1) is all evidence up to the point where the police was able to confirm that Nelline had been murdered: Including the testimony of Nelline's friend, the blood and gun shells in the hallway and Nelline's body in Patrick's locked apartment. E_1 raises and directly confirms expectation M, that Nelline was murdered:

$$p(M|H_i \wedge E_1) = 1$$

Both hypotheses are weak presumptions at this point:

$$p(H_p|E_1) = p(H_{exb}|E_1) > 0$$

Several pieces of E_1, make the police want to bring Patrick in for questioning (Nelline was murdered in his home, found in his apartment and was not surprised to encounter her killer, by testimony of her friend). Patrick however is nowhere to be seen (E_2), making him suspect:

$$p(H_p|E_1 \wedge E_2) > p(H_p|E_1)$$

As the police investigation continues new evidence is discovered which points to Patrick as the murderer: The testimony of Patrick's colleague placing Patrick at the scene of the crime at the right time, the footprint which matches Patrick's shoe type, the various gun attributes and ammunition found in Patrick's house, the testimony of A-27 motorists, who saw Nelline's car crash and someone matching Patrick's description get out,

Patrick's odour print and Nelline's blood in Nelline's crashed car and the wiretapped phone conversations between Patrick's relatives. This evidence taken together is E_3. The chance of Patrick's guilt becomes much greater under $E_1 \wedge E_2 \wedge E_3$. This large change in comparative probability is expressed by the use of an inequality of ratios of conditional probabilities:

$$\frac{p(H_i|E_1 \wedge E_2 \wedge E_3)}{p(H_p|E_1 \wedge E_2 \wedge E_3)} < \frac{p(H_i|E_1 \wedge E_2)}{p(H_p|E_1 \wedge E_2)} \text{ for } i \neq 1$$

At this point, without even considering alternative scenarios the culmination of evidence for H_p (in large part due to E_3), can be considered so overwhelming that it's a certainty that Patrick is the murderer. This is an assumption added to the model in absence of reasonable doubt. It doesn't follow directly from the model thus far:

$$p(H_p|E_1 \wedge E_2 \wedge E_3) = 1$$

The only expectation associated with this is that Nelline was murdered, which is certain. It formally follows that:

$$p(H_p \wedge M|E_1 \wedge E_2 \wedge E_3) = 1$$

In Figure 8 the development of the evidence, hypotheses and expectations is summarized graphically.

5. Evaluation

Both models supported the scenario or hypothesis where Pascal F. is the murderer with a strength correlating with the amount of evidence brought in against him. Which is to say that both models can adequately be used to model this court case.

5.1. Properties of the Bayesian model

The main advantage of the Bayesian network modelling method is the ability to look at different parts of a scenario at exactly the relevant level of detail through unfolding. The unfolding guidelines of this method are very useful clear principles, which also gives a clearly defined stop moment in modelling. When all relevant evidence can be connected the model is done. The unfolding guidelines also have the added benefit of not being binding, which allows for the model maker to divert from them in the case of rare anomalies (such as the fact that there was both direct evidence and seemingly contradictory indirect evidence, for the event of Patrick killing Nelline).

THe graphical structure of Bayesian networks provides very direct and intuitive insight into individual unfolded (sub)scenarios. However the full model of a court case can become very large very easily, especially with all the added evidence nodes. The full model will also most likely contain nodes which appear in both scenarios throughout. This is where the graphical nature loses it benefit, because the resulting spiderweb of connected nodes is very difficult to oversee, especially for an outsider.

The main drawback of this method stems from the fact that is is a Bayesian belief network applied to court cases, which typically have relatively few pieces of evidence with known probabilities attached to them. (nearly none in this case study): Having to elicit almost all conditional probabilities. This is an arduous process. On top of that, some conditional probabilities are very specific (especially the conditional probabilities of nodes with more than two parents) to the point where it sometimes feels arbitrary to assign them a value, even with the help of the verbal-literal scale.

For intuitively assigning a value to the conditional probabilities of nodes with many parents it might have been beneficial to expand the verbal literal scale in order to account for more nuanced estimates of likelihood. But creating even more gradations of likelihood would increase the difficulty of being able to distinguish between them. With the current verbal-literal scale, consisting of four gradations in likelihood (and three complementary ones), it already is difficult in some cases to distinguish between them. (e.g. estimate whether a particular probability is 'likely' or 'quite likely')

Decreasing the amount of dependencies (arrows) in the model would decrease the amount of probabilities which would need to be elicited, but that would also increase the amount of independence assumptions which would need to be done. Some of these independence assumptions can feel as arbitrary as eliciting the numbers.

5.2. Properties of the logico-probabilistic model

The main advantage to using the logico-probabilistic modelling is not having to commit to any assumptions of conditional probabilities or assumptions of independence. The only assumptions which need to be made with this model are assumptions directly related to the court case you're modelling (the assumption of guilt beyond reasonable doubt and the assumptions which go into the choice of scenario's). Moreover assumptions regarding the choice of considered scenarios are no different from the Bayesian model, and due to the scarcity of available numbers concerning a court case, more or less the same assumption of guilt beyond reasonable doubt is also made in the Bayesian model. The difference being that with the Bayesian model the assumption of guilt beyond reasonable doubt is split into a lot of assumptions about the values of conditional probabilities. The chronological order of bringing in evidence provides an orderly framework to deduct the guilt of a suspect, starting from 'no information'. The combination of few variables, a simple visual representation and the commitment to introductory textbook probability theory and propositional logic makes the model in total easier to oversee and more accessible to outsiders than the complete Bayesian model is. With the logico-probabilistic method scenarios (hypotheses) are modelled as elementary propositional variables. This is an inaccurate representation of scenarios, compared to how a scenario is unfolded into subscenarios and events at the relevant level of detail with the Bayesian method. Due to this inaccurate representation, evidence which would specifically apply to one event within a scenario, directly applies to the entire scenario, even though an event in a scenario can be true without the scenario being true. Another drawback of having scenario nodes as elementary variables is that it isn't possible to model internal inconsistency in a scenario, which is when two subscenarios (or events) have a lower probability of occurring together than each one has of occurring individually. The logico-probabilistic method also no clearly defined guideline for the bundling of evidence or for making the certainty assumption. The bundling of evidence could make the model less transparent

for outsiders at first glance, because it isn't explicitly modelled how much weight the model maker lends to each individual piece of evidence.

5.3. Integrating the methods

In summary, the formal approach of the logico-probabilistic method is preferable to the probability elicitation process required when using the Bayesian method. However the unfolding principle of the Bayesian method, provides the resulting case model with a structure of subscenarios and events, which allows for evidence to directly influence the relevant event or subscenario it and for elements within a scenario to contradict each other. To create a model with the advantages of both methods, we will combine the unfolding principle from the Bayesian method with the logico-probabilistic method and define the unfolded structure of subscenarios and events in terms of classical propositional logic.

6. The extended logico-probabilistic method and resulting case study model

6.1. A logico-probabilistic method based around unfolding scenarios

This method is an extension of the logico-probabilistic method of Verheij, and uses all and only the logical connectives and probability functions described in section 2.3. The design of the extended logico-probabilistic method is similar to the design and procedure of the original logico-probabilistic method as described in section 2.4 with the following differences: Events which are represented elementary variables in propositional logic are all part of (sub)scenarios, which will be represented by chains of conjunctions. This chains of conjunctions to represent a (sub)scenario correspond to the (sub)scenario idiom of the Bayesian method and keeps true to the rule of that idiom that all events in a scenario have to be 'True' for the scenario to be 'True', but some events in a scenario can be 'True' when the scenario is 'False'. Variation idioms from the Bayesian method are represented as exclusive disjunctions. Conditional dependence between events in a scenario is modelled as the argument for an event based on a different event. Pieces of evidence will be grouped for convenience, first chronologically and then thematically, and these evidence bundles will be represented as conjunction chains. The first step of the procedure of the extended logico-probabilistic method is to define the scenarios and to apply the unfolding principle (as described in section 2.2) to them. This results in a series of conjunction and possibly exclusive disjunction chains which define the unfolded scenarios. After this the pieces of evidence is added to the model gradually in the order of their chronological and thematic grouping, and the strength of the arguments for the events, (sub)scenarios and expectations is reevaluated based on the new evidence, similarly to the procedure described in section 2.4. For the extended logico-probabilistic method however, each piece of evidence will be looked at individually and will directly influence the argument for the event or (sub)scenario it most specifically applies to (connects to, in Bayesian terms). Larger (sub)scenarios for which there is only indirect evidence, are supported by the evidence when a majority of the events they are composed of is.

6.2. A logico-probabilistic model based on the unfolded scenario of the case study

Seeing as this model is based on the same evidence as the other two models in this paper, the unfolding process done when constructing the Bayesian model will not be reiterated here. Instead the scenario-structure found when constructing the Bayesian model, will be directly translated to it's formal equivalent. All scenarios, subscenarios and bundles of evidence are listed below. The list of definitions for all the propositional values used here is included in appendix 2.

1. $S_p = S_p^1 \wedge S_p^2 \wedge S_p^3 \wedge S_p^4 \wedge S_p^5$
2. $S_p^2 = S_p^{2.1} \wedge S_p^{2.2}$
3. $S_p^3 = S_p^{3.1} \wedge S_p^{3.2} \wedge S_p^{3.3}$
4. $S_p^3.1 = S_p^{3.1.1} \wedge S_p^{3.1.2} \wedge S_p^{3.1.3} \wedge S_p^{3.1.4}$
5. $S_p^3.3 = S_p^{3.3.1} \wedge S_p^{3.3.2} \wedge S_p^{3.3.3}$
6. $S_p^4 = S_p^{4.1} \wedge S_p^{4.2} \wedge S_p^{4.3} \wedge S_p^{4.4}$
7. $S_p^5 = S_p^{5.1} \wedge S_p^{5.2} \wedge S_p^{5.3} \wedge S_p^{5.4}$
8. $S_{exb} = S_{exb}^1 \wedge S_{exb}^2 \wedge S_{exb}^3 \wedge S_{exb}^4 \wedge S_{exb}^5 \wedge S_{exb}^6$
9. $S_{exb}^3 = S_{exb}^{3.1} \wedge S_{exb}^{3.2} \wedge S_{exb}^{3.3}$
10. $S_{exb}^3.2 = (S_{exb}^{3.2.1} \vee S_{exb}^{3.2.2}) \wedge \neg(S_{exb}^{3.2.1} \wedge S_{exb}^{3.2.2})$
11. $S_{exb}^4 = S_{exb}^{4.1} \wedge S_{exb}^{4.2} \wedge S_{exb}^{4.3} \wedge S_{exb}^{4.4}$
12. $S_{exb}^5 = S_{exb}^{5.1} \wedge S_{exb}^{5.2}$
13. $S_{exb}^6 = S_{exb}^{6.1} \wedge S_{exb}^{6.2} \wedge S_{exb}^{6.3} \wedge S_{exb}^{6.4} \wedge S_{exb}^{6.5}$
14. $E = E_1 \wedge E_2 \wedge E_3$
15. $E_1 = E_{1.1} \wedge E_{1.2} \wedge E_{1.3}$
16. $E_3 = E_{3.1} \wedge E_{3.2} \wedge E_{3.3} \wedge E_{3.4} \wedge E_{3.5} \wedge E_{3.6} \wedge E_{3.7}$
17. $E_{3.1} = E_{3.1.1} \wedge E_{3.1.2} \wedge E_{3.1.3} \wedge E_{3.1.4} \wedge E_{3.1.5}$
18. $E_{3.5} = E_{3.5.1} \wedge E_{3.5.2} \wedge E_{3.5.3} \wedge E_{3.5.4}$

The two hypotheses are mutually exclusive:

$$p(S_p \wedge S_{exb}) = 0$$

The only other thing we can say about our scenario's at this point is that we can't exclude any option yet:

$$p(s_i) > 0 \text{ for } i = p, exb$$

The first piece of evidence (termed E_1) represents the initial police investigation. Initially the police was alerted by Nelline's friend, who testified what she had heard on a phone conversation with Nelline which abruptly ended (This is $E_1.1$). This raised the possibility that something happened with or to Nelline, but was not enough to warrant a police investigation by itself. Specifically this supports the events of Nelline doing housework, greeting someone she expected to see and being startled.

$$p(S_p^1) = p(S_{exb}^1) = p(Housework)$$

$$p(Housework|E_1.1) = 1$$

$$p(S_p^{3.3.3}) = p(S_{exb}^{3.3.3}) = p(Scream)$$

$$p(Scream|E_1.1) = 1$$

Only the fact that she greeted someone she expected to see influences the probabilities of events in both scenarios as well as effecting them differently:

$$p(S_p^{3.3.1}|E_{1.1}) > 0$$

$$p(S_p^{3.3.1}|E_{1.1}) > p(S_{exb}^{3.3.1}|E_{1.1})$$

There is also some evidence from absence in the testimony of Nelline's friend. If Nelline let in her killer $S_{exb}^{3.2.2}$, her friend would likely have heard. Seeing as she didn't mention other interaction between Nelline and her presumed killer we get:

$$p(S_{exb}^{3.2.2}|E_{1.1}) < p(S_{exb}^{3.2.2})$$

Later that evening the police is alarmed again by Nelline's roommates who came home to find blood and gun shells in their hallway ($E_1.2$). Giving rise to the possibility that she has been shot, and may have died:

$$p(S_p^{4.1}|E_1.1 \wedge E_1.2) > p(S_p^{4.1}|E_1.1)$$

$$p(S_{exb}^{4.1}|E_1.1 \wedge E_1.2) > p(S_{exb}^{4.1}|E_1.1)$$

$$p(S_p^{4.4}) = p(S_{exb}^{4.3}) = p(Death)$$

$$p(Death|E_1.1 \wedge E_1.2) > p(Death|E_1.1)$$

When the police arrived on the scene they kicked down the door of Patrick's apartment and found Nelline's body there ($E_1.3$).

$$p(Death|E_1) = 1$$

Murder is expected in all scenario's considered at this point:

$$p(Murder|S_i \wedge E_1 \text{ for } i = p, exb) = 1$$

When the police wants to bring him in for questioning, Patrick is nowhere to be found, making him suspect:

$$p(S_p|E_1 \wedge E_2) > p(S_p)$$

The further police investigation (termed E_3) brought seven more pieces of evidence to the table:

$$E_3 = E_{3.1} \wedge E_{3.2} \wedge E_{3.3} \wedge E_{3.4} \wedge E_{3.5} \wedge E_{3.6} \wedge E_{3.7}$$

In the further police investigation a number of gun and ammunition related items were found (termed $E_{3.1}$). These items were a clip holder, a sawed off piece of barrel, ammunition made from modified blank rounds, and drill bits to modify blank rounds into live ammunition (termed $E_{3.1.1}$, $E_{3.1.2}$, $E_{3.1.31}$ and $E_{3.1.4}$ respectively) found in Patrick's apartment, and a bullet made from a modified blank ($E_{3.1.5}$) found at Patrick's parents' house:

$$E_{3.1} = E_{3.1.1} \wedge E_{3.1.2} \wedge E_{3.1.3} \wedge E_{3.1.4} \wedge E_{3.1.5}$$

The finding of the clip holder increases the probability that Patrick acquired a gun in the past via some way ($S_P^{3.1.1}$):

$$p(S_P^{3.1.1}|E_1 \wedge E_2 \wedge E_{3.1.1}) > p(SP^{3.1.1}|E_1 \wedge E_2)$$

The finding of the sawed off piece off barrel increases the probability that Patrick sawed off the barrel of a gun ($p(S_p^{3.1.2}$).

$$p(S_p^{3.1.2}|E_1 \wedge E_2 \wedge E_{3.1.2}) > p(S_P^{3.1.2}|E_1 \wedge E_2)$$

He could only have done this if he had aqcuired a gun previously, therefore:

$$p(S_p^{3.1.1}|S_p^{3.1.2}) = 1$$

$$p(S_P^{3.1.1}|E_1 \wedge E_2 \wedge E_{3.1.1} \wedge E_{3.1.2}) > p(SP^{3.1.1}|E_1 \wedge E_2 \wedge E_{3.1.1})$$

The finding of boxes of modified blanks increases the probability that Patrick bought blanks abroad ($S_p^{3.1.3}$) and modified them into bullets ($S_p^{3.1.4}$):

$$p(S_p^{3.1.3}|E_1 \wedge E_2 \wedge E_{3.1.3}) > p(S_P^{3.1.3}|E_1 \wedge E_2)$$

And the fact that the appropriate drill bits for modifying blanks into bullets were found, and that a modified blank bullet was found at Patrick's parent's house raise the probability that Patrick modified blanks into bullets:

$$p(S_p^{3.1.4}|E_1 \wedge E_2 \wedge E_{3.1.3} \wedge E_{3.1.4} \wedge E_{3.1.5}) >$$

$$p(S_p^{3.1.4}|E_1 \wedge E_2 \wedge E_{3.1.3} \wedge E_{3.1.4}) > p(S_p^{3.1.4}|E_1 \wedge E_2 \wedge E_{3.1.3}) > p(S_P^{3.1.4}|E_1 \wedge E_2)$$

The subscenario formed by these events ($S_p^{3.1}$) is that Patrick acquired a loaded gun at some point in the past:

$$S_p^{3.1} = S_p^{3.1.1} \wedge S_p^{3.1.2} \wedge S_p^{3.1.3} \wedge S_p^{3.1.4}$$

There is an argument to be made for each event of these subscenario, based on the various parts of evidence-bundle $E_{3.1}$. Seeing as every event in this subscenario is supported by this evidence, the complete subscenario is supported by this evidence and a stronger argument can be made for it.

$$p(S_p^{3.1}|E_1 \wedge E_2 \wedge E_{3.1}) > p(S_p^{3.1}|E_1 \wedge E_2)$$

The finding of gun related items also support the subscenario that someone else planted gun related items in Patrick's house($S_{exb}^{5.2}$), but this And this argument strong enough to support the subscenario that Patrick is framed by itself, and also doesn't have an explanation for the bullet at Patrick's parents' house.

$$p(S_{exb}^{5.2}|E_1 \wedge E_2 \wedge E_{3.1}) > p(S_{exb}^{5.2}|E_1 \wedge E_2)$$

The argument for Patrick owning a gun based on $E_{3.1}$ is stronger than the argument for a plant:

$$p(S_p^{3.1}|E_1 \wedge E_2 \wedge E_{3.1}) > p(S_{exb}^{5.2}|E_1 \wedge E_2 \wedge E_{3.1})$$

The autopsy report on Nelline's body confirms that she was shot in the body and subsequently up close in the head. This does not by itself support subscenario that she was either killed by Patrick or by her ex-boyfriend:

$$p(S_p^{4.1} \wedge S_p^{4.3}) = p(S_{exb}^{4.1} \wedge S_{exb}^{4.3}) = p(NellineShot)$$

$$p(NellineShot|E_1 \wedge E_2 \wedge E_{3.2}) = 1$$

The bloody foot prints belonging to Patrick's shoe type found at the crime scene ($E_{3.3}$) are evidence that he was there, and that he dragged Nelline to his kitchen ($S_p^{4.2}$) from the point where she was originally shot:

$$p(S_p^{4.2}|E_1 \wedge E_2 \wedge E_{3.3}) > p(S_p^{4.2}|E_1 \wedge E_2)$$

A this point a stronger argument for the subscenario of Patrick killing Nelline (S_p^4) can be made, because Nelline was shot first in the hallway and then in Patrick's kitchen and there is evidence of Patrick's involvement.

$$p(S_p^4|E_1 \wedge E_2 \wedge E_{3.2} \wedge E_{3.3}) > p(S_p^4|E_1 \wedge E_2)$$

The police didn't find any signs of forced entry ($E_{3.4}$). This means that there was no break in:

$$p(S^3_{exb}.2.1|E_1 \wedge E_2 \wedge E_{3.4}) = 0$$

Because the probability of Nelline letting in her killer has already been lowered, the overall probability of Nelline's ex-boyfriend (or anyone else) entering the house now lowers:

$$p(S^3_{exb}.2|E_1 \wedge E_2 \wedge E_{3.4}) < p(S^3_{exb}.2|E_1 \wedge E_2)$$

This also affects the probability of that person confronting Nelline with a loaded gun (S^3_{exb}) and by extension of him being the murderer (S_{exb}), because those two scenario's can't be True, if one event in them is False:

$$p(S^3_{exb}|E_1 \wedge E_2 \wedge E_{3.4}) < p(S^3_{exb}|E_1 \wedge E_2)$$

$$p(S_{exb}|E_1 \wedge E_2 \wedge E_{3.1} \wedge E_{3.2} \wedge E_{3.3} \wedge E_{3.4}) < p(S_{exb}|E_1 \wedge E_2 \wedge E_{3.1} \wedge E_{3.2} \wedge E_{3.3})$$

The next piece of evidence in the further police investigation is the evidence regarding Nelline's car ($E_{3.5}$). The wreck of Nelline's car was found by the A-27 highway ($E_{3.5.1}$). Upon investigation by the police traces of Nelline's blood ($E_3.5.3$) and Patrick's odour print ($E_{3.5.2}$) were found in Nelline's car, giving rise to the possibility that Patrick (because of $E_3.5.2$) took the keys of Nelline's car from her body ($S^{5.1}_p$):

$$p(S^{5.1}_p|E_1 \wedge E_2 \wedge E_{3.5.2} \wedge E_{3.5.3}) > p(S^{5.1}_p|E_1 \wedge E_2)$$

And that Patrick drove Nelline's car ($S^{5.2}_p$) off the road ($S^{5.3}_p$):

$$p(S^{5.2}_p|E_1 \wedge E_2 \wedge E_{3.5.2}) > p(S^{5.2}_p|E_1 \wedge E_2)$$

$$p(S^{5.3}_p|E_1 \wedge E_2 \wedge E_{3.5.1} \wedge E_{3.5.2}) > p(S^{5.3}_p|E_1 \wedge E_2)$$

Lastly the motorists who reported the accident testified ($E_3.5.4$) they saw a man matching Patrick's description getting out of Nelline's car after it crashed, which would have been Patrick in this event-hypothesis ($S^{5.4}_p$):

$$p(S^{5.4}_p|E_1 \wedge E_2 \wedge E_{3.5.4}) > p(S^{5.4}_p|E_1 \wedge E_2)$$

The four events together form the subscenario S^5_p where Patrick fled the scene of Nelline's murder in her car:

$$S^5_p = S^{5.1}_p \wedge S^{5.2}_p \wedge S^{5.3}_p \wedge S^{5.4}_p$$

The evidence ($E_{3.5}$) each of a number of events which together make up the complete subscenario, therefore:

$$p(S_p^5|E_1 \wedge E_2 \wedge E_{3.5}) > p(S_p^5|E_1 \wedge E_2)$$

The testimony of Patrick's colleague supports the event that Patrick called in sick from work, placing him at his home just before the time of Nelline's murder:

$$p(S_p^{3.2}|E_1 \wedge E_2 \wedge E_{3.6}) > p(S_p^{3.2}|E_1 \wedge E_2)$$

Lastly the wiretapped conversations between Patrick's relatives ($E_{3.7}$) provide evidence that Patrick confessed the murderer of Nelline to his parents, which supports his guilt directly:

$$p(S_p|E_1 \wedge E_2 \wedge E_3) > p(S_p^4|E_1 \wedge E_2 \wedge E_{3.1} \wedge E_{3.2} \wedge E_{3.3} \wedge E_{3.4} \wedge E_{3.5} \wedge E_{3.6})$$

A number of events in the scenario that Patrick is the murderer are now supported by evidence. It has been established by $E_{3.2}$ that Nelline was shot. It has become plausible that Patrick owned the weapon due to $E_{3.1}$, that he was at the scene at the crime at the moment of the crime due to $E_{1.1}$ and $E_{3.6}$, giving him the opportunity to confront and shoot Nelline, who was likely confronted and shot by someone familiar who killed her ($E_{1.1}$ and $E_{1.3}$), while doing housework ($E_{1.1}$). Furthermore it is likely that Patrick dragged Nelline's body to his kitchen from the hallway where she was first shot due to $E_{1.2}$, $E_{1.3}$ and $E_{3.3}$ and it is likely that he fled in Nelline's car due to $E_{3.5}$. On top of this he likely confessed most of this to his parents, indicated by $E_{3.7}$. The events and subscenario's supported by E_3 nearly form the complete scenario of Patrick being the murderer.

The probability that Patrick is the murderer increases largely due to E_3. The large change in comparative probability of Patrick's guilt before and after the addition of E_3 is expressed by the use of an inequality of ratios of conditional probabilities:

$$\frac{p(\neq S_p|E_1 \wedge E_2 \wedge E_3)}{p(S_p|E_1 \wedge E_2 \wedge E_3)} < \frac{p(\neg S_p|E_1 \wedge E_2)}{p(S_p|E_1 \wedge E_2)}$$

At this point it can be considered a certainty that Patrick is the murderer. This is an assumption added to the model in absence of any plausible alternative explanation or reasonable doubt. It doesn't follow directly from the model thus far:

$$p(S_p|E_1 \wedge E_2 \wedge E_3) = 1$$

The only event that isn't conclusively supported by the evidence thus far is Patrick's motive. While we concede that Patrick's motive, and therefore technically the entire scenario may have been something entirely different known only to himself. This is not relevant for the question of Patrick's guilt.

7. Discussion

The extended logico-probabilistic method was designed with the following ideas in mind. It should unfold scenarios to the relevant level of detail, like with the Bayesian method.

This would make it possible to express the scenario structure of subscenarios and events, evidence which applies to subscenarios and inconsistency or conditional dependence between subscenarios. The method should be an extension of the original the logico-probabilistic method, using the same approach, being restricted to classical propositional logic and probability theory, with arguments from evidence to and from scenarios, subscenarios and events with strengths represented as conditional probabilities. The extended method should retain advantages of not having to commit to any undue elicitation of conditional probabilities and having an orderly chronological structure that the original logico-probabilistic method has over the Bayesian method.

7.1. Unfolding scenarios logico-probabilistically compared to the Bayesian way

We were able to define both the (sub)scenario idiom and the variation idiom from the Bayesian design method in a valid way (meaning while keeping true to the respective definitions of those idioms), using classical propositional logic and standard probability theory. With these two idioms defined logically, the scenario structure of the fully unfolded Bayesian model could be expressed using the logico-probabilistic approach and consequently used for the extended logico-probabilistic model. A pieces of evidence directly strengthens the argument for the event it applies to, which in turn indirectly strengthens the argument for the (sub)scenario that event is part of.

Like the Bayesian method, the extended logico-probabilistic method has room to model internal inconsistency in a (sub)scenario. Also like the Bayesian method, the extended logico-probabilistic method allows for a conditional dependence relation between events in a (sub)scenario to bolster evidence. In the model this was used, when there was evidence for Patrick sawing a barrel of a gun, which he could only have done if he had one. Unlike with the Bayesian method, specifying a conditional dependence relation (or making an assumption of independence) between every two events isn't a requirement.

7.2. The extended logico-probabilistic method compared to the original

In the broadest sense, the development of evidence, scenario-hypotheses and expectations, including the certainty assumption, is the same as with the original logico-probabilistic method. The difference being that with the extended logico-probabilistic method we are also able to model the development of subscenario- and event-hypotheses. The original logico-probabilistic method can be seen as an abstraction of the extended method. Another difference between the original logico-probabilistic model and the extended model is that with the latter all hypotheses are introduced in advance, instead of being introduced gradually, when they become relevant in the chronological narrative. This can lead to events or subscenarios being defined (Patrick's memory loss in this model) which aren't needed in order to make the certainty assumption. By unfolding the scenarios a logico-probabilistic model is created that is potentially a lot bigger than the one created by the original method. Depending on the purposes of the model maker, this may be a disadvantage, because a big model with lots of subscenarios and events can be more difficult to oversee than a smaller one. Unfortunately the graph for summarizing the original logico-probabilistic model, can't encompass the entire extended logico-probabilistic method, because hypotheses in this graph are mutually exclusive, unlike subscenario and event hypotheses in the extended model. If the extended method

is to be used in the future, a graphical way to summarize its model would be a necessary addition.

8. Conclusion

In this case-study a murder case was modelled using two different methods: A Bayesian networking method and a formal method using classical proposition logic and probability theory. The models that arose were evaluated: The main advantage of the Bayesian method being it's unfolding principle and resulting scenario structure, and the main advantage of the logico-probabilistic method being its formal approach. To create a method with the advantages of both previous methods, the logico-probabilistic method was extended with the unfolding principle and scenario structure of the Bayesian method. The murder case was modelled with the extended logico-probabilistic method, for the purpose of testing and evaluating it.

The extended logico-probabilistic method provides a viable alternative to the Bayesian method. Due to retaining all of the additional functionality Bayesian method has compared to the original logico-probabilistic method, without having to commit to the process of eliciting conditional probabilities. Compared to the original logico-probabilistic method the extended method trades off some of its former simplicity for more expressive power and nuance. How important those qualities are in making a model largely comes down to personal and situational preference. In conclusion, the extended logico-propositional method has a definite edge on its Bayesian networking counterpart and could potentially be really useful for modelling complicated court cases.

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Appendix 1: Full graph of the Bayesian model

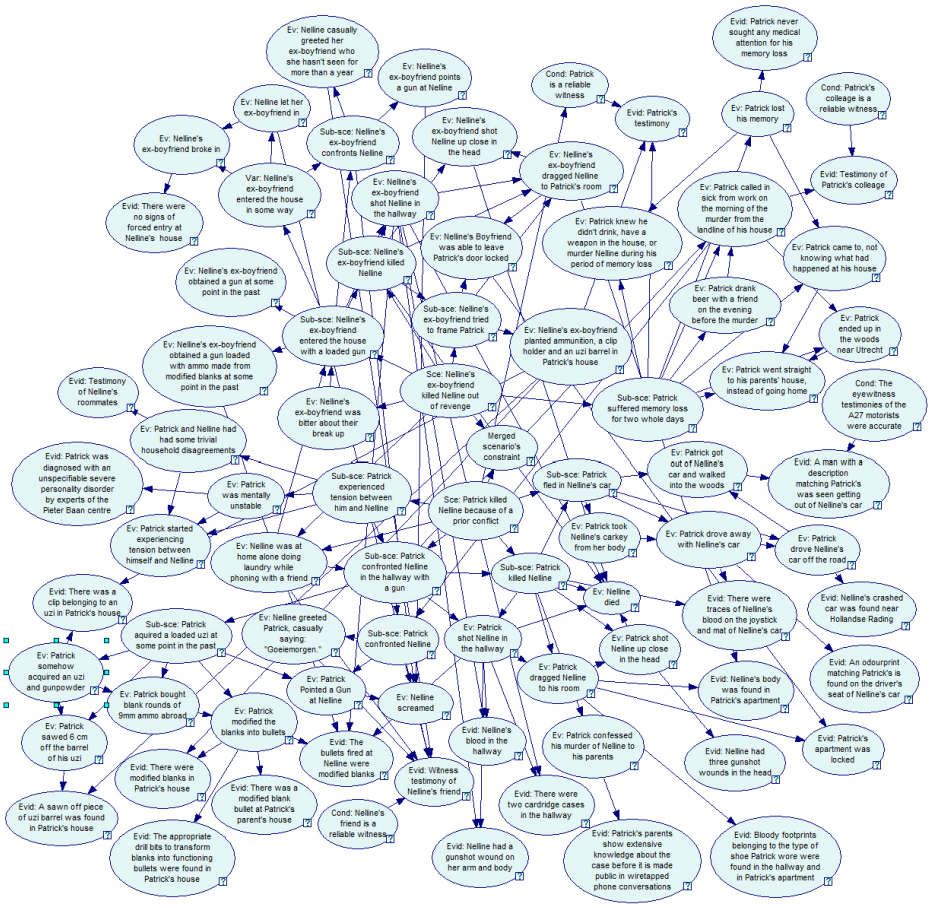

Figure 9. The full Bayesian model of the case study with evidence nodes (Evid:, not to be confused with events which are denoted with Ev:) and conditions (Cond:) attached.

Appendix 2: List of propositional variables in the combined model

1. $S_p = S_p^1 \wedge S_p^2 \wedge S_p^3 \wedge S_p^4 \wedge S_p^5$: The first scenario ($S_p$), consists of Nelline doing housework (S_p^1), Patrick's motive (S_p^2), Patrick confronting Nelline with a gun (S_p^3), killing her (S_p^4) and fleeing in her car (S_p^5).
2. $S_p^2 = S_p^{2.1} \wedge S_p^{2.2}$: Patrick's motive of experiencing tension between him and Nelline (S_p^2) consists of a prior disagreement between them ($S_p^{2.1}$) and his mental instability ($S_p^{2.2}$).

3. $S_p^3 = S_p^{3.1} \wedge S_p^{3.2} \wedge S_p^{3.3}$: Patrick confronting Nelline with a gun (S_p^3) consists of him having acquired a loaded gun ($S_p^{3.1}$), calling in sick from work ($S_p^{3.2}$) and confronting Nelline in the hallway ($S_p^{3.3}$).
4. $S_p^3.1 = S_p^{3.1.1} \wedge S_p^{3.1.2} \wedge S_p^{3.1.3} \wedge S_p^{3.1.4}$: Patrick acquiring a loaded gun ($S_p^{3.1}$) consists of him acquiring gunpowder and a gun ($S_p^{3.1.1}$), sawing off its barrel ($S_p^{3.2}$), buying blanks abroad ($S_p^{3.1.3}$) and modifying those into bullets ($S_p^{3.1.3}$).
5. $S_p^3.3 = S_p^{3.3.1} \wedge S_p^{3.3.2} \wedge S_p^{3.3.3}$: Patrick confronting Nelline in the hallway ($S_p^{3.3}$) consists of her casually greeting him ($S_p^{3.3.1}$), him pointing a gun at her ($S_p^{3.3.2}$) and her screaming ($S_p^{3.3.3}$).
6. $S_p^4 = S_p^{4.1} \wedge S_p^{4.2} \wedge S_p^{4.3} \wedge S_p^{4.4}$: Patrick killing Nelline (S_p^4) consists of him shooting her in the hallway ($S_p^{4.1}$), dragging her to his apartment ($S_p^{4.2}$), shooting her up close in the head ($S_p^{4.3}$) and of Nelline dying ($S_p^{4.4}$).
7. $S_p^5 = S_p^{5.1} \wedge S_p^{5.2} \wedge S_p^{5.3} \wedge S_p^{5.4}$: Patrick fleeing in Nelline's car (S_p^5) consists of him taking Nelline's carkeys from her body ($S_p^{5.1}$), driving away in her car ($S_p^{5.2}$), driving her car off the road ($S_p^{5.1.3}$) and getting out of Nelline's wrecked car and wandering into the woods ($S_p^{5.4}$).
8. $S_{exb} = S_{exb}^1 \wedge S_{exb}^2 \wedge S_{exb}^3 \wedge S_{exb}^4 \wedge S_{exb}^5 \wedge S_{exb}^6$: The alternative scenario (S_exb) consists of consists of Nelline doing housework (S_{exb}^1), her ex-boyfriend's jealousy (S_{exb}^2), Nelline's ex-boyfriend confronting Nelline with a gun (S_{exb}^3), killing her (S_{exb}^4), framing Patrick (S_{exb}^5) and Patrick's memory loss (S_{exb}^6).
9. $S_{exb}^3 = S_{exb}^{3.1} \wedge S_{exb}^{3.2} \wedge S_{exb}^{3.3}$: Nelline's ex-boyfriend confronting her with a gun (S_p^3) consists of him having acquired a loaded gun ($S_p^{3.1}$), entering the house in some way ($S_p^{3.2}$) and confronting Nelline in the hallway ($S_p^{3.3}$).
10. $S_{exb}^3.2 = (S_{exb}^{3.2.1} \vee S_{exb}^{3.2.2}) \wedge \neg(S_{exb}^{3.2.1} \wedge S_{exb}^{3.2.2})$: Nelline's ex-boyfriend entering her house ($S_{exb}^3.2$) consists of him either breaking in ($S_{exb}^{3.2.1}$) or being let in by Nelline ($S_{exb}^{3.2.2}$), (and not both).
11. $S_{exb}^3.3 = S_{exb}^{3.3.1} \wedge S_{exb}^{3.3.2} \wedge S_{exb}^{3.3.3}$: Nelline's ex-boyfriend confronting Nelline in the hallway ($S_{exb}^{3.3}$) consists of her casually greeting him ($S_{exb}^{3.3.1}$), him pointing a gun at her ($S_{exb}^{3.3.2}$) and her screaming ($S_{exb}^{3.3.3}$).
12. $S_{exb}^4 = S_{exb}^{4.1} \wedge S_{exb}^{4.2} \wedge S_{exb}^{4.3} \wedge S_{exb}^{4.4}$: Nelline's ex-boyfriend killing Nelline (S_p^4) consists of him shooting her in the hallway ($S_{exb}^{4.1}$), dragging her to his apartment ($S_{exb}^{4.2}$), shooting her up close in the head ($S_{exb}^{4.3}$) and of Nelline dying ($S_{exb}^{4.4}$).
13. $S_{exb}^5 = S_{exb}^{5.1} \wedge S_{exb}^{5.2} \wedge S_{exb}^{5.3}$: Nelline's ex-boyfriend trying to frame Patrick (S_{exb}^5) consists of him dragging Nelline into Patrick's apartment ($S_{exb}^{5.1}$), planting gun and ammunition related items there ($S_{exb}^{5.2}$) and leaving the door locked ($S_{exb}^{5.3}$).
14. $S_{exb}^6 = S_{exb}^{6.1} \wedge S_{exb}^{6.2} \wedge S_{exb}^{6.3} \wedge S_{exb}^{6.4} \wedge S_{exb}^{6.5}$: Patrick suffering from memory loss for two days consists of S_{exb}^6 him losing his memory ($S_{exb}^{6.1}$), calling in sick from work on the morning of Nelline's murder ($S_{exb}^{6.2}$), ending up in the woods near his home ($S_{exb}^{6.3}$), coming to with no knowledge about the murder at his house ($S_{exb}^{6.4}$) and going straight to his parents' house instead of going home ($S_{exb}^{6.5}$).
15. $E = E_1 \wedge E_2 \wedge E_3$: The evidence ($E$), consists of the initial police investigation (E_1), the fact that Patrick is nowhere to be found (E_2) and the further police investigation (E_3).

16. $E_1 = E_{1.1} \wedge E_{1.2} \wedge E_{1.3}$: The initial police investigation (E_1) consists of the testimony of Nelline's friend ($E_{1.1}$), the bullet shells and Nelline's blood in the hallway ($E_{1.2}$) and the finding of Nelline's body ($E_{1.3}$).
17. $E_3 = E_{3.1} \wedge E_{3.2} \wedge E_{3.3} \wedge E_{3.4} \wedge E_{3.5} \wedge E_{3.6} \wedge E_{3.7}$: The further police investigation (E_3) consists of the gun and ammunition related attributes found by the police ($E_{3.1}$), the autopsy report of Nelline's body ($E_{3.2}$), the bloody footprints which match Patrick's shoe type ($E_{3.3}$), the absence of signs of forced entry at Patrick's house ($E_{3.4}$), the evidence regarding Nelline's crashed car ($E_{3.5}$), the testimony of Patrick's colleague ($E_{3.6}$) and the wiretapped phone conversations between Patrick's relatives ($E_{3.7}$).
18. $E_{3.1} = E_{3.1.1} \wedge E_{3.1.2} \wedge E_{3.1.3} \wedge E_{3.1.4} \wedge E_{3.1.5}$: The gun and ammunition related attributes found by the police ($E_{3.1}$) include a clip holder ($E_{3.1.1}$), a sawed off barrel ($E_{3.1.2}$), bullets made from modified blanks ($E_{3.1.3}$) and drill bits needed to modify blanks into bullets ($E_{3.1.4}$) found in in Patrick's apartment and a bullet made from a modified blank found at Patrick's parents' house ($E_{3.1.5}$).
19. $E_{3.5} = E_{3.5.1} \wedge E_{3.5.2} \wedge E_{3.5.3} \wedge E_{3.5.4}$: The evidence regarding Nelline's crashed car ($E_{3.5}$) includes the wreck of Nelline's car found by the A-27 highway ($E_{3.5.1}$), Nelline's blood in her crashed car ($E_{3.5.2}$), Patrick's odour print in Nelline's crashed car ($E_{3.5.3}$) and the testimony of A-27 motorists who saw a man matching Patrick's description getting out of Nelline's wrecked car ($E_{3.5.4}$).