Rijksuniversiteit Groningen

Department of mathematics and natural sciences

Bachelor thesis

Observing Lorentz Invariance Violation in β-decay

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July 12, 2016

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1 Introduction

Symmetries are a essential part in our understanding of nature. Many elementary theories are defined by their symmetries and their corresponding quantum numbers. One such symmetry is the Lorentz Invariance principle which states that experimental results are independent of the orientation or the boost velocity of the laboratory through space. The Lorentz Invariance can be violated if one can determine a preferred direction in space so then the notion that all reference frames observe the same laws of physics is lifted. Which is saying something as for one this is the basis on which Einstein constructed his special and general relativity theories. Of course Einstein's theories have stood the test of time for over a hundred years so if there is a Lorentz Invariance Violation(LIV) it should be small as nobody has observed it so far. In fact one has already started looking very specifically for a LIV measurable and every more precise measurement has provided stronger constraints on its magnitude.

In this thesis we want to propose a experiment to try to measure one particular Lorentz violating measurable namely a tensor contribution(χ) to the metric tensor in the weak-interaction. One of the ways this χ would manifest itself is that it will modify the angular distribution of a β decay. As the contribution is probably very small a very high number of decays will be necessary and as β particles are stopped easily in a large source it will be difficult to simply measure the angular distribution of β-decays. A solution could be to measure γ-rays following β-decay. The idea is the following: If a polarized particle decays in a way that affects the spin state distribution of the final nucleus then inversely the decay distribution from an unpolarized source can leave the final nucleus in a polarized state. Polarization means a particular substate distribution defined relative to a certain direction. If there are no other directions playing a role but LIV than the distribution can be only along a "preferred direction" defined by LIV.

The purpose of this thesis is to make an assessment on how such a setup would work and if it is suitable to determine components of χ more precisely than currently known bounds.

2 The alphabet soup

The decay rate distribution for β-emission from polarized nuclei as described by the standard model can be written as [2]

$$
\begin{aligned}
&\omega(\langle\vec{J}\rangle|E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu \\
&= \frac{F(\pm Z,E_e)}{(2\pi)^5} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \\
&\quad \times \bar{\xi}\left\{1 + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + b\frac{m_e}{E_e} + c\left[\frac{1}{3}\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} - \frac{(\vec{p}_e\cdot\vec{j})(\vec{p}_\nu\cdot\vec{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle(\vec{J}\cdot\vec{j})^2\rangle}{J(2J-1)}\right]\right. \\
&\quad \left. + \frac{\langle\vec{J}\rangle}{J}\cdot\left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu} + D\frac{\vec{p}_e\times\vec{p}_\nu}{E_e E_\nu}\right]\right\} . \qquad (1)
\end{aligned}
$$

where $E_{e(\nu)}$, $\Omega_{e(\nu)}$, and $p_{e(\nu)}$ denote the total $\beta(\nu)$ energy, direction, and momentum of the $\beta(\nu)$-particle respectively, E_0 is the energy available to the electron and the neutrino, $\langle\vec{J}\rangle$ is the expectation value of the spin of the initial nuclear state, and $\vec{j}$ is the unit vector in this direction; $F(\pm Z, E_e)$ is the Fermi function which modifies the phase space of the electron due to the Coulomb field of the nucleus. Also affecting the phase space is the Fierz interference term, factorized with the coefficient b. This term is zero in the SM. We defined $\bar{\xi} \equiv G_F^2 V_{ud}^2/2\xi$, where ξ gives the strength of the interaction. The remaining terms describe the β-correlation coefficients: the β-neutrino asymmetry a, the β-asymmetry A, the neutrino asymmetry B, and the triple-correlation coefficient D. Also $D = 0$ in the SM (at tree level and neglecting final state interactions). The c coefficient vanishes for non-oriented nuclei and for nuclei with $J = 1/2$, such as the neutron. In this work we also take $F = 1$. The expressions for the correlation coefficients in pure Fermi and GT transitions, assuming the SM applies, are in Table 1.

coefficient	F	GT		
polarized parent parent spin J	$J \to J$	$J \to J-1$	$J \to J$	$J \to J+1$
unpolarized parent daughter spin J	$J \to J$	$J-1 \to J$	$J \to J$	$J+1 \to J$
a	1	$-\frac{1}{3}$		
c	0	-1	$\frac{2J-1}{J+1}$	$-\frac{J(2J-1)}{(J+1)(2J+3)}$
A	0	∓ 1	$\mp\frac{1}{J+1}$	$\pm\frac{J}{J+1}$
B	0	± 1	$\pm\frac{1}{J+1}$	$\mp\frac{J}{J+1}$

Table 1: Values of the relevant correlation coefficients in Fermi and Gamow-Teller transitions. Upper(lower) sign refers to electron(positron)-emission. The second row indicates how to select the parameters when going from an unpolarized parent to a m state distribution in the daughter nucleus.

Eq. (1) then reduces to

$$
\begin{aligned}
&\omega(\langle m\rangle\vec{j}|E_e,\Omega_e,\Omega_\nu)dE_e d\Omega_e d\Omega_\nu \\
&= \frac{1}{(2\pi)^5} p_e E_e (E_0 - E_e)^2 dE_e d\Omega_e d\Omega_\nu \\
&\quad \times \bar{\xi}\left\{1 + a\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} + c\left[\frac{1}{3}\frac{\vec{p}_e\cdot\vec{p}_\nu}{E_e E_\nu} - \frac{(\vec{p}_e\cdot\vec{j})(\vec{p}_\nu\cdot\vec{j})}{E_e E_\nu}\right]\left[\frac{J(J+1)-3\langle m^2\rangle}{J(2J-1)}\right]\right. \\
&\quad \left. + \frac{\langle m\rangle}{J}\vec{j}\cdot\left[A\frac{\vec{p}_e}{E_e} + B\frac{\vec{p}_\nu}{E_\nu}\right]\right\} , \qquad (2)
\end{aligned}
$$

where we also make explicit that one measures the distribution of magnetic substates m on a certain axis. Here it is the axis along which the polarization is made. One is free to choose any polarization axis. For our discussion, we are interested in the Treiman approach where the recoil direction is chosen [7]. This means one can integrate Eq. (2) taking care that $\vec{p}_e$ and $\vec{p}_\nu$ are defined along that axis. It is impossible to know the recoil direction a priori, it has thus no practical meaning. Integrating Eq. (2) for random polarization one gets a value that only depends on E_0/m_e, where m_e is the electron mass. The value is given as X in Treiman [7]. Integrating the same equation with the polarization axis for the term $A\frac{\langle m\rangle}{J}\frac{\vec{j}\cdot\vec{p}_e}{E_e}$ in the recoil direction one gets $-\langle m\rangle/JAX_1 cos\theta_R$ [7] where θ_R is the angle between the recoiling nucleus and the direction of the parent's polarization and X_1 is again a specific integral of E_0/m_e (given incorrectly in Treiman! the first term depending on E_0/m_e squared should be to the fifth power), the B term yields the same integral. The c term yields $\frac{2}{3}c\frac{3\langle m^2\rangle - J(J+1)}{J(2J-1)}X_2(\frac{1}{3} - cos^2\theta_R)$. One can now invert the reasoning and ask what happens if one had started with an unpolarized nucleus. This leads to an expression for the m state distribution

$$
W(m) = \frac{1}{2J+1}\left[1 - (A+B)\frac{X_1}{X}\frac{m}{J} + \frac{2}{3}c\frac{X_2}{X}\frac{3m^2 - J(J+1)}{J(2J-1)}\right] \qquad (3)
$$

With this distribution we can calculate the γ-ray pattern relative to the recoiling nucleus. The derivations above allow us to infer to some extent what happens when one does not measure the recoil direction but there is Lorentz Violation. The equivalent idea going from Eq. (2) to Eq. (3) will play a role for our study of LIV.

3 The case of Lorentz Invariance Violation

The approach in chapter 2 now needs to be adapted for a preferred absolute direction. Treiman's approach requires knowing the recoil direction which we don't measure. We need to change our framework to the $\chi^{\mu\nu}$ framework which allows us to express a decay distribution. The decay rate distribution for pure Gamow-Teller transitions in this framework is given by [1]

$$\begin{aligned} d\Gamma_{GT} = d\Gamma_0 \Bigg\{ &\left[1 - \frac{2}{3}\chi_r^{00} + \frac{2}{3}(\chi_r^{l0} + \tilde{\chi}_i^l)\frac{p^l}{E_e}\right] \\ &\mp \Lambda^{(1)} \left[(1 - \chi_r^{00})\frac{p \cdot \hat{I}}{E_e} + \tilde{\chi}_i^l \hat{I}^l + \chi_r^{lk}\frac{p^l \hat{I}^k}{E_e} - \chi_i^{l0}\frac{(p \times \hat{I})^l}{E_e}\right] \\ &+\Lambda^{(2)} \left[-\chi_r^{00} + (\chi_r^{l0} + \hat{\chi}_i^l)\frac{p^l}{E_e} + 3\chi_r^{kl}\hat{I}^k\hat{I}^l - 3\chi_r^{l0}\hat{I}^l\frac{p \cdot \hat{I}}{E_e} - 3\chi_i^{ml}\hat{I}^m\frac{(p \times \hat{I})^l}{E_e}\right]\Bigg\} . \end{aligned} \tag{4}$$

with

$$d\Gamma^0 = \frac{1}{8\pi^4}|p|E_e(E_e - E_0)^2 dE_e d\Omega_e F(E_e, \pm Z)\xi \tag{5}$$

As we will not measure the electron(positron)'s momentum we should integrate this distribution over $\vec{p}_e$ also. As every term that contains p is odd in p, assuming $\chi^{\mu\nu}$ to be momentum independent, we are left with the simple expression

$$\Gamma_{LIV} = X\left\{1 \mp \Lambda^{(1)}\tilde{\chi}_i^l \hat{I}^l + \Lambda^{(2)} 3\chi_r^{kl}\hat{I}^k\hat{I}^l\right\} \tag{6}$$

$$= X\left\{1 + A\frac{\langle m\rangle}{J}\tilde{\chi}_i^l \hat{I}^l - c\frac{3\langle m^2\rangle - J(J+1)}{J(2J-1)}\chi_r^{kl}\hat{I}^k\hat{I}^l\right\} \tag{7}$$

Here we neglected the contribution from χ_r^{00} and assumed $F = 1$. Again X is some energy integral expressed in E_0/m_e. To keep the discussion simple we will assume the tensor contribution χ_r^{kl} to be diagonal. Now to look at the m-state distribution following the decay of a non-polarized source analogous to Treiman we need to change the A and c values as per Table 1. We also need to choose the axis onto which to project the m-state distribution. As the preferred axes for the alignment and the polarization are not the same we will construct two separate ones each with it's χ vector as the projection axis. It should be safe to separate the two as one will never be able to measure polarization as well as alignment. For the polarization this will be

$$W(m) = \frac{1}{2J+1}\left[1 + A\frac{m}{J}|\vec{\tilde{\chi}}_i|\right] . \tag{8}$$

And for the alignment

$$W(m) = \frac{1}{2J+1}\left[1 - c\frac{3m^2 - J(J+1)}{J(2J-1)}|\vec{\chi}_r|\right] \tag{9}$$

With $\vec{\chi}_r$ being the vector representation of χ_r^{ll} (no summation convention) meaning the real diagonal of the spacial part of $\chi^{\mu\nu}$ and $\vec{\tilde{\chi}}_i$ the vector representation of $\tilde{\chi}_i^l = \epsilon^{lmk}\chi_i^{mk}$. From these we can construct values for a Polarization quantity and a Alignment quantity

$$P \equiv \sum \frac{m}{J}W(m) = \frac{J+1}{3J}A|\vec{\tilde{\chi}}_i| \tag{10}$$

defined in the $\vec{\tilde{\chi}}_i$ direction. Likewise we define a alignment Φ

$$\Phi \equiv \frac{3\sum\left[m^2 W(m)\right] - J(J+1)}{J(2J-1)} = -\frac{2J^2+5J+3}{5J(2J-1)} c|\vec{\chi}_r| \qquad (11)$$

which definition has the same expression as $f_2 N_2$ in [8]. Not coincidentally Φ shows up as a parameter in eqs. (1),(2),(9) and hidden in $\Lambda^{(2)}$ in eq. (8). We defined it as such so that it will be zero for randomly polarized nuclei and one for stretched. However it is not generally -1 for a purely anti-aligned ensemble, anti-aligned meaning having an m-state perpendicular to the alignment-axis so having a spin-projection of zero. But for functionality we'll stick to this definition. If one can measure either the polarization or the alignment with high enough precision one can use these relations to determine (bounds on) either $\tilde{\chi}_i$ or χ_r respectively.

4 Candidate Isotopes

4.1 Decay selection

To study Lorenz Violation by measuring the γ-ray anisotropy one needs to find a radioactive source with the right properties. First of all it needs to either β-decay or electron capture first and then γ-decay. As we are only interested in Gamow-Teller decay [12], as only decays carrying away spin will influence the m-state distribution of the daughter nucleus, we look for a β-decay/EC with $\Delta J = 1$, $\Delta\pi = 0$ and an $\log(ft_{1/2})$ value in the vicinity of 5.5[4]. Also to be able to measure a alignment we need the daughter nucleus to have $J \geq 1$ To be able to do measurements for a longer period of time without changing the source too often the half-life should be somewhere between 1 and a 100 years. Also as discussed in section 4.2 the lifetime of the intermediate states shouldn't be too long but as this constraint is typically very weak and we are also imposing the condition that for detection reasons the γ-rays should have an energy of 100keV or more no nucleus is ruled out by this (see section 4.2). We would like all the decays sensitive to LIV decays to occur with close to 100% intensity but candidates with lower intensities should not be ruled out as the wanted information is still there. However all candidates with suitable decay schemes with an intensity lower than 99% have intensities around 25% if not even lower. Measurements with these kind of nuclei proof to be troublesome as separating the desired signal from the decays that see LIV becomes a very convoluted exercise making high-precision measurement difficult. This is because any anisotropic process causes a background which weakens the asymmetry like in eq. (15). With all these conditions one finds that there are actually just two isotopes suitable for these kind of measurements namely ^{22}Na and ^{60}Co[5]. Their relevant properties are listed in Table 2. Note that the ^{22}Na its β-decay is followed by a γ-decay directly to a ground state whereas the ^{60}Co its β-decay is followed by two successive γ-decays. Also as ^{22}Na does β^+ decay a source of moderate size will also show 511keV photons as the emitted positrons find a electron to annihilate with inside the source or scintillator. These annihilation events won't see the LIV as it is a pure electro-magnetic process. This will weaken the measured asymmetry if the annihilation photons can not be separated from the decay γ-rays reducing the sensitivity.

Isotope	$T_{1/2}$[y]	Decay mode (End-point energy)	Log($ft_{1/2}$)	Transition levels Jπ (β)	$T_{1/2}$ intermediate state[ps]	Radiation energy[keV](mode)
$^{22}_{11}$Na	2,60	β^+ (546 keV)	7.5	$3^+ \rightarrow 2^+$	3.6(5)[3]	1274.5(E2)
$^{60}_{27}$Co	5,27	β^- (317 keV)	7.4	$5^+ \rightarrow 4^+$	3.3 & 0.9	1173(E2+(M3))&1332(E2)

Table 2: Candidate isotopes, unless specified data taken from [5]

4.2 Time evolution of the intermediate state

Because in this experiment the polarization of the intermediate daughter nucleus determines the angular distribution of the following γ-decay the polarization should not change within the lifetime of the intermediate state. A possible problem is the precession of the daughter nucleus in the earth's magnetic field. The relevant quantity here is the Larmor frequency given by

$$\Delta E = 2\mu B = \hbar\omega_L \tag{12}$$

Where μ is the magnetic momentum of the nucleus [3], B the magnatic field strength which is taken to be the earth's field, $\hbar$ is Planck's constant [6] and ω_L the Larmor angular frequency. The latter is the frequency with which a magnetic dipole will precess in a magnetic field [4]. The inverse of this ω_L gives an indication how long a nucleus can precess without having the nucleus' spin direction change appreciably.

$$\frac{1}{\omega_L} = \frac{\hbar}{2\mu B} \tag{13}$$

This can be qualitatively tested by imposing the condition

$$\omega_L T_{1/2} = \frac{2\mu B T_{1/2}}{\hbar} \ll 1 \tag{14}$$

Where $T_{1/2}$ is the intermediate state's half-life time [5]. Values are given in Table 3. These satisfy condition (14) by several orders of magnitude therefore one can conclude that the orientation of the nucleus' spins and thus the nucleus' polarization and alignment do not change appreciably in the intermediate states.

	$^{22}_{10}$Ne	$^{60}_{28}$Ni(2^+)	$^{60}_{28}$Ni(4^+)
μ	0.65	0.32	3*
$T_{1/2}$	3.6ps	0.9ps	3.3ps
B		60μT	
$\omega_L T_{1/2}$	1.3e-8	1.7e-9	5.7e-8

Table 3: List of μ, τ, B and corresponding $\omega_L T_{1/2}$, *$3\mu_N$ is taken as an estimated upper bound as the nuclear magnetic moment of $^{60}_{28}$Ni(4^+) is not known as of yet.

4.3 Estimate of required decay rate

To calculate how much source material we will actually need we look at the present precision for the $\chi^{\mu\nu}$ obtained from forbidden β-decay which is determined as 10^{-6} for χ_r^{kl} and 10^{-8} for $\tilde{\chi}_i^k$. As we can not measure a polarization with our method [12] we will focus on χ_r^{kl}. From [12] we see that the alignment of the nuclei can be determined by measuring the asymmetry in the γ radiation as

$$A_\gamma = \frac{N_1 - N_2}{N_1 + N_2} \approx -\frac{3}{28}|\vec{\chi}_r| \tag{15}$$

Where A_γ is the asymmetry value for the γ-rays and N_1 and N_2 are the count rates of two detectors with an angle of 90° with respect to each other. So the error in the asymmetry should be smaller than 10^{-7} to achieve a bound of order -6, also we would like to improve the precision of χ_r^{kl} by at least one order of magnitude. Therefore we will try to find a count rate for which $\delta A_\gamma < 10^{-8}$. To include all detectors we will use a different asymmetry, see section 6

$$A_i = \frac{N_i - \overline{N}}{\overline{N}} \tag{16}$$

Assuming the error in A_i is dominated by the Poisson distribution in N_i and $N_1 \approx N_2 \approx N_3 \approx \overline{N}$ it can be shown that the error is

$$\delta A_i = \sqrt{\frac{2}{N}} \tag{17}$$

Where N is the total number of decays observed by all detectors. As we have to measure over a finite solid angle we won't measure this maximum asymmetry, we can conclude from [12] that integrated over a solid angle $\Omega = \pi/2$ the asymmetry will be a factor 2 weaker. So to make a significant measurement we would need a N of at least $4 \cdot 10^{16}$. As we can measure two γ-rays from every β decay of ^{60}Co we only need $2 \cdot 10^{16}$ decays of ^{60}Co. Depending on the chosen type of gamma detector a secondary error must be added. We neglect these in this first approximation as to be able to calculate some source properties which will determine what kind of gamma detectors are suitable in this setup. For a assessment of the error including secondary contributions see section 5.3.

4.4 Source temperature

It is important that the source doesn't melt as β particles are stopped inside it. As to estimate the temperature the source will reach we'll approximate it as being a black body. For this we can use the Stefan-Boltzmann law with a background temperature T_c

$$j = \sigma(T^4 - T_c^4). \tag{18}$$

Where j is the thermal energy radiated by the black body per unit time per unit area, σ the Stefan-Boltzmann constant and T the nuclei's temperature. Putting the power radiated by the nucleus $P_r = Aj$ equal to the amount of energy absorbed from the β particles P_β yields a equilibrium temperature

$$T_{eq} = \sqrt[4]{\frac{P_\beta}{A\sigma} + 293^4}. \tag{19}$$

The area A, the mass and the radius of the source can all be related to each other by the geometry of a sphere and it's density ρ.

$$m = \rho V \qquad V = 4/3\pi r^3 \qquad A = 4\pi r^2, \qquad r = \sqrt[3]{\frac{m}{4/3\pi\rho}} \tag{20}$$

To determine P_β one has to know how much energy a β will lose escaping the nucleus which is a function of the source's radius and the particle's energy which has some statistical distribution[4]. Firstly as means of an upper limit we'll assume every β losses all it's energy to the nucleus. In this case the P_β is quite simply $P_\beta = \lambda N \bar{E}_\beta$ where λ is the activity of the nuclei, N the number of nuclei and $\bar{E}_\beta$ the dose of the decay. In Table 4 various values are calculated, where T_{eq}^* is the achieved temperature and m_{max}^* maximum mass for which the nucleus won't melt both assuming total absorption. r is the nuclei's radius and CSDA the "Continuous Slowing Down Approximation" which is an indication to from which point it is safe to assume total absorption. For m_{max}^* to be a good approximation to the limit of how big the nucleus can be before it starts to melt m_{max}^* should be a few orders magnitude larger than the mass at $r = CSDA$.

Source	Dose	Melting point	T_{eq}^* at $r = CSDA$	m_{max}^*	mass at $r = CSDA$
^{22}Na	194.7 keV/decay	371K	426 K	$\sim 100\mu g$	$800\mu g$
^{60}Co	95.7 keV/decay	1768K	301 K	$\sim$1 tonne	$0.6\mu g$

Table 4: Various values for the equilibrium temperature for different sizes/masses of the nucleus.

Table 4 shows that this m_{max}^* is a fine approximation for the m_{max} of ^{60}Co meaning we can use as much of it as we want as far as state stability goes, as 1 tonne of ^{60}Co is undesirable for various other reasons. It also shows that m_{max}^* is a very poor approximation for m_{max} of ^{22}Na meaning a more complex approximation should be made if we'd want to use this, from this table one can safely assume however that sources of more than a few mg will definitely melt if not cooled.

Figure 1 can be used to see that m_{max}^* is a good approximation for ^{60}Co. It shows the "Continuous Slowing Down Approximation" range as a function of β energy. It is good to note that the β-decay has some end-point energy roughly 3 times higher than it's dose which will be less slowed down (will lose less energy per unit distance) from the beginning so their CSDA is disproportionally longer. Meaning that even when r exceeds the CSDA range there will be β-particles which escape the source besides from the fact there will always escape some amount of β-particles from the outer shell.

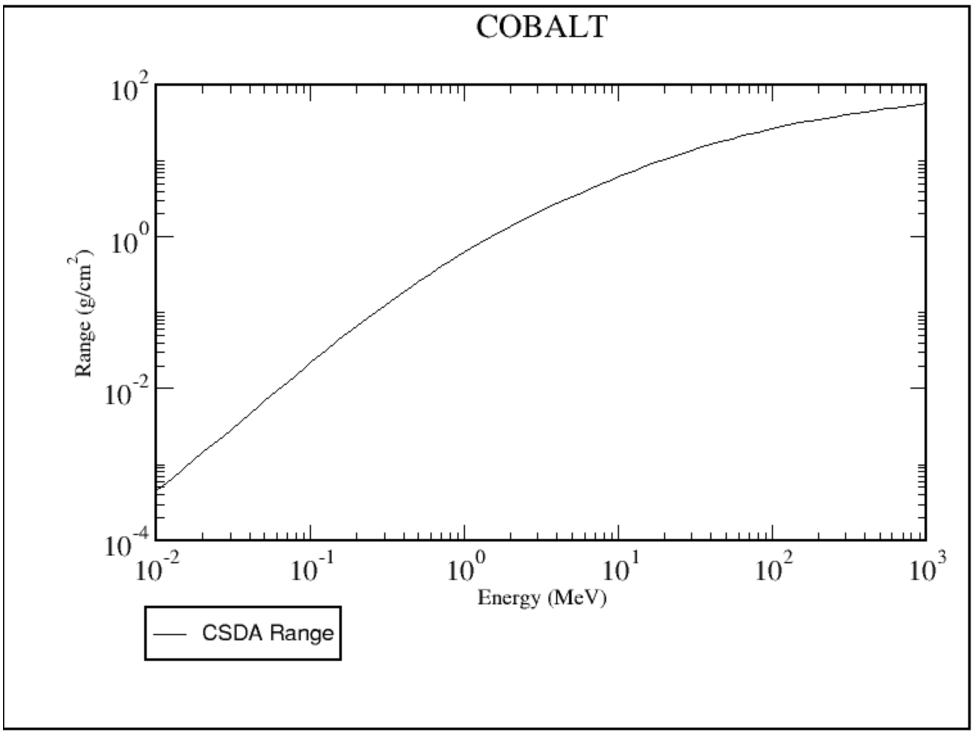

Figure 1: "Continious Slowing Down Approximation" range for β particles in ^{60}Co[9].

4.5 Source mass and activity

The amount of source material needed is determined by the number of counts necessary to determine $\chi^{\mu\nu}$ with sufficient accuracy. As not all decays will be measured we'll need more decays than the necessary number of counts. Assuming N decays are necessary the amount of source nuclei can be determined by

$$N_d = N_0(1 - {}^1\!/_2{}^{t/t_{1/2}}). \tag{21}$$

Where N_d is the number of decays, N_0 the number of starting nuclei, t the measurement time and $t_{1/2}$ the nuclei's half-life time. So putting $N_d = N$ we can determine N_0 as

$$N_0 = \frac{N}{1 - {}^1\!/_2{}^{\frac{t}{t_{1/2}}}}. \tag{22}$$

From this N_0 the needed mass of source material and it's activity at the beginning of the measurements A can be calculated.

$$m = N_0 m_a \qquad A = N_0 \frac{ln2}{t_{1/2}}[Bq] \hat{=} N_0 \frac{ln2}{t_{1/2}} / 3.7 \cdot 10^{10} [Ci] \tag{23}$$

where m_a is the nuclei's atomic mass and $t_{1/2}$ is in seconds.

Table 5 has a few calculated values, assuming one needs $4 \cdot 10^{16}$ γ-rays to make a significant measurement. From this we can see that this is quite a plausible setup as little source material is needed and these sources are by some regulations not even classified as a 'high-activity source' [15]. Also comparing these values to Table 4 we see that both sources are state stable.

	m	A
^{22}Na	6.2 μg	39 mCi
^{60}Co	16 μg	18 mCi

Table 5: Masses and β activities for ^{22}Na and ^{60}Co if $4 \cdot 10^{16}$ decays are necessary for ^{22}Na and $2 \cdot 10^{16}$ for ^{60}Co over the course of one year.

5 Experimental setup

5.1 Scintillators

To measure the asymmetry from the alignment as in [12] via the polarization or alignment respectively a spherical source is surrounded by six pyramid shaped scintillators. We choose to use ZnSe(Te) as scintillation crystal as it has a high luminosity, is radiation hard and has a long emission peak wavelength [11]. This long emission peak wavelength makes for high quantum efficiency for the silicon photodiodes we want to use and the high luminosity decreases the relative error. To determine how large the scintillators needs to be we look at the mass-attenuation coefficient. We approximate photons being stopped in the scintillator by

$$I(x) = I_0 e^{\frac{-x}{\mu}} \tag{24}$$

Where I_0 is the intensity of the incident photons, $I(x)$ the surviving intensity of incident (high energy) photons and μ the crystal's attenuation constant. From [13] we calculate $\mu = 3.59cm$. Assuming stopping 90% of the high energy photons is sufficient the scintillators should have a thickness of

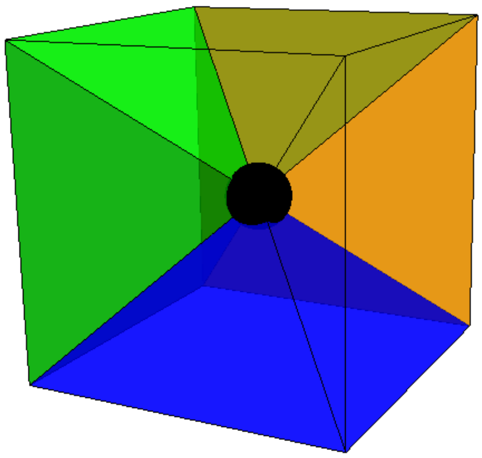

Figure 2: Detector setup. Only the three scintillators in the back are coloured, though slightly transparent.

$$d = -3.59ln(0.1) = 8.3cm \tag{25}$$

So we can conclude that this setup fits easily on a tabletop as with the source being very small this experiment can be smaller than a 20cm sided cube.

5.2 Photodiodes

The scintillation crystal converts each source gamma to $6.2 \cdot 10^4$ low energy gammas on average. The amount of low energy photons is proportional to the amount of incident photons so it can be used to redefine the asymmetry used to determine the alignment. These low energy photons will in turn be converted into a current by silicon photodiodes, the current is proportional to the number of incident photons N_s by [14]

$$I = \frac{\eta e}{h\nu} \cdot P = \eta N_s e \tag{26}$$

Where I is the current supplied by the photodiode, η the diode's quantum efficiency, e the electron charge, h Planck's constant, ν the photon energy and P the wattage of photons applied to the diode's surface. For ^{60}Co we get $I = 2.6\mu A$. In figure 3 a schematic is given how to embed these diodes into a electronic circuit to determine a asymmetry. In practice the circuit should be much more complicated but the exact setup for this circuit goes beyond the scope of this thesis.

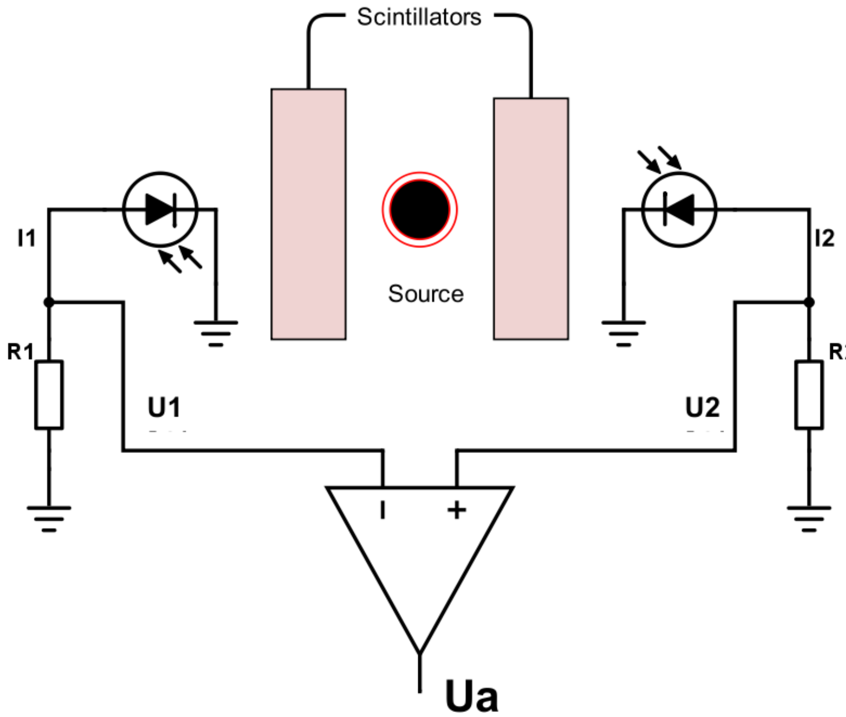

Figure 3: Schematic how to measure the difference between the currents provided by two diodes.

5.3 Error in A_γ

As mentioned in section 4.3 we want to determine the magnitude of $\vec{\chi}_r$ by means of the asymmetry of the γ-rays as

$$A_\gamma = \frac{N_1 - N_2}{N_1 + N_2} \tag{27}$$

Though we don't actually measure the decay rate so a better description of what we will actually measure is

$$A_I = \frac{I_1 - I_2}{I_1 + I_2} = \frac{f(N_1) - f(N_2)}{f(N_1) + f(N_2)} \tag{28}$$

Where I_1 and I_2 are the currents obtained from detector 1 and 2 and the notation $f(N_1)$ is used to denote that I_1 is a function of N_1. The error in A_I

is largely the same as the Poisson error in A_γ discussed in section 4.3 but has a added error originating from the uncertainty in the photon conversion of the scintillator crystals. It can be shown that δA_I can be reduced to

$$\delta A_I = \sqrt{\frac{2}{N}\left(1 + \frac{\sigma^2}{f_0^2}\right)} \tag{29}$$

Where N is the total number of decays and σ and f_0 are the error and expected value of f as a function of N. We'll assume the error in the amount of scintillation photons created per incident γ-ray is negligible compared to its magnitude (at least $\frac{\sigma^2}{f_0^2} \ll 1$), the precision in the current measurement has a similar requirement but I'll refrain from making an assessment on this as the electronics of the setup are beyond the scope of this thesis.

So then in the end we are left with the exact same error we already assumed in section 4.3

$$\delta A_I = \sqrt{\frac{2}{N}} \tag{30}$$

Where N is the total number of decays in the source.

6 Measurements

The magnitude of χ_r is best measured as a asymmetry in the various detectors as it varies over time with the sidereal rotation of the earth [12]. To extract the magnitude and direction of χ_r we'll consider χ as being invariant in the sun centred reference frame and use spherical coordinates with the $\hat{Z}$-axis along the earth's rotation axis, see Figure 4. Here the x-detector points east-west, the y-detector top-bottom and the z-detector north-south. The ζ denotes the colatitude of the laboratory, this is just for demonstration as the phenomena should be translationally invariant one might as well mount the setup under an angle w.r.t. the earth's surface. From [12] we find $W(\theta)$ for alignment as

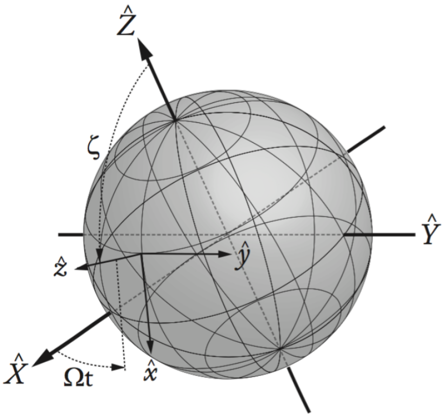

Figure 4: Transformation between the laboratory x, y, z frame and the sun-centred X, Y, Z frame.

$$W(\theta) = \frac{1}{14}(14 + |\vec{\chi}_r|(1 - 3\cos^2\theta) \quad (31)$$

where θ is the angle between the emitted γ-particle and χ_r. Combining this with the sun-centred framework we find the asymmetry for each detector as a function of the sidereal rotation ω

$$\begin{aligned} A_i(\omega) = \frac{N_i(\omega) - \overline{N}}{\overline{N}} = |\vec{\chi}_r| \Bigg\{ &\frac{1}{14}\left((\sin(\zeta)\sin(\theta_n)\cos(\phi_n - \omega) + \cos(\zeta)\cos(\theta_n))^2 - 2(\sin(\zeta)\cos(\theta_n)\right. \\ &\left. - \cos(\zeta)\sin(\theta_n)\cos(\phi_n - \omega))^2 + \sin^2(\theta_n)\sin^2(\phi_n - \omega)\right), \\ &\frac{1}{112}(-3\cos(2(\theta_n - \omega + \phi_n)) - 3\cos(2(\theta_n + \omega - \phi_n)) + 6\cos(2\theta_n) + 6\cos(2\phi_n - 2\omega) + 2), \\ &\frac{1}{14}\left(-2(\sin(\zeta)\sin(\theta_n)\cos(\phi_n - \omega) + \cos(\zeta)\cos(\theta_n))^2 + (\sin(\zeta)\cos(\theta_n)\right. \\ &\left. - \cos(\zeta)\sin(\theta_n)\cos(\phi_n - \omega))^2 + \sin^2(\theta_n)\sin^2(\phi_n - \omega)\right)\Bigg\} \end{aligned} \quad (32)$$

Where $N_i(\omega)$ is the count rate of one specific detector, $\overline{N}$ is the average count rate of all 3 detectors ($\frac{1}{3}\sum N_i$), θ_n is the angle between χ_r and the earth's rotation axis, ϕ_n is the azimuthal angle between χ_r and the $\hat{X}$-axis.

A predicted measurement for a typical case, typical case being χ_r not aligning with the earth's rotation axis and ζ not being near 0 or π, is depicted in Figure 5.

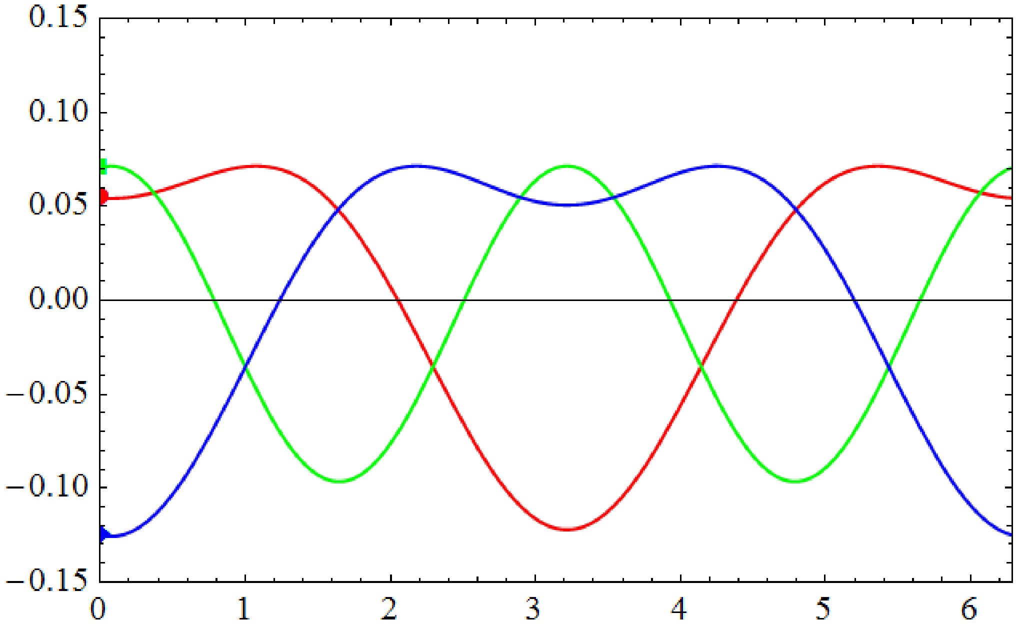

Figure 5: Predicted measurement for the asymmetry A_i. Blue is the top-bottom detector, green east-west and red the north-east one. The y-axis is the measured asymmetry normalised to $|\vec{\chi}_r| = 1$ and the x-axis is the sidereal time ω. For this example $\zeta = 0.8$, $\phi_n = 0$ and $\theta_n = \frac{\pi}{3}$

As we have three equations and three unknowns we can determine the three unknowns $|\vec{\chi}_r|$, θ_n and ϕ_n from looking at the time-dependence of these measurements thus determining the magnitude and direction of $\vec{\chi}_r$. If we are very unlucky however the $\vec{\chi}_r$ might align to the earth's rotation axis so $\theta_n = 0$, then one can see from eq. (32) that any dependence on the sidereal time drops out. In this case if one was to measure these asymmetries one would find three flat lines. Though these three lines probably deviate from zero it would be hard to argue as the effect is extremely small that this signal is anything but a systematic error.

7 Conclusion

We studied the possibility of measuring a Lorentz Invariance Violation in the angular distribution of γ-decay following β-decay in ^{20}Na and ^{60}Co. From these studies we can conclude that this type of setup can give a stronger bound on the real diagonal elements of $X^{\mu\nu}$ by a order magnitude using a moderately active source of only 16mCi measuring over the course of a year. Which is of course promising if one would be able to use a strong source. However this setup requires some second study before being turned into reality as the electronics required must be extremely precise as it needs some kind of processor capable to integrate the difference of two currents in μA range differing maximally by a factor 10^{-6} . Alternatively one is able to bypass the need for such electronics as the γ-rays could also be measured by a (very) large array of single count scintillators though this would be a much larger project then the one proposed in this thesis.

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