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faculty of science
and engineering

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The influence of the Magnus effect in tennis

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Student: S.I.O. Scholte

First supervisor: Prof.dr. ir. R.W.C.P. Verstappen

Second assessor: Prof. dr. H. Waalkens

The influence of the Magnus effect in tennis

Ingmar Scholte

University of Groningen

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Abstract

The Magnus force is a force that acts upon a rotating ball when the ball moves through a fluid. The spinning leads to a deflection in the direction perpendicular to the spinning axis and the velocity vector. Tennis players strike balls with a lot of top- or backspin. In this thesis, we look to the behavior of a ball played with topspin. Different models are created to explain and calculate the Magnus force of a rotating ball. Finally, we use one of the models in an experiment to find the influence that the Magnus force has on the trajectory of a tennis shot.

1 Introduction

In this report we present the results of the research to the deflection of sports balls that are struck with rotation. This deflection is a common phenomenon in sports and seen in for example football, tennis, golf and baseball. Due to the rotation of the ball, the spinning ball experiences a force acting perpendicular to the spinning axis and the velocity vector. This force is called the Magnus Force. During our study, we will investigate the Magnus force using several types of flows. The influence of the Magnus force in the trajectory of a ball is modeled in Matlab. Finally we will discuss our results.

At first we discuss the effects of an ideal flow moving parallel past an smooth sphere. In this ideal flow, there is no viscosity and therefore no boundary layer. The velocity of the flow will first increase and later decrease as it moves along the surface. Using Bernoulli's principle, we are able to show that the pressure of the flow exerting on the sphere also changes due to the changing velocity. When letting the sphere rotate, the velocity of the streamlines at the surface is not the same at the upper and lower part of the sphere. In the case of topspin, the streamlines at the upper side move slower than the lower side, which leads to a pressure difference. This results in a force in the direction of the point with the lower pressure.

From an ideal flow, we modify the problem by considering a viscous laminar flow. The viscosity leads to a boundary layer between the flow and the surface of the sphere. The boundary layer is the reason that the flow separates from the surface at some point. The advancing pressure of the growing boundary layer causes the flow to separate and the separation points are symmetrically at an upper and lower point of the sphere. Adding rotation to the sphere, the velocity at the upper and lower part will change. Therefore it separates earlier at one side of the sphere as its reached its adverse pressure gradient more quickly at that side of the surface. The reverse holds for the other side. The wake behind the sphere now has an deflection upwards or downwards as the separation points are not vertical symmetric anymore. By Newton's 3rd law of motion, the deflected wake has to be compensated by the Magnus force in the opposite direction.

A more mathematical way to approach the problem is to look at potential flows [1]. The theory behind using potential flows is to decouple the calculation of the velocity and pressure. We consider the potential flow to be an ideal flow in a two dimensional velocity field. Being an ideal flow leads to some preconditions on the velocity field. The equations of the velocity are quite similar to the Cauchy-Riemann equations. The curves of the flow and the potential of the flow are connected with those Cauchy-Riemann equations, that play a prominent role in the complex function theory. It is conventional to transform the velocity in a complex velocity. Looking to some examples of typical flows in the complex plane, we can combine them to reconstruct our problem of a parallel flow moving past a rotating sphere. We will use the pressure to calculate the force acting upon the sphere using the first theorem of Blasius, finding that the rotating sphere only experiences a lift force as a result of the rotation.

Before we actually get to a model to plot the trajectory of a rotating ball, we look at last to an engineering approach to the problem[2, 3]. In this approach we consider all forces acting on the rotating ball during his flight. Starting from a point $(x, y) = (0, y_0)$, parametric equations for the coordinates will be introduced. An ordinary differential equation (ODE) is then solved to calculate the velocity of the rotating ball. The drag and Magnus force play an important role in this calculation and they are dependent of their respective drag and lift coefficient. The coefficients are a function of the ratio of the rotating velocity and the velocity of the ball. With a wind tunnel experiment, the dependence of this ratio is measured and a function for the drag and lift coefficients is derived.

Using the model described above, we start an experiment for ourself. We model the flight of the rotating tennis ball to evaluate the dependence of the rotating speed with respect to the angle a tennis player can produce on the tennis court. For our velocity ODE we will use Euler's method to numerically solve it. With the velocity calculated at each iteration, the coordinates of the ball are calculated. The first part of the experiment consists of checking whether the output of our model in Matlab is in line with the literature. After analyzing those output values, we will compare the rotation speed with ball trajectory of several shots. There we find that a ball drops earlier to the ground when hit with more topspin. This gives the possibility for players to hit the ball with a greater angle to get the opponent hit the ball further outside the court.We will change the initial position of the players and look to the difference in angle between shots hit with different rotation speed.

Contents

2 Basic Fluid Mechanics principles

To understand how a rotating tennis ball moving through the air behaves, we first simplify the problem, considering an ideal, laminar flow moving past a sphere which doesn't move nor rotate. The behavior of this simple flow will be studied by introducing some basic concepts of fluid dynamics. The main theorem used in this chapter is Bernoulli's Principle. We use that theorem to explain why the rotation of the ball in an ideal fluid leads to a lift force. The same theorem is also used when we look in the end of this chapter to a viscous fluid. In this more realistic flow, we will again use Bernoulli's principle to see that a lift force occurs when a ball rotates through the air.

2.1 Ideal flow

We start just by understanding the principles of fluid dynamics, considering an ideal flow passing an smooth sphere. The ball is in rest and doesn't rotate. In aerodynamics, this situation is treated similarly to a situation that a sphere is moving through an ideal gas.

So we take an ideal flow, which moves parallel past the sphere. As it is an inviscid flow, it sticks to the surface of the sphere when passing by. This is shown in Figure 1. The streamline of the fluid that hits the front spot (Point A) of the sphere, moves along the surface until it is at the other end (Point D). From there it moves again parallel along the mainstream of the fluid.

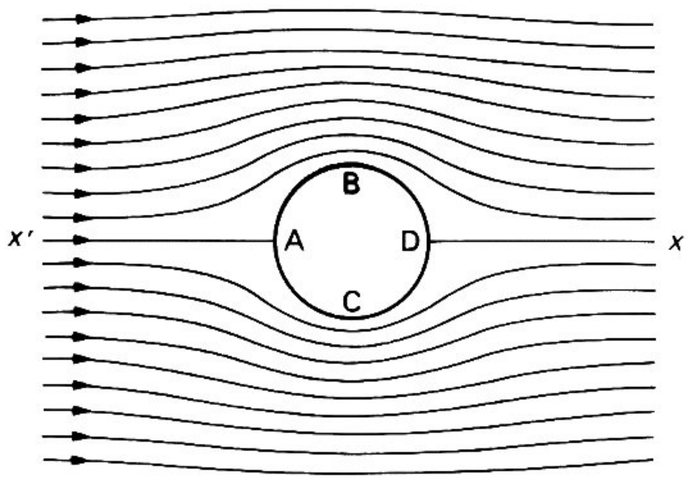

Figure 1: An inviscid flow moving past a sphere

When the fluid makes impact with the object, streamlines of the fluid near the surface get squashed, as they have a more narrow space to move through alongside the sphere. The fluid velocity must therefore increase until it reaches it most narrow space (point B or C). From that point on, those streamlines slow down in speed until it reaches point D, where it has the same velocity as the mainstream again.

2.2 Bernoulli's principle

The increasing and decreasing movement speed of a flow along the surface leads to a change of pressure of the flow on the surface of the sphere. This change of pressure comes forth of a theorem known as Bernoulli's principle. This principle is a commonly used theorem in fluid dynamics. The Mathematician Daniel Bernoulli stated that

$$p + \frac{1}{2}\rho U^2 + \rho g h = \text{constant}, \tag{1}$$

with p the static pressure, ρ the density of the air, U the velocity, h the elevation and g the gravitational acceleration. Bernoulli's principle is derived from the conservation law of energy. So from the principle, we see that if the velocity of the fluid increases, the pressure at the surface has to decrease. In the situation showed in Figure 1, the net pressure at the front and the back of the object is zero. For the top and bottom of the object holds that the velocity of the fluid at both sides are equal (and therefore the pressure is equal). In this situation we therefore have no drag and lift force acting on the sphere.

2.3 Rotating sphere in an ideal flow

We now change the situation by making the object rotate with topspin. So in the case of Figure (1) where the flow goes from left to right, the object is spinning counter-clockwise. We now have created a situation where the speed of the fluid at the top is not equal to the bottom. This is because the rotating speed of the object goes in the opposite direction of the fluid velocity at the top and we therefore have an decreased fluid velocity. At the bottom, the velocity of the rotation and the fluid go in the same direction and we have an increased fluid velocity. Now Bernoulli's Principle tells us that the pressure at the bottom is higher than at the top. The difference in pressure results in a force in the direction of the surface with the lower pressure. This lift force is called the Magnus force.

2.4 Viscous flows

In the previous paragraphs we created an ideal, but very unlikely situation. When we considering air instead of an ideal fluid, we create a whole new and more complex model. Air is for example a viscous flow. The viscosity leads to an additional drag force as the air close to the sphere sticks to surface of the sphere.

In fluid mechanics, the relative motion between the surface of an solid object and a fluid is zero because of this viscosity. The flow velocity being zero at a solid surface is called the no-slip condition. The fluid velocity increases as it moves further away from the surface until it is as far from the surface that it has the same velocity as the main stream. This layer of slower moving flow near the surface of an object is called the boundary layer. The distance between the surface and the air flow velocity reaching the velocity of the mainstream is the thickness of this boundary layer.

As the flow moves past the surface of the sphere, the boundary layer gets thicker. The pressure increases what eventually lead to a separation of the boundary layer from the surface. This is called the adverse pressure gradient. Due to the increasing pressure, the flow velocity decreases by Bernoulli's principle. When separated, the velocity becomes zero and the pressure stays constant from the separation point and on. It has effect upon the pressure at the front and back of the sphere, whereas the pressure over the front is larger. This pressure difference leads to a drag force.

2.5 Rotating sphere in a viscous flow

As seen in the previous paragraph, a sphere moving through the air makes it a more complex problem as when it is moving through an inviscid flow. The boundary layer separates as it is moving past the surface and this creates a wake behind the sphere. In a parallel flow, this wake is directly behind the sphere and the drag force is only operating in the horizontal plane.

Now we let the sphere rotate with topspin and the flow moves again from left to right. Due to the rotation, the boundary layer of the flow rotates alongside the sphere. As already mentioned before, the velocity goes in the opposite direction of the rotation speed on the upper side of the sphere. Therefore the pressure rises faster and the adverse pressure gradient occurs earlier, which means that the flow separates earlier. The reverse happens on the lower side of the sphere. As the flow goes along with the spin of the sphere, the velocity of the boundary layer goes more slowly to zero and it separates later on the lower side of the sphere. This creates a wake which is deflected upwards. This upward deflected wake has to be compensated by a force downward by Newton's third Law of Motion. Therefore there must be downward force acting on the ball, which is again the Magnus force (figure 2).

We now have a better view why a rotating ball is deflected when it moves through the air. In the next chapters we will try to find an expression for the Magnus force exerted from the rotation to use it in finding its influence on the trajectory of a tennis ball.

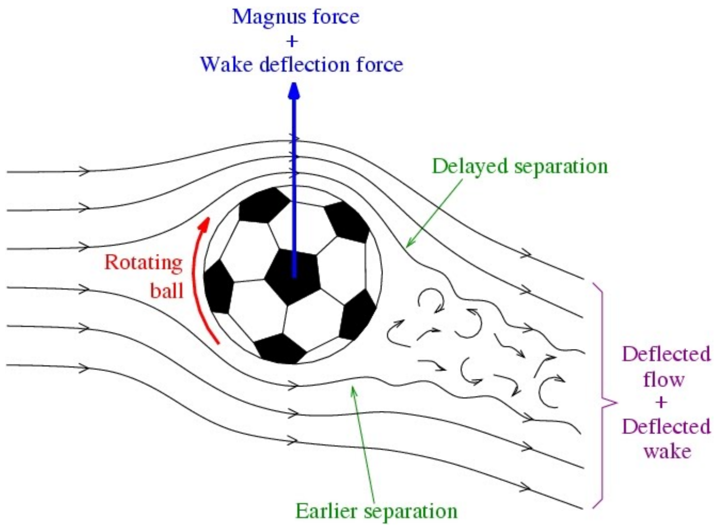

Figure 2: Deflection of the wake behind a football struck with backspin

3 Potential flow

In the previous chapter, an explanation for the Magnus force is given just by using basic principles in fluid dynamics. Now it is our task to find an expression for the lift force of a rotating ball. In order to do this, we will reconstruct the streamlines of the flow over a rotating sphere using potential flows. The main source for this chapter is the dutch dictation about fluid dynamics [1]. First we introduce the properties of a potential flow to find functions for the streamlines of the flow. Then by looking at some examples of typical flows, a reconstruction of the streamlines moving past a rotating sphere is made. We will restrict us to the case of an inviscid fluid in the two-dimensional plane in this chapter.

3.1 Properties and conditions

First we need to define what a potential flow is and what its characterizations are. A potential flow describes the velocity field depending on the velocity potential. This is

$$v = \nabla \Phi, \tag{2}$$

with v the velocity vector and Φ the velocity potential. For a potential flow, the following must hold:

$$\begin{aligned} \nabla \cdot v &= 0 \\ \nabla \times v &= 0. \end{aligned} \tag{3}$$

The first condition means that it is a mass conservation flow. The second condition implies that is an irrotational flow. From $\nabla \cdot v = 0$ in equation

(3) it follows that

$$\nabla \cdot \nabla \Phi \equiv \Delta \Phi \equiv \Phi_{xx} + \Phi_{yy} = 0$$

must hold. If we look at a two-dimensional flow, we pick $(u, v, 0)$ as our three-dimensional velocity vector and $(0, 0, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})$ for the rotation vector. From here, we can write the following equations:

$$\begin{aligned} \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \\ \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0 \end{aligned} \tag{4}$$

and we see that u and v are both potential flows.

From $\nabla \cdot v = 0$, there exists a vector field A such that $v = \nabla \times A$. In other words, v is the rotation of A and we can choose $A = (0, 0, \Psi)$. Ψ is called the flow function and has some important properties. From $v = \nabla \times A = (\Psi_y, -\Psi_x, 0)$, we find $\Psi_y = u$ and $\Psi_x = -v$. Observe that $\Psi_{xx} + \Psi_{yy} = 0$ and therefore Ψ is also a potential flow.

The curves of $\Psi =$ constant are the streamlines of the flow. Computing this, we get

$$\Psi_x + \Psi_y \frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\Psi_x}{\Psi_y} = \frac{v}{u}.$$

So the slope of those curves are equal to the direction of velocity vector.

3.2 Complex velocity

We observe that eq.(4) have the same structure as the Cauchy-Riemann equations. The equations are used for complex functions and by these equations, Φ and Ψ are related to each other via

$$\Phi_x = \Psi_y = u, \; \Phi_y = -\Psi_x = v.$$

By complex function theory, those equations only hold when these functions satisfy the analyticity property, i.e. the complex velocity potential function $\chi = \Phi + i\Psi$ is an analytic function of the complex function $z = x + iy$. Analyticity of χ means that the the derivative of $\chi(z)$ is unambiguous, so that

$$\chi'(z) = \frac{\mathrm{d}\chi}{\mathrm{d}z} = \lim_{z^* \to z} \frac{\chi(z^*) - \chi(z)}{z^* - z}$$

is independent of the way that z^* approaches z. To compute the above limit for a point in the complex plane, it therefore doesn't matter from which direction we take this limit. Computing $\chi'(z)$, knowing it is an analytic function, we can take the derivative in the x-direction and we get

$$\chi'(z) = \frac{\partial \chi}{\partial x} = \frac{\partial \Phi}{\partial x} + i\frac{\partial \Psi}{\partial x} = u - iv \equiv w(z).$$

We could have taken the derivative in the y-direction to get the same outcome:

$$\chi'(z) = \frac{\partial \chi}{\partial (iy)} = \frac{1}{i}\frac{\partial \Phi}{\partial y} + \frac{\partial \Psi}{\partial y} = u - iv.$$

We call $w(z)$ the complex velocity.

3.3 Some examples of different flows

Now we have introduced the complex velocity, we can turn our velocity vector in a complex velocity. First we take a look at parallel streamlines (figure 3). For a parallel flow along the x-axis. We take $v = (U, 0)$ with

$$U = u - iv = w(z).$$

For our complex potential function we then have $\chi = Uz$.

The streamlines are given by Ψ = constant. From $\chi = \Phi + i\Psi$ we get that $\Psi = \Im(\chi) = Uy$ and the streamlines are thus given by the equation y = constant. Looking at parallel streamlines that are moving under an angle, we get

$$U\cos\alpha - iU\sin\alpha = Ue^{-i\alpha} = u - iv$$

and it follows that $\chi = Uze^{i\alpha}$. The stream function is given by $\Psi = Uye^{i\alpha}$ and the streamlines follow from the equation y = constant. Another interesting flow is given by $\chi = z^2 = x^2 - y^2 + 2ixy$. The velocity is $\chi'(z) = 2z = 2x + 2iy$ and the streamlines are given by $\Psi = 2xy = constant$. This flow is called a stagnation flow.

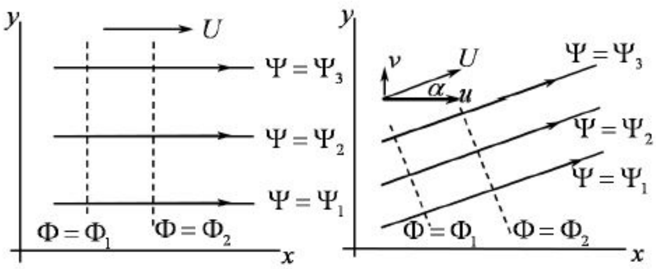

Figure 3: Two examples of a parallel flow.

3.4 Blasius' theorem

In this section we're going to find and expression for the force that a flow is exerting on a stationary body in the complex plane. The expression for this force follows from the first theorem of Blasius. Consider X as the force acting in the x-direction and Y the force acting in the y-direction. For those forces we have the expressions

$$X = \int_S p\,\mathrm{d}y, \quad Y = \int_S p\,\mathrm{d}x,$$

with S the closed contour of the stationary body. Combining the two forces and we get

$$X - iY = -i\int_S p\,(dx - idy).$$

with $p = p_{st} - \frac{1}{2}\rho(u+iv)(u-iv)$ following from Bernoulli's principle. Here p_{st}, standing for stagnation pressure, doesn't contribute to the integral. Continuing our computation gives

$$\begin{aligned}
X - iY &= \frac{1}{2}i\rho\int_S (u+iv)(u-iv)(dx-idy) \\
&= \frac{1}{2}i\rho\int_S (u(1+i\frac{v}{u})(u-iv)dx(1-i\frac{dy}{dx}) \\
&= \frac{1}{2}i\rho\int_S (u(1+i\frac{dy}{dx})(u-iv)dx(1-i\frac{v}{u}) \\
&= \frac{1}{2}i\rho\int_S (u-iv)^2(dx+idy) \\
&= \frac{1}{2}i\rho\int_S [w(z)]^2 dz \\
&= \frac{1}{2}i\rho\int_S (\frac{\mathrm{d}\chi}{\mathrm{d}z})^2\mathrm{d}z.
\end{aligned}$$

The third equality in this computation is true because S is a streamline. On a streamline it holds that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v}{u}$.

3.5 Flow with circulation

We use the first theorem of Blasius to find the force from a circular flow acting on a two-dimensional sphere. In the previous chapter, we saw that this is a similar situation as a flow moving past a rotating sphere. But in order to find the expression for the Magnus force, we have to compute the flow around a cylindrical object with circulation.

First we look at the flow moving past a circular object which doesn't rotate. To compute the streamlines of this flow, we use a special case of flow, namely a flow with a dipole in the origin and combine this with a simple parallel flow. The complex potential function of a dipole in the origin is given by

$$\chi = \begin{cases} \frac{-M}{2\pi z}, & M \text{ is imaginary} \\ \frac{-M}{2\pi}\frac{x-iy}{x^2+y^2}, & M \text{ is real} \end{cases}$$

This function is obtained by considering a well in the origin of the complex plane and a neighboring well at distance dz. Both wells have strength $-Q$, $Q > 0$, in the origin. The strength indicates the amount of fluid that flows through the well. Letting dz approaching zero, we get the complex potential function, setting $Q\mathrm{d}z = M$. The quantity M is called the complex dipole moment.

Now combining a parallel flow with a dipole in the origin with a complex dipole moment $M = -2\pi U a^2$, we get for our complex potential function

$$\chi(z) = U(z + \frac{a^2}{z}).$$

The complex velocity is then given by

$$w(z) = \frac{\mathrm{d}\chi}{\mathrm{d}z} = U(1 - \frac{a^2}{z^2})$$

and leads to the following formula for the streamlines:

$$\Psi = U(y - \frac{a^2 y}{x^2 + y^2}) = \text{constant}.$$

When we look at the streamline for $\Psi = 0$, we see that it consists of the line $y = 0$ and the circle $x^2 + y^2 = a^2$, with a the radius of the circle.

Now we want to add circulation to the problem to recreate the streamlines along a rotating two-dimensional object. To do this, we add a whirl around the origin to our previous complex potential. The complex potential of a whirl in the origin is of the form

$$\chi(z) = \frac{i\Gamma}{2\pi}\ln z.$$

The strength of this whirl is given by Γ. So when we add this function to the one we derived for a flow around a cylindrical object, we get

$$\chi(z) = U(z + \frac{a^2}{z}) + \frac{i\Gamma}{2\pi}\ln z.$$

Using the first theorem of Blasius to find an expression for the force acting upon the cylinder. Computing it gives

$$X - iY = \frac{1}{2}i\rho\int_{r=a}(\frac{\mathrm{d}\chi}{\mathrm{d}z})^2\mathrm{d}z = \frac{1}{2}i\rho\int_{r=a}[U(1 + \frac{a^2}{z^2}) + \frac{i\Gamma}{2\pi z}]^2\mathrm{d}z$$

Now we use the residue theorem for integrals to calculate this integral. The theorem says that the integral is equal to $2\pi i$ times the residue of the integrand in the origin, as $z = 0$ is a singular point. We therefore get $2\pi i$ times the coefficient of $1/z$. Eventually we find

$$X - iY = 2\pi i \cdot \frac{1}{2} i\rho \cdot \frac{i\Gamma U}{\pi} = -i\rho U\Gamma$$

and see that it results solely in a vertical lift force with magnitude $\rho U\Gamma$.

We can conclude that a rotating ball in the air only experiences an extra lift force due to the rotation. So by using potential flows in two dimensions, an expression for the Magnus Force is obtained. In the next chapter we are going to look to the complete picture of an in-flight rotating tennis ball and considering all forces to find the equations for the trajectory of a tennis shot.

4 Tennis ball trajectory

We continue with looking to a more engineering way to approach the problem. The goal is to eventually compute the coordinates of the ball during its whole flight by considering all the forces acting on it. Letting the drag force and the Magnus force attain a certain value dependent on their respective coefficients, it is possible to create a good coordinate system for the trajectory. We will show the ODE of the velocity that needs to be solved for this model and we will discuss how to calculate the drag and lift coefficients.

4.1 Coordinate system

The trajectory of a flying rotating ball is affected by three forces. We have a gravitational force, a drag force and a Magnus Force acting upon the ball. To calculate the flight trajectory, we start with the equilibrium of forces into the normal,

$$mv^2/R = mg\cos(\tau) + M, \tag{5}$$

where m is the mass of the ball, g the gravitational constant, v the velocity of the ball, R the radius of the curve of the ball trajectory, M the Magnus force and τ the angle between v and the horizontal plane. Take s to be the length of the trajectory of the ball. Then $R = -ds/d\tau$ by looking at figure 4 and using that $\sin(\mathrm{d}\tau) \approx \mathrm{d}\tau$.

Recall that

$$dt = ds/v, \quad dx = ds\cos(\tau), \quad dy = ds\sin(\tau).$$

After substitution of R in equation (5) and doing some integration, we find

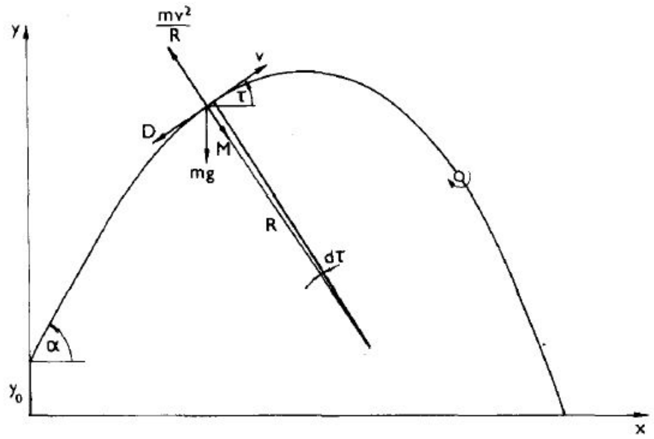

Figure 4: The ballistic trajectory of a ball hit at a point $(0, y_0)$ [2]

the parametric equation for our x-coordinate:

$$\begin{aligned}
& mv^2/R = mg\cos(\tau) + M \\
\Rightarrow \quad & -\frac{\mathrm{d}\tau v^2}{g\mathrm{d}s} = \cos(\tau) + M^* \\
\Rightarrow \quad & -\frac{\mathrm{d}\tau v^2 \cos(\tau)}{g\mathrm{d}x} = \cos(\tau) + M^* \\
\Rightarrow \quad & \mathrm{d}x = -\frac{1}{g}\frac{\mathrm{d}\tau v^2\cos(\tau)}{\cos(\tau) + M^*} \\
\Rightarrow \quad & x = -\frac{1}{g}\int_\alpha^\tau \frac{v^2\cos(\tau)}{\cos(\tau) + M^*}\mathrm{d}\tau.
\end{aligned}$$

Doing the same for our y-coordinate and we have our parametric equations of our coordinate system:

$$\begin{aligned}
x &= -1/g\int_\alpha^\tau \frac{v^2\cos(\tau)}{\cos(\tau) + M^*}\mathrm{d}\tau, \\
y &= y_0 - 1/g\int_\alpha^\tau \frac{v^2\sin(\tau)}{\cos(\tau) + M^*}\mathrm{d}\tau.
\end{aligned} \tag{6}$$

In the above equations, $M^* = M/mg$ is the dimensionless Magnus force and α the angle between the initial direction of the ball with the horizontal axis. The coordinate system is depending on the variable τ. Therefore, the velocity has to be expressed in terms of τ too. Consider the equilibrium of forces in the horizontal space of the trajectory:

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -D - mg\sin(\tau). \tag{7}$$

Expressing dt in terms of R and using equation (5) to substitute R, the above equation turns into

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = \frac{\sin(\tau) + D^*}{\cos(\tau) + M^*} v \tag{8}$$

where $D^* = D/mg$. The ODE (8) is the ODE that needs to be solved to calculate the coordinates of our parametrized equations. In the engineering model, we define D^* and M^* as

$$D^* = C_D(\pi d^2/8mg)\rho v^2,$$
$$M^* = C_L(\pi d^2/8mg)\rho v^2.$$

The drag and lift force of the ball are proportional to the velocity squared. The drag and lift coefficients C_D and C_L will be treated as a function of the ratio of the rotating velocity and the velocity of the ball. The coefficients are also dependent of the Reynolds number, i.e. $C_L = f(w/v, Re)$, where w is the radius of the ball times the angular speed ω. The same holds for the drag coefficient. We now want to make a good approximation of the coefficients for our model.

4.2 Drag and Lift coefficients

The drag and lift coefficients are dependent by the ratio of w and v and the Reynolds number. The Reynolds number is an important dimensionless quantity in fluid dynamics as it can predict the pattern of the flow for example. To get to the right function for the coefficients, an experiment has to be built. A rotating ball gets ejected in a wind tunnel and the values of the coefficients and the different ratios of $\frac{w}{v}$ are plotted against each other. Performing the experiment at different Reynolds numbers, it was found that the influence of the Reynolds number can be neglected. So the regression line of those found values of the coefficients and $\frac{w}{v}$ gives an approximation of both the drag coefficient and the lift coefficient:

$$C_D = 0.508 + \frac{1}{[22,503 + \frac{4,196}{(\frac{w}{v})^{2,5}}]^{0,4}},$$
$$C_L = \frac{1}{2,022 + \frac{0,981}{\frac{w}{v}}}.$$

With the coefficients, we can make a model to calculate the complete trajectory of a tennis ball. We now can proceed to an experiment in finding Our goal in the next chapter is to write the program and look to the different angles that can be played on a tennis court with varying angular velocities.

5 A mathematical experiment

In the previous section we have seen a coordinate system for the x- and y-position of a tennis ball. We will use it for our experiment that will calculate the difference in angles a player can make on a tennis court striking with different rates of topspin. First we will visualize how we want to set up this experiment. The program that we use is MATLAB. Normally the problem is a three dimensional problem, but we want to bring it back to 2D with only a length and height component. To calculate the coordinates, we have to solve the ordinary differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}\tau} = \frac{\sin(\tau) + D^*}{\cos(\tau) + M^*}v$$

and will be solved by Euler's method. We will check if the values for the velocity dependent quantities are similar to what we have seen in the previous chapter, before beginning the experiment. Initial values of some variables will be changed to get a good look to what the difference in spin rate will make.

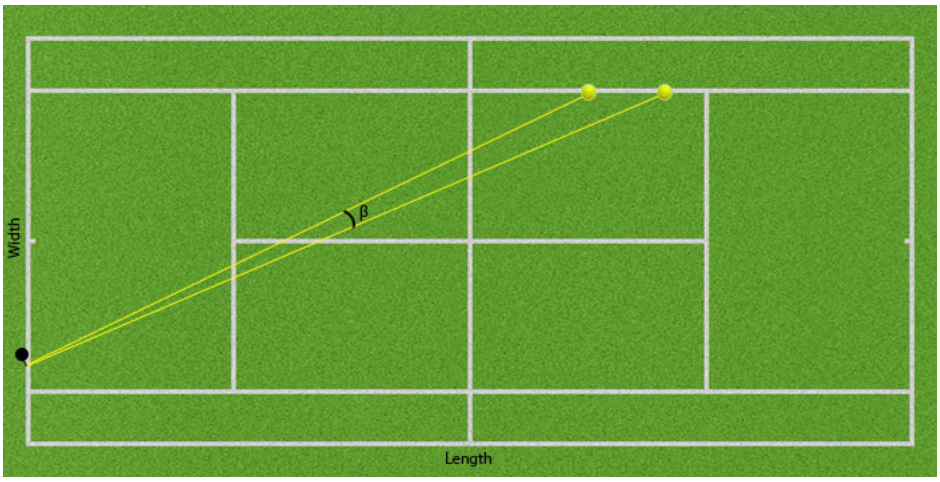

Figure 5: A visualization of our problem looked from above. Our goal is to find the angle β by comparing different spin rates.

5.1 Mathematical model

5.1.1 Setup

Our model consists of solving the ordinary differential equation and from there calculating the coordinates as shown in the previous chapter. Finding the angle between the vertical side line of a tennis court and the trajectory of the ball would be a 3D problem. However, we will consider it as a 2D problem and bring back the results to 3D. we will do this by only considering the height and length of the trajectory of the tennis ball. Before the experiment, we fix the position of where the ball is struck. As we calculate the length of the trajectory and fixing the horizontal displacement of the ball, we can derive the depth of where the ball lands on the court with Pythagorean Law.

In order to solve the ODE, we need to know the values for the Magnus force and the drag force. Those are dependent of their respective coefficients, So at each iteration, we need to calculate those values. The variable τ has to decrease at each iteration and the loop has to stop when the y-coordinate reaches 0, i.e. the ball hits the ground. We assume that the ball lands on the side line of the tennis court.

5.1.2 Euler's Method

In solving the ODE, we use Euler method. Euler's method for this ODE is given by

$$v_{i+1} = v_i + \Delta\tau f(\tau_i, v_i)$$

where we have set $v_i = v(i\Delta\tau)$. At every iteration, the new velocity v_{i+1} is calculated by its current value v_i and the angle step times the ODE function evaluated at that velocity and angle. So

$$f(\tau, v) = \frac{\mathrm{d}v}{\mathrm{d}\tau} = \frac{\sin(\tau) + D^*}{\cos(\tau) + M^*}v.$$

The values for D^* and M^* will be evaluated again at every iteration to calculate the coordinates of the ball.

5.1.3 Variables

To perform the experiment, some quantities has to be fixed. At first we take the mass of a tennis ball to be 0.058kg and diameter 0.067m. Furthermore we know that the density of air is 1.2041kg/m^3 and the gravitational constant 9.81m/s^2.

Then we get to the quantities that will vary during the flight of the ball and the initial values which will be important for the outcome of the experiment. We need an initial velocity, initial angle and an initial position for how the ball is struck. Obviously the spin rate of the ball is also one of the inputs and we want to change that variable to find differences in angle. For the position, it is important to consider a 3D perspective and translate it to a 2D model. So we need an additional variable to choose if the ball is hit in the middle of the court or for example behind one of the side lines. This has also impact on where the ball flies over the net.

For the output, we want the depth on where the ball lands in the court to eventually calculate the difference in angle. We need to plot the trajectory of the ball to look whether the ball lands in the net and also have a good look to the trajectory of the ball.

5.2 Performing the experiment

We want to verify our Matlab code to see if it's calculating the velocity, the forces and the coordinates correctly. We use [2] to compare the different plots. We also need to determine a good $\Delta\tau$ for our Euler's method to solve the ODE. As we've setup and verified our experiment, we need to make choices in certain values to be able to compare different situations. When looking at men's tennis today, the player that has the highest spin rate in his forehand is Rafael Nadal. It is measured that the Spaniard could reach a topspin rate of 5000 rpm in his hitting. The sport is really developing over the last decade, as in the past people as Pete Sampras and Andre Agassi hit their shots with an average around 1800 rpm.

5.2.1 Verification of the program

The code in Appendix A needs to be checked for errors by looking at the plots of the different variables. First we want to know if the program solves the ODE smoothly and if it has the same shape as in [4, 3]. We vary our step size of $\Delta\tau$ and plot it against v and y at some x-coordinate. Looking at figure 7, the velocity change linearly as $\Delta\tau$ decreases. This is consistent to the ODE we are solving. For our experiment, we will choose $\Delta\tau$ to be 0.005. Looking at the figure (6) for the velocity, one can see that it decreases faster at the beginning than at the end. That make perfect sense as at first the gravitational force works against the velocity vector and after it reaches its highest point, the force is in the same direction as the velocity.

Being satisfied with the calculation of the velocity, we would like to see that the figures of the coefficients plotted against the ratio of w/v look similar to those of [2]. So we let w/v vary from 0 till 1.5 with a small step size. Plotting this against the coefficients and we see in figure 8 the outcome, which looks exact like the plots of [2]. Observe from those lines that both can be approximated by two linear functions when splitting the domain. At low ratios of w/v ($w/v < 0.4$ approximately), the values of the coefficient increase faster than at the second domain ($w/v > 0.4$).

5.2.2 Results

Now we're confident that our program solves the ODE in the right way and our outputs are justified we can start our experiment. We want to make good choices for our input values. We will look at four different rotation speeds: $n = 1800$ (average speed 15 years ago), $n = 2700$ (average speed nowadays of players like Roger Federer), $n = 3200$ (average speed of Rafael Nadal) and $n = 4900$ (highest rotation speed of Nadal) [5]. Furthermore we will consider that the shots are played from the baseline and take three positions at the baseline where the ball is struck. The positions are at the

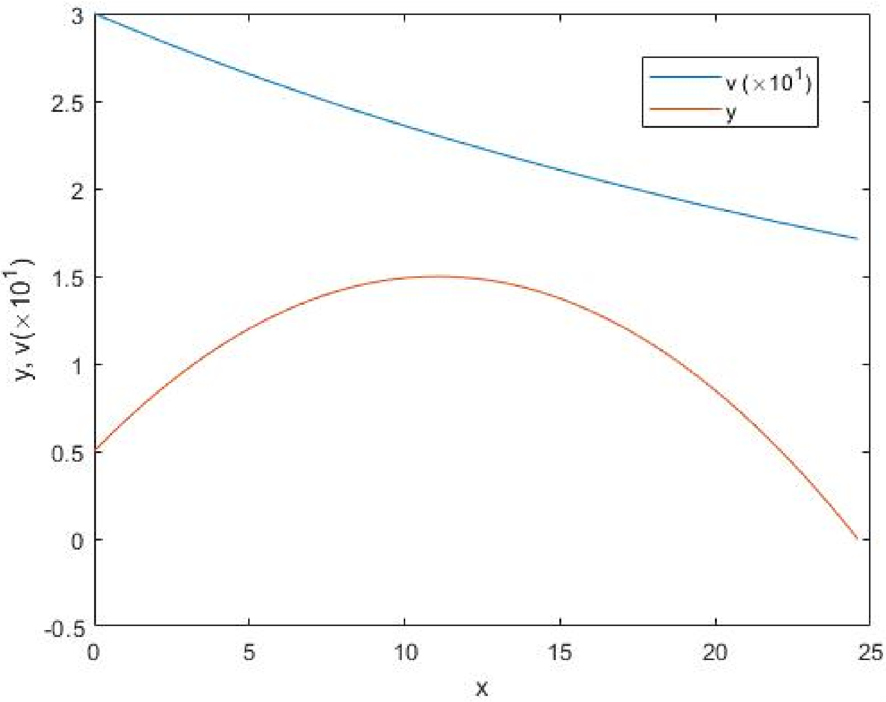

Figure 6: The trajectory of the ball with the velocity at every x.

middle of the court (Position A), at the singles sideline of the court (Position B) and at about 1 meter outside the court (Position C). We will look to the depth of the ball landing on the court between those different rotation speed and with it also calculating the corresponding β from comparing the angle between the shot of $n = 4900$ with the other three rotation speeds. We will use the initial angle α as changing parameter to make sure the ball gets over the net in the first place.

Figure 9 and table 1 show the trajectories of the tennis balls and its accompanying depth hit from the middle of the court. With the depth, one can calculate the angle β between the shot hit with the most topspin and a shot hit with lower topspin using the cosines law:

$$\beta = arccos(\frac{x_{4900}^2 + x_{other}^2 - (depth_{other} - depth_{4900})^2}{2 \cdot x_{4900} \cdot x_{other}}).$$

In the tables we convert the values of β from radians to degrees.

The same is done for the initial position of the player at the sideline of the court and the position of a player standing about 1 meter outside of the court. We also made a difference in initial height of the latter as it would be reasonable that a person who hits a ball outside of the court has to run further and is later in hitting the ball.

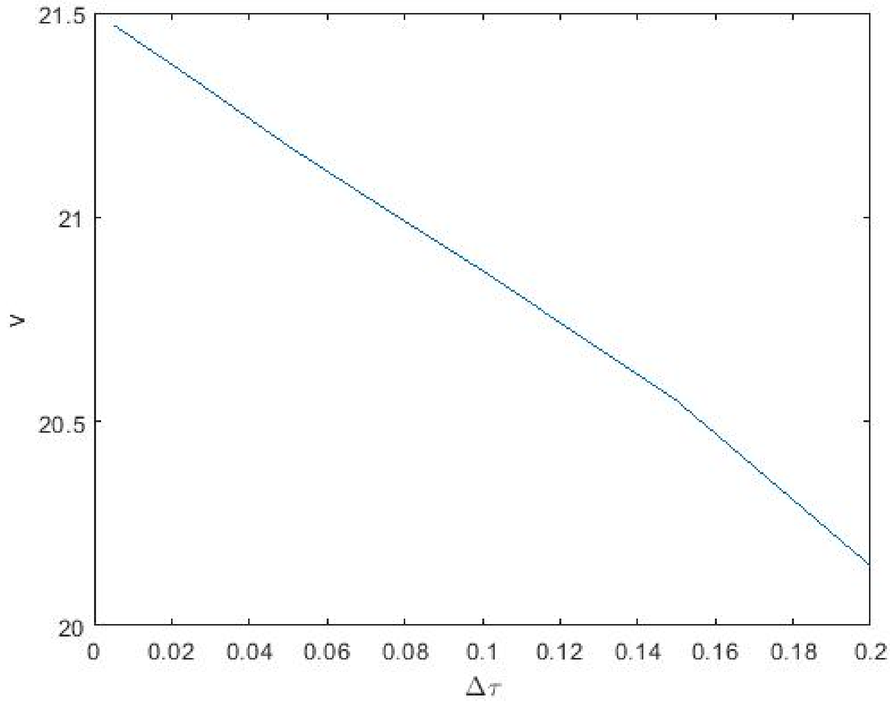

Figure 7: The values of v measured at a point x (We chose it to be at the net) against different $\Delta\tau$

n	**1800**	**2700**	**3200**	**4900**
alpha	6.5	7.1	7.4	8
depth	7.0298	6.4287	6.2540	5.8414
x	19.3747	18.7883	18.6184	18.2163
β	1.4	0.4	0.3	0

Table 1: The depth and difference in angle of the ball played with different values of n at the middle of the court (width = 4.115) with $v = 30$ and $y(1) = 0.8$

n	**1800**	**2700**	**3200**	**4900**
alpha	6.8	7.4	7.7	8.5
depth	5.9938	5.6243	5.4128	5.1858
x	19.6821	19.3470	19.1558	18.9511
β	1.0	0.6	0.3	0

Table 2: The depth and difference in angle of the ball played with different values of n struck at the side line (width = 8.23) with $v = 30$ and $y(1) = 0.8$

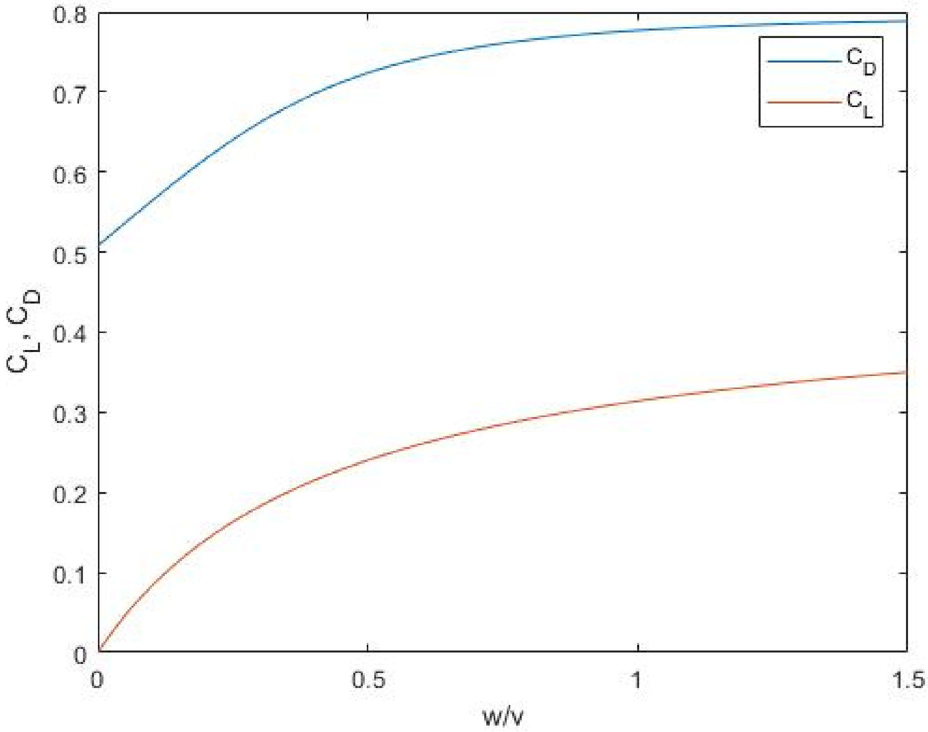

Figure 8: C_D and C_L measured in terms of the ratio w/v.

n	**1800**	**2700**	**3200**	**4900**
alpha	7.4	8.0	8.3	9.0
depth	5.8961	5.0402	4.7734	4.4403
x	20.9085	20.1857	19.9625	19.6843
β	2.2	0.9	0.5	0

Table 3: The depth and difference in angle of the ball played with different values of n struck at the side line (width = 11) with $v = 30$ and $y(1) = 0.8$

n	**1800**	**2700**	**3200**	**4900**
alpha	8.6	9.2	9.5	10.2
depth	6.6173	5.7072	5.4183	5.0330
x	21.5252	20.7482	20.5038	20.1797
β	2.3	1.0	0.6	0

Table 4: The depth and difference in angle of the ball played with different values of n struck at the side line (width = 11) with $v = 30$ and $y(1) = 0.5$

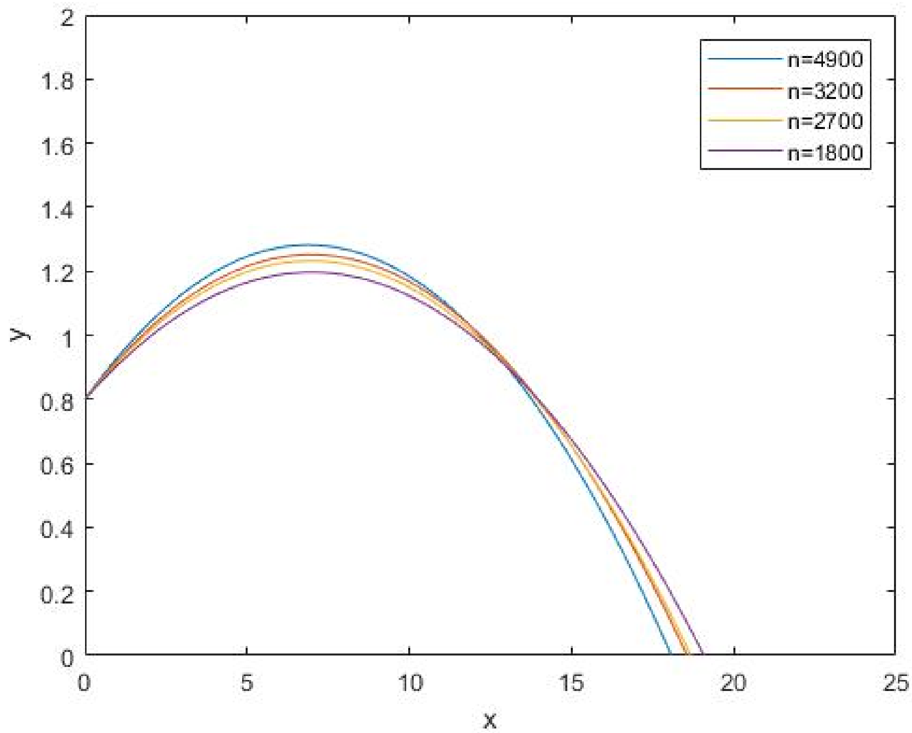

Figure 9: Plots of the trajectories with different values for n at the middle of the court (width $= 4.115$) with $v = 30$

5.3 Observations

From the tables shown above, some observations can be made:

1. Comparing position A and B, it is seen that the difference in angle and the depth between the shots with the lowest and highest rotation speed is smaller for the shot played at position B. This can be explained by the fact that the ball need to go over a higher part of the net. Also the net is reached with a higher velocity in the case of position A. So for the ball to not land outside the court, it is compensated by dropping deeper in the court and creating a bigger angle between the shots.

2. The ratio of w and v is at most around 0.35 for the lowest spinning rate, while for the other spinning rates the ratio between the two values are all at least above 0.3. So as the velocity decreases, the ratio rapidly gets in the domain where the values of C_D and C_L don't change that much. In the case of 4900 rpm, the ratio is solely in the second domain of the graph. This gives an explanation to the bigger difference of β for the lowest spinning rate and the two higher rates of topspin.

6 Discussion

In the thesis we did research to the forthcoming of the Magnus force in tennis. The Magnus force explains a deflection of a rotating ball. For a tennis ball played with topspin, the ball drops earlier to the ground. We used different models to understand why the Magnus force occurs when an object rotates, how to calculate it and finally building an experiment from one of those models to see the influence of the Magnus force in the trajectory of a tennis ball. One could argue that those models are not as precise as in real life it would occur. When using Bernoulli's principle and potential flows, we considered ideal flows. This is far from a realistic approximation to what really happens, but it helped us understand why the Magnus force exists. Also using the Theorem of Blasius, we saw that a rotating sphere in 2D only experiences a lift force. When using the engineering model, it assumes also perfect conditions which you normally won't see when playing a tennis match.

For the experimental part using the engineering model, we could really see some expected differences in the depth of where the ball lands. Although the model doesn't represent realistic circumstances, it gives a nice approximation of the ball trajectories. In particular, when the ball is hit much more at the side of the court, one can really see that the difference in spin rate has a relatively big influence. Still, there are much more factors that influence the behavior of the trajectory of a tennis ball. for instance, the height of hitting the shot we kept nearly the same during the whole experiment. Also the wind and side spin will contribute to another path that the ball will follow. It is convenient to say that tennis nowadays creates more opportunities to play shots like the one we investigated. The top players of the game can hit angled shots with much more pace than about 20 years ago. For further experiments about this topic, a question that may come to mind is to ask where the limit lies in the rate of topspin that can be struck and the angles players can create on the court. An idea could be to let the coordinates be plotted against a wide range of different values of n. Also one could think of a model like they did in [**?**]. Here they measured the initial velocity and the time of the flight by fixing the point of where the ball bounces. Then you can measure the difference in time for players to react to a shot played with a large angle and different rotation speeds.

References

[1] A Velicka A.E.M. Veldman. *Stromingsleer*. Rijksuniversity Groningen, 2011.

[2] Antonin Stepanek. The aerodynamics of tennis balls - the topspin lob. *American Journal of Physics*, 138(56), 1998. doi: https://doi.org/10.1119/1.15692.

[3] Rod Cross Crawford Lindsey Howard Brody. *The Physics and Technology of Tennis*. 2004.

[4] Rod Cross Crawford Lindsey. *Aerodynamic Drag and Lift in Tennis Shots*. 2013.

[5] Christopher Clarey. More and more players deliver slap to classic forehand. *New York Times*, 2006.

[6] R.D. Mehta. Sports ball aerodynamics. *Nrstrud H. (eds) Sport Aerodynamics. CISM International Centre for Mechanical Sciences, Vienna*, 506, 2008. doi: https://doi.org/10.1007/978-3-211-89297-8_12.

[7] R.D. Mehta J.M. Pallis. *Sports Ball Aerodynamics: Effects of Velocity, Spin and Surface roughness*. TMS, 2001.

A Matlab Code

```
clear all
%%The standard mass and diameter of a tennisball, the gravitational
%%constant and the density of air
m = 0.058;
d = 0.067;
g = 9.81;
rho = 1.2041;

%%The input values. n is the revolutions per minute(rpm), alpha the angle
%%between the horizontal axis and the velocity vector, y(1) the initial
%%heigth, x(1) the initial position in the length of the court and v(1)
%%the initial velocity. The width is the width position of the player in
%%3D, with the sideline of where the ball lands as width = 0
n = 4500;
alpha = (10/180)*pi;
i = 1;
y(1) = 0.8;
x(1) = 0;
v(1)= 30;
width = 12;

%%The calculation of the spin velocity w, which is the radius times the
%%spinning speed in radians per second. tau starts the initial angle and
%%will decrease after every iteration. ratio(1) is the intial ratio
w = (d/2)*(n*6.28/60);
tau(1) = alpha;
ratio(1) = w/v(1);

%%Stepsize in the angle for Euler's method in solving the ODE
dtau = 0.005;
```

```
%%%Calculation of the initial value for the coefficients and the drag and
%%%Magnus force
C_D(1)=0.508 + (1/(22.503 + 4.196.*(ratio(1)).^(-2.5)).^0.4);
C_L(1)= 1/(2.202 + 0.981.*(ratio(1)).^-1);
D(1)= (C_D(1)*rho*v(1)^2*pi*d^2)/(8*m*g);
M(1)= (C_L(1)*rho*v(1)^2*pi*d^2)/(8*m*g);

%%%The functions for our coordinates and the ODE
funx = @(tau) (v(i).^2.*cos(tau))./(cos(tau) + M(i));
funy = @(tau) (v(i).^2.*sin(tau))./(cos(tau) + M(i));
func = @(tau,v) v*(sin(tau) + D(i))/(cos(tau) + M(i));

%%%Restriction as the ball lands on the ground
while y(i) > 0

    %%%Calculating the new velocity by Euler's method and letting the angle
    %%%decrease by our stepsize
    v(i+1) = v(i) - dtau*(feval(func,tau(i),v(i)));
    tau(i+1) = tau(i) - dtau;
    i = i + 1;

    %%%Calculating our new values for the ratio, coefficients and forces
    ratio(i) = w/v(i);
    C_D(i) = 0.508 + (1/(22.503 + 4.196.*(ratio(i)).^(-2.5)).^0.4);
    C_L(i) = 1/(2.202 + 0.981.*(ratio(i)).^-1);
    M(i) = (C_L(i)*rho*v(i)^2*pi*d^2)/(8*m*g);
    D(i) = (C_D(i)*rho*v(i)^2*pi*d^2)/(8*m*g);

    %%%Calculating the new x and y coordinate
    y(i) = y(1) -(1/g)*integral(funy,alpha,tau(i));
    x(i) = -(1/g)*integral(funx,alpha,tau(i));


end

%%%Calculating the length of the shot in 3D using pythagoras and the
%%%position of the net using congruence relation of
```

```
    triangles. The depth is
%%the difference between the net and length of the
    shot. The netwidth and
%%netheigth are the place and heigth where the ball
    goes over the net. The
%%heigth of the net is not everywhere the same.
length = sqrt(x(i)^2 - width^2);
net = x(i)*11.885/length;
depth = length - 11.885;
netwidth = depth*width/11.885;
netheigth = 0.914 + (1.042-0.914)*(abs(netwidth -
    4.115)/4.115);

%%Plotting the net, the ground and the ball
    trajectory
x1=net;
y1=0;
x2=net;
y2=netheigth;
plot([x1,x2],[y1,y2])
hold on
plot([0 25],[0 0])
hold on
plot(x,y);
axis([0 25 0 3])
```

B Dimensions of a tennis court

The dimensions of a tennis court are always the same, which are found in figure (10,11). Important to observe is that in singles, both netposts are 1.07 meters away from the singles lines.

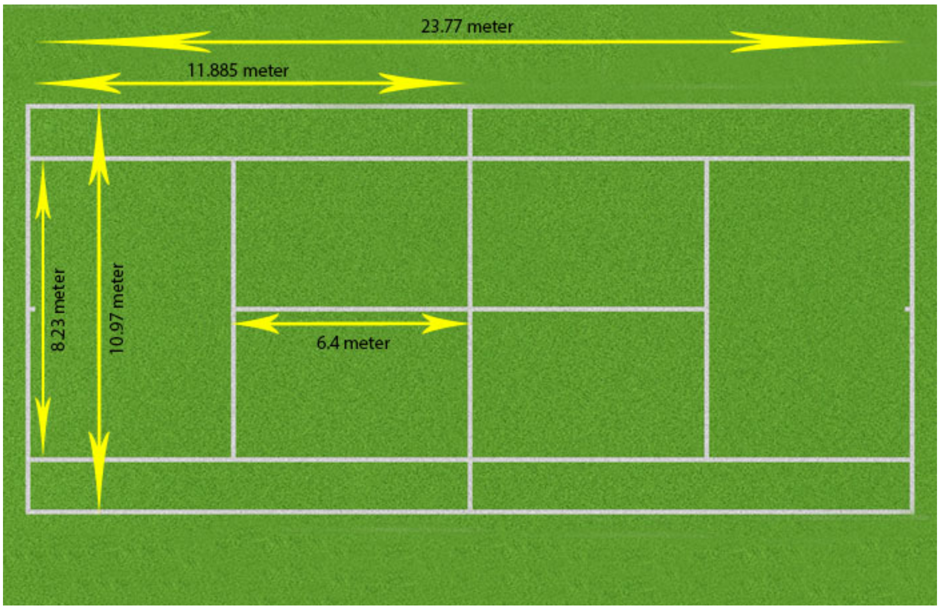

Figure 10: The dimensions on a tennis court

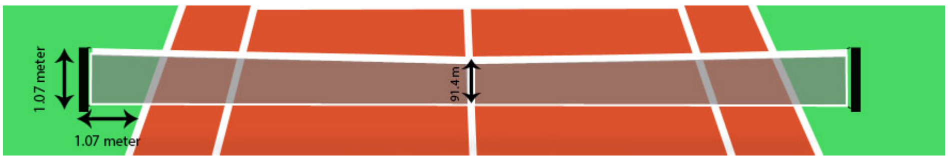

Figure 11: The height of the net at the endpoints and the middle