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Master Thesis

Discovering the Cognitive Factors driving Pupil Dilation: A Model-Based Analysis of an Algebra Task

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Contents

Abstract

Multiple studies have shown pupil dilation to be an indicator of mental effort. However, there is no clear notion of what this mental effort entails. In this thesis, we set out to provide more insight in which cognitive processes contribute to pupil dilation. For this purpose, we collected pupil dilation data in an algebra experiment with equations varying in difficulty. The pupil dilation increased as the equation got more difficult. A cognitive model of the algebra task was created using the cognitive architecture ACT-R. This cognitive model provides a prediction of which cognitive processes are required to perform the algebra task. Each cognitive process has its own specific activity during the task. This activity is then transformed into a pupil dilation prediction, using a convolution method. Linear models based on the predicted pupil dilation of the cognitive processes were fitted to the pupil data. The linear model that explained the pupil data the best, contained the cognitive processes representing long-term, short-term and procedural memory, vision and motor processes. The modules that contribute the most to the pupil dilation are declarative memory, procedural memory and the visual module.

Keywords: pupil dilation; mental effort; cognitive modeling; eye-tracking

Chapter 1

Introduction

It is well known that our pupils adapt themselves to the light in the environment. The pupils constrict in light surroundings or when focused on a close object, and dilate when it is dark (Spector, 1990). In addition to lighting and focus, an additional influence on the pupil size has been identified: mental effort. This influencing factor on pupil dilation has been researched for multiple decades (Hess & Polt, 1964; Beatty, 1982; Steinhauer & Hakerem, 1992; Hoeks & Levelt, 1993; Iqbal, Zheng, & Bailey, 2004; van Rijn, Dalenberg, Borst, & Sprenger, 2012). There is a consensus that the size of the pupil provides an indication of the level of mental investment in the brain. This implies that, as a stimulus or event requires more effort to process, the pupil dilation will become larger. The notion of pupil dilation being an indication of mental effort, however, is only a general overview of effort invested (Mulder, 1986). Though many researchers have contributed to the theory of the correlation between effort and pupil dilation, there is no clear notion of what this effort exactly is. This research tries to provide an overview of the cognitive processes that cause the pupil to change.

Mental effort and pupil dilation

As an early example of research towards pupil dilation, in the nineteen-sixties, an experiment was performed using algebra equations to evoke a pupillary response (Hess & Polt, 1964). The experimenters applied four difficulty levels to determine a possible increase in pupil dilation (e.g., 7 x 8 and 16 x 23). The researchers discovered a close correlation between the equation difficulty and the pupil response: the pupil increases as the algebra problem becomes more difficulty. Hess and Polt (1964) portray pupil dilation as a “direct measure of mental activity” (p. 1190). This is just one of the many descriptions that the brain correlation of pupil dilation has received in the past decades.

Beatty (1982) provided a review of several findings concerning task-related factors that influence the pupil dilation. A short-term memory task with increasing items to be remembered showed that correlation between task difficulty and pupil dilation holds within a task (Kahneman & Beatty, 1966). The pupil increased in size whether the task concerned random words or digits. Furthermore, a character encoding task with multiple difficulty levels showed the same results (Beatty & Wagoner, 1978). This indicates that the pupil dilation and mental effort correlation holds between tasks. In addition, two intellectually different groups performed four cognitive tasks (Ahern & Beatty, 1979, 1981). A significant difference in pupil dilation between the groups was found for three of the four tasks. Beatty concludes that the pupil response induced by the tasks can be used as a physiological measurement for *processing load* or *mental effort*. This provides additional evidence for a correlation between pupil dilation and mental effort between individuals.

Steinhauer and Hakerem (1992) provided a broad overview of the role pupil dilation plays in schizophrenic people and cognitive psychophysiology. They provided an overview of pupil responses to cognitive tasks concerning information processing. Their findings suggest that the dilation of the pupil can be influenced by *cognitive activity* such as *information processing activities*.

An additional term used for the brain-related activity that causes the task-evoked pupil response, is *attentional effort*. The term is used by Hoeks and Levelt (1993), who performed a quantitative analysis of the pupillary reaction to a task, that tries to relate the attentional input to the reaction of the pupil. Their research provided a convolution model that translates attentional input to a pupillary reaction. The convolution method will be described in depth in chapter 3.

Another influencing factor on pupil dilation, used as a measure of mental effort, is the level of masking in speech recognition. A study by Koelewijn, Zekveld, Festen, and Kramer (2012) performed a speech reception threshold task with different masking types. Using the pupil response as a measure of *cognitive processing load*, their results showed that a single-talker mask elicited a larger pupil response than stationary and fluctuating noise. This research provides the cognitive processing of stimuli as a factor influencing the pupil as well.

An additional factor in pupil dilation was found in a study by van Rijn et al. (2012). The experimenters have shown that retrieving information from long-term memory is an additional cognitive process surrounding mental effort and pupil dilation. In this study, participants learned names of brain regions and were tested on the performance of remembering these names. Their results show that, as memory strength increases with the number of repetitions, the pupil dilation decreases. This indicates that the effort of retrieving an item from memory affects pupil dilation as well.

The cognitive processes that elicit a pupillary response have received many names and descriptions over the years. Multiple factors have been discovered that influence the pupil dilation, such as task difficulty, short-term memory load, aural masks and memory strength. However, the current research lacks an overview of the specific cognitive processes that reflect the pupil size. This research attempts to provide this overview.

To achieve this goal, a task had to be selected whose difficulty can be adapted and that consists of several cognitive processes that potentially influence the pupil dilation. An example of a task that can be used to clearly display the effect of difficulty on the dilation of the pupil, is performing algebra equations, such as the experiment performed by Hess and Polt (1964). A step-wise increase of the pupil was clearly visible for the levels of difficulty. Consequently, algebra problems are suitable to induce a pupillary reaction. Five types of equations were distinguished for this experiment to discover the influence of memory retrievals of algebraic facts and mental transformations of the equation. In addition, the equations were either continuously shown to the participants (an *external* representation) or the equation was only shown briefly, requiring the participant to maintain an *internal* representation. After the experiment, a model-based analysis was performed to simulate the experiment and identify the cognitive processes that influence the pupil.

Cognitive processes and pupil dilation in the brain

One of the brain component that influences the dilation of the pupil is the locus coeruleus (LC), a neuromodulatory nucleus in the brainstem (Yoshitomi, Ito, & Inomata, 1985; Loewenfeld & Lowenstein, 1993; Joshi, Li, Kalwani, & Gold, 2016). The LC contains brain cells with the neurotransmitter norepinephrine (NE). This neurotransmitter can be released to innervate brain regions such as the cerebral cortex, midbrain, cerebellum and thalamus (Nieuwenhuis, Gilzenrat, Holmes, & Cohen, 2005), and the LC-NE system is the prime noradrenergic source to innervate the forebrain, neocortex and hippocampus (Nieuwenhuis, Aston-Jones, & Cohen, 2005). Activation in the forebrain is required for multiple processes, one of which is solving algebraic problems (Stocco & Anderson, 2008). Stimulation of the LC-NE system occurs when attending expected or motivationally significant stimuli or events. In addition, salient stimuli can induce activity in the LC. The LC-NE activation from salient stimuli quickly reduces as the frequency of this stimuli increases (Vankov, Hervé-Minvielle, & Sara, 1995).

A theory was devised concerning the role the LC-NE system plays in cognitive processing with respect to pupil dilation (Cohen, Aston-Jones, & Gilzenrat, 2004; Gilzenrat, Nieuwenhuis, Jepma, & Cohen, 2010). The theory distinguishes two modes of LC-NE activity. The tonic mode displays high baseline firing, corresponding with disengagement of the task. In contrast to the tonic mode, the phasic mode shows low baseline LC activity, but high firing rates when responding to task relevant events. The phasic mode allows better performance of the task. The change of the pupil dilation compared to the baseline, closely follows the LC firing rate. As follows, LC activation can indirectly be measured with the pupil dilation. Since the LC-NE system provides activation spreading across the brain, the specific brain regions activated by the release of norepinephrine cannot be determined by pupil dilation alone. However, a model-based analysis can help make predictions of which brain regions are activated during the task-evoked pupil response.

Modelling pupil dilation

The processing of relevant stimuli or events causes an increase of LC activity when engaged with a cognitive task (phasic mode), which actuates an increase of pupil dilation. This LC activity then innervates the brain regions that perform the cognitive processes involved with the task. To determine the specific cognitive processes necessary for a task using pupil dilation, a model-based analysis can be applied (Borst & Anderson, 2017). This form of analysis can be used to make predictions of the cognitive processes that are applied in a task by using a computational model. This type of model simulates the task that needs to be performed and links this to cognitive processes that are required to perform the task. The model activity is then applied to the pupil data, to analyze how the model can explain the data.

The cognitive architecture ACT-R is a computational model that can be used to bridge the gap between pupil dilation and cognitive processes (Anderson et al., 2004). ACT-R is a psychological theory that simulates human cognitive processes and takes psychological theory and physical capabilities (e.g., minimum response time) into account (Anderson, 2005, 2007). This framework for computational modelling consists of multiple modules that represent specific cognitive processes, such as long-term memory, working memory, vision, motor processes and production rules. When a cognitive task is simulated in ACT-R, activation in one of the modules indicates that a human requires the associated cognitive process to perform the task.

For this research, the algebra task will be simulated by ACT-R. Several algebra models have already been created using this architecture (Anderson, 2005, 2007; Stocco & Anderson, 2008). The model gathers data on the module activity present during the task, such as the onset and length of the activity. The activity of each individual module must then be scaled to fit the behavioural results of the participants. This provides an indication of where the cognitive processes occur in a trial and how long these processes are activated. This module activity can then be translated into a potential pupil response to determine the influence of each cognitive process simulated by the ACT-R model.

Hoeks and Levelt (1993) created a system that relates pupil dilation to a series of pulses. These pulses can represent the module activity of the cognitive model. The system by Hoeks and Levelt can convolve the module activity into a predicted pupil response. This convolution method takes the pupillary delay between cognitive activity and dilation into account. In addition, the convolution includes the additive factor of concurrent cognitive activity. This implies that the increase in pupil dilation is linearly related to the increase in cognitive activity. The convolution is described in detail in chapter 4 and is shown in Figure 4.2. This convolution method is able to translate the module activity of ACT-R into a predicted pupil dilation. Analyzing the model's predicted pupil dilation based on its model activity can help determine the influence of specific cognitive factors on the pupil dilation.

Overview of the thesis

This research aims to provide an overview of the cognitive factors influencing the pupil dilation. An experiment was performed where participants performed algebra equations while their pupil was measured. The methods of this experiment can be found in Chapter 2. The results of the experiment are presented in Chapter 3. A cognitive model simulating the experiment was created and is described in Chapter 4. The results of the model-based analysis can be found in Chapter 5. Finally, the results and findings are discussed in chapter 6.

Chapter 2

Algebra task

The algebra task used for this experiment consisted of a series of equations the participant had to solve. The equation types are different compared to the equation used by Hess and Polt (1964). The experiment by Hess and Polt only applied multiplication and increased the difficulty of the multiplication problems. In this experiment, the equations were used to distinguish between long-term memory retrievals and equation transformations. Retrieving an item from long-term memory requires the participant to invest mental effort. This investment increases when retrieving multiple or complex items from memory. Moreover, maintaining a mental representation of an equation in the brain takes up mental resources. Transforming the mental representations demands mental effort, which increases as the number of transformations increase. The question is whether these cognitive processes influence the dilation of the pupil.

The difficulty of the equations was kept at a low level to make sure the participants were able to solve each equation in a reasonable time and without aiding materials (e.g., pen and paper or a calculator). Five types of equations could be distinguished. The equation types differed in their memory retrievals and transformations. For this task, a memory retrieval consists of retrieving a calculus fact from memory. A transformation within an equation requires the user to change the equation on both sides of the equal sign (e.g., subtract 3 on both sides). Combining these two factors resulted in the equation types presented below:

1. $x = 6$ — No retrieval, no transformation
2. $x + 2 = 6$ — Minimal retrieval, average transformation
3. $x = 32/8$ — Average retrieval, no transformation
4. $8x = 32$ — Average retrieval, average transformation
5. $8x + 2 = 34$ — Complex retrieval, complex transformation

As can be seen in the example, each equation has a unique combination of retrieval and transformation level. These types can help provide insight to the influence of memory retrieval and transformation on the pupil, and how this fits with the theory on mental effort causing the pupil to change. It is important to note, that each equation always has an integer for an answer.

We expect equation type 1 to actuate only a small response of the pupil, since this requires no memory retrieval, nor a transformation. Equation types 2 to 4 are expected to induce a larger response than equation type 1. The pupillary response to these three equation types is expected to be close to each other, since all require the participant to retrieve an item from memory or perform a transformation. Since equation type 4 contains both retrieval and transformation, it is expected that this equation type causes a larger response than type 2 and 3. Finally, equation type 5 is expected to have the largest influence on the pupil. Since this type of problem has multiple retrievals and transformations, the pupil is expected to have a larger reaction.

To further examine the mental processes surrounding the pupil dilation, an additional factor is added to the trials in which participants have to solve the equations. To determine the influence of memorizing mental representations of the equations, the equation was either continuously shown to the participant, or the equation disappeared from the screen. In case of the former, the participant can refer back to the presented equation, whereas the latter requires the participant to create a mental representation of the equation in memory if the participant has to think back about the whole equation. It is expected that this internal representation induces an increase of dilation, because participants will have to memorize the whole equation.

To summarize, five different types of equations were created to discover the influence of transformations and memory retrievals on pupil dilation. In addition, each equation type is presented either externally (the participant can always refer back to the equation presented), or internally (the participant has to save the equation in memory, because the equation is only presented briefly).

Participants

For this study, 31 students of the University of Groningen participated in the experiment in exchange for eight euro. Each participant performed the experiment voluntarily. The data of ten participants were not analyzed. For one participant, the eye-tracker stopped recording during the experiment, leaving insufficient data for analysis. Another participant had an accuracy lower than two standard deviations from the group mean and was therefore removed. The data of the remaining eight participants, over 20 percent of the trials consisted of at least 25 percent saccade. These trials were removed due to their unreliability, which left an unreliable number of trials for these participants to be analyzed. This leaves 21 participants (11 female; mean age of 25.4 years; age range 18-30).

Apparatus and stimuli

During the experiment, the pupil size was measured by an Eyelink 1000 eye-tracker from SR Research. The eye-tracker measured the eyes with a frequency of 250 Hz. For this experiment, a nine-point calibration and validation was performed. In order to keep the participant's head movements to a minimum, a customizable head rest was installed in front of the eye-tracker. The eye-tracker was located in a small room containing two desks. The desks were separated by a file cabinet. On one desk, the eye-tracker was installed, and the researcher could monitor the experiment at the other desk.

The experiment was created with the OpenSesame software (Mathot, Schreij, & Theeuwes, 2012), a tool that can be used for setting up an experiment. This software managed the experiment order, provided stimuli such as the fixation cross, connected the experiment with the eye-tracker and logged all the results. In addition, it provides the calibration and validation interface prior to the experiment.

The equations and the corresponding feedback were presented at the centre of the screen in white on a black background, with a font size of 28. All stimuli and text were presented on a Dell Ultrasharp 2007FP monitor, with a resolution of 1600x1200 pixels. The participants used a mouse to provide input. The stimuli can be found in Figure 2.1.

Design

The experiment used a within-subject design. Each participant performed all equation types, with both external and internal representations. The whole experiment consisted of a training block and four experimental blocks. The participants were allowed to have a small break between the blocks to reduce the effect of fatigue.

For each trial, the participants had 10 seconds to determine the answer to the equation. When the participant knew the answer, they could click with the mouse to proceed to the answering screen. The trials with external representation showed the equation until the participant clicked the mouse or the 10 seconds were up. During the trials requiring the internal representation, the equation was removed from the screen after 250 ms. A pilot study showed that this time frame is sufficient to create an internal representation of the equation. See Figure 2.1 for a graphical overview of both an external and internal trial.

To get the participants acquainted with the equation types, 20 training trials were performed. During the first ten training trials the equation remained on the screen (external representation). During the last ten trials the equation disappeared after 250 ms (internal representation). The four experimental blocks alternated between all external trials and all internal trials. Each block consisted of 40 trials. The equation types were evenly distributed within each block and were presented in a randomized order. During the experiment, each equation type was tested 32 times, consisting of 16 internal and 16 external trials.

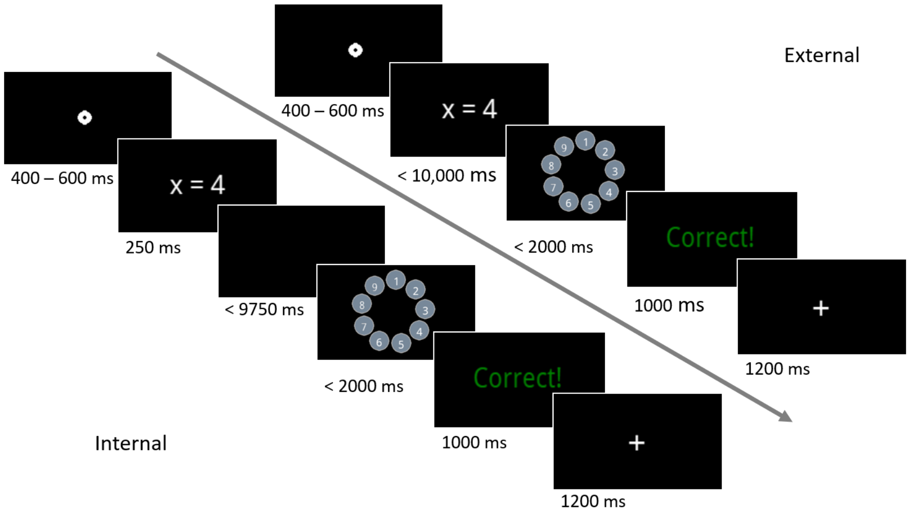

Figure 2.1: Overview of a trial. The participant views a fixation cross presented for a random time between 400 and 600 ms. The problem is shown for a maximum of 10,000 ms and the participant has 2000 ms to select the correct answer. The participant receives feedback for 1000 ms, followed by a fixation plus for 1200 ms.

Procedure

Upon arrival, the participant was asked to read and sign the informed consent. The experiment was verbally introduced with a brief explanation. The participant was asked to sit in the chair in front of the eye tracker. The head rest was adapted to the participant, such that they were able to sit comfortably. A nine-point calibration was performed to measure the left eye. After the calibration, the instructions of the experiment were presented on the screen. The experimenter asked whether the experiment is clear to the participant and if they have any questions. After the participant understood the experiment, they could start the experiment by pressing the left mouse button.

The experiment started with a training block consisting of 20 trials. The equation types were evenly distributed during the training. The participant first performed 10 trials with external equation representation, followed by 10 trials where the participant had to create an internal representation of the equation. The experimenter asked whether the experiment was clear and that the participant could start the first experimental block when they were ready. At the end of each block, the participant was notified that they could take a break, or press the left mouse button to continue to the next block. During the break, the participant was allowed to have a drink or take a small walk. After the final block, the participant was thanked for participating and they were allowed to ask questions about the experiment.

Chapter 3

Experiment results

Prior to the analysis, outliers were removed from the data. First, the accuracy and response times of the participants were examined. The percentage of correct answers of the experimental trials was used as a measure of accuracy. One participant was removed from the data, because the accuracy across the 160 experimental trials was significantly lower (M = 0.50) compared to the mean accuracy (M = 0.91) of all the participants; $t(159) = -10.24$, $p < .001$. Furthermore, for each condition, the trials that deviated more than two standard deviations from the mean response time (time between the problem was shown and the first click) were removed.

Next, the pupil data was analyzed for outliers. For one participant the eye-tracker stopped recording data after the first block. This left an unreliable amount of trials for this participant, resulting in removal from the data set. In addition, for each trial, the change of the pupil size was compared to the baseline of that trial. The baseline was measured during the fixation cross that was presented prior to the problem. The trials with a pupil change of over 2.5 standard deviations compared to the baseline were removed. Furthermore, the proportion of fixation and saccades (eye movement between fixations) was determined for each trial. Trials that consist of 25 % saccade or more were removed due to unreliability of the pupil data. After removing these trials, eight participants were removed from the data set, due to an unreliable amount of remaining trials. The remaining data consists of 2795 trials obtained from 21 participants. A behavioural and pupil analysis was performed on the data.

Behavioural analysis

For the behavioural data, the effects of problem state and problem type on the response time and the accuracy were analyzed. A factorial analysis of variance (ANOVA) was conducted to determine the main effect of problem type and problem state and the interaction effect between problem state and problem type on the response time and on the accuracy of the participants. The mean response times per problem type and both representations can be found in Figure 3.1. The participants' accuracy can be found in Figure 3.2.

Response times

An ANOVA was performed to determine the effect of problem type, problem state and the interaction of the two on the response time. The results show a significant main effect for problem type, which yielded an F ratio of $F(4, 80) = 71.33$, $p < .001$. The main effect for problem state was insignificant with an F ratio of $F(1, 20) = 0.966$, $p = 0.338$. The interaction effect was not significant, $F(4, 80) = 0.598$, $p = 0.665$.

Pairwise t-tests were performed to determine which problem types are significantly different from each other. The p-values of these tests were adjusted using Holm, due to multiple comparisons. There is a significant difference between the response times of problem type 1 and all other response times ($p < .001$). Furthermore, the response times of problem type 5 are significantly different compared to the other problem types ($p < .001$). There is no significant difference between the response times of problem types 2, 3 and 4 ($p > .05$). An extensive overview of the results can be found in Table B.1.

There is no difference in response time between the external and internal representation. Equations of problem type 1 were answered significantly faster than

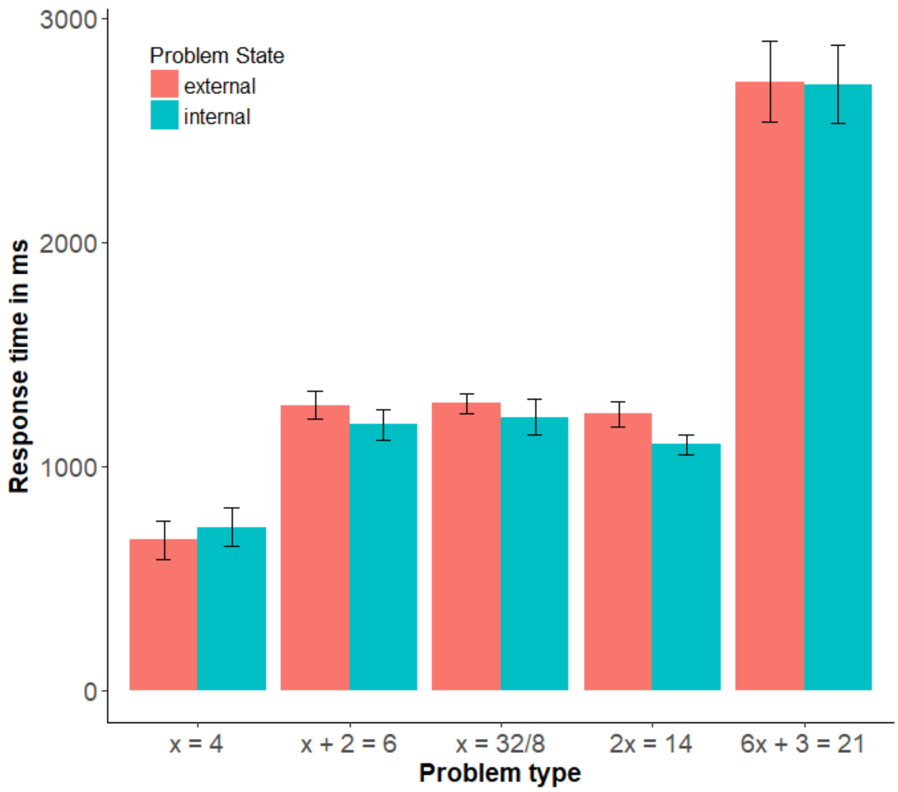

Figure 3.1: Mean response time for all problem types and the two problem states. The response time is measured from the moment the equation is presented to the moment the mouse is clicked first.

the other problem types. Problem types 2, 3 and 4 required similar response times and problem type 5 required significantly more time to answer.

Accuracy

For the accuracy analysis, trials that were incorrect but did not exceed the standard deviation boundaries of saccades, change of the pupil size and response time were used as well. For this analysis, 2948 trials were used. An ANOVA was performed to determine the main effect of problem type and problem state on the percentage of correct answers. The results indicate a significant main effect for problem type, which yielded an F ratio of $F(4, 80) = 39.93$, $p < .001$. The main effect for problem state was also significant with an F ratio of $F(1, 20) = 6.266$, $p = 0.0211$. In addition, the interaction effect of problem type and problem state was significant, $F(4, 80) = 3.957$, $p = 0.00556$.

To discover the accuracy difference between problem types, pairwise t-tests were performed. The results show a significant difference between problem type 5 and the other four problem types ($p < .001$). In addition, a significant difference is found between the accuracy of problem type 1 and the other four problem types ($p < .05$). When looking at the effect of problem state on the accuracy, there

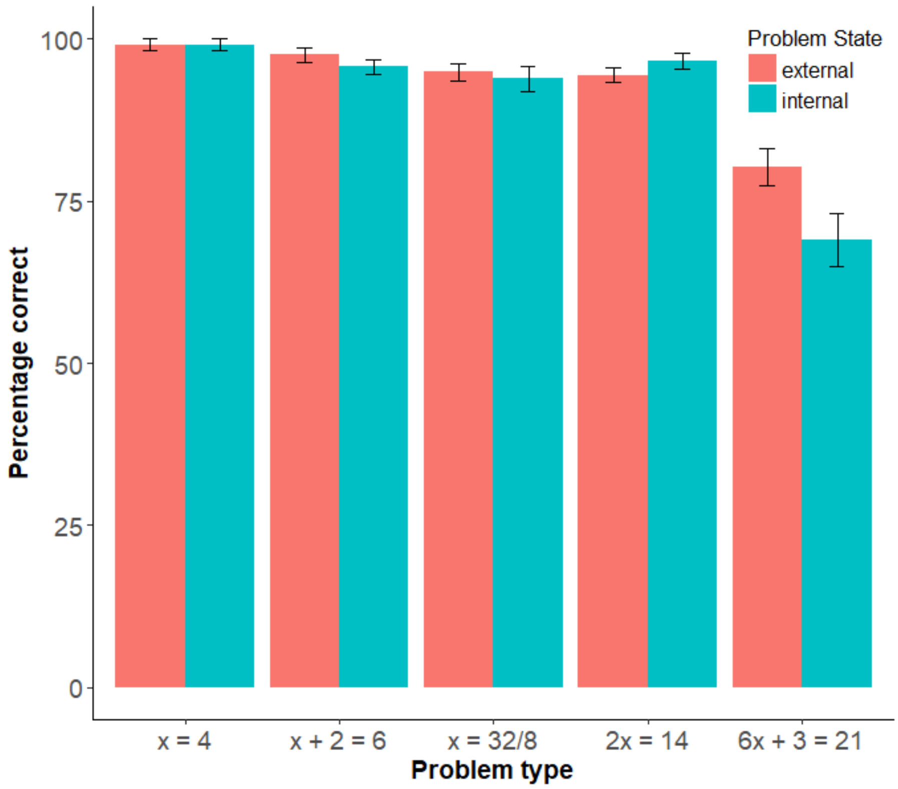

Figure 3.2: Percentage of answers correct for all problem types and problem state.

is no significant difference in internal and external representations between the problem types. Due to multiple comparisons, the p-values are corrected using Holm. All the results can be found in Table B.2 and B.3.

Problem type 1 was almost always answered correctly. Problem type 2, 3 and 4 have almost equal accuracy and problem type 5 was harder for the participants, resulting in a lower accuracy. There seems to be a difference in accuracy for problem type 5 between the external and internal representation, but this is not significant.

Pupil analysis

Fixations

For the pupil analysis the number of fixations per problem type and problem state is analyzed, as well as the change of the pupil. A fixation occurs when the participant maintains their gaze on a single visual location. In this analysis, the number of fixations between displaying the equation to the first mouse click are examined, since this is where a clear distinction can be made between the internal and external representations. The mean amount of fixations during the solving period can be found in Figure 3.3.

An ANOVA was performed to determine the main effect of problem type, problem state and the interaction of the two on the mean amount of fixations. The results displayed a significant main effect of problem type on the mean number of fixations, which yielded an F ratio of $F(4, 80) = 72.6$, $p < .001$. The main effect of problem state is also significant; $F(1, 20) = 22.85$, $p = 1.14e\text{-}04$. In addition, the interaction of problem type and problem state is significant as well; $F(4, 80) = 4.174$, $p = 4.03e\text{-}03$.

Pairwise t-tests were performed to determine the significant differences within the number of fixations between the five problem types. Problem type 1 has significantly less fixations compared to the other four problem types ($p < .001$). In addition, problem type 5 has significantly more fixations compared to the other four problem types ($p < .001$). There was no significant difference in the number of fixation between problem types 2, 3 and 4.

In addition, t-tests were performed to determine the effect of the problem state. For each problem type, the number of fixations of the internal and external representation were compared. With the exception of problem type 1, significantly more fixations occurred within the external representation compared to the internal representation ($p < .05$). All the p-values are corrected using Holm. The results can be found in Table B.4 and B.5.

Problem type 1 received the least amount of fixations while solving the equation. For problem type 2, 3 and 4, the participants had a similar amount of fixations, which were more compared to problem type 1, but less than problem type 5. With the exception of problem type 1, each problem type has significantly more fixation in the external representation compared to the internal representation.

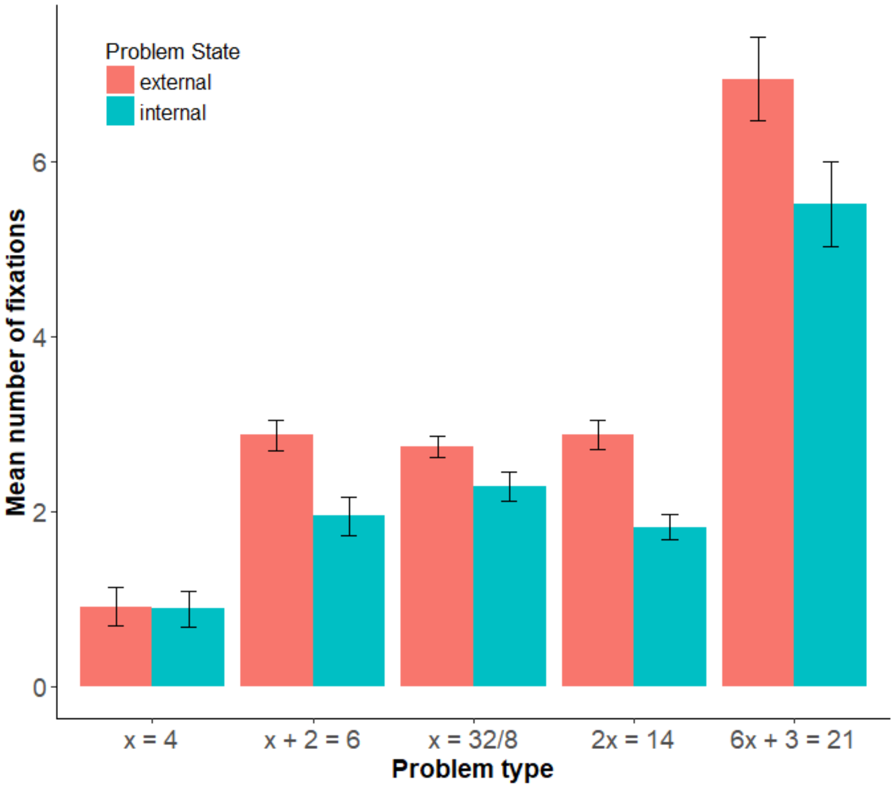

Figure 3.3: The mean number of fixations measured in the period during which the participants solve the equation, from showing the equation to the first mouse click.

Change of the pupil size

The size of the pupil was measured with a frequency of 250 Hz during each trial. Three participants were measured with a frequency of 500 Hz and were down-sampled to 250 Hz. Throughout the experiment, blinks and saccades occurred, during which inaccurate or no pupil measurements were recorded. Trials consisting of 25 % saccade or more were removed due to their unreliability. For the remaining trials, saccades were linearly interpolated, using the 30 measurements (120 ms) before and after the saccade.

Because the pupil responds at a slow pace, the effects of the previous trial must be diminished before the next trial commences. After each trial, a fixation is shown for 1200 ms. This time is sufficient to lose the effect of the previous trial on the pupil. After the 1200 ms, the fixation remains on the screen for 400 to 600 ms. During this period, the baseline of the pupil diameter is measured. The change of the pupil diameter within the trial is then compared to this baseline. For each time-point in each trial, the percentage change of the pupil diameter compared to the baseline is determined. This is referred to as the percentage change of the pupil size (PCPS).

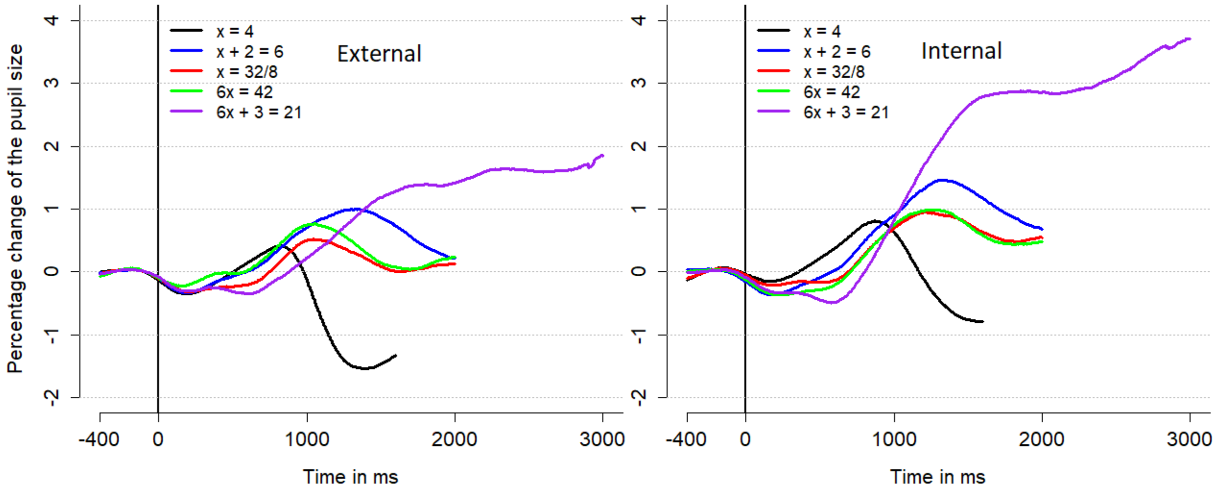

Figure 3.4: The percentage change of the pupil size over time with a stimulus-locked point of view. On the left the trials with external representation are displayed and the internal representation trials on the right. At t = 0 the problem is shown to the participants. The baseline is measured during the 400 ms prior to showing the problem.

Figure 3.4 displays the PCPS of the problem types with a stimulus-locked point of view. The baseline is measured from t = -400 to t = 0. At t = 0 the problem is displayed on the screen. Within the conditions, each trial has a different length. Therefore, the graphs in Figure 3.4 display the course of the pupil as long as there are sufficient datapoints left for a line.

Both the external and internal representations display a negative PCPS when the equation is shown on the screen. This can be explained with the late response of the pupil. In both the external and internal representation the pupil response to the equation is clearly visible for equations type 1 to 4. The pupil increases in size, peaks, and decreases towards the baseline. For equation type 5 there is a large increase of the pupil visible, then levels out, and increases again. A clear maximum cannot be discerned. In addition, there is no decrease of PCPS visible for equation type 5, though they are clearly visible for the other problem types. It is expected that trials with a long response time influence the PCPS at the end of the graph, since the PCPS increases towards the end of solving the equation.

In line with the response times and accuracy results, the PCPS displays a clear difference between the equation types. The first equation elicits a small pupillary response, equation types 2, 3 and 4 are similar, and equation type 5 stands out with a large pupil response. A clear difference in PCPS is visible between the trials with external representation and the trials with internal representation. With the exception of equation type 4, the PCPS of the internal representation is higher compared to the external representation. However, the decrease rate

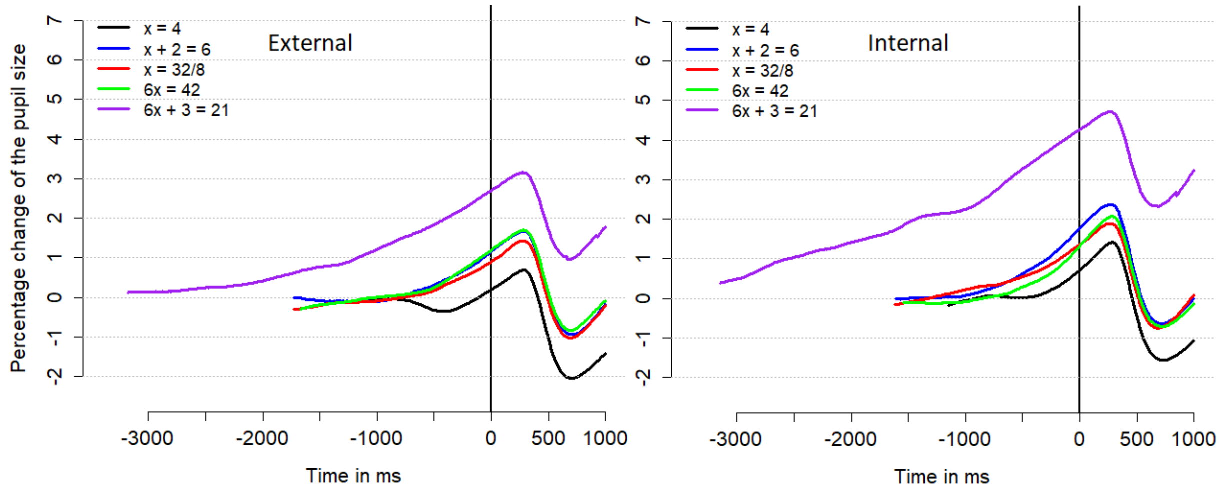

Figure 3.5: The percentage change of the pupil size over time with a response-locked point of view. On the left the trials with external representation are displayed and the internal representation trials on the right. At t = 0 the mouse is clicked, indicating that the participants knew the answer. During the 1000 ms after clicking the mouse the answer was selected.

of the PCPS after the peak is almost equal for equation types 1 to 4, between the external and internal representations.

Figure 3.4 displays the results from a stimulus-locked point of view. The course of the PCPS after exposing the participants to the problem can be viewed through this perspective. However, the participants stop investing mental effort when they have solved the equation. According to theory, the pupil response is additive, therefore the expected maximum PCPS is at the moment the participant has solved the equation and clicks the mouse. Due to the slow response of the pupil, the maximum PCPS is expected just after the first click of the mouse. The course of the PCPS around the first mouse-click can be seen in Figure 3.5.

The results in Figure 3.5 depict the change of the pupil size for the external and internal representations from a response-locked perspective. At t = 0 the participants clicked the mouse, indicating that they solved the equation. As expected, the pupil increases in size during the problem solving, and reaches its maximum pupil response after the first mouse-click. The time of the PCPS peak is also similar along all equation types and both representations. The difference between the equation types can be seen from the response-locked perspective as well. In addition, the decrease of the PCPS for problem type 5 is made visible from this perspective. The rate of decrease of the PCPS is similar for all problem types across the representations. At around 750 ms, the PCPS starts to increase again, which can be caused by finding the correct answer among the answering options. Finally, the influence of holding an internal representation compared to constantly displaying the equation on the screen can be seen. Equation types

1, 2 and 5 have a higher PCPS for the internal trials compared to the external trials. Equation types 2 and 3 show no large differences.

To determine whether there is a significant difference in maximum change in the pupil size between the experimental conditions, an ANOVA was performed. For each participant and every condition, the mean change of the pupil size with a response-locked perspective was calculated. Their maxima were used to determine the effect of problem type and problem state on the maximum change of the pupil size. The results of the ANOVA show a significant main effect for the problem type; $F(4, 80) = 31.4$, $p < .001$. In addition, there was a significant main effect for problem state, yielding an F ratio of $F(1, 20) = 8.67$, $p = 0.00801$. The interaction of problem type and problem state also provided a significant results; $F(4, 80) = 3.40$, $p = 0.0128$.

Pairwise t-tests were performed to determine which problem types have a significant difference in maximum pupil change. The p-values of the tests were corrected using Holm. There is a significant difference between the maximum pupil size of problem type 1 and the other four problem types ($p < .001$). In addition, problem type 5 has a significantly maximum change in pupil size compared to the other problem types ($p < .001$). There was no significant difference in maximum pupil change between problem types 2, 3 and 4 ($p > .05$). When looking at the problem state, only two significant differences were found. The maximum change in pupil size is significantly larger in the internal condition for problem type 1 and 5 ($p < .05$). There is no significant difference in maximum PCPS between the external and internal state for problem types 2, 3 and 4 ($p > .05$). The results of the t-tests can be found in Table C.1 and C.2.

Chapter 4

Cognitive Model

A cognitive model simulates a specific cognitive task and contains assumptions of what cognitive processes are involved in the task (Anderson et al., 2004; Ritter, Anderson, Koedinger, & Corbett, 2007). The model can be used to make predictions on the time humans need to perform the task and their performance (accuracy). One of the methods of creating a cognitive models is by using a cognitive architecture. A cognitive architecture provides a theory on the structure of the brain and how the components of this structure can provide cognitive functions (Anderson et al., 2004; Newell, 1994). The cognitive architecture used to create the cognitive model for the current experiment is the Adaptive Control of Thought - Rational, or ACT-R (Anderson et al., 2004; Anderson, 2005, 2007).

ACT-R

ACT-R is a cognitive architecture that can be used to simulate human cognition. The architecture builds on psychological theories developed through numerous experiments and takes the physical capabilities of humans into account, such as response times and motor actions. The ACT-R architecture is a framework which can be used for various tasks, such as driving a car (Salvucci, Boer, & Liu, 2001), interpreting text (Budiu & Anderson, 2004), calculating (Ritter et al., 2007) and memory tasks (e.g., Elliott & Anderson, 1995). These tasks can be simulated by creating a cognitive model. The ACT-R environment inserts the psychological assumptions to the model. After running the model, the output provides quantitative measures of the task, which can be compared to quantitative measures obtained from participants who performed the task. The structure of the ACT-R cognitive architecture is described below.

The ACT-R architecture consists of a set of so-called modules which represent cognitive functions. With the exception of the procedural module, each module can be accessed through a buffer. This buffer provides information on the state of the cognitive model. The modules can be divided into a group concerned with the external world and a group that is occupied with internal processing. The distinction is made clear in Figure 4.1. The modules that are applied for the algebra model are shortly described.

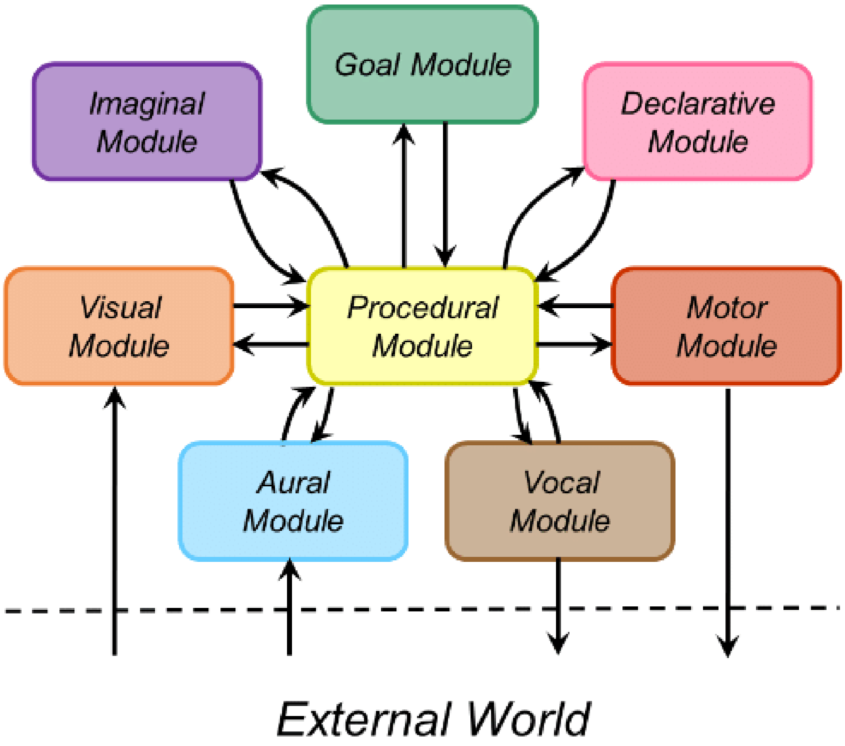

Figure 4.1: The ACT-R modules and their interaction pathways.

- The **visual** module is one of the modules that is able to retrieve information from the external world. The module is used to process visual stimuli presented in the external world. A request can be made to visit and attend a visual location. The visual stimuli within the attended location can then be encoded. The visual module can be used to read and encode text, but also to find a specific stimuli. For the algebra model, this module is needed to attend the fixation cross, read and encode the equations, look for the correct answer and read the feedback.

- The **manual** module is one of the methods ACT-R uses to interact with the external world. Also known as the motor module, the manual module enables performing actions in the physical world. Examples are pointing at a target, holding up a specific number of fingers or moving a mouse. For the algebra model, this module will be used to click on the answer with the mouse.

- Long-term memory is represented in the **declarative** module. Facts can be stored in the form of chunks. A chunk can consist of one or multiple parts. An example of a chunk is:

```
(4times6
isa multiplication-fact
first 4
second 6
answer 24)
```

  Here, "4times6" is the name of the chunk. This chunk is a multiplication consisting of three terms. The multiplication of the value in the "first"-slot with the value in the "second"-slot results in the value in the "answer" slot. A cognitive model can start with a set of facts in declarative memory and is able to store new facts in memory. Each item in memory has an activation level. This level is an indication of the likelihood of being retrieved and, if it can be retrieved, how easy the fact can be retrieved from memory. An easy fact such as "6 x 4 = 24" will have a higher activation than a harder fact such as "13 x 18 = 234". In the current model, each fact starts with a specified activation level. This activation decreases in time and is increased when retrieved from memory. A fact can be requested through the declarative buffer. If the request matches an item from declarative memory, the fact is loaded in the buffer. The activation level of the fact denotes the time it takes to retrieve the fact. If two facts match a memory request, the most active fact of the two will be retrieved.

- The **goal** module, also known as the control module, keeps track of the model's state. The goal buffer contains the "goal" chunk with a state assigned to it:

```
(goal
isa goal
state start)
```

  The buffer contents can be requested to determine the current state of the model. The value of the state in the goal buffer can be changed when the model moves on to a next state.

- The **imaginal** module, which also goes by the name of problem state module, contains a mental representation of the problem. This representation can be requested and updated through imaginal the buffer. As in humans, the number of items that can be stored in this mental representation is limited. The number can be specified in the cognitive model.

- The **procedural** module contains the procedural memory, which is a set of IF-THEN rules. This module is connected with all the other modules and coordinates the processes occurring within ACT-R. The procedural module does not have a buffer, but is able to scan the contents of the buffers of the remaining modules and request the modules to perform a specific task. The procedural module consists of production rules, such as:

```
(p read-equation
  =goal>
    isa           goal
    state         fixation
  =visual>
    isa           visual-object
    value         "+"
  ?visual>
    state         free
==>
  =goal>
    state         start
  +visual>
    isa           clear
)
```

This production consists of two parts, an IF- and a THEN-part. The example above can be read as:

IF the goal is to attend the fixation *and* the visual buffer contains a visual object with the value '+' *and* the state of the visual buffer is free

THEN set the goal state to 'start' *and* clear the visual buffer.

The procedural buffer checks which production rule should be applied based on the information it finds in the modules' buffers. If the state of the buffers satisfy a specific production rule, the production rule will be executed. Only one production rule can be executed concurrently and execution can cause the state of the buffers to change, leading to a new production rule to be executed. The production rules are written by the cognitive modeller. This allows the modeller to program their assumptions of the procedure of a cognitive task, which is then combined with the cognitive theory of ACT-R.

Algebra model

The algebra model performs ten trials, consisting of the five equation types with both internal and external representation. Similar to the experiment, in each trial a fixation cross is displayed, followed by a presentation of the problem, selecting the answer and feedback. At the start of each trial, the declarative memory contains the goal chunks and five mathematical chunks. These five chunks are either subtraction or division facts and look as follows:

```
(14div2 isa math op "/" val1 "14" val2 "2" ans "7")
```

Each mathematical chunk has an operator (e.g., "/" or "–"), two values and an answer. The chunks are necessary to solve the equations presented to the model. Each mathematical chunk has an activation level at the start of the trial. The activation of the subtraction chunks is higher than the division chunks, because division is considered more difficult and therefore requires more time to retrieve from memory (Lebiere & Anderson, 1998). In addition, because the model performs the trials in real time, the activation level of the chunks was adapted to match the model response times to the behavioural data.

At the beginning of each trial, the visual module attends the fixation cross that is displayed. In addition, the goal module state is set to "start". This allows the model to expect an equation next. To read the equation, the visual module first attends the equation. Because the participants were able to read the equation in 250 ms, only one procedure is applied to read the whole equation and store it in the imaginal buffer.

When the equation is stored in the imaginal buffer, the model checks whether the equation has the form "x = y". In this case, the model knows the answer is "y" and triggers the motor module to click the mouse button. If the equation is of problem type 1, the model will click the mouse button after reading the equation. For problem types 2 to 4, the model needs to retrieve one mathematical fact from declarative memory. For example, if the current equation is "2x = 14", the model will request the math-chunk with "14" for the first value, "2" for the second value and division as its operator. The best fitting chunk is the one in the example above. After retrieving this chunk from memory, the model updates the imaginal buffer with the answer that is found in the mathematical chunk. The imaginal buffer now holds the equation "x = 7". This representation is recognized by the model as a solved equation and therefore the mouse will be clicked.

Problem type 5 requires additional procedures to solve. In addition, the model makes no distinction between external and internal representation for problem types 1 to 4, but it does make a distinction for problem type 5. Even though the response times for the external and internal representations are not significantly different for problem type 5, there is a significant difference in the number of fixations. This led to the assumption that participants reread the equation on the screen in the external condition. Because it is assumed the participants reread the equation, the model also applies an additional procedure to update the internal representation.

The first steps the model takes for solving problem type 5 are equal to the procedures used in problem types 2 to 4. For example, if the current equation is "6x + 3 = 21", the model will first request a mathematical chunk that contains "21 - 3 = y". After this chunk is retrieved from declarative memory, the answer is determined and stored in declarative memory. The model now updates the whole equation held in the imaginal buffer. The model updates the left-hand side of the equation by attending the screen (i.e., rereading "6x"). Next, the answer of the first memory retrieval is retrieved from declarative memory and added to the right-hand side of the equation held in the imaginal buffer. Since the updated equation is not in the form of "x = y", the model will continue solving the equation. The model will determine what type of equation is currently in the imaginal buffer ("6x = 18") and retrieves the corresponding mathematical chunk ("18 / 6 = 3"). After this retrieval, the imaginal buffer is updated and holds an equation in the form of "x = y".

For the internal representation, the model stores the equation in the declarative memory after reading it from the screen. After retrieving the first mathematical fact, the model stores the answer of this fact and updates the equation in the imaginal buffer from memory. First, the model retrieves the original equation and updates the left-hand side of the equation. Next, the model requests a retrieval of the answer earlier retrieved from memory and assigns this to the right-hand side of the equation in the imaginal buffer. This equation is then solved further, equal to the external condition.

When the cognitive model holds the answer to the equation in the imaginal buffer, the mouse is clicked and the answer options are displayed. Since the participants have had a training block and therefore know where the answers are displayed, the model also has knowledge on where the answers are presented. The visual module is used to attend the number that corresponds to the answer held inside the imaginal buffer. Next, the motor module is applied to move the cursor to the correct answer and then click the mouse. The display provides feedback, but because the model always answers correctly, the feedback is not processed.

The algebra model contains two types of parameters that were adapted to fit the response times of the participants. The first is the latency factor, which is influences the time it takes to retrieve a chunk from memory. In addition, the activation level of the mathematical chunks was set at the beginning of the experiment. The division facts received a lower activation than the subtraction facts, since division facts are considered to be harder than subtraction facts. These parameters were adapted in order for the model response times to resemble the mean response times of the participants.

Convolution of model activity

The algebra model performed each problem type for both the internal and external condition. At each time-point of these ten model trials, the modules were either active (1) or inactive (0). This results in an activity pattern of each module along the individual conditions. The module activity can be seen as a string of pulses. Hoeks and Levelt (1993) have discovered that these pulses have a linear, additive effect on the pupil response. In addition, they presented a system that convolves this string of pulses into a pupil response. An example of the convolution can be seen in Figure 4.2. In essence, the pupil response follows the Erlang gamma distribution and contains two adjustable parameters. The shape of the response is described by the n-parameter. A high n-value corresponds with a steeper slope and a larger increase of the pupil. The n-value that best fit the data of Hoeks and Levelt was 10.1. The second parameter is the $t.max$. This describes where the maximum pupil response occurs. The maximum pupil response to cognitive activity usually occurs after a delay of approximately 900 ms. The convolution method used by Hoeks and Levelt found an optimal fit for their data at $t.max = 930$ ms.

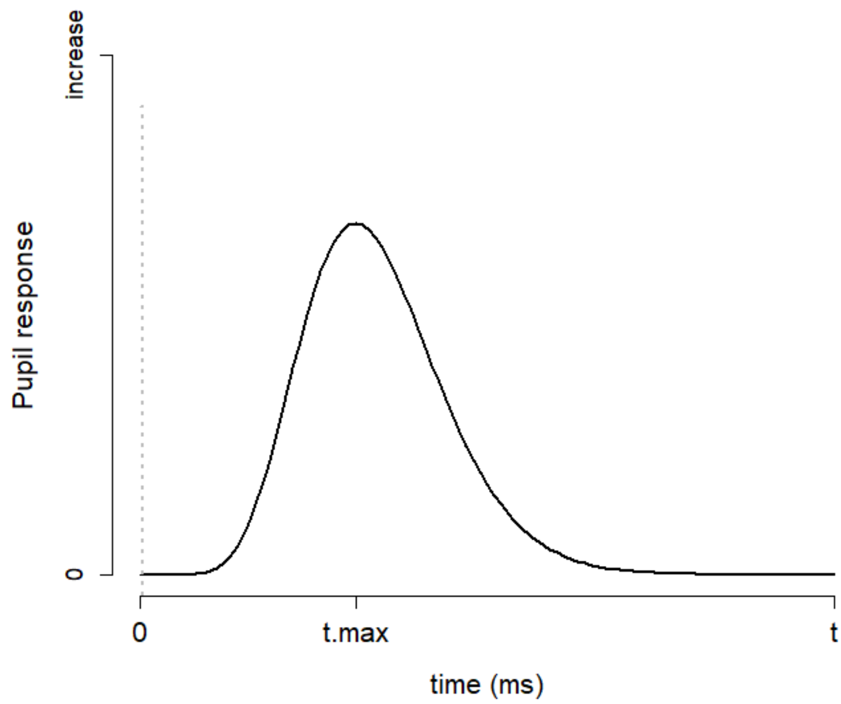

Figure 4.2: The convolution method that transforms a pulse at t = 0 into a pupillary response with $n = 10.1$ and $t.max = 930$ ms. The maximum increase of the pupil occurs at t.max.

The convolution method used here to transform the module activity of the cognitive model into a pupillary response was adapted from Hoeks and Levelt (1993). The rate at which the pupil responds is depicted by l.

$$l = n \; / \; t.max$$

The shape of the response is determined by k:

$$k = n + 1$$

The convolution method provides a pupil response y over a time course in milliseconds. This time course is specified by t. A string of pulses representing module activity can be convolved using:

$$y(t) = \frac{l^k * t^{k-1} * e^{-l*t}}{\gamma(k)}$$

As was mentioned earlier, the pulses have an additive effect on the pupil response. Therefore, a combination of pulses causes a larger pupil response. An example of this phenomenon can be found in Figure 4.3.

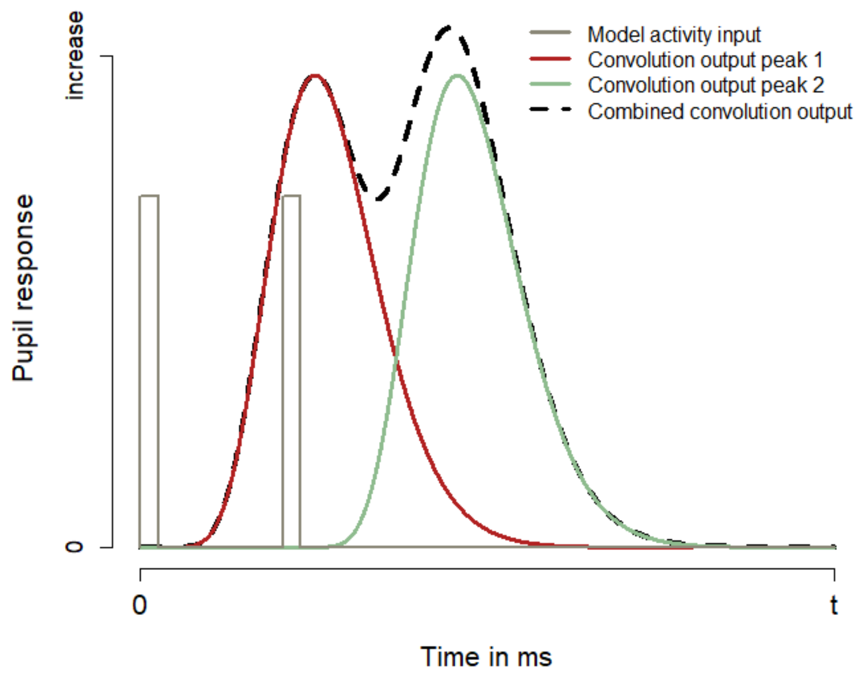

Figure 4.3: If two periods of cognitive activity occur within each other's pupil response, the convolution output is added. The convolution output (red and green) of the two pulses are combined into one accumulated output (black).

Chapter 5

Model results

After the cognitive model was created, the latency factor and the activation levels of the mathematical chunks were adapted for the model response times to approximate the participants' response times. The latency factor was set to 0.95. The chunk activation level of the subtraction facts was set to 2.8 and the division facts to 2.2. All the response times can be viewed in Figure 5.1. For problem types 1 to 4, the model response times are equal for the internal and external representations. As is explained previously, the model only distinguishes

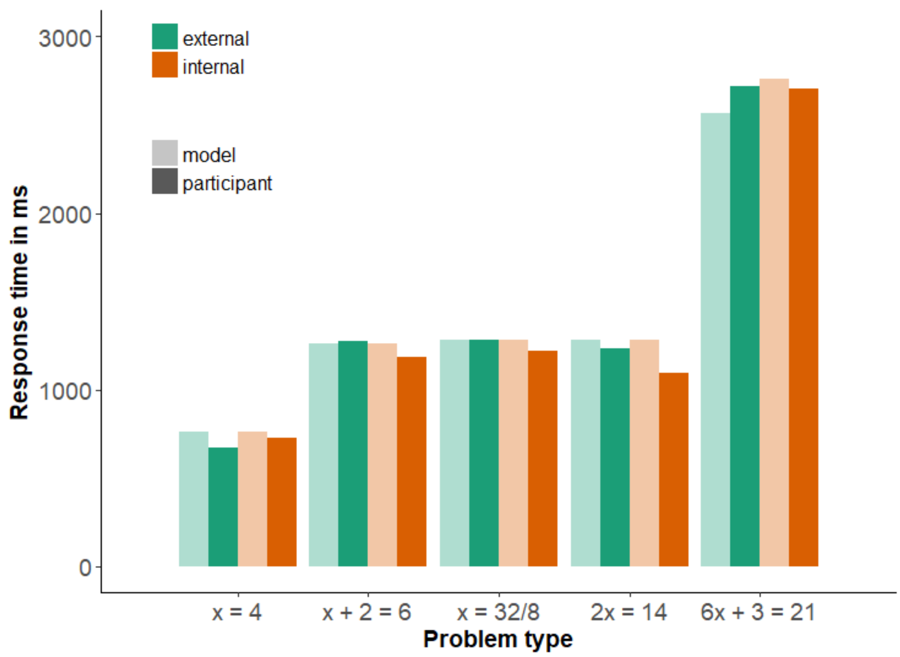

Figure 5.1: The model response times compared to the participants' times. The response time depicts the time between the onset of the equation and the first mouse-click. No error bars are displayed because the model has no variability in answering speed.

between the external and internal representation for problem type 5. Figure 5.1 shows that the model does not have a perfect fit of the participants' response times, but does approach it.

The model provides the activity of the modules for each experimental condition. This module activity is transformed linearly to fit each individual experimental trial. After scaling, the activity of the modules is added to the experimental trials. For each condition, this provides an indication of the onset and duration of the cognitive processes during a trial.

For each separate trial, the activity of the modules is convolved to a predicted pupillary response. The convolution method with $t.max = 930$ and $N = 10$ was applied to add the pupillary response per module to each trial. To determine which modules explain the participants' pupil dilation the best, linear models were fitted. The models provide a β-coefficient for each of the modules within the linear model. The convolved module activity can be multiplied with the β-value to indicate the effect of the module on the pupil dilation. A prediction of the pupil can be made by adding the effects of all the modules. For each time point in all the trials, the convolved module activity is multiplied with the β-value and then summed. This provides a prediction of the pupil size, based on the linear model.

The first linear model contained all five modules of the cognitive model. This large model was compared with models containing only four modules. An ANOVA was performed to determine the difference between the mode;s. The larger model was significantly better than a model with four modules; ΔBIC $>$ 5590, $\chi^2(1) > 5600$, p $<$ 2.2e-16. This indicates that all five modules provide a significant contribution in explaining the pupil dilation. However, the contribution of the motor module was rather low with $\beta = 0.562$, compared to the other modules where the absolute β-values ranged from 2.24 to 13.9.

To investigate the best fit of the linear model to the pupil response, multiple convolution parameters are applied. This results in a pupillary response with a different shape and rate. The correlation of the pupil prediction of the model compared to the actual pupil dilation is used as a measure of fit. Next to the linear model containing all five modules, a linear model containing all modules with the exception of the motor module is tested. Because the motor module had a small effect, the linear model containing the other four modules might have a higher correlation when other $t.max$- and N-values are applied for convolution. The model containing all five modules will be referred to as LM1 and the model containing the procedural, declarative, visual and imaginal module is referred to as LM2.

Figure 5.2 shows the correlation coefficient of the linear models across the two convolution parameters. The $t.max$-parameter does not occur before 750 ms and does not exceed 1100 ms, because this would not be biologically plausible. In addition, the N-value does not go lower than 8 because the resulting shape of the convolution is not in line with the pupil response. An N-value higher than 10 results in a decrease in correlation.

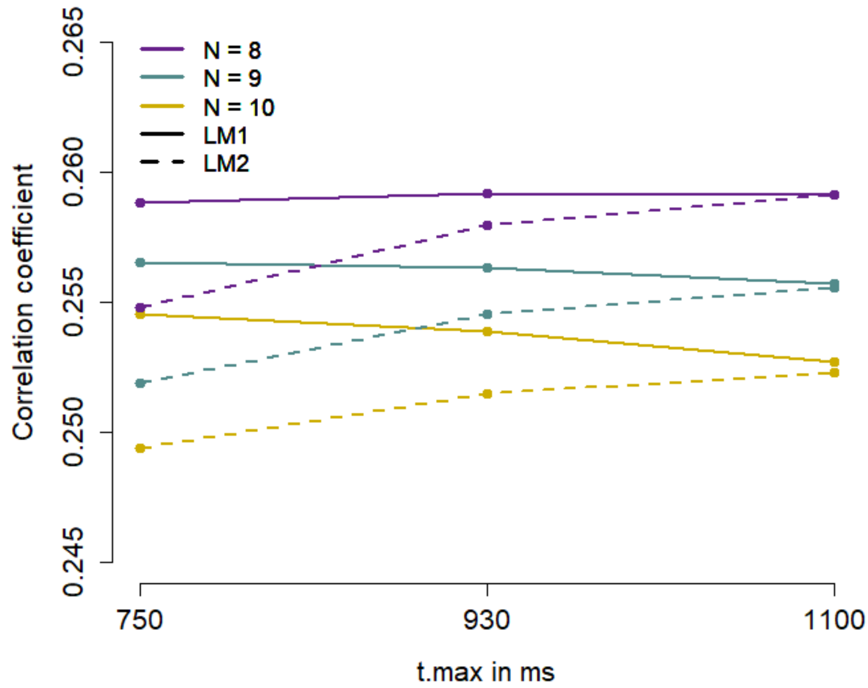

Figure 5.2: The correlation of the predicted pupil response and the participants' pupil dilation. Linear model 1 contains all five modules of the cognitive model and linear model 2 all but the motor module.

The highest correlation coefficients occur with an N-value of 8. LM1 peaks at $t.max$ = 930 and LM2 at $t.max$ = 1100, though the correlation of LM1 is higher. Because the linear model containing the motor module is significantly better than the model without, and the correlation of linear model 1 is best at N = 8 and $t.max$ = 930, the model results of LM1 with the aforementioned convolution parameters are selected for further analysis. The β-coefficients and intercept of the model can be seen in Table 5.1.

Table 5.1: The results of the linear model containing all modules.

Module	Coefficient	Standard error	t-value
Intercept	-0.138	0.182	-0.754
Declarative	9.71	0.0303	320
Procedural	-13.9	0.143	-97.7
Imaginal	2.24	0.0278	80.5
Visual	9.98	0.0994	100
Manual	0.56	0.00751	74.9

Stimulus-locked view

A predicted pupillary response is created by adding the effects of the individual modules determined in the linear model. Just like the pupil size of the participants, the predicted pupil size can be viewed from two perspectives. Figure 5.3 displays the stimulus-locked point of view of the predicted pupil size. Problem type 1 and 5 clearly stand out from the other three problem types. In addition, problem type 5 with internal representation displays a larger increase compared to problem type 5 with external representation. There is no clear distinction between the external and internal representations for problem types 1 to 4, because the cognitive model only took this difference into account for problem type 5. It is observable that the predicted pupil response rises slowly, with the exception of internal type 5, but decreases even more slowly.

To examine the predicted pupil response, the effect of each module can be looked into. Figure 5.4 shows the effect of the modules for problem type 5 for both external and internal representations. The Figure shows a large negative impact of the procedural module and a positive effect of the other four modules. The visual and procedural module are already active prior to the stimulus. This is caused by the model attending the fixation cross. Combined, the two modules make up for a predicted PCPS of 0 during the baseline. When the stimulus is shown, several productions are activated, resulting in a steep negative effect of the procedural module. The problem is attended and read by the visual module, and held by the imaginal module. Next, the equation is examined and the first chunk is requested from declarative memory. From 1000 ms, a clear difference is visible between the external and internal representation. The external graph

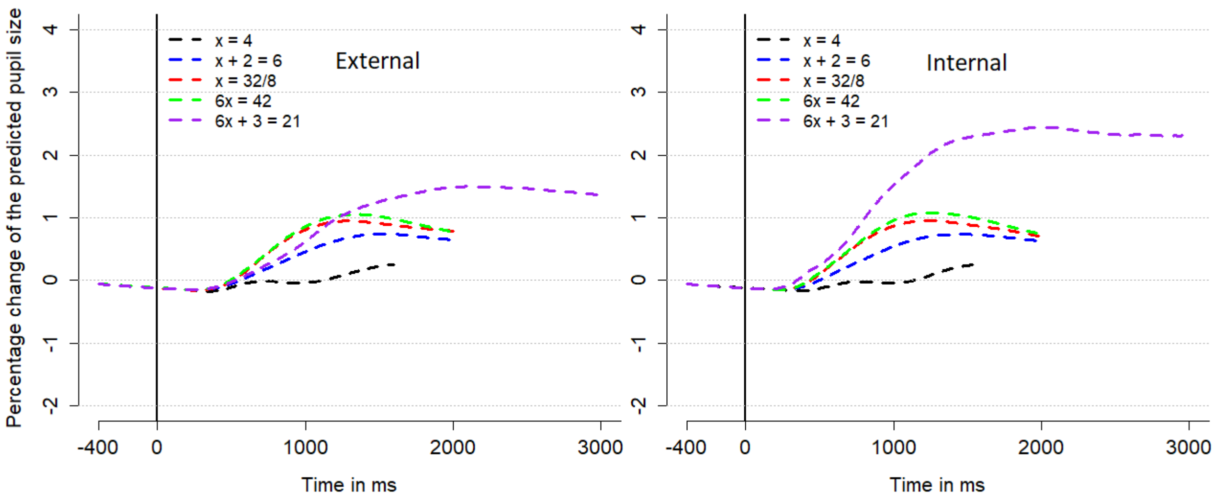

Figure 5.3: The percentage change of the predicted pupil size over time with a stimulus-locked point of view. On the left the model's prediction on trials with external representation are displayed, and the internal representation trials on the right. At t = 0 the problem is shown.

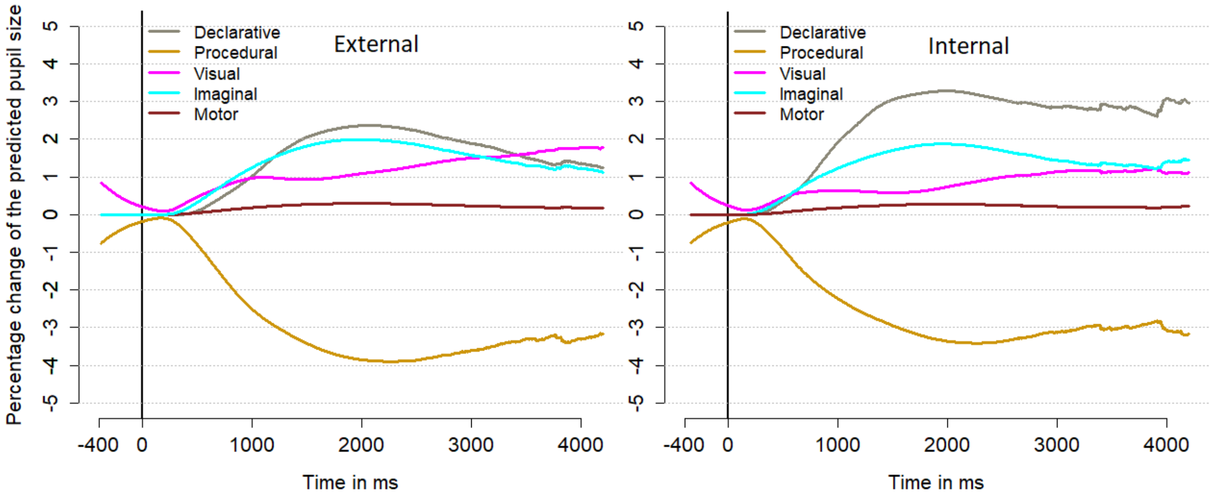

Figure 5.4: The predicted percentage change of the pupil size caused by each individual module over time with a stimulus-locked point of view. The external representation is displayed on the left and the internal representation on the right. At t = 0 the problem is shown.

shows an increase in visual and procedural activity compared to the internal graph. This is because the with the external representation, the cognitive model refers back to the screen, requiring more procedures and the visual module. The internal graph shows a large increase in declarative module activity, because it refers back to the equation through memory. After 2000 ms, the effect of the visual module does increase in the internal module. This is due to finding the answer on the answering screen. The motor module is activated when the mouse is clicked and moved, but it has little effect on the predicted pupil size.

The stimulus-locked module activity of the other problem types can be found in Appendix C. Problem type 1 shows no activity in the declarative module, since no chunk from memory needs to be retrieved. The visual activity is more accentuated because the attending the answering screen follows the equation quickly. Problem types 2, 3 and 4 show similar module activities. Problem 3 and 4 have a higher declarative effect, due to the lower activation of the mathematical chunks.

The course of the predicted pupil size is now shown for the stimulus-locked perspective. With the additional information on how the model's predicted pupil size is formed, the model results can be compared to the results of the participants. Figure 5.5 displays the stimulus-locked view results of the actual and predicted PCPS.

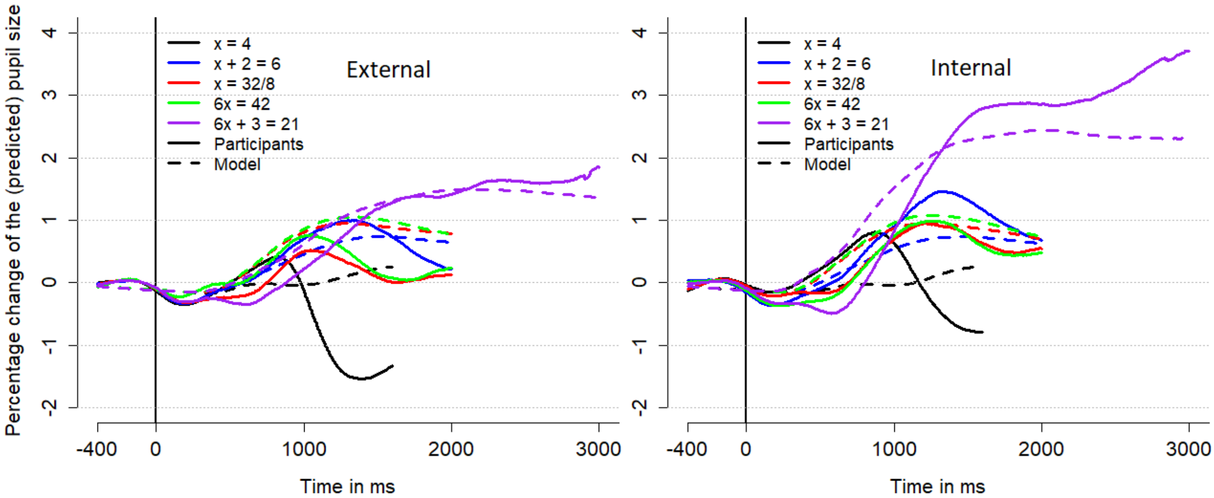

Figure 5.5: The percentage change of the actual and predicted pupil size over time with a stimulus-locked point of view. External representations are displayed on the left and internal representations on the right. At t = 0 the problem is shown.

Several similarities between model and data are observable in Figure 5.5. Both model and data show a small negative PCPS after the problem is shown. For the external representation, the onset of the pupillary response is not perfect, but does show similar slopes. The model closely follows the data mostly for external problem type 5. In the internal representation, the PCPS peaks of problem types 2, 3 and 4 occur almost simultaneously for the model and data. Internal problem type 5 is also clearly distinguishable from the other problem types, though it does not have a perfect fit with the data.

Two dissimilarities are clearly visible. The model shows a small change in PCPS for problem type 1 for both external and internal representations. The, in comparison, large pupil response displayed by the participants' data, is far from recognizable in the model results. In addition, the participants show a fast increase, decrease and another increase in PCPS for problem types 1 to 4. The model only shows an increase, followed by a slow decrease. The model is able to provide a general prediction of the change of the pupil size, but does not follow the course of the pupil size perfectly.

Response-locked view

As was visible in the experiment results, the maximum pupil response occurs just after clicking the mouse. This response-locked perspective provides insight in where the maximum pupil response occurs and which modules affect this maximum response. Figure 5.6 shows the model's predicted change of the pupil size for all conditions.

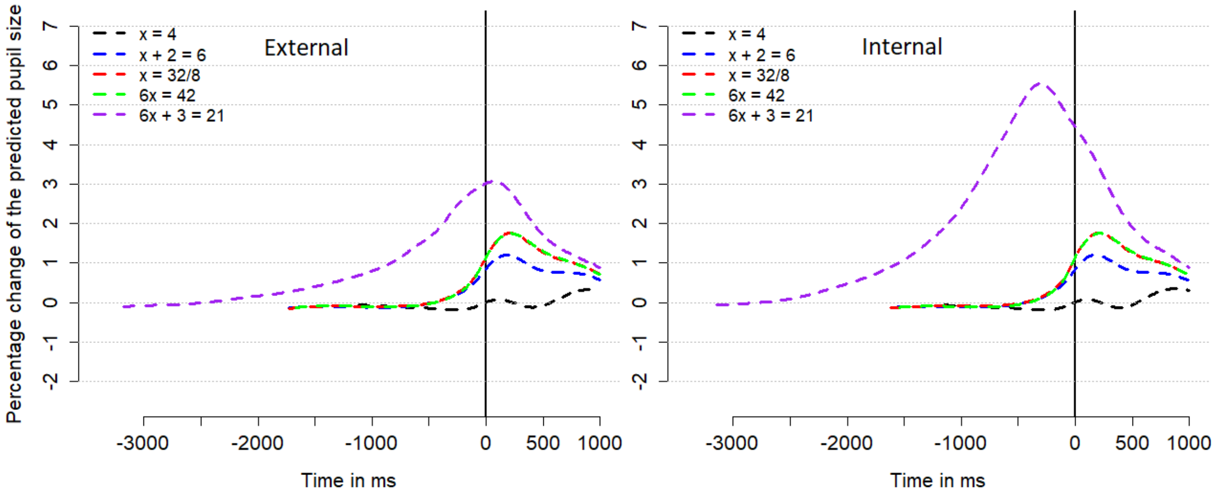

Figure 5.6: The percentage change of the predicted pupil size over time with a response-locked point of view. The external representations are on the left and internal representations on the left. At t = 0 the mouse is clicked.

Figure 5.6 shows a maximum pupil response just after clicking the mouse for almost all conditions. Problem type 2, 3 and 4 show a clear peak after t = 0, but problem type 1 and 5 deviate from this expected time of maximum pupil size. The peak of problem type 5 occurs too early for both the external and internal representation. The peak of problem type 1 is only just visible. Finally, as was visible in the stimulus-locked view, the model displays a slow decrease in PCPS after the peak occurs.

To determine what influences the pupil size predictions of the model, the activity of the individual modules are investigated. Figure 5.7 displays the effect of each module for problem type 5 from a response-locked perspective. Again, the negative effect of the procedural module is visible. Prior to clicking the mouse, the external representation requires the procedural module to perform productions to refer to the equation on the screen. This increases the negative procedural effect and the effect of the visual module. The internal representation requires less productions to refer back to the equation from memory, compared to referring to the equation on the screen in the external representation. This is why the internal procedural and visual effect is smaller than the external effect, but the declarative module effect is larger. This large increase in effect of the declarative memory is what causes problem type 5 to have an early peak in the internal condition. The slow decrease of the model's PCPS in Figure 5.6 can be caused by the increasing effect of the visual and motor module, which start to increase during the decrease of the declarative module.

The module activity of problem types 1 to 4 are displayed in Appendix D. The late peak in problem type 1 is explained by the visual module, which is activated when finding the answer. Problem types 2 to 4 show similar module effects.

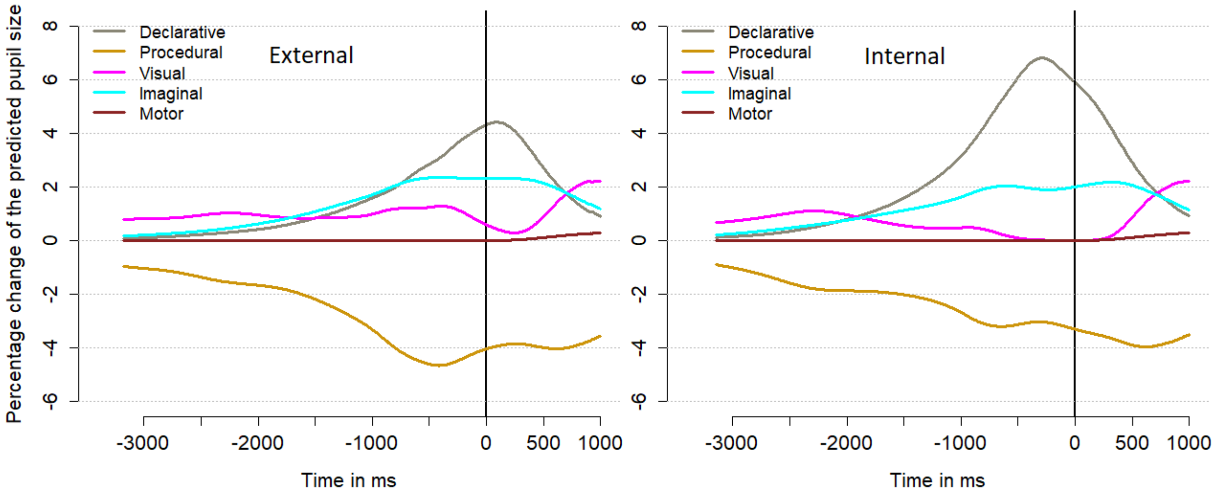

Figure 5.7: The predicted percentage change of the pupil size caused by each individual module over time with a response-locked point of view. The external representation is displayed on the left and the internal representation on the right. At t = 0 the mouse is clicked.

Problem type 2 has a smaller effect of the declarative module, due to the higher activation of the subtraction chunk compared to the division chunks in problem type 3 and 4. The peak of PCPS in problem type 2, 3 and 4 is mainly caused by the declarative and imaginal module, which peak at similar time points. The imaginal buffer is updated prior and after the memory request, which explains the parallel activity.

The predicted pupillary response of the model is compared to the participants' pupil size in Figure 5.8. When looking at the external condition, the model follows the data to just after the peak. Here the model's predicted PCPS does not decrease fast enough and displays no valley as the participants' PCPS does. Again, problem type 1 does not follow the pupil data. The internal conditions show more dissimilarities between the model and the participants. The model predicts a smaller PCPS for problem types 1 to 4. Problem type 5 large follows the effect of the declarative module and peaks higher and earlier than the participants. In both the external and internal condition, the predicted PCPS does not decrease fast enough after reaching the PCPS peak, in order to display valley as in the participants' data.

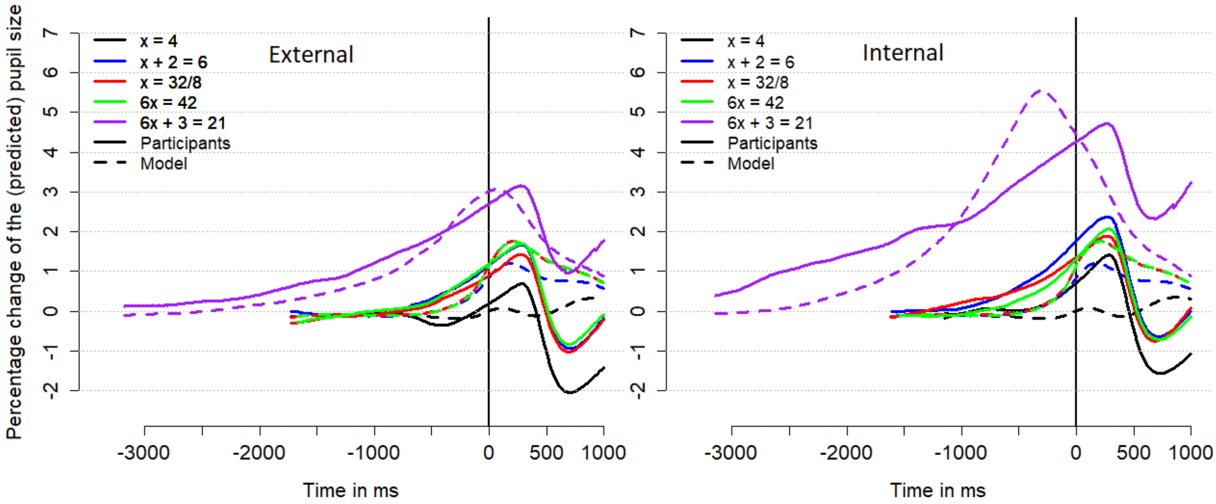

Figure 5.8: The percentage change of the participants' and predicted pupil size over time with a response-locked point of view. The external representations are on the left and internal representations on the right. At t = 0 the mouse is clicked.

Chapter 6

Discussion

Previous literature mentioned terms as mental effort, attentional effort and cognitive activity to influence pupil dilation, but never mentioned what this entails. The goal of the current study was to provide an overview of the cognitive factors that influence the pupil dilation. This research applied a model-based analysis on an algebra task containing five equation types, which were either continually presented externally, or required internal representation. The results in Figure 3.5 show that problem difficulty is reflected in the pupil response. In addition, for one equation type ($6x + 3 = 21$), there is a significant difference in pupil dilation between the problem representations.

Several phenomena are made visible at the start of a trial and after the participants indicate they know the answer. When the stimulus is shown, the pupil decreases in size compared to the baseline. This could be due to the effect of the previous trial. However, this could also be caused by a pupil inhibiting cognitive factor, since the length of this decrease is different between problem types. Figure 3.4 shows that easy problems cause a fast increase in pupil size, whereas the pupil seems to respond slower in harder problems. An additional phenomenon is the large decrease in pupil size after the participant indicates they know the answer. The cause may lie in the fact that people stop performing processes related to problem solving. Another option is the inhibiting factor, which causes the pupil to decrease in size.

The cognitive model of the experimental task is partially based on the pupil results of the participants. A distinction is made for the hardest problem type for the two representation conditions. The module activity of the cognitive model is convolved and with linear models fitted to the experimental data. The modules representing declarative memory, procedural memory, working memory, vision and motor processes have a significant effect on the pupil dilation. The model results in Figure 5.3 display the difference in problem types, as well as the difference in the interpretation for the hardest equation type. Furthermore, the model shows a small decrease in predicted pupil response when the stimulus is shown. This is caused by the influence of the procedural module. According to the cognitive model, procedural memory has a decreasing effect on the pupil dilation. Previous research labeled attentional effort as an influencing factor of pupil dilation (Hoeks & Levelt, 1993). Procedural memory is referred to as an unconscious process, and therefore does not lie within the scope of attentional effort. The exact influence of procedural memory can be found in Figure 5.4 and 5.7.

An important cognitive factor influencing pupil dilation, is declarative memory. The predicted pupil response of problem type 5 mainly follows the convolved activity of the declarative module (see Figure 5.3 & 5.6). The effect of this cognitive process is clearly missing in problem type 1, where the model lacks a clear pupil response. The pupil dilation measured for problem type 1 might have been caused by a cognitive factor lying outside the scope of ACT-R.

This research has revealed some of the cognitive processes influencing pupil dilation through model-based analysis. The model predictions do not show a perfect fit, but do provide an indication of which cognitive processes influence how the pupil responds. It provides evidence that a model-based analysis can help predict which cognitive processes are applied in experimental tasks. This method can be applied in other tasks to further discover the influence of cognitive tasks on pupil dilation. The cognitive architecture ACT-R contains modules that were not necessary for this task, but that might represent cognitive factors that also influence pupil dilation.

Future research can apply a model-based approach to find additional cognitive factors that influence pupil dilation. In addition, studies might focus on factors that are still missing from ACT-R. For example, adding a level of attention to the architecture might provide more accurate predictions. Also, the participants experienced an increase of pressure during the internal representation trials. This might be incorporated in the ACT-R architecture as well. Furthermore, the cognitive processes represented in ACT-R can be linked to the brain. The current research can be extended to fMRI measures to create a link between pupil dilation and activity in specific brain regions. For example, the execution of procedures in the procedural module has been linked to the thalamus (Taatgen, Lebiere, & Anderson, 2006). This brain region can be innervated by the LC-NE system (Nieuwenhuis, Aston-Jones, & Cohen, 2005) and the LC-NE system is theorized to closely follow the change of the pupil size (Gilzenrat et al., 2010). The LC-NE system innervates specific regions in the brain. By applying model-based analysis, change in the pupil dilation might be linked to activation in specific brain regions.

Concluding remarks

A model-based approach provides an indication of the cognitive processes applied in a task, as well as the effect of each of these processes. By applying this type of approach on a simple measurement such as pupil dilation, complex findings can come to light that may help improve future cognitive modeling and our understanding of the mind.

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Appendix A

Experiment instructions

The instructions prior to the experiment are shown on two screens. The participant had to click the left mouse button to move on to the next screen. The two instruction screens were as followed:

Welcome!

In this experiment you are asked to solve algebra questions such as:

$$4x = 8$$

The algebra problem will either remain on the screen,
or is removed after a short time.
When you know the value of X, click the left mouse button.
A circular keypad will appear where you
can select your response.
You can select your response by clicking on the correct answer
with the mouse.

You only have a short time to select your response,
so make sure you already know the answer
before you continue to the keypad!

Click the left mouse button to continue.

The experiment begins with a short practice session,
followed by four experimental blocks.

Click the left mouse button
to start the practice session.

Appendix B

Experimental results

These tables provide results of statistical analyses on the behavioural data. Due to multiple comparisons, the p-values are adjusted using Holm, unless stated otherwise. Three levels of significance are indicated:

$p \leq 0.05$	*
$p \leq 0.01$	**
$p \leq 0.001$	***

Table B.1: The effect of problem type on the response time

Equation Type	Equation Type	t	df	p	Adjusted p
1	2	-7.23	20	5.39e-07	3.23e-06***
1	3	-6.20	20	4.63e-06	2.32e-05***
1	4	-6.13	20	5.44e-06	2.32e-05***
1	5	-9.34	20	9.85e-09	9.85e-08***
2	3	-0.247	20	0.807	0.807
2	4	1.00	20	0.330	0.660
2	5	-8.05	20	1.05e-07	7.35e-07***
3	4	1.80	20	0.0872	0.262
3	5	-8.81	20	1.36e-08	1.23e-07***
4	5	-9.16	20	2.53e-08	2.03e-07***

Table B.2: The effect of problem type on the accuracy (percentage correct)

Equation Type	Equation Type	t	df	p	Adjusted p
1	2	3.25	20	4.00e-03	0.0240*
1	3	2.78	20	0.0153	0.0461*
1	4	3.12	20	5.35e-03	0.0267*
1	5	7.82	20	1.65e-07	1.65e-06***
2	3	1.29	20	0.210	0.631
2	4	0.958	20	0.349	0.698
2	5	7.60	20	2.55e-07	2.29e-06***
3	4	-0.750	20	0.462	0.698
3	5	6.91	20	1.04e-06	7.31e-06***
4	5	7.41	20	3.76e-07	3.01e-06***

Table B.3: The effect of problem state on the accuracy (percentage correct)

Problem	t	df	p	Adjusted p
1	-0.0178	20	0.986	1.00
2	-1.34	20	0.196	0.785
3	-0.528	20	0.603	1.00
4	1.27	20	0.220	0.785
5	-2.40	20	0.0260	0.130

Table B.4: The effect of problem type on the mean amount of fixations

Equation Type	Equation Type	t	df	p	Adjusted p
1	2	-8.34	20	6.10e-08	4.27e-07***
1	3	-7.56	20	2.78e-07	1.11e-06***
1	4	-7.78	20	1.80e-07	1.08e-06***
1	5	-9.68	20	5.49e-09	5.49e-08***
2	3	-0.560	20	0.582	1.00
2	4	0.367	20	0.718	1.00
2	5	-7.76	20	1.85e-07	1.08e-06***
3	4	1.23	20	0.232	0.697
3	5	-8.69	20	3.20e-08	2.65e-07***
4	5	-8.73	20	2.95e-08	2.65e-07***

Table B.5: The effect of problem state on the mean amount of fixations

Problem	t	df	p	Adjusted p
1	-0.320	20	0.752	0.752
2	-4.24	20	0.000405	0.00162**
3	-2.76	20	0.0120	0.0361*
4	-4.83	20	0.000102	0.000512***
5	-2.67	20	0.0148	0.0361*

Appendix C

Pupil results

These tables provide results of statistical analyses on the pupil data. Due to multiple comparisons, the p-values are adjusted using Holm, unless stated otherwise. Three levels of significance are indicated:

$p \leq 0.05$	*
$p \leq 0.01$	**
$p \leq 0.001$	***

Table C.1: The effect of problem type on the maximum change in pupil dilation

Equation Type	Equation Type	t	df	p	Adjusted p
1	2	-5.02	20	6.58e-05	0.000395***
1	3	-3.85	20	9.99e-04	0.000399***
1	4	-4.55	20	1.96e-04	0.000379***
1	5	2.49	20	3.61e-06	3.25e-05***
2	3	-6.32	20	0.0216	0.0648
2	4	0.714	20	0.483	0.483
2	5	-5.38	20	2.91e-05	0.000399***
3	4	-2.09	20	0.0496	0.0991
3	5	-6.51	20	2.38e-06	2.38e-05***
4	5	-5.67	20	1.51e-05	0.000121***

Table C.2: The effect of problem state on the maximum change in pupil dilation

Problem	t	df	p	Adjusted p
1	3.07	20	0.00606	0.0303*
2	1.35	20	0.0518	0.156
3	1.17	20	0.193	0.386
4	2.07	20	0.254	0.386
5	2.93	20	0.00832	0.0332*

Appendix D

Linear models

In this appendix, the influence of the convolution parameters on the linear models are displayed. Two linear models are tested on the paramters: LM1 containing all five ACT-R modules and LM2, containing the declarative, procedural, imaginal and visual modules. The β-coefficients and t-values are reported.

Table D.1: Linear model effects for convolution parameters N = 8, t.max = 750 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.115**	-0.63	**-0.147**	-0.811
Declarative	**8.13**	308	**8.27**	314
Procedural	**-11.1**	-108	**-11.7**	-114
Imaginal	**2.32**	107	**2.39**	110
Visual	**6.61**	93.1	**8.80**	130
Motor	**0.681**	97.5		

Table D.2: Linear model effects for convolution parameters N = 9, t.max = 750 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.115**	-0.628	**-0.137**	-0.755
Declarative	**7.64**	304	**7.74**	307
Procedural	**-9.68**	-103	**-9.83**	-104
Imaginal	**2.13**	106	**2.13**	106
Visual	**5.59**	87.4	**7.47**	122
Motor	**0.700**	104		

Table D.3: Linear model effects for convolution parameters N = 10, t.max = 750 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.115**	-0.628	**-0.128**	-0.706
Declarative	**7.24**	300	**7.28**	301
Procedural	**-8.53**	-98.1	**-8.38**	-96.2
Imaginal	**1.99**	106	**1.93**	102
Visual	**4.79**	82.3	**6.41**	114
Motor	**0.712**	108		

Table D.4: Linear model effects for convolution parameters N = 8, t.max = 930 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.150**	-0.822	**-0.195**	-1.07
Declarative	**10.9**	327	**11.1**	337
Procedural	**-18.9**	-113	**-20.6**	-124
Imaginal	**3.03**	93.7	**3.31**	104
Visual	**13.7**	114	**16.2**	146
Motor	**0.450**	54.7		

Table D.5: Linear model effects for convolution parameters N = 9, t.max = 930 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.143**	-0.783	**-0.187**	-1.03
Declarative	**10.2**	324	**10.5**	332
Procedural	**-16.2**	-105	**-17.5**	-114
Imaginal	**2.59**	86.7	**2.82**	95.0
Visual	**11.7**	107	**14.1**	138
Motor	**0.514**	65.7		

Table D.6: Linear model effects for convolution parameters N = 10, t.max = 930 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.138**	-0.754	**-0.179**	-0.98
Declarative	**9.71**	321	**9.88**	327
Procedural	**-14.0**	-97.8	**-14.9**	-105
Imaginal	**2.24**	80.5	**2.40**	86.2
Visual	**9.98**	100	**12.2**	129
Motor	**0.562**	74.9		

Table D.7: Linear model effects for convolution parameters N = 8, t.max = 1100 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.205**	-1.12	**-0.215**	-1.18
Declarative	**14.0**	338	**14.0**	344
Procedural	**-35.5**	-146	**-35.9**	-152
Imaginal	**5.43**	122	**5.52**	127
Visual	**27.4**	155	**27.9**	173
Motor	**0.0788**	7.99		

Table D.8: Linear model effects for convolution parameters N = 9, t.max = 1100 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.193**	-1.06	**-0.214**	-1.17
Declarative	**13.3**	337	**13.4**	343
Procedural	**-32.1**	-142	**-33.0**	-149
Imaginal	**4.88**	118	**5.04**	124
Visual	**24.8**	153	**26.0**	172
Motor	**0.187**	20.2		

Table D.9: Linear model effects for convolution parameters N = 10, t.max = 1100 ms.

Module	LM1 - β	LM1 - t value	LM2 - β	LM2 - t value
Intercept	**-0.182**	-0.995	**-0.211**	-1.15
Declarative	**12.6**	336	**12.8**	341
Procedural	**-29.0**	-38	**-30.0**	-144
Imaginal	**4.37**	113	**4.55**	119
Visual	**22.4**	150	**23.9**	169
Motor	**0.275**	31.2		

Appendix E

Stimulus-locked module effects

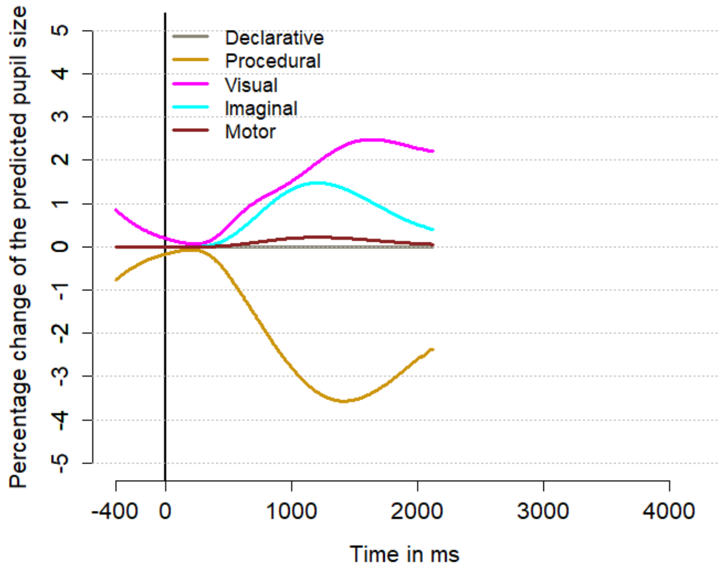

Figure E.1: Stimulus-locked view of the module effects on problem type 1.

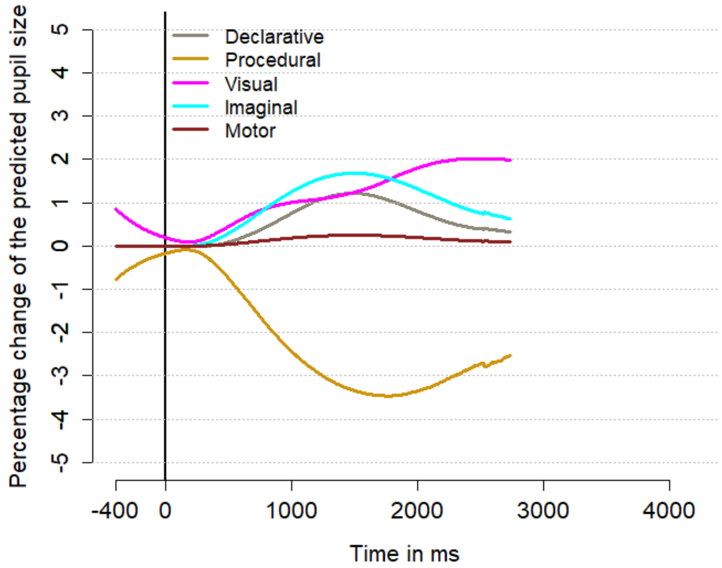

Figure E.2: Stimulus-locked view of the module effects on problem type 2.

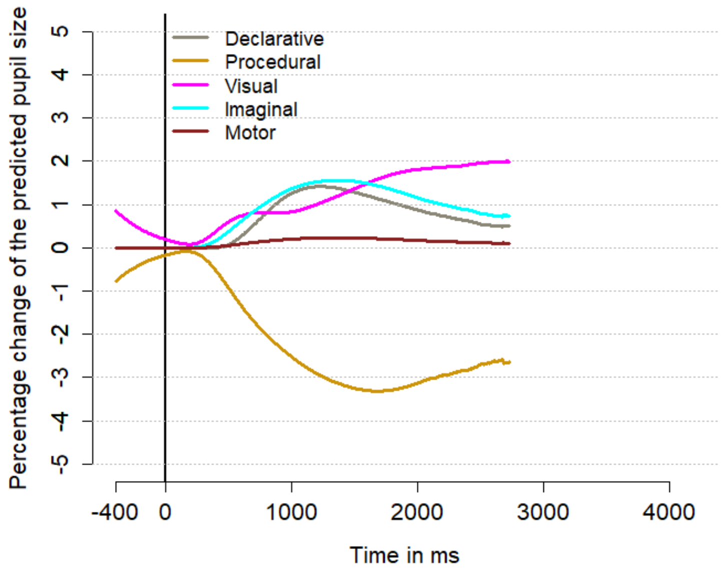

Figure E.3: Stimulus-locked view of the module effects on problem type 3.

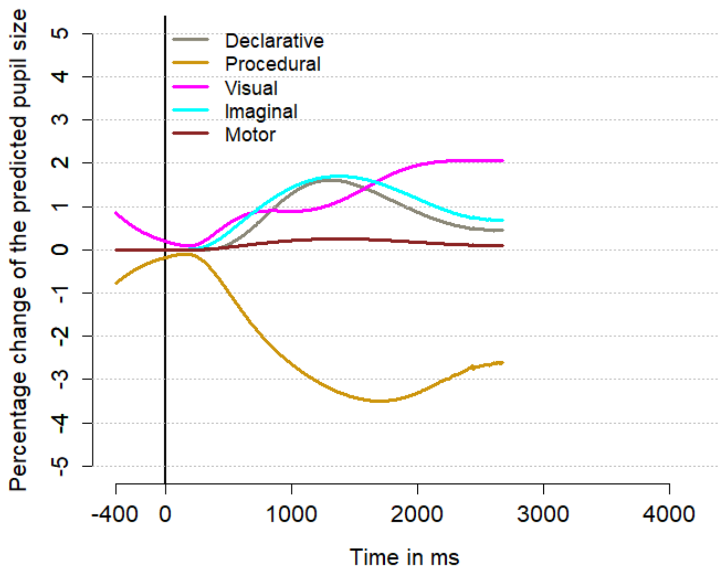

Figure E.4: Stimulus-locked view of the module effects on problem type 4.

Appendix F

Response-locked module effects

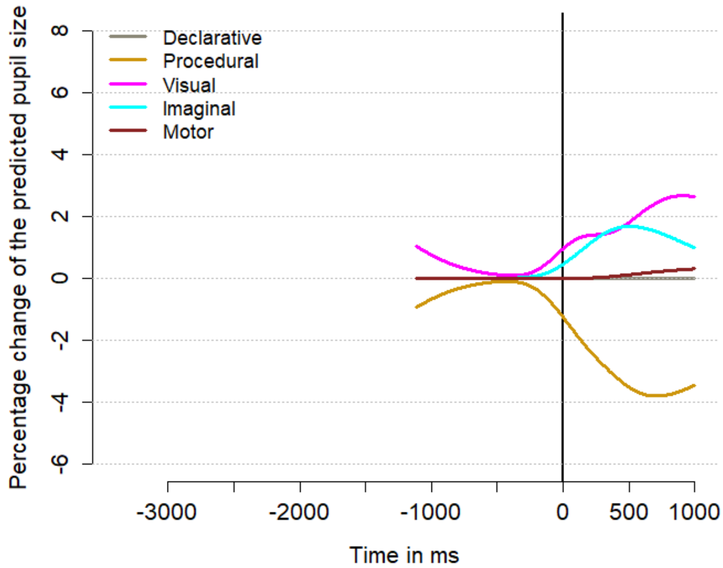

Figure F.1: Response-locked view of the module effects on problem type 1.

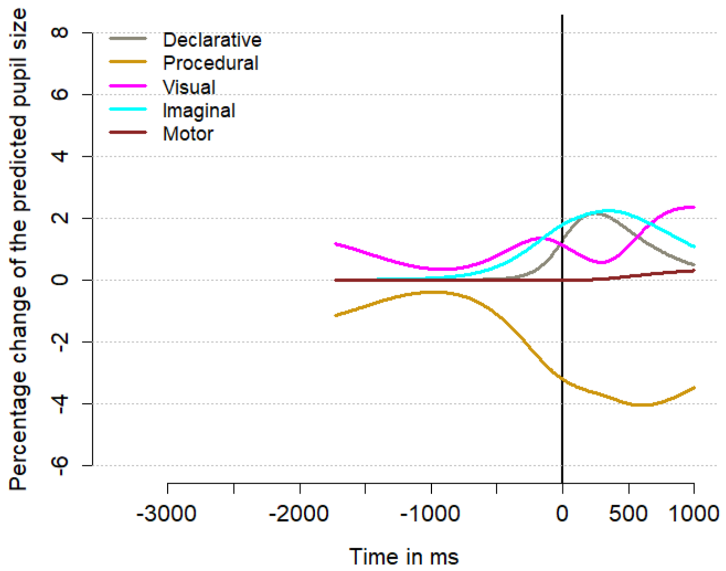

Figure F.2: Response-locked view of the module effects on problem type 2.

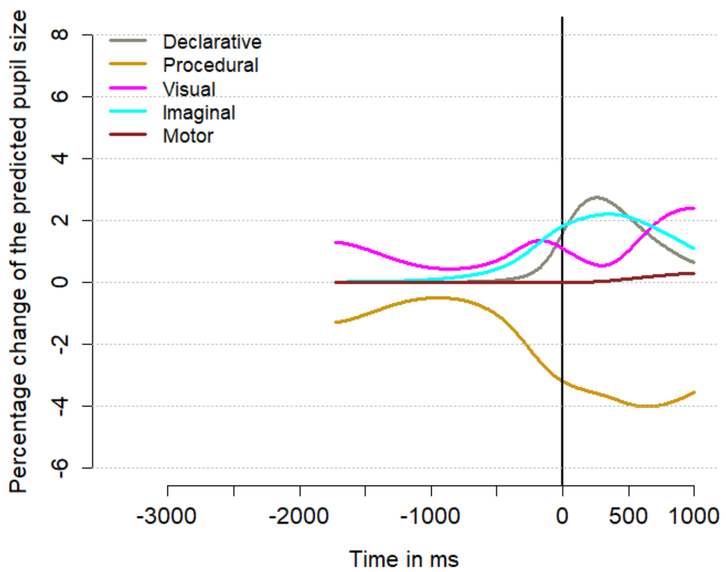

Figure F.3: Response-locked view of the module effects on problem type 3.

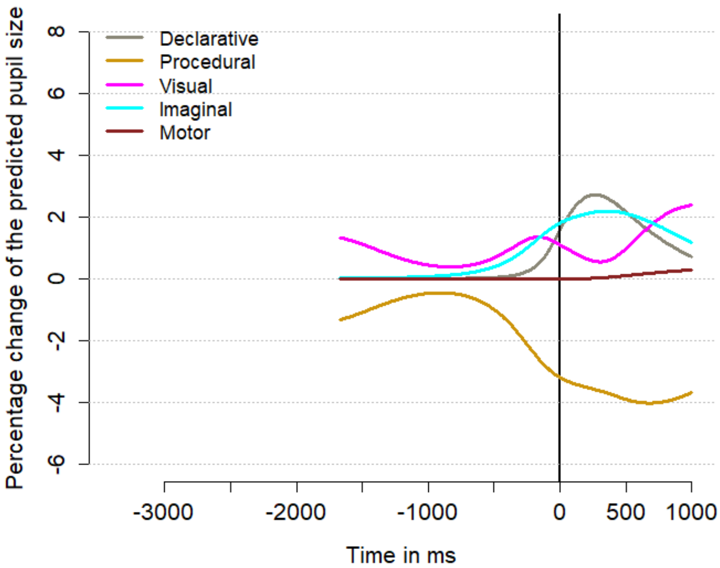

Figure F.4: Response-locked view of the module effects on problem type 4.