MASTER'S THESIS

# Analysis of new models of the three-dimensional bin packing problem for the application to the parcel delivery service industry


*Author:*
Margot KHO

*First supervisor:*
Dr. ir. S. FAZI

*Second supervisor:*
Prof. dr. G. SIERKSMA


September 25, 2018


## Abstract

The unceasing increase of e-commerce volumes worldwide has been putting under pressure the sector of parcel delivery service. Due to a highly competitive market, the different logistic service providers strive to be efficient in every segment of the logistic chain. Among these segments, the packing of parcels into containers is a crucial aspect that, if not done properly, may cause additional costs, such as the use of more resources than the ones actually needed. In this research, we investigate such a packing problem and model it as a three-dimensional bin packing problem (3D-BPP). The 3D-BPP can be defined as the placement of a set of different parallelepiped sized items parallel to the bin dimensions that have to be placed in a minimized number of bins. This problem is a well-known NP-hard problem. We start by discussing the model proposed by Chen et al. (1995), where items may rotate 360° . Next, we develop mathematical models with flexible item rotations and the possibility to include layers, i.e. shelves, by adding constraints to the model of Chen et al. (1995). Even though the new models solve different problems, still their solutions may be used as an indication for the basic model. Our goal is to test the computational performances of an industrial solver, performing a branch-and-bound procedure, for several input classes among which also instances generated using real-world data from a Dutch parcel delivery service provider. Results show that for larger instances we can obtain better results mainly regarding the computational time, with some of the newly developed models. This can potentially open opportunities for enhanced automation and improvement in the packing process.

# Contents

# List of Tables

## List of Figures

## Abbreviations

**3D-BPP** Three-dimensional bin packing problem

**PDS** Parcel delivery service

# 1 Introduction

The e-commerce in Europe has grown substantially in the last decade. In 2015, the sector generated a total online revenue of 455.3 billion euros; whereas, in 2016, it was expected to generate a total of 509.9 billion euros resulting in a 12 percent growth (Ecommerce, 2017). Inevitably, this market is generating high demand for dedicated parcel delivery service providers (PDS), causing increasing difficulties in the last mile logistics (Bortfeldt and Wäscher, 2013).

For PDS, to maintain competitiveness, the efficient use of the available capacity is a key element in such a highly competitive market (Kim et al., 2014). In fact, under-performance in logistic processes can lead to unnecessary costs and unsatisfied customers (Bortfeldt and Wäscher, 2013). For example, unused space in containers, mistakes, and damage to transported goods often occur as a result of poor container loading.

In this research, we explore the problem of loading a set of parcels into (roll-)containers (Figure 1) with the aim of using as efficiently as possible the available capacity, namely using the least amount of containers. The problem is inspired by a Dutch PDS internal logistic system but is common to every competitor in the sector. The company requested anonymity and, henceforth, we will refer to it as PDSNL. Earlier internal research at PDSNL indicated that significant cost savings could be realized if at least, on average, one additional parcel could be transported per roll-container shipped. In essence, this would result in fewer containers needed for transporting the same number of parcels, which, in turn, will reduce the number of trailers needed to transport the containers between depots.


Figure 1: A roll-container

The proposed problem can be linked to the three-dimensional bin packing problem (3D-BPP), defined as the packing of a set of three-dimensional parallelepiped shaped boxes into three-dimensional parallelepiped shaped bins with the aim of minimizing the number of used bins. In Chen et al. (1995) a mathematical model formulation for the classical 3D-BPP was presented. This model assumes that items may be placed in any direction into the bins, as long as their sides are parallel to the bin's sides. As the problem is NP-hard (Martello et al., 2000a), long computational times may be needed for solving to optimality or to find feasible solutions, even with relatively small instances, which is the main drawback for industrial solvers implementations. For this reason, several solving techniques have been developed and investigated with respect to their performance behaviour. Examples of these techniques are; a heuristic based on Tabu search by Lodi et al. (2002a), the exact algorithm for filling a single bin presented by Martello et al. (2000a) and a guided local search of Faroe et al. (2003). Some of these techniques construct solutions by packing the items by layers, namely items are clustered into different groups, i.e. layers, and then inserted in the bin. Specifically, in a bin, the floor of the first layer is the base of the bin itself. Whereas the floors of successive above layers start from the height of the tallest item of the layer below without placing actual physical layers in the bin, hence not all items will be physically supported. Of course, this would result in approximated solutions, however, their implementation would still be possible, since unsupported items would just fall onto below items. Overall, this technique gave positive computational results(Lodi et al., 2002a). However, none of these papers presented a mathematical model for the 3D-BPP that includes physical layers when placing the items in bins.

Based on this information, the aim of this study is to research whether the performances of an industrial solver, performing a branch-and-bound algorithm, can be improved by enhancing the classical 3D-BPP model of Chen et al. (1995) with constraints related to layers; by doing so, the solution state space is changed, but the solutions of the enhanced models will be applicable to the initial model. In this regard, we extend the model of Chen with 4 layered variants and present in this study four new mathematical models.

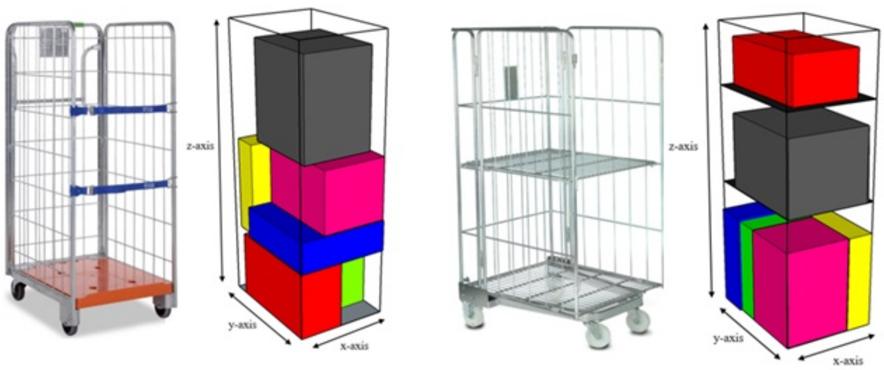



Figure 2: Regular packing and layer packing

Let us explain our concept in more detail. In the real world, layers can be defined with shelves dividing the roll-container into smaller spaces, and they can be placed at any height in the bin, see Figure 2 for an example. Even though, the problems with and without layers are different in essence, for practical implementation, all feasible solutions of the 'layered' model would be feasible for the initial model, namely, they would respect the capacity of the bin. However, the 'layered' models may potentially be solved in less computational time by the industrial solver, due to a reduced amount of feasible configurations. If our hypothesis is successful, we can gain from the new models indications of feasible packing solutions in less time. We do this by extending the model of Chen et al. (1995) with several constraints that include layers when placing the parcels into the roll containers and by varying the rotational freedom of the items. For all models, the bin dimensions are identical. An overview of the four new models and their restrictions is shown in Figure 3. Finally, we test the models by developing an experimental framework with instances based on literature and the real-world case of PDSNL and using a branch-and-bound approach, using CPLEX 12.7, to try to solve them.

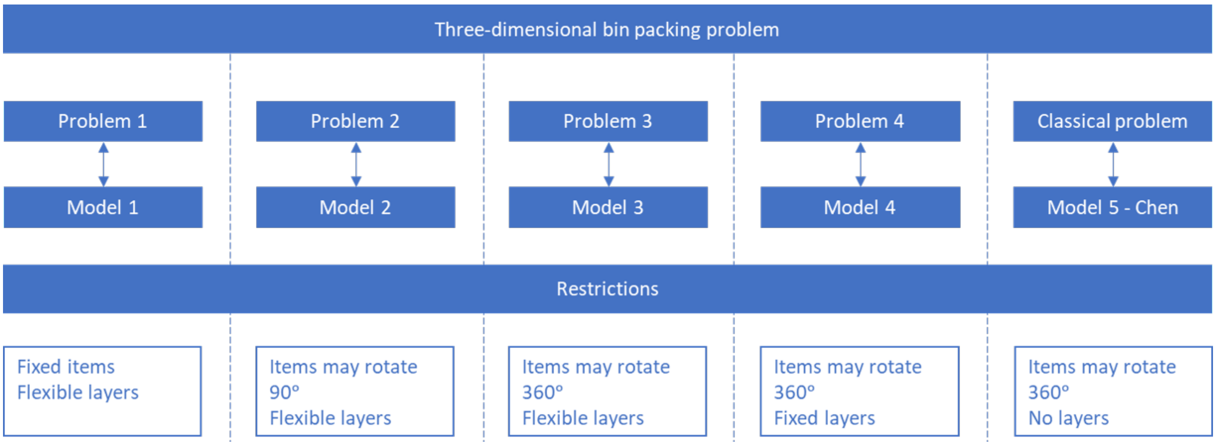



Figure 3: The developed models and their restrictions

Based on this, the research question and goal read as follows:
**Research question:** *Is it possible to find an indication of the solution for the 3D-BPP in lower time with these newly developed models that are an enhancement of the original problem?*
**Research goal:** *The aim of this study is to develop four mathematical models for the 3D-BPP that are restrictions of the classical model in Chen et al. (1995) to find indications of a feasible solution in lower time.*

This research is structured as follows. In Section 2, we review the available literature on the 3D-BPP and related logistic applications. In Section 3, we describe the specific packing problem arising at PDSNL. In Section 4, we develop the mathematical formulations. In Section 5, we present the experimental framework and show and discuss the numerical results. Section 6 ends the research with our conclusions and considerations for future research, and practice-related challenges and opportunities.

# 2 Literature review

Bin packing problems are among the most treated problems in operations research studies, due to the large quantity of related industrial applications. In general, the one-dimensional problem consists of packing a set of weighted items into a set of capacitated bins, with the aim of reducing the number of bins used. Several variants have been introduced. For example, the number of available bins can be fixed or can be heterogeneous in terms of costs and size. The objective function can relate to the minimization of unused space. We refer to Coffman Jr et al. (2018) and to Dowsland and Dowsland (1992) for an extensive bibliography and classifications. Among the most challenging variants, the multi-dimensional packing problem has drawn the attention of many researchers in the last two decades.

The two and three-dimensional packing problems add dimensions to both bins and items, and their complexity limits the implementation of exact methods even for relatively small-scale problems (Crainic et al., 2009). In this review, we will focus on contributions on the three-dimensional case, due to the relevance to the proposed problem; for a survey on the two-dimensional case, we refer to Lodi et al. (2002b).

In terms of variants, typically items and bins are regular parallelepipeds and any placed item's side must be parallel to a bin's side. Except for the model of Chen et al. (1995), most papers exclude rotation, namely an item's edge must always be parallel to the corresponding bin's edge. Wu et al. (2010) propose a variant where the objective is to minimize the variable bin height. With concern to layer packing, to the best of our knowledge, there are no publications that considered this addition in any mathematical model for the three-dimensional case, although it has been considered for the two-dimensional case in Lodi et al. (2004). Except for a few attempts in developing exact methods, see for example Martello et al. (2000b), most papers presented (meta-)heuristic approaches to solve the problems. Zhao et al. (2016) provide a review of developed heuristic, among which: Guided local search, Tabu search, Genetic algorithms.

Although it is often argued that the multi-dimensional packing problems are motivated by industrial applications, scientific papers on real-world examples are lacking. In fact, research in the area of 3D-BPP is mainly focused on the aforementioned standard problems, whereby constraints encountered in practice are often neglected or not applied simultaneously (Bortfeldt and Wäscher, 2013) (Zhao et al., 2016). However, there are some exceptions. In Paquay et al. (2016) a 3D-BPP mixed integer linear programming formulation is used to optimize the loading of boxes into containers, considering realistic constraints that are of interest in the field of air transportation. Specifically, they model the fragility of items, load stability, and weight distribution and consider different shapes for the containers. Sciomachen and Tanfani (2003) and Sciomachen and Tanfani (2007) address the problem of determining stowage plans for containers within ships. Both papers relate the problem to the 3D-BPP and use the similarity to build heuristics.

In terms of the application of 3D-BPP to the packing of parcels in the parcel delivery industry, to date no study is available. This gap should be covered, given the importance of this field. In fact, web-based purchases are huge in volume and critical processes, such as packing, should be investigated for improvement.

# 3 The Problem at PDSNL

At the moment, PDSNL has several depots in the Netherlands where parcels are sorted and packed. For the final delivery, the depots handle the parcels in two phases. In the evening, parcels reach the depots for transportation to other depots. Parcels are placed randomly on a conveyor belt endowed with an OCR scanner that checks the dimensions (length, width and height) and the postal code of destination for the sorting. The belt routes and shoots the parcels to the respective packing area, each associated with a range of postal codes. Here, workers place parcels *manually* into roll-containers. Next, the roll-containers are loaded into trailers, which ship them at night to the destination depot. In the morning, at the destination depots, parcels are again sorted (second phase) and placed in vans for the last mile delivery. An overview of this process is illustrated in Figure 4.

Our focus is on the first phase, namely the allocation of sorted parcels to roll-containers. The roll-container used by PDSNL have uniform size (58 x 78 x 178 cm) and can fit on average 30 parcels. The parcels can have any size and shape as long as it is within the physical limits of the sorting machine. Although the shapes can be as irregular as a bag or any other odd shaped box/envelop, the OCR scanner detects parallelepiped shapes.

There is no formal policy on how the parcels need to be packed. Therefore, workers pack the items according to experience or by rule-of-thumb or with a FIFO policy, without any indication of a desired position of the item in the roll-container. Additionally, fatigue and lack of concentration may lead to a rather random loading, with consequent waste of space. Finally, since the parcels arrive at random, possibilities for better fit might not be exploited manually. For example, small parcels may arrive earlier than larger ones. Given the time pressure, the workers may be forced at some point to place parcels on top of each other, without any sensible scheme.

Thereby, our goal is to develop a modelling framework to support this packing process. Our proposal is that workers are guided towards specific packing patterns by a performing math-

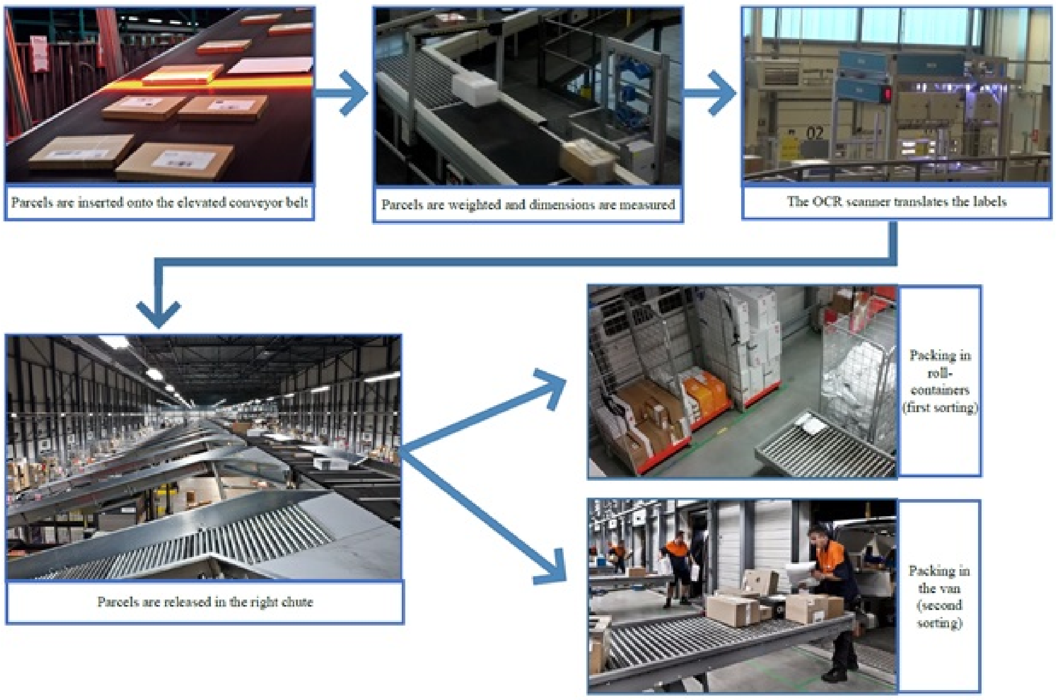



Figure 4: Overview of the sorting process

ematical model. The OCR scanner provides input for the model and workers may follow the pattern supported by a screen, outputting a desirable packing.

In addition, we evaluate a structural extension to the roll-containers in the form of layers to be placed within at variable heights. On one hand, such layers may be useful to increase the stability; to drive the packing process to neater patterns, as shown in Figure 2; and to decrease the computational complexity of the problem. On the other hand, fewer parcels could be accommodated within.

# 4 Mathematical models and formulations

In this section, we provide an extensive explanation of the four models. Besides this, prior to formulating the models, the notations to be used in the formulations are defined.

## 4.1 Methodology

We develop four mathematical models for the 3D-BPP, considering layers and item rotations. These models are an extension of the model for the 3D-BPP of Chen et al. (1995). The first model, model 1, is a 3D layer bin packing model and the orientations of items are fixed and the layer heights are flexible. The second model, model 2, is an extension of model 1, and it additionally allows items to rotate 90° along the z-axes, allowing the item length to be placed parallel along the length or width dimension, and the item width placed parallel along the width or length dimension. Further, the third model allows items to rotate 360°, also referred to as free rotation. Model 4 allows items to rotate 360°, similar to model 3. However, the layer heights for model 4 are fixed.

Table 1 provides an overview of the features of every model in terms of layer types and item rotations.

Table 1: Overview of the four developed models

| Model | Objective | Layers | Rotations of items |
|---|---|---|---|
| 1 | Min number of bins | Flexible | Fixed |
| 2 | Min number of bins | Flexible | 90° |
| 3 | Min number of bins | Flexible | 360° |
| 4 | Min number of bins | Fixed | 360° |
| Chen | Min number of bins | None | 360° |

### Notations

Prior to formulate the models, the notations to be used in the formulations are defined as follows. There are $n$ number of items. The assumption, that there are enough bins available to place all items, results in $n$ bins. Every item has a height, width and length dimension, expressed as $h_i$, $w_i$ and $l_i$, respectively. The bin dimensions are H, W, and L, respectively. In this study, for every model all bins are identical.

$$
\begin{aligned}
&\underline{Parameters:} \\
n &: \text{Total number of items} \\
i,j,k &: \text{Index for an item} \qquad\qquad (i,j,k = 1\dots n) \\
w_i &: \text{Width of item } i \\
h_i &: \text{Height of item } i \\
l_i &: \text{Length of item } i \\
W &: \text{Width of the bin} \\
H &: \text{Height of the bin} \\
L &: \text{Length of the bin} \\
M &: \text{Arbitrarily large number}
\end{aligned}
$$

In addition to the parameters, a list of variables is presented. The Front-Left-Bottom(FLB) approach is used to indicate the placement of an item, the FLB is expressed with the variables $X_i, Y_i, Z_i$. Figure 5 is provided as a clarification of the FLB approach. The term "initializes" is used for both layers and bins. If an item is selected as the leftmost item in a layer, it determines the height of the layer, and we say that the item initialize the layer; if a layer is chosen as the bottom layer in a bin, we say the layer initializes the bin. The binary variables $a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}$ indicate the placement of items relatively to each other. For example, if $a_{ik}$ equals 1, item $i$ is placed on the left side of item $k$.

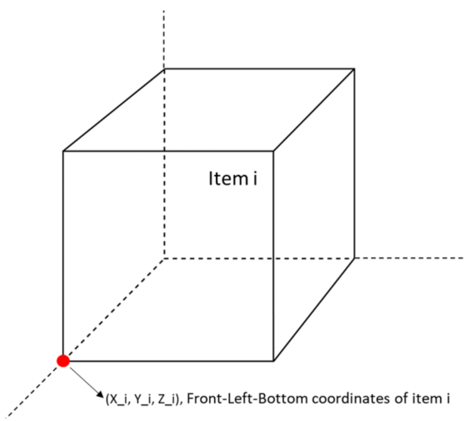



Figure 5: Visualization of the Front-Left-Bottom coordinates of item $i$

$\underline{Variables:}$

$X_i, Y_i, Z_i$ : coordinates of the the Front-Left-Bottom corner of item $i$,

with $X_i \geq 0, Y_i \geq 0, Z_i \geq 0$

$$y_i = \begin{cases} 1 & \text{if item } i \text{ initializes layer } i \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ij} = \begin{cases} 1 & \text{if item } j \text{ is placed into layer } i \\ 0 & \text{otherwise} \end{cases} \qquad j > i$$

$$q_k = \begin{cases} 1 & \text{if layer } k \text{ initializes bin } k \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ki} = \begin{cases} 1 & \text{if layer } i \text{ is placed to bin } k \\ 0 & \text{otherwise} \end{cases} \qquad i > k$$

$$a_{ik} = \begin{cases} 1 & \text{if item } i \text{ is on the left side of item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$$b_{ik} = \begin{cases} 1 & \text{if item } i \text{ is on the right side of item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$$c_{ik} = \begin{cases} 1 & \text{if item } i \text{ is behind item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$$d_{ik} = \begin{cases} 1 & \text{if item } i \text{ is in front of item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$$e_{ik} = \begin{cases} 1 & \text{if item } i \text{ is below item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$$f_{ik} = \begin{cases} 1 & \text{if item } i \text{ is above item } k \\ 0 & \text{otherwise} \end{cases} \qquad i \leq k$$

$i, j, k \in \{1, \ldots, n\}$

The assumptions for all four models are identical, and are mainly based on the early work of Chen et al. (1995) and Lodi et al. (2004). The list of assumptions is defined as follows:

1. All items are shaped parallelepiped.
2. There are enough bins available to place all items.
3. The item dimensions do not exceed the bin dimensions.
4. All items must be placed.
5. All items must be placed orthogonally within the bin, meaning that the item dimensions are placed parallel to the bin dimensions.
6. All bins have identical, fixed, dimension.
7. The item and bin dimensions are known upfront.
8. The items are sorted in non-increasing height.
9. The first item placed in a layer is the tallest item in that layer.
10. The bottom layer in each bin is the tallest layer in that bin.

In the parcel delivery service industry items may have different shapes. However, in this study, the assumption is made that all items are parallelepiped-shaped. Furthermore, it is assumed that all the items to be packed are sorted and indexed based on their height such that $h_1 \geq h_2 \geq ... \geq h_n$. If an item is selected as the leftmost item in a layer, it determines the height of the layer, and we say that the item initializes the layer; if a layer is chosen as the bottom layer in a bin, we say the layer initialize the bin.

## 4.2 Model 1: Three-dimensional layer bin packing model with fixed item orientation

This model does not allow any rotations for items. Besides this, the model does include flexible layers, and the height of every layer depends on the tallest item placed in that layer. Please note, in the model, the index of item, layer and bin have the same dimension, $n$. Layer $i$ is the layer which is initialized by item $i$; and bin $i$ is the bin initialized by layer $i$ which is initialized by item $i$. When the model is solved, not only an optimal solution is found, the decision variables $a_{ik}, bik, c_{ik}, d_{ik}, e_{ik}$, and $f_{ik}$ provide a loading pattern of the items.

The MILP model for three-dimensional layer bin packing model with fixed item orientation is formulated as follows.

$$\min \quad \sum_{k=1}^{n} q_k$$

$$\begin{array}{llllr}
\text{s.t.} & X_i + l_i & \leq & X_k + (1 - a_{ik}) \cdot M & \forall i,k : i < k & (ct1) \\
& X_k + l_k & \leq & X_i + (1 - b_{ik}) \cdot M & \forall i,k : i < k & (ct2) \\
& Y_i + w_i & \leq & Y_k + (1 - c_{ik}) \cdot M & \forall i,k : i < k & (ct3) \\
& Y_k + w_k & \leq & Y_i + (1 - d_{ik}) \cdot M & \forall i,k : i < k & (ct4) \\
& Z_i + h_i & \leq & Z_k + (1 - e_{ik}) \cdot M & \forall i,k : i < k & (ct5) \\
& Z_k + h_k & \leq & Z_i + (1 - f_{ik}) \cdot M & \forall i,k : i < k & (ct6) \\
& a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} & \geq & x_{ji} + x_{jk} - 1 & \forall i,j,k : i < k & (ct7) \\
& \sum_{i=1}^{j-1} x_{ij} + y_j & = & 1 & \forall j & (ct8) \\
& \sum_{i=1}^{j} x_{ij} & = & 1 & \forall j & (ct9) \\
& x_{jj} & = & y_j & \forall j & (ct10) \\
& \sum_{j=i}^{n} x_{ij} & \leq & M * y_i & \forall i & (ct11) \\
& X_i + l_i & \leq & L & \forall i & (ct12) \\
& Y_i + w_i & \leq & W & \forall i & (ct13) \\
& Z_i + h_i & \leq & h_j + (1 - x_{ji}) \cdot M & \forall i,j : j \leq i & (ct14) \\
& \sum_{k=1}^{i-1} z_{ki} + q_i & = & y_i & \forall i & (ct15) \\
& \sum_{i=k+1}^{n} h_i \cdot z_{ki} & \leq & (H - h_k) \cdot q_k & \forall k & (ct16)
\end{array}$$

The model minimizes the number of initial layers, which is, in fact, equivalent to minimize the number of used bins. Constraints 1-6 ensure that there is no overlap between two items. Constraint 7 is included to make sure that the overlapping of any two items is only considered whenever the two items are placed within the same layer. Constraint 8 imposes that an item is placed exactly once, either by initializing a layer or in a layer initialized by a taller item.

Constraints 9-10 ensure that layer initializing items are also taken into account when applying the overlap constraint to the model. Without these constraints, the first item placed in a layer is not part of the loading pattern. Constraint 11 ensures that when an item is assigned to a layer, the layer is considered used. Constraints 12-13 impose that if an item is placed in a bin it has to fit within the bin dimensions. Constraint 14 ensures if an item initializes a layer, the other items placed in this layer cannot be taller than the initializing item. Constraint 15 guarantees the placement of each used layer in bins and constraint 16 makes sure that placed layers do not exceed the total bin height. Table 6 gives an overview of the constraints of model 1, we identified three type of constrains; identical constraints, adapted constraints and new constraints, compared to the classical 3D-BPP of Chen et al. (1995) and the 2D layer BPP of Lodi et al. (2004). NA in the table means not applicable.

| | Classical 3D-BPP | 2D layer BPP | None of both models |
|---|---|---|---|
| Identical constraints | -- | 8, 15, 16 | NA |
| Adapted constraints | 1-7, 9, 11-14 | -- | NA |
| New constraints | NA | NA | 10 |

Figure 6: An overview of the composition of every constraint in model 1

## 4.3 Model 2: Three-dimensional layer bin packing model, items may rotate 90°

To include the 90° rotation of the items in the model, two new binary variables, $ldim_{ai}$ and $wdim_{ai}$, are introduced. These variables indicate whether the length or width of item $i$ is parallel to either the x- or the y-axis. $a$ in the two binary variables can be 1 or 2, corresponding to the x- or y-axis, relatively. For example, if $ldim_{ai}$ is 1 for a=1, the length of item $i$ is placed along the x-axis; otherwise it is equal to 0. If $ldim_{ai}$ equals 1 for a=2, the length of item $i$ is placed along the y-axis.

$$ldim_{ai} = \begin{cases} 1 & \text{if the length of item i is placed along the a-axis} \\ 0 & \text{otherwise} \end{cases}$$

$$wdim_{ai} = \begin{cases} 1 & \text{if the width of item i is placed along the a-axis} \\ 0 & \text{otherwise} \end{cases}$$

$$i \in \{1, \ldots, n\}, \quad a \in \{1, 2\}$$

The MILP model for three-dimensional layer bin packing model with 90° item orientation is formulated as follows.

$$\min \quad \sum_{k=1}^{n} q_k$$

s.t.

$$\begin{aligned}
X_i + l_i \cdot ldim_{1i} + w_i \cdot wdim_{1i} &\leq X_k + (1 - a_{ik}) \cdot M && \forall i, k : i < k && (ct1)\\
X_k + l_k \cdot ldim_{1k} + w_k \cdot wdim_{1k} &\leq X_i + (1 - b_{ik}) \cdot M && \forall i, k : i < k && (ct2)\\
Y_i + l_i \cdot ldim_{2i} + w_i \cdot wdim_{2i} &\leq Y_k + (1 - c_{ik}) \cdot M && \forall i, k : i < k && (ct3)\\
Y_k + l_k \cdot ldim_{2k} + w_k \cdot wdim_{2k} &\leq Y_i + (1 - d_{ik}) \cdot M && \forall i, k : i < k && (ct4)\\
Z_i + h_i &\leq Z_k + (1 - e_{ik}) \cdot M && \forall i, k : i < k && (ct5)\\
Z_k + h_k &\leq Z_i + (1 - f_{ik}) \cdot M && \forall i, k : i < k && (ct6)\\
a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} &\geq x_{ji} + x_{jk} - 1 && \forall i, j, k : i < k && (ct7)\\
\sum_{i=1}^{j-1} x_{ij} + y_j &= 1 && \forall j && (ct8)\\
\sum_{i=1}^{j} x_{ij} &= 1 && \forall j && (ct9)\\
x_{jj} &= y_j && \forall j && (ct10)\\
\sum_{j=i}^{n} x_{ij} &\leq M * y_i && \forall i && (ct11)\\
X_i + l_i \cdot ldim_{1i} + w_i \cdot wdim_{1i} &\leq L && \forall i && (ct12)\\
Y_i + l_i \cdot ldim_{2i} + w_i \cdot wdim_{2i} &\leq W && \forall i && (ct13)\\
Z_i + h_i &\leq h_j + (1 - x_{ji}) \cdot M && \forall i, j : j \leq i && (ct14)\\
\sum_{a=1}^{2} ldim_{ai} &= 1 && \forall i && (ct15)\\
\sum_{a=1}^{2} wdim_{ai} &= 1 && \forall i && (ct16)\\
ldim_{ai} + wdim_{ai} &= 1 && \forall a, i && (ct17)\\
\sum_{k=1}^{i-1} z_{ki} + q_i &= y_i && \forall i && (ct18)\\
\sum_{i=k+1}^{n} h_i \cdot z_{ki} &\leq (H - h_k) \cdot q_k && \forall k && (ct19)
\end{aligned}$$

Constraints 1-4 are adapted to calculate the coordinate of the Back-Right-Top corner of an item, which reflects its rotation. This Back-Right-Top corner is a result of the FLB coordinates and the placement of the item. With this Back-Right-Top corner, the solver knows where the item ends and a new item can be placed and therefore they also ensure appropriate placement of one item on the others. Constraints 12 and 13 ensure packed items can fit bins. Constraints 15-17 ensure the feasible ways to rotate items, these constraints are adjustments of the constraints of the classical model. Table 7 gives an overview of the constraints of model 1, we identified three type of constrains; identical constraints, adapted constraints and new constraints, compared to the classical 3D-BPP of Chen et al. (1995) and the 2D layer BPP of Lodi et al. (2004). NA in the table means not applicable.

| | Classical 3D-BPP | 2D layer BPP | None of both models |
|---|---|---|---|
| Identical constrains | -- | 8, 18, 19 | NA |
| Adapted constraint | 1-7, 9, 11-17 | -- | NA |
| New constraints | NA | NA | 10 |

Figure 7: An overview of the composition of every constraint in model 2

## 4.4 Model 3: Three-dimensional layer bin packing model, items may rotate freely

Model 3 is the further extension of models 1 and 2 in Section 4.2 and 4.3. This third model allows items to rotate 360°, which leads to six possible ways of items placements. Furthermore, the layer heights for this model depend on the item dimension placed along the z-axis. With these new characteristics, the model has fewer restrictions when placing the items. For this reason, model 3 can be seen as a lower bound for models 1 and 2. Compared to the model of Chen et al. (1995) it is an upper bound because this model includes flexible layers.

Besides the binary variables $ldim_{ai}$ and $wdim_{ai}$ introduced in Section 4.3 a new binary variable $hdim_{ai}$ is introduced. $hdim_{ai}$ equals 1 if the height of item i is parallel to either the x-, y-, or z-axis. Consequently, the values of parameter $a$ are extended to equal 1, 2 or 3, corresponding to the x- y- or z-axis, relatively. Constraints 1-6 are adapted to provide the rotational freedom and are similar to the corresponding constraints in Chen et al. (1995). The best composition of every item is selected.

The MILP model is formulated as follows.

$$
\begin{array}{llcll}
\min & \sum_{k=1}^{n} q_k & & & \\
\text{s.t.} & X_i + l_i \cdot ldim_{1i} & & & \\
& +w_i \cdot wdim_{1i} + h_i \cdot hdim_{1i} & \leq & X_k + (1 - a_{ik}) \cdot M & \forall i,k : i < k \quad (ct1) \\
& X_k + l_k \cdot ldim_{1k} & & & \\
& +w_k \cdot wdim_{1k} + h_i \cdot hdim_{1k} & \leq & X_i + (1 - b_{ik}) \cdot M & \forall i,k : i < k \quad (ct2) \\
& Y_i + l_i \cdot ldim_{2i} & & & \\
& +w_i \cdot wdim_{2i} + h_i \cdot hdim_{2i} & \leq & Y_k + (1 - c_{ik}) \cdot M & \forall i,k : i < k \quad (ct3) \\
& Y_k + l_k \cdot ldim_{2k} & & & \\
& +w_k \cdot wdim_{2k} + h_k \cdot hdim_{2k} & \leq & Y_i + (1 - d_{ik}) \cdot M & \forall i,k : i < k \quad (ct4) \\
& Z_i + +l_i \cdot ldim_{3i} & & & \\
& +w_i \cdot wdim_{3i} + h_i \cdot hdim_{3i} & \leq & Z_k + (1 - e_{ik}) \cdot M & \forall i,k : i < k \quad (ct5) \\
& Z_k + l_k \cdot ldim_{3k} & & & \\
& +w_k \cdot wdim_{3k} + h_k \cdot hdim_{3k} & \leq & Z_i + (1 - f_{ik}) \cdot M & \forall i,k : i < k \quad (ct6) \\
& a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} & \geq & x_{ji} + x_{jk} - 1 & \forall i,j,k : i < k \quad (ct7) \\
& & & & \\
& \sum_{i=1}^{j-1} x_{ij} + y_j & = & 1 & \forall j \quad (ct8) \\
& \sum_{i=1}^{j} x_{ij} & = & 1 & \forall j \quad (ct9) \\
& x_{jj} & = & y_j & \forall j \quad (ct10) \\
& \sum_{j=i}^{n} x_{ij} & \leq & M * y_i & \forall i \quad (ct11) \\
& & & & \\
& X_i + l_i \cdot ldim_{1i}+ & & & \\
& w_i \cdot wdim_{1i} + h_i \cdot hdim_{1i} & \leq & L & \forall i \quad (ct12) \\
& Y_i + l_i \cdot ldim_{2i}+ & & & \\
& w_i \cdot wdim_{2i} + h_i \cdot hdim_{2i} & \leq & W & \forall i \quad (ct13) \\
& Z_i + l_i \cdot ldim_{3i}+ & & & \\
& w_i \cdot wdim_{3i} + h_i \cdot hdim_{3i} & \leq & (l_j \cdot ldim_{3j} + w_j \cdot wdim_{3j}+ & \\
& & & h_j \cdot hdim_{3j}) + (1 - x_{ji}) \cdot M & \forall i,j : j < i \quad (ct14) \\
& \sum_{a=1}^{3} ldim_{ai} & = & 1 & \forall i \quad (ct15) \\
& \sum_{a=1}^{3} wdim_{ai} & = & 1 & \forall i \quad (ct16) \\
& \sum_{a=1}^{3} hdim_{ai} & = & 1 & \forall i \quad (ct17) \\
& ldim_{ai} + wdim_{ai} + hdim_{ai} & = & 1 & \forall a,i \quad (ct18) \\
& \sum_{k=1}^{i-1} z_{ki} + q_i & = & y_i & \forall i \quad (ct19) \\
& \sum_{i=k+1}^{n} (l_i \cdot ldim_{3i} + w_i \cdot wdim_{3i}+ & & & \\
& h_i \cdot hdim_{3i}) \cdot z_{ki} & \leq & (H - (l_i \cdot ldim_{3i}+ & \\
& & & w_i \cdot wdim_{3i} + h_i \cdot hdim_{3i})) \cdot q_k & \forall i,k \quad (ct20)
\end{array}
$$

Constraint 1-6 calculate the coordinates of the Back-Right-Top corner and ensures correct overlapping with the other items. Constraint 12 and 13 ensure items can fit bins. Constraint 14 guarantees the limitations on the layer height; whenever an item $i$ is placed in a layer initialized by another item $k$, the item $i$ cannot be taller than item $k$. Constraints 15-18 ensure all items to be placed orthogonally. Constraint 20 is adapted to reflect the fact that the layer height no longer always depends on the items height only. It ensures that for every item placed in a bin-filling layer, the dimension placed along the z-axis cannot be taller than the remaining bin height. Where the remaining bin height is the total bin height minus the height of the layer already placed in the bin. Table 8 gives an overview of the constraints of model 1, we identified three type of constrains; identical constraints, adapted constraints and new constraints, compared to the classical 3D-BPP of Chen et al. (1995) and the 2D layer BPP of Lodi et al. (2004). NA in the table means not applicable.

| | Classical 3D-BPP | 2D layer BPP | None of both models |
|---|---|---|---|
| Identical constrains | 1-6 | 8, 19 | NA |
| Adapted constraint | 7, 9, 11-18 | 20 | NA |
| New constraints | NA | NA | 10 |

Figure 8: An overview of the composition of every constraint in model 3

## 4.5 Model 4: Three-dimensional fixed layer bin packing model, items may rotate freely

The fourth model allows items to rotate 360°, as a result of this, there are six possible positions for every item. This model differs from the models developed in Section 4.2-4.4 in determining the layer height. For model 4, the layer height is fixed and pre-set and is the same for every bin. Lodi et al. (2004) defined two types of layers; bin initializing layers and bin filling layers. Since model 4 has just one fixed layer that divides the bin in two, every bin contains one layer of both types. This model is an adjustment of model 3 and therefore its constraints are adjusted to the new situation. The model can be seen as an upper bound of the original model in Chen et al. (1995). Compared to Model 3, model 4 has one extra condition, which is one fixed shelf, splitting the bin in half.

In model 4, two new parameters, H1 and H2, are introduced. The two parameters define the height of layer 1 and 2 respectively. Of course, on every layer, the items must be placed within the height dimension of that layer.

The MILP model is formulated as follows.

$$
\begin{array}{llcllr}
\min & \sum_{k=1}^{n} q_k & & & & \\
\text{s.t.} & X_i + l_i \cdot ldim_{1i} & & & & \\
& +w_i \cdot wdim_{1i} + h_i \cdot hdim_{1i} & \leq & X_k + (1-a_{ik}) \cdot M & \forall i,k : i<k & (ct1) \\
& X_k + l_k \cdot ldim_{1k} & & & & \\
& +w_k \cdot wdim_{1k} + h_i \cdot hdim_{1k} & \leq & X_i + (1-b_{ik}) \cdot M & \forall i,k : i<k & (ct2) \\
& Y_i + l_i \cdot ldim_{2i} & & & & \\
& +w_i \cdot wdim_{2i} + h_i \cdot hdim_{2i} & \leq & Y_k + (1-c_{ik}) \cdot M & \forall i,k : i<k & (ct3) \\
& Y_k + l_k \cdot ldim_{2k} & & & & \\
& +w_k \cdot wdim_{2k} + h_k \cdot hdim_{2k} & \leq & Y_i + (1-d_{ik}) \cdot M & \forall i,k : i<k & (ct4) \\
& Z_i + +l_i \cdot ldim_{3i} & & & & \\
& +w_i \cdot wdim_{3i} + h_i \cdot hdim_{3i} & \leq & Z_k + (1-e_{ik}) \cdot M & \forall i,k : i<k & (ct5) \\
& Z_k + l_k \cdot ldim_{3k} & & & & \\
& +w_k \cdot wdim_{3k} + h_k \cdot hdim_{3k} & \leq & Z_i + (1-f_{ik}) \cdot M & \forall i,k : i<k & (ct6) \\
& a_{ik} + b_{ik} + c_{ik} + d_{ik} + e_{ik} + f_{ik} & \geq & x_{ji} + x_{jk} - 1 & \forall i,j,k : i<k & (ct7) \\
& \sum_{i=1}^{j-1} x_{ij} + y_j & = & 1 & \forall j & (ct8) \\
& \sum_{i=1}^{j} x_{ij} & = & 1 & \forall j & (ct9) \\
& x_{jj} & = & y_j & \forall j & (ct10) \\
& \sum_{j=i}^{n} x_{ij} & \leq & M * y_i & \forall i & (ct11) \\
& X_i + l_i \cdot ldim_{1i} + & & & & \\
& w_i \cdot wdim_{1i} + h_i \cdot hdim_{1i} & \leq & L & \forall i & (ct12) \\
& Y_i + l_i \cdot ldim_{2i} + & & & & \\
& w_i \cdot wdim_{2i} + h_i \cdot hdim_{2i} & \leq & W & \forall i & (ct13) \\
& Z_i + l_i \cdot ldim_{3i} + w_i \cdot wdim_{3i} + & & & & \\
& h_i \cdot hdim_{3i} & \leq & H1 + (2 - q_j - x_{ji}) \cdot M & \forall i,j : j \leq i & (ct14) \\
& Z_j + l_j \cdot ldim_{3j} + w_j \cdot wdim_{3j} + & & & & \\
& h_j \cdot hdim_{3j} & \leq & H2 + (2 - z_{ki} - x_{ij}) \cdot M & \forall i,j,k : j \geq i & (ct15) \\
& \sum_{a=1}^{3} ldim_{ai} & = & 1 & \forall i & (ct16) \\
& \sum_{a=1}^{3} wdim_{ai} & = & 1 & \forall i & (ct17) \\
& \sum_{a=1}^{3} hdim_{ai} & = & 1 & \forall i & (ct18) \\
& ldim_{ai} + wdim_{ai} + hdim_{ai} & = & 1 & \forall a,i & (ct19) \\
& \sum_{k=1}^{i-1} z_{ki} + q_i & = & y_i & \forall i & (ct20) \\
& q_i + \sum_{k=1}^{i-1} z_{ik} & \leq & 2 & \forall i & (ct21) \\
& \sum_{i=k+1}^{n} (l_i \cdot ldim_{3i} + w_i \cdot wdim_{3i} + & & & & \\
& h_i \cdot hdim_{3i}) \cdot z_{ki} & \leq & (H - (l_i \cdot ldim_{3i} + & & \\
& & & w_i \cdot wdim_{3i} + h_i \cdot hdim_{3i})) \cdot q_k & \forall i,k & (ct22)
\end{array}
$$

Constraints 14 and 15 guarantee the requirement that no item dimension placed along the z-axis can exceed the layer height. If an item is placed in an initializing layer of a bin, the item's height has to be less than, or equal to, the layer height H1. If an item is placed in a bin filling layer its height has to be less than, or equal to, H2. Besides this, constraint 21 is introduced to ensure a maximum of two layers within one bin. Table 9 gives an overview of the constraints of model 1, we identified three type of constrains; identical constraints, adapted constraints and new constraints, compared to the classical 3D-BPP of Chen et al. (1995) and the 2D layer BPP of Lodi et al. (2004). NA in the table means not applicable.

| | Classical 3D-BPP | 2D layer BPP | None of both models |
|---|---|---|---|
| Identical constrains | 1-6 | 8, 20 | NA |
| Adapted constraint | 7, 9, 11-19 | 22 | NA |
| New constraints | NA | NA | 10, 21 |

Figure 9: An overview of the composition of every constraint in model 4

# 5 Numerical experiments

We propose three sets of experiments to test and validate the developed mathematical models. The first experiment is conducted for a small size dataset for validation purposes; in the second experiment we generate instances based on literature to test the models; the third experiment considers real-world instances derived from PDSNL. In this section, we first introduce the experimental setting; next, we report the results of the three different experiments; and finally, we provide a conclusion.

## 5.1 Experiment settings

We test the developed models and the model of Chen et al. (1995)) using IBM ILOG CPLEX 12.7. The optimization software is run on a personal computer, with 2.4 GHz Intel Core i5-6200U processor and 8GB of RAM. In the experiments, the computational time spent on solving a model is limited to 3600 seconds. Imposing the time limit is because computational speed is a crucial aspect in the parcel sorting process, and feasible solutions must be attained in a reasonable time.

To measure the solution quality, we provide the achieved optimality gap (expressed in percentage) for each instance, calculated as the best integer feasible solution minus the best bound, divided by the best integer feasible solution. CPLEX will terminate with four different possible outcomes:

1. Optimal solution, the gap is zero
2. Feasible integer solution with a gap greater than zero, CPLEX exceeds the time limit
3. No solution, CPLEX exceeds computer memory
4. No solution, CPLEX exceeds the time limit

Outcomes 3 and 4 are represented in the results tables by '—'.

## 5.2 Experiment 1: a small scale dataset for model validation

To validate the mathematical models, we design a small test dataset with only six items. The item dimensions are shown in Table ??. The number of bins is equal to the number of items. Table ?? provides an overview of the outputs of each model from CPLEX. In this section, first the results of all models are presented. After this, the optimal solution and loading pattern of model 1 is explained and visualized in more detail.

Table 2: Validation of the models

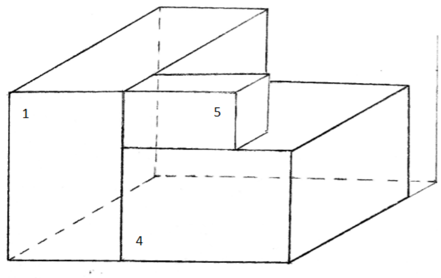
(a) dimensions of test set

| Item | h | w | l |
|---|---|---|---|
| 1 | 6 | 10 | 4 |
| 2 | 4 | 4 | 8 |
| 3 | 4 | 6 | 4 |
| 4 | 4 | 8 | 6 |
| 5 | 2 | 2 | 4 |
| 6 | 2 | 4 | 6 |

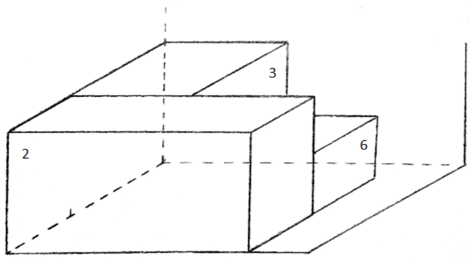
(b) results for test set

| | Model 1 | Model 2 | Model 3 | Model 4 | Model Chen |
|---|---|---|---|---|---|
| Time | 00.53 | 00.39 | 00.71 | 00.60 | 00.28 |
| Constraints | 439 | 351 | 264 | 411 | 246 |
| Variables | 171 | 195 | 225 | 240 | 204 |
| Best integer | 1 | 1 | 1 | 1 | 1 |
| Best bound | 1 | 1 | 1 | 1 | 1 |
| Gap | 0% | 0% | 0% | 0% | 0% |

The six items are sorted in non-increasing order of height. The bin dimensions for this experiment are H, W, D = 10 for all the bins. The solutions of all models equal 1. As mentioned, as an example we will explain the solution of model 1 in more detail. The optimal value of objective function of model 1 equals 1, which represents that all items are placed in one bin ($q_1^* = 1$). Two layers ($y_1^*, y_2^* = 1$) need to be arranged. The bin initializing layer(bottom layer) is initialized by item 1 and a bin filling layer is initialized by item 2 ($z_{12}^* = 1$). Items 4 and 5 are placed in the layer initialized by item 1 ($x_{11}^*, x_{14}^*, x_{15}^* = 1$). Items 3 and 6 are placed in the layer initialized by item 2 ($x_{22}^*, x_{23}^*, x_{26}^* = 1$). The decision variables $a_{ik}, b_{ik}, c_{ik}, d_{ik}, e_{ik}, f_{ik}$ define the loading pattern of the bin. For this particular example: $a_{14}^*, a_{15}^*, a_{36}^*, d_{23}^*, d_{26}^*, f_{45}^* = 1$. The loading pattern per layer is presented in Figure 10a and 10b, respectively. Figure 11 shows the bird's-eye view of the layers placed in the bin, separated by the red line(the layer).




(a) Level 1




(b) Level 2

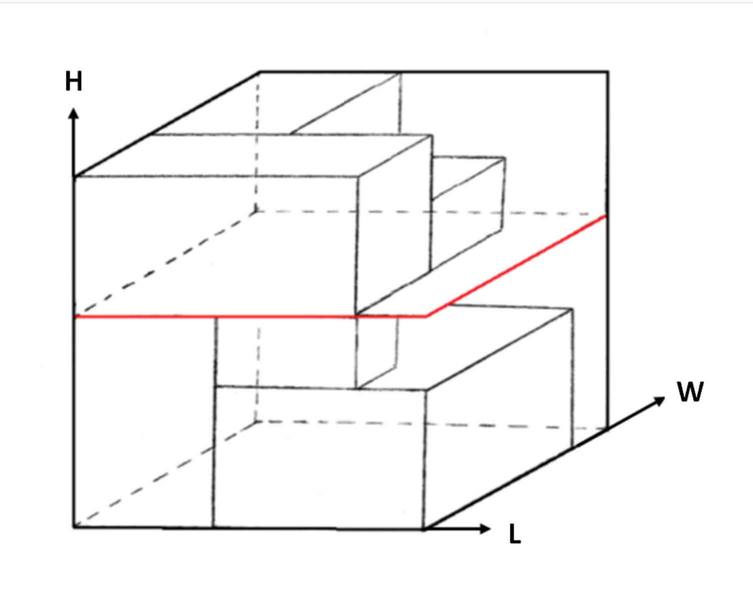



Figure 11: The bird's-eye view of the solution of model 1

## 5.3 Experiment 2: classical instances

### Data generation

In Experiment 2, eleven different classes are considered. A class is a group of items with specific generated dimensions. In the first group of instances, considering 5 classes, we have identical bins with dimensions W,H,L = 100 and we have five possible types of items, each with specific uniform distributions for each dimension as follows:

*Type 1:* $w_j \sim U[1, \frac{1}{2}W]; h_j \sim U[\frac{2}{3}H, H]; d_j \sim U[\frac{2}{3}L, L]$

*Type 2:* $w_j \sim U[\frac{2}{3}W, W]; h_j \sim U[1, \frac{1}{2}H]; d_j \sim U[\frac{2}{3}L, L]$

*Type 3:* $w_j \sim U[\frac{2}{3}W, W]; h_j \sim U[\frac{2}{3}H, H]; d_j \sim U[1, \frac{1}{2}L]$

*Type 4:* $w_j \sim U[1, \frac{1}{2}W]; h_j \sim U[1, \frac{1}{2}H]; d_j \sim U[1, \frac{1}{2}L]$

*Type 5:* $w_j \sim U[\frac{2}{3}W, W]; h_j \sim U[\frac{2}{3}H, H]; d_j \sim U[\frac{2}{3}L, L]$

The generations of the first five classes are as follows. We consider a type of items at a time, henceforth a 'reference type', and for it, we generate 3 instances made of 8, 10 and 30 items. Each item has a probability of 60% to be generated according to the reference type, and 10% possibility to be either one of the other four types(Martello et al., 2000a). Hence, in total we have 3 instances for every class, resulting in 15 instances in total.

For the classes 6-11, the bin dimensions change for every class and all items dimensions have specific uniform distributions for each dimension. For the classes 6-11 these details are presented below.

*Class 6:* $w_j, h_j, l_j \sim U[1, 10]; W, H, D = 10$

*Class 7:* $w_j, h_j, l_j \sim U[1, 10]; W, H, D = 30$

*Class 8:* $w_j, h_j, l_j \sim U[1, 35]; W, H, D = 40$

*Class 9:* $w_j, h_j, l_j \sim U[1, 35]; W, H, D = 100$

*Class 10:* $w_j, h_j, l_j \sim U[1, 100]; W, H, D = 100$

*Class 11:* $w_j, h_j, l_j \sim U[1, 100]; W, H, D = 300$

For the classes 6-11, two instances are generated, consisting of 10 and 30 items, respectively. Resulting in 12 instances in total.

With concern to the fourth model having one fixed pre-set layer per bin, we test it for two different pre-set layer heights. Therefore, henceforth, we refer to model 4 as model 4.1 and 4.2, both with a different fixed layer height. The layer height is a division of the height, with respect to the whole bin.

The height of the layer is determined based on the item composition in every class and represents the majority of the generated sizes. Based on this, for the first five classes, the fixed layer height for model 4.1 and 4.2 was pre-set respectively at 60/40 and 70/30.

Classes 6-11 can be divided into two categories; the first category contains relatively large items, which may have the same size as the bins (Classes 6, 8 and 10). The second category contains relatively smaller items, the dimensions may be one-third of the bin dimensions at most (Classes 7, 9 and 11). The fixed layer height for classes 6, 8 and 10 is based on the experiments for the classes 1-5. The classes 7, 9 and 11 have a fixed layer height based on the largest item dimensions. All fixed layer heights are presented in Table 3.

Table 3: Layer heights for model 4.1 & 4.2.

| | Model 4.1 | | Model 4.2 | |
|---|---|---|---|---|
| Class | Layer 1 | Layer 2 | Layer 1 | Layer 2 |
| 1 | $\frac{6}{10}H$ | $\frac{4}{10}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 2 | $\frac{6}{10}H$ | $\frac{4}{10}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 3 | $\frac{6}{10}H$ | $\frac{4}{10}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 4 | $\frac{6}{10}H$ | $\frac{4}{10}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 5 | $\frac{6}{10}H$ | $\frac{4}{10}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 6 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 7 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{2}{3}H$ | $\frac{1}{3}H$ |
| 8 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 9 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{6}{10}H$ | $\frac{4}{10}H$ |
| 10 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{7}{10}H$ | $\frac{3}{10}H$ |
| 11 | $\frac{1}{2}H$ | $\frac{1}{2}H$ | $\frac{2}{3}H$ | $\frac{1}{3}H$ |

**Results**

This section presents all results obtained for all instances. Every table shows which class is tested, followed by the number of items in the set, the running time CPLEX spent on solving the model in seconds, the best integer, which is shortened into best int, the best bound and the optimality gap in percentage found by CPLEX. Also, if no solution is found, this is represented by '—', as explained in Section 5.1. Besides a table, a bar chart of the results is provided, which presents the best bound and the best integer for every class. Here, we will provide all the results as we see it, whereas the last section of this chapter contains a more in-depth analysis of the results.

**Results of classes 1-5, 8 items**

Table 4 provides an overview of the results. An optimal solution was found for 86.67% of the classes. Exceptions are the solutions of model 4.2 for class 4 and model 4.1 for classes 2, 3 and 4. The optimal solutions of model 3 and Chen's model are identical, however, the CPLEX solver requires significantly more time to solve model 3.

Table 4: Results of classes 1-5 for the input of 8 items

| | | Model 1 | | | | Model 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 8 | 0.30 | 3 | 3 | 0 | 0.72 | 3 | 3 | 0 |
| 2 | 8 | 0.29 | 4 | 4 | 0 | 0.23 | 4 | 4 | 0 |
| 3 | 8 | 0.36 | 3 | 3 | 0 | 0.28 | 2 | 2 | 0 |
| 4 | 8 | 0.20 | 4 | 4 | 0 | 0.18 | 4 | 4 | 0 |
| 5 | 8 | 0.27 | 2 | 2 | 0 | 0.27 | 2 | 2 | 0 |

| | | Model 3 | | | | Model 4.1 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 8 | 2612.23 | 2 | 2 | 0 | 0.55 | 3 | 3 | 0 |
| 2 | 8 | 272.27 | 3 | 3 | 0 | — | — | — | — |
| 3 | 8 | 355.06 | 2 | 2 | 0 | — | — | — | — |
| 4 | 8 | 513.72 | 4 | 4 | 0 | — | — | — | — |
| 5 | 8 | 55.58 | 2 | 2 | 0 | 2.00 | 2 | 2 | 0 |

| | | Model 4.2 | | | | Model Chen | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 8 | 0.94 | 4 | 4 | 0 | 0.41 | 2 | 2 | 0 |
| 2 | 8 | 0.47 | 3 | 3 | 0 | 0.49 | 3 | 3 | 0 |
| 3 | 8 | 0.75 | 3 | 3 | 0 | 0.28 | 2 | 2 | 0 |
| 4 | 8 | — | — | — | — | 0.21 | 4 | 4 | 0 |
| 5 | 8 | 0.47 | 2 | 2 | 0 | 0.19 | 2 | 2 | 0 |

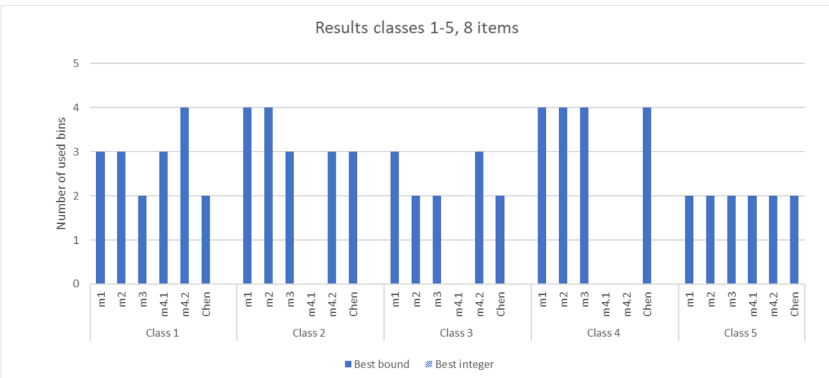



Figure 12: Visualization of results, classes 1-5, 8 items

**Results of classes 1-5, 10 items**

Table 5 gives an overview of the results of the instances with 10 items for classes 1-5. For 76.67% of the cases, the solver is able to find an optimal solution. Models 1, 2 and Chen's model are solved for all classes. The solver found integer solutions of model 3 for input classes 1-4, the solutions are integer and not optimal since the solver exceeds the time limit and the gap is not equal to zero. For model 3 with input class 5 the solution is optimal. Furthermore, the solver found optimal solutions for classes 1-3 of model 4.1 and no solutions for classes 4 and 5. For model 4.2, the solutions are optimal for classes 1-3 and 5, no solution was found for class 4.

Table 5: Results of classes 1-5 for the input of 10 items

| | | Model 1 | | | | Model 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 10 | 0.66 | 4 | 4 | 0 | 0.57 | 4 | 4 | 0 |
| 2 | 10 | 0.53 | 3 | 3 | 0 | 0.23 | 3 | 3 | 0 |
| 3 | 10 | 0.46 | 3 | 3 | 0 | 0.45 | 3 | 3 | 0 |
| 4 | 10 | 0.25 | 7 | 7 | 0 | 0.57 | 6 | 6 | 0 |
| 5 | 10 | 0.63 | 2 | 2 | 0 | 0.61 | 2 | 2 | 0 |
| | | **Model 3** | | | | **Model 4.1** | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 10 | 3600.27 | 3 | 1 | 66.66 | 2.02 | 4 | 4 | 0 |
| 2 | 10 | 3601.26 | 3 | 1 | 66.66 | 1.10 | 4 | 4 | 0 |
| 3 | 10 | 3600.24 | 3 | 1 | 66.66 | 342.42 | 4 | 4 | 0 |
| 4 | 10 | 3600.39 | 6 | 2 | 66.66 | — | — | — | — |
| 5 | 10 | 935.19 | 2 | 2 | 0 | — | — | — | — |
| | | **Model 4.2** | | | | **Model Chen** | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 10 | 3.24 | 4 | 4 | 0 | 1.33 | 3 | 3 | 0 |
| 2 | 10 | 2.68 | 5 | 5 | 0 | 0.75 | 3 | 3 | 0 |
| 3 | 10 | 29.11 | 3 | 3 | 0 | 0.72 | 2 | 2 | 0 |
| 4 | 10 | — | — | — | — | 0.27 | 6 | 6 | 0 |
| 5 | 10 | 73.69 | 2 | 2 | 0 | 0.24 | 2 | 2 | 0 |

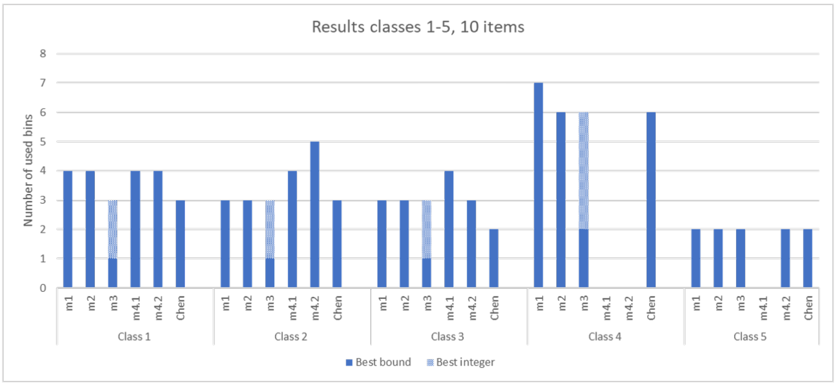



Figure 13: Visualization of results, classes 1-5, 10 items

**Results of classes 1-5, 30 items**

Table 6 provides an overview of the results for the classes 1-5 for the item sets of 30 items. For 25% of the item sets, an optimal solution is found. The results for models 1, 2, 4.2 and Chen's model are presented per class; for models 3 and 4.1 no solutions are found for any class. First, for class 1, no optimal solution is found for any model. For model 1, model 2 and Chen's model, the time limit was exceeded but the solver presented an integer solution. For this class, no solution is found for model 4.2. Second, for class 2, an optimal solution is found for model 1 and model 2. For model 4.2 and Chen's model, the solution is an integer solution. Next, for class 3, the solver is able to find an optimal solution of model 1, of model 2 and Chen's model, the solutions are integer and for model 4.2, no solution is presented. Last, for class 5, the solver found an optimal solution for model 1, model 2 and Chen's model. The solution for model 4.2 is integer.

Table 6: Results of classes 1-5 for the input of 30 items

| | | Model 1 | | | | Model 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 30 | 3601.39 | 7 | 6 | 14.29 | 3607.43 | 7 | 5 | 28.57 |
| 2 | 30 | 1.36 | 9 | 9 | 0 | 6.01 | 9 | 9 | 0 |
| 3 | 30 | 298.17 | 8 | 8 | 0 | 3604.23 | 8 | 5 | 37.50 |
| 4 | 30 | 1.15 | 18 | 18 | 0 | 1.54 | 18 | 18 | 0 |
| 5 | 30 | 30.40 | 4 | 4 | 0 | 55.95 | 4 | 4 | 0 |
| | | Model 3 | | | | Model 4.1 | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 30 | — | — | — | — | — | — | — | — |
| 2 | 30 | — | — | — | — | — | — | — | — |
| 3 | 30 | — | — | — | — | — | — | — | — |
| 4 | 30 | — | — | — | — | — | — | — | — |
| 5 | 30 | — | — | — | — | — | — | — | — |
| | | Model 4.2 | | | | Model Chen | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 1 | 30 | — | — | — | — | 3600.98 | 7 | 5 | 28.57 |
| 2 | 30 | 3606.11 | 12 | 8 | 33,33 | 3601.89 | 8 | 5 | 37.50 |
| 3 | 30 | — | — | — | — | 3604.28 | 8 | 5 | 37.50 |
| 4 | 30 | — | — | — | — | 7.03 | 17 | 17 | 0 |
| 5 | 30 | 3604.93 | 18 | 3 | 83.33 | 3285.93 | 4 | 4 | 0 |

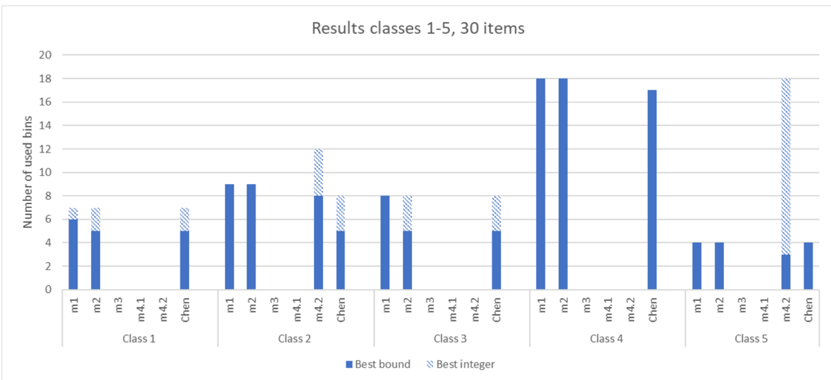



Figure 14: Visualization of results, classes 1-5, 30 items

**Results of classes 6-11, 10 items**

Table 7 presents the results for classes 6-11 with the input sets of 10 items. The best integers and the best bounds are represented in Figure 15. For these classes, 63.89% of the founded solutions is optimal. The similar results for classes 7, 9 and 11 are remarkable. For all models, except for model 4.1, the solver found an optimal solution. For model 4.1 the exception is the integer solution for class 11. The solver found optimal solutions for models 1, 2, 4.2 and Chen's model for the input classes 6, 8 and 10. The solver found for those three classes of model 3 integer solutions, while for model 4.1 no solution was found for classes 6 and 10 and an integer solution for class 8.

Table 7: Results of classes 6-11 for the input of 10 items

| | | Model 1 | | | | Model 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 10 | 0.46 | 4 | 4 | 0 | 0.57 | 4 | 4 | 0 |
| 7 | 10 | 0.62 | 1 | 1 | 0 | 0.22 | 1 | 1 | 0 |
| 8 | 10 | 0.55 | 2 | 2 | 0 | 0.35 | 2 | 2 | 0 |
| 9 | 10 | 0.29 | 1 | 1 | 0 | 0.19 | 1 | 1 | 0 |
| 10 | 10 | 0.61 | 3 | 3 | 0 | 0.33 | 3 | 3 | 0 |
| 11 | 10 | 0.27 | 1 | 1 | 0 | 0.19 | 1 | 1 | 0 |
| | | Model 3 | | | | Model 4.1 | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 10 | 3600.18 | 3 | 1 | 66.66 | — | — | — | — |
| 7 | 10 | 0.48 | 1 | 1 | 0 | 00.48 | 1 | 1 | 0 |
| 8 | 10 | 3600.17 | 2 | 1 | 50 | 3670.84 | 2 | 1 | 50 |
| 9 | 10 | 0.58 | 1 | 1 | 0 | 00.81 | 1 | 1 | 0 |
| 10 | 10 | 3600.63 | 3 | 1 | 66.66 | — | — | — | — |
| 11 | 10 | 0.37 | 1 | 1 | 0 | 3606.60 | 3 | 1 | 66.66 |
| | | Model 4.2 | | | | Model Chen | | | |
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 10 | 00.4 | 4 | 4 | 0 | 0.28 | 3 | 3 | 0 |
| 7 | 10 | 00.66 | 1 | 1 | 0 | 0.19 | 1 | 1 | 0 |
| 8 | 10 | 2.03 | 2 | 2 | 0 | 0.73 | 2 | 2 | 0 |
| 9 | 10 | 00.74 | 1 | 1 | 0 | 0.30 | 1 | 1 | 0 |
| 10 | 10 | 31.52 | 3 | 3 | 0 | 5.84 | 3 | 3 | 0 |
| 11 | 10 | 91.22 | 1 | 1 | 0 | 0.41 | 1 | 1 | 0 |

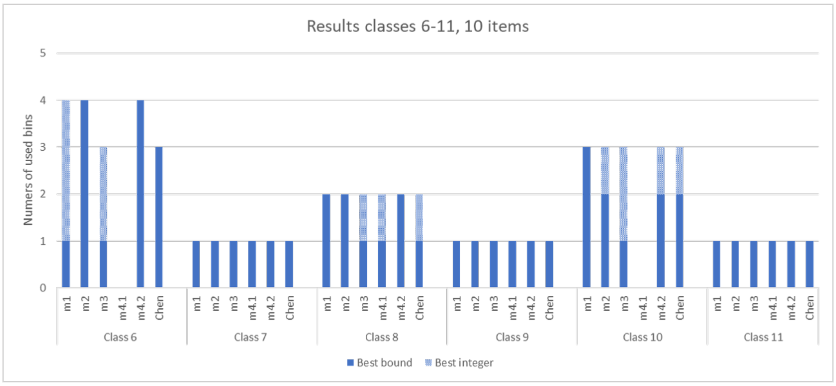



Figure 15: Visualization of results, classes 6-11, 10 items

**Results of classes 6-11, 30 items**

The results for the set of 30 items are presented in Table 8 and visualized in Figure 16. For 38.89% of the cases, the solution is optimal. Model 1 is solved for every class and the solutions are optimal. For model 2, the solver found an optimal solution for classes 6, 7, 8, 9 and 11, and for class 10 the solution is integer. For model 3, no solutions are found for classes 6 and 10, for the remaining classes the presented solutions are integer solutions. For model 4.1 and model 4.2 the solver cannot find an optimal solution. The presented solutions for classes 7, 8, 9 and 11 are integer solution, and for classes 6 and 10, no feasible solution is found. The solutions of Chen's model are optimal for classes 7, 9 and 11 and for classes 6, 8 and 10 integers.

Table 8: Results of classes 6-11 for the input of 30 items

| | | Model 1 | | | | Model 2 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 30 | 37.31 | 7 | 7 | 0 | 69.55 | 6 | 6 | 0 |
| 7 | 30 | 2.51 | 1 | 1 | 0 | 3.72 | 1 | 1 | 0 |
| 8 | 30 | 34.29 | 4 | 4 | 0 | 63.44 | 4 | 4 | 0 |
| 9 | 30 | 2.29 | 1 | 1 | 0 | 3.00 | 1 | 1 | 0 |
| 10 | 30 | 144.40 | 7 | 7 | 0 | 3600.79 | 6 | 5 | 16.67 |
| 11 | 30 | 2.60 | 1 | 1 | 0 | 2.26 | 1 | 1 | 0 |

| | | Model 3 | | | | Model 4.1 | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 30 | — | — | — | — | — | — | — | — |
| 7 | 30 | 3602.09 | 6 | 1 | 83.33 | 3660.06 | 14 | 1 | 92.86 |
| 8 | 30 | 3602.27 | 13 | 1 | 92.31 | 3611.42 | 9 | 1 | 88.99 |
| 9 | 30 | 3601.74 | 6 | 1 | 83.33 | 3611.42 | 9 | 1 | 88.99 |
| 10 | 30 | — | — | — | — | — | — | — | — |
| 11 | 30 | 3600.47 | 2 | 1 | 50 | 3605.34 | 9 | 1 | 88.99 |

| | | Model 4.2 | | | | Model Chen | | | |
|---|---|---|---|---|---|---|---|---|---|
| Classes | Items | Time | Best int | Best bound | Gap | Time | Best int | Best bound | Gap |
| 6 | 30 | — | — | — | — | 3600.71 | 6 | 4 | 33.33 |
| 7 | 30 | 3603.41 | 9 | 1 | 88.89 | 1.73 | 1 | 1 | 0 |
| 8 | 30 | 3605.52 | 7 | 2 | 71.43 | 3601.05 | 4 | 2 | 50 |
| 9 | 30 | 3605.61 | 15 | 1 | 93.33 | 2.00 | 1 | 1 | 0 |
| 10 | 30 | — | — | — | — | 3605.12 | 6 | 4 | 33.33 |
| 11 | 30 | 3604.10 | 15 | 1 | 93.33 | 2.01 | 1 | 1 | 0 |

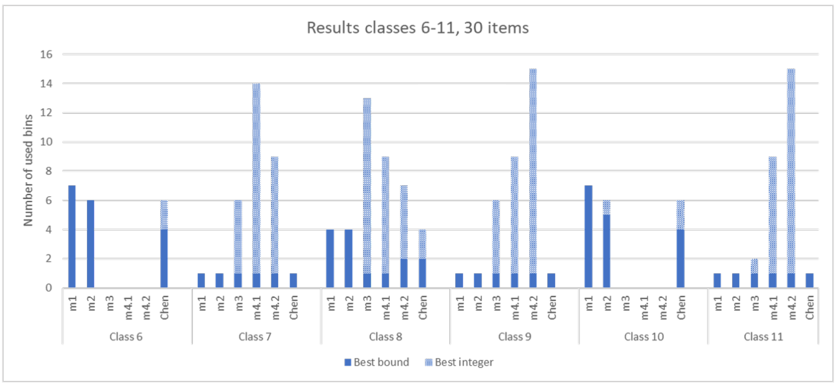



Figure 16: Visualization of results, classes 6-11, 30 items

### 5.4 Experiment 3: real world dataset

Historical data provided by PDSNL is used as input data for the models. The dataset consists of all the parcels handled in one specific depot in three days. This comes down to over 119,000 data entries. For each parcel the dataset gives the date and time of entering the sorting process, its dimensions in millimetres (length, width and height), the lifting conveyor belt on which it enters the sorting machine as well as its destination chute in the sorting machine together with its end destination (depot in this case). It is assumed that this dataset can be generalized over every depot since they all work in the same manner with concern to sorting and all deal with the parcels in the same way.

The dataset is first sorted based on the destination chute to break down the large dataset into manageable parts which will serve as samples. From here, faulty data entries are removed. These are parcels inserted onto the sorting machine but with their dimensions outside the boundaries of the machine (e.g. too small to measure). After transforming every dimension from millimetres to centimetres for generalization, samples are taken from chute belts selected at random and the distributions of their dimensions are drawn. Different distributions are tested for the input data, and this resulted in a log-normal distribution, which is then used to generate all the instances. Four instances have been generated in total. The first two contains 10 items each; and the second two instances contain 30 items each. The size of the bins matches the size of the roll-containers (58 x 78 x 178). The heights for the layer of models 4.1 and 4.2 are pre-set as (89, 89 cm) and (118, 60 cm), respectively. The choice is again driven by the dimensions of the majority of the generated items.

Table 9 presents the results of instance 1 with 10 items. The solver found optimal solutions for models 1, 2, and Chen et al. (1995)'s model, whereas for model 3, 4.1, and 4.2 no optimal solution is found, but only the feasible solution with an optimality gap of 50%.

Table 9: Experiment results for real world dataset of 10 items

| Instance 1: 10 items | | | | | |
|---|---|---|---|---|---|
| **Model** | **Time** | **Best bound** | **Best integer** | **Gap** | **Layer height** |
| 1 | 0.58 | 2 | 2 | 0 | |
| 2 | 0.41 | 2 | 2 | 0 | |
| 3 | 3600.19 | 2 | 1 | 50 | |
| 4.1 | 3601.15 | 2 | 1 | 50 | (89, 89 cm) |
| 4.2 | 3601.98 | 2 | 1 | 50 | (118, 60 cm ) |
| Chen | 133.63 | 2 | 2 | 0 | |
| **Instance 2: 10 items** | | | | | |
| **Model** | **Time** | **Best bound** | **Best integer** | **Gap** | **Layer height** |
| 1 | 0.43 | 2 | 2 | 0 | |
| 2 | 0.73 | 2 | 2 | 0 | |
| 3 | 39.7 | 1 | 1 | 0 | |
| 4.1 | 3600.36 | 2 | 1 | 50 | (89, 89 cm) |
| 4.2 | 3600.39 | 2 | 1 | 50 | (118, 60 cm ) |
| Chen | 0.55 | 1 | 1 | 0 | |

Table 10: Experiment results for real world dataset of 30 items

| Instance 3: 30 items | | | | | |
|---|---|---|---|---|---|
| **Model** | **Time** | **Best bound** | **Best integer** | **Gap** | **Layer height** |
| 1 | 4.04 | 3 | 3 | 0 | |
| 2 | 3600.97 | 3 | 2 | 33.33 | |
| 3 | 3601.54 | 9 | 1 | 88.89 | |
| 4.1 | 3601.9 | 11 | 1 | 90.91 | (89, 89 cm) |
| 4.2 | 3602.45 | 6 | 2 | 66.66 | (118, 60 cm ) |
| Chen | 3608.18 | 3 | 1 | 66.66 | |
| **Instance 4: 30 items** | | | | | |
| **Model** | **Time** | **Best bound** | **Best integer** | **Gap** | **Layer height** |
| 1 | 1.89 | 6 | 6 | 0 | |
| 2 | 3601.45 | 5 | 3 | 40 | |
| 3 | — | — | — | — | |
| 4.1 | 3601.56 | 10 | 1 | 90 | (89, 89 cm) |
| 4.2 | 3602.72 | 6 | 2 | 66.66 | (118, 60 cm ) |
| Chen | 3609.09 | 5 | 1 | 80 | |

Table 9 reports the results for instances 1 and 2. Both instance 1 and 2 contain 10 items, however, for both instances the items are different. Surprisingly, it is found that the composition of items affects the performance of model 3. For instance 2, model 3 has found the optimal solution whereas it could not for instance 1. For the rest of the models, the performance is similar to that of the first set.

Table 10 gives the results of instances 3 with 30 items. This is the most interesting experimentation since the number of items is comparable to the fitted amount in real life. It can be seen that only for model 1 the optimal solution was found, and the solver was for all other models not able to find a feasible solution, but with a gap to the optimal solution. When looking into the optimality gap, for model 2 the gap is 33.33%, which is better than the optimality gap of 66.66% for Chen's model and model 4.2. Models 3 and 4.1 has the poorest optimality gap.

Instance 4 also has 30 items. It is shown in Table 10 that the composition of items does affect the performance of models when compared to the results of Instance 3. Model 1 is solved for instance 4; for model 2 the optimality gap increased to 40% from 33.3

### 5.5 Analysis of the results

In this study, we test the model of Chen et al. (1995) for all the instances. We noticed a difference in the solutions of this model. For the relatively smaller instances, with 10 or fewer items, all the solutions are better or equal to the solutions of other models. Models 1 and 2 are also solved for these instances and an optimal solution is found. However, the model of Chen always requires less or an equal number of bins, which was expected since its solutions yield as a lower bound to the developed models.

For the relatively larger instances, with 30 items, the solver often exceeds the time limit for most models and an optimal solution is not found. This also applies to the solutions of Chen's model, which are often feasible integer solutions instead of optimal solutions and the solver exceeded the time limit of 3600 seconds. In general, for these instances, the optimal solution of model 1 is found in relatively shorter time compared to Chen's model. It is worth mentioning that, for some cases, when for Chen's model no optimal solution is found, the solver can still find the optimal solution for model 1. This applies also for the instances generated using input from PDSNL.

For model 3, often no solution is found or CPLEX exceeds the time limit. If for some instance an optimal solution is found, it requires the same number of bins as the solutions of Chen's model for this instance. Nevertheless, the computational time needed to solve model 3 is often longer than for Chen's model. The solutions of models 4.1 and 4.2 are not comparable to the solutions of Chen's model. Repeatedly no solution is found for these models. This can be caused by the dimensions of the items, which are specific uniform distributions based on the bin dimensions. Therefore, there may be items that do not fit within the fixed layers.

In the parcel delivery services, a key indicator of customer satisfaction is delivering on time. This industry handles large quantities of small items every day. Therefore, the solver should be able to find solutions for a large set of items. Based on the results of Tables 9 and 10, model 1 fits the best for the parcel delivery services. First, this is the only model for which all solutions of experiment three are optimal. Secondly, even though model 1 does not allow for items to rotate, for this industry, this is not a problem since in real practice this might also be a limitation of some items. Thirdly, model 1 uses physical layers to place the items in the bins. Whenever containers are transported, the use of layers may provide extra stability to the containers, which may be an advantage. Furthermore, an advantage may be the possibility to separate different product form each other.

To conclude, Chen's model, which has no layers and allow items to rotate 360°, is a reliable and performing model for small instances. When Chen's model is adapted to layer packing models with fixed height (Models 4.1 and 4.2 or flexible height (model 3), the computational times required increase significantly. Particularly, the fixed height layer packing (model 4.1 and 4.2) has the poorest computational performance in general. However, it is worth mentioning that, when the rotation of items is not allowed, the computational performance is very good as indicated by the outputs of model 1. For the cases with large item sets, the computational performance of model 1 is even better than that of Chen's model. This model, model 1, is also the best fit for the parcel delivery services.

# 6 Conclusions and future extensions

In this research, four new mathematical models of the three-dimensional layer packing problem are developed. Earlier studies that focused on this problem provide mathematical models for the two-dimensional layer packing problem or the classical three-dimensional bin packing problem. By including extra constraints into the classical model, our work has extended this early research to 3-D layer packing; thereafter, four models are developed for 3-D layer packing and tested with literature-based data and real-world dataset and solved with a branch-and-bound procedure.

We may say that our goal, to find indications of a feasible solution in lower time, is achieved. For the relatively smaller instances, the solutions of Chen's model are the best and the solver requires a relatively short computational time, so none of the developed models has better performances than Chen's model. For the larger instances, the optimal solutions of model 1 may be an indication of a feasible solution in lower time. The solutions of model 2 are also often optimal solutions, however, the solver requires more time to find these solutions and therefore model 1 is a better fit.

Based on the information in this study the solutions of the developed models are not exclusively better for industrial use, it depends on the type of items that are to be packed. However, for larger instances, the first two developed models are solved within one hour and the solutions are optimal, the solver is not capable of doing this for the model of Chen et al. (1995). Although, no hard conclusion may be drawn for the practical use of the models, however, it appears that model 1 is best suited, due to its quick performances for larger instances.

## References

Bortfeldt, A. and Wäscher, G. (2013). Constraints in container loading–a state-of-the-art review. *European Journal of Operational Research*, 229(1):1–20.

Chen, C., Lee, S.-M., and Shen, Q. (1995). An analytical model for the container loading problem. *European Journal of Operational Research*, 80(1):68–76.

Coffman Jr, E. G., Csirik, J., Johnson, D. S., and Woeginger, G. J. (2004 (accessed July 21st, 2018)). *An introduction to bin packing*. `www.inf.u-szeged.hu/~csirik/`.

Crainic, T. G., Perboli, G., and Tadei, R. (2009). Ts 2 pack: A two-level tabu search for the three-dimensional bin packing problem. *European Journal of Operational Research*, 195(3):744–760.

Dowsland, K. A. and Dowsland, W. B. (1992). Packing problems. *European journal of operational research*, 56(1):2–14.

Ecommerce (2016 (accessed April 10th, 2017)). *Ecommerce in Europe to reach 509.9 billion Euro in 2016*. `https://ecommercenews.eu/ecommerce-europe-reach-e509-9-billion-2016/`.

Faroe, O., Pisinger, D., and Zachariasen, M. (2003). Guided local search for the three-dimensional bin-packing problem. *Informs Journal on Computing*, 15(3):267–283.

Kim, S.-J., Lim, H., and Park, M. (2014). Analysing the cost efficiency of parcel distribution networks with changes in demand. *International Journal of Urban Sciences*, 18(3):416–429.

Lodi, A., Martello, S., and Vigo, D. (2002a). Heuristic algorithms for the three-dimensional bin packing problem. *European Journal of Operational Research*, 141(2):410–420.

Lodi, A., Martello, S., and Vigo, D. (2002b). Recent advances on two-dimensional bin packing problems. *Discrete Applied Mathematics*, 123(1):379–396.

Lodi, A., Martello, S., and Vigo, D. (2004). Models and bounds for two-dimensional level packing problems. *Journal of Combinatorial Optimization*, 8(3):363–379.

Martello, S., Pisinger, D., and Vigo, D. (2000a). The three-dimensional bin packing problem. *Operations Research*, 48(2):256–267.

Martello, S., Pisinger, D., and Vigo, D. (2000b). The three-dimensional bin packing problem. *Operations Research*, 48(2):256–267.

Paquay, C., Schyns, M., and Limbourg, S. (2016). A mixed integer programming formulation for the three-dimensional bin packing problem deriving from an air cargo application. *International Transactions in Operational Research*, 23(1-2):187–213.

Sciomachen, A. and Tanfani, E. (2003). The master bay plan problem: a solution method based on its connection to the three-dimensional bin packing problem. *IMA Journal of Management Mathematics*, 14(3):251–269.

Sciomachen, A. and Tanfani, E. (2007). A 3d-bpp approach for optimising stowage plans and terminal productivity. *European Journal of Operational Research*, 183(3):1433–1446.

Wu, Y., Li, W., Goh, M., and de Souza, R. (2010). Three-dimensional bin packing problem with variable bin height. *European Journal of Operational Research*, 202(2):347–355.

Zhao, X., Bennell, J. A., Bektaş, T., and Dowsland, K. (2016). A comparative review of 3d container loading algorithms. *International Transactions in Operational Research*, 23(1-2):287–320.

# A Input data

## A.1 Classes 1-5

### A.1.1 8 items

Table 11: Classes 1 and 2, 8 items

| Class 1 | Item | h | w | l | Class 2 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 92 | 4 | 95 | | 1 | 79 | 69 | 67 |
| | 2 | 85 | 47 | 79 | | 2 | 73 | 56 | 56 |
| | 3 | 75 | 4 | 78 | | 3 | 43 | 98 | 77 |
| | 4 | 74 | 36 | 78 | | 4 | 40 | 82 | 69 |
| | 5 | 68 | 93 | 6 | | 5 | 32 | 67 | 81 |
| | 6 | 58 | 67 | 66 | | 6 | 30 | 95 | 82 |
| | 7 | 43 | 73 | 82 | | 7 | 28 | 73 | 93 |
| | 8 | 15 | 88 | 79 | | 8 | 15 | 99 | 71 |

Table 12: Classes 3 and 4, 8 items

| Class 3 | Item | h | w | l | Class 4 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 100 | 92 | 11 | | 1 | 99 | 98 | 14 |
| | 2 | 93 | 4 | 98 | | 2 | 96 | 75 | 62 |
| | 3 | 89 | 71 | 42 | | 3 | 89 | 99 | 75 |
| | 4 | 81 | 84 | 6 | | 4 | 87 | 79 | 79 |
| | 5 | 80 | 68 | 43 | | 5 | 84 | 16 | 76 |
| | 6 | 76 | 67 | 86 | | 6 | 69 | 76 | 59 |
| | 7 | 70 | 70 | 13 | | 7 | 47 | 7 | 13 |
| | 8 | 13 | 45 | 27 | | 8 | 12 | 88 | 82 |

Table 13: Class 5, 8 items

| Item | h | w | l |
|---|---|---|---|
| 1 | 75 | 3 | 90 |
| 2 | 73 | 91 | 14 |
| 3 | 73 | 53 | 67 |
| 4 | 58 | 77 | 68 |
| 5 | 48 | 50 | 13 |
| 6 | 21 | 46 | 7 |
| 7 | 13 | 2 | 12 |
| 8 | 5 | 77 | 74 |

### A.1.2 10 items

Table 14: Classes 1 and 2, 10 items

| Class 1 | Item | h | w | l | Class 2 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 91 | 91 | 24 | | 1 | 95 | 90 | 9 |
| | 2 | 85 | 24 | 96 | | 2 | 68 | 25 | 88 |
| | 3 | 83 | 8 | 68 | | 3 | 53 | 74 | 60 |
| | 4 | 76 | 16 | 79 | | 4 | 50 | 72 | 77 |
| | 5 | 71 | 38 | 68 | | 5 | 49 | 78 | 89 |
| | 6 | 68 | 44 | 98 | | 6 | 43 | 78 | 72 |
| | 7 | 67 | 28 | 74 | | 7 | 37 | 95 | 92 |
| | 8 | 55 | 94 | 71 | | 8 | 32 | 36 | 6 |
| | 9 | 47 | 94 | 85 | | 9 | 5 | 99 | 85 |
| | 10 | 43 | 25 | 27 | | 10 | 4 | 100 | 82 |

Table 15: Classes 3 and 4, 10 items

| Class 3 | Item | h | w | l | Class 4 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 99 | 97 | 13 | | 1 | 95 | 98 | 96 |
| | 2 | 93 | 27 | 79 | | 2 | 85 | 81 | 41 |
| | 3 | 92 | 75 | 17 | | 3 | 80 | 54 | 76 |
| | 4 | 86 | 69 | 35 | | 4 | 75 | 44 | 67 |
| | 5 | 83 | 97 | 15 | | 5 | 57 | 86 | 81 |
| | 6 | 74 | 84 | 12 | | 6 | 56 | 94 | 97 |
| | 7 | 69 | 68 | 10 | | 7 | 55 | 52 | 92 |
| | 8 | 52 | 56 | 56 | | 8 | 54 | 76 | 65 |
| | 9 | 50 | 99 | 88 | | 9 | 33 | 89 | 81 |
| | 10 | 19 | 44 | 50 | | 10 | 11 | 1 | 47 |

Table 16: Class 5, 10 items

| Item | h | w | l |
|---|---|---|---|
| 1 | 94 | 11 | 78 |
| 2 | 94 | 62 | 69 |
| 3 | 73 | 67 | 49 |
| 4 | 45 | 42 | 16 |
| 5 | 30 | 43 | 42 |
| 6 | 29 | 21 | 41 |
| 7 | 21 | 33 | 25 |
| 8 | 18 | 73 | 88 |
| 9 | 9 | 2 | 23 |
| 10 | 6 | 37 | 42 |

### A.1.3 30 items

Table 17: Classes 1 and 2, 30 items

| Class 1 Item | h | w | l | Class 2 | item | h | w | l | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 99 | 33 | 82 | | 1 | 98 | 93 | 50 |
| | 2 | 97 | 12 | 93 | | 2 | 97 | 60 | 74 |
| | 3 | 97 | 70 | 15 | | 3 | 92 | 75 | 42 |
| | 4 | 95 | 16 | 89 | | 4 | 92 | 94 | 48 |
| | 5 | 94 | 35 | 77 | | 5 | 73 | 63 | 57 |
| | 6 | 88 | 62 | 83 | | 6 | 70 | 45 | 92 |
| | 7 | 87 | 7 | 78 | | 7 | 57 | 74 | 57 |
| | 8 | 84 | 22 | 69 | | 8 | 46 | 90 | 93 |
| | 9 | 83 | 5 | 67 | | 9 | 46 | 41 | 7 |
| | 10 | 83 | 53 | 63 | | 10 | 44 | 68 | 81 |
| | 11 | 82 | 33 | 82 | | 10 | 41 | 99 | 73 |
| | 12 | 81 | 36 | 96 | | 10 | 38 | 80 | 86 |
| | 13 | 81 | 2 | 98 | | 10 | 38 | 81 | 68 |
| | 14 | 80 | 34 | 84 | | 10 | 31 | 72 | 76 |
| | 15 | 77 | 50 | 91 | | 10 | 30 | 70 | 100 |
| | 16 | 77 | 18 | 97 | | 10 | 28 | 77 | 76 |
| | 17 | 77 | 32 | 86 | | 10 | 28 | 41 | 27 |
| | 18 | 75 | 11 | 91 | | 10 | 23 | 98 | 98 |
| | 19 | 74 | 89 | 8 | | 10 | 23 | 76 | 75 |
| | 20 | 72 | 62 | 100 | | 10 | 22 | 2 | 20 |
| | 21 | 70 | 13 | 67 | | 10 | 18 | 83 | 79 |
| | 22 | 69 | 28 | 80 | | 10 | 17 | 86 | 100 |
| | 23 | 67 | 21 | 90 | | 10 | 13 | 72 | 86 |
| | 24 | 59 | 62 | 65 | | 10 | 12 | 87 | 90 |
| | 25 | 41 | 24 | 5 | | 10 | 7 | 91 | 75 |
| | 26 | 34 | 99 | 84 | | 10 | 4 | 75 | 92 |
| | 27 | 23 | 30 | 13 | | 10 | 3 | 100 | 83 |
| | 28 | 20 | 89 | 92 | | 10 | 3 | 82 | 75 |
| | 29 | 20 | 98 | 90 | | 10 | 3 | 94 | 99 |
| | 30 | 2 | 81 | 87 | | 10 | 3 | 69 | 84 |

Table 18: Classes 3 and 4, 30 items

| Class 3 | Item | h | w | l | Class 4 | | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 100 | 70 | 3 | | 1 | 100 | 90 | 30 |
| | 2 | 100 | 64 | 73 | | 2 | 95 | 60 | 57 |
| | 3 | 99 | 74 | 8 | | 3 | 91 | 94 | 31 |
| | 4 | 98 | 67 | 29 | | 4 | 88 | 78 | 71 |
| | 5 | 98 | 99 | 98 | | 5 | 87 | 78 | 50 |
| | 6 | 96 | 60 | 88 | | 6 | 86 | 36 | 76 |
| | 7 | 95 | 75 | 32 | | 7 | 86 | 90 | 77 |
| | 8 | 94 | 89 | 36 | | 8 | 81 | 51 | 65 |
| | 9 | 94 | 100 | 35 | | 9 | 79 | 43 | 68 |
| | 10 | 93 | 92 | 46 | | 10 | 78 | 23 | 68 |
| | 11 | 92 | 81 | 20 | | 11 | 77 | 51 | 56 |
| | 12 | 92 | 77 | 86 | | 12 | 74 | 63 | 84 |
| | 13 | 91 | 21 | 88 | | 13 | 73 | 2 | 71 |
| | 14 | 86 | 83 | 11 | | 14 | 72 | 87 | 37 |
| | 15 | 85 | 6 | 87 | | 15 | 71 | 44 | 92 |
| | 16 | 82 | 1 | 92 | | 16 | 70 | 51 | 89 |
| | 17 | 80 | 81 | 25 | | 17 | 70 | 96 | 76 |
| | 18 | 80 | 92 | 23 | | 18 | 68 | 77 | 70 |
| | 19 | 79 | 11 | 67 | | 19 | 68 | 82 | 93 |
| | 20 | 77 | 80 | 1 | | 20 | 66 | 65 | 53 |
| | 21 | 74 | 96 | 34 | | 21 | 63 | 51 | 58 |
| | 22 | 72 | 40 | 97 | | 22 | 62 | 86 | 66 |
| | 23 | 71 | 95 | 9 | | 23 | 61 | 63 | 100 |
| | 24 | 70 | 81 | 37 | | 24 | 59 | 91 | 83 |
| | 25 | 70 | 73 | 29 | | 25 | 53 | 59 | 91 |
| | 26 | 68 | 89 | 36 | | 26 | 50 | 82 | 89 |
| | 27 | 42 | 45 | 17 | | 27 | 40 | 68 | 90 |
| | 28 | 19 | 8 | 34 | | 28 | 33 | 78 | 84 |
| | 29 | 17 | 17 | 2 | | 29 | 33 | 49 | 36 |
| | 30 | 10 | 23 | 11 | | 30 | 3 | 14 | 17 |

Table 19: Class 5, 30 items

| Class 5 | Item | h | w | l |
|---|---|---|---|---|
| | 1 | 100 | 85 | 6 |
| | 2 | 98 | 32 | 78 |
| | 3 | 96 | 26 | 71 |
| | 4 | 92 | 70 | 16 |
| | 5 | 84 | 5 | 84 |
| | 6 | 78 | 67 | 17 |
| | 7 | 76 | 41 | 98 |
| | 8 | 73 | 77 | 34 |
| | 9 | 67 | 67 | 80 |
| | 10 | 64 | 91 | 80 |
| | 11 | 46 | 45 | 4 |
| | 12 | 45 | 30 | 25 |
| | 13 | 44 | 48 | 39 |
| | 14 | 43 | 48 | 34 |
| | 15 | 40 | 1 | 38 |
| | 16 | 37 | 17 | 45 |
| | 17 | 36 | 7 | 16 |
| | 18 | 35 | 5 | 2 |
| | 19 | 31 | 37 | 30 |
| | 20 | 27 | 94 | 99 |
| | 21 | 25 | 42 | 2 |
| | 22 | 23 | 43 | 44 |
| | 23 | 19 | 46 | 45 |
| | 24 | 16 | 29 | 40 |
| | 25 | 13 | 12 | 13 |
| | 26 | 10 | 81 | 67 |
| | 27 | 7 | 44 | 42 |
| | 28 | 5 | 31 | 36 |
| | 29 | 2 | 18 | 41 |
| | 30 | 2 | 30 | 9 |

## A.2 Classes 6-11

### A.2.1 10 items

Table 20: Classes 6 and 7, 10 items

| Class 6 | Item | h | w | l | Class 7 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 10 | 3 | 8 | | 1 | 10 | 2 | 1 |
| | 2 | 10 | 6 | 9 | | 2 | 10 | 7 | 9 |
| | 3 | 9 | 1 | 6 | | 3 | 7 | 2 | 9 |
| | 4 | 8 | 8 | 2 | | 4 | 6 | 2 | 3 |
| | 5 | 6 | 9 | 9 | | 5 | 6 | 8 | 4 |
| | 6 | 6 | 3 | 8 | | 6 | 5 | 2 | 9 |
| | 7 | 5 | 10 | 1 | | 7 | 4 | 2 | 1 |
| | 8 | 5 | 8 | 10 | | 8 | 1 | 1 | 2 |
| | 9 | 4 | 5 | 3 | | 9 | 1 | 5 | 1 |
| | 10 | 3 | 2 | 9 | | 10 | 1 | 5 | 7 |

Table 21: Classes 8 and 9, 10 items

| Class 8 | Item | h | w | l | Class 9 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 32 | 27 | 15 | | 1 | 34 | 18 | 11 |
| | 2 | 22 | 13 | 29 | | 2 | 17 | 1 | 26 |
| | 3 | 22 | 17 | 20 | | 3 | 15 | 9 | 17 |
| | 4 | 18 | 34 | 13 | | 4 | 15 | 14 | 18 |
| | 5 | 14 | 27 | 30 | | 5 | 14 | 12 | 13 |
| | 6 | 12 | 2 | 21 | | 6 | 7 | 29 | 19 |
| | 7 | 8 | 12 | 18 | | 7 | 4 | 8 | 25 |
| | 8 | 7 | 3 | 17 | | 8 | 4 | 20 | 32 |
| | 9 | 7 | 4 | 19 | | 9 | 3 | 20 | 3 |
| | 10 | 4 | 12 | 5 | | 10 | 2 | 12 | 12 |

Table 22: Classes 10 and 11, 10 items

| Class 10 | Item | h | w | l | Class 11 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 98 | 63 | 8 | | 1 | 99 | 43 | 9 |
| | 2 | 93 | 38 | 60 | | 2 | 90 | 24 | 83 |
| | 3 | 83 | 21 | 80 | | 3 | 88 | 40 | 5 |
| | 4 | 80 | 100 | 39 | | 4 | 76 | 80 | 24 |
| | 5 | 75 | 55 | 83 | | 5 | 46 | 8 | 56 |
| | 6 | 52 | 69 | 86 | | 6 | 38 | 87 | 4 |
| | 7 | 29 | 38 | 48 | | 7 | 6 | 15 | 37 |
| | 8 | 27 | 56 | 90 | | 8 | 5 | 10 | 52 |
| | 9 | 21 | 43 | 45 | | 9 | 2 | 85 | 41 |
| | 10 | 14 | 18 | 50 | | 10 | 1 | 80 | 67 |

### A.2.2 30 items

Table 23: Classes 6 and 7, 30, items

| Class 6 | Item | h | w | l | Class 7 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 10 | 6 | 3 | | 1 | 10 | 2 | 6 |
| | 2 | 10 | 9 | 9 | | 2 | 10 | 6 | 5 |
| | 3 | 9 | 7 | 2 | | 3 | 10 | 5 | 8 |
| | 4 | 9 | 1 | 4 | | 4 | 10 | 6 | 7 |
| | 5 | 9 | 1 | 10 | | 5 | 8 | 5 | 1 |
| | 6 | 9 | 8 | 1 | | 6 | 8 | 9 | 5 |
| | 7 | 8 | 8 | 8 | | 7 | 8 | 9 | 10 |
| | 8 | 8 | 9 | 4 | | 8 | 7 | 7 | 1 |
| | 9 | 7 | 7 | 5 | | 9 | 7 | 5 | 5 |
| | 10 | 7 | 10 | 1 | | 10 | 7 | 6 | 1 |
| | 11 | 5 | 6 | 6 | | 11 | 7 | 8 | 5 |
| | 12 | 5 | 5 | 10 | | 12 | 6 | 9 | 1 |
| | 13 | 5 | 7 | 7 | | 13 | 6 | 5 | 6 |
| | 14 | 5 | 9 | 2 | | 14 | 6 | 2 | 8 |
| | 15 | 5 | 9 | 9 | | 15 | 6 | 8 | 4 |
| | 16 | 4 | 10 | 1 | | 16 | 5 | 3 | 6 |
| | 17 | 4 | 9 | 6 | | 17 | 5 | 7 | 6 |
| | 18 | 4 | 6 | 10 | | 18 | 4 | 8 | 10 |
| | 19 | 3 | 8 | 3 | | 19 | 4 | 1 | 8 |
| | 20 | 3 | 2 | 5 | | 20 | 4 | 3 | 2 |
| | 21 | 2 | 3 | 7 | | 21 | 3 | 5 | 3 |
| | 22 | 2 | 2 | 1 | | 22 | 3 | 10 | 9 |
| | 23 | 2 | 1 | 9 | | 23 | 3 | 10 | 3 |
| | 24 | 2 | 2 | 7 | | 24 | 3 | 1 | 1 |
| | 25 | 2 | 2 | 2 | | 25 | 2 | 10 | 2 |
| | 26 | 2 | 4 | 3 | | 26 | 1 | 7 | 3 |
| | 27 | 1 | 6 | 3 | | 27 | 1 | 10 | 6 |
| | 28 | 1 | 3 | 7 | | 28 | 1 | 1 | 4 |
| | 29 | 1 | 3 | 10 | | 29 | 1 | 6 | 10 |
| | 30 | 1 | 2 | 5 | | 30 | 1 | 5 | 5 |

Table 24: Classes 8 and 9, 30 items

| Class 8 | Item | h | w | l | Class 9 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 32 | 4 | 1 | | 1 | 34 | 5 | 10 |
| | 2 | 31 | 26 | 11 | | 2 | 34 | 15 | 19 |
| | 3 | 31 | 12 | 34 | | 3 | 33 | 34 | 18 |
| | 4 | 28 | 1 | 13 | | 4 | 32 | 32 | 34 |
| | 5 | 28 | 20 | 32 | | 5 | 32 | 29 | 3 |
| | 6 | 27 | 4 | 29 | | 6 | 31 | 18 | 18 |
| | 7 | 27 | 26 | 1 | | 7 | 31 | 20 | 7 |
| | 8 | 26 | 5 | 28 | | 8 | 30 | 7 | 14 |
| | 9 | 26 | 24 | 22 | | 9 | 29 | 17 | 25 |
| | 10 | 26 | 6 | 29 | | 10 | 28 | 34 | 10 |
| | 11 | 23 | 4 | 21 | | 11 | 28 | 13 | 30 |
| | 12 | 20 | 23 | 12 | | 12 | 20 | 17 | 15 |
| | 13 | 20 | 2 | 22 | | 13 | 19 | 9 | 23 |
| | 14 | 19 | 18 | 34 | | 14 | 18 | 14 | 26 |
| | 15 | 19 | 21 | 24 | | 15 | 18 | 16 | 15 |
| | 16 | 18 | 2 | 27 | | 16 | 18 | 18 | 17 |
| | 17 | 16 | 8 | 4 | | 17 | 15 | 23 | 32 |
| | 18 | 16 | 30 | 28 | | 18 | 15 | 4 | 14 |
| | 19 | 16 | 12 | 13 | | 19 | 13 | 19 | 8 |
| | 20 | 15 | 26 | 2 | | 20 | 13 | 25 | 9 |
| | 21 | 15 | 10 | 2 | | 21 | 13 | 27 | 19 |
| | 22 | 13 | 3 | 29 | | 22 | 12 | 19 | 19 |
| | 23 | 13 | 23 | 17 | | 23 | 11 | 6 | 10 |
| | 24 | 13 | 4 | 23 | | 24 | 11 | 9 | 13 |
| | 25 | 13 | 10 | 32 | | 25 | 8 | 26 | 17 |
| | 26 | 13 | 12 | 33 | | 26 | 8 | 7 | 29 |
| | 27 | 10 | 27 | 21 | | 27 | 6 | 12 | 29 |
| | 28 | 9 | 29 | 31 | | 28 | 5 | 23 | 26 |
| | 29 | 7 | 8 | 11 | | 29 | 3 | 11 | 17 |
| | 30 | 6 | 8 | 21 | | 30 | 1 | 26 | 29 |

Table 25: Classes 10 and 11, 30 items

| Class 10 | Item | h | w | l | Class 11 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 99 | 28 | 33 | | 1 | 98 | 68 | 75 |
| | 2 | 97 | 45 | 100 | | 2 | 97 | 61 | 13 |
| | 3 | 95 | 64 | 21 | | 3 | 90 | 56 | 85 |
| | 4 | 88 | 32 | 34 | | 4 | 86 | 81 | 56 |
| | 5 | 86 | 62 | 35 | | 5 | 83 | 77 | 65 |
| | 6 | 84 | 98 | 87 | | 6 | 81 | 2 | 15 |
| | 7 | 83 | 4 | 91 | | 7 | 79 | 78 | 77 |
| | 8 | 82 | 74 | 63 | | 8 | 71 | 41 | 19 |
| | 9 | 80 | 76 | 36 | | 9 | 67 | 66 | 61 |
| | 10 | 78 | 7 | 30 | | 10 | 66 | 93 | 27 |
| | 11 | 75 | 27 | 76 | | 11 | 65 | 37 | 81 |
| | 12 | 74 | 30 | 40 | | 12 | 63 | 97 | 11 |
| | 13 | 68 | 67 | 7 | | 13 | 62 | 41 | 38 |
| | 14 | 60 | 9 | 97 | | 14 | 61 | 89 | 13 |
| | 15 | 56 | 22 | 77 | | 15 | 60 | 79 | 12 |
| | 16 | 50 | 70 | 21 | | 16 | 58 | 35 | 65 |
| | 17 | 50 | 12 | 28 | | 17 | 58 | 72 | 15 |
| | 18 | 49 | 97 | 82 | | 18 | 56 | 96 | 8 |
| | 19 | 48 | 20 | 71 | | 19 | 46 | 20 | 13 |
| | 20 | 46 | 87 | 66 | | 20 | 40 | 68 | 10 |
| | 21 | 44 | 74 | 20 | | 21 | 35 | 10 | 30 |
| | 22 | 31 | 66 | 70 | | 22 | 32 | 39 | 67 |
| | 23 | 29 | 37 | 90 | | 23 | 30 | 40 | 23 |
| | 24 | 29 | 94 | 85 | | 24 | 28 | 77 | 65 |
| | 25 | 26 | 12 | 47 | | 25 | 16 | 49 | 96 |
| | 26 | 26 | 91 | 64 | | 26 | 14 | 96 | 70 |
| | 27 | 23 | 41 | 15 | | 27 | 13 | 87 | 57 |
| | 28 | 4 | 93 | 20 | | 28 | 12 | 13 | 59 |
| | 29 | 1 | 26 | 70 | | 29 | 5 | 93 | 45 |
| | 30 | 1 | 44 | 87 | | 30 | 2 | 26 | 20 |

## A.3 Real world data PDSNL

### A.3.1 Instances 1 and 2, 10 items

Table 26: Instances 1 and 2, 10 items

| Instance 1 | Item | h | w | l | Instance 2 | Item | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 77 | 46 | 46 | | 1 | 83 | 24 | 50 |
| | 2 | 62 | 32 | 41 | | 2 | 60 | 24 | 34 |
| | 3 | 62 | 30 | 35 | | 3 | 58 | 29 | 53 |
| | 4 | 60 | 31 | 47 | | 4 | 58 | 22 | 34 |
| | 5 | 58 | 27 | 41 | | 5 | 57 | 27 | 37 |
| | 6 | 55 | 28 | 48 | | 6 | 55 | 33 | 33 |
| | 7 | 50 | 35 | 36 | | 7 | 54 | 30 | 47 |
| | 8 | 47 | 19 | 43 | | 8 | 49 | 24 | 29 |
| | 9 | 38 | 34 | 35 | | 9 | 47 | 19 | 45 |
| | 10 | 37 | 31 | 36 | | 10 | 43 | 10 | 35 |

### A.3.2 Instances 3 and 4, 30 items

Table 27: Instances 3 and 4, 30 items

| Instance 3 | Items | h | w | l | Instance 4 | Items | h | w | l |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 65 | 20 | 26 | | 1 | 77 | 42 | 46 |
| | 2 | 64 | 35 | 63 | | 2 | 72 | 39 | 42 |
| | 3 | 64 | 35 | 55 | | 3 | 72 | 38 | 66 |
| | 4 | 62 | 32 | 61 | | 4 | 64 | 35 | 63 |
| | 5 | 62 | 19 | 25 | | 5 | 62 | 35 | 48 |
| | 6 | 62 | 25 | 28 | | 6 | 62 | 32 | 41 |
| | 7 | 61 | 18 | 30 | | 7 | 60 | 35 | 47 |
| | 8 | 60 | 17 | 41 | | 8 | 59 | 37 | 45 |
| | 9 | 59 | 33 | 38 | | 9 | 58 | 35 | 36 |
| | 10 | 59 | 18 | 39 | | 10 | 58 | 34 | 48 |
| | 11 | 58 | 34 | 42 | | 11 | 58 | 18 | 31 |
| | 12 | 58 | 18 | 31 | | 12 | 58 | 30 | 41 |
| | 13 | 57 | 35 | 42 | | 13 | 57 | 36 | 37 |
| | 14 | 57 | 19 | 40 | | 14 | 56 | 35 | 41 |
| | 15 | 56 | 36 | 43 | | 15 | 56 | 33 | 55 |
| | 16 | 55 | 32 | 38 | | 16 | 56 | 24 | 31 |
| | 17 | 54 | 21 | 33 | | 17 | 55 | 29 | 48 |
| | 18 | 53 | 20 | 31 | | 18 | 51 | 32 | 37 |
| | 19 | 52 | 20 | 33 | | 19 | 50 | 35 | 46 |
| | 20 | 48 | 23 | 36 | | 20 | 48 | 17 | 32 |
| | 21 | 48 | 17 | 32 | | 21 | 47 | 36 | 43 |
| | 22 | 46 | 16 | 30 | | 22 | 44 | 28 | 32 |
| | 23 | 46 | 15 | 32 | | 23 | 43 | 31 | 37 |
| | 24 | 42 | 31 | 41 | | 24 | 43 | 23 | 40 |
| | 25 | 42 | 25 | 30 | | 25 | 42 | 31 | 41 |
| | 26 | 42 | 29 | 40 | | 26 | 41 | 37 | 38 |
| | 27 | 41 | 15 | 23 | | 27 | 38 | 35 | 38 |
| | 28 | 39 | 34 | 37 | | 28 | 38 | 35 | 36 |
| | 29 | 26 | 15 | 24 | | 29 | 35 | 23 | 32 |
| | 30 | 25 | 12 | 22 | | 30 | 25 | 12 | 22 |