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# Non-Relativistic Limits

How the limits of massive gravity theories could hide unexpected physics

Bachelor Research Project

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# 1 Introduction and Motivation

This year marks a hundred years since the first test confirming General Relativity. Over a century after its conception, General Relativity has always served us well. It provided answers to holes present in the theory that was generally accepted at the time, Newtonian Gravity, and it described our world wonderfully. Many of the predictions this theory made have been confirmed through experiments time and time again, starting from the bending of light in 1919 [9] all the way to the more recent discovery of gravitational waves in 2016 [1]. Nevertheless, General Relativity is not a theory without its flaws. In particular, attempts at reconciling General Relativity and Quantum Physics have proved cumbersome [18].

In that spirit several solutions have been put forward, one of them being modifying gravity. General Relativity involves a massless spin-2 particle called the Graviton; therefore, a natural way of modifying it is by adding a small mass to the Graviton. This theory is known as Massive Gravity (See [13]). There are numerous variations of the theory, many solving different issues found in the merger of General Relativity and Quantum Mechanics. Just like General Relativity, Massive Gravity theories are non-linear and thus non-renormalizable [8].

Additionally, in the context of condensed matter, the Fractional Quantum Hall Effect admits a mode called the Girvin-Macdonald-Platzman (GMP) mode [12]. This mode, although non-relativistic, involves a similar massive spin-2 particle yielding a single, parity breaking Schrödinger equation. Therefore, there could be a connection between these two theories [5], which we aim to bridge in this paper.

For that purpose we will examine the non-relativistic limits of scalar and vector fields. We will take both the $c \to \infty$ limit on its own and when we consider the Compton Wavelength is constant, called the force limit. Throughout the paper, we will only include non-interacting fields.

First, we will examine the non-relativistic limit of the real and complex scalar fields. We will also discuss the invariance of the Lagrangian, as well as the algebra used. Next, we will explore the same limits on real and complex vector fields. The spin-2 field will be shortly discussed, explaining how the subtleties and tricks used to take these limits on spin-1 fields will also be valid for these fields [5]. Lastly, we will explain how we can obtain Maxwell's equations from spin-1 fields, as well as take their non-relativistic limits. We will close the paper with a general discussion of the results obtained and concluding remarks.

# 2 Spin 0

## 2.1 The Complex Field

Starting from a complex field with Lagrangian

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \left(\frac{mc}{\hbar}\right)^2 \phi^* \phi \tag{1}$$

where on the left is the kinetic term and on the right the mass term, we obtain the following equation of motion for $\phi$:

$$\frac{1}{c^2}\ddot{\phi} - \nabla^2 \phi + \left(\frac{mc}{\hbar}\right)^2 \phi = 0 \tag{2}$$

If we take the non-relativistic limit immediately, we will run into trouble, since the third term will go to infinity. We therefore must find a way to get rid of this infinity. One option is to set $\frac{mc}{\hbar}$ constant, which will be examined in section 2.3.

The second option is by redefining $\phi$. From the ansatz $\phi = e^{\alpha t}\psi$, we can derive the following redefinition:

$$\phi = e^{-i\frac{mc^2}{\hbar}t}\psi \tag{3}$$

Plugging it into (2) we get

$$\frac{1}{c^2}\ddot{\psi} - \nabla^2\psi - \Big(\frac{2im}{\hbar}\Big)\dot{\psi} = 0 \tag{4}$$

The terms diverging in the non-relativistic limit have canceled, so we can now take the $c \to \infty$ limit, obtaining the Schrödinger equation:

$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\nabla^2\psi \tag{5}$$

## 2.2 The Real Scalar Field

From a real field with Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\Big(\frac{mc}{\hbar}\Big)^2\phi^2 \tag{6}$$

we obtain the following equation of motion:

$$\frac{1}{c^2}\ddot{\phi} - \nabla^2\phi + \Big(\frac{mc}{\hbar}\Big)^2\phi = 0 \tag{7}$$

Just like in the complex case, we wish to get rid of infinities, in particular the one brought on by the third term. This time, we require the field $\phi$ to be real; the most general solution is:

$$\phi = A\sin\Big(\frac{mc^2}{\hbar}t\Big)\psi + B\cos\Big(\frac{mc^2}{\hbar}t\Big)\psi \tag{8}$$

and (7) becomes

$$\begin{aligned}\frac{2m}{\hbar}\Big(A\cos\Big(\frac{mc^2}{\hbar}t\Big) - B\sin\Big(\frac{mc^2}{\hbar}t\Big)\Big)\dot{\psi} + \frac{1}{c^2}\Big(A\sin\Big(\frac{mc^2}{\hbar}t\Big) + B\cos\Big(\frac{mc^2}{\hbar}t\Big)\Big)\ddot{\psi}\\ +\Big(A\sin\Big(\frac{mc^2}{\hbar}t\Big) + B\cos\Big(\frac{mc^2}{\hbar}t\Big)\Big)\nabla^2\psi = 0\end{aligned} \tag{9}$$

We've managed to eliminate the third term in (7). However, our problem is far from solved, since unlike in the complex case, the terms diverging at the limit have not been cancelled out. Because the sine and the cosine are undefined at infinity, so is the $c \to \infty$ limit of the equation. As a result, we cannot obtain the Schrödinger equation. Furthermore, if we consider Schrödinger's is a complex equation, it is unsurprising that we cannot derive it from a real field.

Some authors [10], [19] have taken this limit by scrapping our requirement of also making $\phi$ real. They have made a complex redefinition and proceeded to take the same limit as for the complex field.

## 2.3 The Force Limit

The last term in (2) and (7) can be identified as the inverse of the reduced Compton wavelength, $\lambda_C = \frac{\hbar}{mc}$. We can thus take this term as constant and proceed to take the $c \to \infty$ limit:

$$\nabla^2\phi - \frac{1}{{\lambda_C}^2}\phi = 0 \tag{10}$$

This is a Poisson equation, sometimes called the Screened Poisson Equation. This gives rise to an instantaneous force (hence the name) and hence a potential, called the Yukawa potential. Clearly, this limit is the same for both real and complex fields, since it does not depend on the field itslef.

Note we have lost the time derivative. This means there is no time evolution, and thus no particle.

# 3 Transformations and covariance

In infinitesimal form, Galilean boosts can be written as $\delta_b\psi = tv^i\partial_i\psi$, while translations are $\delta_t\psi = a^i\partial_i\psi$. We would like to check whether the Schrödinger equation is covariant under these transformations. It can be easily checked that whereas this is indeed the case for translations, equation (5) does not transform covariantly under boosts. This is due to the term dependent on time, which is acted upon on the left side of the equation but not on the right. Nevertheless, we can add an additional term that will even this out. From the definition of Lorentz transformations, we know

$$t = \gamma\Big(t' + \frac{v}{c^2}x'\Big), \quad x = \gamma(vt' + x'), \quad y = y', \quad z = z' \tag{11}$$

Using the redefinition of the field given in (3), we require it to be covariant

$$\begin{aligned} e^{-\frac{imc^2}{\hbar}t}\psi(x) &= e^{-\frac{imc^2}{\hbar}t'}\psi'(x') \\ \implies e^{-\gamma\frac{imc^2}{\hbar}t'}e^{-\gamma\frac{imv}{\hbar}x'}\psi(\gamma(vt'+x')) &= e^{-\frac{imc^2}{\hbar}t'}\psi'(x') \end{aligned} \tag{12}$$

But in the non-relativistic limit $\frac{v}{c} \ll 1$, so $\gamma \approx 1$ and

$$\begin{aligned} e^{-\frac{imc^2}{\hbar}t'}e^{-\frac{imv}{\hbar}x'}\psi(vt'+x') &= e^{-\frac{imc^2}{\hbar}t'}\psi'(x') \\ \implies e^{-\frac{imv}{\hbar}x'}\psi(vt'+x') &= \psi'(x') \end{aligned} \tag{13}$$

For an infinitesimal transformation,

$$e^{-\frac{imv}{\hbar}x'} \approx \Big(1 - \frac{imv}{\hbar}x'\Big) \tag{14}$$

and

$$\psi(vt'+x') = \psi(x') + t'v^i\partial_i\psi(x') \tag{15}$$

Up to first order, then,

$$\Big(1 + t'v^i\partial_i - \frac{imv}{\hbar}x'\Big)\psi(x') = \psi'(x') \tag{16}$$

so in infinitesimal form, boosts are now given by

$$\delta_b\psi = tv^i\partial_i\psi - \frac{imv^ix_i}{\hbar}\psi \tag{17}$$

It can be checked that in this form, the Schrödinger equation is covariant under boosts.

It should be noted that boosts and translations do not commute,

$$[\delta_b, \delta_t]\psi = imv^ia_i\psi \tag{18}$$

This will later become relevant.

## 3.1 Four vectors

For a four-vector $(ct, x, y, z)$, (11) can be written as

$$ct = \gamma\Big(ct' + \frac{v}{c}x'\Big), \quad x = \gamma\Big(\frac{v}{c}ct' + x'\Big), \quad y = y', \quad z = z' \tag{19}$$

If $\gamma \approx 1$, for a four vector $(u_0, u_i)$

$$u_0 = u_0' + \frac{v}{c}u_i', \quad u_i = \frac{v}{c}u_0' + u_i' \tag{20}$$

## 3.2 Algebras

From the Galilean algebra with an Inönü-Wigner contraction [14] including a central extension, we can take the non-relativistic limit to get the Bargmann algebra. We will start with the Poincaré algebra. In generator form the commutation relations are given by

$$[P_A, M_{BC}] = 2\eta_{A[B}P_{C]}, \quad [M_{AB}, M_{CD}] = 4\eta_{[A[C}M_{D]B]} \tag{21}$$

where $P_A$ is the generator of spacetime translations and $M_{AB}$ the Lorentz generator [6]. We now want to perform an Inönü-Wigner contraction. For this purpose we can use

$$P_0 = \sqrt{M^2c^4 + P_iP^ic^2} = Mc^2\sqrt{1 + \frac{P_iP^i}{M^2c^2}} \tag{22}$$

But $\frac{P_iP^i}{M^2c^2}$ is small, so

$$\begin{aligned} P_0 &\approx Mc^2\Big(1 + \frac{P_iP^i}{2M^2c^2}\Big) \\ \implies P_0 &= Mc^2 + \frac{P_iP^i}{2M} \end{aligned} \tag{23}$$

And for $\omega = c^{-1}$ and $H = \frac{P_iP^i}{2M}$ we're left with the contracted generator

$$P_0 \to \frac{1}{\omega^2}M + H \tag{24}$$

Likewise,

$$P_i \to \frac{1}{\omega}P_i, \quad J_{i0} \to \frac{1}{\omega}G_i \quad \text{and} \quad \omega \to 0 \tag{25}$$

where $J_{ab}$ is the generator for spatial rotations and $G_a$ the generator for Galilean boosts. So we now get the following commutation relation:

$$\begin{aligned} [P_a, G_b] = \omega^2[P_a, J_{b0}] = \omega^2 2\eta_{a[b}P_{0]} = \omega^2\delta_{ab}P_0 \\ \implies [P_a, G_b] = \delta_{ab}(M + \omega^2 H) \end{aligned} \tag{26}$$

and for $\omega \to 0$,

$$[P_a, G_b] = \delta_{ab}M \tag{27}$$

This relation, added to the equivalent of those of the Poincaré algebra in (21)

$$[P_a, J_{bc}] = 2\delta_{a[b}P_{c]}, \quad [J_{ab}, J_{cd}] = 4\delta_{[a[c}J_{d]b]} \tag{28}$$

and to

$$[G_a, J_{bc}] = 2\delta_{a[b}G_{c]}, \quad [H, G_a] = P_a \tag{29}$$

constitutes the Bargmann algebra. For $M \to 0$ we get the Galilei algebra. But this M is in fact a central extension generator, so we can say that the Bargmann algebra is effectively the Galilei algebra with a central extension [2]. Moreover, (27) and (18) are equivalent, so the Schrödinger equation is covariant in the Bargmann algebra.

# 4 Spin 1

We now move over to the spin 1 case. The lagrangian for these fields is called the Proca Lagrangian. These derivations are based on [4]. For simplicity from now on we'll take $\hbar = 1$.

## 4.1 Proca Lagrangian for the complex field

The 3D Lagrangian for a complex field of spin 1 is given by

$$c^{-1}\mathcal{L} = -\frac{1}{4}F^*_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(mc)^2 A^*_\mu A^\mu \tag{30}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength and the term containing it is the kinetic term, and $A_\mu$ is the vector field and its term the mass term. The $c^{-1}$ factor multiplying the Lagrangian comes from the metric. To find the non-relativistic limit, it is wise to first split the spatial and the time components and then essentially follow the process done for the scalar fields.

After splitting the spatial and time components, we can rewrite the Lagrangian the following way:

$$c^{-1}\mathcal{L} = -\frac{1}{4}F^*_{ij}F_{ij} + \frac{1}{2c^2}F^*_{0i}F_{0i} + \frac{m^2}{2}A^*_0A_0 - \frac{(mc)^2}{2}A^*_iA_i \tag{31}$$

Redefining the complex fields, we set

$$A_0 = e^{-imc^2t}a_0, \quad A_i = e^{-imc^2t}a_i \tag{32}$$

Plugging it in (31) we get

$$c^{-1}\mathcal{L} = -\frac{1}{4}f^*_{ij}f_{ij} + \frac{i}{2}m(a^*_if_{0i} - a_if^*_{0i}) + \frac{m^2}{2}a^*_0a_0 + \frac{1}{2c^2}f^*_{0i}f_{0i} \tag{33}$$

To take the non-relativistic limit, we take $c \to \infty$ to obtain

$$\mathcal{L} = -\frac{1}{4}f^*_{ij}f_{ij} + \frac{i}{2}m(a^*_if_{0i} - a_if^*_{0i}) + \frac{m^2}{2}a^*_0a_0 \tag{34}$$

We have not troubled ourselves with the $c^{-1}$ term in front of the Lagrangian, since it is multiplying the full equation and we can easily redefine it such that this term is gone. We now calculate the Euler-Lagrange equations for $a^*_0$ and $a_0$, given by

$$a^*_0 = -\frac{i}{m}\nabla a^*_i, \quad a_0 = \frac{i}{m}\nabla a_i \tag{35}$$

$a^*_0$ and $a_0$ constitute the auxiliary fields of the equation and add no degrees of freedom, so the above equations can be used to further simplify the Lagrangian. Omitting boundary terms, this is given by

$$\mathcal{L} = \frac{1}{2}a^*_i\nabla^2a_i + \frac{i}{2}m(a^*_i\dot{a}_i - a_i\dot{a}_i{}^*) \tag{36}$$

where we have used $\nabla^2 = \partial_1^2 + \partial_2^2$. From this Lagrangian we obtain the equations of motion:

$$i\dot{a}_i = -\frac{1}{2m}\nabla^2a_i \tag{37}$$

Up until now, this derivation is valid for $n$ dimensions. Specifying $i = 1, 2$, for the eigenfunctions

$$\psi[1] = a_1 + ia_2, \quad \psi[-1] = a_1{}^* + ia_2{}^* \tag{38}$$

(37) can be written as

$$i\dot{\psi}[\pm 1] = \mp \frac{1}{2m}\nabla^2\psi[\pm 1] \tag{39}$$

hence yielding two Schrödinger equations.

## 4.2 Proca Lagrangian for the real field

The Proca Lagrangian for the real field is given by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(mc)^2 A_\mu A^\mu \tag{40}$$

Separating the spatial from the time components, this can be written as

$$\mathcal{L} = \frac{1}{2c^2}|\dot{A}_i|^2 - \frac{1}{2}A_i \cdot \triangle A_i + \frac{1}{2}(\nabla \cdot A_i)^2 + \frac{1}{2c^2}A_0 \triangle A_0 + \frac{1}{c^2}A_0(\nabla \cdot \dot{A}_i) \tag{41}$$

where $\triangle = -\nabla^2 + (mc)^2$. The boundary terms have not been taken into account. From the Euler-Lagrange equation for the auxiliary field $A_0$ given by

$$A_0 = -\triangle^{-1}(\nabla \cdot \dot{A}_i) \tag{42}$$

so (41) becomes

$$\mathcal{L} = \frac{1}{2c^2}|\dot{A}_i|^2 - \frac{1}{2}A_i \cdot \triangle A_i - \frac{1}{2}A_i \partial_i \partial_j A_j + \frac{1}{2c^2}\dot{A}_i \triangle^{-1} \partial_i \partial_j \dot{A}_j \tag{43}$$

Setting

$$A_i = \frac{1}{mc}\triangle^{1/2}\delta_{ij}B_j \tag{44}$$

we can further rewrite the Lagrangian as

$$\mathcal{L} = \frac{1}{2c^2}|\dot{B}_i|^2 - \frac{1}{2}B_i \cdot \triangle B_i \tag{45}$$

where $B_i$ is a real spatial vector.

We know we can always construct a complex field by making a linear combination of two real fields. In fact, we can do the same thing with spatial dimensions. If we set $i = 1, 2$ such that we have two spatial components, we can define a complex field $B = \frac{(B_1 + iB_2)}{\sqrt{2}}$ and use it to express the previous equation as

$$\mathcal{L} = \frac{1}{c^2}|\dot{B}|^2 + B^* \nabla^2 B - (mc)^2 |B|^2 \tag{46}$$

Since $B$ is now complex, we can use the same trick we have used for the complex case and redefine the field

$$B = e^{-imc^2 t}\psi[1] \tag{47}$$

and taking the $c \to \infty$ limit, we get

$$\mathcal{L} = im(\psi^* \dot{\psi} - \dot{\psi}^* \psi) + \psi^* \nabla^2 \psi \tag{48}$$

Using the equation of motion with respect to $\psi$ and replacing $\dot{\psi}^*$ we find

$$-\frac{1}{2m}\nabla^2\psi[1] = i\dot{\psi}[1] \tag{49}$$

which is the Schrödinger equation for spin-1 particles. (47) violates parity in the non-relativistic limit [5], hence why we're left with a single, parity violating Schrödinger equation.

It should be reiterated that the reason why we were able to take the limit for this case and not on the scalar case is because we are now working with vectors with three components. This allows us to use two spatial components to create one complex field on which we can take the limit.

## 5 A word on Spin 2 fields

Spin-2 free fields are described by the Fierz-Pauli Lagrangian [11]. This time, we're dealing with a tensor with 5 components. The same trick that made it possible to obtain a non-relativistic limit from a real Lagrangian is thus also valid here, except instead of it having $1+2$ components it now has $1+2+2$ components [5]. Fortunately, two of the spatial components are auxiliary, so this will still result in a Schrödinger equation [3], this time describing a particle of spin 2. As seen in section 4.2, the real limit yields only one Schrodinger equation instead of two. That the real limit exists for a spin-2 field is relevant, since the GMP mode in condensed matter mentioned earlier has only one Schrödinger equation [3]. This makes the connection between massive gravity and condensed matter theories even more plausible. Moreover, similarly to the lower spin cases, the limit for the complex field can be taken fairly easily, resulting in two Schrödinger equations with two modes in 3D.

But for a correct formulation of massive gravity, we must also include interactions. Here is where the issues begin [5], as we encounter two difficulties. Firstly, massive gravity propagates five degrees of freedom, seemingly making it impossible to reconcile with special relativity, which only propagates two [15]. Secondly, massive gravity theories have an undesired state called a Boulware-Deser ghost to maintain gauge invariance [20], [8]. Thankfully, certain mechanisms and variations of massive gravity have been put forward to solve these problems. Nevertheless, that is beyond the scope of this paper; for more comprehensive reviews, see [8], [13].

## 6 Intermezzo: Maxwell Equations

We can use the Proca Lagrangian to derive the Maxwell equations. Without separating spatial and time components, the equation of motion for both the complex and the real fields is

$$\partial_\mu F^{\mu\nu} = (mc)^2 A^\nu \tag{50}$$

In terms of $F^{\mu\nu}$, the electric and magnetic field can be written as:

$$\vec{E} = c^2 F^{i0}, \quad \vec{B} = F^{ij} \tag{51}$$

Therefore, we can use the equation of motion to obtain the first two Maxwell equations:

$$\vec{\nabla} \cdot \vec{E} = (mc)^2 \phi, \quad \vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + (mc)^2 \vec{A} \tag{52}$$

where $A^\mu = (\phi, \vec{A})$.

The other two Maxwell equations can be derived from $\mathcal{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}F_{\lambda\rho}$ [7].

$$\vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{53}$$

The usual Maxwell equations come from the massless Lagrangian including a term with the 4-current $J^\mu = (\rho, \vec{j})$, sometimes called the Maxwell Lagrangian:

$$\mathcal{L} = -\frac{1}{4\mu_0} F_{\mu\nu} F^{\mu\nu} - J_\mu A^\mu \tag{54}$$

where the term on the left is the kinetic term and the one on the right the interaction term, so (52) becomes

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \vec{\nabla} \times \vec{B} = \frac{1}{c^2}\frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{j} \tag{55}$$

leaving all other equations untouched.

### 6.1 Magnetic and Electric Limits

From section 3.1 it is clear that Lorentz transformations admit two different Galilean limits [16]:

$$u'_0 = u_0, \quad u'_i = u_i - \frac{1}{c} v_i u_0 \tag{56}$$

for timelike vectors with $|u_0| \gg |u_i|$ and

$$u'_0 = u_0 - \frac{1}{c} v_i u_i, \quad u'_i = u_i \tag{57}$$

for spacelike vectors with $|u_0| \ll |u_i|$. In the electric limit, $\vec{E} \gg c\vec{B}$, so using (56),

$$\vec{E}' = \vec{E}, \quad \vec{B}' = \vec{B} - \frac{1}{c^2} \vec{v} \times \vec{E} \tag{58}$$

In the magnetic limit, $c\vec{B} \gg \vec{E}$ and using (57),

$$\vec{E}' = \vec{E} - \vec{v} \times \vec{B}, \quad \vec{B}' = \vec{B} \tag{59}$$

For a somewhat different, yet complete derivation see [17].

## 7 Discussion

We found that once we redefine the field, taking the non-relativistic limit of the complex scalar field is fairly straightforward. It yields the Schrödinger equation, which is covariant under the Bargmann algebra. On the other hand, taking the non-relativistic limit of the real scalar field is not as easy.

For the scalar case, the only way to obtain a limit of the real field is through the force limit. The last term in the Lagrangian is the Compton Wavelength, which we can take as finite, allowing us to take the limit without redefining the field. The terms that diverge as $c \to \infty$ cancel out, and we're left with a Poisson-like equation. Because we have lost all time dependence, this equation is not dynamical and thus cannot describe a particle. Although this limit can also be taken from the complex Lagrangian, this means that the non-relativistic limit of a real scalar field cannot be taken without losing the particle.

We wanted to test whether the Schrodinger equation was covariant under Galilei transformations, and found that it in fact was covariant under the Bargmann algebra. But to obtain this we used the redefinition of the field derived to take the limit of the complex field. This redefinition was complex, like the Schrödinger equation itself. These things explain why we were not able to obtain the Schrödinger equation from the real field.

For vector fields, the derivations were somewhat more complex than that of the scalar case, but obtaining the Schrödinger equation from the complex Lagrangian was still possible by eliminating auxiliary fields and boundary terms. In three dimensions, two Schrödinger equations result from the limit.

Unlike in the scalar case, though, the real vector field does yield a Schrödinger equation. The reason for the discrepancy is that we are now working with three components: one time component and two spatial. This allows us to take these spatial components and combine them to form a complex field, exactly like one would from two real fields. For this reason, only one parity-violating Schrödinger equation will result from the limit.

These circumstances are the same encountered when one tries to take the limit on the spin 2 fields. Remarkably, the single Schrödinger equation obtained from the real tensor field appears in the GMP mode explained in the introduction. So we managed to obtain the same physics seen in condensed matter from the non-relativistic limit of a massive complex field. While it's not the first time the same models have been used for Massive Gravity and the GMP mode ([3], [12]), more research needs to be done on the topic.

Additionally, we briefly mentioned spin 2 fields including interactions. This is an ongoing field of research with many theories proposed, but no concrete answers as of yet.

Lastly, we showed Maxwell's equations can be obtained from the vector field Lagrangian, where the usual notation comes from a massless version with an added current. We can likewise take a non-relativistic limit for these equations, resulting in two different limits. These are Galilei covariant [17].

# 8 Conclusion

We set out to investigate whether there was a way to bridge the two seemingly disconnected theories of Massive Gravity and the Fractional Quantum Hall Effect. Albeit not straightforward, it was the subtleties that taking the limit entailed that allowed us to make this connection. While it's hardly the first time this parallel has been drawn, it is far from being thoroughly explored. Massive Gravity and the Fractional Quantum Hall Effect are currently areas of much interest, but their intersection still needs further investigation. We hope that by conducting this type of research cooperation between the two scientific communities will ensue, bringing along exciting results.

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