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MASTER THESIS - RESEARCH PROJECT

The influence of Value-Of-Time in a greedy ridesharing network

INDUSTRIAL ENGINEERING AND MANAGEMENT

Author:

H.T.L. Meijer, S2507129

First supervisor: A.K. Cherukuri, PhD

Second supervisor: prof. dr. D. Bauso

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Abstract

A growth of human population and an increase in the trading commerce has caused a crowded road network. On the other hand, online information and communication technology has seen rapid growth and has become more affordable. This allowed ridesharing to become an active research topic for both academia and industry. In ridesharing, passengers share the same vehicle to traveling the same direction or reach a common destination and split travel costs such as gas, toll, and parking fees. Much research is done on this topic. However, knowledge on the influence of Value-Of-Time has been lacking. A passenger's Value-Of-Time is its desire to reach its destination as quickly as possible, which is distinct value for an individual.

The goal of this research is to design an optimization model that creates a ridesharing transport schedule and studies the effect of Value-Of-Time on the total travel cost. We consider two scenarios; where riders can and cannot deviate from the schedule. For the former scenario, a greedy transit algorithm is created to calculate the costs for riders not following the schedule. It was concluded that the implementation of Value-Of-Time prevents greedy transits to a certain extent, which has a positive effect on the total travel costs. To test the network where riders cannot deviate from the schedule, the model is tested for a load-sharing schedule for trucks. Sharing cargo has a beneficial effect on the total travel costs. In this situation, it is concluded that the implementation has a substantial beneficial effect on the service quality in terms of arrival time, while the increase in travel costs is limited.

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1 | Introduction

This chapter is divided in three sections. In section 1.1, the problem and its context are investigated and the motivation for this research is explained. This section will describe the system and the stakeholders are identified. Hereafter, the goal is stated and the method suitable for achieving the goal is explained. This section ends with the research questions. Section 1.2 provides more information on the research that is already done on ridesharing. Section 1.3 will clarify what this research will contribute to the general knowledge base.

1.1 Problem and research motivation

1.1.1 Problem background

A growth of human population and an increase in the trading commerce has caused a crowded road network. On the other hand, online information and communication technology has seen rapid growth and has become more affordable. This allowed ridesharing to become an active research topic for both academia and industry (Asirin and Azhari, 2018). Traditionally, there were two main systems to reach a destination (Furuhata et al., 2013); the first system contains a fixed schedule and a fixed geographic route. These fixed-line systems charge the traveller a small amount, but have little convenience. Contrary to the first system, there is the flexible system; private cars or taxis. These come at a higher cost, but are more convenient and are often the faster option. Conceptually, ridesharing falls in the middle these two systems; combining the flexibility and comparable speed of private cars with the reduced cost of fixed-line systems, at the expense of convenience. In ridesharing, passengers share the same vehicle to travel in the same direction or reach a common destination and split travel costs such as gas, toll, and parking fees. A rise of ridesharing platforms can decrease traffic congestion and overall travel costs (Mitchell et al., 2010).

Ridesharing is a branch of research of the traditional vehicle routing problems. These vehicle routing problems have been introduced in 1959 (Dantzig and Ramser, 1959) by finding a method to minimize the mileage of trucks. Concurrent with the rise of the World Wide Web, real-time ridesharing models were first introduced in 1997 (Ferguson, 1997). However, much of the research since then was focused on one switch to a different vehicle (a single hop). Limited research has been done on a multi-hop and multi-rider system (Chen et al., 2019). In 2018, stochastic travel time has been introduced to a ride sharing network (Long et al., 2018) and it showed that the stochasticity of travel time has a significant impact on the transport schedule. The authors also stated that the passenger's Value of Time (VOT) has a significant impact on the service quality of riders, but they did not study to what extent. A passenger's VOT is its desire to reach its destination as quickly as possible. This is an important value in ridesharing, because the travel time of an individual can increase compared to flexible systems (Wei et al., 2019; Furuhata et al., 2013).

Alternative use cases

Ridesharing models have more use cases than just riders sharing a car. Instead of passengers, these models have been used for a network for retrieving messages (Fanelli and Greco, 2015). In this network, the car is regarded to have infinite capacity since messages do not deplete the space of the car. As a second example, a network has been studied where both normal drivers and autonomous vehicles (Wei et al., 2019) pick up passengers. Their study included real-time scheduling between autonomous vehicles and human drivers. Moreover, in a network where goods are transported by trucks, the goods can be seen as the passengers (Unnikrishnan et al., 2009). Instead of passengers switching vehicles, goods can be loaded to a different truck for transportation.

1.1.2 Ridesharing for truck transport

Filling rate of trucks

There is a rising demand in the global trading commerce. Due to its polluting nature, the increase of the use of transportation means has gained interest. Increasing travel costs due to raise in fuel cost, longer distance, faster and on-time deliveries have forced companies to use their transport resources in a more efficient and effective way in order to stay in competitive business environment. According to the European Environment Agency (EEA) report (EEA, 2017), 24% of all CO₂-emissions are linked to road transport in the EU; 23% of those emissions come from medium- and heavy-duty trucks. There is an incentive to increase the efficiency of truck transport and therefore, much research has been done to prevent occurrence from empty trucks (Lin et al., 2016) and creating optimization models to minimize total travel cost (Li et al., 2018). However, not much research is done on partially empty trucks. By sharing shipments of multiple trucks, energy and CO₂-emissions can be decreased. Therefore, loading goods from many trucks to fewer trucks at nodes can be seen as ridesharing problem.

According to Pahlen and Börjesson (Pahlén and Börjesson, 2012), actions increasing the filling rate and efficient use of transport resources are primarily not in the interest of neither the shippers, nor the customers due to increased handling. Related disadvantages of ridesharing models is the inconvenience for passengers, since other passengers have to be taken into consideration, passengers have to switch cars, and ridesharing can lead to increased transport time (Furuhata et al., 2013). The human perspective tends to introduce further requirements leading to balance user inconvenience against minimizing routing costs. This is an existing problem for many vehicle routing problems; a widely-used measure of customer satisfaction in the school bus routing problem is the time comparison of the chosen route with respect to the shortest path to a destination (Park and Kim, 2010).

Business context

In truck transport, the transport schedule is created by the Logistics Service Provider (LSP). The schedule determines what truck delivers which goods to its destination and at what time. It is in their interest to create a cost efficient schedule while fulfilling the wishes of customers. This research aims to provide the next step in incorporating ridesharing in the transport schedule of trucks. It is already proven that this can yield cost savings for the LSP (Lin et al., 2012). Therefore, the LSP is interested in the outcome of this research.

As mentioned in 1.1.1, customers have an VOT, which depends on the desired arrival time. For a truck transport model, the VOT can be seen as cost penalty for not reaching the destination at the desired time. Due to the fact that ridesharing can increase an individual's travel time (Wei et al., 2019; Furuhata et al., 2013) and the arrival time depends will depend on the desires of multiple riders, the effect of a customer's VOT should be known if a ridesharing model is implemented in truck transport scheduling.

1.1.3 Problem statement

The problem statement for this research is formulated as follows:

The LSP has high costs for providing truck transportation of goods due to low filling rate, which can be reduced by implementing a ridesharing model. However, there is a knowledge gap on how a customer's Value-Of-Time influences the performance of such model in terms of total travel costs, filling rate and service quality.

The initial problem is lack of efficiency in the filling rate of the LSP, which is solved by implementing a ridesharing model for scheduling. However, a knowledge gap on the influence of VOT prevents providing excellent service quality, since ridesharing models are indifferent to the exact arrival time.

Scope of research

In this research, a routing schedule will be made for truck transport where goods can be loaded to a different truck at nodes. In this model, trucks and goods are already available at the nodes to start transportation. For a given time frame, information is available for the delivery. For each good, there is information on its availability time, due date, final destination and VOT. This information will be randomized. In this research, a complete directed graph will be used. Only the nodes and links in this graph can be used. Empty trucks that will not be used will leave the system.

1.1.4 System description

In this section, the system is described. The higher system is the delivery of goods by truck and it consists of multiple subsystems as described in Figure 1.1. The inputs to this system are the goods that need to be delivered, the delivery requests with information on the due date, VOT and destination of goods, and the empty trucks. The outputs for this system are the delivered goods and empty trucks. Information on the delivery requests and the empty trucks will enter the first subsystem, where the truck schedule will be created. Its output are the scheduled empty trucks, which will enter the next subsystem 'transfer goods' alongside with the goods. Subsequently, based on the truck schedule, part of the trucks will drive to their final destination and the other trucks will drive to a meeting point. At this meeting point, goods are loaded transferred, which will create empty trucks that will leave the system. Other trucks can either drive to their next meeting point or drive to their final destination. Once the goods are at their final destination, delivered goods and empty trucks will leave the system.

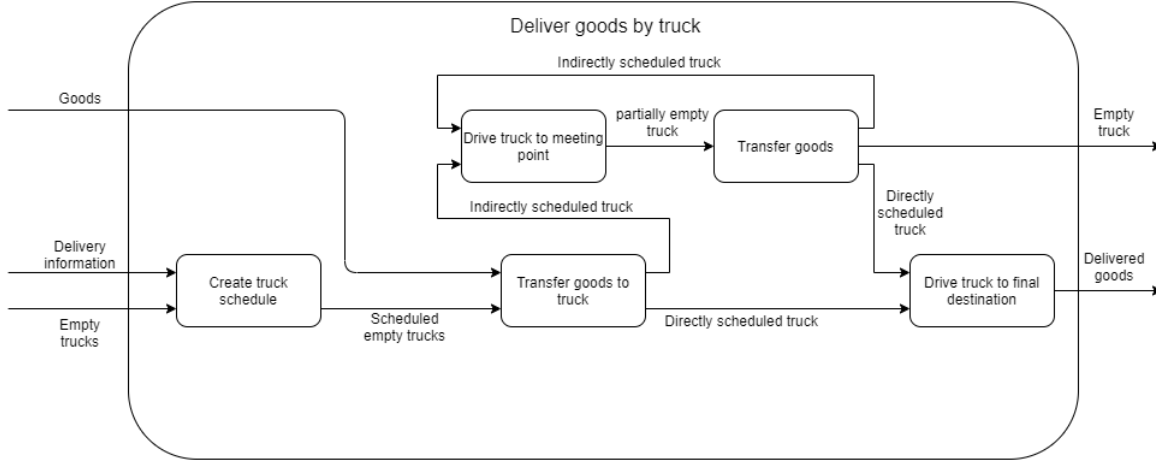


Figure 1.1: Model of the system of the delivery of goods by trucks

1.1.5 Stakeholder analysis

A stakeholder is considered someone who has interest, or is in another way affected by this research. Therefore, the stakeholders might have different requirements regarding knowledge generation (Wieringa, 2014). In this section, the problem owner and other stakeholders are identified. In addition, the different stakeholders are placed in a matrix to show their relative importance based on influence and interest as visualized in Figure 1.2.

Problem owner

The problem owner is scheduler of truck transport, which is the LSP. The LSP has a stake in improving the service quality of the shipping of goods. They are affected by this research, because in order to implement a ridesharing model in practice, it is important to understand the effect of a customer's VOT.

1.1.6 Other stakeholders

Other stakeholders are the truck drivers, since they are affected by the outcome as well. When implementing ridesharing, it is expected that they have to load or deload the goods more often, which takes time and effort. In addition, customers have stake in this research, since LSP can offer lower shipping prices after implementation and customers might have to sacrifice shipping speed.

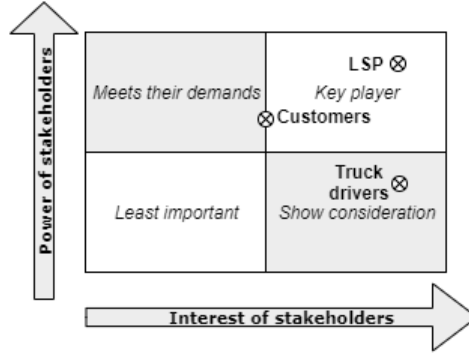


Figure 1.2: Quadrant that describes the relative power and interest of stakeholders

1.1.7 Goal statement

The main goal of this research is to fill a knowledge gap on the influence of VOT on total transportation costs, filling rate and service quality. By studying this effect on a general ridesharing network, the conclusion should also hold for different ridesharing applications.

The goal statement for this research is formulated as follows:

The goal is to design an optimization model that creates a ridesharing schedule in order to determine the influence of Value-of-Time on total travel costs, filling rate and service quality within 5 months.

After this goal has been achieved, a different application of ridesharing will be tested. The goal for the LSP is to implement a ridesharing model that creates a transport schedule for their truck delivery system. They are interested in implementing such a model, because it can have a beneficial effect on the total travel costs. However, it is important that the model provides a high service quality for its customers. Therefore, this research will focus on one aspect of perceived quality, the arrival time. By implementing VOT in a load sharing network for trucks, its service quality towards the customers should improve. It is expected that this closes the presented knowledge gap. A time bound of five months has been selected as the project is due within five months.

1.1.8 Method

From the goal statement it can be concluded that this research is knowledge-oriented, since it is focused on filling a knowledge gap on the influence of VOT. The appropriate cycle for this research is the empirical cycle (see Figure 1.3), since this cycle is used to obtain new knowledge (Heitink, 1999). The first step in this cycle is the observation step. It is already observed from literature that the influence of VOT is important, but not yet studied (Long et al., 2018). For the induction step, an hypothesis is formulated for the main research question, which is done in 1.1.9. The expectation is that the knowledge gap is filled by validating or refuting the formulated hypothesis.

The third step is the deduction step, where an optimization model is created that generates a schedule to transport a fixed number of goods. Subsequently, the experiments will be formulated. The model is created as a tool to generate data in the experiments. The experiments

will be performed as follows. The VOT-parameter will be given a range of values, while the other parameters remain constant. The total travel costs and filling rate of trucks and the service quality will be documented. The next step is testing, where the experiments will be executed and the data is obtained. The evaluation step concludes the empirical cycle. To conclude the cycle the data has to be analyzed and the hypothesis is evaluated.

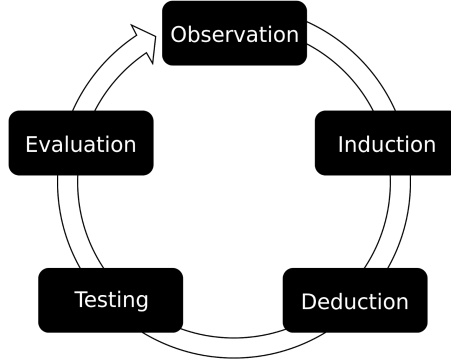


Figure 1.3: Graphical description of the empirical cycle

1.1.9 Research question

The main research question is formulated as follows.

What is the influence of VOT on the total travel costs, filling rate and service quality in a ridesharing network?

Hypothesis

It is expected that a high VOT will result in higher travel costs and a lower filling rate, since the model wants to avoid costs for late arrival and will choose more often for direct transportation to the final destination. The service quality in terms of arrival time will improve significantly.

Sub-questions

1. What is a suitable commercial solver that aids in fulfilling the goal?
2. What is a suitable network for testing the influence of VOT?
3. What is the difference in travel costs between single-hopping and multi-hopping?

In order to validate or refute the hypothesis and achieve the research goal, three sub-questions are formulated. The first two sub-questions regard the setup of the experiments. A network and commercial solver has to be found that provides meaningful results, while keeping computation times within the time frame of this research. The last sub-questions is added due to the limited research on multi-hopping.

1.2 Literature review

In this section, the different directions of research of ridesharing are segregated in section 1.2.1. Subsequently, the literature review will focus on the direction that this research will adopt in section 1.2.2. This chapter will conclude with section 1.2.3, which will dive deeper on optimization of transport services.

1.2.1 Directions of research

Originally, ridesharing became a topic of interest in the 1970s due to the oil crisis, which caused a spike in the petroleum price (Furuhata et al., 2013). Ridesharing is possible due to the low filling rate of cars; this is on average 1.8 riders per vehicle for leisure trips and 1.1 for commuter trips (Peeters et al., 2005). Multiple classes of ridesharing can be identified (Furuhata et al., 2013), each of them outlayed in the upper half of Figure 1.4. The lower half also contains riders sharing a vehicle but do not make use of private vehicles. These are commercial transport services. A property of ridesharing is that the travel cost is determined by the number of riders travelling together. Therefore, public transport and personal transportation are not part of this category. Dial-A-Ride takes requests from riders to go from a to b within a specific area and then provides cars and a transport schedule. These services typically operate from a single or more depot locations. This practice has received significant attention from many researchers (Agatz et al., 2012) due to the increased cost efficiency. It involves a complex vehicle assignment problem, which is a root of the ride-matching optimization problem. However, this pick-up and delivery service provides vehicles themselves instead of the users and therefore, it shares characteristics with both dynamic ridesharing systems and commercial transportation (Berbeglia et al., 2010).

A common characteristic of ridesharing is the fact that the vehicles are delivered by the riders taking part of ridesharing. Five classes ridesharing research are identified. Flexible carpooling is a semi-organized ridesharing practice. The lack of schedule provides flexibility for its users. There exist ridesharing locations where the schedule is formed spontaneously (Levofsky and Greenberg, 2001) at a first-come-first-serve basis in order to gain access to reduced tolls or faster High-Occupancy Vehicle lanes. For this research, creating a schedule to find a optimum is of interest. Therefore, this class of research is not examined further.

General carpooling is a service type for commuters in which participant share a private vehicle. Important participant considerations of this service are work location and the start and end times of work (Teal, 1987; Morency, 2007). Typically, participants have a similar origin and destination and prefer ongoing carpooling. Employer carpool programs, in which employees take turns driving each other to work has peaked in the 1980s (Ferguson, 1997; Morency, 2007). However, this service type does not accommodate unexpected changes of schedule.

One-shot ride-match lets participants input origin, destination and time, and possibly route, well in advance. Then, a match is created by the service or participants can select the match themselves. This service type allows some itinerary flexibility from the user but, on short notice, no changes can be made to the schedule. Although the number of services for this service type has grown considerably (Dailey et al., 1999), the number of participants continues to decline (Ferguson, 1997).

Long-distance ride-match is a matching service for long distance trips with more advanced scheduling. Due to the longer travel distance, more can be gained by ridesharing. Therefore,

this service type requires more flexibility from its participants in terms of origin, destination and travel time (Dailey et al., 1999). Participants specify the departure region, and then they search for the candidates in the list. Therefore, departure time is based on ride availability instead of specifying preferred departure time.

Contrary to the previous service types, dynamic real-time ridesharing can construct a transport schedule on a short notice, which is a key benefit. This is the most recent class of ridesharing and emerged at the same time as the rise of wireless communications (Asirin and Azhari, 2018). A passenger's pick-up and drop-off locations need not be the same as the Origin-Destination-pair of the driver. A passenger can travel with the driver for part of driver's route, which allows multi-hopping for the passengers. In addition, the route of the driver can be adjusted in order to achieve a global optimum in terms of travel costs. Based on these characteristics, this is the main research class for this research. Much research has been done on this service, which will be elaborated on in Section 1.2.2.

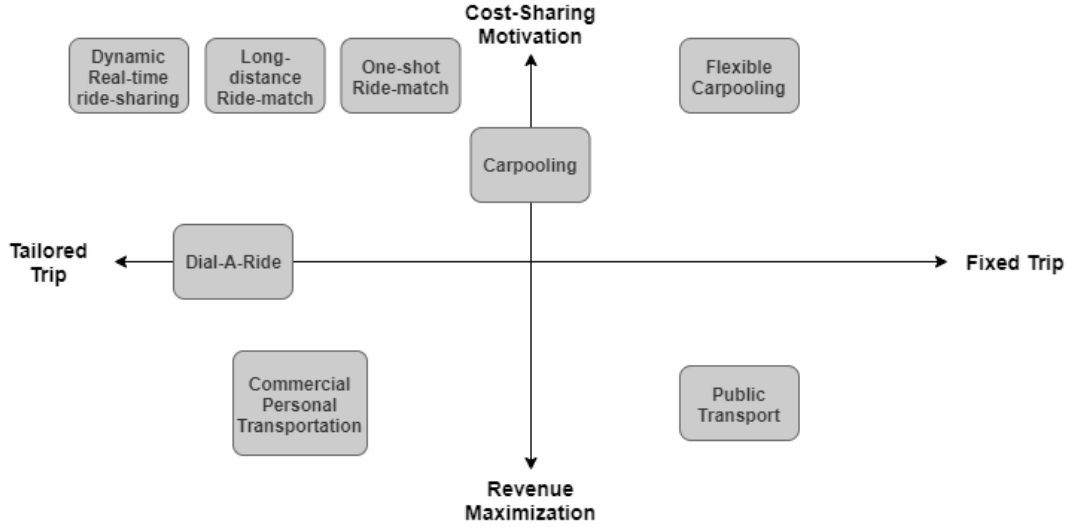


Figure 1.4: Graphical description of matching agencies and other shared vehicle transportation services

1.2.2 Research on dynamic ridesharing

Multiple types of ridesharing systems are researched; an overview can be seen in Table 1.1. Most research is done on passengers travelling with one driver (Furuhata et al., 2013). Researchers focus on either one or multiple passengers travelling with a single driver. However, less research has been done on multi-hopping (passengers travelling with multiple drivers). Multi-hopping allows for a greater global optimum (Lin et al., 2012).

	Single passenger	Multiple passengers
Single driver	Matching of pairs of drivers and passengers	Routing of drivers to pickup and drop-off passengers
Multiple drivers	Routing of passengers to transfer between riders	Routing of drivers and passengers

Table 1.1: Types of ridesharing systems in research

Moreover, dynamic ridesharing contains a minimum cost flow problem, which is a decision problem to find the cheapest possible way of sending a certain amount of flow through a flow network. The foundation for this type of research has been set by Busacher and Gowen (Busacker and Gowen, 1960). In literature, these cost flow problems are solved by means of a construction algorithm or a search algorithm. The construction algorithm start with an empty or incomplete solution and and incrementally make it more complete. A search algorithm starts with one or more complete candidate solutions, and incrementally combine and modify them with the goal of generating improved or more complete solutions. The search algorithm is used by commercial optimization solvers in order to improve the performance of these models. Both types of algorithms are used to study certain aspects of ridesharing. Therefore, the knowledge of the research of both means complement each other. This research will make use of a commercial optimization solver with a search algorithm.

Construction algorithms

Watel and Faye designed an algorithm to transport as many passengers as possible with one taxi (Watel and Faye, 2018). Fanelli and Greco designed a ride sharing model with a vehicle of unlimited capacity (Fanelli and Greco, 2015). Unlimited capacity is possible when the load is considered to be a message and the drive will ride through the network to pick up messages. An algorithm is proposed which creates a routing schedule to pick up all the messages. Subsequently, it is proved mathematically that is in fact the optimal routing schedule. Quadrifoglio et al. and Zhao and Dessouky studied the allowance of one or more detours for each rider to allow for ridesharing (Quadrifoglio et al., 2008; Zhao and Dessouky, 2008). A construction algorithm was used to find the optimum deviation from the base route.

Search algorithms

In search algorithms, an objective function is created, which is minimized or maximized. Generally, there are four types of functions that are maximized or minimized (Agatz et al., 2012);

- Minimizing the total number of travel miles of vehicles.
- Minimizing the total number of travel time of vehicles.
- Maximizing the total number of participant riders in the network
- Maximizing service quality of riders

Often, one objective is chosen based on the criteria of the study. However, multiple studies use multicriteria optimization, such as (Wei et al., 2019) who minimized transport costs and maximized driver’s profits. Herbawi and Weber (Herbawi and Weber, 2012) studied all four types of functions that are listed above.

An important aspect of ridesharing is how travel costs are divided among participants. In some studies, travel costs are divided equally among rideshare partners (Geisberger et al., 2009). Agatz et al. (Agatz et al., 2011) propose a way to allocate the costs of a joint trip proportional to the distances of the separate trips. However, intuitively, more ridesharing occurs if the rider gets a relatively higher compensation, since he will be more willingly to make detours. This effect was studied by Kleiner et al. (Kleiner et al., 2011).

In addition, multiple studies have investigated the effect of adding noise to the system. Long et al. studied the effect of stochastic transport times (Long et al., 2018). In this research, they also studied the effect of uncertain travel costs where the travel cost per hour varies. Chen et al. (Chen et al., 2019) created a ridesharing model that creates a schedule for employees going to work at a fixed time and leaving work at an uncertain time. Correa et al. studied the effect of failing links in a ridesharing network (Correa et al., 2019). Due to extreme congestion or road construction these links are no longer used. A probability density function is used to simulate this occurrence. Similarly, Fu simulated stochastic congestion with the option to find detours in the network (Fu, 2002).

Alternatively, there is research on studying the effects of different types of networks. Wei et al. (Wei et al., 2019) studied a network with mixed autonomy. In this network, there are autonomous vehicles that travel a fixed route and there are human drivers that get a financial compensation for travelling a tailored route. Fahnenschreiber et al. studied the effect of multi-modal transport (Fahnenschreiber et al., 2016a) in which a network is created with drivers and public transport. This research implemented one modality to have an existing schedule and the other modality’s schedule to be created. Naseri Gorgoon et al. studied the effect of network size and passenger’s maximum waiting time (Naseri Gorgoon et al., 2019). As expected, larger networks and greater maximum waiting times result in more ridesharing.

Much research includes a feature to improve the service quality of passengers, for example by including a maximum waiting time (Agatz et al., 2012). However, Gruebele main focus was the influence of different service quality factors (Gruebele, 2008). His model maximizes the Perceived Quality of Service on a user and a system level, which includes factors such as trip cost, social networking, trip time and wait time.

In all ridesharing models, a schedule is created that creates a global optimum. However, this does not mean that it is the optimum situation for every individual. Therefore, it can be concluded that not every individual will follow the created schedule, which will cause a disturbance in the created schedule. This is an example of the price of anarchy, the degradation of system-wide performance if participants act selfishly. In ridesharing, riders can decide to drive themselves instead of travelling as a passenger. This effect was studied by Koutsipas et al. (Koutsopoulos and Papadimitriou, 1999). Its importance was stressed by Long et al. (Long et al., 2018).

In conclusion, studying the effect of more realistic scenarios by introducing stochasticity or different networks has been an active topic. Likewise, the effect of cost penalties to ensure

better service for the passengers, compensations for drivers to allow for additional ridesharing and the effect on the global optimum of acting selfishly has been studied. However, the exact effect of riders choosing to drive themselves due to an unsatisfactory arrival time has not been researched. This can be introduced as a rider’s VOT. The importance of this effect has been recognized in literature (Long et al., 2018), but to what extent this affects the optimality of the schedule has been lacking. Therefore, this research will focus on the VOT in a dynamic real-time ridesharing system.

1.2.3 Optimization of transport services

Much research has been done on general vehicle routing optimization. Theoretically, ridesharing can contribute to many benefits to traditional vehicle routing, such as less traffic congestion, fewer CO₂-emissions and lower travel costs. However, the share of work trips that use ridesharing has decreased by almost 10% in the past 30 years (Furuhata et al., 2013). It appears riders prefer the flexibility of driving themselves, which can also be concluded for transport services for goods (Pahlén and Börjesson, 2012). Intuitively, transportation of goods could receive a greater gain from rideshare optimization, since shippers are expected to follow their schedule. For the shipper, there is no incentive to abandon ridesharing and drive directly to the destination. However, in transport services, customers can often set preferences for transportation. For example, customers set modality constraints on their shipment, while multi-modal transport has proven to allow substantial financial benefits (Fahnenschreiber et al., 2016b). Fahnenschreiber et al. studied a ridesharing network with truck and train transport. Train transportation is static and contains a static schedule. Truck transport uses a flexible schedule that is created by their model.

Hammadi and Ksouri studied many aspects of multi-modal transport, which included an load-sharing optimization model to minimize inventory cost (Hammadi and Ksouri, 2013). In this research, vehicles were travelling from multiple depots to special load-sharing hubs. It was mentioned that sharing cargo is often not in the interest of both shippers and customer due to the increase in handling (Pahlén and Börjesson, 2012). Therefore, it is of significance to present the financial benefits of sharing cargo to create incentive for participants. If we assume cargo to be a rider that needs to be transported from origin to destination, then it demonstrates many similarities to ridesharing. More specifically, it shares most properties of Dial-A-Ride, since cargo does not have the option to drive itself. By incorporating a ridesharing schedule, cargo can travel with the driver for part of driver’s route, which allows for multi-hopping.

1.3 Contribution of research

Before we can answer the main research question, a foundation has to be set. In order to study aspects of ridesharing, a ridesharing model has to be created. This will be done by creating an optimization model that minimizes the total costs. In this model every rider has to travel from origin to destination by either driving themselves or by travelling with a different rider. Subsequently, a study is done that will select a suitable network and a suitable commercial solver for this research. The commercial solver is the software that solves the minimization problem. The commercial solvers CPLEX and Xpress-IVE are studied on computation time for different networks. In addition, pre-processing will be added in order to further decrease the computation time. This is done by removing part of the possible solutions that are in

the feasible set. In these removed solutions, a small selection of drivers transport all riders. Intuitively, these solutions do not make sense, because riders in the network are not taxi drivers.

1.3.1 Improving service quality

After the research foundation has been set, elements on ridesharing can be included in the model in order to study its effects. A constraint will be added to limit the solution to single-hopping. Therefore, the benefit of multi-hopping can be explored. Subsequently, a cost penalty for VOT is included in the objective function. Therefore, the model will regard the exact arrival time of riders. By including VOT in the objective function, the total travel costs will increase. The reason for this is that more riders will arrive closer to their preferred arrival time, which makes it more difficult for ridesharing to occur. After introducing this cost penalty, the improvement in service quality and the increase in travel costs can be measured. This is studied by implementing ridesharing in a transport network for goods. The model is adjusted to fit this type of network. By comparing switching of vehicles with no switching of vehicles, the cost benefit of including ridesharing in goods transport is presented. The VOT is also included for this experiment in order to demonstrate the improvements in service quality.

1.3.2 Preventing greedy riders

Furthermore, a game theory approach will be introduced in order to examine possible cost benefits of including VOT in the objective function. If a rider is not satisfied with the ridesharing schedule in terms of arrival time, then this rider takes a greedy transit. This is phenomena is called price of anarchy and can have considerable travel cost consequences. This research is the first to study the price of anarchy using a game theory approach and also the first to study the exact effect of VOT.

2 | Mathematical model

In this chapter the exact problem is described and the mathematical model will be formulated. Section 2.1 describes the problem that will be modelled. Subsequently, section 2.2 will specify the formulation that will be used. Section 2.3 will define the mathematical model, which includes an objective function and the constraints. Concluding the chapter is section 2.4, which explains the assumptions that are made in the model in order to simplify the model.

2.1 Problem description

A ridesharing network is modeled by an outside scheduler that has perfect information on riders' origin, destination, availability time and due date. It is assumed that every rider follows the schedule that will be created for a given time frame. In reality, this resembles orders from riders that go to the scheduler. After a certain number of orders, a transport schedule is made. Every rider has a car available, but also has the option drive with another rider. However, this is only possible if there is still a seat available in the car. In addition, a rider can also first drive to a node and then leave his car and become a passenger. Once a rider is passenger, he cannot become a driver again. Travel costs will be split equally amongst the riders in a vehicle for the duration they are travelling together. The goal is to find a schedule that minimizes the global cost. Since we optimize a linear function with linear constraints where all variables are continuous, binary or integer, this ridesharing problem can be modeled using a generalized minimum cost flow problem (Busacker and Gowen, 1960).

2.2 Formulation

The mathematical model is formulated as follows. Consider a connected graph $\mathcal{G} = (\mathcal{N}, \mathcal{A})$, where $\mathcal{N} \in \{1, \dots, N\}$ is the set of nodes (denoted by i, j, k). The nodes represent possible locations for departure, arrival or transfer. \mathcal{A} is the set of links that directly connect two nodes. With each link ij , an average travel time t_{ij} is associated based on the travel distance. In this graph, a set of riders $\mathcal{P} \in \{1, \dots, P\}$ are dispatched through the network from their origin o_p to their final destination d_p . Each rider p is available for transport starting at an availability time e_p and must reach his destination d_p before his due date l_p . A rider can choose to drive towards his destination himself or travel as a passenger with another driver in the network. v_p denotes the capacity of passengers the vehicle capacity of rider p has.

At first, only one cost is considered in the objective function. α_1 is the cost per unit of travel time. The model aims to minimize the global travel cost for all riders. Only a cost is associated with the drivers, thus the passengers are transported free of charge. The following variables are involved in this model. The binary variable x_{ij}^p is equal to 1 if rider p is travelling on link ij as a driver and 0 otherwise. The binary variable y_{ij}^{pq} is equal to 1 if rider p is travelling as a passenger with driver q on link ij , 0 otherwise. The integer variable D_{ij}^p represents the departure time of driver p on link ij . In addition, E_{ij}^p is the product of D_{ij}^p and x_{ij}^p , which

is created in order to maintain linearity of the model. Similarly, the variable G_{ij}^{pq} is created, which is the product of D_{ij}^p and y_{ij}^{pq} .

2.3 Mathematical model

In this section, a general ridesharing model will be created. By using a minimum cost flow problem, the costs will be minimized in the objective function, while not violating the constraints. This is done by first creating a shortest route model. Subsequently, constraints will be added or adjusted to create a ridesharing model. Adjusted constraints receive the symbol 'b' to their name. If we want to study aspects of ridesharing, then the objective function has to be adjusted or constraints have to be added. Adjustments to the model will be clarified in the section of the respective experiment. The sets of ridesharing model are listed in Table 2.1, the parameters are listed in Table 2.2 and the variables are listed in Table 2.3.

Summary of sets, parameters and variables

\mathcal{P}	Set of riders
\mathcal{N}	Set of nodes
\mathcal{A}	Set of links that directly connect two nodes

Table 2.1: Sets of the model

<i>Time parameters</i>	
t_{ij}	Travel time from node i to node j , $\forall i, j \in \mathcal{N}$
e_p	Earliest departure time for rider p , $\forall p \in \mathcal{P}$
l_p	Latest arrival time for rider p , $\forall p \in \mathcal{P}$
<i>Spatial parameters</i>	
o_p	Origin of rider p , $\forall p \in \mathcal{P}$
d_p	Destination of rider p , $\forall p \in \mathcal{P}$
v_p	Vehicle capacity for passengers for rider p , $\forall p \in \mathcal{P}$
M	Big M
<i>Cost parameters</i>	
α_1	Travel cost per time unit

Table 2.2: Parameters

x_{ij}^p	1 if rider p travels as driver from node i to node j , 0 otherwise, $\forall p \in \mathcal{P}; \forall i, j \in \mathcal{N}$
y_{ij}^{pq}	1 if rider q rides with driver p on link i, j , 0 otherwise, $\forall p, q \in \mathcal{P}; \forall i, j \in \mathcal{N}$
D_{ij}^p	This is the departure time of driver p travelling on link i, j , $\forall p \in \mathcal{P}; \forall i, j \in \mathcal{N}$
E_{ij}^p	Is the product of D_{ij}^p and x_{ij}^p . This is created in order to maintain linearity in the model, $\forall p \in \mathcal{P}; \forall i, j \in \mathcal{N}$
G_{ij}^{pq}	Is the product of D_{ij}^p and y_{ij}^{pq} . This is created in order to maintain linearity in the model, $\forall p, q \in \mathcal{P}; \forall i, j \in \mathcal{N}$
z_{ij}^q	The number of passengers travelling with driver q , $\forall q \in \mathcal{P}; \forall i, j \in \mathcal{N}$

Table 2.3: Decision variables

Objective function

The objective function is given by equation:

$$\min \left\{ \alpha_1 \sum_{p \in \mathcal{P}} \sum_{i, j \in \mathcal{N}} t_{ij} \cdot x_{ij}^p \right\}. \quad (1)$$

The objective function (1) has one cost. α_1 is the cost per time travelled for each rider p and link ij . It calculates the cost for each link that is used by drivers by the model. The passengers variable y_{ij}^{pq} has no cost associated to it.

2.3.1 Constraints for transport network

This part creates a schedule for every rider travelling from their origin to their destination. It does not include ridesharing and thus, it calculates the shortest route for every rider.

Constraints for network flow

The sum of all travelled nodes from the origin is equal to 1 for all drivers p , which is ensured by equation:

$$\sum_{j \in \mathcal{N}} x_{o_p j}^p = 1, \forall p \in \mathcal{P}; \forall o_p \in \mathcal{N}. \quad (2)$$

This constraints the driver to travel from its origin node to only one other node. Subsequently, driver cannot travel back towards their origin node, which is ensured by constraint:

$$x_{i o_p}^p = 0, \forall i, o_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (3)$$

$x_{i o_p}^p$ must be 0 for every node travelling towards the origin. Together with constraint (2), this assures that all drivers will travel from the origin to one other node without travelling back. Similarly, a driver can only travel once towards his final destination, which is ensured by constraint:

$$\sum_{i \in \mathcal{N}} x_{i d_p}^p = 1, \forall p \in \mathcal{P}; \forall d_p \in \mathcal{N}. \quad (4)$$

Constraint (4) is similar to the previous constraint. The sum of all nodes i that travel to the destination is equal to 1 for all drivers p . Drivers cannot leave their destination node, which is ensured by:

$$x_{d_p i}^p = 0, \forall i, d_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (5)$$

$x_{i o_p}^p$ must be 0 for travelling from the destination to any other node. When drivers enter intermediate nodes, they must also leave these nodes, which is ensured by:

$$\sum_{i \in \mathcal{N}} x_{ij}^p - \sum_{k \in \mathcal{N}} x_{jk}^p = 0, \forall p \in \mathcal{P}; \forall j \in \mathcal{N}; j \neq o_p, d_p. \quad (6)$$

For driver p , the sum of nodes that travel towards j must be equal to the sum of nodes that driver travels from j . It also assures that driver p can only travel to one other node from node j .

Constraints (2-6) construct the flow from the origin to destination for all drivers p . The model must only make use of links that are part of set \mathcal{S} , which is constrained by:

$$t_{ij} - \frac{x_{ij}^p}{M} \geq -M(1 - x_{ij}^p), \forall p \in \mathcal{P}; \forall i, j \in \mathcal{N}. \quad (7)$$

The links part of set \mathcal{S} are the travel time values t_{ij} that are non-zero. This means that x_{ij}^p cannot be 1 if the travel time for that link is 0. This is done by introducing big M . On the left hand side, the x_{ij}^p/M is subtracted from the travel time for link ij . On the right hand side, $-M$ is multiplied with $(1 - x_{ij}^p)$. If x_{ij}^p is 0, then t can have any positive value or 0, since $-M$ will always be smaller. If x_{ij}^p is 1, then t minus x_{ij}^p/M must be bigger than 0. Thus, t must have a value bigger or equal to x_{ij}^p/M . Drivers are also not allowed to travel towards the node they are already in, which is ensured by equation:

$$x_{ii}^p = 0, \forall i \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (8)$$

Travelling towards nodes drivers are in does not have extra cost, thus without this constraint the value can become either a 1 or 0. In addition, loops in the travel schedule is not allowed in this network, which is ensured by:

$$\sum_{i \in \mathcal{N}} x_{ij}^p \leq 1, \forall p \in \mathcal{P}; \forall j \in \mathcal{N}. \quad (9)$$

It ensures that drivers do not revisit nodes. The sum of all the nodes that travel towards j has a maximum of 1, which ensures that drivers can only use 1 link to go to j .

Creating variable E_{ij}^p

The departure time of nodes E_{ij}^p is the product of x_{ij}^p and D_{ij}^p . This is a multiplication of two variables, which makes the model non-linear. The following constraints are created to counteract this problem:

$$E_{ij}^p \leq M \cdot x_{ij}^p, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}, \quad (10)$$

$$E_{ij}^p \leq D_{ij}^p, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}, \quad (11)$$

$$E_{ij}^p \geq D_{ij}^p - M(1 - x_{ij}^p), \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}, \quad (12)$$

$$E_{ij}^p \geq 0, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (13)$$

These constraints create E_{ij}^p while keeping linearity of the model. This variable is needed for the time constraints. Constraint (10) ensures that $E_{ij}^p = 0$, when $x_{ij}^p = 0$. Constraint (11) ensures that E_{ij}^p cannot become larger than D_{ij}^p . Constraint (12) ensures that $E_{ij}^p = D_{ij}^p$, when $x_{ij}^p = 1$. Constraint (13) is the capacity constraint that states that E_{ij}^p is a continuous variable greater or equal to 0. Together, these constraints produce the the product of x_{ij}^p and D_{ij}^p .

Time constraints

To ensure that rider p does not depart from his origin before he is available, the model is constrained by:

$$\sum_{j \in \mathcal{N}} E_{opj}^p \geq e_p, \forall o_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (14)$$

The time at which a driver leaves his origin is the product of D_{opj}^p and x_{opj}^p , which is denoted by the new variable. To ensure that every rider p arrives at his destination before his due date, the model is constrained by:

$$\sum_{i \in \mathcal{N}} (E_{idp}^p + x_{idp}^p \cdot t_{idp}) \leq l_p, \forall d_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (15)$$

E_{idp}^p calculates at which time driver p is starting to transfer to his destination and $x_{idp}^p \cdot t_{idp}$ calculates the travel time to his destination. Riders cannot depart from a node before they have arrived there, which is ensured by constraint:

$$\sum_{k \in \mathcal{N}} E_{jk}^p \geq \sum_{i \in \mathcal{N}} (E_{ij}^p + x_{ij}^p \cdot t_{ij}), \forall j \in \mathcal{N}; j \neq d_p; \forall p \in \mathcal{P}. \quad (16)$$

$(E_{ij}^p + x_{ij}^p \cdot t_{ij})$ is the departure time of driver p to node i plus travelling time to node i , which is the arrival time at node i . E_{jk}^p is the departure time of node j , which should be greater or equal to the arrival time at node j . The destination node is excluded from this constraint, since the left-hand side will be zero in this situation. The following equation is the capacity constraint for the departure time:

$$D_{ij}^p \geq 0, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (17)$$

Constraint (17) states that the departure time is a continuous variable greater or equal to 0.

2.3.2 Creating a ridesharing network

In this part, the constraints for ridesharing are created. Variable y_{ij}^{pq} is introduced to initiate ridesharing for rider p with driver q on link ij . In order to create a ridesharing model, some constraints are adjusted and some new constraints are introduced.

Creating variable G_{ij}^{pq}

Similarly to E_{ij}^p , G_{ij}^{pq} is the product of y_{ij}^{pq} and D_{ij}^q , which is created by the constraints:

$$G_{ij}^{pq} \leq M \cdot y_{ij}^{pq}, \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}, \quad (18)$$

$$G_{ij}^{pq} \leq D_{ij}^q, \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}, \quad (19)$$

$$G_{ij}^{pq} \geq D_{ij}^q - M(1 - y_{ij}^{pq}), \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}, \quad (20)$$

$$G_{ij}^{pq} \geq 0, \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}. \quad (21)$$

G_{ij}^{pq} has to be created with constraints to ensure linearity of the model. This variable is needed for the time constraints of passengers. Constraint (18) ensures that $G_{ij}^{pq} = 0$, when $y_{ij}^{pq} = 0$. Constraint (19) ensures that G_{ij}^{pq} cannot become larger than D_{ij}^q . Constraint (20) ensures that $G_{ij}^{pq} = D_{ij}^q$, when $y_{ij}^{pq} = 1$. Constraint (21) is the capacity constraint that states that G_{ij}^{pq} is a continuous variable greater or equal to 0. Together, these constraints produce the the product of y_{ij}^{pq} and D_{ij}^q .

Constraints for network flow

To ensure that riders cannot carpool with themselves, the following constraint is introduced:

$$y_{ij}^{pp} = 0, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (22)$$

Therefore, when p is equal to p on link ij , the y value should always be zero. Subsequently, the network flow needs to be adjusted for ridesharing. All riders can only travel from the origin to one other node, which is ensured by constraint:

$$\sum_{j \in \mathcal{N}} \left(x_{opj}^p + \sum_{q \in \mathcal{P}} y_{opj}^{pq} \right) = 1, \forall p \in \mathcal{P}; \forall o_p \in \mathcal{N}. \quad (2b)$$

In constraint (2b), the sum of all travelled nodes from the origin must be equal to 1 for all riders. The constraint remains identical for the drivers, but all passengers p driving with all drivers q are added. This assures that all riders in the network travel from the origin node to only one other node. In addition, riders cannot travel back towards their origin node which is ensured by:

$$x_{io_p}^p + y_{io_p}^{pq} = 0, \forall i, o_p \in \mathcal{N}; \forall p, q \in \mathcal{P}. \quad (3b)$$

$x_{io_p}^p$ and $y_{io_p}^{pq}$ and must be 0 for every node travelling towards the origin. Together with constraint (2b), this assures that all drivers and passengers will travel from the origin to one other node without travelling back. Similar to constraint (2b), riders must travel at most once to their destination, which is ensured by:

$$\sum_{i \in \mathcal{N}} \left(x_{id_p}^p + \sum_{q \in \mathcal{P}} y_{id_p}^{pq} \right) = 1, \forall p \in \mathcal{P}; \forall d_p \in \mathcal{N}. \quad (4b)$$

The sum of all nodes i that travel to the destination (as driver or passenger) is equal to 1 for all riders p . All drivers and passengers cannot travel after they have reached their destination, which is ensured by:

$$x_{d_p i}^p + y_{d_p i}^{pq} = 0, \forall i, d_p \in \mathcal{N}; \forall p, q \in \mathcal{P}. \quad (5b)$$

$x_{i o_p}^p$ and $y_{d_p i}^{pq}$ must be 0 travelling from the destination to any other node. When riders arrive at an intermediate node, then they must also leave these nodes, which is ensured by:

$$\sum_{i \in \mathcal{N}} \left(x_{ij}^p + \sum_{q \in \mathcal{P}} y_{ij}^{pq} \right) - \sum_{k \in \mathcal{N}} \left(x_{jk}^p + \sum_{q \in \mathcal{P}} y_{jk}^{pq} \right) = 0, \forall p \in \mathcal{P}; \forall j \in \mathcal{N}; j \neq o_p, d_p. \quad (6b)$$

Riders can also enter a node as driver and leave the node as a passenger, thus in a similar manner as the previous two constraints, the y -values are added. For rider p , the sum of nodes that travel to j must be equal to the sum of nodes that rider p travels from j . Rider p can only rideshare with driver q on link ij if that driver will travel there, which is ensured by:

$$y_{ij}^{pq} \leq x_{ij}^q, \forall p, q \in \mathcal{P}; \forall i, j \in \mathcal{N}. \quad (23)$$

Therefore, the variable y_{ij}^{pq} can only be 1 for passenger p if the variable x_{ij}^q is 1 for driver q . It is allowed for variable x_{ij}^q to be 1 if y_{ij}^{pq} is 0 if no ride sharing occurs on link ij . Rider p can only travel with driver q on link ij if it does not exceed driver q 's vehicle capacity, which is ensured by:

$$\sum_{p \in \mathcal{P}} y_{ij}^{pq} \leq v_q, \forall i, j \in \mathcal{N}; \forall q \in \mathcal{P}. \quad (24)$$

The sum of all passengers driving with q on link ij must be smaller or equal to the vehicle capacity v_p for each driver q and each link ij . Rider p can no longer travel as a driver once he has left his car, which ensured by:

$$\sum_{q \in \mathcal{P}} \sum_{i \in \mathcal{N}} y_{ij}^{pq} + \sum_{k \in \mathcal{N}} x_{jk}^p \leq 1, \forall p \in \mathcal{P}; \forall j \in \mathcal{N}. \quad (25)$$

If a rider is travelling to node j as a passenger ($y_{ij}^{pq} = 1$) then he cannot leave node j as a driver (x_{jk}^p must be 0). If $y_{ij}^{pq} = 0$, then rider p can leave node j as a driver.

Time constraints

All riders can only leave their origin once they are available, which is ensured by:

$$\sum_{j \in \mathcal{N}} \left(E_{o_p j}^p + \sum_{q \in \mathcal{P}} G_{o_p j}^{pq} \right) \geq e_p, \forall o_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (14b)$$

The time at which a driver leaves his origin is the product of $D_{o_p j}^p$ and $x_{o_p j}^p$ and the time at which a passenger leaves his origin is the product of $D_{o_p j}^p$ and $y_{o_p j}^{pq}$. These variables are summed because each rider will leave their origin either as a driver or a passenger. All riders must also arrive at their destination before the due date, which is ensured by:

$$\sum_{i \in \mathcal{N}} \sum_{q \in \mathcal{P}} (G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}) \leq l_p, \forall d_p \in \mathcal{N}; \forall p \in \mathcal{P}. \quad (26)$$

$G_{id_p}^{pq}$ calculates at which time driver q will transfer passenger p to his destination and $y_{id_p}^{pq} \cdot t_{id_p}$ calculates the travel time to his destination. If rider p is a driver to his destination, then this

summation is 0 and the constraint will not be violated. In addition, riders can only leave nodes once they have arrived there, which is ensured by:

$$\sum_{k \in \mathcal{N}} \left(\underbrace{E_{jk}^p}_a + \underbrace{\left(\sum_{q \in \mathcal{P}} G_{jk}^{pq} \right)}_b \right) \geq \sum_{i \in \mathcal{N}} \left(\underbrace{\left(E_{ij}^p + x_{ij}^p \cdot t_{ij} \right)}_c + \underbrace{\sum_{q \in \mathcal{P}} \left(G_{ij}^{pq} + y_{ij}^{pq} \cdot t_{ij} \right)}_d \right), \forall j \in \mathcal{N}; j \neq d_p; \forall p \in \mathcal{P}. \quad (16b)$$

Riders can arrive a node as driver and leave as driver, arrive as driver and leave as passenger and arrive as passenger and leave as passenger. Constraint (16) only holds for entering and leaving drivers. The constraint (16b) assures that drivers and passengers can only leave a node once they are actually there. In the equation, either part a has a value or part b has a value and either part c has a value or part d has a value. In addition, a cannot have a value if d has a value, since a rider cannot leave node j as a driver if he has arrived as a passenger. Therefore, $a \geq c$ if rider p enters a driver and leaves as driver; $b \geq c$ if rider p enters as a driver and leaves as passenger; $b \geq d$ if rider p enters as a passenger and leaves as passenger. The destination node is excluded, since passengers and drivers will not leave after this node, thus a and b will both be 0, while either c or d is not zero. In addition, the departure of passenger p must be the same as his driver q , which is ensured by:

$$E_{ij}^q - G_{ij}^{pq} \leq M(1 - y_{ij}^{pq}), \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}, \quad (27)$$

$$E_{ij}^q - G_{ij}^{pq} \geq -M(1 - y_{ij}^{pq}), \forall i, j \in \mathcal{N}; \forall p, q \in \mathcal{P}. \quad (28)$$

If $y_{ij}^{pq} = 1$, and thus passenger p is riding with driver q , then the right-hand side becomes 0 for constraint (27) and (28). This means $E_{ij}^q - G_{ij}^{pq} \leq 0$ for constraint (27) and $E_{ij}^q - G_{ij}^{pq} \geq 0$ for constraint (28). This means $E_{ij}^q = G_{ij}^{pq}$ when ridesharing occurs. If $y_{ij}^{pq} = 0$, and thus no ridesharing occurs, E_{ij}^p and G_{ij}^{pq} can have any value. Tracking the number of riders in a vehicles is useful information for experiments and is calculated by:

$$\sum_{p \in \mathcal{P}} y_{ij}^{pq} + x_{ij}^q = z_{ij}^q, \forall q \in \mathcal{P}; \forall i, j \in \mathcal{N}. \quad (29)$$

$\sum_{p \in \mathcal{P}} y_{ij}^{pq}$ sums the number of riders that are travelling driver q and x_{ij}^q adds the driver himself. Variables x_{ij}^p and y_{ij}^{pq} are binary variables, which is enforced by:

$$x_{ij}^p, y_{ij}^{pq} \in \{0, 1\}, \forall p, q \in \mathcal{P}; \forall i, j \in \mathcal{N}. \quad (30)$$

2.4 Assumptions

In order to model a real-life ridesharing scenario, assumptions have to be made. This ensures a simplified problem, which narrows the scope of this research and reduces the computation time of the commercial solver. The following assumptions are made.

- The travel time from i to j has a fixed value. Traffic congestion or other travel speed adjustments are not included in this model. Thus, failure of links is also not included.
- Riders can wait at a hub without any additional cost. This is regarded to be outside of the scope of this research.

- All riders must reach their final destination before the due date by driving themselves or travelling with another rider in the model, otherwise the solution is infeasible.
- In this ridesharing network, it is not allowed for drivers to return to a pre-visited node. Although it could be argued that there exists a global optimum solution where drivers return to a certain node, it is unsatisfactory for individual passengers and therefore, these solutions are not considered.
- There is no time penalty included for switching to a different vehicle. This is considered to be outside of the scope of this research.

3 | Enhancing computational load

The goal of this chapter is to find a suitable optimization solver, a suitable network and to adjust the model for experiments. The main focus of this chapter is on computational speed. Solving complex MILP are computationally very heavy. There is currently no solution for this problem (Agatz et al., 2012). Making assumptions on real-life improve computation time, but are less correct. Currently, a balance between both has to be used. For this research, it is important to have a sufficient number of riders in the network, while the computational speed fits the time frame. Section 3.1 tests the network from Lin (Lin et al., 2012) with two different solvers. Section 3.2 tries a more realistic domestic-work network with two highways in between. This network is also tested with both solvers. Section 3.3 adds pre-processing to the model with aims to reduce the computational effort, while creating a more realistic model.

3.1 Experiment network 1

3.1.1 Experimental setup network 1

For this experiment, the experimental road network from Lin (Lin et al., 2012) is used. This connected network consists of 9 nodes and 16 links and is visualized in Figure 3.1. Distances between links are included in the figure.

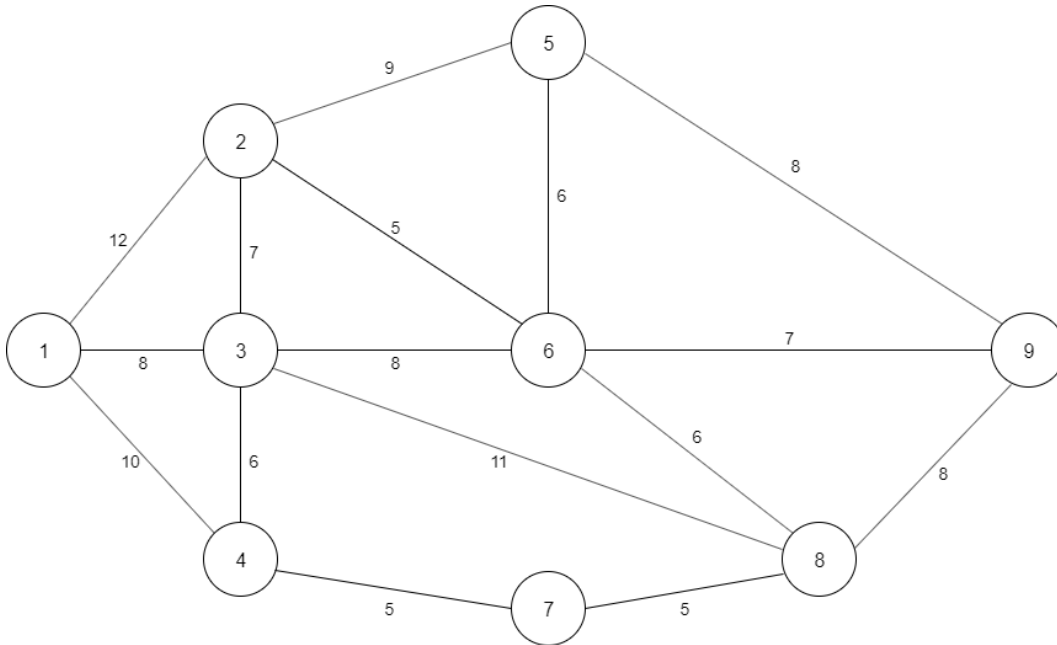


Figure 3.1: Experimental road network with distances between nodes

The parameters are set as follows. Because there is only one cost parameter in the model, α_1

can be arbitrarily chosen and is set to 1. The origin and destination of rider p can be any node in the network and thus, o_p and d_p are randomized with equal probability between 1 and 9. The availability time e_p is randomized with equal probability between 0 and 15 and due date is randomized with equal probability between 45 and 60. The longest travel time between two nodes in the network is 29, thus every rider is able to reach his destination in time. A overview of the parameters is given in Table 3.1.

Subsequently, a travel schedule is created for every rider with ridesharing turned on and turned off. The set of riders are increased incrementally, which is indicated with the powerset \wp . Total travel costs and computation time are recorded. This experiment is performed using the commercial solver Xpress-IVE and CPLEX. Ridesharing is turned off by including the following constraint:

$$y_{ij}^{pq} = 0, \forall p, q \in \mathcal{P}; \forall i, j \in \mathcal{N}. \quad (31)$$

Parameter	$p \in \mathcal{P}$	\wp	α_1	o_p	d_p	e_p	l_p
Value	$\{1, \dots, \mathcal{P} \}$	$\{\mathcal{P}(n) : n \in \{5, 6, \dots, n\}\}$	1	$o_p \sim U(1, 9)$	$d_p \sim U(1, 9)$	$e_p \sim U(0, 15)$	$l_p \sim U(45, 60)$

Table 3.1: Parameter settings for experiment 1

3.1.2 Results experiment network 1

Table 3.2 contains the travel costs of the travel schedule with and without ridesharing. It is clearly evident that ridesharing has a significant cost benefit. Intuitively, the cost saving ratio would increase if the number of riders in the network increases. However, in this experiment, it is not evident yet. Due to the low number of riders in the network, the cost benefit of ridesharing depends mostly on the random origin and destination of riders.

Number of riders	Vehicle capacity	Travel cost	Travel cost without ridesharing	Cost saving due to ride-sharing (in %)	Comp. time Xpress-IVE	Comp. time CPLEX
5	4	46	74	37.84%	03 sec	20 sec
6	4	49	89	44.94%	18 sec	23 sec
7	4	50	98	48.98%	11 sec	43 sec
8	4	58	77	24.68%	01 min 57 sec	01 min 59 sec
9	4	74	102	27.45%	01h 22 min	01 hour 54 min

Table 3.2: Cost savings and computation time for an x number of riders for network 1

In addition, Table 3.2 contains the computation times for the commercial solvers Xpress-IVE and CPLEX. Both solvers were unable to create a schedule for 10 riders within a time frame of three hours. Therefore, adjustments have to be made. Comparing the computation times cannot exclude one solver yet, since both solvers yield similar results.

3.2 Experiment network 2

3.2.1 Experimental setup network 2

A second network is created manually in order to cope with the large computation times. In addition, this network will represent a more realistic setting as ridesharing is most often used for commuting purposes (Ferguson, 1997). In this setting, there is a domestic area (node 1, ..., 4) and an industry area (node 5, ..., 8) where riders work. These nodes and distances between nodes are illustrated in Figure 3.2. Riders have to travel from their domestic hub to their work hub through a highway with greater travel time.

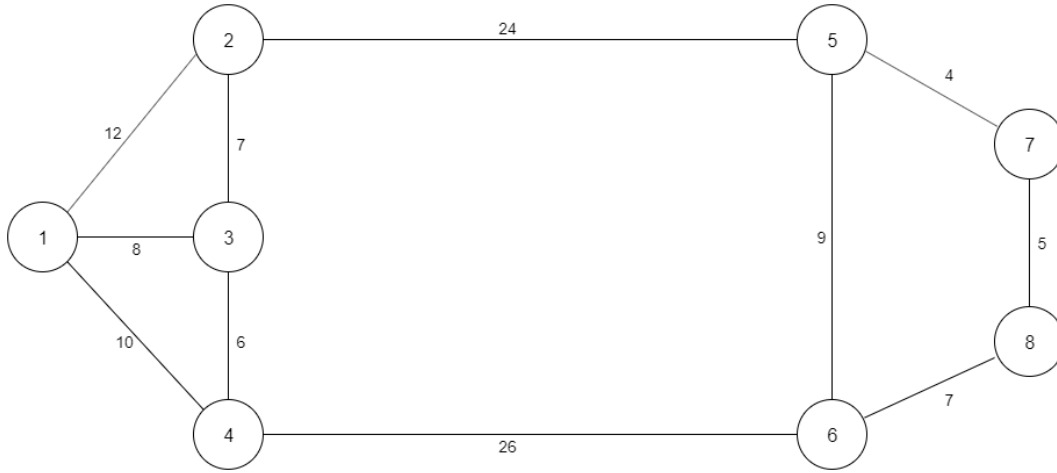


Figure 3.2: Second experimental road network with two highways with distances between nodes

The new parameters are listed in Table 3.3. The due date l_p is changed to a random value between 55 and 70 to ensure that every rider can reach his destination before the due date.

Parameter	$p \in \mathcal{P}$	\mathcal{O}	α_1	o_p	d_p	e_p	l_p
Value	$\{1, \dots, \mathcal{P} \}$	$\{\mathcal{P}(n) : n \in \{5, 6, \dots, n\}\}$	1	$o_p \sim U(1, 4)$	$d_p \sim U(5, 8)$	$e_p \sim U(0, 15)$	$l_p \sim U(55, 70)$

Table 3.3: Parameter settings for experiment 2

3.2.2 Results experiment network 2

Table 3.4 contains the travel costs of the travel schedule with and without ridesharing. The cost saving of this network is considerably larger compared to the previous network. There is a clear explanation for this result. Riders are moving less randomly through this network. There are fewer possible origins and destinations and there is a clear incentive to rideshare on the highways. It is also evident that cost saving ratio is increasing if the number of riders increase. This is up to a certain number of riders, since there is a vehicle capacity that prevents more cost saving. Ideally, more riders are used in the model in order to make an accurate prediction of how ridesharing will perform in a large scale implementation.

Number of riders	Vehicle capacity	Travel cost	Travel cost without ridesharing	Cost saving due to ride-sharing (in %)	Comp. time Xpress-IVE	Comp. time CPLEX
5	4	83	167	50.30%	03 sec	11 sec
6	4	75	183	59.02%	05 sec	16 sec
7	4	91	239	61.92%	12 sec	39 sec
8	4	80	272	70.59%	26 sec	01 min 46 sec
9	4	80	288	72.22%	25 sec	02 min 09 sec
10	4	99	338	70.71%	01 min 48 sec	28 min 49 sec
11	4	75	338	77.81%	01 min 02 sec	–
12	4	98	378	74.07%	02 min 33 sec	–
13	4	118	431	72.62%	01 hour 04 min	–

Table 3.4: Cost savings and computation time for an x number of riders for network 2

In addition, Table 3.4 contains the computation times of the two solvers. For both solvers, there is a clear improvement in computation time. CPLEX was unable to create a schedule for 11 riders within three hours and Xpress-IVE was unable to do this for 14 riders. Due the clear computational benefit of Xpress-IVE, this solver is chosen for further experiments.

3.3 Experiment shortest path

3.3.1 Experimental setup shortest path

In order to further improve the computational efficiency of the model pre-processing is added. Pre-processing aims to remove possible answers that either will not lead to an optimal outcome or create an answer that does not cohere well with flexibility of drivers. Therefore, drivers can only travel from their origin to their destination using their shortest route or their second shortest route. This sets a restriction on the allowable detour of drivers. When riders are travelling as a passenger, there is no restriction for their route. Two sets are introduced to include the pre-processing; \mathcal{SP}_p^1 contains the set of links that consists of the shortest route for rider p and \mathcal{SP}_p^2 contains the set of links that consists of the second shortest route for rider p . These are listed in Table 3.5. Riders are only allowed to make use of these links, which is ensured by the new objective function:

$$\min \left\{ \alpha_1 \sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{SP}_p^{12}} t_{ij} \cdot x_{ij}^p \right\}. \quad (1b)$$

It must be stated that global optimal results could be removed by implementing pre-processing. It is argued that drivers will not be satisfied with making large detours. Therefore, global optima with large detours will not hold in a realistic setting. In addition, all parameters will remain identical for this experiment.

\mathcal{SP}_p^1	Shortest set of links for rider p from his o_p to his d_p , $\forall p \in \mathcal{P}$
\mathcal{SP}_p^2	Second shortest set of links for rider p from his o_p to his d_p , $\forall p \in \mathcal{P}$

Table 3.5: New shortest path sets

3.3.2 Results experiment shortest path

The results of this experiment are displayed in Table 3.6. It is clearly evident that the pre-processing did not have a large impact on the cost saving due to ridesharing. In terms of computation time, there is an evident improvement. Within a realistic time frame (sub three hours), the solver has advanced from 13 riders to 17 riders. Therefore, for forthcoming experiments, the network with two highways and pre-processing will be used.

Number of riders	Vehicle capacity	Travel cost	Travel cost without ridesharing	Cost saving due to ride-sharing (in %)	Computation time
5	4	127	180	29.44%	01 sec
7	4	102	244	58.20%	08 sec
9	4	136	313	56.55%	23 sec
11	4	113	368	69.22%	1 min 32 sec
13	4	119	452	73.67%	19 min 57 sec
15	4	117	449	73.94%	17 min 03 sec
17	4	146	472	76.38%	02 hour 11 min

Table 3.6: Cost savings and computation time for an x number of riders for network 2 with shortest path

4 | Experiments

In this chapter, the experiments are formulated and executed that will answer the main research question. Section 4.1 will explain the setup of the VOT experiments. In addition, it will explain how the mathematical model will be adjusted in order to perform these experiments. In section 4.2, an experiment is executed on VOT-values. Following these experiments, section 4.3 will make an interpretation on the meaning of these results. Section 4.4 gives an explanation on the introduced game theory element that will be used for the next experiment. Section 4.5 contains new experiments with greedy riders. The last experiment is explained in Section 4.6, which makes a transition from transportation of riders to transportation of goods.

4.1 Experimental setup

4.1.1 New set and parameters

An experiment will be done on the VOT of riders in the network. VOT is introduced as a rider's cost penalty for not reaching the destination at the desired time. When riders have a relatively high positive VOT, they aspire to reach their destination as quickly as possible. In case of a relatively high negative VOT, riders want to arrive as close as possible to their due date (but not exceeding it). For a goods transport service, this makes sense due to the inventory costs of goods. In case of a low positive or negative VOT, riders are indifferent to their arrival time, as long as they arrive before their due date. It is important to include the VOT in the mathematical model, because this will better satisfy the individual needs of riders in the network.

The VOT for rider p is specified by α_2^p . It must be stated that the exact value of α_2^p has no meaning. The significance is in the proportional value of α_1 and α_2^p . In the experiments, α_1 has a constant value of 1. Human behavior models are often distributed by a Gaussian distribution (Shen et al., 2016) and we will use this for this research. However, there is no function for Gaussian distribution in Xpress-IVE. Nonetheless, there is a function for binomial distribution. Therefore, we make use of the Central Limit Theorem, which states that the sampling distribution of the sample means approaches a Gaussian distribution as the sample size gets larger, no matter the shape of the population distribution. The Gaussian distribution is created in the following manner. Consider the set $\mathcal{S} \in \{1, \dots, S\}$. For each $s \in \mathcal{S}$, we randomize a value between 0 and 1. Subsequently, we round this value to the nearest integer. In the next step, we count the number of 1's we have in the set \mathcal{S} . This summation gives the VOT for rider p . These steps are repeated for all $p \in \mathcal{P}$. By making the set of \mathcal{S} sufficiently large, we approximate a Gaussian distribution with a normal standard deviation. The mean μ is equal to the size of $S/2$ and can be adjusted by adding or subtracting a value. However, the standard deviation cannot be adjusted since we cannot change the distribution of the initial value between 0 and 1.

In addition, the theoretical minimum transport time for rider p from his origin to destination

is necessary information for the VOT experiment. This is indicated by the parameter ST_p . The mentioned parameters and set are summarized in Table 4.1. The parameters for the experiments are listed in Table 4.2.

α_2^p	An individual's VOT-penalty per time unit, $\forall p \in \mathcal{P}$
ST_p	Shortest travel time from origin to destination for rider p , $\forall p \in \mathcal{P}$
\mathcal{S}	Set of random values between 0 and 1 to create a normal distribution

Table 4.1: New set and parameters

Parameter	Value
$p \in \mathcal{P}$	$\{1, \dots, \mathcal{P} \}$
\wp	$\{\mathcal{P}(n) : n \in \{5, 6, \dots, 15\}\}$
α_1	1
α_2^p	$\alpha_2^p \sim \mathcal{N}(\mu, \sigma^2), 0$
μ	$\mu = \{-13, \dots, 13\}, \mu \neq 0$
σ^2	$\sigma^2 = 1$
o_p	$o_p \sim U(1, 4)$
d_p	$d_p \sim U(5, 8)$
e_p	$e_p \sim U(0, 15)$
l_p	$l_p \sim U(55, 70)$
v_p	4

Table 4.2: Parameter settings for experiment VOT

4.1.2 New objective function

In order to study the effects of positive VOT, it is included in the new objective function:

$$\min \left\{ \underbrace{\alpha_1 \sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{SP}_p^{12}} t_{ij} \cdot x_{ij}^p}_{a} + \underbrace{\sum_{p \in \mathcal{P}} \sum_{i, d_p \in \mathcal{N}} \alpha_2^p \left((E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}) + \sum_{q \in \mathcal{P}} (G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}) - (e_p + ST_p) \right)}_b \right\}. \quad (1bb)$$

This objective function will include riders' VOT in order to create a schedule that will better satisfy the customers. The objective function (1bb) can be divided in two parts. Part a and part b are multiplied with α_1 and α_2^p , respectively; each of which is a cost penalty. α_1 is the cost per time travelled on link ij for each p . It calculates the cost for each link that is used by the model. α_2^p is the VOT-penalty for each individual. This part is calculated by taking the departure time towards the destination and adding the travel time towards the destination. Riders can arrive at their destination as driver or as passenger. $E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}$ calculates the arrival time as a driver and $G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}$ calculates the arrival time as passenger. Thus, one of these values is non-zero. Subsequently, the theoretical minimum arrival time is subtracted from this value. This theoretical minimum arrival time is given by $(e_p + ST_p)$, which is the availability time of rider p and the minimum travel time from origin to destination for rider p .

In a similar manner, objective function (1bbb) is used to study the effect of negative VOT:

$$\min \left\{ \underbrace{\alpha_1 \sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{SP}_p^{12}} t_{ij} \cdot x_{ij}^p}_a + \underbrace{\sum_{p \in \mathcal{P}} \sum_{i,d_p \in \mathcal{N}} -\alpha_2 \left(l_p - (E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}) + \sum_{q \in \mathcal{P}} (G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}) \right)}_b \right\}. \quad (1bbb)$$

There is a cost penalty if a rider arrives earlier than his due date. Therefore, the arrival time is subtracted from the due date l_p . It is assumed during the experiments that a population has either completely positive VOT-values or completely negative VOT-values. This assumption is necessary as positive and negative VOT-values use different objective functions.

In this experiment, we want to compare a network where riders can rideshare with multiple vehicles to a network where riders can travel with one vehicle. In order to ensure that riders can travel with one vehicle, the following constraint is introduced:

$$\sum_{i \in \mathcal{N}} y_{ij}^{pq} - \sum_{k \in \mathcal{N}} y_{jk}^{pq} \leq 0, \forall p, q \in \mathcal{P}; \forall j \in \mathcal{N}; j \neq o_p, d_p. \quad (32)$$

It ensures that passenger p enters node j with driver q , then he must also leave this node with driver q . This constraint can be smaller than 0 if rider p arrives at j as a driver and leaves j as a passenger.

4.2 Experiment different mean VOT's

Table 4.3 contains a selection of the results of the experiments. It lists the total travel costs for a schedule without ridesharing, a ridesharing schedule without VOT and a schedule with a selection of positive VOT's included. This experiment is performed for single-hopping and multi-hopping. The complete set of solutions are provided in Appendix C. Seven different positive mean VOT-values are selected and seven different negative mean VOT-values. This mean value was not extended after a mean of 13 and -13 as no change in schedule was observed with higher values. The results of negative mean VOT-values are not provided in this section, because the results of negative VOT are very similar to their positive counterpart. Figure 4.1, 4.2 and 4.3 contain a graphical description of the results. Only a selection of the mean values are included to improve readability. The model creates a different schedule when VOT is included. A higher mean VOT results in higher total travel costs. When we compare multi-hopping to single-hopping, we observe some situations where single-hopping has higher total travel costs and some situations where multi-hopping has higher total travel costs. This is due to the random aspect of data generation. On average, higher travel costs occur for single-hopping. For $\mu = 1$, average travel costs per rider increased from 12.7 to 13.8 by constraining the model to single-hopping. For $\mu = 5$, average travel costs per rider increased from 18.5 to 21.2 and for $\mu = 9$, average travel costs per rider increased from 21.7 to 23.2. The exact impact of VOT depends on the properties of the network. Intuitively, if the availability time and due dates of riders are very similar, then VOT will have a smaller impact on total travel cost.

Number of riders	Travel cost without ridesharing	Multi-hopping				Single-hopping			
		No VOT	$\mu = 1$	$\mu = 5$	$\mu = 9$	No VOT	$\mu = 1$	$\mu = 5$	$\mu = 9$
5	172	53	81	138	138	88	109	143	143
6	213	78	83	83	118	91	153	181	181
7	252	87	94	155	188	86	86	177	177
8	264	89	117	157	183	125	149	209	209
9	331	94	128	187	226	87	87	179	268
10	346	79	107	209	209	125	138	253	253
11	371	113	132	191	216	129	151	217	217
12	416	124	165	206	266	116	116	255	255
13	447	120	132	287	320	134	150	210	271
14	487	129	188	211	259	148	187	271	304
15	502	134	172	214	262	137	189	239	272

Table 4.3: Travel cost for different VOT-values for multi-hopping and single-hopping

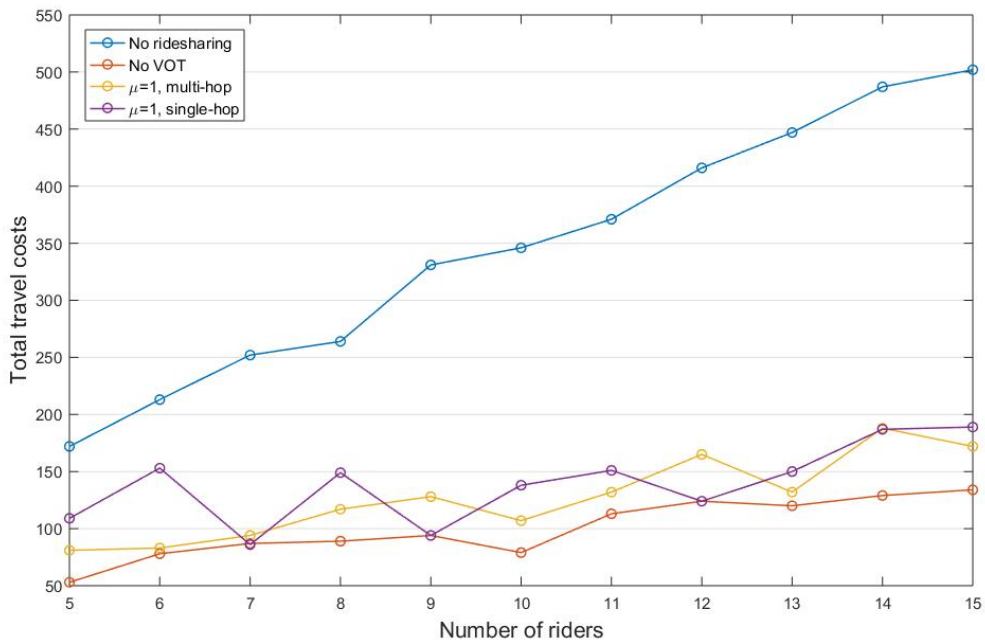


Figure 4.1: Travel costs for a mean VOT-value of 1 and comparison to no ridesharing

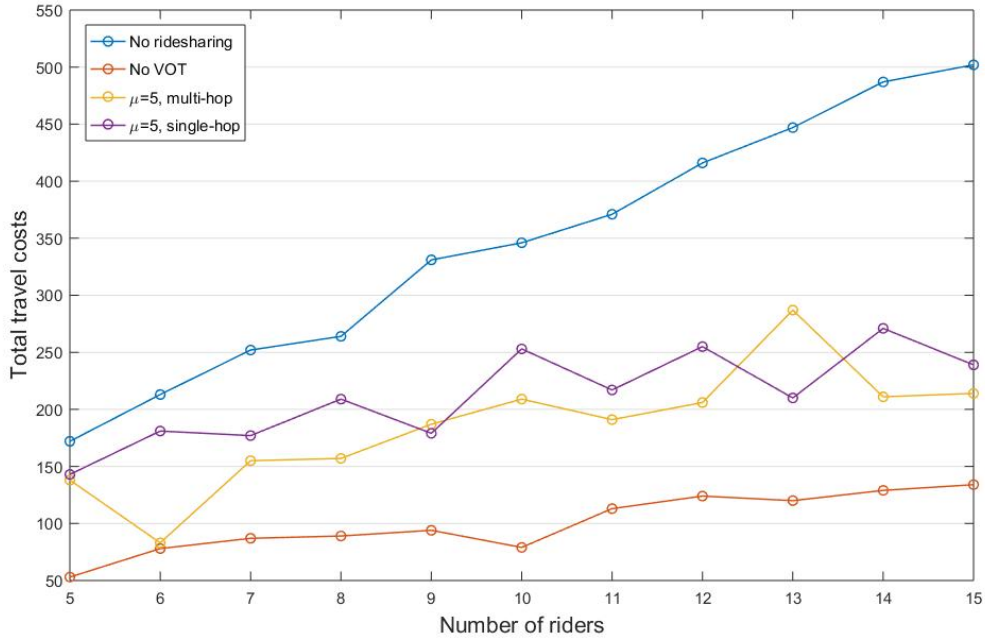


Figure 4.2: Travel costs for a mean VOT-value of 5 and comparison to no ridesharing

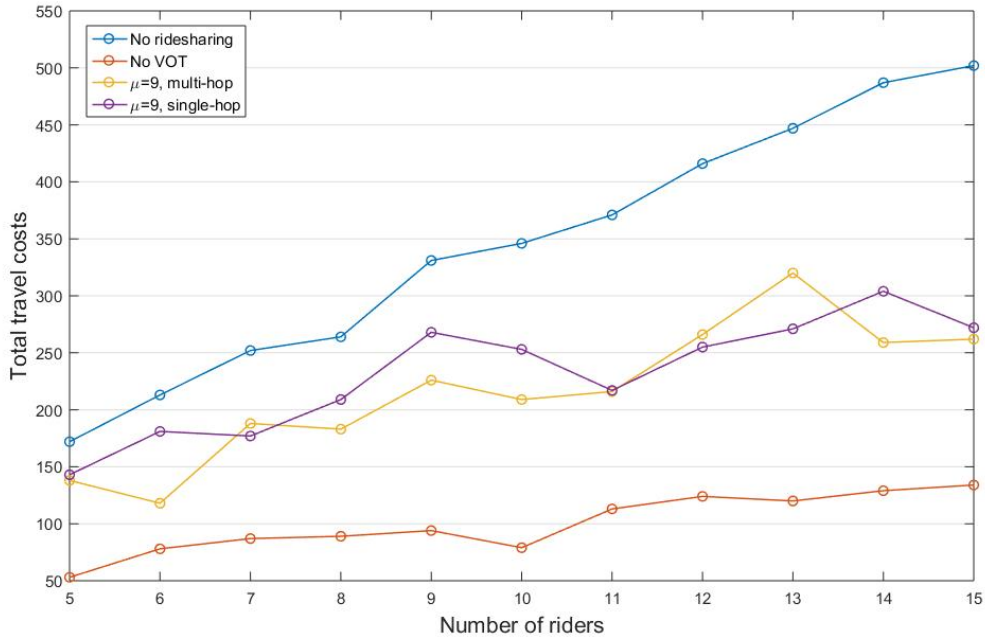


Figure 4.3: Travel costs for a mean VOT-value of 9 and comparison to no ridesharing

The average filling rate of the vehicles is documented in Table 4.4 and a graphical description is given in Figure 4.4, 4.5 and 4.6. In contrast to the travel costs, the schedule without

ridesharing creates a lower bound and the schedule without VOT creates an upper bound for the filling rate. Including VOT in the model lowers the filling rate, since more riders will decide to drive themselves. A higher VOT will result in a lower filling rate, although this is up to certain point. A mean value greater than 13 did not have any additional effect. This created a schedule where ridesharing only occurs if it did not cause a time delay for rider p . In terms of single-hopping and multi-hopping, the average filling rate for single-hopping is lower. For $\mu = 1$ the average filling rate decreased from 2.81 to 2.28 by constraining the model to single-hopping. For $\mu = 5$, this value decreased from 1.66 to 1.51 and for $\mu = 9$, it decreased from 1.40 to 1.39.

Number of riders	Filling rate without ridesharing	Multi-hopping				Single-hopping			
		No VOT	$\mu = 1$	$\mu = 5$	$\mu = 9$	No VOT	$\mu = 1$	$\mu = 5$	$\mu = 9$
5	1.00	2.60	2.00	1.22	1.22	2.25	1.75	1.18	1.18
6	1.00	2.67	2.33	2.33	1.63	2.50	1.33	1.15	1.15
7	1.00	2.50	2.25	1.50	1.29	3.00	3.00	1.55	1.55
8	1.00	2.88	2.10	1.43	1.33	2.45	1.67	1.29	1.29
9	1.00	3.13	2.00	1.50	1.24	2.75	2.75	1.54	1.06
10	1.00	3.57	2.89	1.44	1.44	2.90	2.42	1.39	1.39
11	1.00	3.22	2.33	1.81	1.73	3.00	2.45	1.67	1.67
12	1.00	3.20	2.23	1.63	1.30	2.50	2.50	1.35	1.35
13	1.00	3.10	2.73	1.33	1.22	3.40	2.73	2.14	1.67
14	1.00	3.43	2.59	1.92	1.50	2.81	2.29	1.56	1.40
15	1.00	3.41	2.78	2.17	1.53	3.00	2.14	1.76	1.58

Table 4.4: Filling rate for different VOT-values for multi-hopping and single-hopping

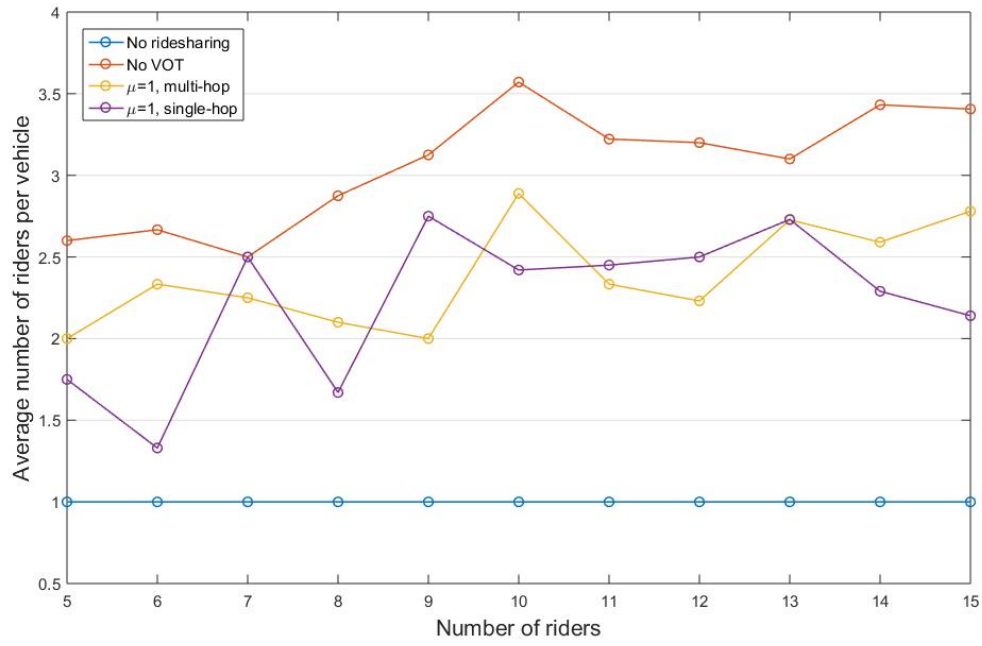


Figure 4.4: Average filling rate of vehicles for a mean VOT-value of 1 and comparison to no ridesharing

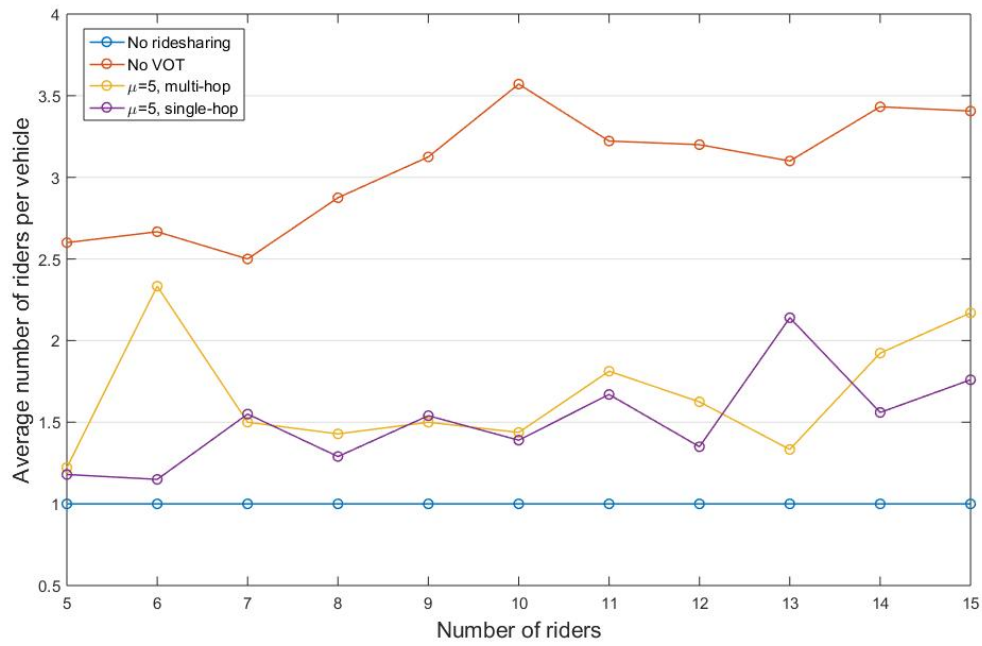


Figure 4.5: Average filling rate of vehicles for a mean VOT-value of 5 and comparison to no ridesharing

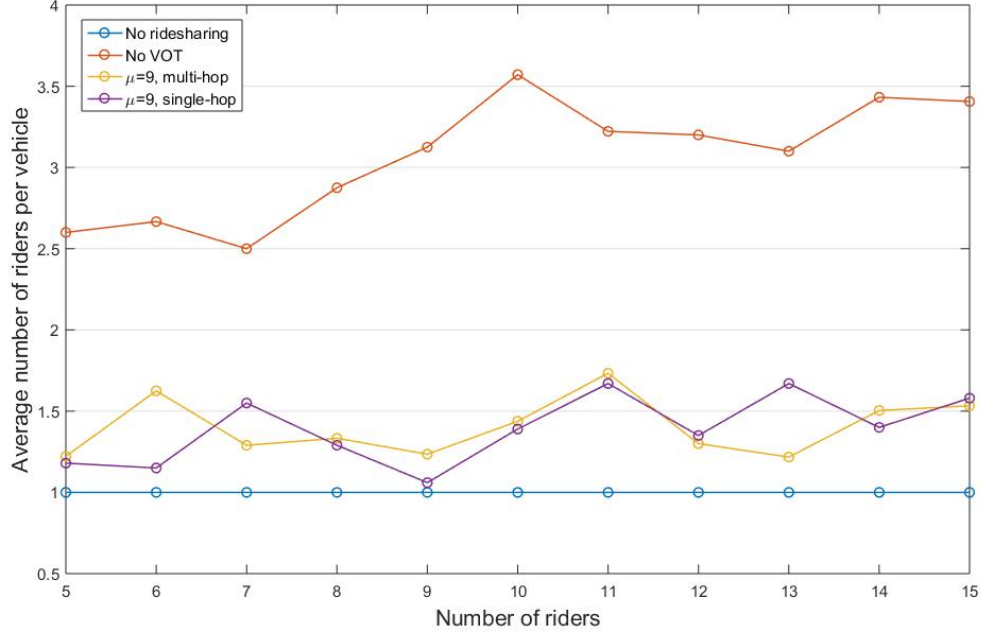


Figure 4.6: Average filling rate of vehicles for a mean VOT-value of 9 and comparison to no ridesharing

4.3 Interpretation of results

From these results, we can conclude that implementing ridesharing in a commuting network yields a very significant reduction in total travel costs. Much existing research focuses on improving the service quality of ridesharing. Therefore, VOT was included in the model to increase riders' satisfaction of their arrival time. In this model, the gain in travel cost due to ridesharing depends on how much freedom riders allow in their arrival time. From the results it was shown that a higher VOT leads to higher travel costs, while the filling rate of vehicles decreases. Currently, it is assumed that the improved service quality outweighs the increase in travel cost. However, it is also possible to show that the implementation of VOT can yield travel cost benefits. As a result, we now consider the goal of implementing VOT to be the evasion of greedy riders. Therefore, the next section will focus on the travel decisions that riders take. Hereby, we will assume that riders will ignore the travel schedule if they believe they can receive a greater payoff. This is done by implementing a game theory approach. Implementing a game theory approach has been done by Li et al. for a taxi sharing service (Li et al., 2016). They assumed taking detours to be a non-cooperative game. Riders in a taxi share travel costs and a detour occurs when it results in lower travel costs for participants. Koutsipap et al. studied the effect on travel cost of greedy riders in the network (Koutsoupias and Papadimitriou, 1999) and named the additional cost the price of anarchy. Taking greedy transits based on arrival time has not been studied in research.

4.4 Game theory

4.4.1 Riders' decisions

Once a ridesharing schedule is created, it is possible that a rider is not satisfied with the schedule, even though he reaches his due date in time. Therefore, the gain in travel costs due to ridesharing is not enough for this rider to stick to the schedule and he will deviate from the schedule to achieve a better payoff. This phenomena, the price of anarchy, has a negative impact on the social optimum of the schedule. In upcoming experiments, it is assumed that riders will behave as players in game theory. Therefore, individuals will play to receive the best payoff possible. It is beneficial to create a situation where each player's Nash equilibrium is also Pareto optimum. In this situation, we achieve the highest possible social optimum.

Table 4.5 contains the payoff matrix of rider p and his driver q for one link in the network. All riders in the network have an identical payoff matrix and can decide to either follow the schedule or take a greedy transit. A greedy transit is defined as the decision to deviate from the schedule and drive alone from origin to destination at the most convenient time. If p 's driver q takes a greedy transit, then the decision of rider p has no significance since he has no other option than to drive himself. The travel costs for rider p is the cost of driving himself from origin to destination. This is also true if rider p decides to take a greedy transit himself. If riders p and q both decide to follow the schedule, then their payoff is equal to their transit costs and their VOT costs. Table 4.6 contains the payoff matrix of rider p and his passenger q . The matrix is identical to 4.5, except for when rider p follows the schedule and passenger q takes a greedy transit. In this situation, rider p will lose a passenger for that link. Thus, there is an additional costs, which is indicated by '+ $z_{ij}^p - 1$ '. It could be argued that rider p 's payoff now changes and it could be in his best interest to take a greedy transit. If we include this fact, then the decisions of riders become a dynamic game. This is considered to be outside of the scope of this research and we assume that rider p will travel with one fewer passenger.

		p 's driver q	
		Follow schedule	Take greedy transit
Rider p	Follow schedule	Transit costs + VOT	$ST_p \cdot \alpha_1$
	Take greedy transit	$ST_p \cdot \alpha_1$	$ST_p \cdot \alpha_1$

Table 4.5: Payoff matrix of rider p and his driver q

		p 's passenger q	
		Follow schedule	Take greedy transit
Rider p	Follow schedule	Transit costs + VOT	+ $z_{ij}^p - 1$
	Take greedy transit	$ST_p \cdot \alpha_1$	$ST_p \cdot \alpha_1$

Table 4.6: Payoff matrix of rider p and his passenger q

4.4.2 Greedy transit algorithm

In order to calculate the additional travel cost of greedy transits and the new filling rate, the greedy transit algorithm is created. The pseudo code is depicted in Algorithm 1. This algorithm makes use of the new parameters that are created for the experiment in Table 4.1. For each rider p , Algorithm 1 first determines whether rider p decides to take a greedy transit. Rider p will do this when his current transit costs added to his VOT-costs are larger than the driving costs himself to his final destination. If this is not true for all riders p , then the schedule will remain unchanged. If rider p takes a greedy transit, there is additional travel cost, because he will not share travel cost with others. If rider p does not take a greedy transit, then algorithm decides whether p 's driver q wants to take a greedy transit. If this is true, then rider p is forced to travel himself.

In the algorithm, it is important to determine the transit costs for rider p . These are the travel costs for each link ij travelled by rider p divided by the number of people travelling with rider p on link ij . This is calculated with equation:

$$\sum_{i,j \in \mathcal{N}} \left(\frac{x_{ij}^p \cdot t_{ij}}{z_{ij}^p} + \sum_{q \in \mathcal{P}} \frac{y_{ij}^{pq} \cdot t_{ij}}{z_{ij}^q} \right). \quad (33)$$

This is only calculated when $z_{ij}^p > 0$ and $z_{ij}^q > 0$. The VOT-costs for positive VOT-values are calculated by part b of the objective function (1bb) and is calculated by:

$$\sum_{i,d_p \in \mathcal{N}} \alpha_2^p \left((E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}) + \sum_{q \in \mathcal{P}} (G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}) - (e_p + ST_p) \right). \quad (34)$$

The VOT-costs for negative VOT-values are calculated by part b of the objective function (1bbb) and is calculated by equation:

$$\sum_{i,d_p \in \mathcal{N}} \alpha_2^p \left(l_p - (E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}) + \sum_{q \in \mathcal{P}} (G_{id_p}^{pq} + y_{id_p}^{pq} \cdot t_{id_p}) \right). \quad (35)$$

Equation (33) and either (34) or (35) calculate the cost of rider p for the created schedule. The cost for deviating from the schedule is $ST_p \cdot \alpha_1$. There will be no VOT-costs since rider p will arrive at his most convenient time. Every rider p will decide to take a greedy transit if his transit costs and VOT-costs combined will be greater than the cost of driving alone. In order to store the additional cost of greedy transits, three new parameters are created: $EC1_p$, $EC2_p$ and $EC3_q$. The added greedy transit cost of rider p consists of the cost of driving yourself minus the transit costs and is calculated by:

$$EC1_p = ST_p \cdot \alpha_1 - \sum_{i,j \in \mathcal{N}} \left(\frac{x_{ij}^p \cdot t_{ij}}{z_{ij}^p} + \sum_{q \in \mathcal{P}} \frac{y_{ij}^{pq} \cdot t_{ij}}{z_{ij}^q} \right), \forall p \in \mathcal{P}. \quad (36)$$

To calculate the new filling rate, the parameters $VC1_p$, $VC2_p$ and $VC3_p$ are created. In addition, the new parameter L_p contains the number of links that is in the shortest path from o_p to d_p , for each rider p . If rider p takes a greedy transit, then he will travel on fewer links or the same amount of links as in the original schedule (as ridesharing can lead to detours). The number of fewer links in the greedy transit for rider p is calculated by:

$$VC1_p = \sum_{i,j \in \mathcal{N}} (x_{ij}^p + \sum_{q \in \mathcal{P}} y_{ij}^{pq}) - L_p, \forall p \in \mathcal{P}. \quad (37)$$

If rider p takes no greedy transit, then this value will be zero. When riders are taking greedy transits, then there will be more drivers in the network. The number of extra links driven is calculated by:

$$VC2_p = L_p - \sum_{i,j \in \mathcal{N}} x_{ij}^p, \forall p \in \mathcal{P}. \quad (38)$$

This concludes the first part of Algorithm 1. However, rider p can also be affected by greedy transits of others. The second part of the algorithm regards the riders that lose their driver. The extra costs rider p has by being forced to drive himself is calculated by an identical equation as a greedy transit and is calculated by:

$$EC2_p = ST_p \cdot \alpha_1 - \sum_{i,j \in \mathcal{N}} \left(\frac{x_{ij}^p \cdot t_{ij}}{z_{ij}^p} + \sum_{q \in \mathcal{P}} \frac{y_{ij}^{pq} \cdot t_{ij}}{z_{ij}^q} \right), \forall p \in \mathcal{P}. \quad (39)$$

Regarding the filling rate, if p 's driver q takes a greedy transit and rider p does not ($VC1_p = 0$), then equation (37) and (38) are calculated again.

The last part of the algorithm regards drivers losing passengers due to a greedy transits of passengers. It is assumed that driver p will follow the schedule with one or more fewer passengers. In this situation, when passenger q takes a greedy transit, then driver p will have extra transit costs due to the lower filling rate. This is calculated by:

$$EC3_q = \sum_{p \in \mathcal{P}} \frac{y_{ij}^{pq} \cdot t_{ij}}{z_{ij}^p}, \forall q \in \mathcal{P}. \quad (40)$$

If p 's passenger q takes a greedy transit on each link ij and rider p is not ($VC1_p = 0$), then the number of passengers he loses on all his links as a driver is calculated by:

$$VC3_p = VC3_p + 1, \forall p \in \mathcal{P}. \quad (41)$$

To calculate the entire greedy travel cost, the additional transit costs of $EC1_p$, $EC2_p$ and $EC3_q$ are summed and added to the travel cost of the original schedule, which is calculated by part a of objective function (1bb) and (1bbb). This is calculated by:

$$\text{Greedy travel cost} = \sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{N}} (x_{ij}^p \cdot t_{ij} + EC1_p + EC2_p + \sum_{q \in \mathcal{P}} EC3_q). \quad (42)$$

The new greedy filling rate of vehicles is calculated by the total total of links travelled by all riders divided the total number links driven by all riders. This is calculated by:

$$\text{Greedy Filling rate} = \frac{\sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{N}} \left(x_{ij}^p + \sum_{q \in \mathcal{P}} y_{ij}^{pq} - VC1_p - VC3_p \right)}{\sum_{p \in \mathcal{P}} \sum_{i,j \in \mathcal{N}} \left(x_{ij}^p + VC2_p \right)}. \quad (43)$$

Algorithm 1: Greedy transit algorithm

Input: VOT and schedule**Output:** Transit costs for rider p

initialization;

if *transit costs + VOT cost > Cost of driving yourself* **then**| rider p drives himself;**else**| **if** *driver q takes a greedy transit* **then**| | rider p becomes a driver;| **else**| | **if** *passenger q takes a greedy transit* **then**| | | driver p loses a passenger;| | **end**| **end**

| No change in transit costs;

end

4.5 Experiment with greedy riders

4.5.1 Experiment setup

In this experiment, the influence of VOT will be tested with a network, where riders behave as players in game theory. Once the travel schedule is presented to all riders, each players will decide to either follow the schedule or to take a greedy transit. Four outcomes are of interest; the travel cost, the number of greedy transit, the filling rate and the VOT cost will be determined. The VOT cost is the sum of the VOT cost of all players. For example, if a rider has an α_2 -value of 2 and he arrives ten time steps from his desired arrival time, then the VOT cost of this rider is 20. In the experiments, the VOT cost will be corrected based on the greedy transits. If a rider takes a greedy transit, then his VOT cost will become 0.

The parameters of the greedy rider experiments are listed in Table 4.7. It is chosen not to include mean α_2^p -values higher than 5 or lower than -5, since little change in schedule was observed with more extreme values in the situation where VOT was not included in the objective function. In the upcoming experiment, we will compare schedules with multiple settings for a set of riders using objective function (1bb) for positive VOT-values and objective function (1bbb) for negative VOT-values. A range of mean VOT-values will be tested. The settings are as follows.

1. The first setting is a schedule without ridesharing. This will be the upper bound for travel costs and a lower bound for filling rate.
2. The second setting is a ridesharing network where VOT is not included and it is assumed that every riders will follow the schedule, which is called the sheep network. This will create a lower bound for travel costs and an upper bound for filling rate.
3. In the upcoming settings, a mean VOT is introduced. A ridesharing schedule is created. However, riders that can receive a greater payoff will deviate from this schedule. The total travel costs, including the greedy transit costs, will be calculated.

4. In this condition, VOT-costs are included in the objective function, as was done in the experiments of Section 4.2. The total travel costs, including the greedy transit costs, will be calculated.

Parameter	Value
$p \in \mathcal{P}$	$\{1, \dots, \mathcal{P} \}$
\mathcal{O}	$\{\mathcal{P}(n) : n \in \{5, 6, \dots, 15\}\}$
α_1	1
α_2^p	$\alpha_2^p \sim \mathcal{N}(\mu, \sigma^2), 0$
μ	$\mu = \{-5, \dots, 5\}, \mu \neq 0$
σ^2	$\sigma^2 = 1$
o_p	$o_p \sim U(1, 3)$
d_p	$d_p \sim U(4, 7)$
e_p	$e_p \sim U(0, 15)$
l_p	$l_p \sim U(55, 70)$
v_p	4

Table 4.7: Parameter settings for experiment greedy riders

4.5.2 Results experiment greedy riders

The results of all the experiments are presented in Appendix D. A selection of the results are displayed in this section. The experiments with negative VOT-values are not displayed in this section, but yielded very similar results to their positive counterpart. Table 4.8 and Figure 4.7 contain the results for $\mu = 1$. It is evident that the implementation of VOT in the objective function yields significant travel cost benefits. The greedy transits of riders cause great additional cost. It must be noted that the implementation of VOT in the objective function did not have a positive effect in every situation. In some cases, the VOT caused a change of schedule, while no rider had the incentive to take a greedy transit. On average, the travel cost per rider decreased from 16.93 to 11.89. In terms of VOT costs, the implementation of VOT had a large benefit for all situations. This greatly improves the service quality of riders in terms of arrival time. On average, the VOT costs per rider decreased from 14.64 to 4.86.

	No VOT		$\mu = 1$, VOT not in obj. function			$\mu = 1$, VOT in obj. function		
Number of riders	Travel cost without ridesharing	Sheep network	Travel cost	Number of greedy transits	Corrected VOT costs	Travel cost	Number of greedy transits	Corrected VOT costs
5	177	54	54	0	132	54	0	29
6	202	80	80	0	175	80	0	56
7	258	90	228	2	17	118	0	42
8	258	84	202	1	35	84	0	25
9	291	98	236	3	14	112	0	45
10	326	92	120	1	104	113	0	74
11	393	123	123	0	117	126	0	15
12	408	119	119	0	230	134	0	52
13	458	120	270	2	80	156	0	59
14	507	113	113	0	127	150	0	38
15	524	133	241	1	182	154	0	58

Table 4.8: Travel cost and service quality in terms of VOT costs using $\mu = 1$

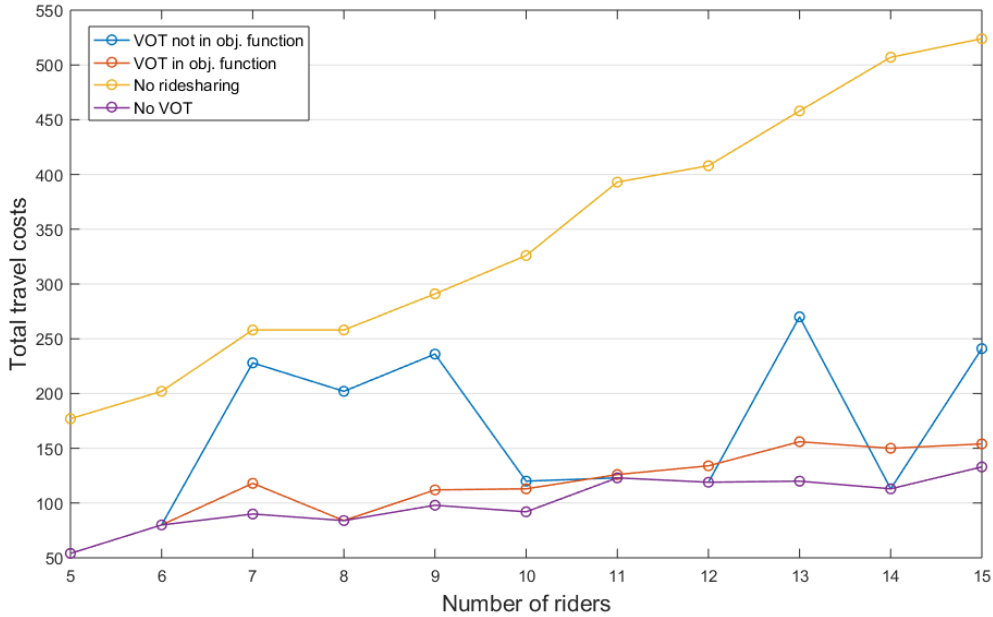


Figure 4.7: Travel cost for $\mu = 1$, comparing to sheep network and no ridesharing

Table 4.9 and Figure 4.8 contain the results for $\mu = 3$. For these settings, including VOT in the objective function yields travel cost benefits for every number of riders. Implementing VOT in the objective function did not prevent greedy transits from occurring, but they did occur less often. On average, the travel cost per rider decreased from 30.70 to 19.27 by implementing VOT. In terms of VOT costs, the large number of greedy transits caused many riders to drive alone. Therefore, the VOT costs are 0 for those drivers. On average, the VOT costs per rider increased from 3.91 to 5.58.

	No VOT		$\mu = 3$, VOT not in obj. function			$\mu = 3$, VOT in obj. function		
Number of riders	Travel cost without ridesharing	Sheep network	Travel cost	Number of greedy transits	Corrected VOT costs	Travel cost	Number of greedy transits	Corrected VOT costs
5	177	54	177	2	0	107	0	16
6	202	80	173	3	46	147	0	17
7	258	90	217	4	27	146	0	80
8	258	84	258	3	0	225	2	28
9	291	98	291	7	0	239	2	39
10	326	92	326	4	0	145	0	76
11	393	123	308	4	64	126	0	73
12	408	119	215	4	224	188	1	59
13	458	120	396	3	73	240	1	79
14	507	113	504	9	2	206	0	71
15	524	133	493	5	21	236	0	88

Table 4.9: Travel cost and service quality in terms of VOT costs using $\mu = 3$

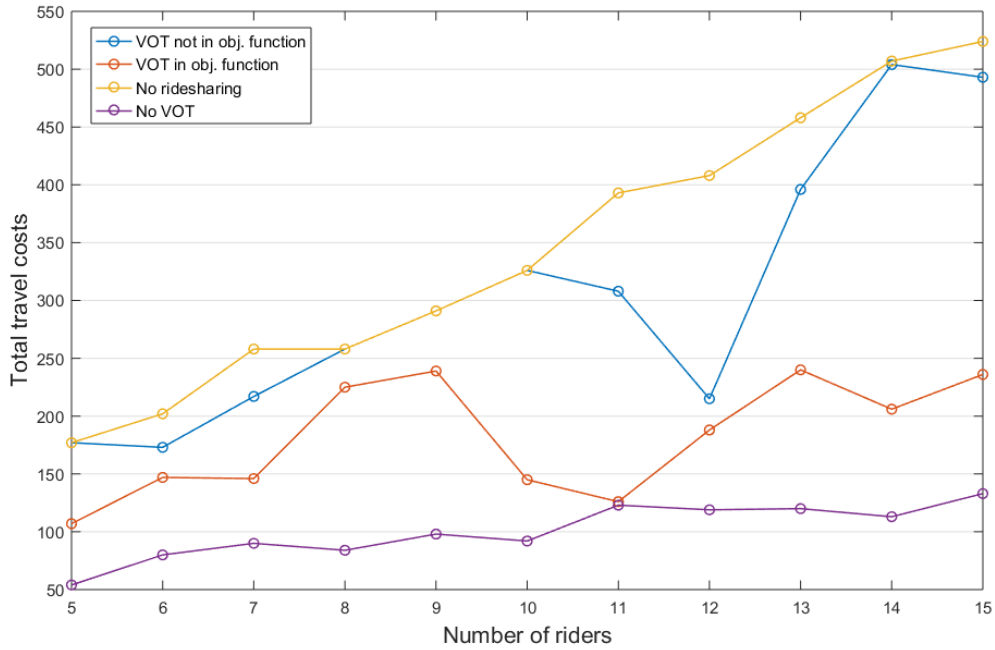


Figure 4.8: Travel cost for $\mu = 3$, comparing to sheep network and no ridesharing

Table 4.10 and Figure 4.9 contain the results for $\mu = 5$. It is evident that the setting, where VOT is not included in the objective function, is very similar to a situation without ridesharing. Most riders choose to take a greedy transit. On average, the travel cost per rider decreased from 34.04 to 21.38 by implementing VOT, which is a decrease of 37.2%. In terms of VOT costs, the same conclusion holds compared to $\mu = 3$. Due to the many greedy transits, the VOT costs are 0 for many riders. On average, the VOT costs per rider increased from 1.00 to 5.89.

	No VOT		$\mu = 5$, VOT not in obj. function			$\mu = 5$, VOT in obj. function		
Number of riders	Travel cost without ridesharing	Sheep network	Travel cost	Number of greedy transits	Corrected VOT costs	Travel cost	Number of greedy transits	Corrected VOT costs
5	177	54	177	4	0	107	0	19
6	202	80	173	3	66	147	0	27
7	258	90	258	6	0	215	1	77
8	258	84	258	5	0	229	0	65
9	291	98	291	9	0	207	0	53
10	326	92	326	8	0	172	0	55
11	393	123	396	9	0	180	1	76
12	408	119	408	9	0	228	0	46
13	458	120	458	11	0	275	0	36
14	507	113	504	10	4	258	0	63
15	524	133	524	9	0	222	0	122

Table 4.10: Travel cost and service quality in terms of VOT costs using $\mu = 5$

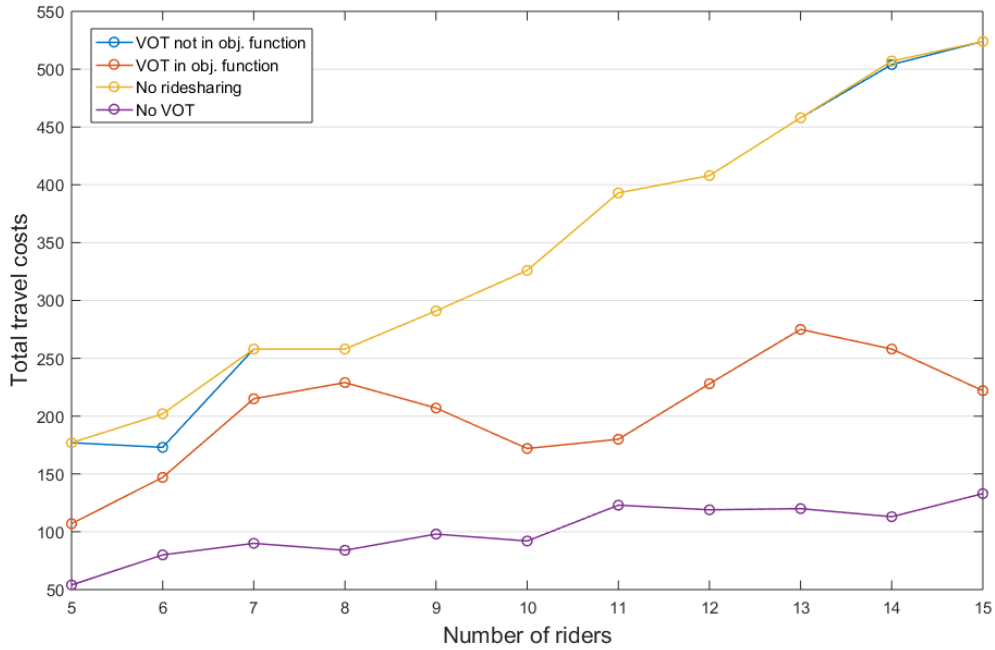


Figure 4.9: Travel cost for $\mu = 5$, comparing to sheep network and no ridesharing

Table 4.11 contains the filling rate of $\mu = 1$, $\mu = 3$ and $\mu = 5$. Figure 4.10 and 4.11 give a graphical representation of the filling rate for $\mu = 1$ and $\mu = 5$, respectively. There is a negative correlation between travel cost and filling rate and this is clearly evident from the results. For $\mu = 1$, the average filling rate increased from 2.33 to 2.52. For $\mu = 3$, the average filling rate increased from 1.19 to 1.77 and for $\mu = 5$, it is increased from 1.02 to 1.51.

Number of riders	No VOT		VOT not in obj. function			VOT in obj. function		
	Filling rate without ridesharing	Sheep network	$\mu = 1$	$\mu = 3$	$\mu = 5$	$\mu = 1$	$\mu = 3$	$\mu = 5$
5	1.00	3.20	3.20	1.00	1.00	3.20	1.86	1.86
6	1.00	2.17	2.17	1.18	1.18	2.17	1.30	1.30
7	1.00	2.88	1.24	1.24	1.00	2.10	1.75	1.19
8	1.00	2.86	1.29	1.00	1.00	2.86	1.13	1.13
9	1.00	3.00	1.41	1.00	1.00	2.67	1.18	1.33
10	1.00	3.50	2.80	1.00	1.00	2.56	2.18	1.77
11	1.00	3.00	3.00	1.21	1.00	2.20	2.20	1.16
12	1.00	3.50	3.50	2.19	1.00	2.67	1.93	1.67
13	1.00	2.89	1.39	1.15	1.00	2.09	1.53	1.33
14	1.00	3.63	3.63	1.00	1.00	2.69	2.25	1.71
15	1.00	3.54	1.96	1.11	1.00	2.51	2.20	2.11

Table 4.11: Average filling rate of vehicles for different mean VOT-values

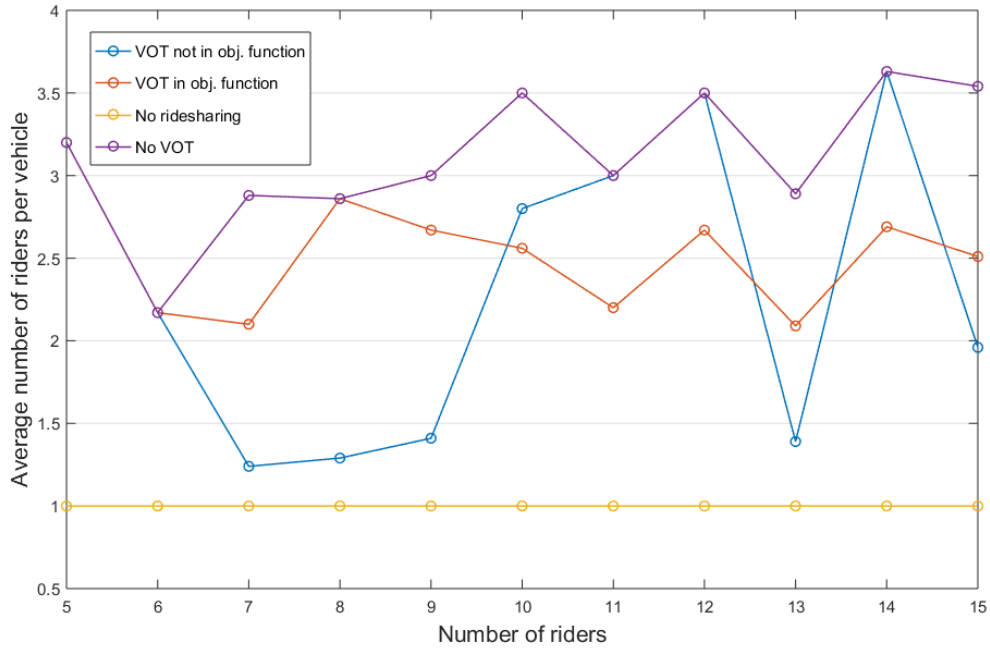


Figure 4.10: Filling rate for $\mu = 1$, comparing to sheep network and no ridesharing

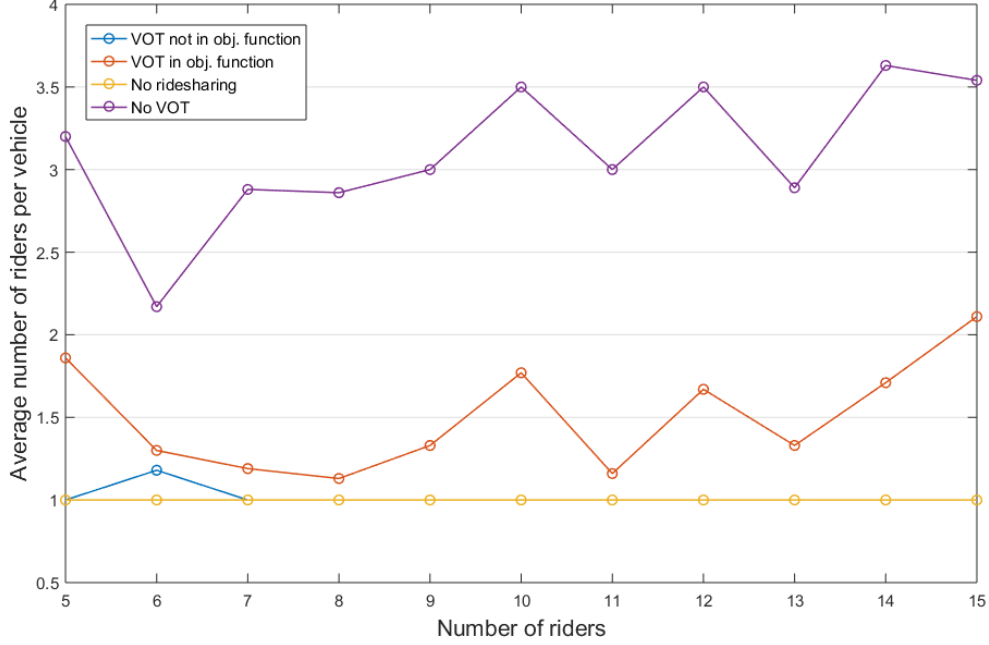


Figure 4.11: Filling rate for $\mu = 5$, comparing to sheep network and no ridesharing

4.6 Truck transport experiments

In this section, the ridesharing model is transformed to a truck transport model. This model shows many similarities to the Dial-A-Ride class, where passengers are matched with vehicles. In this situation, trucks are matched to goods. An assumption is made where all goods have an identical fixed size. In addition, goods can be transferred to different vehicles without additional cost or time.

4.6.1 Model adjustments

A subset $\mathcal{T} \in \{1, \dots, T\}$ is created, where $\mathcal{T} \subseteq \mathcal{P}$. This subset \mathcal{T} contains the number of trucks that are used to transport the goods. The subset $\mathcal{G} = \mathcal{P} - \mathcal{T}$ contains the number of goods that have to be transported. Subsequently, the mathematical model has to be adjusted in order to create a Dial-A-Ride model. It is assumed that every truck driver will follow the schedule without protest and therefore, pre-processing is removed. The truck drivers have an origin, but no restriction on a destination.

The ridesharing model is adjusted to a truck transport model by constraining the solution in the following manner. In this network trucks are free to travel, but the travel restrictions still apply for the goods. Therefore, goods cannot travel towards their origin, can only travel towards their destination once and do not leave their destination, which is ensured by constraints:

$$x_{i o_p}^p + y_{i o_p}^{pq} = 0, \forall i, o_p \in \mathcal{N}; \forall p \in \mathcal{G}; \forall q \in \mathcal{P}, \quad (3bb)$$

$$\sum_{i \in \mathcal{N}} (x_{id_p}^p + \sum_{q \in \mathcal{P}} y_{id_p}^{pq}) = 1, \forall p \in \mathcal{G}; \forall d_p \in \mathcal{N}, \quad (4bb)$$

$$x_{d_pi}^p + y_{d_pi}^{pq} = 0, \forall i, d_p \in \mathcal{N}; \forall q \in \mathcal{P}; \forall p \in \mathcal{G}. \quad (5bb)$$

Trucks do not have a due date and that they do not have to arrive at a destination. Only goods have these constraints, which is ensured by:

$$\sum_{i \in \mathcal{N}} (E_{id_p}^p + x_{id_p}^p \cdot t_{id_p}) \leq l_p, \forall d_p \in \mathcal{N}; \forall p \in \mathcal{G}. \quad (15b)$$

Trucks are not able to rideshare, which is enforced by:

$$y_{ij}^{pq} = 0, \forall i, j \in \mathcal{N}; \forall q \in \mathcal{P}; \forall p \in \mathcal{T}. \quad (44)$$

Moreover, goods do not have the ability to drive, which is enforced by:

$$x_{ij}^p = 0, \forall i, j \in \mathcal{N}; \forall p \in \mathcal{G}. \quad (45)$$

4.6.2 Experimental setup

For the truck experiments, a new network is created with three depots ($i = 1, 2, 3$) where four trucks $\mathcal{T} \in \{1, \dots, 4\}$ are located and a variable amount of goods. The new network is visible in Figure 4.12.

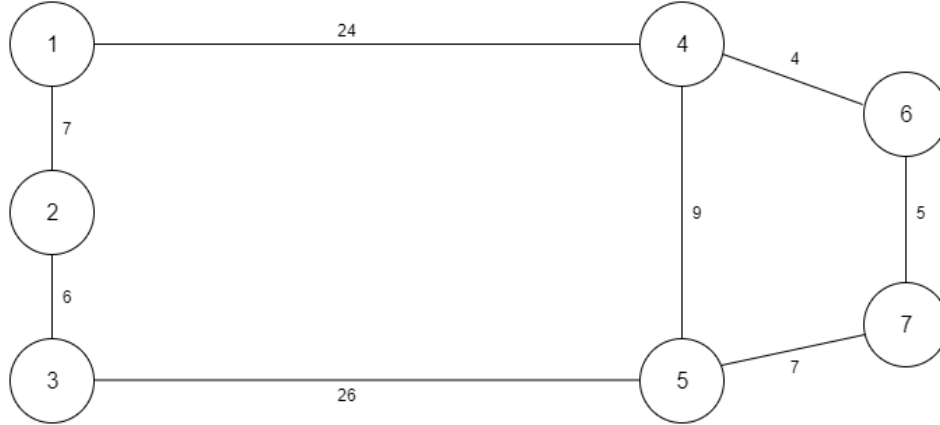


Figure 4.12: Experimental road network for trucks with a ride-sharing hub

The parameters of the greedy rider experiments are listed in Table 4.12. The number of riders has been increased to 23 due to the lower computational difficulty of the new model. This is a result of trucks not being to rideshare and goods not being able to drive. It is chosen not to include mean α_2^p -values higher than 5 or lower than -5, since little change in schedule was observed with more extreme values. In the forthcoming experiment, we will compare schedules with multiple settings for a set of riders using objective function (1bb) for positive VOT-values and objective function (1bbb) for negative VOT-values. A range of mean VOT-values will be tested. The settings are as follows.

1. The first setting is a ridesharing network where VOT is not included in the objective function. This will create a lower bound for travel costs and an upper bound for filling rate.
2. In the upcoming settings, a mean VOT is introduced. A ridesharing schedule is created and the travel costs, filling rate and VOT costs are calculated.
3. In this condition, VOT-costs are included in the objective function, as was done in the experiments of Section 4.2. The total travel costs, filling rate and VOT costs will be calculated.
4. All settings above are used for a network which allows multi-hopping and one that does not allow multi-hopping by using constraint (32). It must be stated that single-hopping has a different meaning in this context. Since goods cannot drive themselves, single-hopping means that no ridesharing occurs, while multi-hopping allows for switching of vehicles.

Parameter	Value
$p \in \mathcal{P}$	$\{1, \dots, \mathcal{P} \}$
\mathcal{O}	$\{\mathcal{P}(n) : n \in \{9, 10, \dots, 23\}\}$
α_1	1
α_2^p	$\alpha_2^p \sim \mathcal{N}(\mu, \sigma^2), 0$
μ	$\mu = \{-5, \dots, 5\}, \mu \neq 0$
σ^2	$\sigma^2 = 1$
o_p	$o_p \sim U(1, 3)$
d_p	$d_p \sim U(4, 7)$
e_p	$e_p \sim U(0, 15)$
l_p	$l_p \sim U(55, 70)$
v_p	8

Table 4.12: Parameter settings for experiment truck transport

4.6.3 Results truck experiment

Appendix E contains all the results with multi-hopping allowed and Appendix F contains all the results where switching of vehicles is not allowed. Hereafter, a selection of the results will be explained and graphed. It is chosen not to include negative VOT-values in this selection, since the results are very similar to their positive counterpart.

Multi-hopping allowed, $\mu = 1$

Table 4.13 and Figure 4.13 contain the results for $\mu = 1$ and a reference state where no VOT is involved. Compared to the reference state, the average travel costs increased from 7.08 per good to 8.59 per good. The filling rate of vehicles decreased as the travel costs increased. The inclusion of VOT in the objective function did have a significant effect on the VOT costs of goods. On average, this decreased from 13.66 per good to 4.71 per good. Therefore, it can be concluded that there is significantly better service quality for a relatively small increase of travel costs.

Single-hopping, $\mu = 1$

Average travel costs increased from 8.30 to 9.35 per good by including VOT. Similarly to the previous experiments, the average filling rate slightly decreased. For this setting, the inclusion of VOT in the objective function had a smaller benefit on the VOT costs of goods. On average, this decreased from 11.05 per good to 9.35 per good. Therefore, it can be concluded that the VOT cost benefit exceeds the additional travel costs. Nevertheless, the cost benefit has declined significantly by not allowing switching of vehicles.

The constraint that ensures single-hopping, has a significant effect on the travel costs. In the setting where no VOT is introduced, the average travel cost per good is 7.08. This increased to 9.35 when switching between vehicles was not allowed, which is an increase of 32%. When a VOT-value was introduced with $\mu = 1$, the average travel cost per good increased from 8.59 to 9.35 when switching of vehicles was not allowed. This is an increase of 9%. Thus, there is a significant cost benefit by allowing multi-hopping.

Number of riders	Multi-hopping						Single-hopping					
	VOT not in obj. function			VOT in obj. function			VOT not in obj. function			VOT in obj. function		
	Travel cost	Filling rate	VOT cost $\mu = 1$	Travel cost	Filling rate	VOT cost $\mu = 1$	Travel cost	Filling rate	VOT cost $\mu = 1$	Travel cost	Filling rate	VOT cost $\mu = 1$
9	46	4.40	40	57	3.00	27	80	2.67	127	72	3.00	23
11	83	3.13	180	84	3.00	41	89	3.25	163	96	3.11	63
13	70	5.00	143	130	2.62	31	79	5.14	100	85	4.00	43
15	71	7.17	203	73	5.83	45	88	4.28	199	92	4.00	88
17	85	5.43	160	83	4.38	62	120	3.80	145	126	4.25	67
19	89	5.78	187	131	4.25	52	98	5.00	186	123	4.09	63
21	115	5.60	191	128	4.33	111	121	4.30	292	174	3.59	90
23	121	5.43	207	139	4.22	83	122	5.09	258	130	4.54	70

Table 4.13: Comparing travel costs, filling rate and VOT costs for multi-hopping and single-hopping using $\mu = 1$

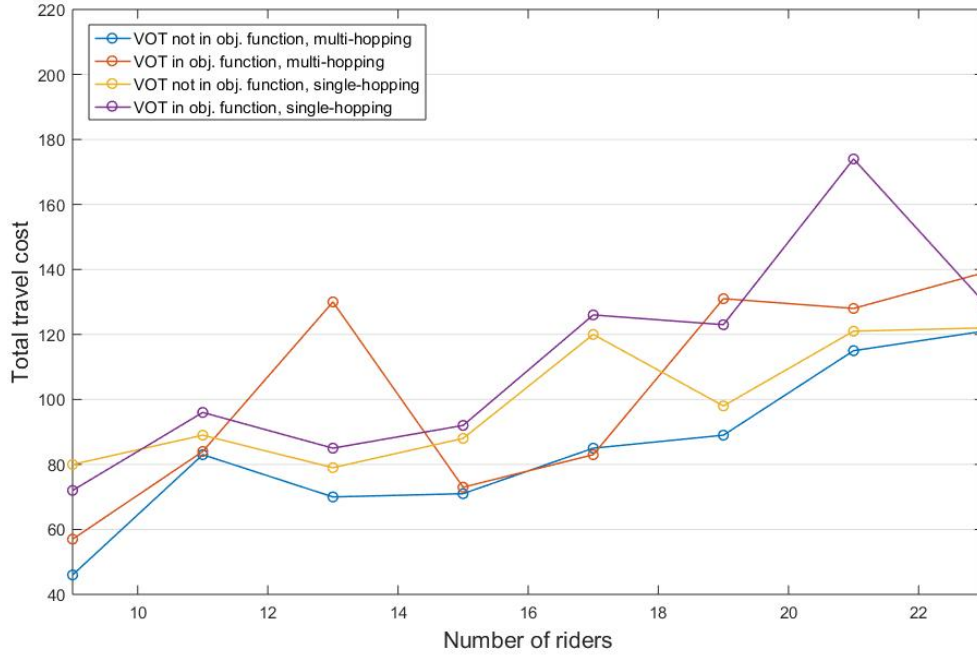


Figure 4.13: Comparing travel costs of a network for multi-hopping and single-hopping using $\mu = 1$

Multi-hopping allowed, $\mu = 3$

Table 4.14 and Figure 4.14 contain the results for $\mu = 3$ and a reference state where no VOT is involved. Compared to the reference state, the average travel costs increased from 7.08 per good to 12.44 per good. The filling rate of vehicles decreased as the travel costs increased. The inclusion of VOT in the objective function did have a significant effect on the VOT costs of goods. On average, this decreased from 33.19 per good to 7.72 per good. Therefore, it can be concluded that there is significantly better service quality for a relatively small increase of travel costs.

Single-hopping, $\mu = 3$

Average travel costs increased from 8.30 to 12.80 per good by including VOT. Similarly to the previous experiments, the average filling rate slightly decreased. For this setting, the inclusion of VOT in the objective function had a smaller benefit on the VOT costs of goods. On average, this decreased from 27.91 per good to 9.10 per good. Therefore, it can be concluded that the VOT cost benefit exceeds the additional travel costs. Nevertheless, the cost benefit has declined significantly by not allowing switching of vehicles.

The constraint that ensures switching of vehicles is not allowed, has a smaller impact on the travel costs when VOT is included in the objective function. When a VOT-value was introduced with $\mu = 3$, the average travel cost per good increased from 12.44 to 12.80 when switching of vehicles was not allowed. This is an increase of 2.9%.

Number of riders	Multi-hopping						Single-hopping					
	VOT not in obj. function			VOT in obj. function			VOT not in obj. function			VOT in obj. function		
	Travel cost	Filling rate	VOT cost $\mu = 3$	Travel cost	Filling rate	VOT cost $\mu = 3$	Travel cost	Filling rate	VOT cost $\mu = 3$	Travel cost	Filling rate	VOT cost $\mu = 3$
9	46	4.40	164	119	2.00	20	80	2.67	80	110	2.25	34
11	83	3.13	356	117	2.56	33	89	3.25	73	131	2.50	66
13	70	5.00	251	165	2.40	74	79	5.14	365	127	2.84	43
15	71	7.17	495	121	3.27	79	88	4.28	277	181	2.44	115
17	85	5.43	503	172	2.59	71	120	3.80	391	165	3.20	130
19	89	5.78	437	157	3.25	160	98	5.00	536	170	3.06	118
21	115	5.60	469	179	3.63	168	121	4.30	434	174	3.59	191
23	121	5.43	511	164	4.03	136	122	5.09	523	171	3.75	177

Table 4.14: Comparing travel costs, filling rate and VOT costs for multi-hopping and single-hopping using $\mu = 3$

Figure 4.14: Comparing travel costs of a network for multi-hopping and single-hopping using $\mu = 3$

Multi-hopping allowed, $\mu = 5$

Table 4.15 and Figure 4.15 contain the results for $\mu = 5$ and a reference state where no VOT is involved. Compared to the reference state, the average travel costs increased from 7.08 per good to 13.09 per good. The filling rate of vehicles decreased as the travel costs increased. The inclusion of VOT in the objective function did have a significant effect on the VOT costs of goods. On average, this decreased from 55.00 per good to 14.58 per good. Therefore, it can be concluded that there is significantly better service quality for a relatively small increase of travel costs.

Single-hopping, $\mu = 5$

Average travel costs increased from 8.30 to 13.57 per good by including VOT. Similarly to the previous experiments, the average filling rate slightly decreased. For this setting, the inclusion of VOT in the objective function had a smaller benefit on the VOT costs of goods. On average, this decreased from 47.07 per good to 14.13 per good. Therefore, it can be concluded that the VOT cost benefit exceeds the additional travel costs. Nevertheless, the cost benefit has declined significantly by not allowing switching of vehicles.

The constraint that ensures switching of vehicles is not allowed, has a smaller impact on the travel costs when VOT is included in the objective function. When a VOT-value was introduced with $\mu = 5$, the average travel cost per good increased from 13.09 to 13.57 when switching of vehicles was not allowed. This is an increase of 3.7%.

Number of riders	Multi-hopping						Single-hopping					
	VOT not in obj. function			VOT in obj. function			VOT not in obj. function			VOT in obj. function		
	Travel cost	Filling rate	VOT cost $\mu = 5$	Travel cost	Filling rate	VOT cost $\mu = 5$	Travel cost	Filling rate	VOT cost $\mu = 5$	Travel cost	Filling rate	VOT cost $\mu = 5$
9	46	4.40	404	119	2.00	29	80	2.67	112	110	2.25	44
11	83	3.13	303	124	2.27	51	89	3.25	154	170	2.06	53
13	70	5.00	618	172	2.35	100	79	5.14	645	168	2.5	52
15	71	7.17	461	159	2.71	81	88	4.28	451	181	2.44	150
17	85	5.43	703	172	2.59	127	120	3.80	648	166	3.38	209
19	89	5.78	869	157	3.25	290	98	5.00	974	163	3.20	235
21	115	5.60	907	179	3.63	406	121	4.30	633	174	3.59	321
23	121	5.43	863	175	3.56	316	122	5.09	902	171	3.75	292

Table 4.15: Comparing travel costs, filling rate and VOT costs for multi-hopping and single-hopping using $\mu = 5$

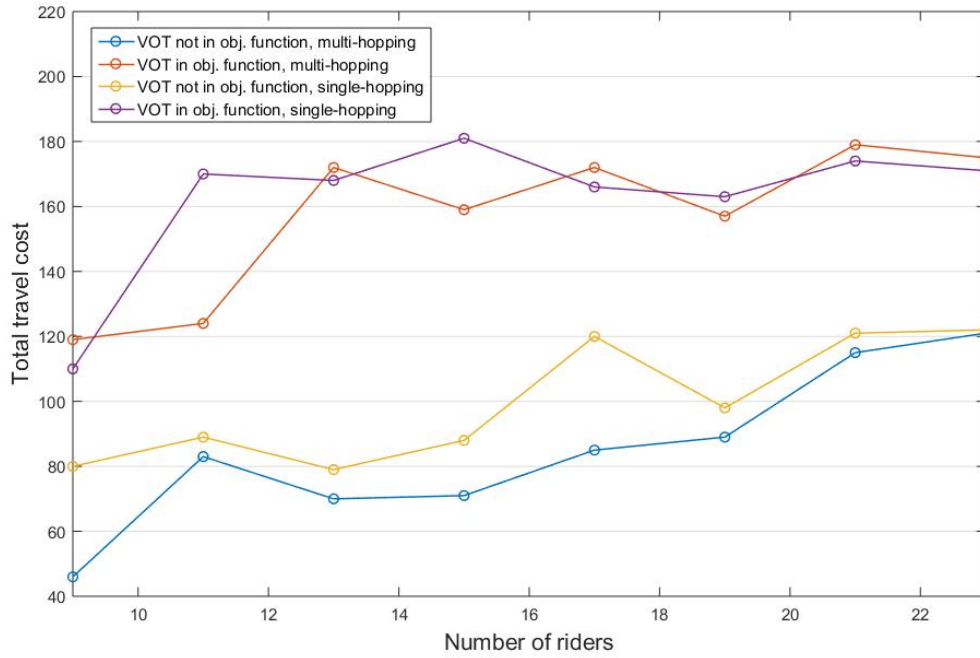


Figure 4.15: Comparing travel costs of a network for multi-hopping and single-hopping using $\mu = 5$

5 | Concluding remarks

In this chapter, a conclusion to the research will be given. In section 5.1, future directions of research will be explored. Future research consists of exploring assumptions of this research and elements of ridesharing that were not investigated in this research. Section 5.2 contains the conclusions of this research, answering the research questions and discussing whether the goal is reached.

5.1 Areas for further research

In this research, a mathematical model is created that functions as a tool for future research. Due to the ease of adding features or constraints, new elements of ridesharing can be studied. This research focused on adding game theory elements to ridesharing, which can be further explored. A topic for future research is to include Subgame Perfect Equilibria, since the decisions of riders could be viewed as a dynamic game. This would create a more precise approximation of the costs for greedy transits. In this research, it is assumed that drivers will travel with one fewer passenger if that passenger has decided to take a greedy transit. However, driving alone could yield a better payoff for this driver in order to avoid high VOT costs. A strategy profile could be created for each rider in the network. Another topic of game theory could be included. The Stackelberg leadership model is a strategic game in economics in which the leader moves first and then the followers move sequentially. This creates a different strategy profile. After the leader announces his strategy, the optimization model could create a new schedule given this information. Subsequently, the next leader announces his strategy. However, in this research it is assumed that players make their decision at the same time.

As mentioned, much research focuses on either increasing the service quality of riders or creating a model that strives to model real-life. Waiting time of passengers, cost of switching to different vehicles and the possibility of failing links have not been incorporated. This research has shown the service quality benefit of switching vehicles in goods transportation. Cost of switching to different vehicles is an important addition to verify whether switching vehicles is worth the effort. In goods transport, switching of vehicles typically does not occur due to the increase in handling. Implementing cost of switching vehicle could verify this. The maximum allowable detours have been included in the form of pre-processing. Riders choose their shortest or second shortest path. This assumption on the behavior of riders could be further explored. In addition, it is assumed that the travel time for links is static. They could be stochastic by nature or it could be a function of the number of people travelling on that link. When travel time depends on the number of people travelling on that link, then a switch towards nonlinear optimization has to be made. Typically, nonlinear optimization problems are much harder to solve. Adding a nonlinear constraint in this research does allow for a better solution. By adding a constraint that every rider's travel cost plus its VOT-cost are lower than the cost for travelling alone, no rider has incentive to take a greedy transit. By ensuring no greedy transits, a better global optimum is found. In this research, it is assumed

that riders' VOT was either positive for all riders or negative for all riders. The distribution of riders' VOT-values could be further explored by having both positive and negative values and adjusting both the mean and variance of the distribution.

Due to the difficulty of solving ridesharing problems, this research has focused on linear optimization. Numerous assumptions has been made to decrease computation time, since solving complex MILP are computationally very heavy. There is currently no solution for this problem and thus, computational efficiency is an important topic for future research. Making assumptions on real-life improves computation time, but are less correct. Currently, is is necessary to use a balance between both. Switching to a (partly) static model decreases computation time. Existing research already constrains the solution to an existing schedule.

5.2 Discussion

Much research on ridesharing focuses on of two goals; either a ridesharing model's accuracy is improved with regards to real-life or the service quality to participants is improved. This research aims to improve both goals. A game theory methodology was introduced in order to measure the effect of VOT in practice. This is done by means of the greedy transit algorithm. Once the ridesharing schedule is created, each rider will have information on its transit costs and its arrival time. This rider will have a cost in mind for not arriving at his convenient time. Subsequently, he will decide whether deviating from the schedule gives him a greater payoff. An optimization model is created to create a transport schedule and generate the data. A commuters network has been tested on computation time for different solvers. An acceptable solving time was found for the commercial solver Xpress-IVE. After the schedule is created, greedy transit algorithm adjusts the schedule based on greedy transits and calculates the new travel costs and filling rate. The generated data shows that the greedy transits of riders can have harmful consequences on the global travel costs of the system. To counter the excessive travel costs of greedy transits, a VOT cost penalty is introduced in the objective function. As a result, the number of greedy transits has decreased significantly. On average, this yielded substantial travel cost improvements and increased the average filling rate of vehicles. Nonetheless, not for all situations did VOT decrease travel cost. For a distinct minority of situations, no greedy transits occurred, while VOT in the objective function changed the schedule to create a sub-optimal schedule. In addition, the satisfactory level regarding rider's arrival time has grown considerably. This conclusion holds when we assume riders to behave as players in game theory.

The influence of VOT was also tested in a sheep network, where riders do not have the option to take greedy transits. This was performed using a truck transport network, where truck drivers do not have incentive to deviate from the schedule and the customers of the goods do not have this option. In this experiment, we first show that implementing a ridesharing network for trucks yields significant cost benefits. This is done by comparing single-hopping to multi-hopping. Subsequently, the VOT of goods is introduced to increase service quality. It is concluded that multi-hopping leads to lower travel costs and higher filing rate. The addition of VOT in the objective function could increase the travel costs by up to 63%. However, the cost benefits in terms of VOT-costs exceeded the increase of travel costs for all settings considerably. It can be concluded that this data fulfills the goal and answers the main research question.

On a concluding note, when looking at the implementation of ridesharing, it is advised to study the features of the network. Subsequently, it is necessary to decide which features are important to implement in the model. This research has shown that is beneficial to incorporate a ridesharing model for multiple scenarios. Looking at a commuters network, the implementation of VOT decreased travel cost significantly due to the avoidance of greedy transits. Incorporating ridesharing in a truck transport network, multi-hopping has shown to reduce travel costs and the implementation of VOT has a considerable benefit in the service quality.

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Appendices

Appendix A: Optimization data

```

1  %@encoding CP1252
2  model "Optimization ride-sharing"
3  uses "mmxprs","mmjobs","mmive","mmmodbc","advmod"; %gain access to the Xpress-
    Optimizer solver
4
5  declarations
6
7  modSub: Model
8
9
10 P=1..5 %set of persons
11 %SP= 1..80 %set of arcs belonging to the shortest path from origin to
    destination
12 N= 1..8 %set of nodes
13 S= 1..10 %set of numbers for binomial distribution
14
15
16 x: array (P,N,N) of mpvar %decision variable driver's allocation
17 y: array (P,P,N,N) of mpvar %decision variable for passenger's allocation
18 alpha1: integer %travel cost per time unit
19 alpha2: array(P) of real %VOT-penalty per time unit per person
20 t: array(N,N) of real %travel time from i to j
21 D: array (P,N,N) of mpvar %departure time from node i for person p
22 E: array (P,N,N) of mpvar %Is x*D, created in order to make the model linear
23 G: array (P,P,N,N) of mpvar %Is y*D, created in order to make the model linear
24 e: array(P) of integer %earliest departure time for person p
25 l: array(P) of integer %latest arrival time of person p
26 o: array(P) of integer %origin of person p
27 d: array(P) of integer %destination of person p
28 m: array(P) of integer %maximum wait time for person p
29 v: array(P) of integer %vehicle capacity for for person p
30 M: integer %big M
31 n: integer %maximum number of drivers
32 destination: array(N) of string %array that contains all possible destinations
33 z: array (P,N,N) of mpvar %denotes the number of persons travelling with driver
    p on i,j
34 O: array(S,P) of integer
35 SP: array(P) of integer %this is shortest travel time for rider p
36 L: array(P) of integer %this counts the number of links in the shorest path
    for rider p
37 end-declarations
38
39 %

```

```

40 alpha1:=1
41 % Travel time

```

```

42 forall (i,j in N) do
43   if (i=1 and j=2) then
44     t(i,j):=12
45     t(j,i):=12
46   elif (i=1 and j=3) then
47     t(i,j):=8

```

```

48     t(j,i):=8
49
50     elif (i=1 and j=4)then
51         t(i,j):=10
52         t(j,i):=10
53
54     elif (i=2 and j=3)then
55         t(i,j):=7
56         t(j,i):=7
57
58     elif (i=2 and j=5)then
59         t(i,j):=24
60         t(j,i):=24
61
62     elif (i=3 and j=4)then
63         t(i,j):=6
64         t(j,i):=6
65
66
67     elif (i=4 and j=6)then
68         t(i,j):=26
69         t(j,i):=26
70
71     elif (i=5 and j=6)then
72         t(i,j):=9
73         t(j,i):=9
74
75     elif (i=5 and j=7)then
76         t(i,j):=4
77         t(j,i):=4
78
79     elif (i=6 and j=8)then
80         t(i,j):=7
81         t(j,i):=7
82
83     elif (i=7 and j=8)then
84         t(i,j):=5
85         t(j,i):=5
86
87     end-if
88     end-do
89
90     %

```

```

91     %this creates random origin and destination for each person (between node 1 and
92     %10)
93     %it also creates an earliest departure time and due time for person p, vehicle
94     %capacity and maximum wait time for person p
95     forall (p in P) do
96         o(p):=round(random*4+0.5)
97         d(p):=round(random*4+4.5)
98         l(p):=round(random*15+55.5)
99         e(p):=round(random*15)
100        %if o(p)=d(p) then %origin cannot be the same as destination
101        %o(p):=round(random*3+1)
102        %d(p):=round(random*3+5)

```

```

101 %end-if
102 v(p):=4
103 m(p):=100
104
105 end-do
106
107 n:=20
108
109 %

```

```

110 setparam("mmxprs.xprs_loadnames", true)
111
112
113 %

```

```

114 %this creates the shortest path matrix
115 forall (p in P) do
116   if (o(p)=1 and d(p)=5) then
117     SP(p):=36
118     L(p):=2
119   elif (o(p)=1 and d(p)=6) then
120     SP(p):=36
121     L(p):=2
122   elif (o(p)=1 and d(p)=7) then
123     SP(p):=40
124     L(p):=3
125   elif (o(p)=1 and d(p)=8) then
126     SP(p):=43
127     L(p):=3
128   elif (o(p)=2 and d(p)=5) then
129     SP(p):=24
130     L(p):=1
131   elif (o(p)=2 and d(p)=6) then
132     SP(p):=33
133     L(p):=2
134   elif (o(p)=2 and d(p)=7) then
135     SP(p):=28
136     L(p):=2
137   elif (o(p)=2 and d(p)=8) then
138     SP(p):=33
139     L(p):=3
140   elif (o(p)=3 and d(p)=5) then
141     SP(p):=31
142     L(p):=2
143   elif (o(p)=3 and d(p)=6) then
144     SP(p):=32
145     L(p):=2
146   elif (o(p)=3 and d(p)=7) then
147     SP(p):=35
148     L(p):=3
149   elif (o(p)=3 and d(p)=8) then
150     SP(p):=39
151     L(p):=3
152   elif (o(p)=4 and d(p)=5) then
153     SP(p):=35

```

```

154 L(p):=2
155 elif (o(p)=4 and d(p)=6) then
156 SP(p):=26
157 L(p):=1
158 elif (o(p)=4 and d(p)=7) then
159 SP(p):=38
160 L(p):=3
161 elif (o(p)=4 and d(p)=8) then
162 SP(p):=33
163 L(p):=2
164 end-if
165 end-do
166
167 initializations to 'Data_Model.dat'
168 alpha1 t e l o d m v SP L
169 end-initializations
170
171
172 % Compile the model file
173 if compile("Optimization4.mos")<>0 then exit(1); end-if
174 load(modSub, "Optimization4.bim") % Load the bim file
175 run(modSub) % Start model execution
176 wait % Wait for model termination
177 dropnextevent % Ignore termination event message
178 end-model

```

Appendix B: Optimization model

```

1  %@encoding CP1252
2  model "Optimization ride-sharing"
3  uses "mmxprs","mmive","mmodbc","advmod"; %gain access to the Xpress-Optimizer
   solver
4
5  declarations
6
7  P=1..5 %set of persons
8  N= 1..8 %set of nodes
9  S= 1..40 %set of binomial instances (size(S)=2*mean)
10
11 x: array (P,N,N) of mpvar %decision variable drivers allocation
12 y: array (P,P,N,N) of mpvar %decision variable for passengers allocation
13 alpha1: integer %travel cost per time unit
14 alpha2: array(P) of real %VOT-penalty per time unit per person
15 t: array(N,N) of integer %travel time from i to j
16 D: array(P,N,N) of mpvar %departure time from node i for person p
17 E: array(P,N,N) of mpvar %Is x*D, created in order to make the model linear
18 G: array(P,P,N,N) of mpvar %Is y*D, created in order to make the model linear
19 e: array(P) of integer %earliest departure time for person p
20 l: array(P) of integer %latest arrival time of person p
21 o: array(P) of integer %origin of person p
22 d: array(P) of integer %destination of person p
23 m: array(P) of real %maximum wait time for person p
24 v: array(P) of integer %vehicle capacity for for person p
25 M: integer %big M
26 n: integer %maximum number of drivers
27 destination:array(N) of string %array that contains all possible destinations
28 z: array(P,N,N) of mpvar %denotes the number of persons travelling with driver
   p on i,j
29 O: array(S,P) of integer %creates random values for normal distribution
30 SP: array(P) of integer %contains the minimal travel time for rider p
31 EC: array(P) of real %these array track the additional costs of greedy
   transits
32 EC1: array(P) of real
33 EC2: array(P,P) of real
34 EC4: array(P) of real
35 EC3: array(P,P,N,N) of real
36 VC1: array(P) of real %tracks loss in filling rate due to your greedy transit
37 VC2: array(P) of real %tracks loss in filling rate due to losing your driver
38 VC3: array(P) of real %tracks loss in filling rate due to losing a passenger
39 L: array(P) of integer %this counts the number of links in the shorest path
   for rider p
40 end-declarations
41
42 %var declariation

```

```

43 forall(p in P, i in N, j in N) x(p,i,j) is_binary
44 forall(p in P, q in P, i in N, j in N) y(p,q,i,j) is_binary
45 forall(p in P, i in N, j in N) z(p,i,j) is_integer
46 forall(p in P, i,j in N) D(p,i,j) is_integer
47 forall(p in P, i,j in N) E(p,i,j) is_integer
48 forall(p,q in P, i,j in N) G(p,q,i,j) is_integer
49

```

```

50 %forall(p in P, i in N) A(p,i) is_integer
51 initializations from 'Data_Model.dat'
52 alpha1 alpha2 t e l o d m v SP L
53 end-initializations
54 %-----
55 %Creating normal distribution
56 forall (s in S, p in P) do
57 O(s,p):=round(random*1)
58 end-do
59 forall (p in P) do
60 alpha2(p):=0.5*(sum(s in S)O(s,p)-(20-3))
61 end-do
62 %-----
63 M:=500
64
65 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
66 %forall(p in P) alpha2(p):=0
67 %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
68 destination(1):="Node 1"
69 destination(2):="Node 2"
70 destination(3):="Node 3"
71 destination(4):="Node 4"
72 destination(5):="Node 5"
73 destination(6):="Node 6"
74 destination(7):="Node 7"
75 destination(8):="Node 8"
76 %destination(9):="Node 9"
77 %

```

```

78
79 %Objective function

```

```

80 OBJ:= alpha1* (sum(p in P, i,j in N)(t(i,j)*x(p,i,j))) % + %for positive VOT
81 % sum(p in P)alpha2(p)*((sum(i,j in N|j=d(p))(((E(p,i,j)+x(p,i,j)*t(i,j)+sum(q
82 in P)(G(p,q,i,j)+y(p,q,i,j)*t(i,j)))))))-(e(p)+SP(p)))
83 %OBJ:= alpha1* (sum(p in P, i,j in N)(t(i,j)*x(p,i,j))) + %for negative VOT
84 %sum(p in P)(alpha2(p)*(1(p)-sum(i,j in N|j=d(p))(((E(p,i,j)+x(p,i,j)*t(i,j)+
85 sum(q in P)(G(p,q,i,j)+y(p,q,i,j)*t(i,j)))))))
86 %Spatial constraints

```

```

87 %forall(p,q in P,i,j in N) do %turn off ridesharing
88 %y(p,q,i,j)=0
89 %end-do
90
91 forall (i,j in N, p in P)do
92 y(p,p,i,j)=0
93 end-do
94
95 forall (p in P,i in N|i=o(p)) do %cannot enter the origin node as driver (
96 you can only depart once from there)
97 sum(j in N)(x(p,i,j)+sum(q in P)y(p,q,i,j))=1
98 end-do

```

```

98
99 forall (p,q in P, i,j in N|j=o(p)) do
100 x(p,i,j)+y(p,q,i,j)=0
101 end-do
102
103 forall (p in P,j in N|j=d(p)) do %you can only reach your destination once
104 sum(i in N)(x(p,i,j)+sum(q in P)y(p,q,i,j))=1
105 end-do
106
107 forall (p,q in P, i,j in N|i=d(p)) do %dont leave the destination
108 x(p,i,j)+y(p,q,i,j)=0
109 end-do
110
111 forall (p in P, j in N|j<=o(p) and j<=d(p)) do %number of drivers entering
    intermediate nodes equals leaving drivers
112 sum(i in N)(x(p,i,j)+sum(q in P)y(p,q,i,j))-sum(k in N)(x(p,j,k)+sum(q in P)y(p
    ,q,j,k))=0
113 end-do
114
115 %Start possible routes-----
116 forall (p in P) do
117 %Origin is node 1
118 if (o(p)=1 and d(p)=5 ) then
119 x(p,1,2)=1 and x(p,2,5)=1 or x(p,1,3)=1 and x(p,3,2)=1 and x(p,2,5)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,1,3)=1 or x(p,3,2)=1 and x(p,1,3)=1 or x(p,1,2)=1
120 elif (o(p)=1 and d(p)=6 ) then
121 x(p,1,4)=1 and x(p,4,6)=1 or x(p,1,3)=1 and x(p,3,4)=1 and x(p,4,6)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,1,4)=1 or x(p,1,3)=1 or x(p,1,3)=1 and x(p,3,4)=1
122 elif (o(p)=1 and d(p)=7 ) then
123 x(p,1,2)=1 and x(p,2,5)=1 and x(p,5,7)=1 or x(p,1,3)=1 and x(p,3,2)=1 and x(p
    ,2,5)=1 and x(p,5,7)=1 or sum(i,j in N)x(p,i,j)=0 or x(p,1,3)=1 or x(p,3,2)
    =1 and x(p,1,3)=1 or x(p,1,2)=1
124 elif (o(p)=1 and d(p)=8 ) then
125 x(p,1,4)=1 and x(p,4,6)=1 and x(p,6,8)=1 or x(p,1,2)=1 and x(p,2,5)=1 and x(p
    ,5,7)=1 and x(p,7,8)=1 or sum(i,j in N)x(p,i,j)=0 or x(p,1,4)=1 or x(p,1,2)
    =1
126 %Origin is node 2
127 elif (o(p)=2 and d(p)=5 ) then
128 x(p,2,5)=1 or x(p,2,3)=1 and x(p,3,4)=1 and x(p,4,6)=1 and x(p,6,5)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,2,3)=1 or x(p,2,3)=1 and x(p,3,4)=1
129 elif (o(p)=2 and d(p)=6 ) then
130 x(p,2,5)=1 and x(p,5,6)=1 or x(p,2,3)=1 and x(p,3,4)=1 and x(p,4,6)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,2,3)=1 or x(p,2,3)=1 and x(p,3,4)=1
131 elif (o(p)=2 and d(p)=7 ) then
132 x(p,2,5)=1 and x(p,5,7)=1 or x(p,2,5)=1 and x(p,5,6)=1 and x(p,6,8)=1 and x(p
    ,8,7)=1 or sum(i,j in N)x(p,i,j)=0
133 elif (o(p)=2 and d(p)=8 ) then
134 x(p,2,5)=1 and x(p,5,7)=1 and x(p,7,8)=1 or x(p,2,5)=1 and x(p,5,6)=1 and x(p
    ,6,8)=1 or sum(i,j in N)x(p,i,j)=0
135 %Origin is node 3
136 elif (o(p)=3 and d(p)=5 ) then
137 x(p,3,2)=1 and x(p,2,5)=1 or x(p,3,4)=1 and x(p,4,6)=1 and x(p,6,5)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,3,2)=1 or x(p,3,4)=1
138 elif (o(p)=3 and d(p)=6 ) then
139 x(p,3,4)=1 and x(p,4,6)=1 or x(p,3,2)=1 and x(p,2,5)=1 and x(p,5,6)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,3,2)=1 or x(p,3,4)=1
140 elif (o(p)=3 and d(p)=7 ) then

```

```

141 x(p,3,2)=1 and x(p,2,5)=1 and x(p,5,7)=1 or x(p,3,4)=1 and x(p,4,6)=1 and x(p
    ,6,8)=1 and x(p,8,7)=1 or sum(i,j in N)x(p,i,j)=0 or x(p,3,2)=1 or x(p,3,4)
    =1
142 elif (o(p)=3 and d(p)=8 ) then
143 x(p,3,4)=1 and x(p,4,6)=1 and x(p,6,8)=1 or x(p,3,2)=1 and x(p,2,5)=1 and x(p
    ,5,7)=1 and x(p,7,8)=1 or sum(i,j in N)x(p,i,j)=0 or x(p,3,2)=1 or x(p,3,4)
    =1
144 %Origin is node 4
145 elif (o(p)=4 and d(p)=5 ) then
146 x(p,4,6)=1 and x(p,6,5)=1 or x(p,4,3)=1 and x(p,3,2)=1 and x(p,2,5)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,4,3)=1 or x(p,3,2)=1 and x(p,4,3)=1
147 elif (o(p)=4 and d(p)=6 ) then
148 x(p,4,5)=1 or x(p,4,3)=1 and x(p,3,2)=1 and x(p,2,5)=1 and x(p,5,6)=1 or sum(i,
    j in N)x(p,i,j)=0 or x(p,4,3)=1 or x(p,3,2)=1 and x(p,4,3)=1
149 elif (o(p)=4 and d(p)=7 ) then
150 x(p,4,6)=1 and x(p,6,8)=1 and x(p,8,7)=1 or x(p,4,6)=1 and x(p,6,5)=1 and x(p
    ,5,7)=1 or sum(i,j in N)x(p,i,j)=0
151 elif (o(p)=4 and d(p)=8 ) then
152 x(p,4,6)=1 and x(p,6,8)=1 or x(p,4,6)=1 and x(p,6,5)=1 and x(p,5,7)=1 and x(p
    ,7,8)=1 or sum(i,j in N)x(p,i,j)=0
153 end-if
154 end-do
155 %End possible routes

```

```

156
157 forall (p in P, i,j in N) do %this ensures that only non-empty links are used
158 t(i,j)-x(p,i,j)>=M*(1-x(p,i,j))
159 end-do
160
161 forall (p in P, j in N) do %do not travel to nodes multiple times
162 sum(i in N)x(p,i,j)<=1
163 end-do
164
165 forall (i in N, p,q in P) do %no travelling to current location
166 x(p,i,i) + y(p,q,i,i)=0
167 end-do
168 %Creating variable E

```

```

169 forall(p in P, i,j in N) E(p,i,j) <= M*x(p,i,j)
170 forall(p in P,i,j in N) E(p,i,j) <= D(p,i,j)
171 forall(p in P, i,j in N) E(p,i,j)>= D(p,i,j)-(1-x(p,i,j))*M
172 forall(p in P, i,j in N) E(p,i,j)>=0
173 %Time constraints

```

```

174 forall (p in P, i in N|i=o(p)) do %departure cannot be before availability
175 sum(j in N)(E(p,i,j)+sum(q in P)G(p,q,i,j))>=e(p)
176 end-do
177
178 forall (p in P, j in N|j=d(p)) do %arrival at destination must be before due
    date
179 sum(i in N)(E(p,i,j)+x(p,i,j)*t(i,j))<=l(p)
180 end-do
181
182 forall (p in P, i,j in N) do %non-negativity constraints

```



```

183 D(p,i,j)>=0
184 end-do
185 %Creating variable G

```

```

186 forall(p,q in P, i,j in N) G(p,q,i,j) <= M*y(p,q,i,j)
187 forall(p,q in P,i,j in N) G(p,q,i,j) <= D(p,i,j)
188 forall(p,q in P, i,j in N) G(p,q,i,j)>= D(p,i,j)-(1-y(p,q,i,j))*M
189 forall(p,q in P, i,j in N) G(p,q,i,j)>=0
190 %Ride-sharing constraints

```

```

191
192
193 forall (q in P, i in N,j in N) do    %Z contains the number of people driving
    with p from i to j
194 (sum (p in P) y(p,q,i,j))+x(q,i,j)=z(q,i,j)
195 end-do
196
197 forall (p in P, j in N) do    %A person cannot drive anymore if he leaves his
    car
198 sum(q in P, i in N) y(p,q,i,j) + sum(k in N) x(p,j,k) <= 1
199 end-do
200
201 forall (p in P,q in P,i in N,j in N) do %Carpooling can only occur from i to j
    if the driver goes there
202 y(p,q,i,j) <= x(q,i,j)
203 end-do
204
205 forall (q in P,i in N,j in N) do %Capacity constraints for the driver
206 sum(p in P) y(p,q,i,j) <= v(q)
207 end-do
208
209 forall (p in P, j in N | j=d(p)) do %passengers due date constraint
210 sum(i in N, q in P) (G(p,q,i,j)+y(p,q,i,j)*t(i,j)) <= l(p)
211 end-do
212
213 forall (p in P, j in N | j > d(p)) do %driver/passenger cannot leave before it
    has arrived there
214 sum(k in N) (E(p,j,k)+sum(q in P) G(p,q,j,k)) >= sum(i in N) ((E(p,i,j)+x(p,i,j)*t(i,j))+sum(q in P) (G(p,q,i,j)+y(p,q,i,j)*t(i,j)))
215 end-do
216
217 forall (p,q in P, i,j in N) do %departure driver is departure passenger
218 E(q,i,j)-G(p,q,i,j) <= M*(1-y(p,q,i,j))
219 E(q,i,j)-G(p,q,i,j) >= -M*(1-y(p,q,i,j))
220 end-do
221
222 forall (p in P,i,j in N) do
223 D(p,i,j) >= 0
224 end-do
225
226 minimize (OBJ)
227
228 %print optimum solution

```

```

229
230 writeln("Simulation with ",getsize(P)," riders")
231
232 forall(p in P) do
233   writeln("Rider ",p,":   origin ",o(p), "   destination ",d(p), "   alpha2 ",
           alpha2(p),"   e(p) ",e(p))
234
235 end-do
236 writeln(" _____ ")
237 procedure print_status
238   declarations
239     status: string
240   end-declarations
241
242   case getprobstat of
243     XPRS_OPT: status:="Optimum found"
244     XPRS_UNF: status:="Unfinished"
245     XPRS_INF: status:="Infeasible"
246     XPRS_UNB: status:="Unbounded"
247     XPRS_OTH: status:="Failed"
248     else status:="???"
249   end-case
250
251   writeln("Problem status: ", status)
252 end-procedure
253
254 print_status
255
256 writeln(" Drivers: ")
257 forall (p in P, i in N, j in N) do
258   if(x(p,i,j).sol = 1) then
259     writeln("Driver ", p , ": path ", destination(i)," —> ",destination(j), "
           by driver ", p," leaves at ", x(p,i,j).sol*D(p,i,j).sol," , travel time ",
           x(p,i,j).sol*t(i,j), " , due date ", l(p))
260   end-if
261
262 end-do
263 writeln(" Passengers: ")
264 forall (p,q in P, i in N, j in N) do
265   if(y(p,q,i,j).sol = 1) then
266     writeln("Passenger ", p , ": path ", destination(i)," —> ",destination(j),
           " by driver ", q," leaves at ", y(p,q,i,j).sol*D(p,i,j).sol," , travel
           time ", y(p,q,i,j).sol*t(i,j), " , due date ", l(p))
267   end-if
268 end-do
269 writeln(" _____ ")
270 writeln("value obj. function ",OBJ.sol )
271 writeln("Total sheep travel cost ",sum(p in P, i,j in N)x(p,i,j).sol*t(i,j))
272 writeln(" _____ ")
273 writeln("Average #of riders in vehicle: ",(sum(p in P, i,j in N)x(p,i,j).sol+
           sum(p,q in P, i,j in N)y(p,q,i,j).sol)/sum(p in P, i,j in N)x(p,i,j).sol)
274
275
276 %Greedy transit algorithm


---


277 forall (p in P) do

```

```

278 %If rider p takes a greedy transit
279 if (sum(i,j in N|z(p,i,j).sol>0)( x(p,i,j).sol*t(i,j)/z(p,i,j).sol+sum(q in P
    |z(q,i,j).sol>0)(y(p,q,i,j).sol*t(i,j)/z(q,i,j).sol )) + alpha2(p)*(sum(i,j
    in N|j=d(p))( E(p,i,j).sol+x(p,i,j).sol*t(i,j) +sum(q in P)(G(p,q,i,j).sol
    +y(p,q,i,j).sol*t(i,j))) -(e(p)+SP(p)))>SP(p)*alpha1 ) then
280 writeln("Rider ",p," takes a greedy transit , extra cost: ",(SP(p)*alpha1)- (
    sum(i,j in N|z(p,i,j).sol>0)( x(p,i,j).sol*t(i,j)/z(p,i,j).sol)+sum(i,j
    in N,q in P|z(q,i,j).sol>0)(y(p,q,i,j).sol*t(i,j)/z(q,i,j).sol ) ) )
281 EC1(p):=(SP(p)*alpha1)- ( sum(i,j in N|z(p,i,j).sol>0)( x(p,i,j).sol*t(i,j)/
    z(p,i,j).sol)+sum(i,j in N,q in P|z(q,i,j).sol>0)(y(p,q,i,j).sol*t(i,j)/z(
    q,i,j).sol ) ) )
282 end-if
283 EC(p):=EC1(p)
284 end-do
285
286 %If p's driver q takes a greedy transit
287 forall (p in P) do
288     forall (q in P) do
289         if (sum(i,j in N)y(p,q,i,j).sol>=1 and EC(q)>0.01 and EC(p)<0.01) then
290             EC2(p,q):= (( sum(i,j in N|z(p,i,j).sol>0)( x(p,i,j).sol*t(i,j)/z(p,i,j).
                sol)+sum(i,j in N|z(q,i,j).sol>0)(y(p,q,i,j).sol*t(i,j)/z(q,i,j).sol )
                ))
291         end-if
292     end-do
293     if (EC(p)<0.1 and sum(q in P)EC2(p,q)>0 ) then
294         EC4(p):=(SP(p)*alpha1)-sum(q in P)EC2(p,q)
295         writeln("Extra cost of riders having to drive themselves for driver ",p," : ",
            EC4(p))
296         EC(p):=EC(p)+EC4(p)
297     end-if
298 end-do
299 %if p's passenger takes a greedy transit
300 forall(p in P) do
301     forall (q in P,i,j in N|z(p,i,j).sol>0) do
302         if (y(q,p,i,j).sol=1 and EC(q)>0.01 and EC(p)<0.01) then
303             EC3(p,q,i,j):=(y(q,p,i,j).sol*t(i,j))/z(p,i,j).sol
304         end-if
305     end-do
306     writeln("Extra cost due to lower filling rate for rider ",p," : ", sum(q in P,i
        ,j in N)EC3(p,q,i,j))
307     EC(p):=EC(p)+sum(q in P,i,j in N)EC3(p,q,i,j)
308 end-do
309 writeln("-----")
310
311 writeln("Total greedy travel cost: ",sum(p in P)EC(p)+sum(p in P, i,j in N)x(p,
    i,j).sol*t(i,j))
312
313 %greedy vehicle capacity


---


314 forall (p in P) do
315     if (EC1(p)>0.01 or sum(q in P)EC2(p,q)>0.01) then
316         VC1(p):= (sum(i,j in N)x(p,i,j).sol+sum(q in P, i,j in N)y(p,q,i,j).sol)-L(p)
317         VC2(p):=L(p)-sum(i,j in N)x(p,i,j).sol
318     end-if
319     if (sum(q in P, i,j in N)EC3(p,q,i,j)>0.01) then
320         VC3(p):=VC3(p)+1

```

```

321  end-if
322  end-do
323
324  writeln("Greedy vehicle capacity: ",(sum(p in P, i,j in N)x(p,i,j).sol+sum(p,q
      in P, i,j in N)y(p,q,i,j).sol-sum(p in P)VC1(p)-sum(p in P)VC3(p))/(sum(p in
      P, i,j in N)x(p,i,j).sol+sum(p in P)VC2(p)) )
325
326  end-model

```

Appendix C: VOT experiments

Positive VOT experiments

Number of riders	Travel cost								
	without ridesharing	No VOT	$\mu = 1$	$\mu = 3$	$\mu = 5$	$\mu = 7$	$\mu = 9$	$\mu = 11$	$\mu = 13$
5	172	53	81	81	138	138	138	138	138
6	213	78	83	83	83	83	118	118	164
7	252	87	94	127	155	178	188	188	188
8	264	89	117	129	157	157	183	212	236
9	331	94	128	154	187	226	226	226	226
10	346	79	107	174	209	209	209	237	237
11	371	113	132	158	191	249	216	280	280
12	416	124	165	206	206	266	266	302	302
13	447	120	132	212	287	287	320	320	344
14	487	129	188	188	211	211	259	298	298
15	502	134	172	214	214	256	262	323	323

Table C.1: Travel cost using different positive mean VOT-values and comparison to a network without ridesharing, multi-hopping

Number of riders	Travel cost								
	without ridesharing	No VOT	$\mu = 1$	$\mu = 3$	$\mu = 5$	$\mu = 7$	$\mu = 9$	$\mu = 11$	$\mu = 13$
5	176	88	109	143	143	143	143	176	176
6	221	91	153	181	181	181	181	181	181
7	265	86	86	143	177	177	177	177	265
8	264	125	149	173	209	209	209	209	209
9	331	87	87	179	179	235	268	268	268
10	346	125	138	193	253	253	253	296	332
11	371	129	151	179	217	217	217	253	291
12	416	116	116	172	255	255	255	255	255
13	447	134	150	186	210	238	271	271	348
14	487	148	187	214	271	304	304	332	332
15	502	137	189	215	239	272	272	272	272

Table C.2: Travel cost using different positive mean VOT-values and comparison to a network without ridesharing, single-hopping

Number of riders	Filling rate without ridesharing	No VOT	$\mu = -1$	$\mu = -3$	$\mu = -5$	$\mu = -7$	$\mu = -9$	$\mu = -11$	$\mu = -13$
5	1.00	2.60	2.83	1.63	1.63	1.63	1.63	1.63	1.63
6	1.00	2.67	2.20	1.80	1.80	1.80	1.50	1.50	1.5
7	1.00	2.50	2.67	2.67	1.92	1.92	1.92	1.92	1.92
8	1.00	2.88	3.00	2.64	2.64	2.64	2.08	2.08	2.08
9	1.00	3.13	3.11	3.11	1.83	1.83	1.83	1.67	1.67
10	1.00	3.57	2.89	2.45	2.00	1.80	1.80	1.80	1.88
11	1.00	3.22	2.90	2.42	2.42	2.42	2.42	2.42	2.42
12	1.00	3.20	3.56	3.08	2.43	2.43	2.79	2.28	1.85
13	1.00	3.10	3.45	3.00	3.00	3.00	2.25	2.05	2.00
14	1.00	3.43	3.45	2.38	1.81	2.05	1.86	1.93	1.93
15	1.00	3.41	3.15	2.51	1.82	2.04	1.87	1.81	1.39

Table C.3: Filling rate using different negative mean VOT-values and comparison to a network without ridesharing, multi-hopping

Number of riders	Filling rate without ridesharing	No VOT	$\mu = -1$	$\mu = -3$	$\mu = -5$	$\mu = -7$	$\mu = -9$	$\mu = -11$	$\mu = -13$
5	1.00	2.25	2.00	1.71	1.80	1.80	1.80	1.80	1.80
6	1.00	2.50	1.67	1.67	1.67	1.60	1.60	1.60	1.60
7	1.00	3.00	2.00	1.38	1.38	1.21	1.21	1.21	1.21
8	1.00	2.45	2.45	2.15	2.15	2.00	1.76	1.76	1.76
9	1.00	2.75	2.75	2.00	1.69	1.57	1.57	1.57	1.25
10	1.00	2.90	2.42	2.31	1.75	1.75	1.72	1.58	1.58
11	1.00	3.00	2.92	2.29	2.13	2.13	1.76	1.76	1.50
12	1.00	2.50	2.80	2.14	2.14	1.79	1.72	1.35	1.35
13	1.00	3.40	3.40	2.43	1.68	1.52	1.52	1.52	1.41
14	1.00	2.81	2.27	2.24	1.94	1.94	1.94	1.65	1.65
15	1.00	3.00	2.54	2.13	1.83	1.74	1.74	1.58	1.61

Table C.4: Filling rate using different negative mean VOT-values and comparison to a network without ridesharing, single-hopping

Negative VOT experiments

Number of riders	Travel cost without ridesharing	No VOT	$\mu = -1$	$\mu = -3$	$\mu = -5$	$\mu = -7$	$\mu = -9$	$\mu = -11$	$\mu = -13$
5	168	53	56	113	113	113	113	113	113
6	206	78	100	125	125	125	158	158	158
7	235	87	99	99	135	135	135	135	135
8	283	89	101	129	129	129	157	157	157
9	320	94	95	95	154	154	154	194	194
10	330	79	107	118	152	163	163	163	186
11	390	113	118	146	146	146	146	146	146
12	456	124	113	127	160	160	166	210	245
13	469	120	122	150	150	150	190	222	222
14	487	129	141	186	234	252	270	298	298
15	502	134	152	182	240	267	284	284	323

Table C.5: Travel cost using different negative mean VOT-values and comparison to a network without ridesharing, multi-hopping

Number of riders	Travel cost without ridesharing	No VOT	$\mu = -1$	$\mu = -3$	$\mu = -5$	$\mu = -7$	$\mu = -9$	$\mu = -11$	$\mu = -13$
5	176	88	95	104	117	117	117	117	117
6	221	91	117	117	117	123	123	123	123
7	265	86	121	182	182	210	210	210	210
8	264	125	125	142	142	155	194	194	194
9	331	87	87	122	155	179	179	179	216
10	346	125	153	160	203	203	219	247	247
11	371	129	142	175	190	190	215	215	253
12	416	116	118	160	160	216	224	273	273
13	447	134	132	199	264	302	302	302	335
14	487	148	188	203	225	225	225	258	258
15	502	137	164	195	228	252	252	283	295

Table C.6: Travel cost using different negative mean VOT-values and comparison to a network without ridesharing, single-hopping

Number of riders	Filling rate without ridesharing	No VOT	$\mu = 1$	$\mu = 3$	$\mu = 5$	$\mu = 7$	$\mu = 9$	$\mu = 11$	$\mu = 13$
5	1.00	2.60	2.00	2.00	1.22	1.22	1.22	1.22	1.22
6	1.00	2.67	2.33	2.33	2.33	2.33	1.63	1.63	1.17
7	1.00	2.50	2.25	1.80	1.50	1.43	1.29	1.29	1.29
8	1.00	2.88	2.10	1.67	1.43	1.43	1.33	1.19	1.12
9	1.00	3.13	2.00	1.75	1.50	1.24	1.24	1.24	1.24
10	1.00	3.57	2.89	1.77	1.44	1.44	1.44	1.28	1.28
11	1.00	3.22	2.33	2.23	1.81	1.56	1.73	1.35	1.35
12	1.00	3.20	2.23	1.63	1.63	1.30	1.30	1.18	1.18
13	1.00	3.10	2.73	1.88	1.33	1.33	1.22	1.22	1.17
14	1.00	3.43	2.59	2.59	1.92	1.92	1.50	1.36	1.36
15	1.00	3.41	2.78	2.35	2.17	1.57	1.53	1.30	1.30

Table C.7: Filling rate using different positive mean VOT-values and comparison to a network without ridesharing, multi-hopping

Number of riders	Filling rate without ridesharing	No VOT	$\mu = 1$	$\mu = 3$	$\mu = 5$	$\mu = 7$	$\mu = 9$	$\mu = 11$	$\mu = 13$
5	1.00	2.25	1.75	1.18	1.18	1.18	1.18	1.00	1.00
6	1.00	2.50	1.33	1.15	1.15	1.15	1.15	1.15	1.15
7	1.00	3.00	3.00	2.00	1.55	1.55	1.55	1.55	1.00
8	1.00	2.45	1.67	1.50	1.29	1.29	1.29	1.29	1.29
9	1.00	2.75	2.75	1.54	1.54	1.19	1.06	1.06	1.06
10	1.00	2.90	2.42	1.79	1.39	1.39	1.39	1.19	1.09
11	1.00	3.00	2.45	2.08	1.67	1.67	1.67	1.47	1.25
12	1.00	2.50	2.50	1.85	1.35	1.35	1.35	1.35	1.35
13	1.00	3.40	2.73	2.31	2.14	1.88	1.67	1.67	1.30
14	1.00	2.81	2.29	1.88	1.56	1.40	1.40	1.27	1.27
15	1.00	3.00	2.14	1.88	1.76	1.58	1.58	1.58	1.58

Table C.8: Filling rate using different positive mean VOT-values and comparison to a network without ridesharing, single-hopping

Appendix D: Results greedy transit experiments

No VOT experiments

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	0	0	0	1.00
6	202	0	0	0	1.00
7	258	0	0	0	1.00
8	258	0	0	0	1.00
9	291	0	0	0	1.00
10	326	0	0	0	1.00
11	393	0	0	0	1.00
12	408	0	0	0	1.00
13	458	0	0	0	1.00
14	507	0	0	0	1.00
15	524	0	0	0	1.00

Table D.1: No ridesharing

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	54	0	0	0	3.20
6	80	0	0	0	2.17
7	90	0	0	0	2.88
8	84	0	0	0	2.86
9	98	0	0	0	3.00
10	92	0	0	0	3.50
11	123	0	0	0	3.00
12	119	0	0	0	3.50
13	120	0	0	0	2.89
14	113	0	0	0	3.63
15	133	0	0	0	3.54

Table D.2: Ridesharing, no VOT

Positive VOT experiments

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	54	0	132	132	3.20
6	80	0	175	175	2.17
7	228	2	130	17	1.24
8	202	1	124	35	1.29
9	236	3	76	14	1.41
10	120	1	131	104	2.80
11	123	0	117	117	3.00
12	119	0	230	230	3.50
13	270	2	170	170	1.39
14	113	0	127	127	3.63
15	241	1	182	182	1.96

Table D.3: $\mu = 1$, VOT not in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	54	0	29	29	3.20
6	80	0	56	56	2.17
7	118	0	42	42	2.10
8	84	0	25	25	2.86
9	112	0	45	45	2.67
10	113	0	74	74	2.56
11	126	0	15	15	2.20
12	134	0	52	52	2.67
13	156	0	59	59	2.09
14	150	0	38	38	2.69
15	154	0	58	58	2.51

Table D.4: $\mu = 1$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	1	195	0	1.00
6	154	1	264	86	1.30
7	227	2	232	31	1.24
8	235	3	273	23	1.13
9	271	6	143	5	1.15
10	326	5	226	0	1.00
11	293	4	223	52	1.41
12	358	4	393	45	1.23
13	287	4	333	117	1.58
14	473	4	310	19	1.04
15	493	5	362	19	1.11

Table D.5: $\mu = 2$, VOT not in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	71	0	4	4	2.60
6	123	0	24	24	1.44
7	180	1	74	64	1.50
8	112	0	35	35	2.11
9	235	1	47	41	1.17
10	148	0	61	61	1.92
11	126	0	18	18	2.20
12	144	0	49	49	2.38
13	164	0	60	60	2.08
14	164	0	64	64	2.83
15	178	0	64	64	2.89

Table D.6: $\mu = 2$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	2	289	0	1.00
6	173	3	383	46	1.18
7	217	4	259	27	1.24
8	258	3	359	0	1.00
9	291	7	160	0	1.00
10	326	4	389	0	1.00
11	308	4	319	64	1.21
12	215	4	504	224	2.19
13	396	3	514	73	1.15
14	504	9	592	2	1.00
15	493	5	402	21	1.11

Table D.7: $\mu = 3$, VOT not in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	107	0	16	16	1.86
6	147	0	17	17	1.30
7	146	0	80	80	1.75
8	225	2	51	28	1.13
9	239	2	89	39	1.18
10	145	0	76	76	2.18
11	126	0	73	73	2.20
12	188	1	65	59	1.93
13	240	1	85	79	1.53
14	206	0	71	71	2.25
15	236	0	88	88	2.20

Table D.8: $\mu = 3$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	4	320	0	1.00
6	173	3	458	54	1.18
7	217	4	324	34	1.24
8	258	4	468	0	1.00
9	291	9	232	0	1.00
10	326	7	536	0	1.00
11	393	7	488	0	1.00
12	408	8	653	0	1.00
13	458	5	643	0	1.00
14	504	10	592	0	1.00
15	524	9	582	0	1.00

Table D.9: $\mu = 4$, VOT not in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	107	0	12	12	1.86
6	176	0	20	20	1.09
7	146	0	70	70	1.75
8	229	0	67	67	1.13
9	259	2	54	32	1.11
10	160	0	65	65	1.77
11	238	1	65	59	1.29
12	238	0	59	59	2.07
13	221	1	82	75	1.53
14	206	0	97	97	2.19
15	222	0	101	101	2.11

Table D.10: $\mu = 4$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	4	419	0	1.00	5	107	0	19	19	1.86
6	173	3	558	66	1.18	6	147	0	27	27	1.30
7	258	6	410	0	1.00	7	215	1	90	77	1.19
8	258	5	547	0	1.00	8	229	0	65	65	1.13
9	291	9	281	0	1.00	9	207	0	53	53	1.33
10	326	8	729	0	1.00	10	172	0	55	55	1.77
11	396	9	673	0	1.00	11	180	1	84	76	1.16
12	408	9	810	0	1.00	12	228	0	46	46	1.67
13	458	11	837	0	1.00	13	275	0	36	36	1.33
14	504	10	800	0	1.00	14	258	0	63	63	1.71
15	524	9	832	0	1.00	15	222	0	122	122	2.11

Table D.11: $\mu = 5$, VOT not in obj. functionTable D.12: $\mu = 5$, VOT in obj. function

Negative VOT experiments

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	54	0	99	99	3.20	5	71	0	6	6	2.60
6	111	1	196	122	1.63	6	97	0	20	20	1.88
7	90	0	190	190	2.88	7	119	0	22	22	1.90
8	84	0	177	177	2.86	8	91	0	17	17	2.67
9	122	1	91	71	2.70	9	122	0	19	19	2.40
10	92	0	114	114	3.50	10	92	0	29	29	3.50
11	137	0	50	47	2.60	11	133	1	15	14	2.42
12	180	2	116	76	2.27	12	123	0	20	20	2.80
13	311	2	169	62	1.20	13	138	0	15	15	3.08
14	124	0	168	163	3.80	14	146	0	27	27	3.00
15	204	1	193	147	2.87	15	158	0	27	27	3.06

Table D.13: $\mu = -1$, VOT not in obj. functionTable D.14: $\mu = -1$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	54	0	141	141	3.20	5	71	0	12	12	2.60
6	169	3	94	13	1.20	6	80	0	28	28	2.17
7	136	1	302	188	1.92	7	113	0	40	40	2.08
8	231	3	246	24	1.19	8	118	0	33	33	2.10
9	150	2	107	61	2.25	9	122	0	41	41	2.40
10	92	0	253	253	3.50	10	110	0	46	46	2.78
11	293	3	118	32	1.41	11	118	0	34	34	3.10
12	330	3	326	66	1.19	12	123	0	31	31	2.80
13	458	6	344	0	1.00	13	157	0	22	22	2.92
14	279	3	272	124	1.85	14	146	0	45	45	3.00
15	346	4	389	130	1.98	15	161	0	41	41	2.96

Table D.15: $\mu = -2$, VOT not in obj. functionTable D.16: $\mu = -2$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	2	150	0	1.00	5	71	0	21	21	2.60
6	133	2	230	87	1.50	6	113	0	42	42	1.63
7	161	2	380	157	1.69	7	114	0	36	36	2.10
8	231	4	371	29	1.19	8	118	0	55	55	2.10
9	271	2	178	14	1.15	9	203	1	34	30	1.40
10	312	2	340	16	1.13	10	113	0	59	59	2.78
11	214	3	221	106	1.92	11	140	0	40	40	2.46
12	335	5	490	72	1.20	12	123	0	5	5	2.80
13	458	8	475	0	1.00	13	157	0	36	36	2.92
14	254	4	368	169	2.24	14	207	1	55	51	2.56
15	432	5	522	82	1.16	15	176	0	48	48	2.88

Table D.17: $\mu = -3$, VOT not in obj. functionTable D.18: $\mu = -3$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	2	237	0	1.00	5	71	0	37	37	2.60
6	202	5	274	0	1.00	6	151	0	24	24	1.27
7	161	2	505	208	1.69	7	114	0	28	28	2.10
8	231	4	620	48	1.19	8	211	1	15	13	1.31
9	181	3	198	75	1.86	9	185	0	38	38	1.67
10	326	4	466	0	1.00	10	113	0	28	28	2.78
11	393	6	326	0	1.00	11	164	1	29	26	2.29
12	408	7	619	0	1.00	12	182	0	8	8	2.12
13	458	10	780	0	1.00	13	157	0	44	44	2.92
14	411	9	506	44	1.32	14	179	0	49	49	2.53
15	524	9	720	0	1.00	15	181	0	51	51	2.61

Table D.19: $\mu = -4$, VOT not in obj. functionTable D.20: $\mu = -4$, VOT in obj. function

Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate	Number of riders	Costs	Number of greedy transits	VOT costs	Corrected VOT	Filling rate
5	177	2	317	0	1.00	5	71	0	41	41	2.60
6	202	5	341	0	1.00	6	151	0	6	6	1.27
7	258	6	699	0	1.00	7	114	0	36	36	2.10
8	231	5	629	37	1.19	8	142	0	38	38	1.91
9	303	5	321	0	1.00	9	185	0	31	31	1.67
10	326	7	521	0	1.00	10	113	0	39	39	2.78
11	393	6	427	0	1.00	11	238	1	43	39	1.69
12	408	9	844	0	1.00	12	182	0	10	10	2.12
13	458	11	934	0	1.00	13	157	0	71	71	2.92
14	419	10	670	43	1.24	14	205	0	48	48	2.39
15	524	11	842	0	1.00	15	213	0	53	53	2.44

Table D.21: $\mu = -5$, VOT not in obj. functionTable D.22: $\mu = -5$, VOT in obj. function

Appendix E: Truck transport experiments: switching between trucks allowed

VOT not in objective function

Number of riders	No VOT			VOT not in obj. function									
	Travel cost	Filling rate	VOT cost	$\mu = 1$	$\mu = 2$	$\mu = 3$	$\mu = 4$	$\mu = 5$	$\mu = -1$	$\mu = -2$	$\mu = -3$	$\mu = -4$	$\mu = -5$
9	46	4.40	0	40	82	164	196	230	122	122	196	325	404
11	83	3.13	0	180	256	356	522	682	61	104	147	217	303
13	70	5.00	0	143	142	251	314	403	170	368	386	497	618
15	71	7.17	0	203	292	495	648	820	103	174	296	369	461
17	85	5.43	0	160	279	503	714	852	66	259	379	530	703
19	89	5.78	0	187	298	437	495	687	176	336	449	708	869
21	115	5.60	0	191	310	469	678	808	185	464	574	788	907
23	121	5.43	0	207	321	511	701	798	181	346	502	693	863

Table E.1: VOT not in obj. function

VOT included in objective function

Number of riders	$\mu = 1$			$\mu = -1$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	57	3.00	27	53	4.80	12
11	84	3.00	41	91	2.78	34
13	130	2.62	31	88	4.33	9
15	73	5.83	45	91	4.63	45
17	83	4.38	62	113	4.17	39
19	131	4.25	52	103	5.64	30
21	128	4.33	111	129	4.92	27
23	139	4.22	83	114	5.04	23

Table E.2: $\mu = 1, \mu = -1$

Number of riders	$\mu = 2$			$\mu = -2$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	79	2.50	31	53	4.80	29
11	98	2.40	44	106	2.73	22
13	165	2.40	48	88	4.33	18
15	114	3.80	51	98	4.10	49
17	154	2.93	72	115	3.46	33
19	160	3.93	89	128	4.40	17
21	158	4.15	127	139	4.71	45
23	164	4.03	103	114	5.04	31

Table E.3: $\mu = 2, \mu = -2$

Number of riders	$\mu = 3$			$\mu = -3$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	119	2.00	20	76	3.33	15
11	117	2.56	33	134	2.31	15
13	165	2.40	74	88	4.33	25
15	121	3.27	79	116	3.58	46
17	172	2.59	71	131	3.67	79
19	157	3.25	160	128	4.4	52
21	179	3.63	168	139	4.71	71
23	164	4.03	136	144	4.12	36

Table E.4: $\mu = 3, \mu = -3$

Number of riders	$\mu = 4$			$\mu = -4$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	119	2.00	28	62	4.17	31
11	124	2.27	36	134	2.31	16
13	172	2.35	87	88	4.33	31
15	159	2.71	69	123	3.21	63
17	160	2.87	119	163	3.71	43
19	146	3.64	220	128	4.53	69
21	162	3.86	312	139	4.71	90
23	175	3.56	201	144	4.12	48

Table E.5: $\mu = 4, \mu = -4$

Number of riders	$\mu = 5$			$\mu = -5$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	119	2.00	29	88	3.38	97
11	124	2.27	51	137	2.50	17
13	172	2.35	100	88	4.33	36
15	159	2.71	81	122	3.82	78
17	172	2.59	127	163	3.71	65
19	157	3.25	290	128	4.40	74
21	179	3.63	406	139	4.71	117
23	175	3.56	316	148	4.01	79

Table E.6: $\mu = 5, \mu = -5$

Appendix F: Truck transport experiments: no switching between trucks

VOT not in objective function

Number of riders	No VOT			VOT not in obj. function									
	Travel cost	Filling rate	VOT cost	$\mu = 1$	$\mu = 2$	$\mu = 3$	$\mu = 4$	$\mu = 5$	$\mu = -1$	$\mu = -2$	$\mu = -3$	$\mu = -4$	$\mu = -5$
9	80	2.67	0	29	52	80	64	112	127	153	262	349	400
11	89	3.25	0	28	63	73	114	154	163	329	446	618	756
13	79	5.14	0	103	279	365	557	645	100	167	275	370	541
15	88	4.28	0	132	173	277	390	451	199	353	424	677	878
17	120	3.80	0	166	243	391	484	648	145	349	494	730	1005
19	98	5.00	0	225	320	536	741	974	186	209	428	585	745
21	121	4.30	0	154	211	434	577	633	292	557	834	1216	1362
23	122	5.09	0	224	367	523	724	902	258	444	543	838	1150

Table F.1: VOT not in obj. function

Number of riders	$\mu = 1$			$\mu = -1$			Number of riders	$\mu = 2$			$\mu = -2$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs		Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	72	3.00	23	72	3	11	9	80	2.86	48	72	3	13
11	96	3.11	63	95	3.11	13	11	131	2.50	19	95	3.11	43
13	85	4.00	43	81	4.71	41	13	85	4.00	78	85	3.88	47
15	92	4.00	88	94	4.56	36	15	139	2.93	73	120	4.2	26
17	126	4.25	67	126	4.25	24	17	119	4.45	116	125	4.08	44
19	123	4.09	63	145	4.27	36	19	165	3.25	79	166	3.56	55
21	174	3.59	90	126	5.42	50	21	174	3.59	129	167	4.47	44
23	130	4.54	70	164	4.27	44	23	165	4.00	108	185	3.84	58

Table F.2: $\mu = 1, \mu = -1$

Table F.3: $\mu = 2, \mu = -2$

Number of riders	$\mu = 3$			$\mu = -3$			Number of riders	$\mu = 4$			$\mu = -4$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs		Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	110	2.25	34	100	2.43	11	9	110	2.25	32	108	2.44	15
11	131	2.50	66	95	3.11	60	11	170	2.06	53	95	3.11	81
13	127	2.84	43	102	4.30	75	13	127	2.84	77	128	3.55	74
15	181	2.44	115	120	4.20	43	15	174	2.56	123	120	4.20	75
17	165	3.20	130	132	3.92	55	17	166	3.38	165	165	3.27	53
19	170	3.06	118	176	3.69	59	19	163	3.20	202	176	3.69	79
21	174	3.59	191	126	5.33	101	21	175	3.50	253	190	4.18	86
23	171	3.75	177	191	3.60	103	23	171	3.65	229	185	3.84	150

Table F.4: $\mu = 3, \mu = -3$

Table F.5: $\mu = 4, \mu = -4$

Number of riders	$\mu = 5$			$\mu = -5$		
	Travel costs	Filling rate costs	VOT costs	Travel costs	Filling rate costs	VOT costs
9	110	2.25	44	108	2.44	17
11	170	2.06	53	137	2.46	59
13	168	2.50	52	161	2.64	48
15	181	2.44	150	120	4.20	85
17	166	3.38	209	165	3.27	62
19	163	3.20	235	197	3.50	62
21	174	3.59	321	190	4.18	105
23	171	3.75	292	185	3.84	175

Table F.6: $\mu = 5, \mu = -5$