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Artificial Life Simulation of the Majority Vote

Bachelor's Project Thesis

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Abstract: In this thesis the results of a cellular automaton simulation of majority voting are reported. A bi-dimensional cellular automaton of 100×100 cells is used to simulate the effects of the majority vote rule with initial majority proportions p in the domain of $\{0.5, 0.505, ..., 0.995\}$. Three topologies are compared, Von Neumann neighbourhood and Moore neighbourhoods with radius 1 and 2. The results show that an increasing size of the initial majority population, results in a proportionally larger majority in the final state for initial majority proportions up to 0.56 with the radius 2 Moore neighbourhood, 0.62 with the radius 1 Moore neighbourhood and 0.70 with the Von Neumann neighbourhood. Both Moore neighbourhoods exhibit a stable majority convergence, always unanimous with initial majorities from $p = 0.6$ with radius 2 and $p = 0.82$ with radius 1. The Von Neumann topology only achieves a 100% consistent convergence at around $p = 0.98$. The Moore topology Cellular Automata correspond to the Class 1 Cellular Automaton converging to a stable pattern, while the Von Neumann majority vote cellular automaton is Class 2 converging to an Oscillating pattern with initial majorities of up to 71%.

1 Introduction

Cellular Automata are formal computational models of artificial life structured as closed worlds of discrete space and time and locally defined interactions. These were initially envisioned as potential carriers of self-reproducing behaviour. Their very first inception is frequently attributed to John von Neumann and his axiomatic definition presented in (Von Neumann, 1951). He describes the starting point for Cellular Automaton as black boxes that *'[...] react to certain unambiguously defined stimuli, by certain unambiguously defined responses.'* , and which can be modelled as part of larger organisms formed of such components. Broadly he describes a (1) scalable computational model represented in a (2) n-dimensional discrete space (3) where agents in (4) a set of possible states (5) evolve by an update function (6) over discrete time steps.

Arguably an earlier expression of the same proposition is shown in Alan Turing's paper (Turing, 1990) originally published in 1951. As mentioned in (Copeland, 2004), Turing phrases a similar construction to explain morphogenesis in nature. Using a mathematical definition, he proves that naturally observable spatial patterns can arise from the diffusion of chemicals reacting with each other. A bio-chemical expression of patterns behaving like a Fibonacci series.

The next largely relevant advancement in the field was Wolfram's publication describing the Elementary Cellular Automata as a utensil to model complex dynamic systems (Wolfram, 1983): "Any physical system satisfying differential equations may be approximated as a cellular automaton by introducing finite differences and discrete variables."

Plenty of research has been done since then in the field of Cellular Automata. A historical overview of which can be found in (Sarkar, 2000). Some real life application examples are: the simulation of forest fire spreading (Karafyllidis and Thanailakis, 1997), road traffic (Maerivoet and De Moor, 2005) and applications of socio-physics to the financial markets (Vilela et al., 2019). All of these use the described artificial life simulation to model microscopically observed behaviour at a macroscopic scale. The aforementioned features, make Cellular Automaton a suitable model to simulate the majority vote.

Not only Cellular Automaton but also comparable implementations of dynamic modelling have been used to simulate the effects of several aspects of the majority voting problem. Examples extracted from the literature are: a Probabilistic Cellular Automata (Słowiński and MacKay, 2015), a dynamic small world network (Stone and McKay, 2015) and (Campos et al., 2003), or an implementation with different agents such as in (Vilela and Moreira, 2009). Most of the research in this field includes a noise parameter in the transition rule that adds stochasticity to the models.

The majority vote problem addresses the convergence of a population of agents to a unanimous state which is the most frequent in the initial configuration (at $t = 0$). There should be at least 2 mutually exclusive states, and a transition rule by which each population unit adopts the state of the majority of agents to which it is connected. Sometimes this is described as a majority classification problem, for example when the application is used to solve a complex computing challenge as in (Andre et al., 1996) and (Moore, 1997). In computing science the simulations are used to search for the best performing transition rule in achieving the highest rate of correct classifications. In such setting, the classification is correct when the system achieves the unanimous state that had the largest initial population.

A contemporary real-life application of the majority voting problem exists in political science. Some electoral systems use a form of majority vote to form their representative organs, this is still in practice for example in The United States of America and United Kingdom. The population is distributed in districts, each of which will select one candidate by majority (not unanimity), where each voter casts one vote to one candidate. This electoral setting, also called First-Past-the-Post, has been under scrutiny because of an observed phenomenon labelled gerrymandering.

Gerrymandering is a process by which the districts of an election are deliberately mapped in such a way that the partitions benefit one of the electoral parties (also called Partisan Gerrymandering). The technique looks at how the electoral bias is distributed locally, and attempts to maximise the number of districts where candidates of one specific party will achieve a majority, see for example (Shotts, 2001). In this case, the microscale majority proportion is misrepresented in the macroscale outcome by manipulating the underlying topology.

The local partitions that are created then as the outcome of this mapping become the smallest unit of democratic representation. Underlying, the majority dynamics within each of those groups will respond to human opinion formation and decision making. From social sciences, cognitive psychology and behavioural sciences, some indicators can be extracted to look for behavioural markers of the majority dynamics.

Independently of the social interaction itself, it seems that humans are more prone to express an opinion that is shared with a majority. In (Koriat et al., 2016) researchers show that people would share the opinion they adhere to when it is aligned with a perceived majority. The larger the size of the majority, the more confidently an affiliate opinion would be communicated. In the absence of a clear majority, however, one would prefer to compare himself to the group whose opinions or skills aligns the most (Festinger, 1954).

This thesis reports on the outcomes of a cellular automaton simulation of the majority vote. The majority vote is implemented in line with the above mentioned principles: at each time step, all cells will adopt the state of the majority of its neighbours. In the presence of a tie, the current state of the cell is maintained. Three different neighbourhood topologies are tested, the Von Neumann neighbourhood and Moore neighbourhoods with radius 1 and 2.

The purpose of the cellular automaton simulation is to investigate the effects of the initial proportion of the majority in the final majority proportion in a fully deterministic setup.

The investigation reports on the convergence curve of the Cellular Automata as a function of the initial majority ratio. Furthermore, it studies the conditions under which the initial majority always takes over the complete grid and adopts the initial majority as a unanimous state.

Finally this thesis also reports on what possible structures remain in the end state. What permanent patterns appear that do not transform (boundaries, islands and oscillators) and in which initial set-ups these take place.

Section 2 explains the details of the cellular automaton model and the parameters used in the simulations. Section 3 shows the outcomes, including the convergence curve and snapshots of the encountered patterns. Finally, section 4 circles back to the Cellular Automata theory and the socio-economic and cognitive aspects of the majority vote introduced in this section.

2 Methods

In this section all the parameters of the simulation and the cellular automaton implementation are explained. A formal definition of the cellular automaton is given in the next section together with the description of the three neighbourhoods tested.

The simulation is an adaptation of an open source python software. The original code can be found in `https://gitlab.com/DamKoVosh/cellular_automaton`. The software is published under Apache License 2.0 and therefore is free to use. The original code has been modified, and a new main file was created to implement the majority vote rule and run the simulation with the different parameters (neighbourhood and initial bias). The cellular automaton in this package can be run with a single process or multiple processes. Only the single process settings have been used in this research.

2.1 Cellular automaton: formal definition

The formal definition of a cellular automaton is popularly defined as in (Burguillo, 2018) and (Delorme, 1999):

A d-dimensional cellular automaton A, is a 4-tuple as described in Expression 2.1:

$$A = (\mathbb{Z}^d, S, N, f) \tag{2.1}$$

where:

- $\mathbb{Z}^d$ is the finite or infinite d-dimensional lattice,
- S is a finite set of cell states for A,
- N is a finite cell subset of $\mathbb{Z}^d$, $N = \{ c_j \mid c_j = (x_{1j}, ..., x_{dj})$, j ϵ $\{1, ..., d\}$ $\}$, and denotes the neighbourhood of A,
- f : $(S, N) \rightarrow S$ is the local transition function, or local rule, of A.

Circling back to the initial definitions, the neighbourhood N denotes the finite set of unambiguously defined stimuli to which each cell (black box) reacts and the transition function f defines the response. The cells are simultaneously updated by means of the local transition function at each clock tick. It is common practice that the neighbourhood subset does not include the cell itself, neither does so in this case. However, in the transition rule, when there is no majority amongst neighbours, the cell's self state is used to break the symmetry.

2.2 Model

In this simulation a bi-dimensional grid or lattice $\mathbb{Z}^2$ is used with dimension length of 100. In practice this is implemented as a two dimensional array of cells c with two indexes. The array is periodically bounded where the edge cells of each side have as neighbours the edge cells of the opposite side, as mapped onto a torus.

Each cell in the lattice $\mathbb{Z}^d$ has a list of neighbours referenced by relative coordinates. The states of these neighbours are then used as input for the transition function to recalculate each cell's new state. At every point in time, each cell can be at one of 2 states in $S = \{0, 1\}$.

In the model, three different neighbourhood settings of neighbours are tested. In neither case does the self state form part of the neighbourhood. The three topologies are:

- Moore neighbourhood with radius 1: consisting of the 8 adjacent neighbours of cell c as shown in Table 2.1.

X	X	X
X	C	X
X	X	X

Table 2.1: Moore neighbourhood with radius 1 of cell C consisting of cells X

- Moore neighbourhood with radius 2: also called extended Moore neighbourhood, consisting of the 8 adjacent neighbours of c as shown in table 2.1, plus the 16 adjacent neighbours to that perimeter, as show in 2.2

X	X	X	X	X
X	X	X	X	X
X	X	C	X	X
X	X	X	X	X
X	X	X	X	X

Table 2.2: Moore neighbourhood of cell C consisting of cells X

- Von Neumann neighbourhood: consisting of the 4 neighbouring cells of c at a Manhattan distance of 1 as shown in table 2.3.

E	X	E
X	C	X
E	X	E

Table 2.3: Von Neumann neighbourhood of cell C consisting of cells X and excluding cells E

In each update the whole lattice is updated at once. At each time step, the entire grid is first calculated and then updated. In practice, each cell has a current state and a future state. Once all the cell's new states have been defined, the new states become the current states.

The transition rule f together with the initial configuration are used to model the majority vote as defined in the following subsection.

2.3 Majority Vote

In line with what is explained in the introduction, at each time step the cells will adopt the vote of the majority of their neighbours. When the initial configuration of the cellular automaton is drawn, each of the cells is at one of two states. This is similar to a voting process where each voter must choose one of two candidates. Each state (or party) has then a proportion of the initial vote. Two elements are then used as varying parameters of the majority vote in the simulation:

- A majority proportion of the population p in the initial configuration.
- The transition rule $f : (S, N) \to S$

In the initial configuration the ratio of each of the state populations is defined. To draw the proportions a probability is used as a parameter. Each cell will be initiated in the majority state S_1 with a probability equivalent to the desired initial majority proportion p. The simulation is run 200 times per each value of p in the domain {0.5, 0.505, ..., 0.995} in steps of 0.005. One correction has been made to label the majority always as the initially larger group. If the initial majority is larger for the cells with S_0 (can happen when $p \approx 0.5$), this group is taken as the initial majority.

The transition function $f : (S, N) \to S$ is applied to each cell in the cellular automaton simultaneously. At each time step the new states are calculated for each cell and then the whole cellular automaton is updated at once. The update rule has been implemented as shown in algorithm 2.1 for all neighbourhoods:

Algorithm 2.1 Calculate $A_t \to A_{t+1}$

```
Input: A_t
Output: A_{t+1}
  for each c in Z^d_A do
    a = 0
    d = 0
    for each n in N_(c) do
      if S_n == 1 then
        a = a + 1
      else
        d = d + 1
      end if
    end for
    if a == d then
      S_{c,t+1} ⇐ S_{c,t}
    else if a > d then
      S_{c,t+1} ⇐ 1
    else S_{c,t+1} ⇐ 0
    end if
  end for
```

The model implements an efficiency check: cells are set as inactive when neither itself nor any of its neighbours has been updated in the previous time step. The opposite is true when this changes, setting the cell back to active. The transition rule is only applied on active cells, leaving stable areas of the grid out of the recalculation.

After a few runs, it was observed that the final stable state was achieved in at most 40 iterations. In the case of the Moore neighbourhoods the cellular automaton would arrive to a final stable state. The Von Neumann neighbourhood set up, however, shows cyclic two phase oscillating patterns in the stable state. For safety, the simulation continues for 100 iterations, after which, the world is redrawn to a new initial configuration.

2.4 Feature Summary and Analysis

One simulation run is considered as one initial configuration drawn, to which the transition rule is applied for 100 iterations. The majority proportions are tested for every proportion p in the domain {0.5, 0.505, ..., 0.995} (100 values). The simulations are completely symmetric, so the results of the initial majorities are equally applicable for initial minority proportions of 1-p. Each proportion is then run for 200 simulation. The whole process is repeated for each of the topology settings with Moore and Von Neumann neighbourhoods. The model also allows to vary the size of the lattice $\mathbb{Z}^d$, but this has not been tested in this research.

The random initialiser used for the parameter p has a constraint: the proportion is implemented as a probability and the values are approximate. Although it can be considered that by running each simulation 200 times the differences in the initial majority are evened out, in the case of the 0.5 proportion, there is a small limitation. The majority is 99% of the times one or the other, which means that the effects of strict 50% are difficult to explore.

A summary of the constants of the simulation can be found in Table 2.4. In Table 2.5 the different tested parameters are displayed.

Name	Definition	Value
Runs	Number of simulations performed with the same parameters	200
Iterations (t)	Number of discrete time steps	100
Dimension size	length of each of the dimensions	100

Table 2.4: Constants implemented in the simulation

The results of the simulation runs are then averaged for each setting. This means that the results of each run have an equal weight in the totals displayed per majority proportion and per topology setting.

In the next section the results are shown in a graph as a mapping of the average proportion of

Name	Definition	Values	Total Settings
Majority Proportion	initial population bias of the majority	{0.5, 0.505, ..., 0.995}	100
Topologies	Shape of the neighbourhood	Moore radius 1, Moore radius 2, Von Neumann	3

Table 2.5: Parameters and values tested

the initial majority population against the average proportion of the final majority for both topologies. When the average final proportion is 1, the initial majority always takes over.

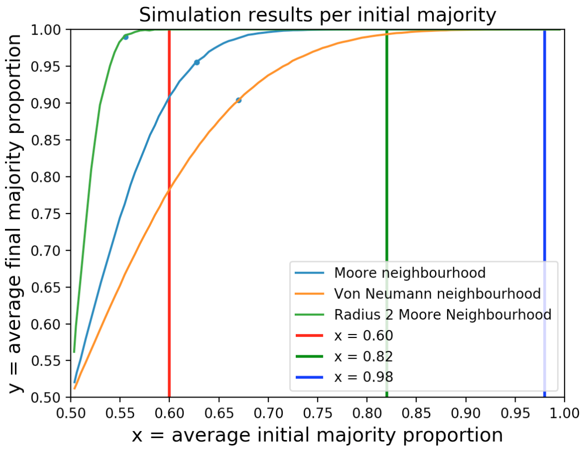


Figure 3.1: The correlation of initial majority proportion and final majority proportion overlaid with the tangent lines where the curve shows a 100% of majority takeover for each of the topologies. The dots on each of the curves indicate the start of a negative marginal final majority per increase in initial majority.

3 Results

Two aspects of the simulation are analysed in this section: (1) key findings and (2) remaining patterns observed in the end state.

3.1 Key Findings

The results show that an increase in the initial majority proportion yields a larger increase in the final majority proportion for initial majorities from 50% till 55.5% with the Moore neighbourhood with radius 2, from 50% up to 62% with the Moore neighbourhood with radius 1 and up to 67% with Von Neumann neighbourhoods.

After these inflection points, the results show a decreasing marginal conversion rate, such that an increase in the initial majority proportion yields a smaller increase in the final majority proportion. These results are summarised in Figure 3.1 below.

Intuitively, because the self state is used to break the tie when there is no majority in the neighbourhood, a higher proportion of active cells in the initial state should increase the marginal convergence of the initial majority. The fact that the self state decides in the event of a tie between neighbouring cells, directly increases the chances of the majority taking over. As a consequence, a higher number of ties will be broken in favour of the majority group, and the initial majority amplified. However, the topology puts limits to the amplification effect of the initial majority proportion, since minority groups also cluster and create stable islands of resistance, or oscillators. These results show the limit to that amplification power of the initial bias.

The results also show that the islands and oscillators do not always remain. In some scenarios the initial majority always, 100% of the times takes over the full cellular automaton. In the implementation with the radius 2 Moore neighbourhood this effect is observed when the initial majority is at least 60%, and with radius 1 Moore neighbourhood with 82% of the initial population. In the case of the Von Neumann neighbourhood, this only happens at 98%. Figure 3.1 shows the tangent lines over the convergence curves where the initial majority proportion always (without exception) converges to 100% unanimity.

Figures 3.2, 3.3 and 3.4 show in more detail the number of runs in which (1) there is at least one cell in a minority state (non-unanimous end state), and (2) there are no remaining cells in the minority end state (unanimous end state) per initial majority proportion. As mentioned above, with a Moore neighbourhood of radius 2, Figure 3.2 shows a constant unanimous state with an initial majority proportion of exactly 0.60. The Moore neighbourhood of radius 1 shows 0 non-unanimous end states with an initial majority proportion starting at 0.82 as seen in Figure 3.3. Figure 3.4 shows the results with a Von Neumann neighbourhood. In this case, consistent 100% unanimous outcomes appear with an initial majority proportion of 0.98.

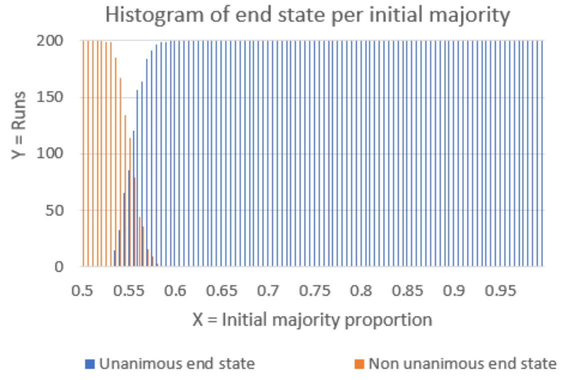


Figure 3.2: Histogram showing the count of unanimous and non-unanimous end states per initial majority proportion with radius 2 Moore neighbourhood

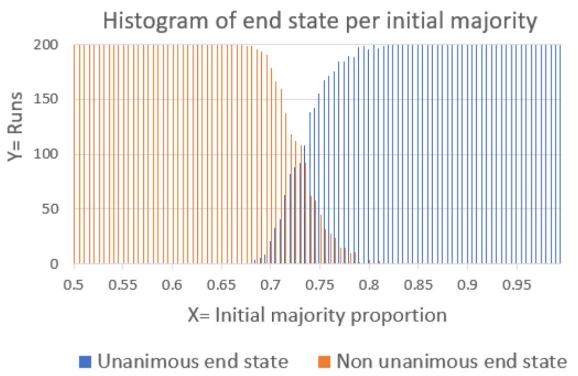


Figure 3.3: Histogram showing the count of unanimous and non-unanimous end states per initial majority proportion with a Moore neighbourhood

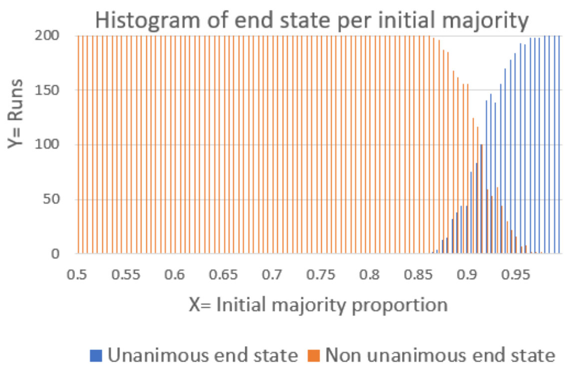


Figure 3.4: Histogram showing the count of unanimous and non-unanimous end states per initial majority proportion with a Von Neumann neighbourhood

3.2 Patterns, oscillators and boundaries

The different settings of the simulation yield a variety of landscapes of state clusters. When the proportions of the initial populations are close to each other, more islands and sizzling boundaries appear in both topologies. The higher the initial bias, the smaller and more sparse the minority clusters will be.

At first sight, it hits the eye that:

- The boundaries of Moore neighbourhood implementations show smoother lines and defined clusters or islands as seen in Figure 3.5. Only one oscillator has been found at ≈ 0.5 initial majority proportion 1 out of 10 times with radius 1 Moore neighbourhood. In Figure 3.6 shows the final state with the radius 2 Moore neighbourhood where the boundaries between clusters are the smoothest amongst the 3 cases.

- The boundaries of Von Neumann neighbourhood final states are a lot rougher and they frequently include oscillators see Figure 3.7

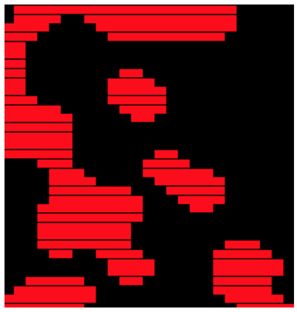


Figure 3.5: Final stable state with radius 1 Moore neighbourhood and an initial majority of 50%

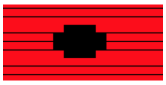
Figure 3.6: Final stable state with radius 2 Moore neighbourhood and an initial majority of 50%

When the initial majority proportion is somewhere between 0.5 and 0.7, concrete patterns are seen, different per topology. With the radius 1 Moore neighbourhood settings, these patterns are (almost) always stable. Figure 3.8 shows a recurrent resistant pattern occurring with a Moore neighbourhood of radius 1 with initial majority up to 73%. With the radius 2 Moore neighbourhood only one symmetric form has been observed with initial majority proportion of 0.53 as shown in Figure 3.9.

Figure 3.7: Final state with Von Neumann neighbourhood and an initial majority of 50%

Final state patterns are considerably different with a Von Neumann neighbourhood, which frequently yields a bi-phasic oscillating final state with initial majorities of up to 71%.

(a) 4×4 resistant cross-pattern



(b) 4×5 resistant cross-pattern

Figure 3.8: Resistant cross patterns observed with radius 1 Moore neighbourhood with initial proportion up to 0.73

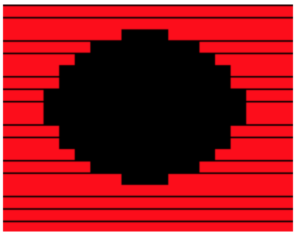
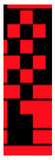


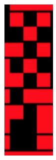
Figure 3.9: Symmetric island pattern observed with radius 2 Moore neighbourhood and 0.53 initial majority proportion.

The remaining oscillation patterns observed are also smaller when the initial majority is higher. Figure 3.10 shows a vertical blinker of 3×7, that appears at initial majority proportions of 0.5. Figure 3.11 shows the blinkers that appear when the initial majority goes up to 60%, these are of sizes 3×3 and 3×2.

Although with low frequency, the simulation with Moore neighbourhood of radius 1 with an initial majority population around 50% also shows one oscillator. The pattern is a two cycle blinker, alternating a vertical by a horizontal 3-cell strip. Figure 3.12, shows a snapshot of the superposition of the two phases.



(a) Vertical blinker in phase 1



(b) Vertical blinker in phase 2

Figure 3.10: 3×7 vertical oscillation observed with Von Neumann neighbourhood at 0.5 initial proportion

(a) Squared 3×3 blinker

(b) Horizontal 3×2 blinker

Figure 3.11: Small oscillation patterns observed with Von Neumann neighbourhood with initial majority proportion of 0.6

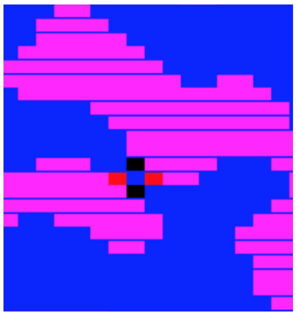


Figure 3.12: Oscillating cross pattern observed with a Moore neighbourhood and an initial majority of 50%

Finally, when we increase the initial population of the majority up to 70% , 80% and 90% with the Von Neumann neighbourhood, there is still some resistance. A stable 2×2 block keeps coming up with decreasing density at increasing initial majority proportions.

According to the classification of Cellular Automata described by Wolfram in (Wolfram, 1983), a class one cellular automaton characteristically has a stable final state and a class two automaton ends in an oscillating cycle. The majority vote Moore neighbourhood cellular automaton fits in the former class and the Von Neumann version in the latter.

4 Conclusions and Discussion

The cellular automaton simulations of the majority vote report some conditions under which a majority can become a unanimous state. Furthermore, it also shows a threshold under which the initial majority is amplified more with a small initial bias than with a larger one. These effects are more pronounced with the Moore neighbourhood of radius 2, showing bigger effects when each cell has a larger set of connections.

These statements align with previous research such as findings in (Mossel et al., 2014) and (Słowiński and MacKay, 2015). These researches however, implement far more complex rules and worlds than those used in the model reported here. The advantage of it being that this Artificial Life simulation simplifies the micro-scale behaviour as much as possible and takes advantage of the power of the cellular automaton to observe the results in an emerging complex system.

Furthermore, these findings are highly compatible with existing theories of human thinking and decision making linked to majority dynamics at different levels. Opinions that affiliate to a perceived majority are endorsed faster, independently of the social interactions (Koriat et al., 2016). Notwithstanding what the total majority really is, the majority that is available to the individual is the one that will play a role in the cognitive process (Lerman et al., 2016). This is also reflected as the islands and clusters that are formed in the final state of the majority vote cellular automata, where the local majority is the less popular state globally.

In line with the aforementioned, it follows from dynamical psychology that the decision mechanisms of oneself are in continuous interaction with the decision making processes of those in one's network (or in this case neighbourhood). As a result, consolidation, clustering and continuing diversity will arise as described in (Kenrick et al., 2003).

Finally, frequency also plays a role in the cognitive process of opinion formation. Recurrent exposure to information reinforces the perception of validity as found in (Hasher et al., 1977). Which could be an interpretation of the effect of iteratively exposing the individual (or the cell) to the majority vote, which consolidates the polarisation of the initial bias.

In terms of finding an optimum state, the majority vote rule shows vulnerability in the presence of local minima. This phenomenon has been labelled *Madness of the Crowds*, when a group aligns on an answer which seems the most popular, although erroneous (Lorenz et al., 2011) by removing the variance or diversity in the individual answers as a result of social interaction. However, when the correct answer is the one that strengthens, the same phenomena has a positive impact and can approach expert performance (Mavrodiev et al., 2013). Then, the same behaviour receives the name of *Wisdom of the Crowds.*

This is a particularly delicate topic when it is put in the light of democratic elections. One research used a cellular automaton to study the effects of electoral surveys with a majority vote model, see (Alves et al., 2002). Alves et al. argue that this could indicate that by repeatedly exposing voters to the electoral surveys, majorities strengthen, and in extreme cases democratic systems would be vulnerable to totalitarianism.

The risk becomes more pronounced when the method to elect representatives is First-Past-the-Post, since in these cases minorities could end up having no representation at all in the democratic organs. Furthermore, if tools like partisan gerrymandering are available to one of the candidate parties, it becomes fairly easy to target those clusters that are of a differing opinion, or split them in different districts, in order to manufacture a favourable bias ratio per district. This is specially relevant taking in consideration that even in network based social organisations local connections seem to have a larger impact (Bond et al., 2012).

Further thorough research would be needed to validate the extent to which the simulation with cellular automata is applicable to this challenge. Larger investigations in the precise amplification of the smallest discrete increases in bias would be interesting to see how sensitive this majority vote systems could be in the large scale.

Finally, the model implemented in this thesis is strictly deterministic, and still the results show alignment with cognitive processes and social dynamics. It would be interesting to explore other implementations with agents, behaviours or rules where some systemic method of avoidance is included with a deterministic implementation (Berto and Tagliabue, 2017). The great majority of the literature found while conducting this research (if not all) uses complex stochastic variables to model such effects. However, what other methods are available to simulate similar behaviours without statistically manipulating the parameters? A comparison between those dynamic representations and alternative deterministic models would further demonstrate the potential of artificial life as a computing tool.

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