Optimal Experimental Design for Calibration

Internship MSc Applied Mathematics

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1 Introduction

1.1 Xsens

Xsens Technologies B.V. is a company that specializes in 3D motion tracking technology, using inertial measurement units (IMUs). Their products are used in fields such as film industry, sports science and rehabilitation centers. The company was founded in 2000, in Enschede,The Netherlands. Since then, it has expanded to four locations in Los Angeles, USA , in Hong Kong and in Shanghai. Since 2017, Xsens is part of mCube, a company that provides the world's smallest and lowest power MEMS motion sensors.
The company focuses on capturing movement, both for industrial applications and human movement. Xsens portfolio is divided into three parts [1]. First, are the 3D body motion capturing solutions: MVN Awinda and MVN Link, Figure 1a. MVN AWinda, consists of 17 wireless sensors that can be attached to the body with straps. The MVN Link uses the same 17 sensors however, the sensors are placed on a full body Lycra suit, achieving even more accurate data recording. Second, are the inertial sensor modules called the MTi series, Figure 1b. The MTi series includes a variety of products having several integration levels and are mostly used for industrial applications. Last, the Xsens DOT, Figure 1c, a wearable sensor module meant to analyse and report human kinematics. Moreover, the user can combine different DOTs for more accurate complex motion tracking data.

(a) MVN Awinda - MVN Link (b) MTi series. (c) Xsens DOT

Figure 1: Xsens portfolio.

1.2 Problem description

As mentioned above Xsens is based on IMUs systems for motion capturing motion. An IMU consists of clusters of accelerometers, which measure static and/or dynamic forces of acceleration, gyroscopes, which measure angular velocities, and sometimes (even) magnetometers, which measure the magnetic field. The IMU's components are normally Micro Electro Mechanical Systems (MEMS), which are suitable for many application because of their small size, their low power consumption, as well as their low cost. Unfortunately, these systems have deterministic errors, due to their physical nature . These errors include non zero biases, non accurate scaling, cross axis sensitivity and sensor axis misalignments. The process of identifying these quantities is referred to as IMU calibration (parameter estimation). As defined in [2] *Calibration is the process of comparing instrument outputs with known reference information and determining coefficients that force the output to agree with the reference information over a range of output values*.

Currently, Xsens relies on a calibration method using industrial robotic arms. Each sensor is being placed on the robotic arm calibration, and then the arm follows a predetermined trajectory. Afterwards, we determine the coefficients that force the sensor's output to agree with the known trajectory within an acceptable range.The inconvenience of this procedure is that the process takes much time to complete and needs to be repeated several times. Furthermore, the choice of the trajectory that the robot follows is based on experience. Hence, as the production of sensors and sensor variants increase, there is a need for a new calibration method for each type, which not only satisfies our accuracy requirements but also using the robot resource as little as possible. In other words, we want to be able to calculate an optimal calibration procedure, satisfying sensor, and system-specific constraints. In literature, this procedure is referred to as optimal experimental design (OED).
Generally, the aim of OED is the design of optimal experimental setups which results in obtaining more "informative" data during an experiment, that is, data that can reveal more information on the desired parameters, in comparison to a less educated input. For more details about OED please refer to [3] and [4].

The objective of this work is to provide an optimal signal trajectory for calibration of MEMS sensors, that yield measurement data resulting in estimates of the unknown model parameters with a minimal statistical uncertainty. The procedure that we will follow is summarized by the flow chart in Figure 2. First, we will perform the parameter estimation of the sensor using a random signal trajectory. Given the estimated parameters, the OED is implemented to determine the optimal signal input. Finally, we repeat the parameter estimation using the optimal input signal and we compare the performance of the estimations.

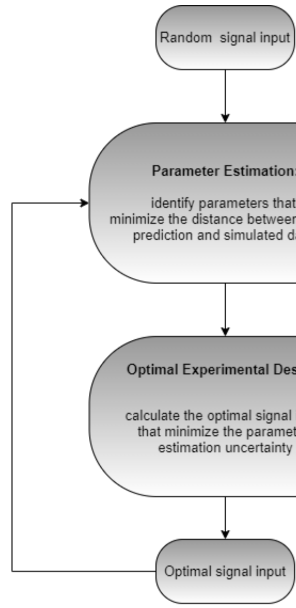


Figure 2: Flow chart of the parameters estimation and optimal experimental design process.

The internship report is structured as follows. In Chapter 2 we will introduce a mathematical framework for parametric estimation. Chapter 3 introduces the mathematical model and a calibration method of the sensor. In Chapter 4 the optimization method based on the covariance matrix is presented and the benefits of the proposed method are highlighted with simulated results. Chapter 5 summarizes the obtained results.

2 Mathematical background

In this chapter, the method of least squares is introduced for the estimation of system parameters based on [5]. Least squares is an estimation procedure that was developed in 1795, by Carl Friedrich Gauss, and mainly used for astronomical computations. Since that time the method has been applied for the solution of many problems with the most important application in data fitting. In general, least squares method attempts to estimate parameters by minimizing the squared discrepancies between observed data, on the one hand, and their expected values on the other.

2.1 Least squares approach

A non-linear least-squares parametric estimation problem is an optimization problem in the form

$$\begin{aligned} &\underset{p \in \mathbb{R}^{n_p}}{\text{minimize}} && f(p) \\ &\text{subject to} && g_i(p) = 0, \quad i = 1, \ldots, n. \end{aligned} \tag{1}$$

The function f is called an objective function, with $p \in \mathbb{R}^{n_p}$ is denoted the unknown parameters that need to be estimated and the functions g_i are called constraint and are the condition that the solution must satisfy.

Let the function $h(p)$ represent the model response, and y represent the measurement values. The difference between the model function and the measured value called residual and is denoted by $r(p)$. Let M be the number of samples, then r is the vector function

$$r(p) = \begin{bmatrix} r_1(p) & r_2(p) & \ldots & r_M(p) \end{bmatrix}^T.$$

Our purpose is to find the value of p, denoted by $\hat{p}$, that minimizes the sum of squares of the residuals. Therefore, the objective function can be written as

$$f(p) = \frac{1}{2}\sum_{i=1}^{M}(y_i - h_i(p))^2 = \frac{1}{2}\sum_{i=1}^{M} r(p)^2 = \frac{1}{2} r(p)^T r(p) = \frac{1}{2}||r(p)||^2,$$

Hence, the optimization problem (2) can be formulated as

$$\hat{p} = \min_{p \in \mathbb{R}^{n_p}} \frac{1}{2}||r(p)||^2,$$

and the least-squares problem can be interpreted as trying to find the point $\hat{p}$ in parametric space $\mathbb{R}^{n_p}$ that corresponds to the point $r(\hat{p})$ in observation space $\mathbb{R}^m$ that is the closest to zero.

2.2 Weighted least-squares problem

In many cases the measurement values $y \in \mathbb{R}^m$ are polluted by additive noise, i.e. $\tilde{y}_i = y_i + \epsilon_i$, where the ϵ_i are assumed to be independent and zero - mean Gaussian distributed $\mathcal{N}(0, \Sigma_y)$ with $\Sigma_y \in \mathbb{R}^{m \times m}$ the covariance matrix. We define the weighting matrix Σ_y^{-1} as the inverse of the covariance matrix of the noise. Then, the weighted least-squares is of the form

$$
\begin{aligned}
\underset{p \in \mathbb{R}^{n_p}}{\text{minimize}} \quad & \frac{1}{2}||r(p)||^2_{\Sigma_y^{-1}} \\
\text{subject to} \quad & g_i(p) = 0, \quad i = 1, \ldots, n.
\end{aligned}
\tag{2}
$$

where g_i are the constraints imposed by the problem. For instance, in our problem, we need the optimal trajectory to be physically realizable.

3 Calibration - Parameter Estimation

This chapter introduces a calibration method of a sensor. In this project, we consider only accelerometers. This calibration method is based on least squares estimation and the goal of this process is to determine the bias, the scale factor along with the state of the sensor. First, the sensor model is described followed by the formulation of the parameter estimation problem. Thereafter, the results are illustrated and lastly, an evaluation of the results is performed to assess the quality of the calibration.

3.1 Sensor model

This section presents the error model for the accelerometer. The sensor error model describes the process of measurement from the actual physical quantity to the sensor output.

The simplest acceleration model for a single axis sensor considers only a scale factor and a constant bias. Then the single axis sensor model can be described as,

$$y(t) = k\alpha(t) + b,$$

where $y(t) \in \mathbb{R}$ represents the output of the accelerometer, $k \in \mathbb{R}$ is the scale factor, $b \in \mathbb{R}$ denotes the bias, and $\alpha(t) \in \mathbb{R}$ stands for the acceleration.
A MEMS triaxial accelerometer is composed of three single axis orthogonally mounted accelerometers, each of them measures the acceleration on each axis. The generalization of the single axis model axis, for triaxial accelerometers, which accounts for scale factors and bias reads as,

$$y(t) = K\boldsymbol{\alpha}(t) + \mathbf{b},$$

where $K \in \mathbb{R}^{3\times3}$ is a diagonal matrix that includes the scale factors, $\mathbf{b} \in \mathbb{R}^3$ represents the bias. $\boldsymbol{\alpha}(t) \in \mathbb{R}^3$ denotes the acceleration and $y(t) \in \mathbb{R}^3$ represents the measured acceleration that coming out of the sensor.
In the ideal case, the matrix K is the identity matrix, the $\mathbf{b}$ is a null vector. Moreover, in practice the set of single axis accelerometer is not orthogonally mounted, however we assume that there is. To complete the sensor error model we assume that the output measurements are affected by Gaussian noise. Thus the model employed is

$$y(t) = K\boldsymbol{\alpha}(t) + \mathbf{b} + \nu,$$

where $\nu \sim \mathcal{N}(0, \Sigma_\nu)$.

3.1.1 System model

In this project, we used the derivative of acceleration, jerk, to generate smooth trajectories for the robotic arm. Hence, our system model is *jerk - controlled*. In order to generate and control the sensor's movement, we discretize the time horizon of the motion into N equal time intervals, $[\tau_i, \tau_{i+1}]$ for $i = 0, 1, \ldots, N$ with time period $\Delta\tau$. Let $u_i \in \mathbb{R}^3$ the control input that is applied to the sensor on each axis, on the interval $[\tau_i, \tau_{i+1}]$ and $x_i \in \mathbb{R}^3$ denote the state of the sensor with $i = 0, 1, \ldots, N$. Considering that, the acceleration follows a linear piece-wise trajectory on each axis, where on each interval $[\tau_i, \tau_{i+1}]$ the acceleration is given by

$$\boldsymbol{\alpha_i}(\tau) = x_i + u_i\tau, \qquad \text{for } i = 0, 1, \ldots, N.$$

Moreover, since we want the acceleration trajectory to be continuous there is a need to set some constraints, that is

$$x_i - x_{i+1} = (u_{i+1} - u_i)\tau_i, \qquad \text{for } i = 0, 1, \ldots, N.$$

An illustration of such a *jerk - controlled* acceleration trajectory, with $N = 15$ time intervals is given on Figure 3.

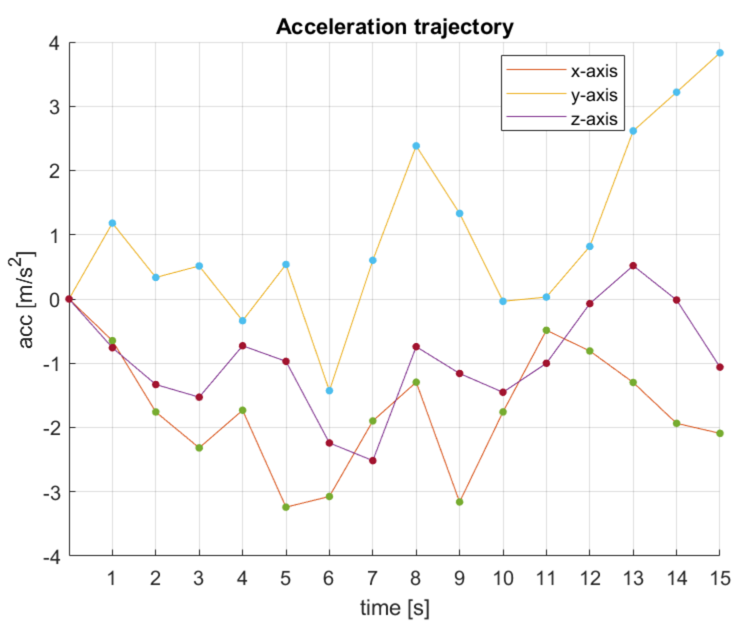


Figure 3: Jerk-controlled acceleration trajectory, with $N = 15$ and $\Delta\tau = 1$s.

3.1.2 Measurement model

Consider M be the total number of measurements of the experiment, at a sampling rate of f_s. We define the vector of the noisy observed data $y \in \mathbb{R}^{3\times M}$ as follows,

$$y = \begin{bmatrix} y_1 & y_2 & \dots & y_M \end{bmatrix},$$

and the model response $h(\cdot) \in \mathbb{R}^{3\times M}$ is defined as

$$h(\cdot) = \begin{bmatrix} h_1 & h_2 & \dots & h_M \end{bmatrix} = \begin{bmatrix} K\boldsymbol{\alpha}(t_1) + b & K\boldsymbol{\alpha}(t_2) + b & \dots & K\boldsymbol{\alpha}(t_M) + b \end{bmatrix}$$

where the value of the acceleration $\boldsymbol{\alpha}(t)$, is given by the functions $\boldsymbol{\alpha}_i(\tau)$, depending on the time interval measurement $[\tau_i, \tau_{i+1}]$. More specifically, when $t_j \in [\tau_i, \tau_{i+1}]$ then the actual acceleration is given by

$$\boldsymbol{\alpha}(t_j) = x_i + u_i t_j \quad \text{for} \quad i = 0, 1, \dots, N \quad \text{and} \quad j = 1, 2, \dots, M.$$

Finally, for simplicity we define the vector $p \in \mathbb{R}^6$ which include the unknown parameters that have to be estimated,

$$p = \begin{bmatrix} b_x & b_y & b_z & K_x & K_y & K_z \end{bmatrix}.$$

3.1.3 Problem formulation

Parameter estimation is formulated as a non-linear optimization problem, whose objective is to estimate the parameter values, that minimise a scalar measure of the distance between model predictions and simulated data. To solve the optimization problem we use the weighted least square approach which introduced in Section 2, with weights set to the inverse of the experimental noise. Summing up, the parameter estimation problem can be formulated as follows

$$\begin{aligned} \underset{p,x,u}{\text{minimize}} \quad & \frac{1}{2}\sum_{k=1}^{M} ||y_k - h_k(p,x,u)||_W^2 \\ \text{subject to} \quad & x_i - x_{i+1} = (u_{i+1} - u_i)\tau_i, \quad i = 0, \dots, N-1, \\ & \boldsymbol{\alpha}(t_1) = \boldsymbol{\alpha}_{\text{initial}}, \\ & \boldsymbol{\alpha}(t_M) = \boldsymbol{\alpha}_{\text{end}}, \\ & \boldsymbol{\alpha}(\boldsymbol{t_1}) \neq \boldsymbol{\alpha}(t_M). \end{aligned} \tag{3}$$

where the weighting matrix W is the inverse of the diagonal matrix Σ_ν, which contains the covariance of the measurements. With $\boldsymbol{\alpha}(t_1)$ and $\boldsymbol{\alpha}(t_M)$ are denoted the initial and the final acceleration, respectively. The problem is subject to constraints, the first imposed by the continuity of the acceleration trajectory. Moreover, in order to make the estimation solvable, we have to reduce the degrees of freedom of the state, therefore the initial and the final acceleration of the sensor must be known and not equal.

3.2 Numerical Results

Within this section, the simulation results from the parametric estimation are studied. The optimization problem (3) was implemented in MATLAB with the use of CasADi, an open - source software tool [6]. CasADi provides the possibility to formulate complex symbolic expressions and generate derivative information efficiently using algorithmic differentiation.
We used the data sheet of the accelerometer Memsic MXD2020U/WTo [7] to simulate the results, so the bias and the scale factor are assumed Gaussian with standard deviations 0.1 m/s^2 and 0.005 respectively. In addition to that, zero-mean additive white Gaussian noise was added to the acceleration measurements,with standard deviation of $\sigma_\nu = 0.0002$ m/s^2. In the following, we provide the calibration results, obtained by solving the optimization problem (3).

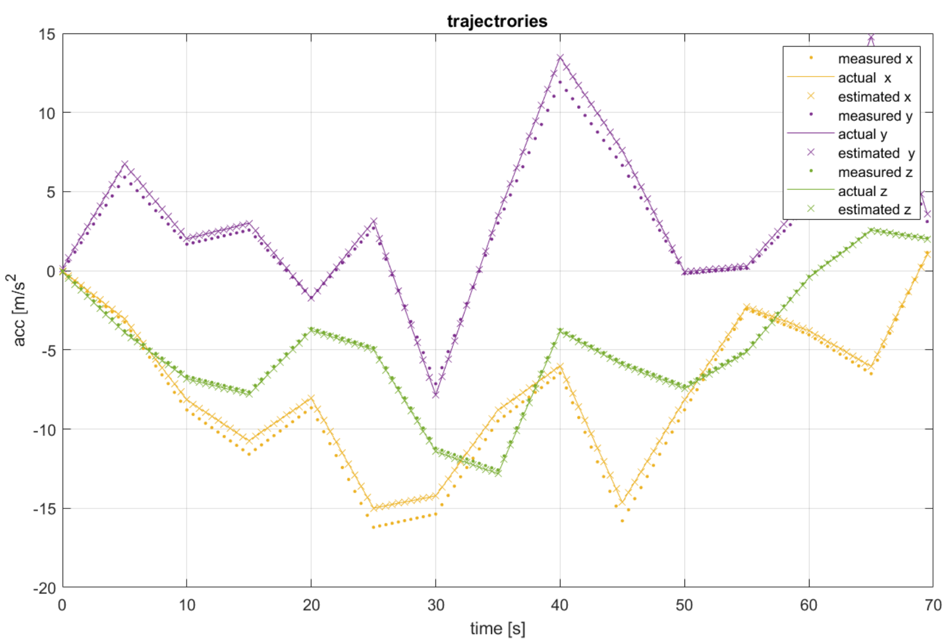


Figure 4: Comparison of actual, measured and estimated acceleration, on frequency of 2Hz and $\Delta\tau = 5s$.

In Figure 4 the trajectory in each axis is illustrated. The solid lines depict the true acceleration of the sensor, while the dots show the sensor's output sampled on the frequency of 2Hz. As observed in the plot the estimated trajectory converges to the actual trajectory. Figure 5 illustrates in detail the convergence of the estimated motion to the actual. In the upper graph the states x_i are depicted, while in the lower graph the controls u_i are plotted for $i = 0, 1, \dots, 14$. Therefore, we estimate the motion of the accelerometer with an uncalibrated sensor.

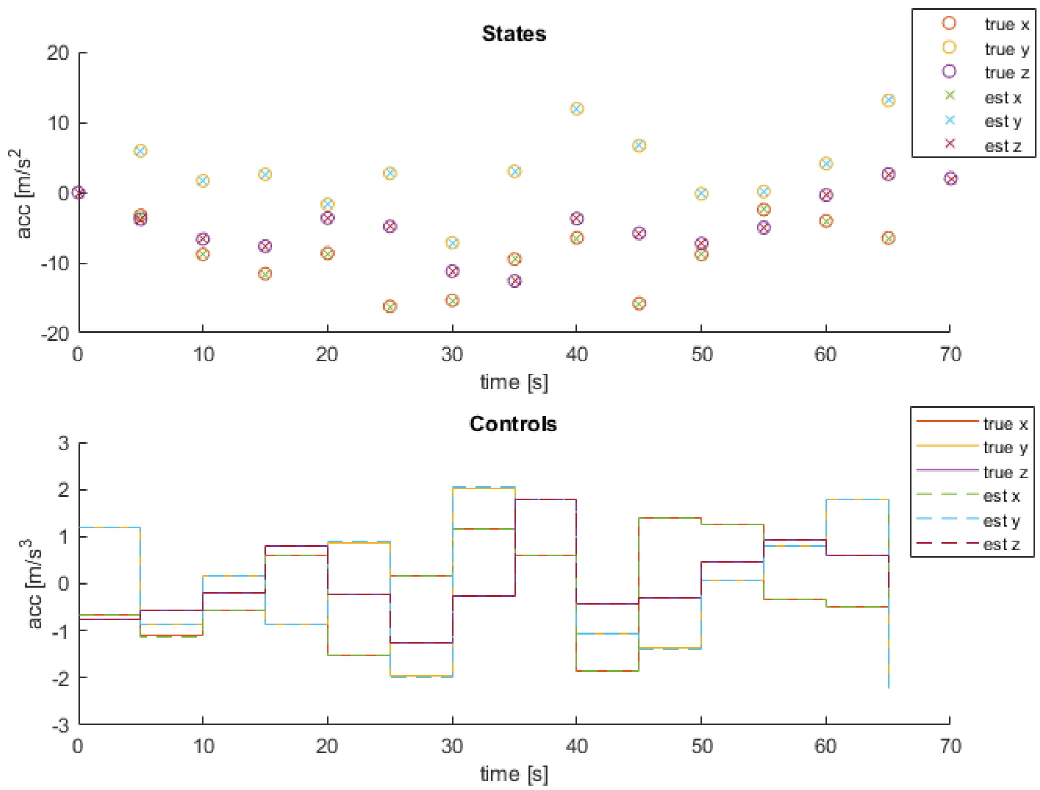


Figure 5: Comparison of actual and estimated state and inputs on frequency of 10Hz and $\Delta\tau = 5s$.

Next, in order to assess the calibration, we perform the calibration on 50 accelerometers following piece-wise linear trajectory with $N = 14$ linear curves, on the frequency of 2Hz. The comparison between the actual and the estimated parameter errors are depicted in the following graphs. Figure 6 illustrates that the estimated parameter errors, the bias in the left graph and the scale factor in the right graph, lie very close to the actual values.

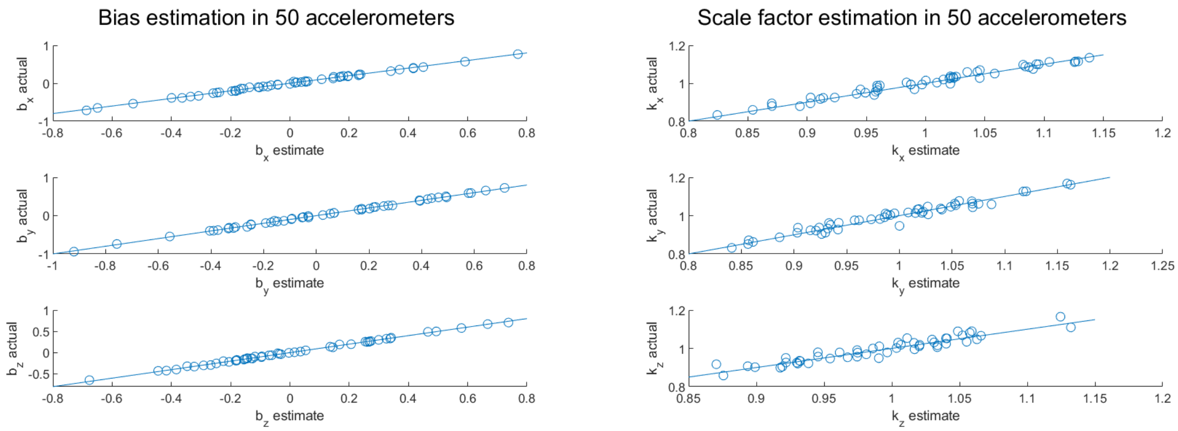


Figure 6: Comparison of actual and estimated parametric errors.

Hence, without knowing the exact motion of the accelerometer we can calibrate both the bias and the scale factor of the sensor. However, there is still scope for improvement in the accuracy of the estimation especially in the estimation of the scale factor.

4 Optimal Experimental Design

In this section, an introduction to an optimal input signal for accelerometer calibration is given and the formulation of an OED problem for the sensor is provided. The formulation and the implementation of the OED problem was based on [8], [9] and [10] .

4.1 OED formulation

In sensor calibration, the design of the signal input provided for the system during the procedure is crucial for the accuracy of the parameter estimation. If a signal is not suitable, the data obtained during the calibration might not contain enough information on the desired parameters and the estimation might be inaccurate.

Our goal is to reduce the parameter estimation uncertainty. A way to evaluate the quality of the estimation results without knowing the true signal is through the covariance matrix [11]. The estimated parameter covariance matrix is a square, positive semi-definite matrix and its diagonal contains the parameter variances. Intuitively, higher values of variance entail higher uncertainty.
Hence, the main idea of OED is to use an information function of the covariance matrix $\Phi(\Sigma_p)$ as the objective of an optimization problem. Thus, the optimal experimental design problem can be written generally as

$$\begin{aligned} \underset{x,\, u}{\text{minimize}} \quad & \Phi(\Sigma_p(\cdot)) \\ \text{subject to} \quad & g(\cdot) = 0, \\ & x_{\min} \leq x \leq x_{\max}, \\ & u_{\min} \leq u \leq u_{\max}. \end{aligned}$$

where $x_{\min}, u_{\min}, x_{\max}, u_{\max}$ are the lower and the upper bounds of the input and the state. The constraint $g(\cdot)$ is based on the system dynamics.
There exist several optimality criteria that can be used in the optimization problem with different features [12]. In this project, we use the so called *A-optimality*, within the objective of the OED problem, which seeks to minimize the trace of the covariance matrix of the model coefficient estimates that obtained from the calibration using random input signal. Therefore, our optimal

signal input can be obtained from the solution of the following optimization problem

$$
\begin{aligned}
\underset{x,\,u}{\text{minimize}} \quad & \text{Trace}(\Sigma_p(x,u,\bar{p})) \\
\text{subject to} \quad & x_i - x_{i+1} = (u_{i+1} - u_i)\tau_i, \quad i = 1, \ldots, N-1, \\
& x_{\min} \leq x \leq x_{\max}, \\
& u_{\min} \leq u \leq u_{\max}.
\end{aligned}
\tag{4}
$$

In (4), $\bar{p}$ is an initial guess of sufficient quality for the true value of p, which can be obtained by solving the parameter estimation problem that was analyzed in Section 3. However, since in this problem we are concerned only about the parameters errors, the optimization variables are the bias and the scale factor and not the movement of the sensor. Thus, the parametric estimation is of the form

$$
\begin{aligned}
\underset{p,\,x}{\text{minimize}} \quad & \frac{1}{2}\sum_{k=1}^{M} ||y_k - h_k(p,x,u,t_k)||_W^2 \\
\text{subject to} \quad & x_i - x_{i+1} = (u_{i+1} - u_i)\tau_i, \quad i = 0, \ldots, N-1
\end{aligned}
\tag{5}
$$

where u is a known random initial input signal.

In case further improvement is desired, the OED and the calibration could be repeated based on the improved estimation results.

4.1.1 Motivation

Before the implementation of the OED is needing to study the behaviour of the objective function of (4) and examine if indeed there is exist an optimal solution.

In order to visualise the objective, we choose a single axis accelerometer following trajectory with two time intervals. Therefore the model response is of the form

$$
h(t_k) = \begin{cases} k(u_1 t_k + x_1) + b, & \text{if } t_k \in [\tau_0 \ \tau_1] \\ k(u_2 t_k + x_2) + b, & \text{if } t_k \in [\tau_1 \ \tau_2]. \end{cases}
$$

for $k = 1, 2, \ldots, M$.

Next, for several combinations values of input signals u_1, u_2 we implement the parameter estimation (5) and we calculate the trace of the covariance matrix, the result is represented in Figure 7.

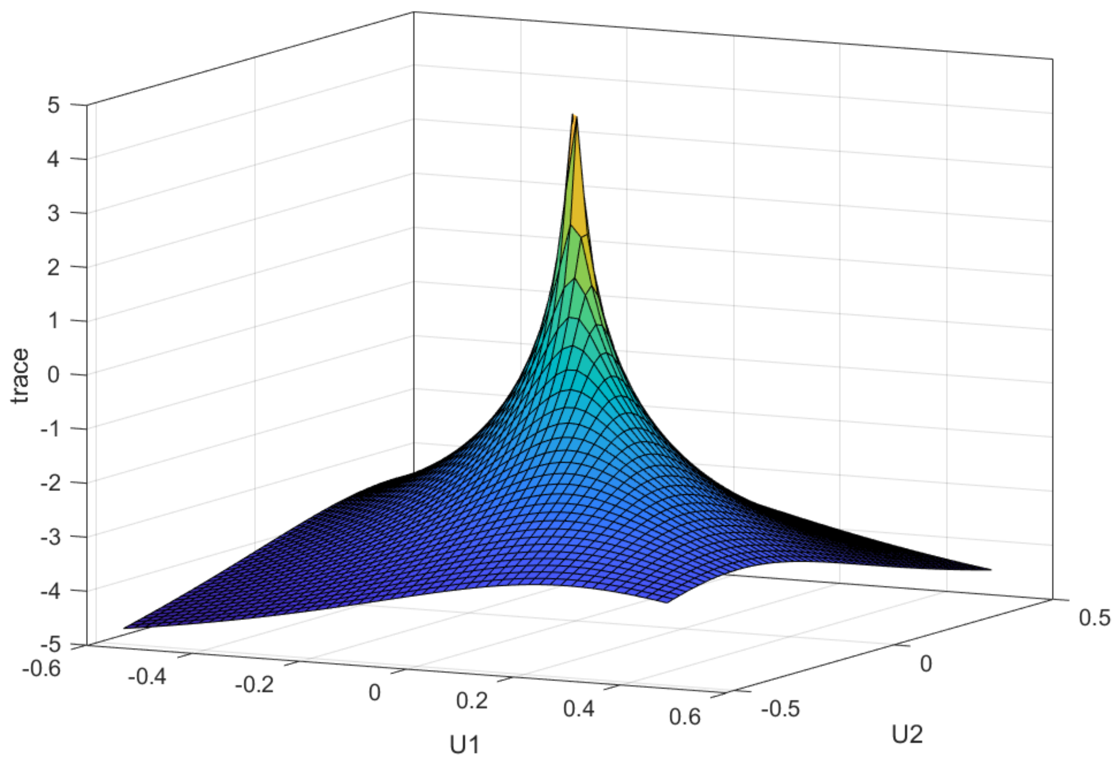


Figure 7: The logarithmic trace of the covariance matrix for various input signals u_1, u_2.

The surface illustrates the logarithmic trace calculated for various inputs signals u_1, u_2 taking values within the interval $[-0.5 \; 0.5]$. We choose an area that is close enough to zero to focus on the interesting parts of the surface. First, we see that the objective is continuous, except the point $(0, 0)$, and not constant, thus there are input signals that minimize the trace. Next, we observe that when both the inputs are close to zero the trace is increased, meaning that the estimation becomes inaccurate, and specifically when $u_1 = u_2 = 0$ the estimation is impossible since the sensor must be moving to estimate the scale factor. On the other side, when the inputs reach their maximum absolute values the trace becomes minimum.
In the following section, we analyze in more detail how the optimal signals behave and if indeed they minimize the accuracy of the calibration.

4.2 Numerical Results

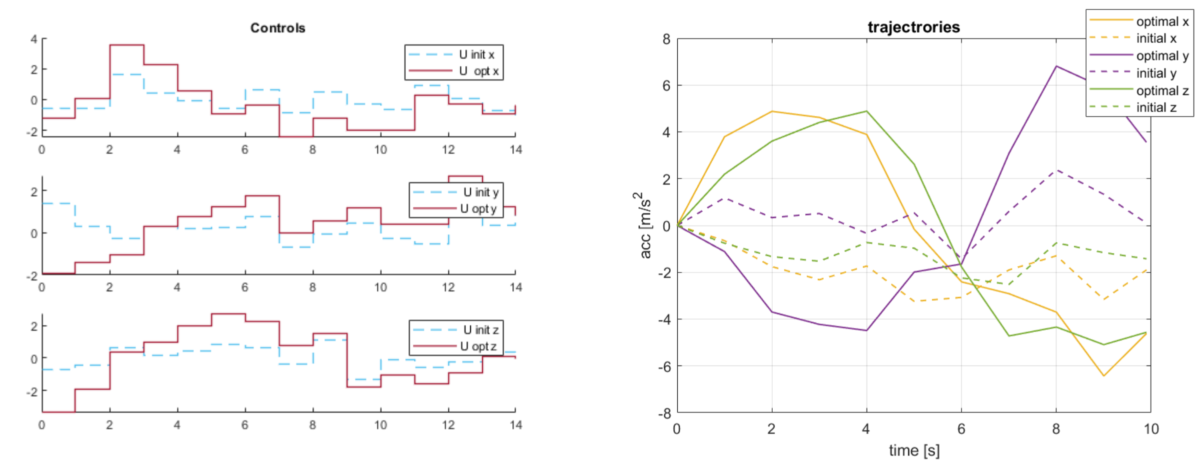
In this section, we used the data-sheet values for accelerometer Memsic MXD2020U/WTo to simulate the OED described above. The optimization problem (4) were solved using the open-source

CasADi, in MatLab. The upper and lower bounds on the state and on the input are

$$x_{\min} = u_{\min} = -30 \quad \text{and} \quad x_{\max} = u_{\max} = 30.$$

Moreover, the initial trajectory is divided into $N = 10$ time intervals with period $\Delta\tau = 1s$. Lastly, the measurements results obtained at sampling rate of 10Hz.

In the following, we provide a comparison between the optimal signal and the initial one. Figure 8a shows the optimized inputs in comparison to the initial inputs. As it can be noticed, the absolute values of the optimal inputs are higher than the initial, meaning that the rate of change of the acceleration with time is greater than the one in the initial signal. This can be visualized in Figure 8b where the optimal acceleration trajectory is compared with the initial trajectory. We observe that the absolute value of the optimal acceleration is larger and the rate of change is higher, showing how the optimized signal is used to intentionally excite different parts of the system and increase the information content.



(a) Comparison of initial and optimal inputs.

(b) Comparison of initial and optimal trajectory.

Figure 8: Results of OED

Next, in order to examine if there is a unique optimal input signal, we repeat the implementation of OED on 10 accelerometers. The resulting 10 optimal trajectories on each axis are illustrated in Figure 9. As shown on the graphs, every two optimal signals follow either approximately the same sinusoidal curve or each symmetric curve with respect to the time axis. Intuitively, since the optimal acceleration is sinusoidal, the optimal displacement and velocity of the sensor are changing sinusoidally. Hence, the robotic arm should be initialized at zero speed, then increases

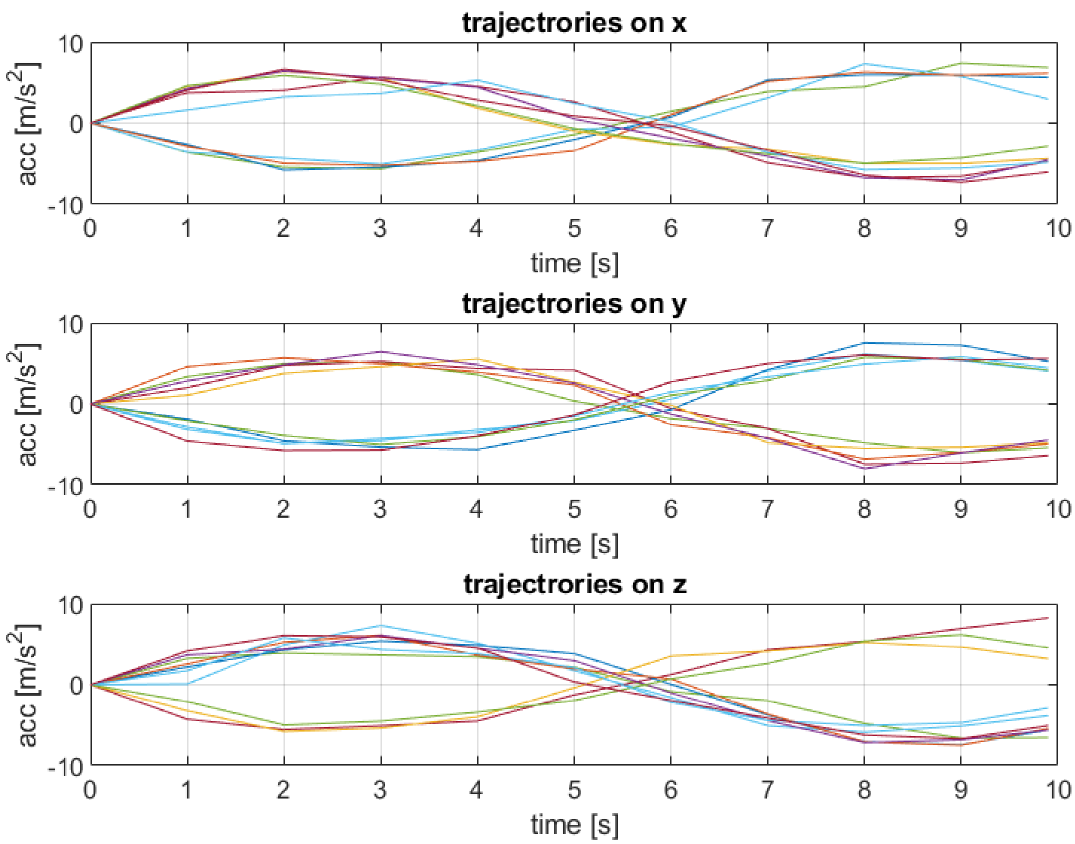


Figure 9: Optimal trajectories on each axis.

to maximum speed and gradually, its velocity decreasing. When the velocity reaches zero then the robotic arm changes direction and speeds up to reach the maximum velocity on the opposite side.

The increase in certainty achieved by the experimental design can be illustrated in the following graphs. We used the optimal signal to generate optimized measurement data and we repeated the calibration according to Section 3 with otherwise unchanged settings. We implemented the procedure in 30 accelerometers and we compared the optimal with the initial signal inputs in terms of variances reduction and convergence of the estimated parameter vector to the true value. Figure 10 presents the trace of the covariance, which is the sum of the variances of the regression coefficients. We observe the OED reduces the sum of the bias and scale factor variances to a single value independent of the initial signal, meaning that the optimization reaches a minima. Furthermore, the reduction of the uncertainty can clearly be visualized in Figures 11 which depict the errors of the estimation. Figure 11a, shows the absolute difference between the simulated and the estimated value of bias on each axis, while Figure 11b depicts the corresponding result for

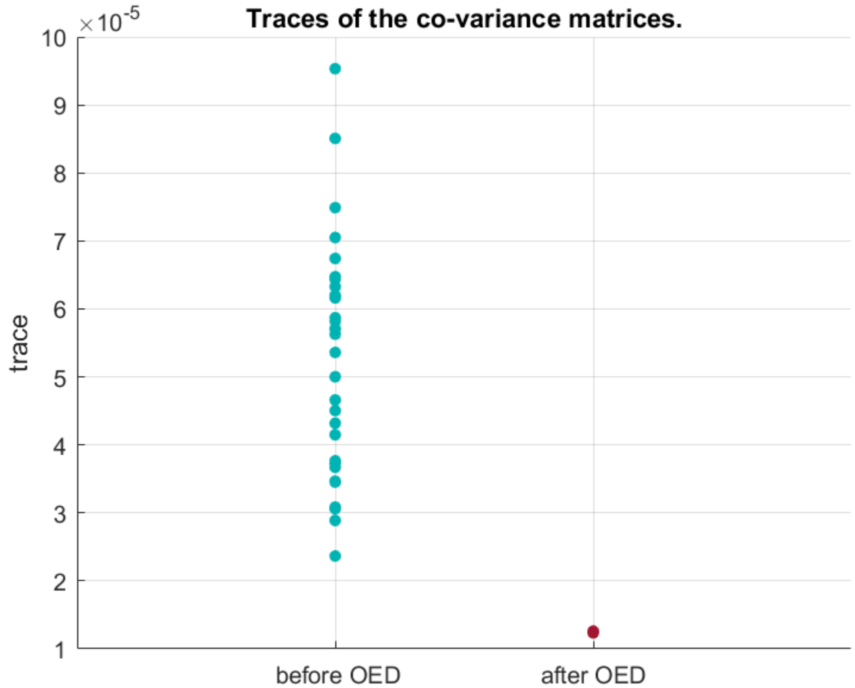
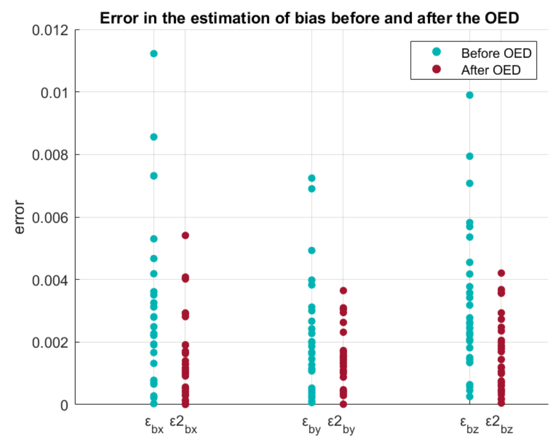
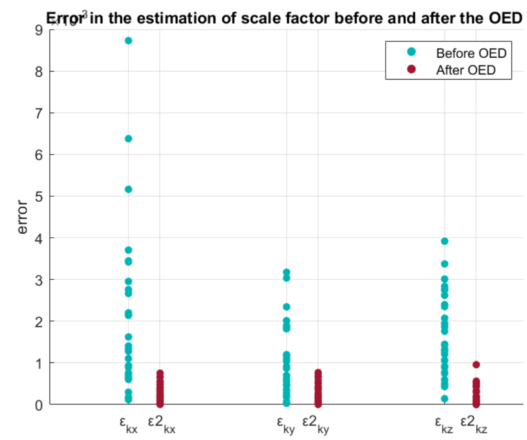


Figure 10: Trace of the covariance matrices. Blue colour indicates the results of the calibration using random signal while red colour indicates the results that obtained using the optimal signal.

the scale factor, where the reduction is greater. These results verify that an optimal design of the experiment can provide an estimation of parameters with high accuracy both in terms of converging parameter values and variance.



(a) Absolute difference between estimated and simulated bias vector.



(b) Absolute difference between estimated and simulated scale factor.

Figure 11: Comparison between the initial and the optimal signal. Blue colour indicates the results of the calibration using random signal while red colour indicates the results that obtained using the optimal signal.

4.3 Addition of process noise

To improve the model's fidelity for real-world applications, in this section we take into account the system noise.

The process that is followed is similar to the aforementioned steps the only difference is that in this case, for each motion interval, the input signal that is sent to the system is subject to noise. This input noise is a special kind of process noise. The procedure is summarized in the schematic diagram in Figure 12.

As a remedy to this, as proposed in [8], additional degrees of freedom can be introduced in the optimization problem. At each interval we add the process noise $w_i \in \mathbb{R}^3$ with $i = 0, 1, \ldots, N$ which depends on the corresponding input signal u_i. Specifically, w_i follows zero-mean Gaussian distribution with standard deviation a function of the signal i.e. $w_i \sim \mathcal{N}(0, \sigma_u(u_i))$.
Therefore, the inputs that actually excite the system are

$$\bar{u}_i = u_i + w_i, \quad \text{for} \quad i = 0, \ldots, N.$$

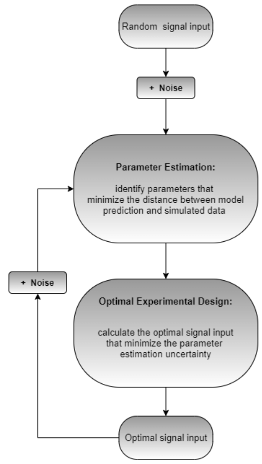


Figure 12: Flow chart of the parameters estimation and optimal experimental design process under system noise.

Taking everything into account, the parameter estimation problem becomes

$$\begin{aligned} \underset{p,x,w}{\text{minimize}} \quad & \frac{1}{2}\sum_{k=1}^{M} ||y_k - h_k(p,x,\bar{u})||^2_{\Sigma_y^{-1}} + ||w||^2_{\Sigma_w^{-1}} \\ \text{subject to} \quad & x_i - x_{i+1} = (u_{i+1}-u_i)\tau_i, \quad i = 1,\dots,N-1. \end{aligned} \tag{6}$$

where Σ_w a diagonal matrix with entries the $\sigma_u(u_i)$.
Consequently, the OED is of the form

$$\begin{aligned} \underset{x,u}{\text{minimize}} \quad & \text{Trace}(\Sigma_p(x,u,\bar{p},\bar{w})) \\ \text{subject to} \quad & x_i - x_{i+1} = (u_{i+1}-u_i)\tau_i, \quad i = 1,\dots,N-1, \\ & x_{\min} \leq x \leq x_{\max}, \\ & u_{\min} \leq u \leq u_{\max}. \end{aligned}$$

where $\bar{p}, \bar{w}$ the results of the optimization problem (6).

Next, we implement the OED under process noise on 30 accelerometers and we set the lower and upper bounds as

$$x_{\min} = u_{\min} = -20 \quad \text{and} \quad x_{\max} = u_{\max} = 20.$$

In order to evaluate the quality of the results, we compared the performance of the optimal with the initial signals. Figure 13 depicts the traces of the covariance matrices obtained with the initial signal and with the optimal one. We observe that also in this case, where our system model is more complicated, the uncertainty was decreased by the OED. This result is verified in the plots in Figure 14, where the absolute differences between the estimated and the actual values of the bias and the scale factor are illustrated. These results show that it is also possible to retrieve confident parameter estimates in a more complex model.

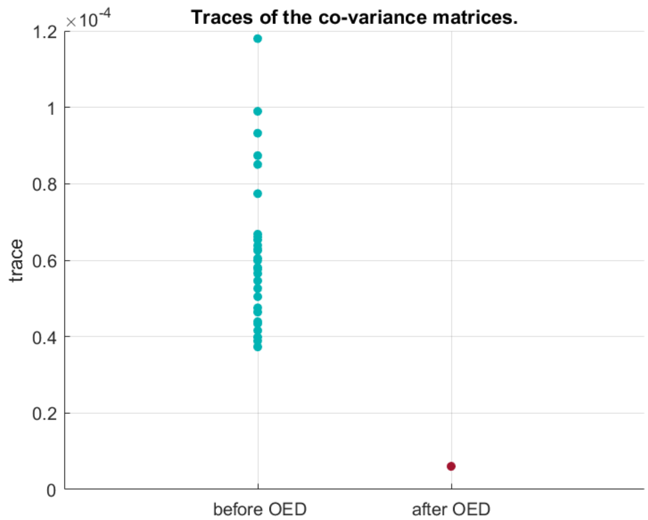
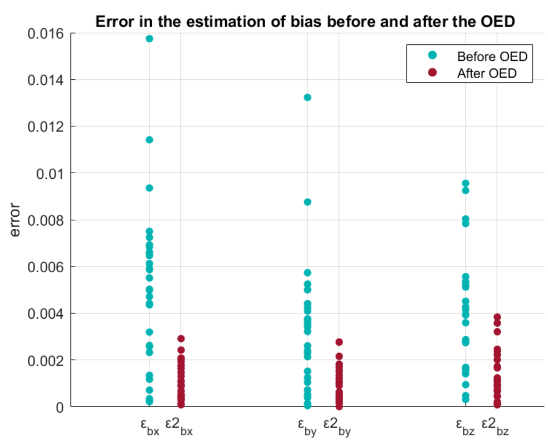
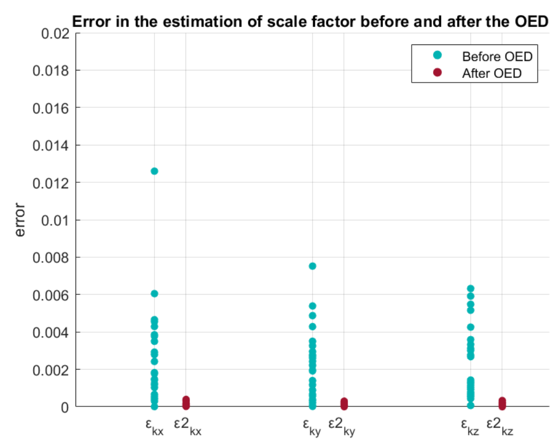


Figure 13: Trace of the covariance matrices. Blue colour indicates the results of the calibration using random signal while red colour indicates the results that obtained using the optimal signal.



(a) Absolute difference between estimated and simulated bias vector.



(b) Absolute difference between estimated and simulated scale factor.

Figure 14: Comparison between the initial and the optimal signal. Blue colour indicates the results of the calibration using random signal while red colour indicates the results that obtained using the optimal signal.

5 Conclusion

This work can provide a further theoretical understanding of the OED for sensor calibration and its potential application in realistic experiments. In this project, we have presented an optimal signal trajectory that maximizes parameters identifiability, by reducing the covariance matrix. The OED has been carried out for the calibration of a triaxial MEMS sensor. Thus, a suitable non-linear mathematical model for calibration related to *jerk-controlled* accelerometer was introduced.
First, we determined the bias and the scale factor of the sensor and identify the sensor motion, simultaneously. Afterwards, we moved on to the formulation of the OED where the optimized signal trajectory was obtained by minimized the trace of the covariance matrix of the estimated parameters. Lastly, for a more realistic model we included process noise.
The simulation results showed that the optimal signals follow similar or symmetric on the time axis sinusoidal curve. The results obtained comparing the initial signals with the optimal signals highlight an increase in the accuracy of the parameters identification, which is guaranteed by the decrease of the variance of the estimated parameter.
From the simulation, we observed minor improvement in the accuracy, this can be interpreted due to the simplicity of the model. Nevertheless, the results from the implementation of the OED under process noise, give us confidence that in a more complex model the improvement will be more significant.

An interesting direction of future work is to broaden these results in more realistic sensor models. For example, by adding the sensor axis misalignment's error and include the variation of accelerometer biases with temperature. In addition, more constraints are necessary both for the motion of the sensor and for a more detailed system. Moreover, the IMU sensors consist of clusters of gyroscopes and accelerometers, therefore there is room for further research to simultaneous calibration of gyroscope. Lastly, it would be interesting to implement the OED using different optimization criteria, in order to examine if different criteria give better estimation accuracy.

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