Quadratic Voting and how it performs regarding Social Welfare

Bachelor's Project Thesis

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Abstract: Quadratic Voting (QV) is a voting rule designed to allow voters to express the intensity of their preferences. This research investigates how QV performs with regard to utilitarian and egalitarian social welfare by simulating voting under QV and comparing the results to the voting outcome under majority voting. Data is generated to create voters' preference intensities, voting strategies are developed representing different voter behavior, ballots are cast according to the various voting strategies and the voter welfare is computed to compare the performance of the voting rules. The results of this research show that QV yields a significant welfare improvement compared to majority voting, when voters spend their votes in proportion to their preference intensities. Furthermore, a growing population size negatively impacts the voter welfare and a larger amount of decisions significantly improves the welfare distribution among all voters when voters vote according to the best performing voting strategies.

1 Introduction

Voting and collective decision-making frequently adheres to the principle 'one-person-one-vote' (1P1V) by applying the majority rule. Countries use referenda allowing the population to have a direct influence on political decisions, like changing policy or realizing public projects, and legislators pass bills, all using majority voting (Goeree & Zhang, 2017; Posner & Weyl, 2015).

Under the majority rule, the 1P1V principle seems fair since every voter receives one vote and therefore has an equal chance to influence the outcome of a decision, however the principle prevents voters from signaling the intensity of their preference (Weyl, 2013). Additionally, voters are not able to increase their influence on a certain decision, which is more important to them, by giving up influence on some other decision, which matters less in their opinion (Lalley & Weyl, 2018). The majority rule also promotes the disregard of legitimate interests of the minority by the majority, the so-called 'tyranny of the majority' (Posner & Weyl, 2015). These drawbacks entailed by majority voting lead to the question whether a socially optimal result is achievable through voting under the majority rule, especially when voters do not only disagree in their favored outcome but also in their intensity of how much they care (Goeree & Zhang, 2017; Posner & Weyl, 2015).

To tackle the previously described deficiencies with majority voting, new voting rules have been suggested that allow voters to express their preference intensities in decisions. One voting rule implementing such properties was recently proposed and is called Quadratic Voting (QV) (Lalley & Weyl, 2018; Posner & Weyl, 2015) . With QV, every voter receives a budget of 'voice credits', an artificial currency, which can be spend freely influencing the outcome of binary decisions (Lalley & Weyl, 2018). Voters are able to allocate voice credits in the direction of their favored alternative (Lalley & Weyl, 2019) and additional voice credits can be assigned when a voter possesses a strong preference regarding a decision (Lalley & Weyl, 2019). Thus, under QV, 101 voters with very weak preferences are no longer able to outvote 99 opponents who passionately care about the decision.

Previous research and experiments using QV have demonstrated that QV is able to improve social welfare compared to majority voting, which suffers from insufficient social welfare through i.a.

the tyranny of the majority (Casella & Sanchez, 2019; Lalley & Weyl, 2018; Quarfoot et al., 2017). However, most studies focused on minority victories (Casella & Sanchez, 2019) and examined in most cases only sparsely utilitarian efficiency (Casella & Sanchez, 2019; Lalley & Weyl, 2019) as measures of the population well-being. Research is lacking that concentrates on the detailed analysis of QV regarding multiple social welfare metrics. Thus, this research aims attention at the question how QV performs concerning various social welfare metrics where the results are compared to the majority rule as a base case.

To answer the research question, a program was created that generates voting data synthetically under QV as well as majority voting to assess the influence of QV on the population well-being and whether the performance of QV varies with regard to different social welfare metrics. Three distinct voting strategies under QV were developed and implemented to picture diverse voting behaviors. After creating, processing and evaluating the voting data, the program computes the voter welfare to assess the performance of QV under three welfare metrics: two variants of utilitarian social welfare and egalitarian social welfare. Statistical analyses are carried out to interpret the results and to determine the performance of QV under the social welfare metrics while also comparing QV to the base case of majority voting.

2 Preliminaries

This research considers a population of N voters $i = 1, ..., N$ that vote on a set of $D > 1$ binary collective decisions. Each voter i possesses a preference for every decision d which is reflected by the preference valuation p_{id}, where $p_{id} > 0$ illustrates that i favors alternative A of the decision and $p_{id} < 0$ indicates that i prefers alternative B. Thus, the sign of p_{id} represents the direction of i's preference whereas the value of p_{id} signals i's preference intensity (Casella & Sanchez, 2019; Lalley & Weyl, 2018). This means, with $|p_{i1}| > |p_{i2}|$, voter i possesses a larger intensity of preference for decision 1 compared to decision 2.

Every voter i votes genuinely, without voting costs and casts votes on decision d in the direction of the preference p_{id}, where the amount of votes spend is denoted by v_{id}. Every decision is determined in the direction that is favored by the majority of votes (Lalley & Weyl, 2018; Posner & Weyl, 2017). Consequently, alternative A is implemented when $\sum_{i=1}^{N} v_i > 0$ and alternative B prevails when $\sum_{i=1}^{N} v_i < 0$. A tie occurs when the sum of votes equals 0. A random tie breaker is used to distribute wins and losses fairly while avoiding a preference for one alternative.

In this research two voting rules are examined: Quadratic Voting and majority voting. The two voting rules specify different principles under which votes can be spend as well as define various voting strategies representing the voter behavior.

2.1 Quadratic voting

With QV, every voter receives a budget of 'voice credits' C, an artificial currency (Lalley & Weyl, 2018). Voters are able to distribute these voice credits freely over all decisions, where the amount of voice credits spend by voter i on decision d, in the direction of i's preferred alternative, is denoted by c_{id}.

Voice credits spend on a decision are converted into votes at a quadratic cost according to the vote pricing rule of QV (Lalley & Weyl, 2018; Posner & Weyl, 2017):

$$v_{id} = \sqrt{c_{id}} \tag{2.1}$$

Formula 2.1 shows the vote casting principle of QV, where v_{id} represents the amount of votes cast by voter i on decision d given c_{id} distributed voice credits. It outlines that v^2 voice credits need to be spend, to cast v votes on a decision (Posner & Weyl, 2015). The appeal to accumulate votes is bounded by the quadratic cost, however the opportunity to spend more than one vote on a decision allows voters to signal their preference intensity and strengthens their position on the decision outcome.

Since the equilibrium properties of QV for more than one decision have not been theoretically analyzed (Casella & Sanchez, 2019) and because voice credits can be cast freely, various voting strategies have been implemented to model different voting behavior. This research developed three voting strategies under QV: all-or-nothing (AON) voting, lottery all-or-nothing (LAON) voting, and proportional voting. All voting strategies adhere to the voting rules of QV but display various vote spending principles.

2.1.1 All-or-nothing voting strategy

According to the AON voting strategy, a voter i cumulates all voice credits c_i on one decision d. The votes are cast on the decision for which the voter possesses the largest absolute preference intensity $|p_{id}|$, thus on the decision that matters most to the voter. All other decisions receive no votes.

The AON voting strategy is implemented because an experiment conducted by Casella & Sanchez (2019) using QV showed a strong tendency of voters towards spending all votes on one decision (40% of all participants cumulated all their votes on one decision under the QV voting rule).

2.1.2 Lottery all-or-nothing voting strategy

The LAON voting strategy is similar to the AON voting strategy (see section 2.1.1) to the extent that a voter i casts all voice credits c_i on one decision d, however, the decision on which all votes are spend is determined through a lottery. The base of the lottery consists of a probability distribution over all decisions, where the probabilities are proportional to the absolute preference intensities $|p_i|$ of the voter. Thus, decisions for which a voter possesses a larger absolute intensity of preference, receive a greater probability to be selected in the lottery and respectively, decisions with a lower preference intensity obtain a smaller probability to be chosen.

The LAON voting strategy is used to introduce noise into the voting process. The reason for this is that previous research suggests that randomness might have an influence on voter behavior as voting can be noisy and seems to be never totally efficient (Goeree & Zhang, 2017).

2.1.3 Proportional voting strategy

With the proportional voting strategy, a voter i spends voice credits c_i in proportion to the voter's absolute preference intensities $|p_i|$ across all decisions. Thus, a voter casts more voice credits on a decision for which a greater absolute intensity of preference exists and fewer votes on a decision for which the absolute preference intensity is lower.

The proportional voting strategy is implemented since previous research on QV implies that the optimal strategy for voters to maximize their welfare consists in distributing votes approximately proportional to their preferences (Casella & Sanchez, 2019; Goeree & Zhang, 2017; Lalley & Weyl, 2019).

2.2 Majority voting

Majority voting means that each voter receives an amount of votes, which is equal to the number of decisions. Every voter i needs to cast one single vote v_i on every decision d, thus with majority voting, an accumulation of votes is not possible.

Majority voting consolidates the amount of support present for each alternative of a decision because voters can spend votes in the direction of their preference, however voters are not able to express the intensity of that preference. As the options of casting votes under majority voting is limited, one voting strategy is implemented to represent voter behavior: one-decision-one-vote (1D1V) voting.

2.2.1 One-decision-one-vote voting strategy

The 1D1V voting strategy models the vote casting under majority voting. Under this voting strategy, every voter i is allowed to cast one vote v_i on each decision d in the direction of their preference p_{id}.

2.3 Modelling voter preferences

Each voter i possesses a preference for every decision d that reflects the direction as well as the intensity of the preference. The preferences of every voter i $\{p_{i1}, ..., p_{id}\}$ are known privately and are defined over [-1, 1], being symmetric around 0. When voter i's preference for decision d, p_{id}, is close to the extreme values (thus to 1, respectively -1), then the voter i owns a strong intensity of preference (for alternative A, respectively alternative B). A preference value p_{id}, which is located around 0 indicates a weak preference of voter i for decision d.

2.4 Evaluating voter welfare

The welfare of voters is measured to determine, which voting strategy under which voting rule yields the largest population well-being. This research considers three social welfare metrics that focus on various aspects of the voter welfare: two variants of utilitarian social welfare, and egalitarian social welfare (Sen, 1995).

2.4.1 Utilitarian social welfare: classical

Utilitarian social welfare measures the welfare of the whole population of voters. Thus, the higher the utilitarian social welfare, the happier the voters overall regarding the voting outcome. Two variants of utilitarian social welfare are covered in this research, which differ in the determination of the realized utility of a voter.

The classical approach specifies that the realized utility u_{id} of voter i for decision d is $+u_{id}$ when the outcome is in favor of alternative A and $-u_{id}$ when alternative B wins, where the value of u_{id} is equal to i's valuation p_{id} (Lalley & Weyl, 2019). This results in a positive utility u_{id}, when the direction of i's preference and the outcome of decision d match. Respectively, voter i obtains a negative utility u_{id}, when i's preference differs from the voting result.

$$W_u = \sum_{i=1}^{N} U_i \tag{2.2}$$

Formula 2.2 shows that the voter welfare U over all decisions for every voter i is summed up to obtain the utilitarian social welfare W_u.

The explanatory power of utilitarian social welfare is important for this research as it allows the analysis of the performance of the various voting strategies under QV as well as under majority voting.

2.4.2 Utilitarian social welfare: strictly positive

Also the second variant of the utilitarian social welfare, considered in this research, computes the wellbeing of the entire population.

The difference compared to the classical utilitarian social welfare approach consists in only using positive voter utilities. Thus, if decision d is decided in the direction favored by voter i, then i's realized utility u_{id}, amounts to i's absolute preference intensity $|p_{id}|$, otherwise the utility is normalized to 0 (Casella & Sanchez, 2019).

The computation of the strictly positive utilitarian social welfare adheres to the same formula 2.2 as the classical utilitarian social welfare, where the voter welfare U over all decisions for every voter i is summed.

The strictly positive approach regarding utilitarian social welfare was included in this research to, on the one hand, allow for an additional metric to assess the performance of QV and, on the other hand, to implement a social welfare metric that has been used in a previous QV experiment (Casella & Sanchez, 2019).

2.4.3 Egalitarian social welfare

Egalitarian social welfare focuses on the least satisfied voter of the population. Thus, the welfare of the whole population is determined by the welfare of the worst-off voter. The realized utility u_{id} of voter i for decision d is defined, for the computation of the egalitarian social welfare, as in the classical utilitarian social welfare approach.

$$W_e = min(U_1, U_2, ..., U_N) \tag{2.3}$$

Formula 2.3 represents that the egalitarian social welfare W_e is depicted by the lowest welfare U over all decisions of the the i^{th} voter.

The measurement of egalitarian social welfare is valuable when a voting rule should ensure that every voter displays a minimum of welfare concerning the voting outcome. Additionally, linking egalitarian social welfare to utilitarian social welfare demonstrates how even the voter welfare is distributed among all voters of the population. Thus, a consistent utilitarian social welfare over time in combination with an increasing egalitarian social welfare shows that the welfare is more evenly distributed among all voters.

3 Methods

The aim of this research is to examine the performance of QV regarding the population well-being, yet previous studies on QV suggest that various factors next to the developed voting strategies (see section 2.1) might influence the welfare of voters when voting under QV (Chandar & Weyl, 2019). Thus, various parameters are examined as independent variables in this research: the voting strategies, the population size, and the amount of decisions. To measure how these factors affect the voter welfare, the three social welfare metrics, explained in section 2.4, represent the dependent variables in this research.

Previous research indicates that the population size has an effect on the voter welfare under QV and that majority voting might outperform QV in some settings where the amount of voters is not large enough (Chandar & Weyl, 2019). Thus, this research utilizes four different settings for the population size to assess its influence on the population well-being: 5 voters, 50 voters, 500 voters, and 1000 voters.

Similar to the population size, it seems likely that also the number of decisions has an influence on the outcome of ballots and thus on the welfare of voters. Previous studies on QV focused often on settings with one single decision or examined a vote buying system using real currencies (where voters can buy as many votes as they want) (Posner & Weyl, 2014, 2015; Weyl, 2017). Thus, examining multiple decisions is interesting, especially as the budget of voice credits in QV is limited and therefore introduces an aspect of scarcity for the voters (Quarfoot et al., 2017). This research uses four settings for the amount of decisions to determine its impact on voter welfare: 5 decisions, 10 decisions, 25 decisions, and 50 decisions.

The budget of voice credits, a voter receives when casting votes under QV, does not represent an independent variable in this research. The reason is that if a voter holds a very large number of voice credits, then the votes will be cast according to the same voting strategies, regardless of the total budget of voice credits, and thus will probably result in very similar outcomes. Also, if a voter receives very few voice credits, then some voting strategies might be impossible to implement by the voter, which would represent an unwanted result in any real life setting of QV. In this research, the budget of voice credits is determined by the square of the number of decisions. Thus, each voter can spend 25 voice credits on 5 decisions and respectively obtains 100 voice credits for 10 decisions. Thus, when all votes are cumulated on one decision, the voter casts in total the same amount of votes as the voter would spend with majority voting over all decisions, which increases comparability across both voting rules.

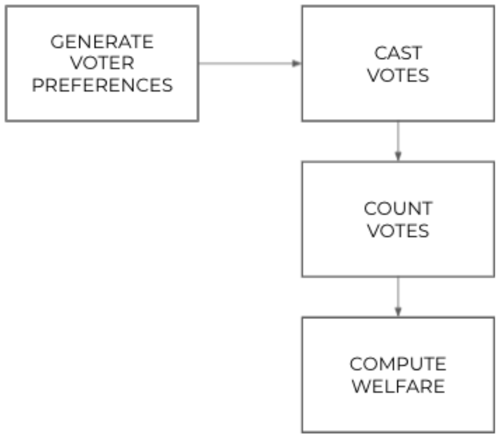

Figure 3.1: Flowchart representing the structure of the program that is used in this research.

To asses the performance of QV regarding the voter welfare, a program was created that generates voter preferences, casts votes based on these preferences according to all voting strategies, counts the votes and finally, computes the voter welfare for all social welfare metrics. Figure 3.1 shows a flowchart depicting the structure of these parts of the program which are explained in detail in the following subsections.

The different settings for the number of voters and the amount of decisions result in 16 experiments that were run in this research. Each experiment was repeated 100 times and the average values were taken to allow for the generalization of the results. Each repetition consists of the process described in figure 3.1.

3.1 Generating voter preferences

Firstly, the program creates the voter preferences which are then used by the rest of the program to examine the direction and the intensity each voter possesses for every decision. To generate these voter valuations, the preferences are randomly sampled from a truncated standard normal distribution (from -1 to 1) for each voter and every decision. The value for the standard deviation of the normal distribution is set to 1 and the mean equals 0. These standard values are chosen for the normal distribution as no indications exist that suggest a shift or stretch of the distribution.

A normal distribution of preferences is used due to a previous study by Casella & Sanchez (2019), who registered the participants' preference distribution over various binary decisions showing predominantly normally distributed preferences.

3.2 Casting the votes

This part of the program casts the votes for each voter in the direction of the voter's preference and according to every voting strategy separately to produce a ballot for every voter voting according to all voting strategies under QV and majority voting: AON voting, LAON voting, proportional voting, and 1D1V voting. Thus, the performance of the voting strategies can be compared based on the same decisions and equal preference valuations of the voters.

3.2.1 All-or-nothing vote casting

To cast votes according to the AON voting strategy under the QV voting rule (see section 2.1.1), the program determines for each voter the decision with the largest absolute preference intensity. All votes available are then cast on this decision and the ballots for all voters are returned.

3.2.2 Lottery all-or-nothing vote casting

The program creates a probability distribution for every voter with probabilities proportional to the voter's absolute preference intensities, to distribute votes after the LAON voting strategy under QV (see section 2.1.2). A lottery based on the probability distribution determines the winning decision on which all votes are cast and the ballots are returned for all voters.

3.2.3 Proportional vote casting

The algorithm 3.1 represents how votes are cast after the proportional voting strategy under QV (see section 2.1.3) for one voter. The procedure described in the algorithm is repeated for every voter of the population.

The algorithm shows how voice credits are distributed proportionally to the preference valuations of a voter. When voice credits are cast on decisions,

Algorithm 3.1 Proportional vote casting algorithm per voter

```
D ⇐ amount of decisions
s_d ⇐ preference share for decision d
B ⇐ budget of voice credits
c_d ⇐ voice credits spend on decision d
v_d ⇐ votes spend on decision d
for all decisions d = 1, ..., D do
  c_d ⇐ s_d * B
  c_d ⇐ round to closest quadratic value
  v_d ⇐ √c_d
end for

C ⇐ total number of voice credits spend
while C > B do
  l ⇐ decision with lowest absolute preference
  if c_l > 0 then
    v_l ⇐ decrease votes by 1
    c_l ⇐ v_l^2
    C ⇐ update total number of credits spend
  else
    l ⇐ decision with next lowest absolute preference
  end if
end while

continue ⇐ true
while C < B and continue do
  h ⇐ decision with largest absolute preference
  v_h ⇐ increase votes by 1
  c_h ⇐ v_h^2
  C ⇐ update total number of credits spend
  if C > B then
    v_h ⇐ decrease votes by 1
    if decisions left to examine for h then
      h ⇐ decision with next largest absolute preference
    else
      continue ⇐ false
    end if
  end if
end while
```

the amount of voice credits spend might need to be rounded up or down to the nearest quadratic value to obtain an integral number when converting the voice credits back to votes later. The rounding procedure as well as the proportional distribution of voice credits can lead to cases where a voter assigns more voice credits than available or where voice credits remain. The program accounts for both cases.

When the budget of voice credits is exceeded, the amount of voice credits spend needs to be decremented, starting with the decision which is least important to the voter.

This research assumes that a rational voter would want to distribute all voice credits available to increase their impact on all decisions. Thus, when a voter has not yet exhausted the voice credit budget, the program tries to increase the amount of voice credits spend, starting with the decision which is most important to the voter. Finally, the voice credits are translated into votes according to the quadratic vote pricing rule. After this procedure is repeated for every voter, the ballots of all voters are returned.

3.2.4 One-decision-one-vote vote casting

To cast votes according to the 1D1V voting strategy under majority voting (see section 2.2.1), the program spends for every voter regarding every decision one vote. Then the ballots of all voters are returned.

3.3 Counting the votes

This part of the program receives the ballots of all voters for every decision and according to all voting strategies. The vote counting process consists of summing the votes of the ballots to determine the prevailing alternative for every decision. The procedure is equal for all voting strategies, however the vote counting is performed separately for every strategy to obtain a result according to each voting strategy.

3.4 Computing voter welfare

To evaluate the performance of QV by means of its implemented voting strategies and to compare it to majority voting, the social welfare metrics discussed in section 2.4 are used. The welfare of a voter regarding the outcome of all decisions is determined by summing up the voter's utilities for each decision. This voter welfare is computed for every voter, thus it can be used to calculate the two kinds of utilitarian and the egalitarian welfare according to the formulas 2.2 and 2.3.

3.5 Data analysis

The program used in this research was created in the programming language R (version 3.6.1). The data analysis, which uses various kinds of analysis of variance as statistical tests, as well as the graphical visualization of results were performed in R (version 3.6.1) and run in RStudio (version 1.4.1103).

4 Results

This research uses different types of analysis of variance to examine the performance of the various voting strategies under QV in comparison to majority voting with the aim to determine which strategies yield the greater population well-being. Additionally, the analyses investigate the effect of the population size and the influence of the number of decisions on the welfare of voters.

4.1 Welfare differences of voting strategies

It is investigated how the mean social welfare per voter per decision differs between the voting strategies regarding all three welfare metrics.

To examine whether differences in performance of the voting strategies exist, a one-way repeated measures analysis of variance is used. A repeated measures analysis is appropriate since the experiment set-up lets every population vote under all four voting strategies. A Shapiro-Wilk test ensured for a normal distribution of the data and the assumption of sphericity was tested through a Mauchly's test.

4.1.1 Classical utilitarian social welfare

The mean utilitarian welfare after the classical approach per voter per decision for all voting strate-

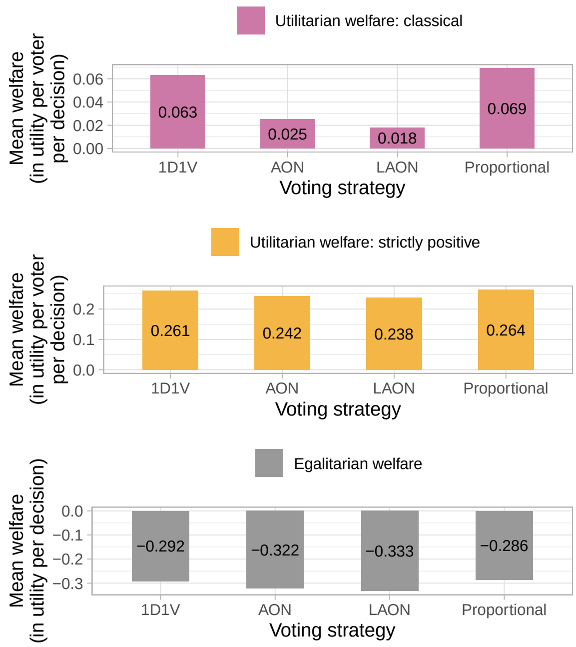

Figure 4.1: Graphs showing the mean voter welfare for all voting strategies for every social welfare metric separately.

gies is shown in the top graph in figure 4.1. It can be detected that the population overall seems to be the most satisfied when voting according to the proportional voting strategy under QV. Followed by voting under majority voting with its 1D1V voting strategy, with some distance the AON voting strategy under QV and finally, the LAON voting strategy under QV.

The analysis of variance shows that the classical utilitarian social welfare is statistically significantly different regarding the four voting strategies with a p-value < 0.0001 ($F(1.45, 2317.7) = 802.6$, $p < 0.0001$, generalized eta squared $= 0.129$). Post-hoc analyses with a Bonferroni adjustment reveal that all the pairwise differences, between voting strategies regarding the classical utilitarian social welfare, are statistically significant ($p \leq 0.05$) showing that the performance distinctions observed in figure 4.1 are significant.

4.1.2 Strictly positive utilitarian social welfare

The middle graph in figure 4.1 displays the average utilitarian social welfare per voter and decision following the strictly positive concept for all voting strategies. The pattern of the mean utilitarian social welfare regarding the strictly positive approach looks similar compared to the classical utilitarian social welfare: the proportional voting strategy under QV yields the largest average welfare and second best performs the 1D1V voting strategy under majority voting, followed by the AON and LAON voting strategy under QV.

The analysis of variance shows that the average strictly positive utilitarian social welfare is statistically significantly different regarding the various voting strategies with a p-value < 0.0001 ($F(1.45, 2317.7) = 802.6$, $p < 0.0001$, generalized eta squared $= 0.117$). The post-hoc analyses with a Bonferroni adjustment reveal that all pairwise differences, between the voting strategies concerning the strictly positive utilitarian social welfare, are statistically significant ($p \leq 0.05$).

4.1.3 Egalitarian social welfare

The bottom graph in figure 4.1 represents the mean of the egalitarian social welfare per decision for all voting strategies. For the egalitarian welfare the worst-off voter is considered, thus for all strategies the mean egalitarian welfare has a negative value. Voters voting according to the proportional voting strategy under QV show the greatest mean egalitarian welfare, followed by the 1D1V strategy under majority voting, the AON strategy under QV and the LAON strategy under QV. This order corresponds to the results obtained for the two utilitarian social welfare measures.

The analysis of variance states that the egalitarian social welfare scores are statistically significantly different regarding all four voting strategies with a p-value < 0.0001 ($F(2.38, 3801.75) = 177.144$, $p < 0.0001$, generalized eta squared $= 0.007$). The post-hoc analyses with a Bonferroni adjustment reveal that all the pairwise differences, between the voting strategies regarding the mean egalitarian welfare, are statistically significant ($p \leq 0.05$) and thus confirming the observations made from figure 4.1.

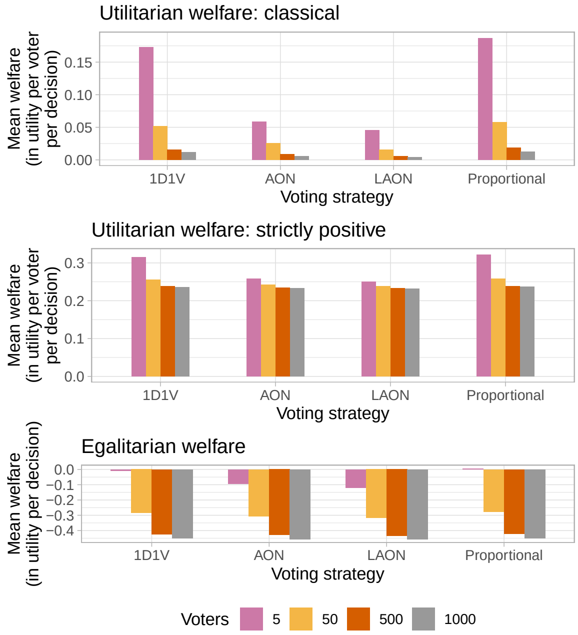

Figure 4.2: One graph for every social welfare metric, displaying the mean welfare regarding all voting strategies for the various population sizes.

4.2 Effect of population size on social welfare

This research investigates whether the population size has an influence on the population well-being and whether the impact differs regarding the voting strategies. The experiments tested four population settings: 5, 50, 500, and 1000 voters. For the different population sizes, the mean welfare is considered to account for differences occurring because of the varying number of voters (the welfare of 50 voters would otherwise be 10 times larger compared to the welfare of 5 voters).

A two-way mixed analysis of variance is conducted to evaluate the effect of the number of voters on social welfare. A Shapiro-Wilk test ensured for the normal distribution of the data, the assumption of sphericity was tested by a Mauchly's test and a Levene's test was performed to guarantee the homogeneity of variance.

4.2.1 Classical utilitarian social welfare

The mean utilitarian social welfare after the classical approach per voter per decision is displayed in the top graph in figure 4.2 comparing various population sizes for all voting strategies. As a general trend over all strategies it can be seen from figure 4.2 that the mean classical utilitarian social welfare decreases when the population size increases. A population voting under the proportional voting strategy under QV obtains for each population size the greatest welfare score per decision. Followed by voting under majority voting with the 1D1V voting strategy, then with some distance the AON and the LAON strategy, both under QV.

The analysis of variance shows that a statistically significant two-way interaction between the voting strategies and the population size on the classical utilitarian social welfare exists ($F(6.27, 3336.11) = 775.224$, $p < 0.0001$). Because of the significant two-way interaction, the effect of the population size on classical utilitarian social welfare for every voting strategy is investigated. The resulting Bonferroni adjusted p-values reveal that the simple main effect of population size is significant for all strategies (with $p < 0.001$). Pairwise comparisons display that the mean utilitarian welfare after the classical approach is significantly different for the majority of population sizes over all voting strategies. Except the comparison of 500 vs. 1000 voters is not significant, but only for the voting strategies AON under QV and LAON under QV.

4.2.2 Strictly positive utilitarian social welfare

The middle graph in figure 4.2 shows the mean utilitarian social welfare according to the strictly positive method per voter and per decision. The figure 4.2 displays that the general pattern of the mean strictly positive utilitarian welfare over all voting strategies is similar to that of the classical utilitarian welfare: strictly positive utilitarian welfare decreases, while population size increases. Also the order of the voting strategies regarding their performance stays equal: proportional voting under QV yields the highest mean strictly positive utilitarian social welfare value over all population sizes, then the 1D1V voting strategy under majority voting follows. The AON strategy and LAON strategy under QV represent the worst performing voting strategies.

The analysis of variance results in a statistically significant two-way interaction between voting strategy and population size on utilitarian social welfare according to the strictly positive approach with a p-value < 0.0001 ($F(6.27, 3336.11) = 775.224$, $p < 0.0001$). Thus, the effect of the population size on strictly positive utilitarian social welfare is investigated and yields, considering the Bonferroni adjusted p-values, that the simple main effect of the population size is significant for all voting strategies ($p < 0.001$). Pairwise comparisons show that the mean utilitarian welfare after the strictly positive method is significantly different for the majority of population sizes for all voting strategies, except for 500 vs. 1000 voters for which no significant difference in strictly positive utilitarian social welfare for all voting strategies exists.

4.2.3 Egalitarian social welfare

The bottom graph in figure 4.2 displays the mean egalitarian welfare per decision for all voting strategies over all population sizes. Also for the egalitarian welfare, the figure 4.2 shows a similar pattern compared to the two utilitarian social welfare metrics: over all strategies, the mean egalitarian welfare scores decrease with increasing population sizes. Again, proportional voting under QV performs best with the greatest mean egalitarian welfare values for all population sizes, followed by the 1D1V strategy under majority voting, the AON strategy and the LAON strategy under QV.

The analysis of variance confirms that a statistically significant two-way interaction exists between voting strategy and population size on egalitarian welfare ($F(7.58, 4029.95) = 72.741$, $p < 0.0001$). Because of the significant two-way interaction, the effect of the population size on egalitarian welfare is examined for every voting strategy. The resulting Bonferroni adjusted p-values show that the simple main effect of population size is significant for all voting strategies ($p < 0.001$). Pairwise comparisons show that the mean egalitarian welfare is significantly different for most population sizes regarding all voting strategies, except for population size 500 vs. 1000 voters.

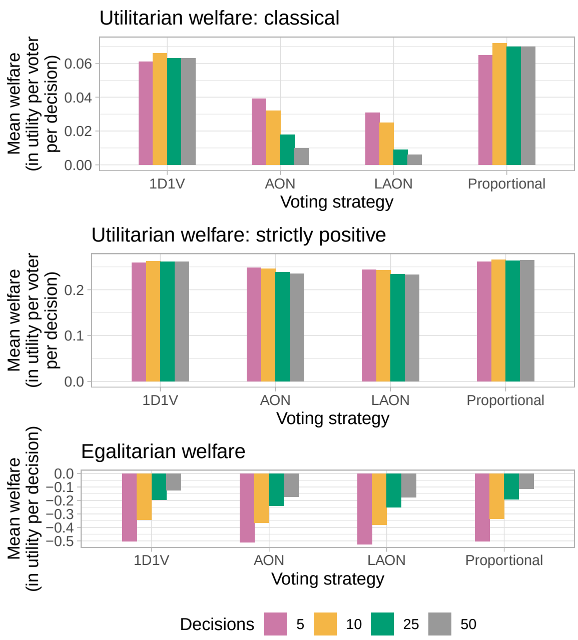

Figure 4.3: One graph for every social welfare metric, displaying the mean welfare regarding all voting strategies for the various number of decisions.

4.3 Effect of decision size on social welfare

This research explores whether the number of decisions has an impact on the voter welfare and whether the influence varies regarding the voting strategies. The experiments were conducted with four different decision sizes: 5, 10, 25, and 50 decisions. To account for differences in voter welfare because of various numbers of decisions, the mean welfare is considered (otherwise the welfare for 10 decisions would be twice as high as for 5 decisions).

A two-way mixed analysis of variance is carried out to evaluate the effect of the amount of decisions on social welfare. A Shapiro-Wilk test ensured for the normal distribution of the data, the assumption of sphericity was tested by a Mauchly's test and a Levene's test was conducted to guarantee the homogeneity of variance.

4.3.1 Classical utilitarian social welfare

The top graph in figure 4.3 displays the average utilitarian social welfare according to the classical approach per voter and per decision for all voting strategies. Proportional voting under QV yields the greatest welfare values for every decision size, followed by the 1D1V voting strategy under majority voting. With some distance follows the AON voting strategy under QV and the LAON voting strategy under QV. For the AON and LAON voting strategies the figure 4.3 shows that the classical utilitarian welfare decreases with an increasing number of decisions. This pattern cannot be noticed for the two best performing voting strategies.

The analysis of variance reveals that a statistically significant two-way interaction between voting strategy and decision size on classical utilitarian social welfare exists ($F(4.4, 2338.18) = 18.512$, $p < 0.0001$). With the Bonferroni adjusted p-values, it is visible that the simple main effect of decision size is significant ($p < 0.001$) for the voting strategies AON under QV and LAON under QV, however the effect is not significant for the 1D1V strategy under majority voting and the proportional voting strategy under QV. Pairwise comparisons confirm this result, as no differences are significant for neither proportional voting under QV nor for the 1D1V strategy under majority voting, confirming the observations made in figure 4.3. For the AON and LAON voting strategy under QV, the pairwise comparisons yield significant differences regarding classical utilitarian social welfare between all decision sizes, except for 25 vs. 50 decisions.

4.3.2 Strictly positive utilitarian social welfare

The mean utilitarian social welfare after the strictly positive approach per decision and per voter is represented in the middle graph in figure 4.3 for all voting strategies. The figure 4.3 displays the same pattern as observed for the classical utilitarian welfare: proportional voting under QV performs best for every decision size, followed by the 1D1V strategy under majority voting, the AON strategy under QV and the LAON voting strategy under QV. The trend observed for the classical utilitarian welfare, of a decreasing welfare with an increasing decision size for the AON and the LAON voting strategy, is also observable, although not as notable for the strictly positive utilitarian welfare. This pattern is not observable for the proportional voting strategy and the 1D1V voting strategy regarding the strictly positive utilitarian welfare.

The analysis of variance shows a statistically significant two-way interaction between voting strategy and decision size on the strictly positive utilitarian welfare ($F(4.4, 2338.18) = 18.512$, $p < 0.0001$). Considering the Bonferroni adjusted p-values, it can be seen that the simple main effect of decision size is significant ($p < 0.001$) for the strategies AON and LAON, but not significant for the 1D1V and the proportional voting strategies. The pairwise comparisons reveal that the mean strictly positive utilitarian welfare is significantly different for the AON and the LAON voting strategies for all decision sizes, except for 5 vs. 10 and 25 vs. 50 decisions. For the proportional voting strategy and the 1D1V strategy, no pairwise comparison regarding the decision size is significant.

4.3.3 Egalitarian social welfare

The bottom graph in figure 4.3 displays the mean egalitarian welfare per decision to compare the voter welfare for various decision sizes across all voting strategies. It shows that proportional voting under QV yields the greatest mean egalitarian welfare for every decision size, followed by the 1D1V strategy under majority voting, the AON and LAON voting strategies under QV. A general pattern is visible from the figure 4.3: while the amount of decisions grows, also the mean egalitarian welfare for each voting strategy increases.

The analysis of variance reveals that a statistically significant two-way interaction between voting strategy and decision size on egalitarian welfare is present ($F(7.22, 3839.08) = 12.113$, $p < 0.0001$). The Bonferroni adjusted p-values indicate a significant simple main effect of decision size ($p < 0.001$) for all voting strategies. Pairwise comparisons demonstrate that the mean egalitarian welfare is significantly different for all voting strategies across all decision size settings.

5 Conclusion

In conclusion, the voter welfare differs significantly between all four voting strategies with the result that the proportional voting strategy under QV performs best by yielding the greatest voter well-being regarding all social welfare metrics across all population and decision sizes. The second best performing voting strategy is represented by the 1D1V strategy under majority voting, followed by the AON strategy under QV and the LAON strategy under QV. Thus, with its most advantageous voting strategy, QV is able to outperform majority voting concerning utilitarian and egalitarian welfare. Additionally, this research confirms previous studies that indicated spending votes proportional to the voter's preferences leads to an optimal strategy to maximize voter welfare (see section 2.1.3) (Casella & Sanchez, 2019; Goeree & Zhang, 2017; Lalley & Weyl, 2019).

Regarding the influence of the population size on voter welfare, it can be stated for all voting strategies and for both forms of utilitarian social welfare as well as for egalitarian social welfare that a significant interaction effect of the voting strategy and the number of voters exists. Thus, the impact the voting strategy has on the voter welfare depends on the population size (and vice versa). A general trend for the majority of voting strategies and most social welfare metrics could be observed: while the population size increases, the voter welfare decreases significantly until a population size of 500 voters is reached.

Also, the number of decisions has a significant interaction effect with the voting strategies on the three social welfare metrics. Thus, the influence of the voting strategies on voter welfare depends on the decision size (and vice versa). However, the main effect of the amount of decisions on the two forms of utilitarian social welfare is only significant for the AON and the LAON voting strategies under QV, but not for proportional voting under QV and 1D1V under majority voting. Regarding the egalitarian social welfare, the welfare of the worst-off voter increases significantly with a growing number of decisions for all voting strategies. Therefore, for the proportional and the 1D1V voting strategy, the well-being of the whole population of voters does not change significantly with an increasing number of decisions, but the welfare of the worst-off voter grows. This means that, the voter welfare is more evenly distributed among the voters of the population when the decision size increases.

6 Discussion

Some limitations might affect the generalization of QV in a real world voting setting. Additionally, QV is still in its early beginning phase and thus has not been used often in a voting context which leaves many possibilities for future research.

6.1 Limitations

First, this research does not allow for communication among voters before voting. However, this aspect is essential in a real world voting setting (Weyl, 2017). Modelling voter communication predecision, with a potentially consequential preference change, would exceed the framework of this research and was therefore not included. A consequence of the absence of communication is that this research gives no possibility for fraud, collusion or strategic voting, although representing a possible option in a real world voting setting. However, previous studies indicated that QV seems to be fairly robust concerning collusion as well as fraud (Weyl, 2017).

Second, this research assumes that every voter of the population votes on all decisions. But realistically, voter turnout in a real world voting context will not amount to 100 percent under QV (Kaplow & Kominers, 2017).

Third, for this research the assumption was made that the voter population consists of a homogeneous group where every voter votes according to the same voting strategy. However, a population of voters will probably exhibit a combination of different kinds of voting strategies for various voter types. But since previous studies have not yet analyzed what voting strategies voters utilize under QV and especially in what proportions various strategies are represented, this simplification of a homogeneous voter population was used in this research.

Fourth, the proportional voting strategy under QV implements a rounding up or down mechanism (see section 3.2.3) that ensures integral values after converting voice credits into votes. However, this rounding method might lead to a loss of proportionality because differences between decisions might diminish.

6.2 Future research

The results of this research suggest that with QV, voters voting in proportion to their preference intensities yield the most satisfied population overall and a more equal welfare distribution among all voters. However, a proportional voting strategy was not investigated in a voting experiment with real voters. Thus, this might be interesting to examine in a future research, especially to find out whether voters are able to assess their own preference intensities as well as to proportionally distribute voice credits, particularly when the decision size is large.

This research indicates that the well-being of the population seems to be more evenly distributed with a larger number of decisions. The experiments of this research examined the voter welfare for up to 50 decisions, however future research might want to investigate what decision size is optimal and realistic in a real world voting context with QV.

Additionally, future research might inspect how QV could realistically be implemented in decision making processes in the real world. The reason is that QV, compared to majority voting, represents a rather complex voting rule that involves mathematical skills and might therefore be difficult to use by an average voter population (without any prior knowledge). Previous studies tested QV with real voters, however these experiments either simplified QV (Casella & Sanchez, 2019) or QV was supported by a sophisticated digital interface (Quarfoot et al., 2017). Thus, future studies might study how QV can be implemented in a real world voting setting as well as how long a population needs to be trained until voters can vote under QV. In addition, it might be interesting to research whether real voters enjoy voting under QV in comparison to majority voting, whether a population would accept voting under QV and thus, whether QV has an impact on voter turnout.

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