State Estimation in District Heating Systems

Integration Project

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July 1, 2022

Abstract

The goal of this Integration Project is to compare two different state estimation strategies (SES) for a stratified storage tank in a District Heating System (DHS) based on simulations and corresponding results. First, a system description with the according Ordinary Differential Equation (ODE) is established and the SES that will be compared are gathered from literature and adapted to be compatible with the system. The SES used in this research is the Observer Design (OD) from Sandoval et al [1] and the Unscented Kalman Filter (UKF) from Kreuzinger et al [2], which uses chosen sigma points and average temperatures to estimate the temperatures of neighboring control volumes. The goal of these SES is to estimate the temperature of the different control volumes as precisely as possible to ultimately have a good estimation of the State of Charge (SoC) of the storage tank.

In the established Ordinary Differential Equation (ODE) the changes in temperature caused by buoyancy, conduction, convection, and losses to the environment are taken into account. These adapted SES are simulated over time using MATLAB and Simulink and the different states (loading, tapping, or idle) are used to get the most relevant results. From these simulations, it can be established that the UKF has a low error margin and settling time and has medium computational effort while the OD has a high computational effort and settling time.

When looking at the comparison criteria and the differences in estimated SoC make the UKF the better SES for this stratified storage tank and controller.

Contents

Abbreviations

CHP Combined Heat and Power. 8, 10

DHS District Heating System. 2, 8–10, 17, 26

EKF Extended Kalman Filter. 21, 22

GRV Gaussian Random Variable. 21, 22

OD Observer Design. 2, 18, 19, 28, 29, 31–35, 44–48

ODE Ordinary Differential Equation. 2, 10, 21

SES State Estimation Strategy. 2, 9, 15–18, 21, 27–35

SoC State of Charge. 2, 9, 17, 26–35

UKF Unscented Kalman Filter. 2, 21–25, 28–35, 40–42

UT Unscented Transformation. 21–24

List of Variables

The next list describes several variables that will be later used within the body of this Integration Project report

α Fluid diffusivity [m$^2/s$]

λ_w Heat conductivity [W/(m $\cdot K$)]

ρ Density [kg/m^3]

A Area [m^2]

c Heat capacity [J/K]

c_p Specific heat [J/(kg$\cdot K$)]

d Diameter [m]

h Specific enthalpy [kJ/kg]

k_w Heat transfer coefficient [W/(m$^2 \cdot K$)]

P Perimeter [m]

T Temperature [$^\circ$Celsius]

u Mass flow rate [kg/s]

1. Introduction

Energy is being used to heat spaces in buildings and District Heating Systems (DHS) is a system that uses hot water to heat buildings more efficiently so fewer resources are needed to provide the same amount of energy [3]. A visualization of a 4th generation DHS is shown in Figure 1. An important aspect is that DHS makes use of fuel or heat resources that would be thrown away by others such as industrial processes and power plants [4]. Next to that, DHS systems can also be run with different sustainable energy resources (including solar power and biomass) which DHS can be used to heat the water [5].

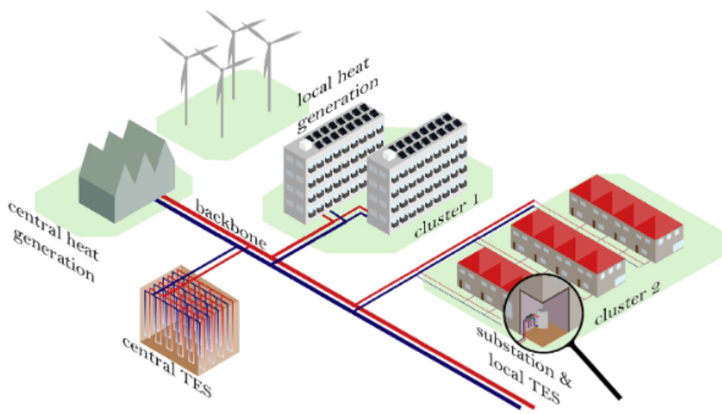

Figure 1: Example structure of a 4th generation district heating system [6]

With the current climate problems, it is necessary for DHS to be optimized in its operations and heat losses to have the largest positive effect on the climate problems possible. DHS makes use of a connection between buildings in a district by using pipes which ensures that different sources of heat can be used [3]. DHS are generally connected to Combined Heat and Power (CHP) plants which provide both heat and energy to the DHS [4]. In DHS, storage tanks can be used so water can be stored until it is needed to be used somewhere in the system. In stratified storage tanks hot and cold water are stored. Hot water stays at the top of the storage tank and cold water at the bottom and the two different temperatures are separated by a so-called thermocline which is a (non-solid) line in the storage tank in which the temperature changes drastically [7].

This thermocline is a line that divides the hot and cold water in the storage tank. The location of this thermocline is measured with sensors that are present along the length of the storage tank and provide information on how much hot water is present in the storage tank and thus also how much heat is present [5]. This calculation can be used to find the percentage of the State of Charge (SoC) for the storage tank, providing information about of energy present in the storage tank and thus the amount of tapping or loading that can occur in the near future [8]. For the storage tank, controllers are important to manage the in- and outflow to get the SoC to reach the desired value [5]. These controllers use the values that are estimated by a state estimation strategy on which this thesis will be focusing.

A state estimation uses equations and input values (gathered from sensors) to estimate the location, amount/level of some or multiple variables in a system. It evaluates the internal state of the system and provides directions about the necessary inputs for the controller to determine the inflow of the storage tank [9].

In DHS, it is important to make sure the most relevant factors are taken into account to be able to make a comparison between state estimation strategies (SES) that is useful. These factors include the amount of water flow, temperatures of the water that is incoming and outgoing, heat power, and heat losses [9]. A state estimation based on customer demand has been made by [10] but the presented research project focuses on the SoC in storage tanks and the SES that estimates its value. This thesis will compare two different SES with the focus on the SoC to see which strategy estimates the value of the SoC most precisely. These requirements will be found and used to make a good comparison. This thesis will also develop a controller to change the in- and outflow according to the desired value of the SoC.

In order to achieve this, first a system description will be established and two SES gathered from literature will be altered according to the system description to be compatible. Then a controller will be added to the SES to change the inflow according to a desired value of the SoC. Lastly, simulations will be run from which the results of both SES will be compared according to previously established comparison criteria. This will determine which of the two SES performs better on the chosen storage tank and controller.

2. System Description

In this research, the system is the stratified storage tank used in DHS. As discussed in Section 1 this stratified storage tank is filled with hot and cold water. As this storage tank is involved in DHS, inflows and outflows are present in the system of the storage tank and the most relevant factors that make sure that energy is added to the system or where energy is lost have to be taken into account to get a relevant and realistic model.

A visualization of the storage tank and the in- and outflow is shown in Figure 2. In this figure the Combined Heat and Power (CHP) is shown as well as the in- and outflow $u(t)$ with the respective temperatures T_{in} and T_{out}. A heat exchanger is present at the consumers which will gather the heat from the hot water to help heat up the buildings. At last, the environment temperature is described as T_∞.

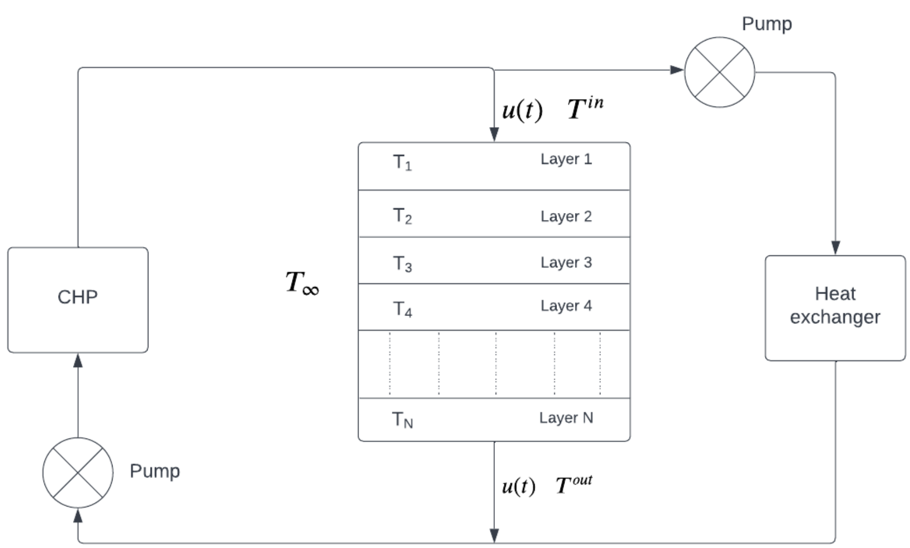

Figure 2: Schematic representation of a stratified storage tank derived from Sandoval et al [1]

2.1 *System Equations*

The energy equation 1 is the ODE system equation that is used in this research.

$$\frac{dT_i(t)}{dt} = \alpha\frac{T_{i+1} + T_{i-1} - 2T_i}{\Delta z^2} + \frac{P_i k_i}{\rho A_i c_p}(T_\infty - T_i) + u(t)\frac{T_i^{in} - T_i}{\rho A_i \Delta z}, \\ + (\theta_{i-1}\frac{1}{\mu}log(e^0 + e^{\mu(T_{i-1}-T_i)}) - \theta_{i+1}\frac{1}{\mu}log(e^0 + e^{\mu(T_i - T_{i+1})})\Delta t \quad (1)$$

$$With\ \theta_{i-1} = \frac{A_{i-1}\Delta z_{i-1}}{A_i\Delta z_i + A_{i-1}\Delta z_{i-1}}\ \varepsilon[0,1] \tag{2}$$

In these equations the following variables and parameters are used:

- α: fluid diffusivity $[m^2/s]$
- T_i: temperature of the control volume $[°C]$
- T_∞: temperature of the environment $[°C]$
- u: mass flow rate $[kg/s]$
- ρ: density of the water $[kg/m^3]$
- P_i: perimeter $[m]$
- k_i: heat conductance of the isolation wall $[W/(m^2 \cdot K)]$
- A_i: surface area $[m^2]$
- Δz: difference in height $[m]$
- μ: parameter which value is 10
- d_i: diameter of the control volume $[m]$
- c_p: specific heat $[J/kg * K]$

The index i describes the respective control volume for the equation.

These system equations are gathered from Lago et al [11] and Kreuzinger et al [2] and describe the relevant factors that influence the temperature changes in the storage tank. equation 1 describes a one-dimensional model of a storage tank which includes conduction, convection, energy loss to the environment, and buoyancy. It was chosen to use a 1D-model for simplicity as two or three-dimensional models are too complex to be used for optimization purposes [11]. One drawback of 1D-models is that the energy changes are not as detailed as it is described in 2D- or 3D models.

A 1D model that is often used only focuses on the energy balance equation is quite similar to the model of Kreuzinger et al [2]. This model is as follows:

$$\frac{dT_i(t)}{dt} = \frac{\lambda_w}{\rho c}\frac{T_{i+1}(t) - 2T_i(t) + T_{i-1}(t)}{\Delta z^2} - u\frac{T_{i+1/2}(t) - T_{i-1/2}(t)}{\Delta z} - \frac{4k_w}{d_i \rho c} \cdot (T_i(t) - T_{amb})$$
$$For\ i = 1, ..., N \quad (3)$$

With the additional parameters:

- λ_w: heat conductivity $[W/(m \cdot K)]$
- c: heat capacity $[J/K]$
- c_p: specific heat $[J/(kg \cdot K)]$

This equation considers the conduction and convection of the storage tank and the energy losses to the environment. Kreuzinger et al [2] reveal that a storage tank can have three different states in which the storage tank is either loading (more hot water is added), tapping (hot water is leaving), or is idle (no in- or outflows are operating). In a system in which the mass of the inflow is equal to the mass of the outflow the states can be determined by the average temperature of the in- and outflow [2]. If the temperature of the inflow is higher than the temperature of the outflow the system is in a loading state. These three different states are shown with the variable u.

$$u = \begin{cases} > 0, system\ is\ in\ loading\ state \\ = 0, system\ is\ idle \\ < 0, system\ is\ in\ tapping\ state \end{cases} \quad (4)$$

One important aspect of this model is that it is missing the buoyancy term. This term must be included to make the system more detailed and valid [11]. Buoyancy effects are important as, for example, the top layer will experience more heat losses to the environment because the contact area to the environment is larger than the layer below. Therefore, this top layer will eventually have a temperature that is lower than the temperature of the layer below, and mixing will occur as a water with a lower temperature has a higher density and instead of the water floating it will mix with the layer below [11]. The article of Lago et al [11] suggests manners to include the buoyancy term in a 1D model. They show how this term came to exist and how it can be added to the energy balance equation 7. Important to note is that, even though the system is simplified in a 1D model, the model equations of [11] have been simulated against real data and have shown to be valid and useful for optimization purposes.

Comparing equation 1 to equation 3 it can be seen that the first three terms are very similar as they describe the same aspects of the model (convection and conduction). In equation 1 two terms have been added which are continuous smooth functions that add the behavior of buoyancy to the model [11]. This term is taken from an algorithm used to include buoyancy in the energy balance equation [11].

One important variable of this buoyancy term is the θ_{i-1} and θ_{i+1} which are the ratio between the volume of one layer and the total volume of the layer above or below. This ratio is important for the rate of the energy exchange [11]. In Lago et al [11] it was also found that the value for the scaling factor μ is reasonable to have a value of 10. This constant μ makes sure that the max approximation of the buoyancy is steeper.

Lago et al [11] has simulated this model against real-time data and found a low error margin making this model to be useful for further research. Lago et al [11] use layers with different volumes but for the simplification of the model it has been chosen to use control volumes with equal dimensions as is done in other papers [2] [1]. Having control volumes with equal dimensions causes the variables θ_{i-1} and θ_{i+1} to become constants with an equal value.

2.2 *Balance Equations*

The equation 1 is found from balance equations that are used to describe a 3D-model [12]. To obtain the model, three principles from physics are considered, namely: [13]

1. Conservation of mass
2. Conservation of momentum
3. Conservation of energy

When using all three equations together a 3D-system can be established Yaïci et al [12]. However, to simplify the model a 1-dimensional system will be used as 2D or 3D systems are too complex to be used in optimization research [11]. Next to that, research has shown that a 1D model, even though it is simplified, accurately describes the behavior of a storage tank [1]. One problem with 1D models is that they do not include the buoyancy that occurs in the storage tank [11]. As a 1D-model has been chosen, these balance equations need to be rewritten to 1D balance equations which are done according to Hauschild et al [14]. It is important to understand these equations to understand the behavior that the storage tank shows. All three equations lead to the Navier-Stokes equations [13].

Starting with the conservation of mass, the following equation can be used [12] [13] and is rewritten to 1D equations according to Hauschild et al[14]:

$$\vartheta_x u = 0 \tag{5}$$

With this equation, important assumptions are made that the inflow to the storage tank is the same as the outflow and that the density is constant over time. Consequently, the mass of fluid that is present in the system will not change over time [2].

The equation for conservation of momentum is made according to [12] and is rewritten to 1D equation according to Hauschild et al[14]:

$$\rho\frac{\delta u}{\delta t} + (\rho u \vartheta_x u) = -\vartheta_x p + \vartheta_\tau - \rho\beta(T - T_{ref})g \tag{6}$$

The goal of this equation is to provide information about the body and surface forces acting in the system [13]. This equation involves the stress (τ) (caused by the surface forces) and buoyancy ($\rho\beta(T - T_{ref})g$) terms. In the buoyancy term, the temperature is exchanged because of mixing that occurs between layers and is influenced by the difference in temperatures, the density (ρ), and the weight (g). Other terms in this momentum balance equation include the mass flow rate (u) and pressure (p) which also influence the momentum in a system.

The equation for the conservation of energy is also rewritten to 1D equations according to [14] [13] and can be described as follows:

$$\rho C_p\frac{\delta T}{\delta t} + \rho C_p u \cdot \vartheta_x T = \vartheta_x \cdot (\lambda \cdot \vartheta_x T) \tag{7}$$

It includes the mass flow rate (u) with its respective temperature changes and the changes in energy because of its surroundings ($\lambda \cdot \vartheta_x T$) but the term for thermal conduction can also be derived from this term regarding energy loss to the surroundings.

It can also be seen that the buoyancy term that is present in Equation 3 can not be found in Equation 7 as this term belongs in the balance equation of momentum 6. Lago et al [11] [11] found a term through an algorithm that could be included in the energy balance equation to add the effects of buoyancy during simulations. This term must be added as it influences the temperature in a layer because of mixing water between layers. Consequently, by implementing the buoyancy term the model because more detailed and valid to be used for simulation purposes.

2.3 *Storage Tank*

2.3.1 *Control Volumes*

Control volumes can be established in the system as as energy changes occur because of convection and conduction within the water that is present in the storage tank. Article [15] explains the stratification process and the different zones that exist because of stratification in a storage tank. Karim et al [16] describe more in-depth the behavior of the thermocline according to differences in dimensions (aspect ratio) and inlet velocities. All these articles make use of control volumes in the stratified storage tank which simplifies the model as it can be assumed that the temperature is the same in the entire control volume. It also simplifies the model as it discretizes the storage tank in a 1D or 2D model in which only vertical changes in energy occur [2]. This spatial discretization is done in order to minimize numerical diffusion meaning that, during simulations, the diffusivity will have a higher value than the true value of diffusivity [2].

The schematic representation of a stratified storage tank in Figure 2 shows the control volumes as layers with their respective temperatures [1].

2.3.2 *Aspect Ratio*

Karim et al [16] researches the impact of different variables on a stratified storage tank. One of these variables is the height/diameter difference (aspect ratio). From its research can be concluded that a higher aspect ratio makes sure that the width of the thermocline is smaller as well as the surface area of the environment. This smaller width of the thermocline is caused by a lower amount of mixing that occurs in such a storage tank which is preferable as less energy loss will take place. The smaller surface area will make sure that less energy is lost to the environment [11]. The influence of such variables is important in choosing the right dimensions for the storage tank as energy losses will be minimized and the SES can give a more specific location for the thermocline.

2.3.3 *Sensors*

Sensor placement is important to limit the error margin of the SES. These temperature sensors are usually placed vertically along the height of the storage tank [1]. The error margin of an SES can be lowered by increasing the number of sensors alongside the storage tank. However, in reality, a limited amount of sensors is being used [1]. This is mostly done to reduce costs.

Kreuzinger et al [2] use three sensors in their simulations in which they concluded that it is important to locate two sensors close to the bottom and top but far enough from the mixing regions. It is required to use a third sensor to remove uncertainties [2].

According to Sandoval et al [1], it is ideal to have a temperature sensor present in each control volume but as this is not always available they also performed simulations with limited temperature sensors. When only two sensors are used these sensors should be stated at the top and bottom of the storage tank for low error results [1].

For this research, it has been chosen to use three sensors, one at the bottom, middle, and top of the storage tank. This way the SES should have a low error margin in estimating the temperatures of the respective control volumes while minimizing the number of sensors in use.

3. Comparison Criteria

Now that the system has been developed, some criteria need to be established according to which the different SES will be compared.

SES is used to evaluate the system and provide directions for changes that the Control Strategy should implement [9]. In order to work properly, the SES needs to have a low error margin in the evaluation of the system and this is one of the main comparison criteria that this research will focus on. This low error margin is important for optimal performance of the DHS with regards to sustainability as a more precise estimation of the temperature makes sure that energy losses can be limited by the controller [10].

As the SES needs to be used by DHS it is important that the usage of the SES is not too difficult but can be easily programmed and used. This is thus another comparison criterion in this research.

Another important criterion of the SES is the settling time to make the system reach the desired value of the SoC. The value of the desired SoC changes over time as the required energy by the consumers changing over time. Because of this, the settling time is an important comparison criterion which value can be approximated by looking at the graphs and seeing when the desired value is reached and is stable.

4. State Estimation Strategies

The observer design (OD) of Sandoval et al [1] is followed as one of the SES for the comparison in this research. This OD is chosen as it is a very recently published article. Necessary changes will be made to the model of the OD to make it compatible with the model that is used in this research.

Another SES that has been chosen for the comparison is that of Kreuzinger et al [2]. This SES has been chosen because it is well established and this article is currently still being cited. Next to that, this article provides all information necessary to make the required changes to the OD to be suitable for the model that is being used in this research.

The goal of an OD is to solve the observation problem is to estimate the temperature $\hat{T}$ with the given input values of the inflow u and the true temperature received from the three sensors T.

4.1 *Observer Design Sandoval et al*

According to Sandoval et al [1], a stratified storage tank is a non-linear system and therefore needs a non-linear model for the OD. Sandoval et al [1] took their basis for their OD from the discrete Luenberger observer structure [1]. A non-linear discrete-time system has a general form that looks as follows [17]:

$$\begin{cases} T_{t+1} = F(T_t, u_t) \\ y_t = h(T_t) \end{cases} \tag{8}$$

Which can be visualized in Figure 3.

This visualization only describes the system that is used in Sandoval et al [1] but as our system in Equation 1 is more precise. regarding the energy losses, some changes need to be made and some matrices have to be added to the OD to make it compatible with the system.

A more detailed form of that of Equation 8 that is used by Sandoval et al [1] and adapted to the system of Equation 1 is as follows:

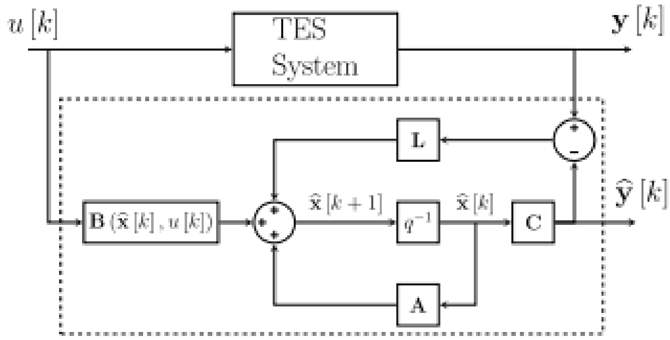

Figure 3: Visualization of the initial Observer Design by Sandoval et al [1]

$$\dot{\hat{T}}[t] = A\hat{T}[t] + B(\hat{T}[t], u[t])u[t] + \Gamma[t] \tag{9}$$

$$\hat{T}[t+1] = \dot{\hat{T}}[t]\Delta t + \Theta[t] \tag{10}$$

$$\hat{y}[t] = C_0\hat{T}[t] \tag{11}$$

In equations 9 and 10 the terms $\Gamma[t]$ and $\Theta[t]$ represent buoyancy and energy losses to the environment respectively and are not present in the model of Sandoval et al considered in Figure 3 and thus have to be added to the OD of Sandoval et al [1]. Here the vector $u[t]$ is again the mass flow rate of the system.

The entries in the vector $y[t]$ are dependent on the number of sensors and their placement as they show where the measurements are from within the storage tank. The entries will have value y_i if there is a sensor present in the respective control volume.

The variables that are used in the OD are explained as follows:

- t = time during the simulation

- A and C = output matrices in which C is a 3 $\times Nidentitymatrixand$A$isaN \times Nsystemmatrix$B$=$ $controlorinputmatrixwithentriesconsistingoftemperatureofthelayers$(T= $statevector)andtheope$ $massflowrate)$

- $\hat{T}[t]$ and $\hat{y}[t]$ = the estimated outputs at time t.

- L = diagonal matrix used for adding the error to the system

- Γ is at matrix that adds the energy loss to the environment to the OD

- Θ is a matrix but adds the buoyancy terms of the system description to the OD

The matrices that Sandoval et al [1] have used in these equations are adapted to the system described in Section 2. The matrices used in this research are as follows:

System matrix A:

$$A = \begin{bmatrix} a_1 - \alpha & c_1 & 0 & \cdots & 0 \\ b_2 & a_2 & c_2 & \cdots & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & b_N & a_N - \alpha \end{bmatrix} \tag{12}$$

With the following entries:

- $a_i = 2\alpha - \frac{P_i k_i}{\rho A_i \Delta z}$
- $b_i = \alpha$
- $c_i = \alpha$

The control matrix B depends on the value of the input of u which expresses the state in which the system operates (loading or tapping) and has the following form:

$$B(T,u) = \begin{cases} \begin{bmatrix} \frac{(T_i^{in} - T_1)}{A_i \Delta z \rho} \\ \frac{(T_1 - T_2)}{A_i \Delta z \rho} \\ \vdots \\ \frac{(T_{N-1} - T_N)}{A_i \Delta z \rho} \end{bmatrix} \forall\ u > 0 \\ \\ \begin{bmatrix} \frac{(T_1 - T_2)}{A_i \Delta z \rho} \\ \frac{(T_2 - T_3)}{A_i \Delta z \rho} \\ \vdots \\ \frac{(T_N - T_i^{in})}{A_i \Delta z \rho} \end{bmatrix} \forall\ u < 0 \end{cases} \tag{13}$$

The buoyancy term is added through matrix $\Theta[t]$ its explicit form is as follows:

$$\Theta = \begin{bmatrix} \theta_2 \frac{1}{\mu} log(e^0 + e^{\mu(T_1 - T_2)}) \\ \theta_1 \frac{1}{\mu} log(e^0 + e^{\mu(T_1 - T_2)}) - \theta_3 \frac{1}{\mu} log(e^0 + e^{\mu(T_2 - T_3)}) \\ \theta_2 \frac{1}{\mu} log(e^0 + e^{\mu(T_2 - T_3)}) - \theta_4 \frac{1}{\mu} log(e^0 + e^{\mu(T_3 - T_4)}) \\ \vdots \\ \theta_{N-1} \frac{1}{\mu} log(e^0 + e^{\mu(T_{N-1} - T_N)}) \end{bmatrix} \tag{14}$$

The last term that is added is the energy loss to the environment which is added through matrix $\Gamma[t]$.

$$\Gamma = \begin{bmatrix} \frac{P_1 k_1}{\rho A_1 \Delta z} T_\infty \\ \frac{P_2 k_2}{\rho A_2 \Delta z} T_\infty \\ \vdots \\ \frac{P_N k_N}{\rho A_N \Delta z} T_\infty \end{bmatrix} \tag{15}$$

Matrix L is a diagonal matrix whose main diagonal components are $[l_1, ..., l_N]$ and C is an identity matrix with a size 3x5 as we have three layers in which a sensor is present and there are five control volumes in total. Important to note is that the entries of the C matrix are dependent on the number of sensors and their placement. In the matrix, the entry will have a value of 1 if there is a sensor present in that layer.

After $y[t]$ has been measured the estimation error ($\widetilde{e}[t]$) can be calculated by looking at the difference of $y[t]$ and $\hat{y}[t]$ [1]. With this value the error dynamics $\widetilde{e}[t+1]$ can be calculated as follows:

$$\widetilde{e}[t] = y[t] - \hat{y}[t] \tag{16}$$

$$\widetilde{e}[t+1] = (A - LC)\widetilde{e}[t] + B(\widetilde{e}[t], u[t])u[t] + \Gamma[t] \tag{17}$$

The value of $\widetilde{e}[t]$ will be used when estimating the next value of $\dot{\hat{T}}$ as it will update the error made in the previous estimation step. The equation for the $\dot{\hat{T}}$ now looks as follows:

$$\dot{\hat{T}}[t] = A \cdot \hat{T}[t] + B(\hat{T}[t], u)u[t] + L \cdot \widetilde{e}[t] + \Gamma \tag{18}$$

$$\hat{T}[t+1] = \hat{T}[t] + \dot{\hat{T}}[t]\Delta t + \Theta \tag{19}$$

4.2 *Unscented Kalman Filter*

Another SES that will be used in the comparison is the Unscented Kalman Filter (UKF) that Kreuzinger et al [2] describe in their article and which is suitable for any nonlinear ODE [2]. The UKF is an extension of the Extended Kalman Filter (EKF) but uses an Unscented Transformation (UT) which is used to calculate the true mean and covariance of the Gaussian Random Variables (GRV) meaning that the propability density function can be written in the general form [18]. The EKF also makes use of the GRV but not of the UT which causes the EKF to experience problems with calculations of the true mean and covariance of the samples. The true covariance and true mean are used to estimate the temperature values for the following time step and these true covariance and true mean are updated after each time step t_k.

The basis equations of the UKF are stated as follows:

$$\dot{x} = f(x(t), u(t)) + w(t) \tag{20}$$

$$y(t) = h(x(t)) + v(t) \tag{21}$$

In this basic UKF the $w(t)$ is the process noise and the $v(t)$ is the observation noise. The goal of the UKF is to estimate the so-called noisy states by putting an earlier made estimation of the temperature through a nonlinear transformation and using this to solve a Ricatti equation [2]. The UKF goes through two types of steps when estimating the output value which are the prediction step and the update step. All equations that Kreuzinger et al [2] used to generate the UKF for their system are already established equations for the UKF. Therefore, only variable names need to be adjusted to make the UKF compatible with the system equation 1.

In the UKF the entries in the x matrix are the temperatures in the different layers and thus the matrix has size N as in this system there are N amount of control volumes in the storage tank.

$$x = \begin{bmatrix} T_1 & T_2 & ... & T_N \end{bmatrix}^T \tag{22}$$

Kreuzinger et al [2] propose to use a discrete-continuous-time framework to use this UKF in a stratified storage tank with time interval t_k. They have found that this increases the accuracy of the UKF [2]. Another important variable in the UKF is the matrix $\mathscr{X}_{k-1}$ which refers to the set of vectors of sigma points which are the weighted points that represent the distribution and are from an initial mean and covariance [19] and these sigma points have respective weights W_i with which the next mean and covariance are calculated [20]. The matrix $\mathscr{X}_{k-1}$ is established by the vectors $\mathscr{X}_{i,k-1}$ in which the entries are calculated with prediction steps as follows:

$$\mathscr{X}_{0,k-1} = \hat{x}_{k-1} \tag{23}$$
$$\mathscr{X}_{i,k-1} = \hat{x}_{k-1} + (\sqrt{(L+\lambda)P_{k-1}})_i, \quad i = 1, ..., N \tag{24}$$
$$\mathscr{X}_{i,k-1} = \hat{x}_{k-1} - (\sqrt{(L+\lambda)P_{k-1}})_{i-N}, \quad i = N+1, ..., 2N \tag{25}$$
$$\lambda = \alpha^2(N + \mathbf{k}) \tag{26}$$

Equations 23-25 is the first step of the UT in the UKF. The index of zero describes the sigma points around the mean values and the index i reports the ith column of the covariance matrix. λ is a scaling parameter that indicates the distribution around the mean $\hat{x}_k$. α is the distribution of the sigma points around the mean $\hat{x}$ and has a value of 0.001 according to Kreuzinger et al [2], **k** is also a scaling parameter whose value is zero according to Kreuzinger et al [2], and N is another scaling parameter which value is the dimension of the state vector, which is, in this case, the number of control volumes in the storage tank [18]. P is the covariance matrix in the UKF.

The initial conditions for equations 23-25 are:

$$\hat{x}_0 = E[x(t_0)] \tag{27}$$
$$P_0 = E[(x(t_0) - \hat{x}_0)(x(t_0) - \hat{x}_0)^T] \tag{28}$$

In these equations the E is the expected value.

The variables $\hat{x}_k$, P_k and (k) are updated or predicted after each time step as follows:

$$\mathscr{X}_k = \mathscr{X}(t_k) \tag{29}$$
$$\hat{x}_k = \sum_{i=0}^{2L} W_i^{(m)} \mathscr{X}_{i,k} \tag{30}$$
$$P_k = \sum_{i=0}^{2L} W_i^{(c)} [\mathscr{X}_{i,k} - \hat{x}_k][\mathscr{X}_{i,k} - \hat{x}_k]^T + Q(t_k) \tag{31}$$

$Q(t_k)$ is the covariance matrix of the process noise.

Important to note is that the assumption is made that the noise signals are white, normal distributed and zero-mean noise signals [2]. W_i^c and W_i^m are the respective weights for the covariance and mean respectively [18] and are calculated in the second step of the UT as follows:

$$W_0^m = \frac{\lambda}{N+\lambda} \tag{32}$$

$$W_0^c = \frac{\lambda}{N+\lambda} + (1+\alpha^2+\beta) \tag{33}$$

$$W_i^c = W_i^m = \frac{\lambda}{2(N+\lambda)}, \quad i = 1, ..., 2N \tag{34}$$

In these equations β is a constant which optimal value is 2 [2] [18].

After the covariance and mean are updated these are used again in equations 23-25 to estimate the following set of temperature values.

After these steps the UT is completed and the remaining steps of the UKF are to be completed. First, the sigma points are selected from the matrix $\mathscr{X}_k$ and the predicted output values by the following equations:

$$\mathscr{Y}_k = C\mathscr{X}_k \tag{35}$$

$$\hat{y}_k = \sum_{i=0}^{2N} W_i^{(m)} \mathscr{Y}_{i,k} \tag{36}$$

In this equation, matrix C is an identity matrix with the size NxN.

As the future temperature values are now predicted all other vectors and values have to be evaluated at time t_k by calculating the correction and Kalman gain as follows:

$$P_{y_k y_k} = \sum_{i=0}^{2N} W_i^{(c)} [\mathscr{Y}_{i,k} - \hat{y}_k][\mathscr{Y}_{i,k} - \hat{y}_k]^T + R(t_k) \tag{37}$$

$$P_{x_k y_k} = \sum_{i=0}^{2N} W_i^{(c)} [\mathscr{X}_{i,k} - \hat{x}_k][\mathscr{Y}_{i,k} - \hat{y}_k]^T \tag{38}$$

$$K_k = P_{x_k y_k} \cdot P_{y_k y_k}^T \tag{39}$$

$$\tag{40}$$

In this equation, the matrix $R(t_k)$ is another covariance matrix but this time about the measurement noise with constant entries that describe the precision of the measurements [21].

After the correction and the Kalman gain are calculated the true estimated values can be determined:

$$\hat{x}_k = \hat{x}_k + K_k[y(t_k)] - \hat{y}_k] \tag{41}$$

$$P_k = P_k - K_k P_{y_k y_k} K_k^T \tag{42}$$

With Equations 23-42 the UKF is completed and can be used in the system that was established in Section 2.

5. State of Charge

State of Charge (SoC) is the ratio between the amount of energy currently stored in the storage tank and the amount of energy that is stored in a fully charged storage tank [22]. Estimating the SoC is important as it gives information about the state in which the system is operating which is necessary to provide directions to optimize the systems operations and the possibilities for loading and tapping shortly [8]. The value of SoC is given as a percentage.

A fully charged storage tank only contains hot water that has not experienced any heat losses. The temperature of this hot water will thus be the same as the temperature of the inflow.

In this research, a charged control volume has a temperature of 80°C as the DHS nowadays work with temperatures below 100 °C [3]. Therefore, a control volume for which this temperature is estimated is counted as a fully charged control volume and for the control volumes with a temperature lower than 80°C the percentage of charge needs to be calculated. According to Sandoval et al the energy available in a control volume can be calculated as follows [1]:

$$U_i = m_i h_i T_i \tag{43}$$

In this equation U_i is the amount of energy in a control volume in kj, the m_i is the mass of water present in the control volume and can also be rewritten as $A_i \cdot z \cdot \rho$, h_i is the specific enthalpy of water and T_i is the temperature of the water inside the control volume [1]. As the control volumes have equal size and the amount of water inside the storage tank does not change over time, the mass of water inside a control volume is and the assumption of a constant specific enthalpy of water is made. This causes the amount of energy stored in the control volume to only be dependent on the temperature.

The value of the SoC is calculated as [22]:

$$SoC_{layer} = \frac{U_i - U_{discharged}}{U_{fullycharged} - U_{discharged}} \cdot 100\% \tag{44}$$

$U_{fullycharged}$ is the amount of energy inside a layer when this layer has a temperature of 80°Celsius and with a specific enthalpy of water having the value 418 kj/Kg this value for the chosen storage tank will be approximately 115.38 kj/Kg. This temperature is chosen as it is the temperature used for the inflow and so if the water in the storage tank experiences no heat losses this will be the maximum value for the temperature.

A fully discharged layer inside the storage tank is when this layer has a temperature of 10°C as this is the same as the chosen temperature for the environment and as the temperature of the inflow has the same or a higher temperature, 10°C will be the minimum value the storage tank can have. When the layer has this temperature the energy stored has a value of 14.423 KJ/Kg and this is subtracted from both the current and fully charged energy to get the correct percentage for the SoC.

The SoC of the storage tank is calculated as follows:

$$SoC_{tank} = \frac{(\sum U_i) - U_{tankdischarged}}{U_{tankfullycharged} - U_{tankdischarged}} \cdot 100\% \tag{45}$$

The values for $U_{tankfullycharged}$ and $U_{tankdischarged}$ are five times the value of $U_{fullycharged}$ and $U_{discharged}$ as there are five control volumes present in the storage tank.

It is important to see how the SoC changes over time when the in- and outflow are different and if the SES can estimate the SoC with a low error margin.

6. Controller

A PI-controller is added to the system to determine the inflow that is calculated using the estimated temperature values of the SES as its inputs. This controller uses the proportional gain k_p and the integration gain k_i together with the current and desired SoC to control the amount of inflow u such that the temperature of the layers increases or decreases to achieve the desired amount of SoC in the storage tank. The controller thus directs the system to a desired SoC [23] which changes over time as at different times a different amount of SoC is desired as, for example, consumers require higher amounts of energy when it is cold than when it is warm. The error from the goal is denoted as $e(t)$ and is calculated as $SOC_{current} - SOC_{desired}$. A general equation for a controller looks as follows[23]:

$$u(t) = k_p e(t) + k_i \int_0^t e(t)dt \tag{46}$$

In this equation, the $e(t)$ is the difference between the current situation and the desired situation at time t.

The controller causes the system to overshoot and oscillate around the desired value of the SoC and it is the goal to choose values for k_p and k_i such that the overshoot is low but also the settling time (the time it takes for the system to be stable around the desired value of the SoC) to be low [23]. The value of k_i depends on the value of k_p because of the eigenvalues of the matrix of the controller shown below:

$$\begin{bmatrix} -k_p & 1 \\ -k_i & 0 \end{bmatrix} \tag{47}$$

Setting the eigenvalues of this matrix to have real negative values the value of $k_i = \frac{k_p^2}{4}$ and the value for k_p is chosen by simulating different values and looking for which value of k_p the system exhibits small overshoots and a low settling time. This value is found to be 0.3 for the UKF and to be 0.01 for the OD.

7. Results

Multiple simulations were conducted to assess the performance of the UKF and the OD-based estimation strategy. It was found that the timestep necessary for the OD to have relatively good results had to be lower than the time step that was necessary for the UKF to have a better response. This is the case as the OD needs to have a more continuous calculation to estimate the temperature of the control volumes with a lower error.

The UKF was simulated for a system that includes noise and the OD was simulated with an initial condition that has a value of $0.95 \cdot T0$. This is done to make sure that OD has to take the error into account when starting its estimations to the true temperature of the storage tank without and for the UKF to see how well the SES would function when having random white noise added to the system as this is likely to occur in real-life as well.

7.1 *System*

The results of the System Description can be found in Figures 4 and 14, in Appendix A and B and show the temperature changes of the different layers over time. For these results the UKF and OD have been simulated with their controller to show how the inflow changes over time due to the desired value of the SoC and the temperature of the different layers respond. The system shows a stratified storage tank over time due to the in- and outflows. Due to the controller and the changing value of the desired SoC, the amount of in- and outflow of the system keeps changing over causing the temperature levels of the different control volumes in the system to be changing.

The results of the system shows that the storage tank stays stratified and shows the right behavior for changing inflows.

7.2 *Unscented Kalman Filter*

The results of the UKF can be found in Appendix A and show the trajectory of the estimated values of the respective control volumes against the true temperature and the measured temperature. The measured temperature is different because of the noise that occurs due to the sensors that are used and therefore only occur in control volume 1, 3, and 5 as these are the control volumes in which a sensor is present.

Figures 6-10 in Appendix A show the true temperature against the estimated temperature of the UKF and from these figures can be seen that the estimated temperature exhibits the same behavior as the system does. The error of the estimation temperatures over time is shown in Figure 11 in Appendix A shows that the error has an increase when the mass inflow rate u is changed by the controller. This error is then quickly solved by the UKF as the error continues having a low value afterward.

The average error of the estimated temperature depends on the different layers and the values of these errors are shown in Table 1 below.

The overall mean absolute error of all the layers by the UKF is 0.1365 meaning that on average the estimated value that is calculated by the UKF is 0.1365 $^{\circ}$Celsius higher or lower than the true value of the temperature in the storage tank.

From Table 1 can also be seen that the mean error in layers 2 and 4 are higher than the mean error of layers 1, 3, and 5. This is the case because in layers 2 and 4 there is no sensor present. The SES thus estimates these values without having prior knowledge of the temperature or being able to correct these temperatures with measurements of these layers. It can thus be expected that the estimations of the layers, in which no sensor is present, have a higher mean error value.

The max error value for the UKF is found to be -14.7925 for layer 5. Meaning that at a specific time the estimated value of the temperature in layer 5 was 14.7925 $^{\circ}$Celsius lower than the true temperature of the layer.

Another aspect of the SES that needs to be taken into account is the estimation of the SoC of the storage tank. The behavior of the estimated and the true SoC over time is shown in Figure 12 in Appendix A.

The average error on the SoC of the storage tank has a value of 0.2704 meaning that on average the UKF estimates the SoC to be 0.2704% higher or lower than the value the SoC of the storage tank has in real life.

The max value for the error of the estimated SoC is 5.0315 meaning that at a specific time the estimated value for the SoC was 5.0315% higher than the true value for the SoC at that time.

Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
-0.0002	0.8320	-0.0009	-0.0525	0.0001

Table 1: The average error value of the respective layers for the UKF

The next comparison criteria to be looked at is the settling time of the UKF which is found to be approximately 50 seconds, meaning that when the value of the desired SoC changes the system takes approximately 50 seconds to become stable around that value for the SoC.

The last part of the comparison criteria that needs to be discussed for the UKF is the computational effort of the SES. The computational effort of the UKF is medium as there already exist built-in functions in MATLAB that help compute the UKF predictions. Important is the code for the System Description and the Controller as values that are calculated in these files are used within the code of the UKF. The medium computational effort is mostly caused by the medium size of the time step that is being used for the simulations.

An overview of the performance of the UKF according to the comparison criteria is shown in Table 3.

7.3 *Observer Design*

The results of the OD can be found in Appendix B and show the estimated temperature values over time as well as the error margin.

From the graphs (Figures 14-20) can be seen that the system and the estimated temperatures by the OD behave with many oscillations. This is caused by the oscillations of the estimated temperature of the OD that is used for the controller to determine the inflow. The amount of inflow that is being put into the system shows high oscillations causing the temperature of all layers to oscillate as well.

The error for the estimated temperature of the layers can be seen in Figure 21 and show many oscillations over time and a high value. The average absolute error values of the different layers can be found in Table 7.3. In this table can, again, be seen that the error values for layers 2 and 4 are higher than the error values of layers 1, 3, and 5. This is again because of the sensors that are present in layers 1, 3, and 5. The total average absolute error value of the OD is 421.3343, meaning that on average the OD estimates the temperature in the tank to be 421.3343 °C higher than the temperature truly is. The maximum absolute value for the error has been found to be 4069.9 for layer 4 in the storage tank.

It must be noted that no noise has been added to the OD as the error value of the estimated temperature by the OD was already too high. The noise would have only increased the error values as it causes a random difference between the measured tem-

Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
102.5327	818.1775	103.9696	946.0725	134.9189

Table 2: Error of layers OD

peratures by the sensors and the true temperature in the system.

Figure B.3 shows the estimated SoC by the OD over time. This figure also shows that the estimated SoC oscillates over time which can be expected as the estimated temperature shows many oscillations over time. Figure 23 shows the error value of the estimated SoC by the OD over time which has a mean absolute value of 117.9347 and a maximum absolute value of 500.6063%. This means that on average the OD estimates the SoC with a value that is 117.9347% lower or higher than the true value of the SoC.

From the graphs can be seen that the estimated temperatures by the OD do not become stable over time but keep showing oscillations. The controller does cause the real SoC value to become close to the desired value but does continuously show oscillations. From Figure 14 can be seen that the temperatures in the different layers become somewhat stable approximately 250 seconds after the value of the desired SoC has been changed. This long settling time can be derived from the speed of the controller and observer as the controller needs a smaller kp and ki value than in the UKF to make the OD exhibit the right behavior causing the settling time to increase [24].

The computational effort of the OD is high. The time step in which this SES functions best is lower than the time step for which the UKF already shows good results and therefore it takes longer to simulate the SES over time. Also, the tuning of the OD was much more difficult than the tuning of the UKF and still, the optimal values for the OD have not been found as the convergence to zero error takes too long.

With these many and high oscillations it can be concluded that, even thought the matrices have real negative eigenvalues, that the OD is not stable. The estimated temperatures do not converge to the true temperature values but continue to oscillate around that values.

An overview of the performance of the OD according to the comparison criteria can be found in Table 3.

	UKF	OD
Mean error temperature	0.1365°C	421.3341°C
Mean error SOC	0.2704%	117.9347%
Settling time	50 seconds	250 seconds
Computational effort	Medium	High

Table 3: Overview of the performance on the comparison criteria of the UKF and the OD

8. Conclusion

This research was focused on comparing two SES on a stratified storage tank to determine the SoC as precisely as possible. First, equation 1 was established that describes the system including conduction, convection, environmental heat losses and buoyancy terms. The UKF and the OD have been altered to be compatible with the system description and a PI-controller has been added to control the amount of in- and outflow in the storage tank with the goal of reaching the desired value of the SoC.

The first conclusion that can be made is that the system is exhibiting correct behavior. The model shows that the storage tank will keep being stratified over time and exhibits the right energy gains and losses when the mass flow rate changes. One point of attention is the buoyancy term in the system model as this is a computational artifice that was determined by Lago et al [11]. When using this term in the system Equation 1 the system did not show the right behavior. It was found that this behavior was caused by the value for μ which Lago et al [11] found to be best at value 10. However, when altering this value to be -10 the system did show the correct behavior regarding the energy gains, losses, and stratification. Without the buoyancy term, the system did not show much different behavior but it is recommended to keep using such a buoyancy term to include this part of the heat losses in the system.

The results in Section 7 show that the UKF has a mean absolute error in temperature of 0.1365°C and a mean absolute error of 0.2704% for the SoC. The computational effort of the UKF is medium and the computational effort of OD of high. Overall the UKF is a good SES to be used. The mean error values are low and the settling time of the UKF has a value of approximately 50 seconds. For the OD no noise could be added as the computational effort was already high for the controller and the observer to exhibit the correct behavior as the tuning was especially difficult for the OD and still, the behavior of this observer is not very precise as the settling time is approximately 250 seconds which is very long compared to the UKF. The lowest settling time is preferred as than the desired value of the SoC is reached earlier. The average absolute error values are 421.3342°C for the temperature and 117.9347% for the SoC. More research is necessary to find the values for the parameters for the eigenvalues of this observer and the values for the ki and kp values for the controller that results in more precise estimations and lower settling times [].

From these results can be concluded that, even though the UKF SES was initially supposed to be used on a different system equation, it is also able to function properly and with a low error margin on other system equations. It can, therefore, be expected that the UKF will also be able to work on a more advanced model, for example, a 3-dimensional model. The same thing, however, cannot be said for the OD as this SES exhibits a high settling time, high error values, and requires much computational effort. However, if the controller and observer are better tuned the results of the OD will probably be better.

The overall conclusion is that, according to the simulations in this research, the UKF is a more suitable SES to be used in stratified storage tanks than the OD. It has a low error margin, low settling time, and the computational effort is medium.

9. Discussion

This research focused on comparing two different SES on a stratified storage tank with the goal of finding which SES provides the best estimation on the temperature of the control volumes, has a low settling time and computational effort. Even though multiple assumptions were made to create a simplified model equation (buoyancy term and a 1-dimensional model) the results of this research are still relevant as it shows the differences in computational effort and settling time well.

Improvements can be made in the system equation by creating a 3-dimensional model which includes the buoyancy term according to physics instead of using a computational artifice. In order to do this more research is needed that focuses on the 3-dimensional model itself but keeps the computational effort of this model low so that it is useful for optimization purposes. Having a 3-dimensional model would improve the realistic aspect of simulations as all heat losses are described more in detail and the temperature changes on all the dimensions of the storage tank are described as well. A 3D model would also solve the problems that occurred with the value of μ in the buoyancy term of the system equation as the computational artifice of the buoyancy term would not be necessary anymore.

Another point of further research is to run simulations of the UKF and the system against a real-life storage tank to see if this system and the UKF work any differently in real life. Not only will these real-life simulations help further research in SES but also in the development of stratified storage tank model equations.

For the OD more research is necessary on the tuning of this SES with a controller. In this research, the results of the OD showed a large settling time and the computational effort was high as well. More research on tuning is necessary to limit the settling time and to get a lower computational effort that is necessary.

At last, further research can focus on trying different controller designs to both SES to see how they work together can be recommended. In this Integration Project, only a simple PI-controller has been used to get realistic data from the system and the SES but other controller designs might work differently with the respective SES, for example, a PID-controller that includes the derivative term [25] or a specifically made controller for the SES and its system. Next to that, the controller can become more detailed and the desired level of SoC can be dependent on more variables. For example, the weather will have an effect on the required heat necessary for heating up buildings and this could also be included in the controller for the SoC.

10. References

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Appendix A - Unscented Kalman Filter

A.1 *System*

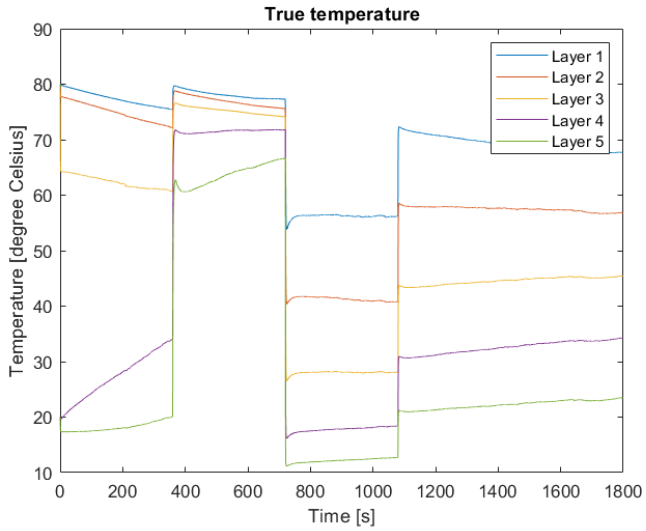

Figure 4: True temperature of the different layers in the storage tank

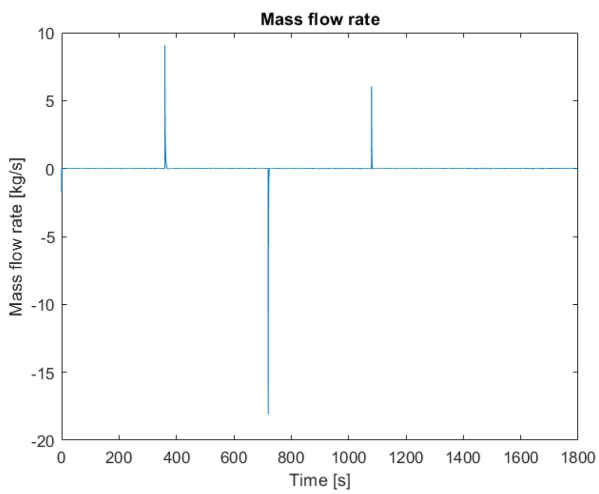

Figure 5: The inflow over time

A.2 *Estimation of the layers*

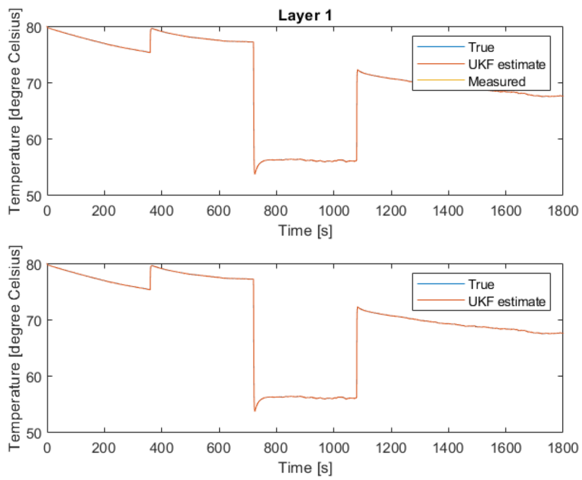

Figure 6: True temperature and estimation of the UKF of layer 1

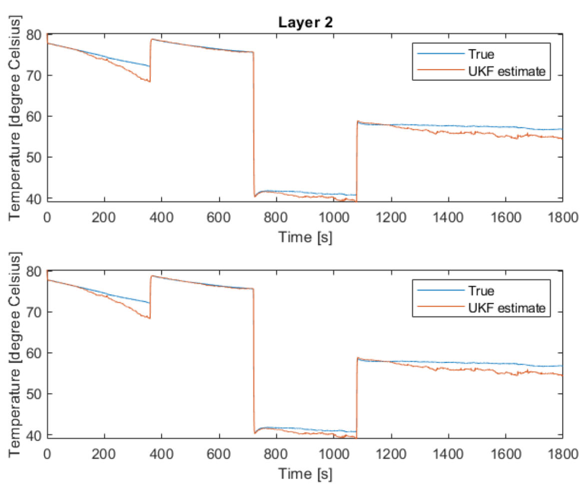

Figure 7: True temperature and estimation of the UKF of layer 2

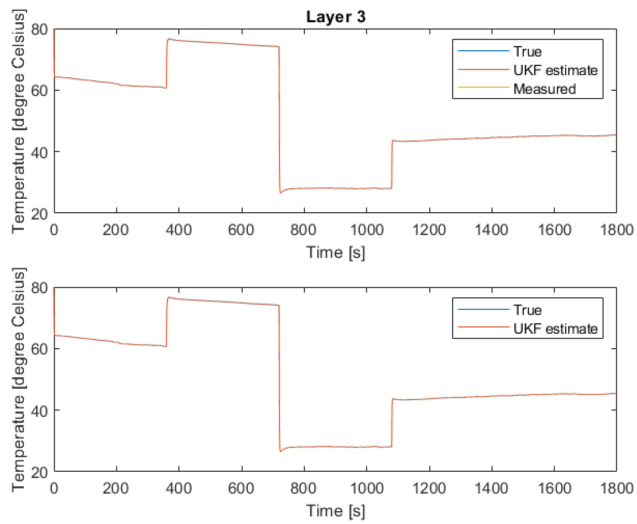

Figure 8: True temperature and estimation of the UKF of layer 3

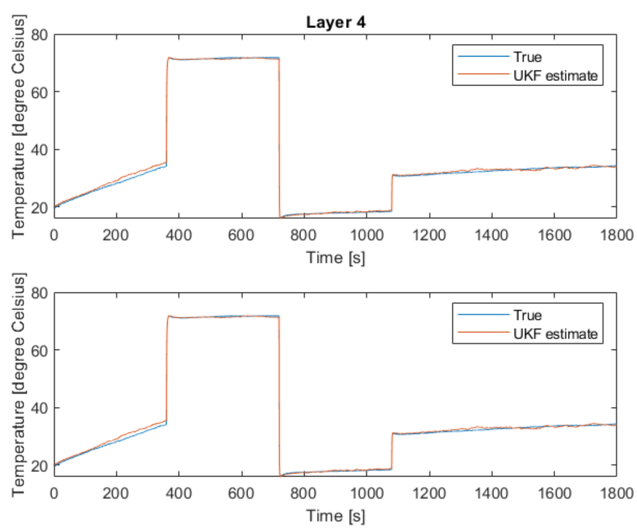

Figure 9: True temperature and estimation of the UKF of layer 4

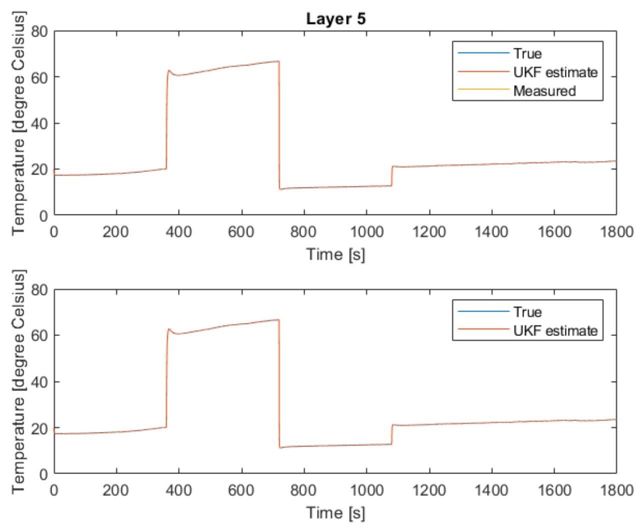

Figure 10: True temperature and estimation of the UKF of layer 5

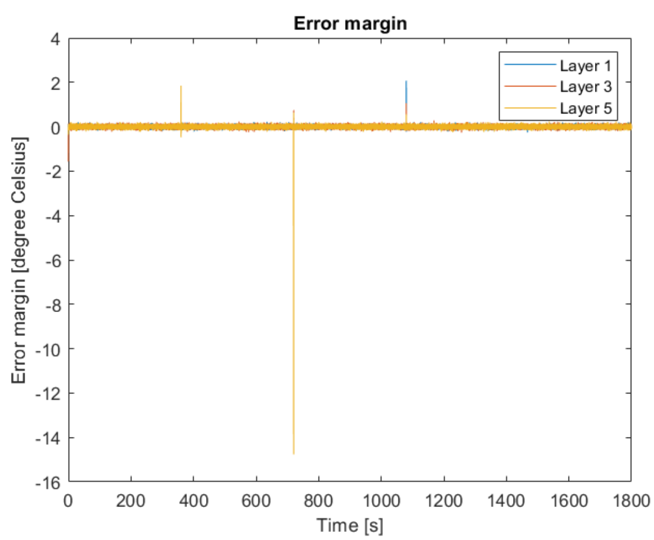

Figure 11: The error of the estimated temperature of the different layers over time

A.3 *State of Charge*

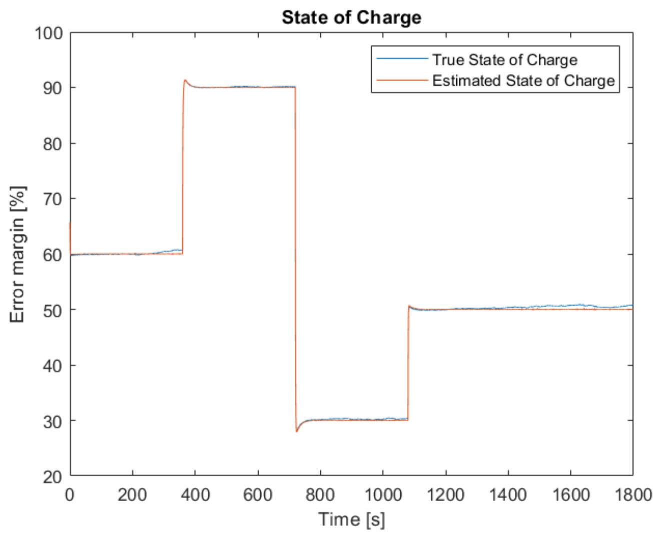

Figure 12: The true and estimated State of Charge over time

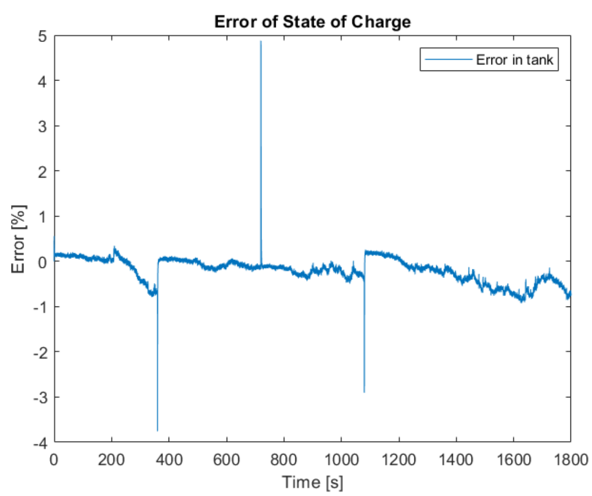

Figure 13: The error of the estimation in the State of Charge

Appendix B - Observer Design

B.1 *System*

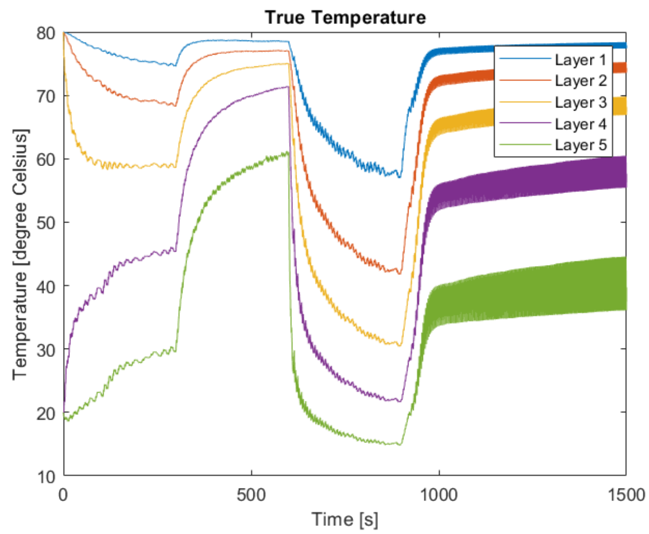

Figure 14: True temperature of the different layers in the storage tank with the OD

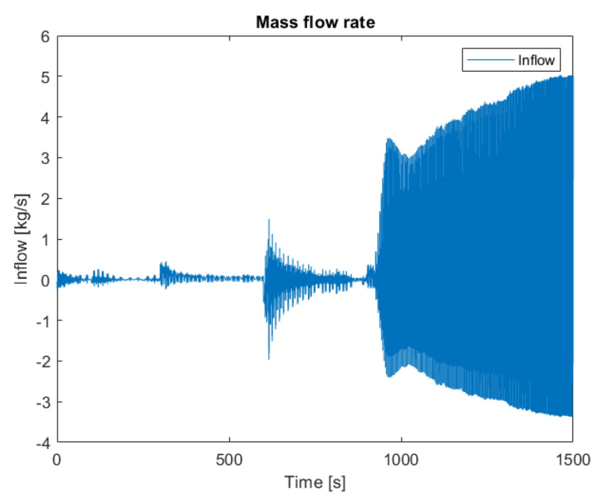

Figure 15: The inflow over time during the OD

B.2 *Estimation of the layers*

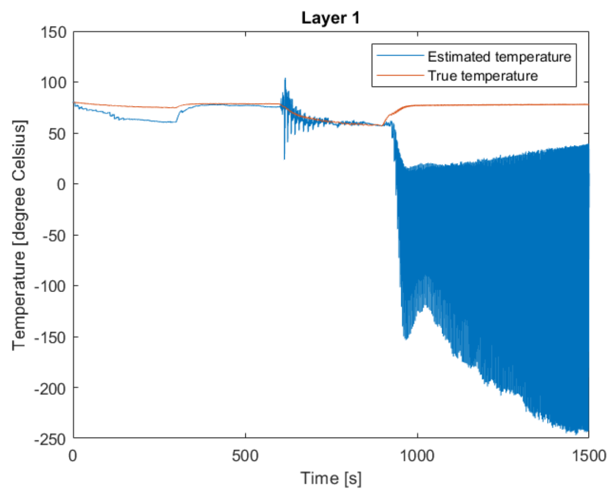

Figure 16: True temperature and estimation of the OD of layer 1

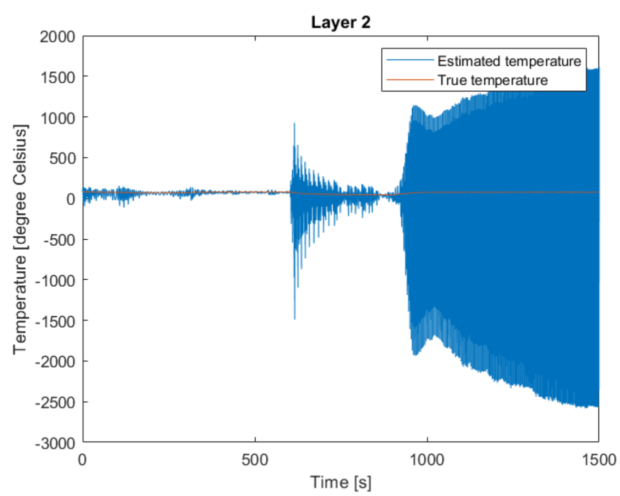

Figure 17: True temperature and estimation of the OD of layer 2

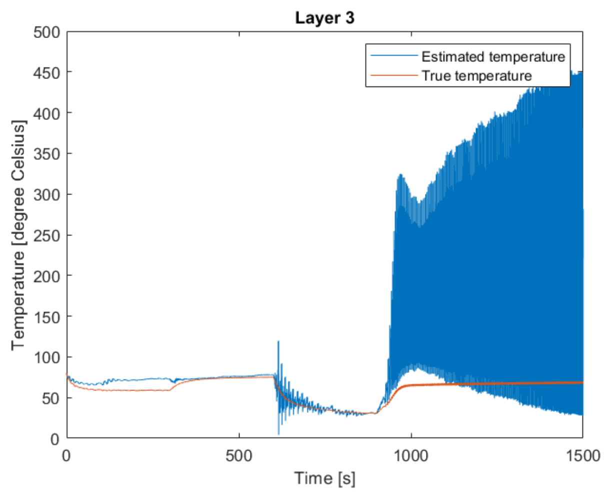

Figure 18: True temperature and estimation of the OD of layer 3

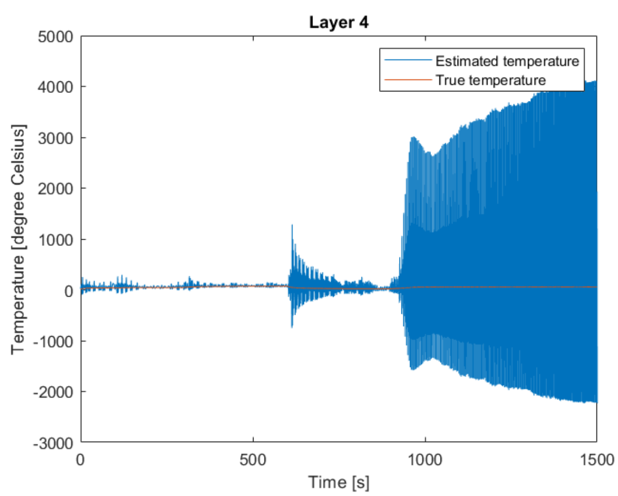

Figure 19: True temperature and estimation of the OD of layer 4

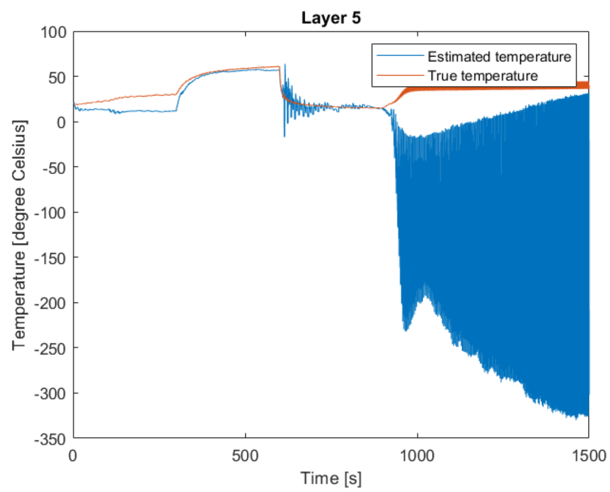

Figure 20: True temperature and estimation of the OD of layer 5

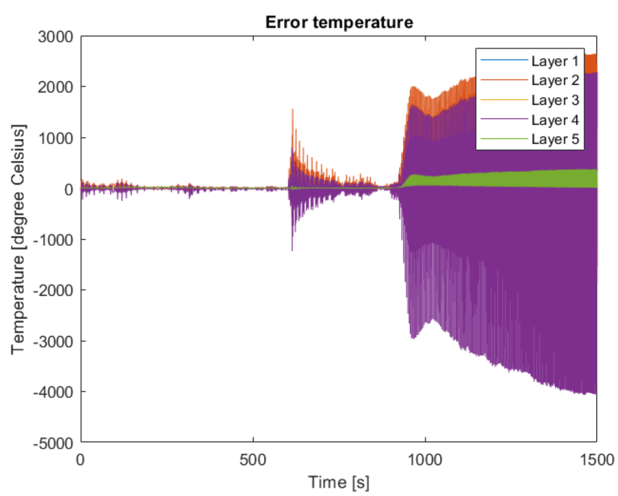

Figure 21: The error of the temperature in different layers during the OD

B.3 *State of Charge*

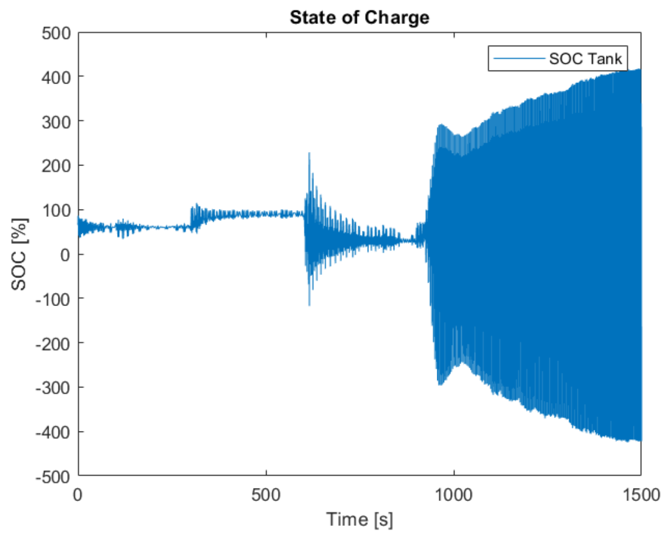

Figure 22: The true and estimated State of Charge over time during the OD

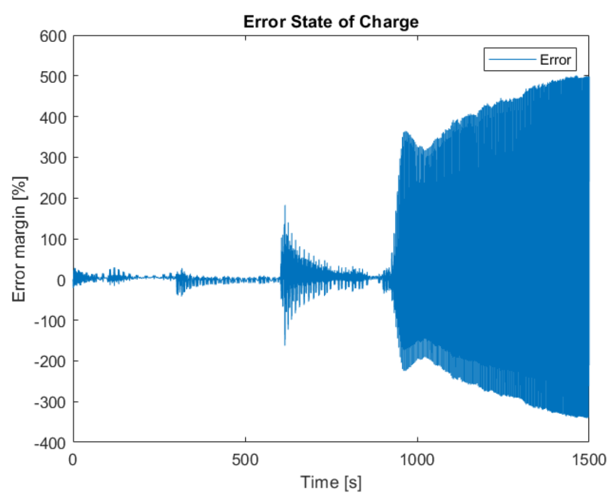

Figure 23: The error of the estimation in the State of Charge during the OD