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How does the Limited Voting rule perform on 'almost party-list' elections?

Bachelor's Project Thesis

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Abstract:
This research investigates the performance of Limited Voting (LV), a voting rule that allows voters to cast fewer votes than the number of candidates in the winning committee, using the 'almost party-list profile' representation. LV's performance is compared to the Multi-Winner Approval Voting (AV) voting rule. The intuitive assumption is that LV can effectively represent minority groups in the winning committee, thereby enhancing diversity. To test this intuitive assumption, we simulate elections and measure the outcomes in terms of diversity (CC-score) and proportionality (PAV-score). The results demonstrate that LV outperforms AV in terms of both the CC-score and the PAV-score, indicating that LV is a promising voting rule for promoting diversity and proportionality in 'almost party-list' profile elections.

1 Introduction

As there exist various types of elections, it is essential to implement multiple types of voting rules that are tailored to a specific situation. All these voting rules have their properties, advantages, and disadvantages. For instance, such properties could relate to the diversity or proportionality of the winning committee, which is achieved by applying the voting rule. Consider an election, where each voter indicates the candidates that they deem acceptable for a particular position. This type of election defines as an approval-based election, which separates candidates into approved and disapproved sets, like a binary classification (Lackner & Skowron (2023)). Approval-based elections are used for various purposes, such as selecting parliament members, choosing competition finalists, or picking members of a scientific organization. The outcome of such an election is determined by a voting rule. For example, in a parliament election, a voting rule determines how seats are allocated among the parties.

1.1 Related Work

The multi-winner voting rules that coincide with approval-based elections are called approval-based committee (ABC) rules. The definition of the voting rules that are considered in this paper are based on the definitions described in Los et al. (2023). The plainest ABC-rule is Multi-Winner Approval Voting (AV) (see Definition 1.2). This rule will select the k candidates that are approved by most voters. Besides AV there exist various other ABC-rules that can determine how candidates are selected. Limited Voting (LV) (see Definition 1.3), is such a voting rule, where voters may approve a committee of size k with at most l candidates where $l < k$ (Lackner & Skowron (2023)). In Janson (2018) however, the author states that voters may approve a committee of size k with at most $l \leq k$. In this research, we assume that $l < k$. Currently, LV is used in some electoral systems, for instance, in Spain to elect its senators. In Spain, each province elects four senators, yet the voters may cast only three votes ("Composición del Senado. Elección y Designación de Senadores" (n.d.)). Reasons to believe that LV may be an interesting rule to conduct further research on is stated in for instance Allouche et al. (2022). The authors describe that more weight must be assigned to smaller ballots and they consider smaller ballots to be more reliable. Take, for instance, the scenario where voters are granted the freedom to cast votes for numerous candidates. In such cases, voters might find it challenging to discern the truly optimal choice among the candidates. On the contrary, if the ballot size would be decreased, it will enable voters to carefully consider their choices and select the candidates they believe to be the most suitable in a specific situation. Therefore, smaller ballots can lead to more informed and deliberate voting decisions. Besides LV, there also exist other voting rules that are based on ballot-length restrictions. Consider Block Vote (Janson (2018)); where each voter votes for at most l candidates, the l candidates having the most votes will be elected. This resembles LV, the key difference is that in block voting l can be equal to k. Therefore, if $l = k$ in LV, it would be Block Voting. Another voting rule is Single Non-Transferable Vote (SNTV), which is the same as LV, only $l = 1$ (Lackner & Skowron (2023)).

1.2 Preliminaries:

Let C be a set of candidates $C = \{c_1, ..., c_n\}$, N be a set of voters $N = \{v_1, ..., v_n\}$, k be the size of the committee and let l be the ballot limit. Altogether, we can form the election instance $E = (N, C, k, l)$. For all voters there is an approval profile, which describes all voters' preferences $A = \{A_i : i \in N\}$, and a ballot profile $L = \{L_i : i \in N\}$, which is the set of all ballots. Furthermore, $A_i \subseteq C$ is the approval ballot of voter i and $L_i \subseteq C$ is the limited ballot of voter i where $|L_i|$ is at most l. Moreover, $L_i \subseteq A_i$, if $|L_i| \leq |A_i|$, or $A_i \subseteq L_i$, if $|L_i| > |A_i|$.

1.2.1 Definitions:

Below we provide the definition of party-list profiles in Definition 1.1, which is a fundamental concept to understand 'almost party-list' profiles. Additionally, the definition of the two voting rules that are relevant throughout this paper will be presented in Definition 1.2 and Definition 1.3. A voting rule will return a winning committee W given the election instance E.

Definition 1.1 (**Partly-list profiles (Peters & Skowron (2019))**). *An approval profile $A = (A_1, ..., A_n)$ is a party-list profile if for all $i, j \in N$ either $A_i = A_j$ or $A_i \cup A_j = \emptyset$. The election instance (A, k) is a party-list instance if A is a party-list profile, and for each voter $i \in N$, $|A_i| \geq k$.*

Definition 1.2 (**Multi-Winner Approval Voting, AV**). *This rule elects the committee $AV(E) = W$ with the k candidates that are approved by most voters. The AV-score of a candidate $c \in C$ is defined as follows:*

$$score_{AV}(A, c) = |\{i \in N : c \in A_i\}| \tag{1.1}$$

The outcome of the election is selecting committee W by maximizing

$$score_{AV}(A, W) = \sum\nolimits_{c \in W} score_{AV}(A, c) \tag{1.2}$$

In case there are multiple candidates with a similar score, and the winning committee cannot accommodate all of them, some candidates will be added randomly. This will be discussed in more detail in Section 2.

Definition 1.3 (**Limited Voting, LV**). *This rule elects the committee $LV(E) = W$ with the k candidates that are approved by most voters, where a voter may cast at most l votes. The LV-score of a candidate $c \in C$ is defined as follows:*

$$score_{LV}(A, c) = |\{i \in N : c \in L_i\}| \tag{1.3}$$

The outcome of the election is selecting committee W by maximizing

$$score_{LV}(A, W) = \sum\nolimits_{c \in W} score_{LV}(A, c) \tag{1.4}$$

Again, in case of ties between candidates randomness is applied, this will be discussed in more detail in Section 2.

1.3 Motivation:

In this research, we are interested to find out how limited voting performs in elections of the type 'almost party-list'. Before, we defined a party-list profile in Definition 1.1. More specifically, party-list profiles are like political parties that we find in political elections where voters are presented to parties and they vote for precisely one party. In the case that approval profile A is a party-list profile, we have sets of voters $N = N_1 \cup ... \cup N_g$, and a set of parties $C \supseteq (C_1 \cup ... \cup C_g)$ that can be divided into g disjoint groups, where all voters from N_i, $i \in [g]$ approve the candidates from only the party C_i (Peters & Skowron (2019)). However, for this research, we are interested in the 'almost party-list' profiles. This means that even though a voter is in favour of some party C_i, a voter may also cast votes for candidates from a party different from C_i, and does not specifically need to approve all candidates from C_i. 'Almost party-list' profiles are relevant as it is adopted by multiple countries such as Switzerland and Luxembourg. In these countries, voters can cast their votes over candidates from multiple party-lists. This results in the voters having more choices and it encourages more personal voting (Mustillo & Polga-Hecimovich (2020)).

Moreover, it is worth noting that LV may not always be a satisfactory voting rule. For instance, if a voter is in favour of more than three candidates but is only allowed to cast three votes, their options are restricted. Nonetheless, LV is particularly beneficial for promoting diversity by giving a voice to minority groups and limiting the power of the majority. The following example will demonstrate this and involves a party-list profile type of election. Suppose there exist four groups of voters, each group of voters is in favour of a specific party, all consisting of k candidates. The winning committee will also consist of k candidates. For AV, the number of votes a voter may cast is unlimited. Assume all groups of voters will vote for the candidates of the party they adhere to. As a result, the winning committee will consist of all the candidates that are a member of the party that the largest group of voters adhere to. This means that the largest group of voters will consistently have the largest impact. In contrast, when we apply LV, the winning committee will look different. Say a voter may cast two votes ($l = 2$) this time and $l < k$. Assume that the voters vote for the first two candidates of the party they adhere to. The candidates in the winning committee will now consist of the first two candidates of the biggest party, the first two candidates of the second biggest party, and so on, until we have reached a winning committee of k candidates. In conclusion, this winning committee is much more diverse compared to the winning committee of AV.

To evaluate LV's performance on 'almost party-list' elections, we will compare its performance to plain AV. We will first simulate elections. Simulations are used since the elections modeled involve dynamic behaviour, on which we test varying sets of inputs. The dynamic behaviour is caused by the high degree of randomness, which makes it impossible to predict the election outcome. Furthermore, in an election, votes must be generated. This is done by making use of the $(p, \phi, g)-$disjoint model described in Szufa et al. (2022). Here we draw a random partition of candidates divided over g parties. Next, we sample the vote from a (p, ϕ)-resampling model. Now it will be decided with a certain probability whether a candidate is approved. This procedure will generate an approval profile on which we apply the LV rule. Subsequently, we can establish LV's performance. As mentioned before, the performance of LV is compared to the performance of AV. First, the performance is measured by measuring proportionality, where disjoint groups of voters should be granted a fair and proportional representation of the voters' preferences (Janson (2018)). A proportional representation assures that the diversity of the voters' opinions is reflected (Aziz & Lee (2019)). To measure proportionality, we will use the PAV-score. Second, the other measure used to evaluate the performance of LV is diversity. For diversity, we attempt to maximize the number of voters who have at least one candidate that they approve in the elected committee (Lackner & Skowron (2023)). The diversity is measured using the CC-score.

In summary, to answer the research question "How does the limited voting rule perform on the 'almost party-list' profile type of election?" we discuss the following. Firstly, in Section 2 this paper describes the architecture of the model, and the setup used to simulate the elections. Secondly, this paper demonstrates the performance of LV and AV in Section 3. Lastly, this paper attempts to answer the research question, discuss the limitations of this research, and makes suggestions for future work in Section 4.

2 Methods

To obtain an answer to the research question we measure the performance of LV in terms of the PAV-score and CC-score. These scores are obtained by building a model that simulates elections, on which we apply the LV-rule. We eventually compare the performance of LV to plain AV. The model can be divided into different components. First, the parties are generated. Based on these parties, the votes are created. After that, the LV-rule and the AV-rule are applied. Finally, the performance is measured. An overview of the model is shown in Figure 2.1.

The elections are repeated 10 times, for different configurations of g, l, k, p, and ϕ. Meanwhile, the number of voters (nv) and the number of candidates (nc) are incremented.

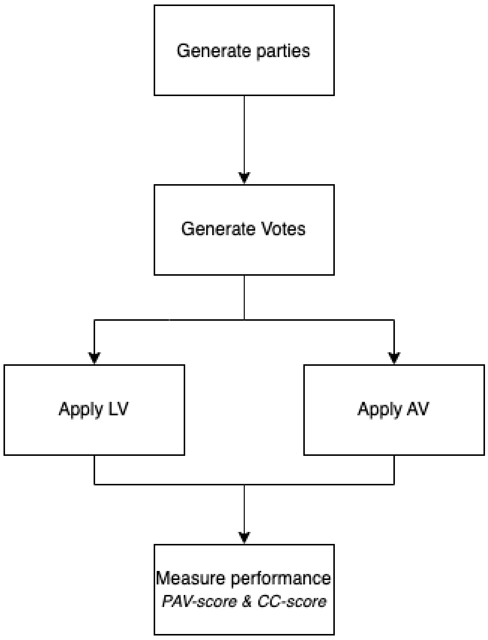

Figure 2.1: Overview of the program

2.1 Generating parties

The parties are generated as follows. We divide the number of candidates randomly over the g parties. This is done in a way such that the size of the party is chosen randomly, containing at least one candidate. Moreover, there should be at least one candidate left for each party that still has to be assigned candidates. The parties are represented as follows: $[[0], [0, 0, 0, 0], [0, 0, 0, 0, 0]]$ where each sub-list represents a party and all items of the sub-lists represent a candidate.

2.2 Generating votes

To generate votes, the (p, ϕ, g)-disjoint model from Szufa et al. (2022) is implemented. Here, one first draws a random partition of candidates C into g sets, resulting in $C_1, ..., C_g$, where C_i represents a party. A vote is generated by choosing $i \in [g]$ uniformly at random. Next, the vote is sampled from a (p, ϕ)-resampling model. The (p, ϕ)-resampling model requires a central vote u and generates a new vote by first setting $A(v) = A(u)$. The central vote u its initial setting is set such that all candidates from C_i are approved. For every candidate $c_i \in C$, with probability $100 - \phi$, the approval remains the same and with probability ϕ its value is resampled. If resampling takes place, c_i is approved with probability p. Note that p and ϕ are two integers in $[0, 100]$ and g is a non-negative integer. This procedure yields the approval ballot of each voter. An example of an approval ballot would be $[[0], [1, 1, 1, 0], [0, 0, 0, 1, 0]]$. Where a 1 indicates that a candidate is approved and a 0 indicates that a candidate is not approved.

2.3 Applying LV

For applying LV, there needs to be established which candidates received the most votes. Based on this, a ranking is created. Subsequently, the creation of the ranking will allow for generating the limited vote. How this has been conducted is described in Section 2.3.1 and 2.3.2.

2.3.1 Score and ranking

To find out which candidates have the highest number of approvals, a score needs to be calculated. The score of each candidate represents the total amount of votes received. The score of a candidate for AV is defined in Definition 1.2 Equation 1.1. For LV, the generation of the limited vote will be based on these AV scores, and the scores and ranking will be generated later on. This will be further clarified in Section 2.3.2. The score of each candidate in AV is represented in this fashion: $[[8], [11, 11, 11, 7], [8, 3, 2, 8, 5]]$. Subsequently, a ranking is created to prioritize the more popular candidates over the less popular candidates. The list containing the scores is sorted, having the highest score upfront and the lowest last. Simultaneously, there has been kept track of the index of the corresponding party and candidate.

2.3.2 Generating the limited vote

For each voter, a limited vote is created. A candidate is included in the limited vote if i) the candidate is present in the approval ballot of the voter ($c_i \subset A_i$), and ii) the candidates in the approval ballot will be included based on having the highest number of approvals. For LV, the score of a candidate is defined in Definition 1.3 Equation 1.3. The score of each candidate is displayed and the rank is created in the same manner as described in Section 2.3.1. Selecting the candidate with the highest number of approvals favours the most popular candidates over the less popular ones. In the real-world, the most popular candidates also have a higher chance to be elected. In the event of a tie in the number of approvals among candidates, all candidates are added if there are sufficient votes in the limited vote left. If not, the remaining candidates are added randomly until we reach the maximum number of votes in the limited vote. The choice to implement randomness can be motivated by considering that for each setting multiple runs (10) are performed, which results in an average of all conceivable situations.

2.4 Winning committee

The winning committee is selected by choosing the candidates that received the most votes. The total number of votes for each candidate has been added up for both AV and LV. In addition, a rank has also been created. For AV the total amount of votes for each candidate is based on the approval ballots A_i and the winning committee is selected as described in Definition 1.2 Equation 1.2. Then, for LV the total amount of votes is based on the limited vote of each voter, and the winning committee is selected as described in Definition 1.3 Equation 1.4. If there is a tie in the scores between candidates and there is enough space in the winning committee, all tied candidates are selected. However, if there is not enough space in the winning committee and a tie occurs, the remaining candidates are randomly chosen for the committee. As mentioned in Section 2.3.2, the randomness yields an average of all conceivable situations.

2.5 Measuring performance

The performance of LV and AV is measured in terms of the PAV-score and the CC-score. For each configuration of variables, which is run 10 times per configuration, the PAV-score and CC-score are calculated. In the end, a mean PAV-score and CC-score of those 10 runs is computed. To establish whether or not LV outperforms AV, calculating the gain between the two voting rules is introduced. Note that if LV performs better than AV in terms of either the CC-score or the PAV-score, the gain will be positive. Otherwise, the gain will be negative.

2.5.1 PAV-score

The PAV-score is as mentioned before, used to express proportionality. Proportionality grants more fairness in voting. The preference of the voters should represent a fair share in the winning committee. For instance, if 30% of the voters have a certain preference, then 30% of the members of the winning committee should represent this preference. For the PAV-score, we check for each voter how many approved candidates are in the winning committee. For all j approved candidates that are also in the winning committee, the voter contributes to the PAV-score the value of the j-th harmonic number as shown in Equation 2.1 and 2.2. To find out how much gain LV yields with respect to AV, the PAV-gain is calculated as shown in Equation 2.3.

$$h(x) = \Sigma_{j=1}^{x} \frac{1}{j} \tag{2.1}$$

$$s_{PAV}(A, W) = \Sigma_{i \in N} h(|W \cap A_i|) \tag{2.2}$$

$$gain_{PAV}(E) = s_{PAV}(A, LV(E)) - s_{PAV}(A, AV(E)) \tag{2.3}$$

2.5.2 CC-score

The CC-score ensures a degree of fairness as well and is used to express diversity. A winning committee having a high CC-score is desirable, as we attempt to maximize the voters' satisfaction. This is done as the CC-score will increase when a voter has at least one candidate in the winning committee that this voter approved. The CC-score of a winning committee is calculated as shown in Equation 2.4. The CC-score will be incremented by one if for a voter there is at least one approved candidate in the winning committee. To find out how much gain LV yields with respect to AV, the CC-gain is calculated as shown in Equation 2.5.

$$s_{CC}(A, W) = |\{i \in N : W \cap A_i \neq \emptyset\}| \tag{2.4}$$

$$gain_{CC}(E) = s_{CC}(A, LV(E)) - s_{CC}(A, AV(E)) \quad (2.5)$$

2.6 Configuration of variables

As mentioned before, there are 10 runs performed for all possible configurations of g, l, k, p, and ϕ. Meanwhile, the number of candidates and voters are incremented. To simulate the elections, it is necessary to determine the values that the variables can take. These values are chosen in such a way that they reflect the characteristics of elections occurring in the real-world. However, different types of elections exist, some involving a large number of participants, such as a parliament election. There exist elections that are smaller in scale as well, such as an election for a scientific organization. Conducting simulations for large-scale elections can be computationally expensive. Therefore, for this research, we focus on elections that are not too large in scale, such as those for scientific organizations. For this type of election, an overview of the variables along with their values can be seen in Table 2.1. We will also further clarify that simulating parliament elections is unfeasible for this research.

Table 2.1: Overview of variables

nc	20	40	80								
nv	80	160	320								
g	2	3	4	5	6						
p	0	10	20	30	40	50	60	70	80	90	100
ϕ	0	10	20	30	40	50	60	70	80	90	100
l	1	2	3	4							
k	5	6	7	8							

Consider the number of candidates in a parliament election, in such a setting the number of candidates can be vast. The computational effort would be simply too big to represent parliament elections. For electing members of some sort of scientific organization, smaller numbers can be worked with. Therefore, $nc \in \{20, 40, 80\}$ could represent such an election.

In the real-world, the number of voters is bigger than the number of candidates. Therefore, the number of voters is chosen such that the number of voters is four times as large as the number of candidates. Therefore, $nv \in \{80, 160, 320\}$.

For g, $g \in \{2, 3, 4, 5, 6\}$ is chosen. The number of parties can vary from a very small number to a large number in for example parliament elections. In the Netherlands for instance, at this moment there are 20 parliament parties in the Second Chamber (*Parliamentary parties* (n.d.)). On the other hand, some countries have two-party systems, such as England. For an election of a scientific organization, having a large number of parties would not be appropriate. Therefore, $g \in \{2, 3, 4, 5, 6\}$ would be a fair choice.

l is representing the number of votes a voter may cast for LV. As it is undesirable to present a voter with a high cognitive load, it is unfeasible to allow a voter to vote for too many candidates. Therefore, $l \in \{1, 2, 3, 4\}$ is chosen.

The number of candidates in the winning committee is of size k. Again, consider the example of some sort of scientific organization. It would not be straightforward to end up with a committee size that involves hundreds of candidates, as in parliament elections. In reality, such a committee will be much smaller. Therefore, $k \in \{5, 6, 7, 8\}$.

3 Results

This section provides a visual overview of the data that was generated by the model that simulated the elections. The data contains 29040 observations in total. First, the data of the CC-gain and PAV-gain is displayed in terms of their distribution. Second, the performance of LV and AV is compared with regard to their mean CC-scores and PAV-scores. Third, we consider the configurations in which LV performs optimally. Finally, we will analyze the impact of the variables on the CC-gain and PAV-gain by exploring their correlations. To visualize these correlations, a heatmap will be presented.

3.1 Distribution of the data

To gain a better understanding of the performance of LV, we assessed the nature of the distributions that the data of the CC-gain and PAV-gain follow. More specifically, the performance of LV is related to the skewness of the distribution of the CC-gain and the PAV-gain. This assessment can be achieved by analyzing the density plots for the CC-gain and the PAV-gain. In Figure 3.1 and 3.2 one can observe that the data is not normally distributed. In Figure 3.1 a positive skew with an exponential nature can be observed, meaning the CC-gain is predominantly positive. In Figure 3.2 a similar skewness can be observed. The assumption that the data for the CC-gain and PAV-gain demonstrate skewed distributions and do not follow a normal distribution is further supported by the results of an Anderson-Darling normality test. The outcome of the test indicates that $p < 0.05$, which means we have enough evidence to reject the null hypothesis that the data is normally distributed.

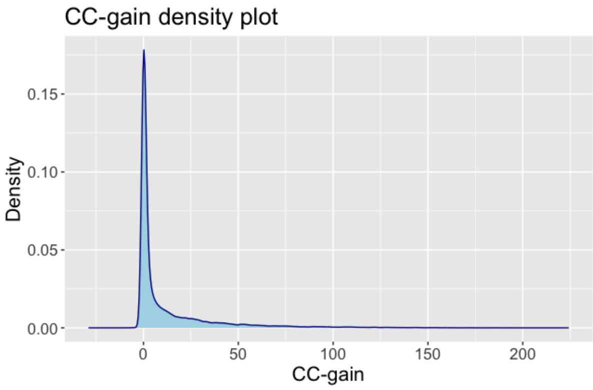

Figure 3.1: Distribution of the CC-gain represented in a density plot

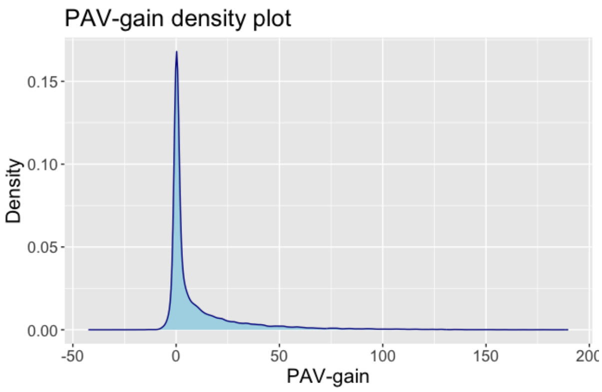

Figure 3.2: Distribution of the PAV-gain represented in a density plot

3.2 Mean scores

To assess whether LV performs better than AV, the mean CC-scores and PAV-scores are used for comparison. A Welch Two Sample t-test is performed for $nc = 20$ and $nv = 80$, $nc = 40$ and $nv = 160$, and $nc = 80$ and $nv = 320$ to establish whether there is a difference in means between LV and AV. First, we perform an Anderson-Darling normality test to check the nature of the distribution. We find that the data of the CC-scores and PAV-scores do not follow a normal distribution ($p < 0.05$). This raises questions about the validity of performing a statistical test that assumes normality, such as the Welch Two Sample t-test. However, note that the data is of a very large size. For this reason, a Welch Two Sample t-test is still a reliable test. This is due to the fact that this test is based on mean values. In addition, when one considers the central limit theorem, the sample means of large samples will converge to a normal distribution, even if the data itself does not follow a normal distribution.

The comparison of the mean CC-scores of LV and AV yielded the results as shown in Table 3.1. Moreover, for all CC-scores $p < 0.05$, meaning we can reject the null hypothesis of no difference and say that the true difference in means between LV and AV is not equal to zero. Furthermore, the scores increase in proportion to nc and nv. Consequently, it would be appropriate to measure the performance differences between LV and AV by examining their percentage increase. Table 3.1 also provides the percentage increase between LV and AV.

Table 3.1: Mean CC-scores of AV and LV and the percentage increase

	AV	**LV**	**% increase**
nc = 20, nv = 80	67.29	72.24	7.35
nc = 40, nv = 160	130.54	141.18	8.16
nc = 80, nv = 320	255.30	275.60	8.00

Next, the mean PAV-scores of AV and LV are computed. We again performed a Welch Two Sample t-test to examine whether there is a significant difference in the means between LV and AV. The resulting mean values are presented in Table 3.2. Moreover, $p < 0.05$ for all PAV-scores meaning we can reject the null hypothesis of no difference and say that the true difference in means is not equal to zero. Furthermore, the scores increase in proportion to nc and nv. Hence, we again calculated the percentage increase between LV and AV to further examine their performance differences. This increase is also presented in Table 3.2.

Table 3.2: Mean PAV-scores of AV and LV and the percentage increase

	AV	LV	% increase
nc = 20, nv = 80	123.05	127.08	3.27
nc = 40, nv = 160	241.44	250.79	3.87
nc = 80, nv = 320	473.32	491.87	3.92

Additionally, it is of interest to determine the configurations of variables for which the CC-gain is optimal. To accomplish this, we calculated the mean values of nc, nv, g, l, k, p, and ϕ for the top 10 highest CC-gain values. The resulting mean values are presented in Table 3.3. Moreover, we computed the standard deviations of these variables to assess the amount of variability for the optimal CC-gain. These are reported in Table 3.4.

Table 3.3: Mean values of top 10 highest CC-gain

CC-gain	nc	nv	g	p	ϕ	l	k
191.85	80.00	320.00	4.7	3.00	10.00	1.10	6.30

Table 3.4: Standard deviations (CC-gain)

nc	nv	g	p	ϕ	l	k
0.00	0.00	0.95	4.83	0.00	0.32	0.95

Furthermore, we seek to determine the optimal configurations of variables for which the PAV-gain is optimal. To accomplish this, we computed the mean values of nc, nv, g, l, k, p, and ϕ for the top 10 highest PAV-gain values. These mean values are presented in Table 3.5. We also computed the standard deviations of these variables to assess the variability associated with the optimal PAV-gain. These standard deviations are reported in Table 3.6.

Table 3.5: Mean values of top 10 highest PAV-gain

PAV-gain	nc	nv	g	p	ϕ	l	k
177.73	80.00	320.00	3.00	1.00	11.00	2.20	7.70

Table 3.6: Standard deviations (PAV-gain)

nc	nv	g	p	ϕ	l	k
0.00	0.00	0.94	3.16	3.16	1.03	0.48

3.3 Correlation

In Figures 3.3 and 3.4, we present heatmaps displaying the Pearson correlation coefficients between the CC-gain and PAV-gain and all the variables. The heatmaps provide a visual representation of the strength and direction of the correlations, where the colour red indicates a positive correlation and purple a negative one. The brightness of the colours corresponds to the strength of the correlation. All correlations in the heatmaps are significant, as $p < 0.05$ for all correlations.

Before we consider the correlation between all the different variables and the gains, the correlation between the CC-gain and the PAV-gain is computed. The results show that the two gains are highly positively correlated, with a correlation coefficient being 0.96, being significant as $p < 0.05$.

When considering the correlation of the CC-gain with all the different variables, the heatmap in Figure 3.3 shows that the strongest correlation is a negative correlation between the CC-gain and ϕ. Subsequently, p demonstrates the second strongest correlation, being negative as well. Next, nc and nv show an equally strong correlation, yet a positive one. Furthermore, l exhibits a low negative correlation with the CC-gain. Finally, the CC-gain shows a very weak positive correlation with k and a very weak negative correlation with g. The correlation between nc and nv is equal to 1 due to the experimental setup, where the number of candidates and voters increases in steps.

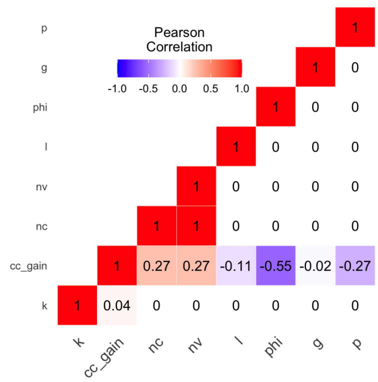

Figure 3.3: Correlation coefficients for the CC-gain represented in a heatmap

Concerning the correlations between the variables and the PAV-gain, one can observe in Figure 3.4 that the strongest correlation is again a negative correlation between the PAV-gain and ϕ. Additionally, nc and nv demonstrate a moderate positive correlation with the PAV-gain. Next, the negative correlation between p and the PAV-gain is the strongest. k and g show weak correlations with the PAV-gain, k shows a positive correlation and g a negative correlation. Lastly, l has the weakest correlation with the PAV-gain and is negatively correlated. Again, the correlation between nc and nv is equal to 1, due to the experimental design.

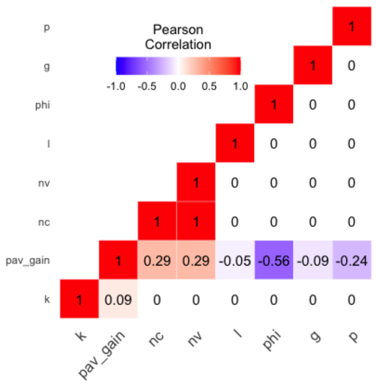

Figure 3.4: Correlation coefficients for the PAV-gain represented in a heatmap

4 Discussion

In the first part of this section, in Subsection 4.1, the performance of LV is discussed and compared to the performance of AV by considering multiple aspects presented in Section 3. Next, the limitations of this research are discussed in Subsection 4.2. Finally, there are made suggestions for possibilities to conduct future research in Subsection 4.3.

4.1 Interpretation of the results

First, the density plots in Figure 3.1 and 3.2 reveal skewed distributions for both the CC-gain and PAV-gain. The density plot for the CC-gain shows a predominance of positive values, with a large peak near zero. Similarly, the density plot for the PAV-gain shows a similar trend, yet, with fewer positive outliers and more negative outliers. In short, the distribution of the gains demonstrate a predominantly positive trend, which implies that the gains are predominantly positive, and LV outperforms AV in most cases.

Second, the mean CC-scores and PAV-scores of LV and AV are computed as shown in Table 3.1 and Table 3.2. This is done for each configuration of the number of candidates and voters. All differences in means between LV and AV are significant. The mean CC-scores of LV are in every setting of the number of candidates and voters significantly higher than the CC-scores of AV. The mean PAV-scores also show that LV has significantly higher scores than AV for every setting of the number of candidates and voters. Furthermore, when the number of candidates and voters is increased, there is observed a positive effect on both scores. The reason this occurs is due to the fact that both scores lie in a certain range, with 0 being the minimum. This range becomes larger for a larger number of candidates and voters. As more voters have a chance to have a larger number of their approved candidates in the winning committee. For example, the CC-score's maximum value is equal to the number of voters. In addition, as the increase in the number of voters and candidates also implies an increase in the scores, the percentage increase of LV with respect to AV has been computed. In Table 3.1 one can observe that LV's CC-scores are $7.35 - 8.16\%$ higher than the CC-scores of AV. The PAV-scores of LV are $3.27 - 3.92\%$ higher than the PAV-scores of AV as reported in Table 3.2. These results suggest that LV outperforms AV, particularly in terms of its CC-scores.

Finally, an interpretation of the effect the different variables have on the CC-gain and PAV-gain will be provided in the following paragraphs. This is done by considering the settings in which LV reaches its optimal performance which is reported in Table 3.3 and 3.5. Next, the standard deviations of the optimal gains are considered, which are given in Table 3.4 and 3.6. Lastly, the correlations of the CC-gain and the PAV-gain with the different variables are discussed, displayed in Figure 3.3 and 3.4. Bear in mind that the direction of the correlations is identical for the CC-gain and the PAV-gain. This is due to the fact that the two are strongly positively correlated with a correlation coefficient of 0.96.

Starting with nc and nv, by considering Table 3.3 and 3.5, one observes that the CC-gain and PAV-gain are the greatest when the number of candidates and voters are at their maximum value. The reason that the gain increases when the number of candidates and voters is higher is related to the explanation provided previously. If the CC-score for both LV and AV increases proportional to nc and nv, the difference between the two, the gain, increases as well. Additionally, Tables 3.4 and 3.6 indicate that the standard deviation of the gain is 0, meaning that LV performs optimally for only those values of nc and nv. Decreasing the number of voters and candidates will cause the range of the scores to shrink drastically. The presence of a moderate positive correlation observed in Figures 3.3 and 3.4 between nc, nv, and the CC-gain and PAV-gain further supports these findings.

Next, we will consider p and ϕ. When one considers Table 3.3 and 3.5, very low values of p and ϕ are observed for an optimal performance of LV. Note that when p and ϕ are low, the elections are resembling a party-list profile type of election. We have established before (in the example in the Introduction in Section 1.3) that LV performs better in terms of diversity in the case of a party-list profile election. This is also supported by the findings of Los et al. (2023), where the authors state that under some assumptions on the voters' ballots, LV will lead to a more diverse winning committee. Thus, AV performs very poorly in the case of a low p and ϕ, meaning the gain will increase. This can be explained by first considering the central ballot, where all candidates are approved. The chance of resampling is very low (ϕ is low), and the chance to approve a candidate if resampling does take place, is close to 0 (p is low). This means that the approval ballot of a voter is very likely to be practically identical to the central ballot. As a result, almost all candidates within a party will approximately have the same amount of approval votes. Therefore, first, the k candidates from the most popular party or parties will be selected for the winning committee. The candidates in the party or parties that are less popular (or were not or barely assigned the central vote) will not be represented in the winning committee. This affects both scores in a way such that only the voters who voted for the candidates in the most popular party or parties, will contribute to the scores. For LV, the amount of votes a voter may cast are limited. This will result in the limited vote not being similar to the central ballot. The number of votes that the candidates of a certain party receive will for this reason also vary more, as it is highly unlikely that all the limited votes are similar. Therefore, the winning committee will be represented by candidates from different parties, positively affecting both the CC-gain and PAV-gain. Furthermore, when one observes Table 3.4 and 3.6, one can observe that the standard deviation is very close to 0. As a consequence, this also supports that LV performs optimally only when p and ϕ are of low values. Furthermore, in Figure 3.3 and 3.4, it is visible that p and ϕ have a moderate and strong negative correlation with the gains. This is also consistent with the rather strong effect of a low p and ϕ described before. If p and ϕ decrease, the performance of AV will decrease, meaning the gain will increase. The correlation of ϕ with the gains is stronger than the correlation of p with the gains, this is due to the fact that this variable plays a role in whether or not resampling will take place. If this is not the case, the probability p that a candidate is approved does no longer matter. As a result, the correlation between the gains and ϕ is stronger than the correlation between the gains and p.

Additionally, one can observe in Table 3.3 and 3.5 that for LV's optimal performance, the values of l are rather low with regards to the CC-gain, and rather average for the PAV-gain. Concerning the CC-gain, because l is close to 1, only a few candidates will eventually be represented in the limited vote. As the limited vote will consist of a low number of approved candidates, the odds that all the limited votes are different increase. This, in turn, leads to more variability in the number of votes each candidate receives, resulting in a more diverse winning committee being selected. Consequently, the CC-score for LV increases, while the CC-score of AV remains low due to the current optimal settings of variables such as a low p and ϕ. For the PAV-score, a lower l is also desirable, yet, the effect of l is lower. This is consistent with the expectations since we know that decreasing l has a positive effect on the CC-gain and that the CC-gain has a strong positive correlation with the PAV-gain. Furthermore, when considering the standard deviations of the optimal settings of the CC-gain in Table 3.4, one can observe a very low standard deviation for l. This means that for only low values of l, LV performs optimally, not deviating much from the mean value of l. Concerning the standard deviation of l with respect to the optimal values of the PAV-gain, a significantly higher standard deviation is observed in Table 3.6. This implies that LV performs well for multiple values of l, and l is thus less meaningful with regard to the PAV-gain. Lastly, l is slightly negatively correlated with the CC-gain as can be observed in Figure 3.3. If l decreases, there will be less overlap in all the limited votes, which yields more diverse votes. Besides, the larger the value of l, the more LV resembles AV. This is undesirable as we already established that AV performs worse in terms of the CC-score. In addition, the negative correlation of l with the PAV-gain is very weak as can be seen in Figure 3.4. This weak correlation is also consistent with the fact that the standard deviation of l is very high, implying that l can take multiple values and is thus barely meaningful.

Subsequently, one can observe in Table 3.3 and 3.5 that for LV's optimal performance, the value of g differs substantially for both gains. Considering the current optimal values of the CC-gain, g is rather high. If g is higher, the votes are distributed over more parties. This will result in the candidates of the parties receiving relatively fewer votes, as it is more likely that the central vote will be assigned to more different parties. This leaves out even more voters that can contribute to the CC-score of AV, where only the candidates of the most popular party will be represented in the winning committee (caused by the low values of p and ϕ in the current settings). Also, due to the settings of the variables (e.g. the low value of l), LV is able to introduce some variance which increases its performance compared to AV substantially. On the contrary, AV does not have the possibility to introduce this variance. Furthermore, the optimal value of g is lower for the PAV-gain than the CC-gain. This, however, contradicts the expectation that LV performs optimally when g is rather high. The expectation of the value of g is similar to that of the value of g with regard to the optimal CC-gain. The fact that g does not meet this expectation could be clarified by observing the high standard deviation in Table 3.6. This means that for multiple values of g LV's performance is optimal regarding the PAV-gain. Moreover, g shows a very weak negative correlation in Figure 3.4, this implies that g is not very meaningful. With respect to the standard deviation of g when considering the optimal variable configuration of the CC-gain in Table 3.4, one can again observe a high standard deviation. This in combination with the low negative correlation of g with the CC-gain as shown in Figure 3.3, implies again that g is not very meaningful. For both gains, observe that if g decreases, the gains will increase. This is due to the fact that when g is low, a relatively high number of candidates are approved initially (and specifically when p and ϕ are low, the approvals are likely to remain intact). This is favourable as we have a higher chance of more approved candidates within an approval ballot. Therefore, the chances of overlap between the winning committee and the approval ballots increase, resulting in a positive effect on both the CC-score and PAV-score. This effect is not exclusive to LV and applies to AV as well, which explains the low correlations of the gains with the variable g. The correlation of g with the PAV-gain is slightly stronger due to the fact that the number of resemblances between the approval ballot and the winning committee is an important factor for PAV-scores. When we have a relatively higher number of approvals, it will thus affect the PAV-scores more.

Lastly, k will be considered. Observe the optimal values of k for LV in Table 3.3 and 3.5. One can see that k takes rather large values regarding the PAV-gain, and a slightly lower k is observed for the CC-gain. For the optimal CC-gain, when the k value is not too small, intuitively the chances that a voter has at least one candidate in the winning committee also increase. However, a larger value of k also positively contributes to the CC-score of AV, meaning increasing k does not have a large effect on the CC-gain. Furthermore, for the optimal PAV-gain, k has almost reached its maximum value. Naturally, k has a stronger effect on the PAV-scores than the CC-scores. This is because for the PAV-scores the number of approved candidates of a voter that also occurs in the winning committee matters. For the CC-score on the other hand, the number of approved candidates of a voter that also occur in the winning committee is insignificant. Therefore, for a larger k, the probability that the resemblance between the winning committee and the approval ballots increases is higher, increasing the PAV-scores. Observe that this also applies to the PAV-score of AV, and hence the value of k does also not have a major impact on increasing the PAV-gain. Concerning the standard deviations for the optimal CC-gain, one can observe that k shows a rather high standard deviation in Table 3.4. This implies that for multiple values of k the CC-gain is optimal, and thus less meaningful. On the other hand, the standard deviation of the optimal PAV-gain, is much lower, as reported in Table 3.6. Therefore, the PAV-gain is only optimal for fewer values of k, and is more meaningful. In short, the standard deviations are consistent with the previously described effect of k on the gains. Furthermore, when one takes the correlations into account, there is observed a very weak positive correlation of the CC-gain with k. Again, this supports that k is barely meaningful. Bear in mind that the positive correlation of the PAV-gain with k is slightly higher. This again proves that k is more meaningful with respect to the PAV-gain, yet, the correlation is still weak.

In summary, this paper aimed to investigate how LV performs in 'almost party-list' profile elections. Based on the results, it can be concluded that LV outperforms AV in terms of both diversity (the CC-score) and proportionality (the PAV-score). Therefore, we can conclude that LV is a more effective voting system than AV for achieving diverse and proportional outcomes in the 'almost party-list' type of elections.

4.2 Limitations

While the aim of this research was to simulate real-world elections, it may not always be a completely accurate representation. For instance, the number of voters and candidates are limited to a rather low value. This is not representative of larger-scale elections. However, increasing the number of voters and candidates would require a significant amount of computational power and is thus unfeasible for this research. Additionally, the fixed ratio of voters to candidates $(1:4)$ in the simulation is highly unlikely to occur in real-world elections, where this ratio can vary greatly. The generation of votes using randomness is another limitation to consider. While random generation allows for exploring different scenarios, it fails to fully capture the complexity of real-world voting behaviour. In reality, voting decisions are influenced by various factors such as a voter's personal beliefs. Besides, external factors may influence a voter's decision, such as the event of a political scandal occurring. Furthermore, this research does not include the fact that in some cases voters may vote strategically. More specifically, voters may cast their votes for a candidate they do not fully support, solely to prevent another candidate from winning. Finally, the fact that the votes were generated randomly implies that all candidates have an equal probability to be elected. However, this is often not the case in real-world scenarios where certain candidates may have advantages such as greater resources or more effective campaign strategies.

4.3 Future Research

LV is not a voting mechanism on which a lot of research is conducted yet. It may therefore be interesting to compare LV to other existing voting mechanisms. This would provide a more comprehensive understanding of LV's strengths and weaknesses compared to other voting mechanisms. It would be specifically interesting to compare LV to voting mechanisms that are frequently used, to find out whether LV would be a reasonable alternative. Moreover, it could be interesting to apply LV to elections that have already taken place, and where the outcome has already been established. This might provide insight into how LV affects the outcome, and how the performance in terms of the CC-gain and PAV-gain is affected. Furthermore, as the current research only involves a limited number of candidates and voters, future studies could explore how LV performs with larger populations. This would provide a more realistic understanding of its applicability to larger-scale elections. Lastly, this research only involves the 'almost party-list' type of election. There also exist other list representations that might be interesting to consider to find out how it affects the performance of LV. An example of such a list representation is the 'open-list' representation. There are different versions of this list representation; one of them is that a voter is allowed to cast a variable number of votes on multiple candidates of one specific party (Barrett et al. (2014)).

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