The Effect of Change in Temperature and Rainfall upon the Spread of Malaria

Bachelor Project Applied Mathematics

University of Groningen

Student: Siddhanta Mishra

First Supervisor: Prof. Dr. J.G. Peypouquet

Second Supervisor: Prof. Dr. M.K. Camlibel

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ABSTRACT

This thesis attempts to understand the relation between the changes in the temperature and rainfall, and the spread of malaria. This will be carried out by first introducing a multi-layered SIR model, with a layer looking at a constant human population, while the second would look at the fluctuating mosquito population. This fluctuation would be defined using the factors of temperature and rainfall. Finally, a programme would be constructed to simulate the model based on temperature and rainfall. Using this programme and the data in regard to both of the previously stated factors, simulations of the countries Libya, the Central African Republic, and South Africa for the years 2001, 2011, 2021 will be carried out. After a comparison between the predicted number of cases and actual number of cases, we will look to methods to improve the model by the inclusion of human efforts that alter various factors concerning the mosquito population and its interaction with the human population.

CONTENTS

1 INTRODUCTION

Climate change has in recent years resulted in unforeseen changes to the environment, including but not limited to the spread of diseases to change. Which for example is effected by temperature and rainfall. This thesis will focus on how the changes in these two factors affect the spread of diseases, more specifically the spread of malaria. We will also look to understand the extent of human efforts to reduce the spread of malaria.

Malaria is a disease caused by a protozoan parasite of the genus Plasmodium. There are five parasite species within this genus which infect humans, of which Plasmodium falciparum and Plasmodium vivax have the highest number of infection cases[6]. These parasites have a two part life cycle, the first occurs within the body of a female mosquito which acts as the vector of spread, before being transmitted to a human. Due to the requirements in regard to the growth of the parasite and its vector, it is most prevalent in the Sub-Saharan region[8].

Malaria is one of the most researched and modelled infectious diseases, beginning with the Ross model in the 1890s. In these various models, each attempts to understand the spread of malaria based upon different aspects, such as age, socio-economic status, immunity, or the weather. This is done by simplifying complex biological processes into mathematical approximates of them, something that we will be encountering throughout the thesis[5].

In this thesis, we will be using the Parham-Michael model, to gain an understanding of the effects of the change in rainfall and temperature upon the spread of malaria. Parham-Michael model is a set of 6 ODEs that track the growth of the human population through an SIR model and of the mosquito population through the SEI model, while also establishing the relation between the two. Then we will be extending the model to include human efforts in the reduction of the number of infection cases based on the actual number of cases. Which will showcase that the number of cases simulated with human intervention is closer to the real number of new infection cases.

The outline of the thesis is as follows. In Section 2, we will be covering the preliminaries in the SEIR models. In Section 3, we shall see how biological processes are translated into the Parham-Michael mathematical model, which will be followed by its application to three different countries in three years at intervals of a decade in Section 4. Based on real data, we will alter the model to consider human intervention in Section 5, before applying them to the same cases. Finally, in Section 6, we will conclude the thesis while looking to areas to improve the model. Thereon, we have the Appendix with the equations and programmes used throughout the thesis and the Acknowledgements.

2 PRELIMINARIES

2.1 *SEIR Models*

There have been many model types that have been used to mathematically describe the growth and spread of various infectious diseases. Among them is the family of SEIR models, that look upon the spread and recovery of an infectious disease in a population through the movement of the aforementioned population in-between the individual compartments. These individual categories are categorized as Susceptible (S), Exposed (E), Infected (I), Recovered (R). The susceptible population is the segment of the population that may be infected by the disease. Exposed category refers to a subgroup that has been infected but is not yet capable of infecting others, which is often included for diseases that have an incubation period within the subject.

However, this does not imply that every single model, uses every single category, for example, in some cases there is a possibility that once the population recovers from the infection, they once

SI Model
$$\begin{tikzcd} S \arrow[r] & I \end{tikzcd}$$

SIR Model
$$\begin{tikzcd} S \arrow[r] & I \arrow[r] & R \end{tikzcd}$$

SIS Model
$$\begin{tikzcd} S \arrow[r] & I \arrow[l, bend right] \end{tikzcd}$$

SIRS Model
$$\begin{tikzcd} S \arrow[r] & I \arrow[r] & R \arrow[ll, bend right] \end{tikzcd}$$

SEIRS Model
$$\begin{tikzcd} S \arrow[r] & E \arrow[r] & I \arrow[r] & R \arrow[lll, bend right] \end{tikzcd}$$

SEIS Model
$$\begin{tikzcd} S \arrow[r] & E \arrow[r] & I \arrow[ll, bend right] \end{tikzcd}$$

Figure 1: The various possible S-E-I-R based models

more become susceptible either immediately or after a certain duration of time, giving rise to SEIS and SEIRS models. We can see, in Figure 1, that the states of Susceptible and Infected are present in any model of such a type.

The Basic Reproduction number (R_0) is defined as the he average number of secondary cases arising from an average primary case in an entirely susceptible population. When $R_0 = 1$ is maintained, then the infection will continue to exist within a population without any additional infection from an external source. Below that, we can see the possibility of an infection-free population, while above it, the infection is likely to spread [4].

3 MODEL CONSTRUCTION

3.1 *Model Background*

Malaria is a disease that is transmitted in the majority of cases through female *Anopheles* mosquitoes. Yet not all the various Anopheles species are effective transmitters of the disease. *A. gambiae* is a variety of mosquitoes, native to Africa, that are often the subject of mathematical modelling. To properly model such a vector, the model gains a secondary level, when it is applied to the mosquito population. In the mosquito population, as they never recover from the infection, what we see is either SI or SEI models. Then, these models interact when infected humans infect susceptible mosquitoes, which then infect susceptible humans.

How often these bites happen is dependent on the gonotrophic cycle. The gonotrophic cycle is divided into three parts, the first would be the bite and consumption of blood. The second part is the digestion of blood and the maturation of the eggs, which in stage three are deposited in an appropriate water body [1]. We can then model the entire cycle of which stage two is temperature dependent, the inverse of this can be used to calculate the temperature-dependent biting rate ($a(T)$) as follows:

$$a(T) = \frac{T - T_1}{D_1}, \tag{1}$$

Where D_1 and T_1 are constants that were collected from based upon experimental data from the European *A. maculipennis*[1].

Constant	Value [7]	Unit
T_1	19.9	$^\circ$C
D_1	36.5	$^\circ$CDays

Table 1: Bite Rate Constants

The first of such models, which attempted to explore the spread of malaria through mathematical modelling, would be the Ross Model, developed by Ronald Ross in the early 20th century. It is a coupled SIS (for humans) and SI (for mosquitos) model. While being very simple in comparison to many of the malaria models that are available, this model was an invaluable first step towards the modelling of the spread of malaria.

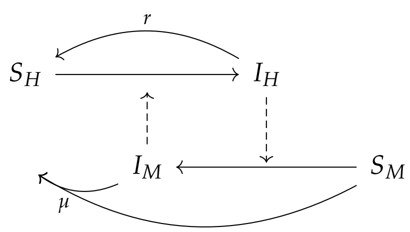

Figure 2: Visualization of the Ross Model

Solid lines represent transition of populations, and Dashed lines represent influence

One of the factors that were not considered in the Ross model was the latency period of the malaria virus in the mosquito. The following model, the Macdonald model, was constructed to address this factor, this was done so by including the Exposed category for the mosquitos in the model. Hence, it looks at the development and change of three different categories [5].

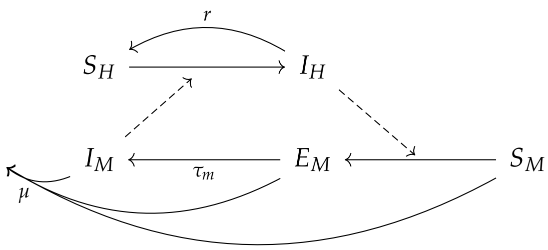

Figure 3: Visualization of the Mcdonald Model

We know that the exposed phase of the mosquito is dependent on the sporogonic cycle of the virus in question. The sporogonic cycle is the duration it takes the virus to infect, then reproduce within the mosquito, before finally resulting in the mosquito producing infectious saliva [1]. We can use the following equation to model this cycle.

$$\tau_m(T) = \frac{DD}{T - T_{min}}, \tag{2}$$

Again, DD and T_{min} are constants that have been based upon experimental data and differ for different varieties of viruses that cause malaria [7].

Constant	Value [7]	Unit
DD (P. falciparum)	111	°C
DD (P. vivax)	105	°C
T_{min} (P. falciparum)	16	°CDays
T_{min} (P. vivax)	14.5	°CDays

Table 2: Sporogonic Cycle Constants

While the above two models on their own would be able to give us the changes in the proportion of the different categories in the system, it does not tell us directly on what aspects to focus on to reduce the spread of malaria. We can look at the basic reproduction numbers for a clearer picture. With this in mind, when we go back to the above two models, we are presented with the following R_0:

$$R_0 = \frac{ma^2 b_1 b_2}{r\mu}, \tag{3}$$

Here b_1 is the proportion of bites from an infectious mosquito to a susceptible human that produce infection in the human, while b_2 is the proportion of bites from a susceptible mosquito to an infected human that cause infection. The r is the average recovery rate of humans. Here μ is the per capita rate of mosquito mortality, which is defined as a constant in this instance but is dependent on temperature for the final model.

Constant	Value [7]	Unit
b_1	0.09	Dimensionless
b_2	0.04	Dimensionless
r	$\frac{1}{120}$	Days^{-1}

Table 3: General Constants

Here, (3) is the R_0 for the Ross Model. We can see, from the equation, that a decrease in the number of mosquito bites would be better able to lower the basic reproduction number, when compared to the increase in recovery rate of humans, in these models. You will note that a is not dependent on the temperature in this model, since it was not considered so in the Ross model, similarly for the mortality rate of mosquitoes.

$$R_0 = \frac{ma^2 b_1 b_2}{r\mu} e^{-\mu\tau_m}, \tag{4}$$

Now we have introduced (4) as the R_0 for the McDonald Model. The only difference between this equation and the previous, is the introduction of the exponential. This exponential implies that the McDonald model gives greater importance to increasing the mortality rate of mosquitoes in comparison to the reduction of the number of bites, even though that too would be reduced when increasing the mortality rate.[5]

To begin discussing the mortality rate of mosquitos, we need to first understand the life cycle of mosquitoes, as they have different dangers at various stages of life. After first being laid in an appropriate water body, the eggs of the Anopheles mosquitoes go through four stages of larvae, before turning to pupae, beyond which they become adult mosquitoes. The four larvae stages are together are dependent on the temperature, we can ascertain the duration of the larvae period ($\tau_L(T)$) as follows:

$$\tau_L(T) = \frac{1}{\alpha T + \beta}, \tag{5}$$

Where α and β are based upon a study of the development of *A. gambiae* [1].

Constant	Value [7]	Unit
α	0.00554	$(^\circ\text{CDays})^{-1}$
β	-0.06737	$(\text{Days})^{-1}$

Table 4: Larvae Development Constants

After discussing the lifecycle of a mosquito, it is important to note that both the egg stage and pupae state are short enough that mortality due to temperature during this period is insignificant. Hence, we have two stages where we must know the mortality rate of the collective larvae stage and the adult stage. For the model that we will use, we define the probability of daily survival of Larvae (p_L), and the per capita death rate of adult mosquitoes. They are defined as follows:

$$p_L(T) = e^{-(\alpha T + \beta)} \tag{6}$$

$$\mu(T) = \frac{1}{AT^2 + BT + C} \tag{7}$$

Here once more, A, B, and C, are constants that are dependent on experimental data [7].

Constant	Value [7]	Unit
A	-0.03	$(^\circ\text{C}^2\ \text{Days})^{-1}$
B	1.31	$(^\circ\text{CDays})^{-1}$
C	-4.4	$(\text{Days})^{-1}$

Table 5: Mosquito Mortality Constants

More recently, one model, which considered the effect temperature could have on the mosquito population and hence the spread of malaria, was constructed in 2004 [3]. This was the Hoshen and Morse model, which integrated the sporogonic cycle and the gonotrophic cycle into the model. They did this by introducing a Larval stage, which accounted for all three stages of the development in immature mosquito progeny. The introduction of new eggs was constructed to be based upon the rainfall in the last 10 days. The larval stage would determine the number of adult mosquitoes in the SEI model of the population, which would then affect the SEIS model for the human population [1].

Something that is not considered in the model above is the possibility of rainfall flooding the progeny away, regardless of the stage they are in. The model, we will use, considers the daily probability of mortality for all three stages, with a similar equation, which is as follows:

$$p_i(R) = (\frac{4p_{Mi}}{R_L^2})R(R_L - R), \quad i = \{E, L, P\} \tag{8}$$

p_{Mi}: Peak daily probability of survival

R_L: Rainfall Threshold beyond which no immature mosquitoes survive[7]

Constant	Value [7]	Unit
p_{ME}	0.9	Dimensionless
p_{ML}	0.25	Dimensionless
p_{MP}	0.75	Dimensionless
R_L	50	mm

Table 6: Daily Progeny Survival Constants

After having introduced the various building blocks of the model, both historically and mathematically, we can introduce the model itself, the Parham-Micheal Model.

3.2 *Model*

Before, we can go into the mathematical equations of the model, it is important to understand the simpler fundamentals of the model. Firstly, in the human population, the exposed phase has been integrated into the equation for the change in the infected population, hence it disappears as a stage in the model. Then, instead of a loop where once the infected recover, they are once more susceptible, the model assumes that the population gains permanent immunity from the illness, hence there is no transition from recovered to susceptible.

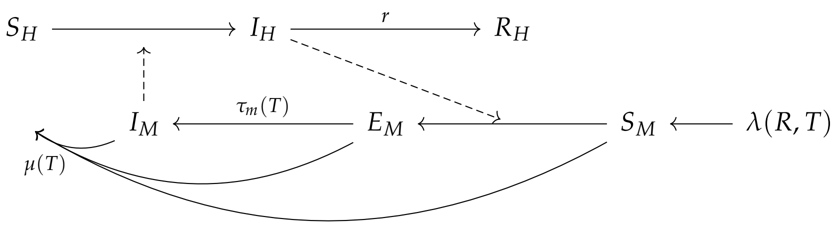

Figure 4: Visualization of the Parham-Micheal Model

Next we have to look at the mosquito population, which unlike the human population does not remain static. Here, in this model, unlike the Hoshen and Morse model, there is no additional larval stage, instead an adult mosquito population is directly introduced into the model depending on the rainfall and temperature. Beyond that, it is a rather standard model of the mosquito population. We can see a visualization of the model in Figure 4.

To begin constructing the model, we can first look at how the influx of adult mosquitoes is calculated. The Parham-Micheal model does so by using a total number of eggs, then calculating the total probability of daily survival before using the total time spent in the immature phase.

Having defined the various temperature and rainfall-dependent factors, we get the following relationship:

$$\lambda(R,T) = \frac{B_E p_E(R) p_L(R) p_L(T) p_P(R)}{\tau_E + \tau_L(T) + \tau_P}, \tag{9}$$

Here we know that the duration of the egg stage (τ_E), and the duration of the (τ_P) are both stated to be 1 day long, while B_E is defined as the number of eggs layed [7].

We can start looking at the equations of the model that describe the changes in the population of the two parties. We will first start with the human population, as it is simpler. The change in population for the susceptible humans is only negative, as there is no influx via births, or through a loss of immunity in the recovered population. Next we will consider the infected human population. We have an influx from the susceptible population. Both of these are dependent on the population of infected mosquitoes. The other factor is the recovery of the infected humans, which is also the only factor that affects the recovered population. Hence, we get the following model:

$$\frac{\mathrm{d}S_H}{\mathrm{d}t} = -a(T) b_1 I_M \frac{S_H}{N}, \tag{10}$$

$$\frac{\mathrm{d}I_H}{\mathrm{d}t} = a(T) b_1 I_M \frac{S_H}{N} - r I_H, \tag{11}$$

$$\frac{\mathrm{d}R_H}{\mathrm{d}t} = r I_H, \tag{12}$$

Here N is the total human population that we are considering [1].

Now we can move onto the fluctuating mosquito population. We have already established the influx of adult mosquitoes, which determines the positive change in the susceptible population. Then a certain number of the population enter the exposed phase after being exposed to the virus. The mosquitoes then travel from the exposed phase to the infectious population based on the sporogonic cycle, and the chances of survival through the exposed phase. In all three of these phases, there exists a certain probability of mosquito mortality, which we established in the previous section.

$$\frac{\mathrm{d}S_M}{\mathrm{d}t} = \lambda(R,T) - a(T) b_2 S_M \frac{I_H}{N} - \mu(T) S_M, \tag{13}$$

$$\frac{\mathrm{d}E_M}{\mathrm{d}t} = a(T) b_2 S_M \frac{I_H}{N} - a(T) b_2 (t - \tau_M(T)) l_M(t) S_M \frac{I_H(t - \tau_M(T))}{N} - \mu(T) E_M, \tag{14}$$

$$\frac{\mathrm{d}I_M}{\mathrm{d}t} = a(T) b_2 (t - \tau_M(T)) l_M(t) S_M \frac{I_H(t - \tau_M(T))}{N} - \mu(T) I_M, \tag{15}$$

Where $l_M(T) = e^{-\mu(T)\tau_M(T)}$, is the probability of an adult female mosquito surviving the sporogonic cycle [1].

Having established the model, we also must look to the basic reproduction number. It will be able to provide us with a better look as to what the potential of an epidemic, the virus has under different rainfall and temperatures.

$$R_0 = \frac{M(R,T) a(T)^2 b_1 b_2}{N r \mu(T)} l_M(T), \tag{16}$$

Now this equation should look very familiar, if we look at (4), we will notice that R_0 is the same. Only here, we showcase certain aspects to be dependent on the rainfall and temperature. Here $M(R,T)$ represents the total population of the mosquito, which we are given has a Poisson distribution given with a mean of $\frac{\lambda(R,T)}{\mu(T)}$ regardless of the initial population[7].

4 MODEL SIMULATION

4.1 *Programme Construction*

For the simulation, we will be creating a program that solves ODEs that we have stated above numerically. We will be using MATLAB for this. To begin constructing the programme, much like we did for the actual model, we will have to start with the foundational equations. These foundational equations are things such as bite rate, sporogonic cycle, duration of the larval period, death rate of adult mosquitoes, along with the daily probability of survival for the various stages of mosquito progeny. All these factors have been shown to be dependent on the temperature and rainfall. One thing to note, is that due to the equations for the biting rate and the sporogonic cycle, we need to create a safety net should the temperature go below the specified T_1 and T_{min}. We set up a similar for the daily probability of survival dependent on rainfall.

Now we need to determine the number of the mosquitos that survive to adulthood depending on the number of eggs laid, the survival rate at different rainfall and temperatures, and the duration of the larval period. We also need to determine the rainfall and temperature, which we base upon the time of year and location we would consider. For this purpose we will be using Libya, The Central African Republic, and South Africa, in the years 2001, 2011, and 2021. We are using these specific years and countries to broadly cover development across Africa over the years. Finally, we also consider the likelihood of a mosquito's survival over the period of the sporogonic cycle.

Finally, we can set up the ODEs, which we will set up in one programme. We do this because this allows us to only define the constants once, taking less time computationally. Using this, we can set up the Runge-Kutta method RK4 for the duration of the year. This method is useful as it lowers the global error when compared to the Euler's method or Heun's Method. We will also be calculating the R_0 as we go, so that we don't need additional if-loops due to the fact that some variables are dependent on temperature and rainfall, which in turn are dependent on time.

For the numerical method, we will be using a higher-order Runge-Kutta Method, as stated earlier. In this case, we will use the RK-4 which works with four estimates. These four estimates are then used to calculate the next step using the following equation.

$$y_{n+1} = y_n + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4) \tag{17}$$

Where K_i, represent an estimate. These estimates are calculated based on, (x_n, y_n) as well as, the previous estimates. The exact equations can be found below.

$$K_1 = hf(x_n, y_n), \tag{18}$$

$$K_2 = hf(x_n + \frac{h}{2}, y_n + \frac{K_1}{2}), \tag{19}$$

$$K_3 = hf(x_n + \frac{h}{2}, y_n + \frac{K_2}{2}), \tag{20}$$

$$K_4 = hf(x_{n+1}, y_n + K_3), \tag{21}$$

Where h is the step size, and f represents the function of the ODE.

We will be using the RK4 method, over the Euler method because, firstly, it allows us to have a larger step count, which is very important as we will be running the model over a period of 365 days. Secondly, it gives us a lower error for our values, as it uses 4 estimates, unlike the Euler method which uses only one.

4.2 *Model Data & Constants*

Now having seen the programmes, we can move onto the data. The data was gathered from Climate Change Knowledge Portal. We can observe certain patterns in the temperature and rainfall. The three countries each have vastly different climate profiles.

Year	2001	2011	2021
Jan	13.18	13.28	13.27
Feb	14.23	15.06	15.06
Mar	20.63	18.08	18.42
Apr	23.85	23.47	23.91
May	27.62	26.7	28.09
Jun	29.23	29.84	30.47
Jul	30.69	30.44	30.94
Aug	30.81	30.11	31.11
Sep	29.84	28.37	28.4
Oct	24.56	23.6	23.76
Nov	19.2	17.55	19.31
Dec	13.73	13.43	13.43

Table 7: Temperature Data (°C) for Libya [2]

Year	2001	2011	2021
Jan	3.97	5.62	3.73
Feb	6.98	5.95	2.35
Mar	1.19	4.17	2.18
Apr	1.53	2.68	0.62
May	1.03	1.26	0.61
Jun	0.23	0.79	0.33
Jul	0.31	0.33	0.33
Aug	1.04	1.1	1.04
Sep	0.44	1.53	0.91
Oct	1.07	3.96	3.27
Nov	7.49	4.29	3.55
Dec	11.14	7.04	11.57

Table 8: Precipitation Data (mm) for Libya [2]

Libya tends towards more fluctuating temperatures throughout the year, approaching 30 (°C) during the summer months, and going below 15 (°C) in the winter months. This, in addition to the lack of rainfall throughout the year, leads to the assumption that the introduction of new mosquitoes will likely be low. The gonotrophic cycle and sporogonic cycle will also both be lengthened during the winter months.

Year	2001	2011	2021
Jan	23.89	23.92	25.72
Feb	26.23	27.13	26.81
Mar	27.26	27.7	27.87
Apr	27.57	27.62	27.32
May	26.76	26.61	26.73
Jun	25.02	25.58	25.66
Jul	24.44	24.88	24.42
Aug	23.98	24.28	24.28
Sep	24.32	24.59	24.62
Oct	24.68	24.56	25.25
Nov	24.67	24.51	25.65
Dec	24.52	24.18	25.12

Table 9: Temperature Data (°C) for the Central African Republic [2]

Year	2001	2011	2021
Jan	6.06	7.01	13.67
Feb	14.44	15.6	17.67
Mar	47.46	58.56	69.42
Apr	100.12	95.77	81.21
May	122.39	145.35	148.65
Jun	148.78	169.86	165.22
Jul	189.1	202.94	213.93
Aug	214.52	244.79	265.04
Sep	226.02	224.75	250.65
Oct	224.39	166.57	171.65
Nov	49.21	36.22	54.64
Dec	8.08	6.75	7.79

Table 10: Precipitation Data (mm) for the Central African Republic [2]

The Central African Republic, unlike Libya, maintain a stable temperature between the 23 to 28, this implies that there is no major change in temperature dependent factors, such as the per capita death rate, and duration of the larval period. Hence, the change in the influx of adult mosquitoes in mainly dependent on the rainfall, which we see increase considerably in the months between June and October.

In South Africa, both the temperature and rainfall fluctuate. Both of them reach favourable levels for mosquito and virus development. The optimal period of time occurs at the two ends of the year, January to March and October to December. We enter this data into the RK4 to get the

Year	2001	2011	2021
Jan	23.21	23.12	23.33
Feb	22.87	23.35	23.01
Mar	21.96	22.67	21.64
Apr	17.79	17.78	19.64
May	14.67	14.74	15.2
Jun	12.25	11.21	13.37
Jul	10.79	10.7	10.76
Aug	13.21	13.26	13.17
Sep	15.31	16.93	17.14
Oct	19.7	18.86	18.32
Nov	20.45	20.27	20.14
Dec	21.88	22.13	21.46

Table 11: Temperature Data (°C) for South Africa [2]

Year	2001	2011	2021
Jan	39.04	98.46	114.23
Feb	58.05	70.53	70.21
Mar	55.63	69.2	44.9
Apr	51.78	57.2	22.41
May	22.69	43.2	10.3
Jun	8.4	31.61	11.44
Jul	19.56	16.5	8.26
Aug	20.43	17.37	18.64
Sep	43.05	8.17	11.82
Oct	46.71	30.25	35.36
Nov	98.84	49.59	57.45
Dec	82.89	59.89	121.59

Table 12: Precipitation Data (mm) for South Africa [2]

values for those years.

We will be taking a static human population of 1000, 900 of whom would be susceptible, while 100 would be already infected. We will have an initial population of 2000 for each of the mosquito categories. Finally, we will be using the *P. falciparum* as the malaria strain. While we know that the population can only exist in integer values, to better understand the changes, we will leave them unrounded.

4.3 *Model Results*

4.3.1 *Libya*

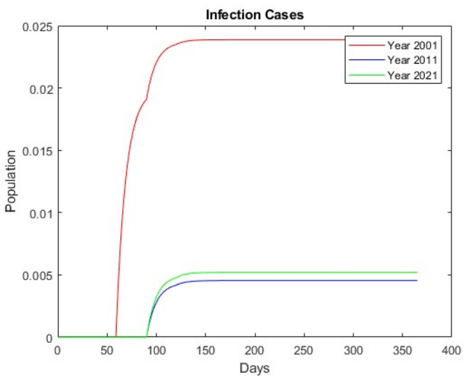

Figure 5: Number of cases in a population of 1000 in Libya by the Parham-Micheal Model

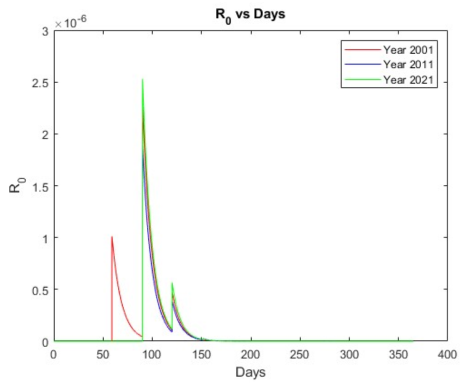

Figure 6: R_0 of Libya by the Parham-Micheal Model

Due to the dry nature of Libya, mosquitoes quickly die out, with negligible introduction of adult mosquito. This results in the spread of malaria being very low, near zero. This can be seen in Figure 5, where there's not even a loss of one human to the infection. This is further emphasized when we look at the R_0 in Figure 6 and see how small the scale is. While we do not have any direct data from the country itself, we do have data from its two neighbours, Algeria and Egypt, to show that the number of cases in a 1000 are zero [8]. Hence, we can assume that this holds true for Libya. This would be primarily due to the extremely dry condition of the country that remains consistent, throughout the year. As a result, even when the temperatures rise to more mosquito-friendly levels, there simply is not enough moisture to support an abundant enough population of mosquitoes for there to be an impact.

4.3.2 *The Central African Republic*

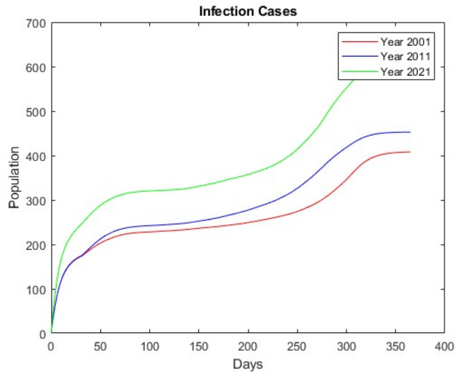

Figure 7: Number of cases in a population of 1000 in The Central African Republic by the Parham-Micheal Model

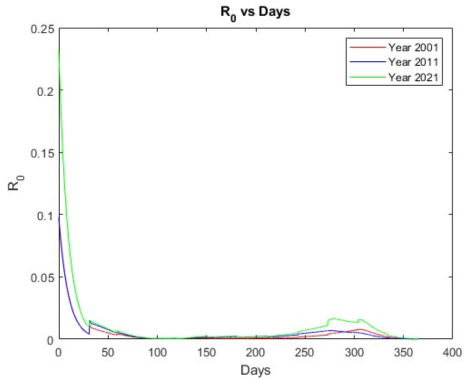

Figure 8: R_0 of The Central African Republic by the Parham-Micheal Model

The Central African Republic showcases the highest cases of infected individuals, it also gives us a higher R_0 when compared to the other two graphs. This is primarily due to the consistently hospitable temperature, as a result, the influx of adult mosquitoes is primarily dependent on the rainfall experienced by the country. The latter months, result in a high influx of adult mosquitoes, which can be seen in the increase of R_0 around these months in Figure 8. In the three years, we do not see a drastic change in the temperature that would explain differences between number of cases. Hence, we should look at the differences in rainfall, which shows an increase in later years. However, this change does not correspond with the data that we have, where we see a reduction of new cases and not an increase [8].

4.3.3 *South Africa*

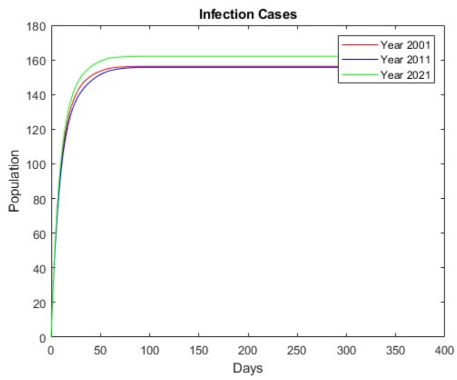

Figure 9: Number of cases in a population of 1000 in South Africa by the Parham-Micheal Model

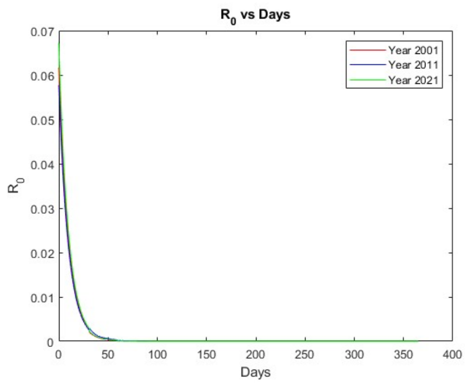

Figure 10: R_0 of South Africa by the Parham-Micheal Model

When we run the model for South Africa, we see an interesting pattern, where the number of infections that occur, only do so in the initial period. This is because the temperature starts to fall below the hospitable level after this period, both for mosquitoes and the malaria virus. In addition, one interesting fact that must be noted is that the model showcases over 140 cases of infection in all three years. However, the data that we have showcases that there were at most 6 cases in a 1000 people out of the three years. This implies that even before this period, much care was taken to eliminate the disease in the region, a trend that seems to have continued in the two decades, with the number of cases falling to below 1 out of 1000 people in 2021 [8].

5 ALTERED MODEL

5.1 *Model*

As we saw after running the simulations, the model, while telling us what are the effects of the rainfall and temperature on the spread of malaria, it does not paint a complete or accurate picture. Hence, we will attempt to construct a secondary model that will account for the various methods used to minimize the spread of the disease. We will be taking inspiration from both the Hoshen and Morse model, as well as the Parham-Michael Model. Firstly, we will introduce an exposed phase for the human population, for which we use a 14-day period as introduced in the Hoshen and Morse model [3]. In this model, we will not consider the immunity or resistance, that the human population may have developed, as that would increase the complexity of the model beyond the bounds of the paper.

$$\frac{\mathrm{d}S_H}{\mathrm{d}t} = -a(T)b_1 I_M \frac{S_H}{N}, \tag{22}$$

$$\frac{\mathrm{d}E_H}{\mathrm{d}t} = a(T)b_1 I_M \frac{S_H}{N} - \frac{1}{14}E_H, \tag{23}$$

$$\frac{\mathrm{d}I_H}{\mathrm{d}t} = \frac{1}{14}E_H - rI_H, \tag{24}$$

$$\frac{\mathrm{d}R_H}{\mathrm{d}t} = rI_H, \tag{25}$$

One thing to note, is that in this case the introduction of an exposed phase does not have an effect on the R_0, this is because we do not consider the possibility of death in the human population, hence the likelihood of someone surviving the exposed phase is 1.

To start with, we must construct a relation between the number of current mosquitoes and the number of eggs deposited. For this, we will look towards the Hoshen and Morse model, which does have such a relation. Briefly discussed earlier, this relationship is dependent on the rainfall over the period of the last 10 days (R_d), we can then state the number of eggs layed by mosquitoes(B_E) as follows.

$$B_E(R_d) = \gamma R_d a(T)M, \tag{26}$$

Where γ is defined as 1 egg/mm, we use the biting rate as it is the inverse of the gonotrophic cycle[1]. M represents the current mosquito population

There are three major avenues through which the spread of malaria can be effectively controlled when it concerns mosquitoes, which are reducing the influx of adult mosquitoes, reducing the rate of mosquito bites, and finally increasing mosquito mortality. We will start with the first, where we need to control the influx of adult mosquitoes. Recent years have showcased a multitude of preventive measures, such as the reduction of available water bodies for the mosquitoes to oviposition and making the remaining water bodies as inhospitable as possible.

$$\lambda(R,T) = \frac{B_E(R_d)p_E(R)p_L(R)p_L(T)p_P(R)}{\tau_E + \tau_L(T) + \tau_P}\lambda', \tag{27}$$

Where λ' is the percentage of the population that survive any measures taken to reduce the number of immature mosquitoes.

Secondly, We need to establish a reduced bite rate, that accounts for the increase in usage of items such as insect nets, and insect repellents. The usage of such measures has become increasingly common in areas with heavy mosquito presence [8].

$$a(T) = \frac{T - T_1}{D_1}\gamma, \tag{28}$$

Where γ represents the percentage of individuals using such means to avoid insect bites.

Finally, we will include an induced mortality that is dependent on the country and year. This variable allows us to account for measures including but not limited to insecticides.

$$\mu(T) = \frac{1}{AT^2 + BT + C} + \mu' \tag{29}$$

The μ' is the induced mortality rate, that we need to quantify at a later period.

Now we can establish the ODEs for the change in the population of mosquitoes. Here, much will be kept the same, besides the transition between the exposed population to the infected population. Here, we take the approach adopted by the Hoshen and Morse model.

$$\frac{\mathrm{d}S_M}{\mathrm{d}t} = \lambda(R,T) - a(T)b_2 S_M \frac{I_H}{N} - \mu(T)S_M, \tag{30}$$

$$\frac{\mathrm{d}E_M}{\mathrm{d}t} = a(T)b_2 S_M \frac{I_H}{N} - \frac{1}{\tau_M(T)}E_M - \mu(T)E_M, \tag{31}$$

$$\frac{\mathrm{d}I_M}{\mathrm{d}t} = \frac{1}{\tau_M(T)}E_M - \mu(T)I_M, \tag{32}$$

5.2 *Programme*

Due to the fact that we are building upon the previous model, we do not need to change many aspects of the code. The three primary codes for the influx, biting rate, and per capita will be changed to introduce the new element based on the year and country, which can be seen in the Appendix. Furthermore, we only need to change the inputs in the programme for the survival probability of a mosquito in through the sporogonic cycle.

The next set of changes occur in the ODE programme, where we add in the Exposed human phase, and change the equations for the Exposed and Infected mosquito populations to the ones used in this model. Finally, in the RK4, we must look at include the additional variable.

Before We begin modelling, we must also be aware of the real life data, that we have been provided with. The following table shows the number of cases per thousand people [8]. A

Year Country	Libya	The Central African Republic	South Africa
2001	< 0.01	433.94	5.82
2011	<0.01	385.81	1.90
2021	<0.01	335.99	0.75

Table 13: Incidence of Malaria per 1000 people

thing to be noted is that, the value for the number of cases in Libya is estimated based upon the incidence rate of malaria is the neighbouring countries. Secondly, this programme will assume that at the start of the year the ratio of mosquitoes capable of carrying the malaria virus to humans will be 6 to 1, wherein the mosquitoes are equally divided into three populations. This does not reflect the actuality of the environment, hence the variables may also need to account for this deviation.

The initial attempt was done by fine-tuning the variables, until we received an appropriate number of infection cases. After this first attempt gave us a good estimate as to what the weight should look like, we can rearrange the Ro to get:

$$\ln\left(\frac{NR_0 r\mu(T)^2}{b_1 b_2 a(T)^2 \lambda(R,T)}\right) + \mu(T)T = \ln\left(\frac{\lambda'\gamma^2}{(\mu(T)+\mu')^2}\right) - \mu'T \tag{33}$$

Here μ, λ, and a represent the original equations for these variables. Using how the R_0 changes through the months, as well as what the previous attempt showed us what the accumulative R_0, we can now use multivariate regression.

5.3 *Model Results*

5.3.1 *Libya*

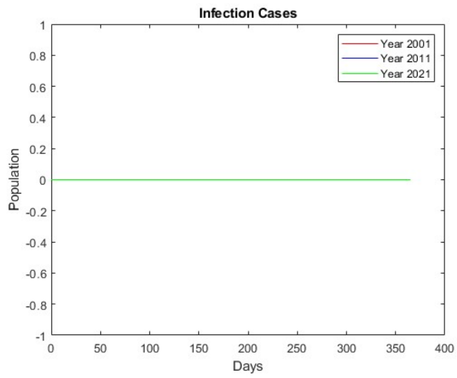

Figure 11: Number of cases in a population of 1000 in Libya by Altered Model

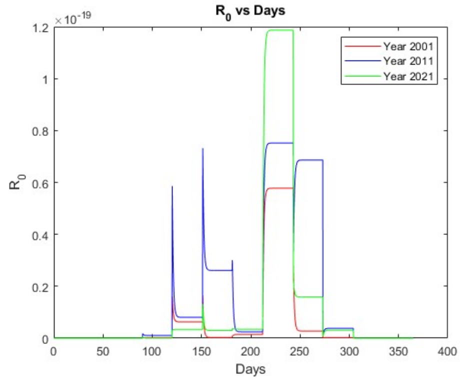

Figure 12: R_0 of Libya by Altered Model

Year & Variable	λ'	γ	μ'
2001	0.0267	0.0267	0.8691
2011	0.0312	0.0312	0.8663
2021	0.0321	0.0321	0.8679

Table 14: Variables for Libya in the Altered Model

First, we should look at the Figure 11, here we see that the number of cases is 0, unlike in Figure 5. Secondly, we can have a look at Figure 16, here we see that the summer months still have an amplifying effect on the R_0, even if the resultant R_0 is significantly smaller than the one shown in Figure 6. Finally, when we look at Table 14, we can see a significant reduction in birth and biting rates, while there is a high induced death rate. These changes maybe due to the high starting mosquito population.

5.3.2 *The Central African Republic*

Year & Variable	λ'	γ	μ'
2001	0.0703	4.6314	0.1524
2011	0.0773	4.5921	0.1794
2021	0.8856	1.1950	0.023

Table 15: Constants for the Central African Republic in the Altered Model

When we look at Figure 13, we notice that unlike the previous figure for the Central African Republic, it plateaus before the first month is over in 2001 and 2011, with the cases in 2021 only extending this period slightly. This would presumably be due to the increase in the weights on the birth rate, and the decrease in the induced death that we see in Table 15. Yet, despite this plateau in the number of cases, we see two bumps in the R_0 around day 100 and 300, of which only the second one was seen in Figure 8. The height of these bumps relate back to the difference

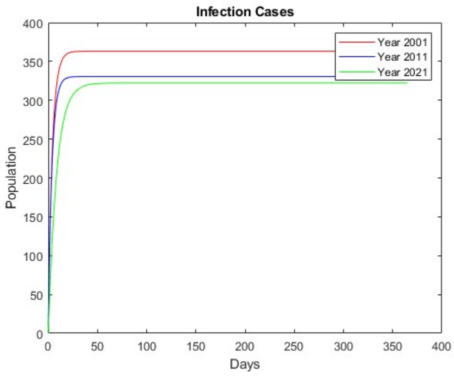

Figure 13: Number of cases in a population of 1000 in The Central African Republic by Altered Model

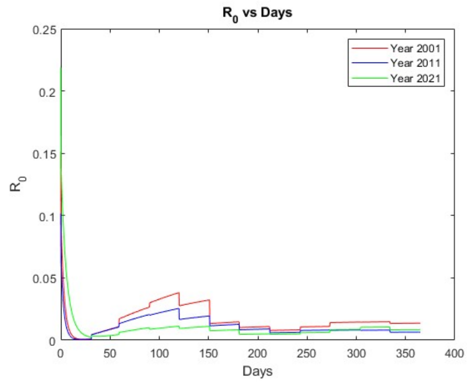

Figure 14: R_0 of Libya by Altered Model

in the biting rate. Here we can see that the biting rate go over 1, which implies that the initial mosquito population may be too small in comparison to the actual number of mosquitos.

5.3.3 *South Africa*

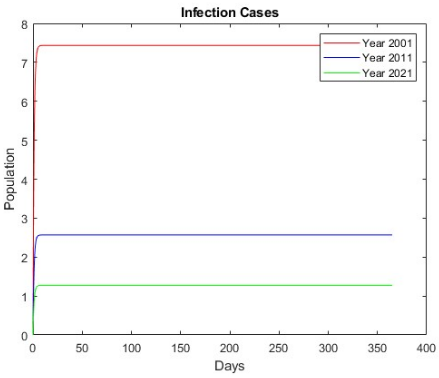

Figure 15: Number of cases in a population of 1000 in South Africa by Altered Model

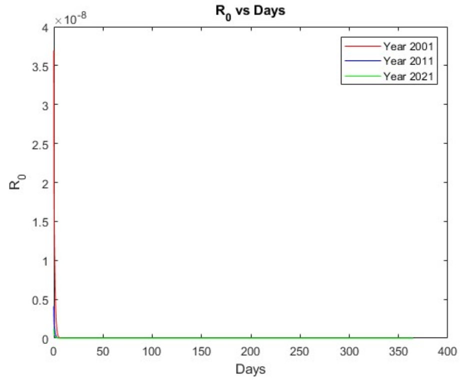

Figure 16: R_0 of South Africa by Altered Model

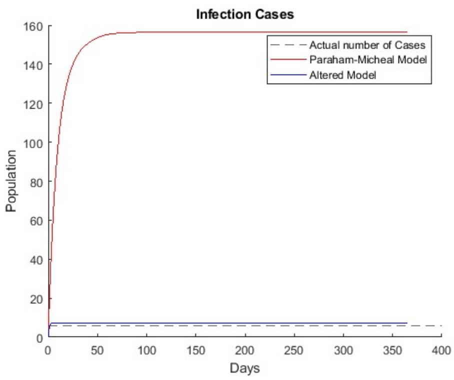

Figure 17: Comparison of the number of cases in 2001

Year & Variable	λ'	γ	μ'
2001	0.0147	0.4038	0.8523
2011	0.1424	0.1424	0.8476
2021	0.5378	0.0668	0.8556

Table 16: Constants for South Africa in the Altered Model

The number of cases, in South Africa, is the one that see the most drastic changes. To properly illustrate this, we have included the Figure 17. In Table 16, we can see that the values for weights of the Adult mosquito Influx, do not have as great of an impact on the period during which the number of cases rise, when compared to the ones in the Central African Republic, this may be due to the lower number of final infection cases. This along with the similar induced death rate implies that the main impact on the final number of cases happens to be the biting rate. The behaviour of the R_0 that is seen in the Figure 16, matches very well with what was seen in the Figure 10.

6 CONCLUSION & FURTHER IDEAS

In this thesis, we have looked at the construction of SEIR models, and the specifics of Malaria-based SEIR models. We have seen how biological processes, such as the gonotrophic cycle and sporogonic cycle, have been converted into mathematical equations that allow us to utilize them to give further accuracy to such mathematical models. Beyond this, we looked at the basic reproduction number and how that can give us insight beyond what the number of cases show us. We have also looked at the Paraham-Micheal model [7], with a focus on how it includes rainfall and temperature into the model. It told us that without human intervention, those factors would result in an upward trend in the number of infection cases. Finally, we began to alter the model, to relate the eggs deposited and mosquito population, the exposed phase for the human population, and finally the human influence on the mosquito population, and its interaction with the human population. This model, showed us that heavy human intervention would be required, especially in South Africa, for us to see the data.

If we were to revisit the model, the following points would help to expand it and make it a better approximation of the real life situation. First, we could introduce, human birth and death rate, which would have a significant impact on the model as it would change the R_0 of the model. Secondly, we could introduce the human immunity factor. This could be done in several ways, one of which would be to factor in the degree of immunity that vaccines provide to susceptible individuals. Another method would be to consider for how long individuals have immunity before they are once more susceptible, unlike in this model.

7 APPENDIX

7.1 *Equations*

Name	Equation
Bite Rate	$a(T) = \frac{T-T_1}{D_1}$
Sporogonic Cycle	$\tau_m(T) = \frac{DD}{T-T_{min}}$
Duration of the Larval Period	$\tau_L(T) = \frac{1}{\alpha T+\beta}$
Daily Larvae survival prob.	$p_L(T) = e^{-(\alpha T+\beta)}$
Per capita Mosq. Death rate	$\mu(T) = \frac{1}{AT^2+BT+C}$
Daily Egg survival prob.	$p_E(R) = (\frac{4p_{ME}}{R_L^2})R(R_L - R)$
Daily Larvae survival prob.	$p_L(R) = (\frac{4p_{ML}}{R_L^2})R(R_L - R)$
Daily Pupae survival prob.	$p_P(R) = (\frac{4p_{MP}}{R_L^2})R(R_L - R)$
Influx of Adult Mosq.	$\lambda(R,T) = \frac{B_E p_E(R) p_L(R) p_L(T) p_P(R)}{\tau_E+\tau_L(T)+\tau_P}$
Sporogonic Cycle survival prob.	$l_M(T) = e^{-\mu(T)\tau_M(T)}$
Parham-Micheal Model	$\frac{dS_H}{dt} = -a(T)b_1 I_M \frac{S_H}{N}$ $\frac{dI_H}{dt} = a(T)b_1 I_M \frac{S_H}{N} - rI_H$ $\frac{dR_H}{dt} = rI_H$ $\frac{dS_M}{dt} = \lambda(R,T) - a(T)b_2 S_M \frac{I_H}{N} - \mu(T)S_M$ $\frac{dE_M}{dt} = a(T)b_2 S_M \frac{I_H}{N}$ $-a(T)b_2(t-\tau_M(T))l_M(t)S_M \frac{I_H(t-\tau_M(T))}{N} - \mu(T)E_M$ $\frac{dI_M}{dt} = a(T)b_2(t-\tau_M(T))l_M(t)S_M \frac{I_H(t-\tau_M(T))}{N} - \mu(T)I_M$
Eggs deposited based on Rainfall	$B_E(R_d) = \gamma R_d a(T) M$
Altered Adult Mosquito Influx	$\lambda(R,T) = \frac{B_E(R_d) p_E(R) p_L(R) p_L(T) p_P(R)}{\tau_E+\tau_L(T)+\tau_P}\lambda'$
Altered Bite Rate	$a(T) = \frac{T-T_1}{D_1}\gamma$
Altered Death Rate	$\mu(T) = \frac{1}{AT^2+BT+C} + \mu'$
Altered Model	$\frac{dS_H}{dt} = -a(T)b_1 I_M \frac{S_H}{N}$ $\frac{dE_H}{dt} = a(T)b_1 I_M \frac{S_H}{N} - \frac{1}{14}E_H$ $\frac{dI_H}{dt} = \frac{1}{14}E_H - rI_H$ $\frac{dR_H}{dt} = rI_H$ $\frac{dS_M}{dt} = \lambda(R,T) - a(T)b_2 S_M \frac{I_H}{N} - \mu(T)S_M$ $\frac{dE_M}{dt} = a(T)b_2 S_M \frac{I_H}{N} - \frac{1}{\tau_M(T)}E_M - \mu(T)E_M$ $\frac{dI_M}{dt} = \frac{1}{\tau_M(T)}E_M - \mu(T)I_M$

Table 17: Equations

7.2 *Programmes*

```
function a = bite_rate(T)
%Input
%T = Temperature

%Output
%a = biting rate

%Constants
T_1 = 19.9;
D_1 = 36.5;
```

```

if T>=T_1
    a = (T-T_1)/D_1;
else
    a=0;
end
```

Listing 1: Bite Rate

```
function tau_m= spor_cycle(T)
%Input
%T = Temperature

%Output
%tau_m = Duration of the Sporogonic cycle

%Constants P. falciparum
T_min = 16;
DD = 111;

%Constants P. vivax
%T_min = 14.5;
%DD = 105;

if T>=T_min
    tau_m = DD/(T-T_min);
else
    tau_m = 10000;
end
```

Listing 2: Sporogonic Cycle

```
function tau_l = lar_dur(T)
%Input
%T = Temperature

%Output
%tau_l = Duration of the Larvae Stage

%Constants
alpha = 0.00554;
beta = -0.06737;

tau_l = 1/((alpha*T)+beta);
```

Listing 3: Duration of the Larval Period dependent on the Temperature

```
function p_l = prob_ds_lar(T)
%Input
%T = Temperature

%Output
%p_l = Daily Survival probability of the Larvae Stage based on Temp

%Constants
alpha = 0.00554;
beta = -0.06737;

p_l = exp(-((alpha*T)+beta));
```

Listing 4: Probability of daily survival of a Larvae depending on the Temperature

```
function mu = death_rate(T)
%Input
%T = Temperature

```

```
%Output
%mu = Per capita death rate for adult mosquitoes

%Constants
A = -0.03;
B = 1.31;
C = -4.4;

mu = 1/((A*(T^2))+(B*T)+C);
```

Listing 5: Death Rate per capita

```
function p_E = prob_ds_rain_egg(R)
%Input
%R = Rainfall

%Output
%p_E = Daily Survival probability of the Egg Stage based on Rain

%Constants
p_M = 0.9;
R_L = 50;

if (R<=50)
    p_E = ((4*p_M)/(R_L^2))*R*(R_L-R);
else
    p_E = 0;
end
```

Listing 6: Probability of daily survival of an egg depending on the Rainfall

```
function p_L = prob_ds_rain_larvae(R)
%Input
%R = Rainfall

%Output
%p_L = Daily Survival probability of the Larvae Stage based on Rain

%Constants
p_M = 0.25;
R_L = 50;

if (R<=50)
    p_L = ((4*p_M)/(R_L^2))*R*(R_L-R);
else
    p_L = 0;
end
```

Listing 7: Probability of daily survival of a Larvae depending on the Rainfall

```
function p_P = prob_ds_rain_pupae(R)
%Input
%R = Rainfall

%Output
%p_P = Daily Survival probability of the Larvae Stage based on Rain

%Constants
p_M = 0.75;
R_L = 50;

if (R<=50)
    p_P = ((4*p_M)/(R_L^2))*R*(R_L-R);
else
    p_P = 0;
end
```

Listing 8: Probability of daily survival of a Pupae depending on the Rainfall

```
function lambda = Mosq_Influx(R,T,B)
%Input
%T = Temperature
%R = Rainfall
%B = Number of mosquito eggs deposited

%Output
%lambda = Influx of Adult Mosquito

%Constants
tau_e = 1; %Duration of Egg stage
tau_p = 1; %Duration of Pupae stage

%Building Blocks
tau_l = lar_dur(T); %Duration of the Larvae Stage
p_l = prob_ds_lar(T); %Daily Survival probability of the Larvae Stage based on Temp
p_E = prob_ds_rain_egg(R); %Daily Survival probability of the Egg Stage based on Rain
p_L = prob_ds_rain_larvae(R); %Daily Survival probability of the Larvae Stage based on
    Rain
p_P = prob_ds_rain_pupae(R); %Daily Survival probability of the Pupae Stage based on
    Rain

lambda = (B*p_l*p_E*p_L*p_P)/(tau_l+tau_p+tau_e);
```

Listing 9: The Influx rate of adult mosquitoes

```
function A = Rain_Temp(t,C,Y)
%Input
%t = Time
%C = Country
%   0 = Libya
%   1 = Central African Republic
%   2 = South Africa
%Y = Year
%   0 = 2001
%   1 = 2011
%   2 = 2021

%Output
%R = Average Rainfall
%T = Average Temperature

switch C
    case 0
        switch Y
            case 0
                load('Lib_2001.mat');
            case 1
                load('Lib_2011.mat');
            case 2
                load('Lib_2021.mat');
        end
    case 1
        switch Y
            case 0
                load('CAR_2001.mat');
            case 1
                load('CAR_2011.mat');
            case 2
                load('CAR_2021.mat');
        end
    case 2
        switch Y
```

```
            case 0
                load('SA_2001.mat');
            case 1
                load('SA_2011.mat');
            case 2
                load('SA_2021.mat');
        end
end


if t<=31
    R = Rain(1)/31;
    T = Temp(1);
elseif t<=59
    R = Rain(2)/28;
    T = Temp(2);
elseif t<=90
    R = Rain(3)/31;
    T = Temp(3);
elseif t<=120
    R = Rain(4)/30;
    T = Temp(4);
elseif t<=151
    R = Rain(5)/31;
    T = Temp(5);
elseif t<=181
    R = Rain(6)/30;
    T = Temp(6);
elseif t<=212
    R = Rain(7)/31;
    T = Temp(7);
elseif t<=243
    R = Rain(8)/31;
    T = Temp(8);
elseif t<=273
    R = Rain(9)/30;
    T = Temp(9);
elseif t<=304
    R = Rain(10)/31;
    T = Temp(10);
elseif t<=334
    R = Rain(11)/30;
    T = Temp(11);
else
    R = Rain(12)/31;
    T = Temp(12);
end
A = [R;T];
```

Listing 10: Calculate the Temperature and Rainfall depending on the Month, Year, and Country

```
function l_M = Sur_Spor_Cyc(T)
%Input
%T = Temperature

%Output
%l_M Likelihood of mosquito survival through the Sprorgonic cycle

mu = death_rate(T); %Per capita death rate for adult mosquitoes
tau_m= spor_cycle(T); %Duration of the Sporogonic cycle

l_M = exp(-mu*tau_m);
```

Listing 11: Probability of a mosquito surviving through the Sporogonic Cycle

```
function ddt = ODE(t,S_H,I_H,N,S_M,E_M,I_M,C,Y)
%Input
```

```
%t        = Time
%S_H      = Susceptible Human Population
%I_H      = Infected Human Population
%N        = Total Human Population
%S_M      = Susceptible Mosquito Population
%E_M      = Exposed Mosquito Population
%I_M      = Infected Mosqutio Population
%C        = Country
%             0 = Libya
%             1 = Central African Republic
%             2 = South Africa
%Y        = Year
%             0 = 2001
%             1 = 2011
%             2 = 2021

%Output
%ddt      = Change in different populations

%Constant
b_1       = 0.09; %Proportion of bites to Susceptible Humans that produce infection
b_2       = 0.04; %Proportion of bites to Susceptible Mosquitoes that produce infection
r         = 1/120; %Recovery Rate
R_T       = Rain_Temp(t,C,Y); % Average Rainfall and Temperature
l_M       = Sur_Spor_Cyc(R_T(2)); %Likelihood of mosquito survival through the Sprorgonic
    cycle
a         = bite_rate(R_T(2)); %Biting Rate
B         = 200; %Number of egg laid
lambda    = Mosq_Influx(R_T(1),R_T(2),B); %Influx of Adult Mosquito
mu        = death_rate(R_T(2)); %Per capita death rate for adult mosquitoes
tau_m     = spor_cycle(R_T(2)); %Duration of the Sporogonic cycle

%Equation
dS_H = -a*b_1*I_M*(S_H/N);
dI_H = (a*b_1*I_M*((S_H)/N))-(r*I_H);
dR_H = r*I_H;
dS_M = lambda - (a*b_2*S_M*(I_H/N)) - mu*S_M;
dE_M = (a*b_2*S_M*(I_H/N)) - (a*b_2*S_M*l_M*(t-tau_m)*((I_H*(t-tau_m))/N)) - mu*E_M;
dI_M = (a*b_2*S_M*l_M*(t-tau_m)*((I_H*(t-tau_m))/N)) - mu*I_M;

ddt = [dS_H,dI_H,dR_H,dS_M,dE_M,dI_M];
```

Listing 12: Paraham-Micheal Model ODE

```
function y = RK4(q,N,C,Y)
%Input
%q = step size
%N = Total Human Population
%C = Country
%    0 = Libya
%    1 = Central African Republic
%    2 = South Africa
%Y = Year
%    0 = 2001
%    1 = 2011
%    2 = 2021

%Output
%y = All Populations at different times

b_1       = 0.09; %Proportion of bites to Susceptible Humans that produce infection
b_2       = 0.04; %Proportion of bites to Susceptible Mosquitoes that produce infection
r         = 1/120; %Recovery Rate
F         = 365/q;
x         = zeros(F,6);
z         = zeros(F,1);
R         = zeros(F,1);
```

```
x(1,1)   = N/2;
x(1,2)   = N/2;
x(1,4)   = 2*N;
x(1,5)   = 2*N;
x(1,6)   = 2*N;
R_T      = Rain_Temp(z(1),C,Y); % Average Rainfall and Temperature
l_M      = Sur_Spor_Cyc(R_T(2)); %Likelyhood of mosquito survival through the Spororgonic
    cycle
a        = bite_rate(R_T(2)); %Biting Rate
mu       = death_rate(R_T(2)); %Per capita death rate for adult mosquitoes
M        = 6*N;
R(1)     = (M*b_1*b_2*(a^2)*l_M)/(N*r*mu);

for i = 1:F
    z(i+1)       = i*q;
    K1           = q*ODE(z(i),x(i,1),x(i,2),N,x(i,4),x(i,5),x(i,6),C,Y);
    K2           = q*ODE(z(i)+(q/2),x(i,1)+(K1(1)/2),x(i,2)+(K1(2)/2),N,x(i,4)+(K1(4)/2),
    x(i,5)+(K1(5)/2),x(i,6)+(K1(6)/2),C,Y);
    K3           = q*ODE(z(i)+(q/2),x(i,1)+(K2(1)/2),x(i,2)+(K2(2)/2),N,x(i,4)+(K2(4)/2),
    x(i,5)+(K2(5)/2),x(i,6)+(K2(6)/2),C,Y);
    K4           = q*ODE(z(i+1),x(i,1)+K3(1),x(i,2)+K3(2),N,x(i,4)+K3(4),x(i,5)+K3(5),x(i
    ,6)+K3(5),C,Y);
    K            = x(i,:)+ (1/6)*(K1+2*K2+2*K3+K4);
    for j =4:6
        if K(j) <0
            K(j) = 0;
        end
    end
    x(i+1,:) = K;
    R_T      = Rain_Temp(z(i+1),C,Y); % Average Rainfall and Temperature
    l_M      = Sur_Spor_Cyc(R_T(2)); %Likelyhood of mosquito survival through the
    Spororgonic cycle
    a        = bite_rate(R_T(2)); %Biting Rate
    mu       = death_rate(R_T(2)); %Per capita death rate for adult mosquitoes
    M = K(4)+K(5)+K(6);
    R(i+1) = (M*b_1*b_2*(a^2)*l_M)/(N*r*mu);
end
y=[z,x,R];
```

Listing 13: Paraham-Micheal Model RK4

```
Pop_2001 = RK4(1/24,1000,1,0);
Pop_2011 = RK4(1/24,1000,1,1);
Pop_2021 = RK4(1/24,1000,1,2);
Pop_2001(:,2) = 1000-Pop_2001(:,2);
Pop_2011(:,2) = 1000-Pop_2011(:,2);
Pop_2021(:,2) = 1000-Pop_2021(:,2);
figure(1)
plot(Pop_2001(:,1),Pop_2001(:,2),'red',Pop_2011(:,1),Pop_2011(:,2),'blue',Pop_2021(:,1),
    Pop_2021(:,2),'green')
title('Infection Cases')
xlabel('Days')
ylabel('Population')
legend({'Year 2001','Year 2011','Year 2021'})
figure(2)
plot(Pop_2001(:,1),Pop_2001(:,8),'red',Pop_2011(:,1),Pop_2011(:,8),'blue',Pop_2021(:,1),
    Pop_2021(:,8),'green')
title('R_0 vs Days')
xlabel('Days')
ylabel('R_0')
legend({'Year 2001','Year 2011','Year 2021'})
```

Listing 14: Paraham-Micheal Model Cases and Ro Plot

```
function Z2= R_0(C)
switch C
    case 0
```

```
        load('LIB.mat');
    case 1
        load('CAR.mat');
    case 2
        load('SA.mat');
end


b1 = 0.09;
b2 = 0.04;
r = 1/120;
N = 1000;
M = 6000;
Z = zeros(264,3);
Try = zeros(12,1);
Z2 = zeros(264,1);
for i=1:264
    Z(i,1) = i;
    a = bite_rate(Temp(i));
    B = 6000*10*Rain(i)*a;
    lambda = Mosq_Influx(Rain(i),Temp(i),B);
    mu = death_rate(Temp(i));
    T = spor_cycle(Temp(i));
    M = lambda/mu;
    Z(i,3) = M;
    l_M = Sur_Spor_Cyc(Temp(i));
    Up = M*a*b1*b2*l_M;
    Down = N*r*mu;
    Z(i,2) = Up/Down;
end
Z1 = Z(:,2);
for j = 0:21
    for i= 1:12
        n = i+j*12;
        Try(i,1) = Z(n,2);
    end
    Try(:,1) = Try(:,1)/max(Try);
    Try(:,1) = Try(:,1)*(Ro(j+1)/sum(Try));
    Z2((1+j*12):(12+j*12),1) = Try(:,1);
end
```

Listing 15: Expected R_0 for real data

```
C= 1;
switch C
    case 0
        load('LIB.mat');
    case 1
        load('CAR.mat');
    case 2
        load('SA.mat');
end

b1 = 0.09;
b2 = 0.04;
r = 1/120;
N = 1000;
M = 6000;
Y = zeros(12,1);
X = zeros(12,2);
X(:,1) = 1;
Z = R_0(C);
J = zeros(12,1);
for i = 1:12
    f = i+1*12; %11*12; %21*12
    a = bite_rate(Temp(i));
    B = 6000*10*Rain(i)*a;
```

```
    lambda = Mosq_Influx(Rain(i),Temp(i),B);
    mu = death_rate(Temp(i));
    T = spor_cycle(Temp(i));
    if Z(f) ==0
        Z(f) = 1*10^(-16);
    end
    M = lambda/mu;
    J(i,1) = M;
    Up = N*(Z(f))*r*mu^2;
    Down = b1*b2*(a^2)*lambda;
    Comp = Up/Down;
    Y(i) = log(Comp)+(mu*T);
    X(i,2) = -T;
end
L = mvregress(X,Y);
L(1) = ((mu+L(2))^2)*exp(L(1));
L
```

Listing 16: Parameter Estimations

```
function lambda = Mosq_Influx(R,T,B,alpha)
%Input
%T = Temperature
%R = Rainfall
%B = Number of mosquito eggs deposited
%alpha = Weight

%Output
%lambda = Influx of Adult Mosquito

%Constants
tau_e = 1; %Duration of Egg stage
tau_p = 1; %Duration of Pupae stage


%Building Blocks
tau_l = lar_dur(T); %Duration of the Larvae Stage
p_l = prob_ds_lar(T); %Daily Survival probability of the Larvae Stage based on Temp
p_E = prob_ds_rain_egg(R); %Daily Survival probability of the Egg Stage based on Rain
p_L = prob_ds_rain_larvae(R); %Daily Survival probability of the Larvae Stage based on
    Rain
p_P = prob_ds_rain_pupae(R); %Daily Survival probability of the Pupae Stage based on
    Rain

lambda = (alpha*B*p_l*p_E*p_L*p_P)/(tau_l+tau_p+tau_e);
```

Listing 17: Altered Mosquito Influx

```
function a = bite_rate(T,alpha)
%Input
%T = Temperature
%alpha = Weight
%C = Country
%   0 = Libya
%   1 = Central African Republic
%   2 = South Africa
%Y = Year
%   0 = 2001
%   1 = 2011
%   2 = 2021

%Output
%a = biting rate

%Constants
T_1 = 19.9;
D_1 = 36.5;
```

```

if T>=T_1
    a = ((T-T_1)/D_1)*alpha;
else
    a=0;
end
```

Listing 18: Altered Bite Rate

```
function mu = death_rate(T,alpha)
%Input
%T = Temperature
%alpha = Induced Death Rate

%Output
%mu = Per capita death rate for adult mosquitoes

%Constants
A = -0.03;
B = 1.31;
C = -4.4;
%a = bite_rate(T,Co,Y);

mu = 1/((A*(T^2))+(B*T)+C)+alpha;
```

Listing 19: Altered Death Rate

```
function l_M = Sur_Spor_Cyc(T,alpha)
%Input
%T = Temperature
%alpha = Induced Death Rate

%Output
%l_M Likelyhood of mosquito survival through the Spororgonic cycle

mu = death_rate(T,alpha); %Per capita death rate for adult mosquitoes
tau_m= spor_cycle(T); %Duration of the Sporogonic cycle

l_M = exp(-mu*tau_m);
```

Listing 20: Altered Probability of a mosquito surviving through the Sporogonic Cycle

```
function ddt = ODE(t,S_H,E_H,I_H,N,S_M,E_M,I_M,C,Y)
%Input
%t      = Time
%S_H    = Susceptible Human Population
%E_H    = Exposed Human Population
%I_H    = Infected Human Population
%N      = Total Human Population
%S_M    = Susceptible Mosquito Population
%E_M    = Exposed Mosquito Population
%I_M    = Infected Mosqutio Population
%C      = Country
%           0 = Libya
%           1 = Central African Republic
%           2 = South Africa
%Y      = Year
%           0 = 2001
%           1 = 2011
%           2 = 2021

%Output
%ddt    = Change in different populations

switch C
    case 0
```

```
        switch Y
            case 0
                alpha=[0.0267;0.0267;1-0.1309];
            case 1
                alpha=[0.0312;0.0312;1-0.1337];
            case 2
                alpha=[0.0321;0.0321;1-0.1321];
        end
    case 1
        switch Y
            case 0
                alpha=[0.0703;4.6314;0.1524];
            case 1
                alpha=[0.0773;4.5921;0.1794];
            case 2
                alpha=[0.8856;1.1950;0.023];
        end
    case 2
        switch Y
            case 0
                alpha=[0.0147;0.4038;1-0.1477];
            case 1
                alpha=[0.1424;0.1424;1-0.1524];
            case 2
                alpha=[0.5378;0.0668;1-0.1444];
        end
end

%Constant
b_1      = 0.09; %Proportion of bites to Susceptible Humans that produce infection
b_2      = 0.04; %Proportion of bites to Susceptible Mosquitoes that produce infection
r        = 1/120; %Recovery Rate
R_T      = Rain_Temp(t,C,Y); % Average Rainfall and Temperature
%l_M      = Sur_Spor_Cyc(R_T(2),C,Y); %Likelyhood of mosquito survival through the
    Sprorgonic cycle
a        = bite_rate(R_T(2),alpha(2)); %Biting Rate
M        = S_M+E_M+I_M;
B        = eggs_layed(t, M,C,Y, alpha(2))+200; %Number of egg layed
lambda   = Mosq_Influx(R_T(1),R_T(2),B,alpha(1)); %Influx of Adult Mosquito
mu       = death_rate(R_T(2),alpha(3)); %Per capita death rate for adult mosquitoes
tau_m    = spor_cycle(R_T(2)); %Duration of the Sporogonic cycle
J        = 1/14;

%Equation
dS_H = -a*b_1*I_M*(S_H/N);
dE_H = (a*b_1*I_M*((S_H)/N))-(J*E_H);
dI_H = J*E_H - r*I_H;
dR_H = r*I_H;
dS_M = lambda - (a*b_2*S_M*(I_H/N)) - mu*S_M;
dE_M = (a*b_2*S_M*(I_H/N)) - (1/tau_m)*E_M - mu*E_M;
dI_M = (1/tau_m)*E_M - mu*I_M;

ddt = [dS_H,dE_H,dI_H,dR_H,dS_M,dE_M,dI_M];
```

Listing 21: Altered Model ODE

```
function y = RK4(q,C,Y)
%Input
%q = step size
%N = Total Human Population
%C = Country
%   0 = Libya
%   1 = Central African Republic
%   2 = South Africa
%Y = Year
%   0 = 2001
%   1 = 2011
```

```
%    2 = 2021

%Output
%y = All Populations at different times
switch C
    case 0
        switch Y
            case 0
                alpha=[0.0267;0.0267;1-0.1309];
            case 1
                alpha=[0.0312;0.0312;1-0.1337];
            case 2
                alpha=[0.0321;0.0321;1-0.1321];
        end
    case 1
        switch Y
            case 0
                alpha=[0.0703;4.6314;0.1524];
            case 1
                alpha=[0.0773;4.5921;0.1794];
            case 2
                alpha=[0.8856;1.1950;0.023];
        end
    case 2
        switch Y
            case 0
                alpha=[0.0147;0.4038;1-0.1477];
            case 1
                alpha=[0.1424;0.1424;1-0.1524];
            case 2
                alpha=[0.5378;0.0668;1-0.1444];
        end
end
b_1     = 0.09; %Proportion of bites to Susceptible Humans that produce infection
b_2     = 0.04; %Proportion of bites to Susceptible Mosquitoes that produce infection
r       = 1/120; %Recovery Rate
F       = 365/q;
x       = zeros(F,7);
z       = zeros(F,1);
R       = zeros(F,1);
N       = 1000;
x(1,1)  = N;
x(1,5)  = 2*N;
x(1,6)  = 2*N;
x(1,7)  = 2*N;
R_T     = Rain_Temp(z(1),C,Y); % Average Rainfall and Temperature
l_M     = Sur_Spor_Cyc(R_T(2),alpha(3)); %Likelyhood of mosquito survival through the
    Spororgonic cycle
a       = bite_rate(R_T(2),alpha(2)); %Biting Rate
mu      = death_rate(R_T(2),alpha(3)); %Per capita death rate for adult mosquitoes
R(1)    = (6*b_1*b_2*(a^2)*l_M)/(r*mu);

for i = 1:F
    z(i+1)      = i*q;
    K1          = q*ODE(z(i),x(i,1),x(i,2),x(i,3),N,x(i,5),x(i,6),x(i,7),C,Y);
    K2          = q*ODE(z(i)+(q/2),x(i,1)+(K1(1)/2),x(i,2)+(K1(2)/2),x(i,3)+(K1(3)/2),N,
    x(i,5)+(K1(5)/2),x(i,6)+(K1(6)/2),x(i,7)+(K1(1)/2),C,Y);
    K3          = q*ODE(z(i)+(q/2),x(i,1)+(K2(1)/2),x(i,2)+(K2(2)/2),x(i,3)+(K2(3)/2),N,
    x(i,5)+(K2(5)/2),x(i,6)+(K2(6)/2),x(i,7)+(K2(7)/2),C,Y);
    K4          = q*ODE(z(i+1),x(i,1)+(K3(1)),x(i,2)+(K3(2)),x(i,3)+(K3(3)),N,x(i,5)+(K3
    (5)),x(i,6)+(K3(6)),x(i,7)+(K3(7)),C,Y);
    K           = x(i,:)+ (1/6)*(K1+2*K2+2*K3+K4);
    for j =1:6
        if K(j) <0
            K(j) = 0;
        end
    end
```

```
    x(i+1,:) = K;
    R_T      = Rain_Temp(z(i+1),C,Y); % Average Rainfall and Temperature
    l_M      = Sur_Spor_Cyc(R_T(2),alpha(3)); %Likelyhood of mosquito survival through
     the Spororgonic cycle
    a        = bite_rate(R_T(2),alpha(2)); %Biting Rate
    mu       = death_rate(R_T(2),alpha(3)); %Per capita death rate for adult mosquitoes
    M        = K(4)+K(5)+K(6);
    R(i+1)   = (M*b_1*b_2*(a^2)*l_M)/(N*r*mu);
end
y=[z,x,R];
```

Listing 22: Altered Model RK4

```
Pop_2001 = RK4(1/24,1,0);
Pop_2011 = RK4(1/24,1,1);
Pop_2021 = RK4(1/24,1,2);
Pop_2001(:,2) = 1000-Pop_2001(:,2);
Pop_2011(:,2) = 1000-Pop_2011(:,2);
Pop_2021(:,2) = 1000-Pop_2021(:,2);
figure(1)
plot(Pop_2001(:,1),Pop_2001(:,2),'red',Pop_2011(:,1),Pop_2011(:,2),'blue',Pop_2021(:,1),
    Pop_2021(:,2),'green')
title('Infection Cases')
xlabel('Days')
ylabel('Population')
legend({'Year 2001','Year 2011','Year 2021'})
figure(2)
plot(Pop_2001(:,1),Pop_2001(:,9),'red',Pop_2011(:,1),Pop_2011(:,9),'blue',Pop_2021(:,1),
    Pop_2021(:,9),'green')
title('R_0 vs Days')
xlabel('Days')
ylabel('R_0')
legend({'Year 2001','Year 2011','Year 2021'})
figure(5)
```

Listing 23: Altered Model plot for Infection Cases and Ro

8 ACKNOWLEDGEMENT

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REFERENCES

[1] Steffen E. Eikenberry and Abba B. Gumel. "Mathematical modeling of climate change and malaria transmission dynamics: a historical review". In: *Journal of Mathematical Biology* 77 (Oct. 2010), pp. 857–933. DOI: http://dx.doi.org/10.1007/s00285-018-1229-7.

[2] World Bank Group. *Climate Change Knowledge Portal*. URL: https://climateknowledgeportal.worldbank.org/. (accessed: 04.07.2023).

[3] MB Hoshen and AP Morse. "A weather-driven model of malaria transmission". In: *Malaria Journal* 3 (Sept. 2004). DOI: http://dx.doi.org/10.1186/1475-2875-3-32.

[4] Matt J Keeling and Pejman Rohani. *Modeling Infectious Diseases in Humans and Animals*. Princeton University Press, 2011. ISBN: 9781400841035.

[5] Sandip Mandal, Ram Rup Sarkar, and Somdatta Sinha. "Mathematical models of malaria - a review". In: *Malaria Journal* 10 (July 2011). DOI: http://dx.doi.org/10.1186/1475-2875-10-202.

[6] World Health Organization. *Malaria*. URL: https://www.who.int/health-topics/malaria. (accessed: 04.07.2023).

[7] Paul Edward Parham and Edwin Michael. "Modeling the Effects of Weather and Climate Change on Malaria Transmission". In: *Journal of Mathematical Biology* 118 (May 2010), pp. 620–626. DOI: http://dx.doi.org/10.1289/ehp.0901256.

[8] Max Roser and Hannah Ritchie. *Malaria*. 2019. URL: https://ourworldindata.org/malaria. (accessed: 04.07.2023).