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Optimization within a local energy hub network

Research Project

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Abstract

Efficient energy infrastructure solutions are essential in response to rising global energy needs, limited fossil fuel supply, and the need for sustainable energy sources. A possible solution is the creation of local energy markets through energy hubs. The creation, conversion and consumption of energy are made easier by these hubs, which act as connectors between various energy systems. Energy hubs are established to solve energy shortages and surpluses in the local market by tying together multiple energy transporters and loads. The objective of the project is to create an algorithm that balances the needs of various energy hubs while maximizing network benefits generally. The algorithm's structure encourages hubs to work together to find solutions that achieve a balance between their objectives, preferences, and constraints. Based on real-world inputs, optimal energy exchange values are calculated for each hub, representing the convergence of energy market dynamics. Moreover, the corresponding price is determined as well. The simulation results show how prices react to different energy situations, demonstrating that the model is consistent with the original hypotheses.

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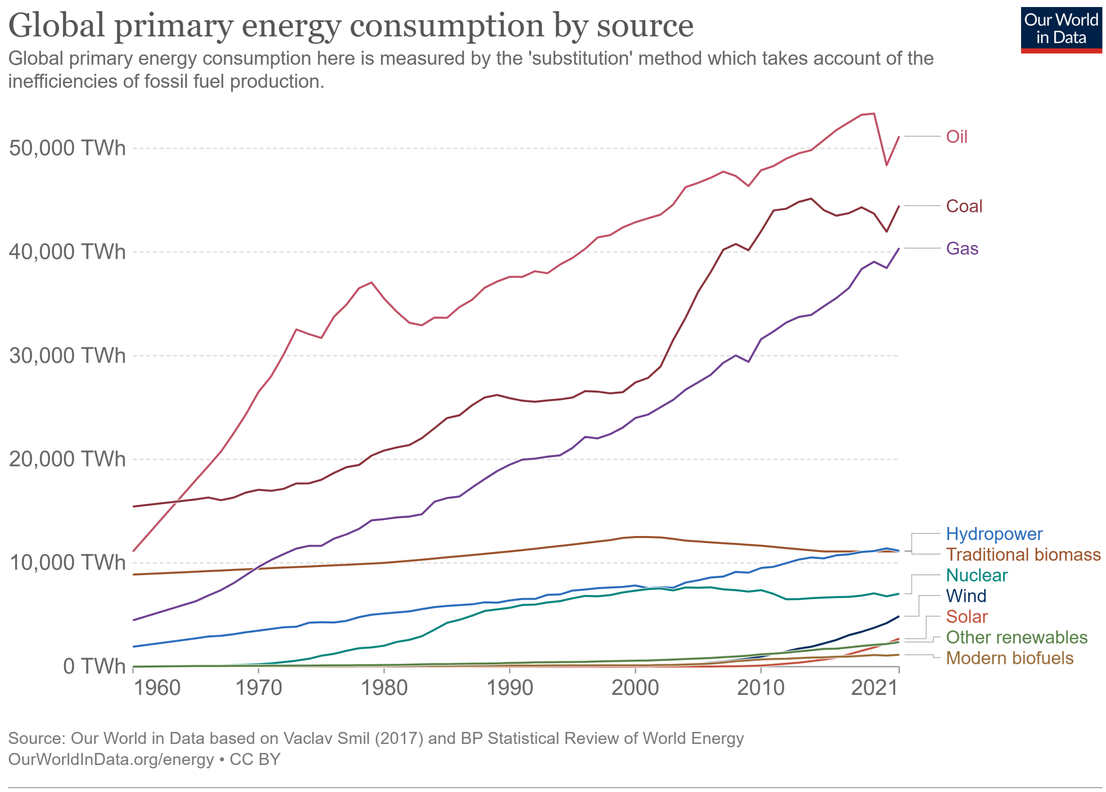
1 Introduction

Global energy consumption is increasing almost every day, resulting in an energy crisis and contamination of the environment [1]. The world's energy demand is increasing significantly because of population growth and industrial evolution [2]. Noticeably, developing nations have significantly contributed to the population growth of more than 2 billion people within one generation [3]. As the global population continues to grow and industrialization spreads throughout the globe, the need for energy to power homes, companies, transportation, and industries has increased significantly. The rise in energy consumption caused by industrial development is resulting from technical improvements and the expansion of industry and production [4]. Manufacturing industries require lot of energy for their operations, machinery, and production processes. Furthermore, when economies develop, so does the demand for products and services, resulting in increased energy consumption across the supply chain.



Global primary energy consumption by source

Global primary energy consumption here is measured by the 'substitution' method which takes account of the inefficiencies of fossil fuel production.

Source: Our World in Data based on Vaclav Smil (2017) and BP Statistical Review of World Energy
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Figure 1: Global Energy Demand visualized over a period of 60 years [5]

In order to fulfill all the energy requirements of this growing population, it is necessary to go for reliable, low cost and eternal renewable energy sources for energy demand arising in the future [3]. This implies eco-friendly strength sources for the betterment of the future world [6]. To accomplish these requirements, taking into account renewable energy sources including solar, wind, hydropower, and geothermal, will be necessary [7].

Together with the shrinking amount of fossil energy available, congested transmission systems and the aim of utilizing more sustainable and environmentally friendly energy sources, it is questionable if the energy requirements of tomorrow can be met [2]. Traditional energy systems, such as power, natural gas, and thermal, are typically planned and operated individually [8]. The lack of coordination of traditional energy systems impedes the entire system's economical and efficient operation. With the advancement of the energy internet [9], [10], the coupling of various energy sources (electricity, gas, and heat) has been tightened, and the interaction between power sources, power grid, and load is constantly strengthened [8].

The benefits of integrated energy infrastructure, such as electricity, natural gas, and district heating networks, are gaining popularity thanks to the development of technologies like efficient multi-generation systems[11]. This has sped up the transition to multi-energy systems (MES). By combining several energy sources, multi-energy systems can improve utilization efficiency [12]. Within a MES, various energy carriers interact in a cooperative manner. The supply and conversion of these various energy resources can increase the overall system's flexibility and stability [8]. However, finding a path that leads to successful collaboration is challenging [11].

1.1 Motivation & Contribution

The main motivation for me is the possible significant advantages this project can have in the real world. Currently, there any many cases of unbalanced and inaccurate energy supply which can lead to disruptions in daily operations, reduced productivity, and an overly-congested wholesale network. A steady and trustworthy energy supply can be ensured by adopting optimum scheduling and commitment mechanisms for energy hubs. As a result, it decreases disutility. This problem-solving and algorithmic design method fulfills me since it allows me to use my analytical skills and creativity to actual energy challenges.

This project contributes significantly to the field of energy management and optimization due to the fact that it presents a model for resolving the complex problem of optimal dispatch in energy hubs within local markets. A conceptual framework for optimizing energy hubs is proposed, including all the considerations and constraints to take into account. It connects demand-side management, renewable energy sources, and local energy markets to provide a full knowledge of the problem's complexities. The study investigates theoretical principles while also increasing the algorithm's practical usability by applying it to real-world data.

1.2 Structure of the report

After the brief introduction, this report follows up with a literature review. This section reviews the literature that has been published on energy hubs, their function in local and wholesale markets, optimization issues, and various methods for solving them. Moreover, a description of the specific problem that the study attempts to optimize is given, as well as information on the scope and objectives of the project. The optimization component of energy hub dispatch in the local markets is also explored, followed by a design and description of decentralized algorithms for resolving the optimization issue. Finally, the results of using the optimization techniques and algorithms are given and analysed, a conclusion is drawn and a discussion is written.

2 Literature Review

2.1 Energy Hubs

A possible solution for the integrated management of MES is the energy hub. An energy hub (EH) is a location where the production, conversion, storage, and consumption of MES occur [11], [13]. An EH represents an interface between different energy infrastructures and/or loads [2]. An EH connects several energy carriers by way of its multiple input and output ports. The input port, that is connected to e.g. electricity and other (renewable) energy infrastructures, is where energy hubs consume power. On the other hand, they provide energy for their internal load and energy services including electricity, heating, cooling, and compressed air at the output ports [11]. Energy is transformed inside the hub utilizing converters such as compressors, heat exchangers, transformers, power-electronic devices, and combined heat and power technologies [11]. A visualization of an EH can be seen in Figure 2.

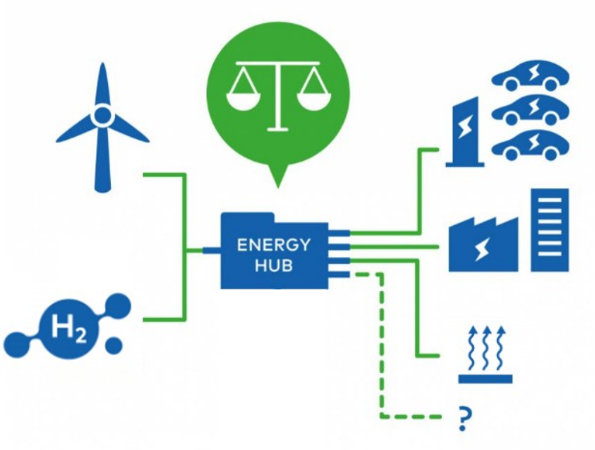

Figure 2: Outline of an energy hub. Inputs are (renewable) energy sources. Outputs are provided energy services [14].

Energy is converted inside each energy hub, with each sort of conversion having a particular efficiency rate. A schematic illustration of the inside of an energy hub is shown in figure 3. A conversion matrix, also known as a coupling matrix, lists the conversion rates between the different energy types. The links between energy inputs, outputs, and the accompanying conversion processes are shown in this matrix [15].

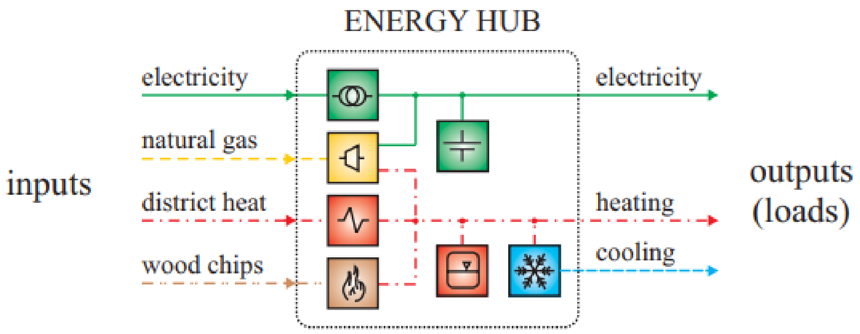

Figure 3: A schematic visualization of a multi-source multi-product energy hub [2]

2.2 Energy hubs in a local market

A local network is introduced which contains several energy hubs. These hubs are interconnected and have the ability to buy/sell directly from/to each other. Two distinct benefits result from the redundancy in the supply of energy resources provided by the use of a network that consists of local energy hubs. Firstly, since the input is no longer completely dependent on one energy network (the wholesale market), the reliability of the energy supply may be improved [8]. The load is less prone to interruptions and breakdowns in a single infrastructure by diversifying the energy sources and dispersing them over several networks. Moreover, this can lead to a lower energy price of the local market compared to the energy price of the wholesale market. Fewer interruptions and breakdowns as well as the fact that renewable energy use is offered will stimulate energy hubs to buy and within the local network [8]. Secondly, The deployment of energy hubs also adds a new degree of flexibility that enables improved supply management [8]. As a result, the hub can effectively optimize the allocation of energy carriers entering the system based on various criteria such as cost, emissions, availability, and other relevant factors. This makes it possible to distribute energy inputs optimally, guaranteeing effective resource use and distribution. Overall, utilizing energy hubs may increase resource allocation efficiency and supply reliability, which has a positive impact on the entire energy system [8].

Energy hubs have a number of essential characteristics that need more explanation. First, the algorithm assumes that the inputs are predefined numbers since they relate to the forecasted amount of renewable energy production. Moreover, the load of the energy hub is an accurate representation of how much energy the hub gathers for internal use. Furthermore, the amount of energy sold and bought from other energy hubs is a variable affected by the price of energy set by the market operator. The market operator notifies each energy hub of the price at each time step, and the energy hub changes its optimal load in response. As a result, the energy price is updated before the start of the following time step. Each energy hub's output, namely different ideal loads for internal use, can be more than the input that was initially anticipated.

In these situations, the input of each energy hub is the energy transaction with other energy hubs. It would be ideal if other hubs had an excess of energy, which the hubs with a shortage could buy to lower the overall disutility within the local market.

Social Welfare The overall balance of energy availability within a local network is very important. When there is a mismatch between the distribution of resources, opportunities, and results and the ideal or desired condition that would promote the highest overall societal welfare, social welfare problems occur. The social welfare problem aims to find a collective solution that balances the goals of individual energy hubs while ensuring that the overall benefits for the entire local network are maximized [16]. In this project, the disutility represents the mismatch between the distribution of resources. It represents the negative effects or costs of failing to fulfill the energy hub's necessary demand.

2.3 Different Energy Converters

There are several possibilities for the architectural structure of an energy hub. They can contain various energy input and output types, as well as the types of converters used inside the hub. Electric machines, gas turbines, internal combustion engines, fuel cells, thermoelectric converters, pumps, transformers, inverters, and heat exchangers are used to change the input carriers' quality, quantity, and mode so that they can be used in the output. This section will shed more light on some of the different energy converters used in such hubs.

Micro (gas) turbine Micro-turbines can generate both electricity and heat [17]. These tiny gas turbines are small heat and power-producing devices with fast starting and load-following capabilities. Micro gas turbines' outstanding characteristics, including their dependability, low maintenance requirements, fuel flexibility, and load-following capability, have positioned them as a good candidate for combined heat and power (CHP) generation [18].Researchers have long worked to enable tiny gas turbines to run on hydrogen-blended fuels. A noteworthy milestone was achieved in May 2022 when the first micro gas turbine running solely on pure hydrogen was successfully developed in Stavanger, Norway, and subsequently launched [18].

Heat Pump Heat pumps extract thermal energy from low temperatures and raise the temperature to the necessary level for practical usage [19]. Within these heat pumps, electricity is used to power the compressors that do the labor-intensive operation of concentrating and transporting thermal energy [19]. Heat pumps are flexible and eco-friendly climate control solutions since they are very effective and can supply both heating and cooling in one device. When the cost of energy is low, using a heat pump is economically favorable [20].

Combined Heat and Power Unit A combined heat and power (CHP) system can be seen as a fuel cell that simultaneously generates electricity and useful heat, and therefore, is implemented in this project. Pressure plants are often used to produce CHP, allowing for the modification of heat and power generation within set limits [21]. A combined heat and power plant is shown in figure 4. Additional energy sources, such as condensing plants, hydropower, and heat plants, as well as procurement contracts for externally produced electricity or heat, may be included in a comprehensive CHP system. As the use of renewable energy grows, CHP facilities are becoming more important in easing the connectivity of various energy sectors within an integrated energy system (IES) [21]. This is mostly due to the high efficiency of CHP facilities. Furthermore, the system's flexibility is increased by its ability to adjust and respond to both demand and supply fluctuations [21]. CHP production is controlled by a least-cost optimization approach in accordance with the heat or electricity demand [22]. The operation of a CHP system is often strategically planned using an optimization model to ensure cost-effectiveness. In that instance, the goal is to minimize costs while fulfilling both local electricity and heat demand. This is accomplished by importing or exporting power based on energy costs. Using optimization approaches can result in significant savings. Based on the favorable characteristics of a CHP, it will be considered within this project.

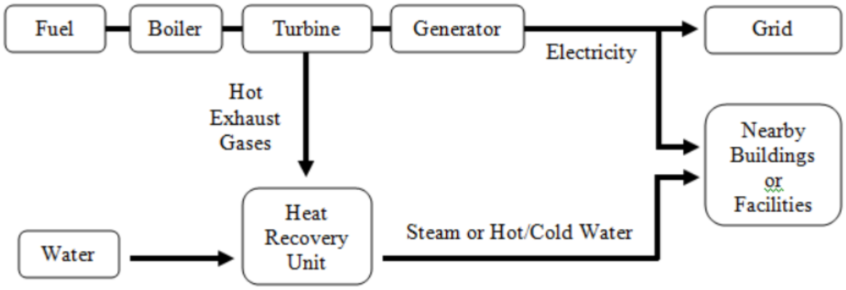

Figure 4: Schematic overview of a CHP unit [21]

Combined Cooling, Heat and Power Unit Trigeneration, also known as Combined Heating, Cooling, and Power (CCHP), is an integrated system that combines CHP technology with other components such as heat pumps or absorption cooling systems [23]. The goal of this system is to produce cooling by utilizing the heat and power generated by CHP. When these systems are properly configured, they can attain high overall efficiencies of up to 90% [24]. The primary goal of CCHP systems is to segregate heating, cooling, and electrical production and consumption. Several academics have stressed the importance of energy prices and loads in determining the techno-economic feasibility of combined thermal and electric energy systems.

Step-Down Transformers In the context of electricity, a transformer is a device that uses electromagnetic induction to transmit electrical energy between two or more circuits. The voltage level from the primary winding to the secondary winding is lowered using a step-down transformer [25]. These transformers are frequently used to lower the high voltage generated by power lines to levels that are safe for domestic or commercial use. Within this project, the voltage levels generated by renewable energy sources such as wind turbines are usually between 575 and 690 V [26]. As these cannot be used within the energy hubs themselves, the voltage has to be transformed into lower voltages, which is done by the step-down transformers considered in this project.

Inverters An electronic device called an inverter changes direct current (DC) into alternating current (AC) [27]. This conversion can power AC equipment from DC sources like batteries or solar panels, among other uses. The inverter is a crucial component of many applications, including electronic devices, backup power systems, and renewable energy systems, since it can transform DC energy into AC power [27]. Where AC electricity is the norm, inverters are frequently employed in residential, commercial, and industrial settings.

Furnace A lot of generators work by using the heat to turn water to steam. The expanding volume of gas turns a crank, and the crank moves a magnet past wires to generate electricity [2].

2.4 Distributed Energy Resources (DER)

Within this project, the system of DER (Distributed Energy Sources) is introduced. DER can be defined as a system for producing energy at or near the place of consumption [28]. The development of these systems will reduce the waste of primary energy, reduce transmission losses and thus reduce operating costs. DER have the ability to use different technologies such as fuel cells, micro gas turbines, waste heat recovery equipment, and renewable technologies such as small wind turbines and PV [28]. This is applicable to this project as local energy hubs with their own renewable energy production are considered in this project. These types of on-site energy generation resources can be one of the main sources of energy for an energy hub. Renewable resources can play an essential role in DER and their share is increasing rapidly. The inefficiency of fossil-fuel-based energy systems has led to the integration of RES with these systems and the move towards 100 percent renewable energy systems [29].

2.5 Control Measures

When contemplating smart energy hubs, it is common practice to use one of the following approaches to solve management and control issues. These include optimization, bidding or game theory.

Optimization In mathematics and engineering, the term ”optimization” refers to the process of selecting the ideal response to a problem from a range of workable options. As long as certain restrictions and limitations are followed, optimization's goal is to either maximize or minimize the objective function (sources to be added).

In mathematical terms, an optimization problem can be formulated as follows:

$$\text{minimize (or maximize)} \quad f(x)$$

subject to:

$$g_i(x) \leq 0, \quad i = 1, 2, \ldots, m$$
$$h_j(x) = 0, \quad j = 1, 2, \ldots, p$$

where:

- $f(x)$ represents the objective function, which is either to be minimized or maximized (depends on the case).
- x is the vector of decision variables, representing the possible solutions to the problem.
- $g_i(x)$ are inequality constraints, which specify limitations on the feasible region of solutions.
- $h_j(x)$ are equality constraints, which represent conditions that must be satisfied by the solutions.

An optimization algorithm's goal is to identify the values of the decision variables (x) that fulfill all constraints and result in the optimal value of the objective function ($f(x)$).

Engineering, economics, operations research, finance, and data science are all fields that have optimization difficulties [30]. They are used to handle a broad variety of real-world problems, such as determining the most cost-effective manufacturing plan, optimizing structure design, maximizing return in financial portfolios, and determining the shortest path in transportation networks. Within this project, a minimization problem is present. Analytical approaches (for simple problems with closed-form solutions), numerical methods (for more difficult problems), and heuristic algorithms (for large-scale, combinatorial problems where an exact solution is unfeasible) are all used to solve optimization problems [31], [32]. Among the most common optimization approaches include gradient-based methods, linear programming and quadratic programming.

Bidding This method aims to maximize energy sales by determining the optimal bid that an energy hub can place on the day-ahead market for both buying and selling energy. The objective is to find the bid strategy that maximizes the energy hub's overall revenue by carefully considering the market conditions and optimizing the bidding process (sources to be added).

The bidding process is frequently launched by the buyer or client issuing a Request for Proposal (RFP) or Request for Quotation (RFQ) [33]. The RFP or RFQ specifies the requirements, project specifics, evaluation criteria, and terms and conditions that prospective bidders must address in their bids.

Game Theory Game Theory applies a comprehensive framework for optimal planning of competing energy hubs [34]. The Nash equilibrium, a key notion in game theory, is concerned with decision-making. It states that each agent participating in the game will attempt to attain the best possible outcome depending on the behaviors of its competitors and the other way around. At the Nash equilibrium, no agent has a reason to abandon its chosen strategy after considering the choices of its competitors [35]. As a result, by finding the Nash balance, the equilibrium state may be identified, which is the optimal planning of competing energy hubs.

2.6 Optimization Problems

Based on their properties, optimization issues can be defined and solved in a variety of ways. Linear problems (LP), nonlinear problems (NLP), mixed-integer linear problems (MILP), and mixed-integer nonlinear problems (MINLP) are some examples of frequent forms. Each composition has unique strengths and applications in various settings.

When handling optimization issues, numerous methodologies can be used in addition to the various problem formulations. In this project, the Lagrangian method is employed.

2.6.1 Convex Optimization Problems

When a line segment connecting any two points on a function or set sits wholly above or on the function/set itself, the function or set is said to be convex [36]. Convex function local minima are also global minima, which facilitates optimization [37]. Due to the constant behavior of convex functions and sets, convexity also enables effective solutions to a variety of optimization issues.

2.6.2 Lagrangian and Primal-Dual

The Lagrangian method involves relaxing certain constraints to simplify the problem and find feasible solutions. Primal-dual optimization is an effective approach for resolving restricted optimization issues [38]. It's a technique for determining the best answer to a problem by concurrently taking into account the dual issue (a reformulation of the primary problem) and the primal problem (the initial optimization problem). When applying a primal-dual method, both the primal variable and the dual variables are updated every time step in order to satisfy the optimality conditions [39]. Within this project, the primary problem is formulated by a minimization function. A simplified explanation is given of how to formulate the primal-dual problem [38]. The first step is to construct the original optimization issue, which consists of an objective function that may either be maximized or reduced within specific bounds. As an example, a main objective can be expressed as:

$$\min_{x} \quad f(x) \quad \text{s.t.} \quad g(x) \leq 0, \quad h(x) = 0.$$

- $f(x)$ is the objective function
- $g(x)$ are inequality constraints.
- $h(x)$ are equality constraints.
- x is the optimization variable.

By adding Lagrange multipliers (dual variables) that correspond to the restrictions, the Lagrangian function is created [39]:

$$\mathcal{L}(x, \mu, \lambda) = f(x) + \mu g(x) + \lambda h(x).$$

where:

- μ are the Lagrange multipliers associated with the inequality constraints.
- λ are the Lagrange multipliers associated with the equality constraints.

The dual issue then consists of determining the lower bound of the Lagrangian function with regard to the Lagrange multipliers:

$$\max_{\mu > 0, \lambda} \quad \theta(\lambda, \mu) = \max_{\mu > 0, \lambda} \inf \mathcal{L}(x, \lambda, \mu).$$

The dual function $\theta(\lambda, \mu)$ represents the minimum value of the Lagrangian when looking at all possible x for fixed λ and μ. In order to find the optimal Lagrange multipliers λ^* and μ^* that maximize the dual function $\theta(\lambda, \mu)$, the dual problem needs to be solved.

The primal and dual solutions are related at optimality under specific circumstances, called the Karush-Kuhn-Tucker (KKT) conditions [39]. They guarantee that there is no more workable option and that the primal and dual solutions are compatible [38]. The optimal value of the primary problem ($f(x^*)$) is equal to the optimal value of the dual problem ($\theta(\lambda^*, \mu^*)$), and the ideal primary solution (x^*) may be obtained from the ideal Lagrange multipliers. Primal-Dual optimization may effectively solve challenging restricted optimization problems by using the interactions between the primal and dual issues. When the primary problem is challenging to directly tackle but its secondary problem is manageable, it is especially helpful to use the primal-dual optimization method [38].

2.6.3 Centralised, Decentralised vs Distributed Approach

Another consideration is the choice between centralized and decentralized approaches, where centralized optimization involves solving the problem as a whole, while decentralized optimization involves dividing the problem into sub-problems and optimizing them independently or collaboratively [40]. Only one decision-making agent is taken into account when the optimization issue is tackled centrally. In this situation, a centralized algorithm makes use of a thorough understanding of the whole system. In contrast, numerous decision agents with unique localized information are present in a decentralized network [40].

3 Problem Description

In the contemporary electricity market, a distributed generation system is present where various energy-producing technologies, such as wind turbines, solar panels, hydroelectric power, or natural gas engines, are used at energy hubs. By combining different energy sources, these energy hubs provide a more regular and predictable source of energy. This is achieved by balancing out changes in energy supply and demand. The local energy market consists of a number of energy hubs, each having its own predicted inputs, energy converter types, and internal loads. In the local market, this configuration enables energy hubs to buy or sell any excess or shortfall of energy to other energy hubs. Because the loads in the system are schedulable, their energy requirements can be controlled and improved over time. Diverse energy sources must be integrated, and their outputs must be managed to fit demand. This needs complex control systems and advanced algorithms. Energy supply or demand mismatches might still cause operational problems like frequency changes or blackouts without effective coordination and management. To guarantee a reliable and consistent power system, the total generation from all energy sources must be balanced with the total load demand at any given time.

4 Research focus

4.1 System

This section will discuss a few characteristics of the system of local energy hubs.
To guarantee effective energy distribution, energy hubs are strategically situated throughout the region or country they serve. Energy demand centers, closeness to energy supplies (e.g., renewable resources), and existing infrastructure are used to select sites.

A local control center may be included in the system to monitor and supervise the operation of all energy hubs. For optimization and decision-making, this control center can make use of modern technologies like supervisory control and data acquisition (SCADA) systems, real-time monitoring, and artificial intelligence [41].

Energy hubs are linked by a transmission and distribution network to ease energy transfer between hubs and end customers. High-voltage transmission lines provide energy across great distances, whilst low-voltage distribution lines bring power to homes and businesses.

Various converters play critical roles in the infrastructure of an Energy hub. These technologies, including combined heat and power (CHP) systems, combined cooling, heating, and power (CCHP) systems, and transformers, help to generate heat and electricity. These converters generate power and heat by using energy in the form of electricity and/or hydrogen.

The energy hubs are connected to various energy sources. In this project, renewable energy sources (solar, wind, hydrogen), and possibly distributed energy resources (DERs) like rooftop solar panels are considered to be the input to the energy hubs. One of the primary functions of the energy hub's conversion units is to create electricity. This electric power supports the energy hub's internal load, which is the electricity required by the hub's own activities. This internal load indicates the energy consumption required to keep the energy hub running.

The energy hub may efficiently create both heat and power by utilizing technologies such as fuel cells, CHP, CCHP, transformers or tiny gas turbines. The electricity and heat generated by these conversion units is used to power the hub's internal activities, delivering a steady and self-sufficient energy source.

The overall goal of the system can be fulfilled by the implementation of smart grid technologies. Smart grid technologies are used in the system, allowing for two-way communication between energy hubs and the central control center [42]. This makes it possible to monitor and adjust loads in real time, respond to demand and optimize energy usage while reducing peak demand. The structure of the energy hub applied in this project is visualized in figure 5

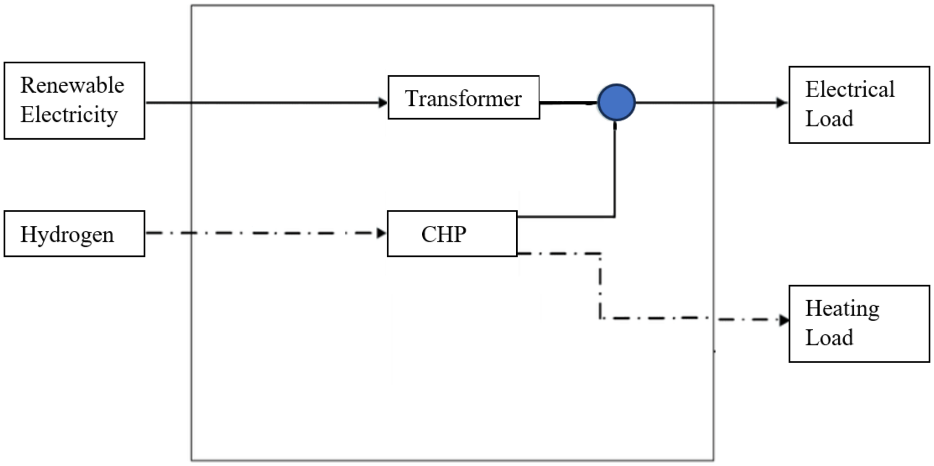

Figure 5: Visualization of the structure of an energy hub

Electricity and hydrogen are the two main inputs to the energy hub. The grid or renewable energy generators (such as solar or wind turbines) are two examples of the many places renewable electricity can come from. A hydrogen source with no marginal cost is considered. An instance of such a source is water electrolysis driven by renewable energy sources. Electricity is used in water electrolysis to break water molecules into hydrogen and oxygen [43]. When using electrolysis, renewable energy sources provide the energy input from sources like sunshine or wind, which once the necessary infrastructure is in place, have no fuel expenses. The cost of manufacturing more hydrogen after these systems are established becomes small, resulting in a nearly zero marginal cost. This is because the expense is related to installing and maintaining the electrolysis equipment and the renewable energy system.

The hydrogen-to-electricity converter is a device that converts the hydrogen that has been stored into usable energy and heat using fuel cell-like technology. This technique is a clean approach for producing power because it is effective and only produces water vapor as a side product. The electricity generated by the fuel cell's (in this project a CHP) reaction with the hydrogen can be used internally by each energy hub. The chemical reaction also results in the release of heat. The system's overall energy efficiency can be increased by capturing and using this heat for various tasks including space heating, water heating, or industrial processes. The fuel cell and solar panels are anticipated to produce power at a voltage greater than the typical residential voltage of 230V. As a result, the voltage is stepped down to an appropriate level for distribution by the use of a transformer. The 230V converted electricity is now available for usage and distribution inside the energy hub.

In the higher level system, shown in figure 6, energy hubs are connected to each other and form a local network. The thought of connected energy hubs establishing a local network represents a significant development in building a more effective energy system. Based on elements like as regional weather conditions, the availability of renewable resources, and demand patterns, each energy hub within the network may have varied levels of energy generation. Energy hubs that are connected can exchange excess energy with other hubs that might have higher demand than what their input fulfills. This makes it possible to use resources more effectively and lessens the need for individual hub overcapacity.

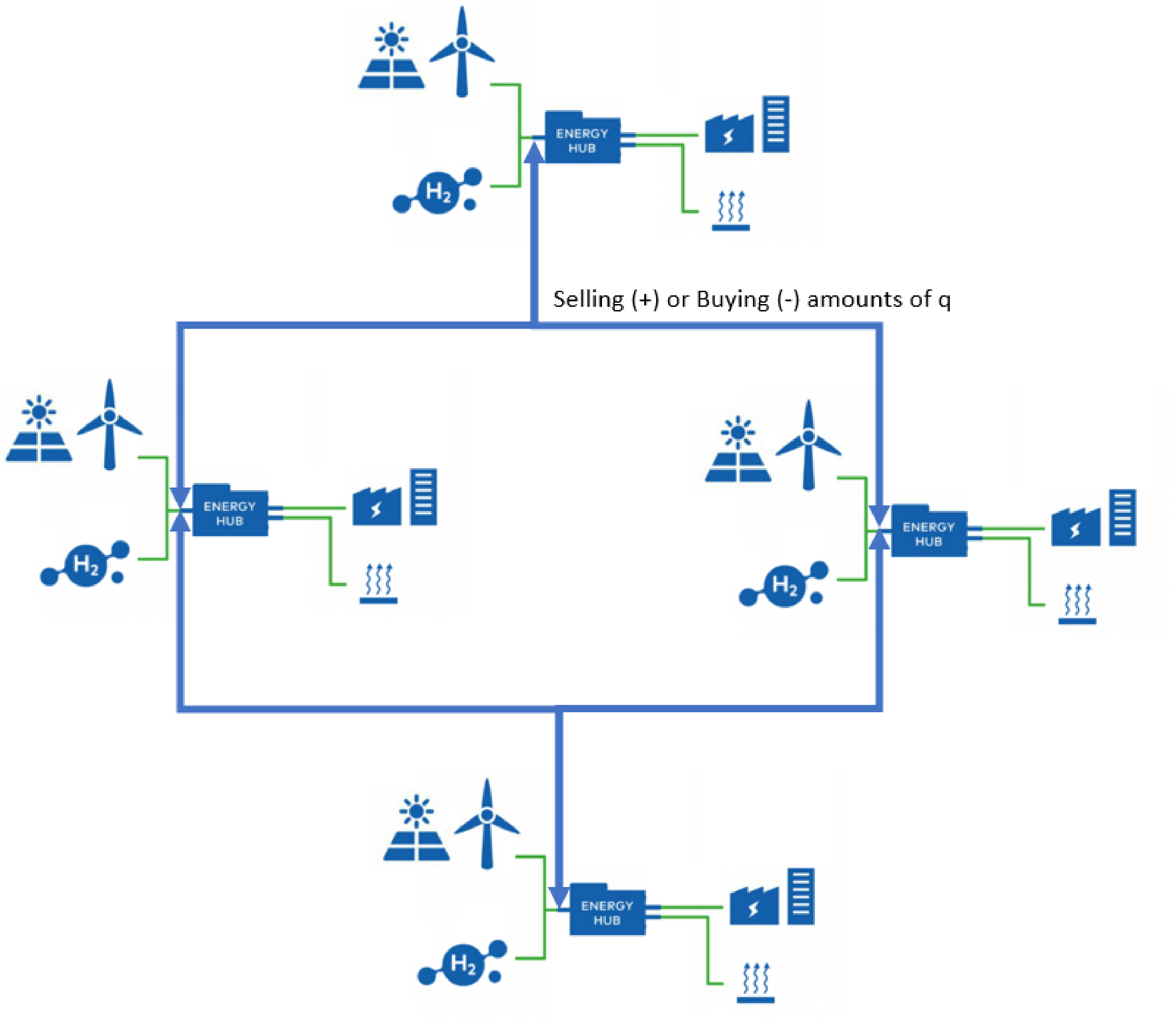

Figure 6: Higher level system

4.2 Scope

The scope of this study encompasses the optimization of a distributed generation system with schedulable loads in the context of the current energy market. The goal of the study is to effectively meet loads' energy demands while minimizing the disutility caused by energy shortages or disruptions. Therefore, the utility of schedulable loads should be increased as much as possible. The inputs are based on forecasted data. It is assumed that the input of both energy types is always available for the energy hubs. The operational constraints placed on generating units and loads will also be taken into account. These limitations are necessary to keep the distributed generating system stable and secure. By balancing the entire generation from energy sources with the total load demand at any given time, the study will make sure that the overall power balance of the system is maintained. In order to prevent imbalances that can result in frequency variations or potential blackouts, this limitation is essential.

4.3 Research Objective

In a distributed generation system with schedulable loads, the general objective is the development of the best schedule and degree of commitment for each generating unit and load. This optimization aims to minimize the running costs of the generating units and maximize the utility of the loads [44], [16]. To formulate the optimization problem, the objective can be reformulated as ”minimizing the disutility of energy hubs within the local market.” The disutility can be thought of as the negative impact or cost associated with not meeting the required energy demand of the loads [44]. For instance, if particular loads are interrupted or do not receive the required energy, it may result in reduced productivity, discomfort, or other negative effects. In order to operate the distributed energy system as cheaply and reliably as possible, the minimization problem must be solved using advanced optimization algorithms and methodologies that take into account the intricate relationships between various generating units, loads, and operational limitations. Therefore, each energy hub's major objective is to reduce its disutility while abiding by its own operational constraints and making sure the system's overall power balance is maintained.

5 Optimal Dispatch of Energy Hubs in Local and Wholesale markets

5.1 Mathemathical Modelling of an Energy Hub

In the context of energy hubs, the model represents an optimization challenge. Energy hubs are systems that combine different energy sources, such as renewable energy generation, and traditional energy sources, to deliver a consistent and efficient energy supply [45]. The optimization problem's goal is to minimize a certain cost or disutility function. The cost function often provides a measure of inefficiency, cost, or environmental effect connected with energy sources and conversions. In this project, the disutility of the local network of energy hubs is minimized.

To define the problem, the model takes into account many parameters:

- The number n denotes the number of energy hubs. Each hub is a node in the energy network that is in charge of controlling energy supply and demand.
- The number of input energy sources within this project is fixed to two different energy types. These sources, which can include renewable sources like solar or wind power as well as conventional sources like natural gas or coal, give energy to the hubs. In this project, two renewable energy sources with zero marginal cost are taken into account: electricity and hydrogen.
- The number of output energy sources is represented by two different energy types, namely electricity and thermal energy (heat). These sources are the hubs' energy outputs, which can be delivered to users or used for other reasons. In this project, the output is seen as the variable internal load l.

An energy hub comprising multiple converting devices is considered that converts P_j, $j \in \{1, 2\}$, input power flows into L_i, $i \in \{1, 2\}$, output power flows. Note that in these sets, "e" will stand for electricity, "h" for hydrogen and "th" for thermal energy. Moreover, the system is considered to be in a steady state [46].

The energy hub exhibits the characteristic of having many inputs and outputs for power flows. According to the number of converters involved, a corresponding number of variables are added for each input power flow. Therefore, for input power flow p_α, two variables are introduced for each hub. As a result, the hub model is linear. Additionally, $c_{\alpha\beta}$ represents the converter efficiency while converting an input power of type α into an output power of type β.
The multi-input multi-output converter's resultant formulation is as follows:

$$\begin{bmatrix} l_e \\ l_t \end{bmatrix} = C * \begin{bmatrix} p_e \\ p_{th} \end{bmatrix} \tag{1}$$

where matrix C is called the converter coupling matrix which was introduced in section 2.1. It accounts for the efficiency and conversion losses that occur when one energy source is converted into another. Overall, if a converter converts an input power p_j into output power l_i, the corresponding element in the matrix C is equal to the efficiency $c_{i,j}$. Multiple conversion matrices C are specified for the energy hubs, and they are integrated into a block diagonal matrix C that represents the conversion matrix for all hubs. Within this project, which considers the case of two inputs and outputs, C will have the following form:

$$C = \begin{bmatrix} c_{e,e} & c_{e,th} \\ c_{h,e} & c_{h,th} \end{bmatrix} \tag{2}$$

In the case of similar input and output units, the conversion matrix consists of values between zero and one. If element $c_{i,j}$ equals zero, it implies that no conversion exists between p_j and l_i. If the value equals one, it means there is no conversion (e.g. electricity is both an input and an output). Any value in between shows the efficiency of the conversion process. The efficiency can differ for each energy hub within the local network because the transformer that is present may have different efficiencies as well. The efficiency of a transformer is around 70-90% [25]. In the case of different units, the conversion matrix consists of any value larger or equal to zero.

Within this project, besides transforming electricity, hydrogen conversion into electricity and heat is considered as well. One kilogram of hydrogen can be converted into 33 kWh of electricity. The average efficiency of fuel cells is between 70–90%. From 1 kg of hydrogen, an average converter is able to produce about 23-30 kWh of electricity [47]. For the conversion from hydrogen to heat, the heating value should be used. The high heating value (HHV) is the total energy released when the fuel is burned in the air along with the latent heat contained in the water vapor, and as such, it is the highest amount of energy that could possibly be recovered from a specific biomass source [48]. The low heating value (LHV) is however the appropriate value to use for the energy available because, in practice, the latent heat contained in the water vapor cannot be used effectively. The lower heating value of 120 MJ represents the amount of heat created when converting one kilogram of hydrogen [49].

5.2 Optimal Dispatch of Energy hubs in Local markets

The most effective management of energy resources within local markets has emerged as a crucial challenge in the quest for a sustainable and resilient energy future. The dynamic integration of renewable energy sources and demand-side control are crucial for achieving efficiency, cost-effectiveness, and environmental goals as decentralized energy systems gain popularity [3], [6]. A model has been created to address this challenging optimization challenge, and it is poised to completely alter how local markets distribute their energy centers. Energy hubs can mutually buy or sell amounts of input energy types for a local price. It implies that the energy input type can be bought or sold before it is converted to the output energy type. This process is visualized for two hubs in figure 7

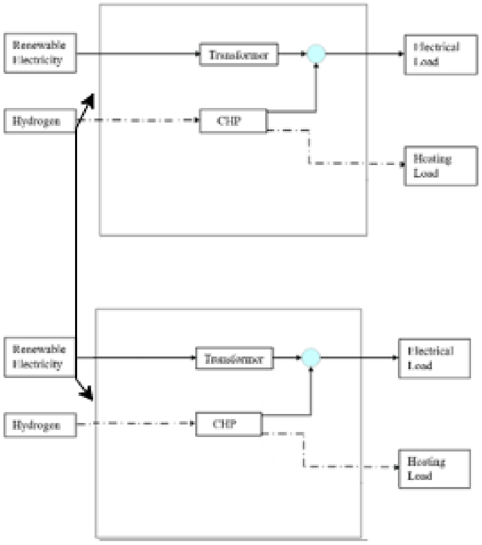

Figure 7: Two schematic energy hubs that can buy or sell energy directly from/to each other

The optimization problem for a hub $i \in \{1, ..., n\}$ takes the following form:

$$min \quad \frac{1}{2}(l_i - l_i^0)^\top Q_i(l_i - l_i^0) - \lambda^\top q_i \tag{3}$$

s.t.

$$l_i = C_i(p_i - q_i) \tag{4}$$

$$q_i \leq p_i \tag{5}$$

$$l_i \leq \bar{l_i} \tag{6}$$

Since the social problem aims to find a collective solution, the sum of all these disutilities will be minimized [16]. This is the most beneficial situation for the entire local network while balancing the goals of individual energy hubs. Moreover, in the social welfare problem, the following constraint will be added: $1^\top \cdot q = 0$ represents the fact that the balance between buying and selling energy within the local market should equal zero. Therefore, as the sum of q is zero, $\lambda^\top q_i$ will equal zero as well and is therefore not present in the social welfare problem. The objective function would look as follows:

$$min_{\{q_1,...,q_n\}} \quad \frac{1}{2}\sum_{i=1}^{n}(l_i - l_i^0)^\top Q_i(l_i - l_i^0) \tag{7}$$

Consequently, the social welfare problem can be reformulated by substituting the first constraint into the function itself:

$$min \quad \frac{1}{2}\sum(c_i(p_i - q_i) - l_i^0)^\top Q_i(c_i(p_i - q_i) - l_i^0) \tag{8}$$

s.t.

$$\sum_{i=1}^{n} q_i = 0 \tag{9}$$

$$q \leq p \quad \forall i \in \{1, ..., n\} \tag{10}$$

$$C_i(p_i - q_i) \leq \bar{l_i} \quad \forall i \in \{1, ..., n\} \tag{11}$$

Multiple parameters and decision variables are used in the minimization problem. They are listed below.

Variables:

- $q_i = \begin{pmatrix} q_{i,e} \\ q_{i,h} \end{pmatrix}$
  The purchase or sale of input energy sources is represented by the optimization variable q. q is a column vector representing the amount of each energy type bought or sold. It calculates the quantity of energy purchased or sold from each input source for each hub. The objective is to discover the ideal q values that minimize the cost function.
- $l_i = \begin{pmatrix} l_{i,e} \\ l_{i,th} \end{pmatrix}$
  The hub demand for output sources is represented by the vector l_i. It provides the quantity of energy required from each output source for each hub. l_i is also reshaped into a column vector for each energy hub i.
- $\lambda^\top = \begin{pmatrix} \lambda_e \\ \lambda_h \end{pmatrix}$
  It represents the price of the energy bought and sold within the local market.

Parameters:

- $p_i = \begin{pmatrix} p_{i,e} \\ p_{i,h} \end{pmatrix}$
  The supply of each input energy source per hub is represented by the vector p. It indicates how much energy is available from each source to each hub.
- $l_i^0 = \begin{pmatrix} l_e \\ l_{th} \end{pmatrix}$
  The nominal electricity and thermal load of the hub i is represented by the vector l_i^0.

- Q_i is a 2-by-2 block matrix that represents the discomfort factor of hub i. It shows the relative weights or penalties associated with the various optimization problem components. The discomfort can differ for each individual energy hub.
- $\bar{l}_i$ represents the maximum internal load of the hub i
- C_i defines the conversion matrix of the hub i

The first component in the objective function 3 characterizes the discomfort experienced by the energy hub as it deviates from its nominal load. The second part of the individual optimization problem captures the revenue/costs made from selling/buying excess/shortage energy to/from other energy hubs. This part is only present in the individual minimization problem because the sum of all the amounts that are bought and sold between energy hubs equals zero.
Simplifying and rephrasing the previously mentioned problem in terms of q_i yields the subsequent optimization problem:

$$\begin{aligned} min \quad & \frac{1}{2}\sum q_i^\top \cdot m_i \cdot q_i + b_i^\top \cdot q_i + e_i \qquad (12)\\ \text{s.t.} \quad & \end{aligned}$$

$$\sum_{i=1}^{n} q_i = 0 \qquad (13)$$

$$q \leq p \quad \forall i \in \{1, ..., n\} \qquad (14)$$

$$C_i \cdot q_i \leq d_i \quad \forall i \in \{1, ..., n\} \qquad (15)$$

where:

- $m_i = C_i^\top \cdot Q_i \cdot C_i$
- $b_i = C_i^\top \cdot Q_i \cdot l_i^0 - C_i^\top \cdot Q_i \cdot c_i \cdot p_i$
- $d_i = C_i \cdot p_i - \bar{l}_i$
- $e_i = \frac{1}{2} l_i^{0\top} \cdot Q_i \cdot l_i^0 + \frac{1}{2} p_i^\top \cdot m_i \cdot p_i - l_i^{0\top} \cdot Q_i \cdot c_i \cdot p_i$

5.2.1 Property Analysis

Equality Constraints The equation $\sum_{i=1}^{n} q_i = 0$ represents the equality constraint. The optimization variable vector q, represents the amount of hub purchase/sales of input sources. The balance of input and output energy sources for the network of energy hubs is captured by this limitation. It assures that the entire quantity of bought and input energy that is traded within the local market equals zero. The dual variable λ is introduced as the Lagrange multiplier for this constraint. It should be noted that this dual variable represents the price of the energy carrier in each market.

Inequality Constraints Secondly, the set of inequality constraints can be analyzed. There are two inequality constraints present: $q \leq p \quad \forall i \in \{1, ..., n\}$ and $C_i \cdot q_i \leq d_i \quad \forall i \in \{1, ..., n\}$. The first constraint requires that the energy sold to other hubs is less than or equal to the available supply of input energy (p_i) at each hub. Additionally, the second constraint introduces a load capacity $\bar{l_i}$, which implies that the converted energy within each energy hub is below or equal to the upper bound of the internal load capacity of the energy hub.

To effectively manage inequality restrictions, projected functions are a mathematical concept utilized in optimization and numerical approaches and are therefore included in this model. In the context of optimization, boundaries or restrictions that the solution must meet are represented as inequality constraints. Projected functions enable the inclusion of these constraints in the objective function to direct the optimization process while still enabling the algorithm to explore the feasible region [37], [50]. For the model introduced, the projected functions would look like this:
Constraint $q \leq p \quad \forall i \in \{1, ..., n\}$:

$$\mu = \begin{cases} \mu - q_i - p_i & for\ \mu > 0 \\ \mu + max(0, q_i - p_i) & for\ \mu = 0 \end{cases}$$

Constraint $C_i \cdot q_i \leq d_i \quad \forall i \in \{1, ..., n\}$:

$$\nu = \begin{cases} \nu - C_i \cdot q_i - d_i & for\ \nu > 0 \\ \nu + max(0, -C_i \cdot q_i - d_i) & for\ \nu = 0 \end{cases}$$

It implies that for the inequality function, Lagrange multipliers μ and ν (dual variables) are introduced which will be updated every time step with either zero or an addition or subtraction of the inequality constraint.

Convexity In this section, an elaboration on the convexity of the optimization problem 3 is presented. When breaking down the different components of the objective function, it can be seen if it satisfies the convexity property. Convexity is a desired attribute in optimization issues because it ensures that every local minimum is also a global minimum, making it easier to discover the best solution [37].
The objective function is given as:

$$min \quad \frac{1}{2} \sum q_i^\top \cdot m_i \cdot q_i + b_i^\top \cdot q_i + e_i \tag{16}$$

s.t.

$$\sum_{i=1}^{n} q_i = 0 \tag{17}$$

$$q \leq p \quad \forall i \in \{1, ..., n\} \tag{18}$$

$$C_i \cdot q_i \leq d_i \quad \forall i \in \{1, ..., n\} \tag{19}$$

where:

- q_i is the optimization variable vector.
- Q and C_i are semi-positive definite square matrices.
- p_i and $\bar{l_i}$ are semi-positive definite vectors.

The equality constraint $\sum_{i=1}^{n} q_i = 0$ is linear, and linear constraints are always convex. The inequality constraints $q_i \leq p_i \quad \forall i \in \{1, ..., n\}$ and $c_i(p_i - q_i) \leq \bar{l_i} \quad \forall i \in \{1, ..., n\}$ are linear combinations of semi-positive definite matrices and vectors and, therefore, both convex. Matrices and vectors are semi-positive definite if all their eigenvalues are non-negative [51]. Consequently, as both the equality and inequality constraints are convex, it suffices to demonstrate the convexity of just the objective function shown in equation 16.

Convexity characterizes the three sets of restrictions outlined above. When a problem is strictly convex, optimization using the Lagrangian is a very useful method. The problem function, shown in equation 16, is strictly convex if matrix Q is positive definite. Note that these solutions assume that the matrix c_i has full column rank and that the inverse of $c_i^\top \cdot Q \cdot c_i$ exists. A matrix $Q \in \mathrm{R}^{ixj}$ is said to be full column rank if its column vectors are linearly independent, which means that no column vector can be represented as a linear combination of the others [52]. In other words, the columns of a full-column rank matrix encompass the matrix's whole column space, establishing a foundation for that space. Formally, Q is full column rank if and only if the rank of Q is equal to the number of columns, j, i.e., rank(Q)=j. If these assumptions do not hold, the solutions may not exist or may be different.

Since the objective function is a sum of a convex term and linear terms, and all three terms are convex or concave, the overall objective function is convex.

Convexity plays a crucial role in optimization because every local minimum obtained while minimizing a convex function over a convex set is also the global minimum [53]. In addition, saddle points—points where the gradient is zero but the point is not a local minimum or maximum—do not exist for convex functions. This increases the predictability of optimization algorithms and reduces their propensity to become entangled in undesirable areas of the function space. Convex optimization is frequently more resistant to noise in the data or the goal function. This is due to the fact that a convex function's global minimum is frequently less sensitive to minor changes [54].

5.3 Decentralized Algorithm Design

A decentralized algorithm is suggested to execute optimal energy hub dispatch within local markets. Using local data, this method enables each energy hub to make autonomous decisions that advance the network's overall optimization objective. The decentralized algorithm architecture that is presented below demonstrates how energy hubs can cooperate and enhance their dispatch plans within the larger framework of a local market.

Each energy hub uses data on local energy production, consumption, storage, and predictions to start its internal model. Hubs specify their goals, such as cost reduction, emission reduction, or profit maximization, as well as their limitations, such as storage and power restrictions. Energy hubs independently improve their dispatch plan based on internal models and goals. Hubs assess future energy generation, demand, and market pricing using historical data and AI-based forecasting. In order to produce a set of Pareto-optimal solutions taking into account their goals and restrictions, hubs use local optimization methods, such as explained in section 2.5. Energy hubs examine trade-offs between several optimization goals and overall network effectiveness in collaboration. In an effort to match their activities with overall optimization objectives, hubs modify their dispatch tactics in response to the updated prices they receive from the operator of the local network. Based on real-time data feeds, energy hubs regularly update their internal models and plans to ensure adaptability to shifting energy generation, consumption, and market situations. Before incorporating information into their decision-making process, each hub checks it for consistency and veracity.

A primal-dual algorithm is used in the optimization process, which is an iterative approach for addressing restricted optimization problems. As explained in section 2.6.2, the method changes the primal variable (q) and dual variables (λ, μ and ν) depending on certain update criteria in each iteration. During the primal-dual iterations, these multipliers are adjusted repeatedly. The constraint, indicated by $1^\top \cdot q = 0$, is represented by the Lagrange multiplier λ, which stands for the price of the energy types that are bought and sold within the local market. The constraint connected to the variables' non-negativity is represented by the Lagrange multiplier μ. Finally, the constraint connected to the production capacity is represented by ν. The capacity constraint is modeled by introducing an upper bound on the internal load (converted production). It implies that the internal load of each individual hub cannot exceed the pre-defined upper bound. The dual variable λ corresponds to the equality constraint and μ and ν represent the first and second inequality constraint, respectively. Deriving the Lagrangian from the objective function shown in equation 12 results in the following function:

$$\mathcal{L}(q_i, \lambda, \mu, \nu) = \sum_{i=1}^{n} \left(\frac{1}{2} q_i^\top \cdot m_i \cdot q_i + b_i^\top \cdot q_i + e_i - \mu_i (q_i - p_i) - \nu_i^\top (C_i \cdot q_i - d_i)\right) + (-\lambda)^\top \sum_{i=1}^{n} q_i \quad (20)$$

To solve this problem, the values of q_i, λ, μ and that minimize the Lagrangian $\mathcal{L}(q_i,\lambda,\ \mu,\ \nu)$ should be found. To do this, the partial derivatives of $\mathcal{L}$ with respect to q_i, λ, μ and ν can be used. Given the Lagrangian function above, the following continuous dynamics converge to the optimal solution of the optimization problem shown in equation 12 due to its convexity properties:

Each hub $i \in \{1, ..., n\}$:

- $\dot{q_i} = -m_i \cdot q_i - b_i - \mu_i + C_i^\top \cdot v_i + \lambda$
- $\dot{\mu_i} = \begin{cases} q_i - p_i & if\ \mu_i > 0 \\ max(0, q_i - p_i) & if\ \mu_i = 0 \end{cases}$
- $\dot{\nu_i} = \begin{cases} d_i - C_i \cdot q_i & if\ \nu_i > 0 \\ max(0, d_i - C_i \cdot q_i) & if\ \nu_i = 0 \end{cases}$

The market operator:

- $\dot{\lambda} = \sum_{i=1}^{n} q_i$

It should be noted that the dual variable of the equality constraint, λ, represents the price of the energy carrier in each market.

6 Results

6.1 Preliminary notes

Before analyzing the results, it is necessary to grasp the meaning of negative and positive values for q, the relationship between load l and the conversion matrix multiplied by the input (converted input energy), and the concept of energy deficit and surplus inside the energy hub network.

Negative q values imply that a hub must acquire a specific quantity of energy due to scarcity. This signifies that the internal load of the hub exceeds the energy supplied by its input sources. To satisfy its demand, the energy hub in this example wants to purchase the missing amount of energy from other energy hubs in the local network.

Positive q values imply that an energy hub has surplus energy and desires to sell it to other energy hubs. This happens when the internal load of the energy hub is less than the available energy from its input sources. By selling the excess energy to other energy hubs with higher demand, the individual energy hub hopes to lower the costs related to the excessive input they have.

Prices are zero when the overall system is balanced, which means that the sum of the load l equals the conversion matrix times the sum of the input ($l = C * p$). Individually, energy hubs might have an excess or shortage of energy, but as the demand for energy equals the amount offered, the system will be balanced after buying or selling specific values for q with the price equal to zero. The absence of a price suggests that energy is distributed effectively across the network. This balance demonstrates that the energy supply precisely fits all energy hubs' internal load demands. Energy hubs will not have an extra burden (shortage or excess after optimizing) since there is neither a shortage nor a surplus of energy in the overall local network.

If the overall load l in the network of energy hubs is less than the converted input ($l < C * p$), it indicates that the overall energy intake is insufficient to meet the overall internal loads of all the energy hubs within the local network. In this case, it results in positive pricing. To compensate for the energy shortfall, hubs must acquire energy from other hubs. The shortfall is dispersed across the energy hub network, which means that each hub receives slightly less energy than its demand. This distribution seeks to reduce the disutility induced by the scarcity at each hub as much as possible, but overall, a slight burden will be present due to the overall shortage of energy within the local network.

Conversely, if the overall load l in the network of energy hubs exceeds the converted input ($l > C * p$), it implies an excessive energy intake. This extra energy is expensive because there is insufficient internal load demand to use. In this example, hubs want to sell the extra energy, which results in negative prices and positive values for q. However, no hub is willing to purchase the surplus energy. The extra energy is divided across all hubs to guarantee a fair allocation of the network's excess expenses. This means that each individual hub will end up with a slightly higher energy input than needed resulting in higher costs and disutility.

As a result, while assessing the model's results, negative and positive values of q represent energy shortages and surpluses, respectively. The link between load l and the conversion matrix multiplied by the input $C * p$ offers information about the balance or imbalance of energy supply and demand in the energy hub network. The prices for q represent the costs and incentives for hubs to buy or sell energy based on internal load and available energy from input sources.

6.2 Results of the Model

6.2.1 Basic Run

The proposed algorithm is validated using a straightforward example. It makes it simpler to check if the constraints are being properly applied and helps to assure the model's correctness and dependability. Consider a system comprised of four energy centers. Each hub has its own set of features and energy needs. The system has four energy hubs with each two energy input sources and two energy output sources, visualized in figure 9. The conversion matrix c depicts the rates of energy conversion between sources. The conversion matrix is specified as an identity matrix in this situation, showing that the conversion between energy sources is direct and without losses. This implies that electricity is produced from electricity and heat from hydrogen. There is no conversion from electricity to heat and hydrogen to electricity (just for simplicity in this example). This is visualized in figure 8. These hubs can only buy and sell energy from/to each other. The total input of energy within the energy hub system equals the total output of energy of the system. This implies that the sum of all the bought and sold amounts per energy type should equal zero, this is the first constraint presented. Moreover, another constraint added is that hubs cannot sell more energy than the amount of energy present within that hub. It makes sense that the amount of sold energy will always be equal to or lower than the amount of input energy. The capacity constraint is for simplicity of the numbers not taken into account within this example. It will be taken into account within the upcoming case study. This initial run is performed to see if the constraints are satisfied. Table 1 describes the input and internal demand of each input energy source per hub:

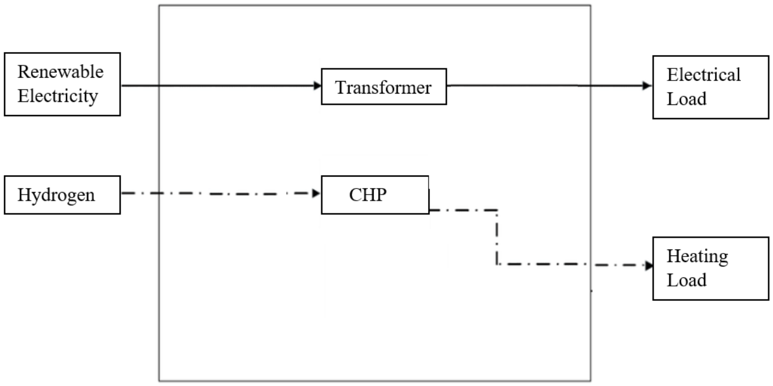

Figure 8: Simple structured Energy Hub

Table 1: Inputs and Internal Loads for each energy hub

Energy Hub	Input p_e (kWh)	Input p_h (kg)	Load l_e^0 (kWh)	Load l_{th}^0 (MJ)
1	300	350	200	250
2	200	250	250	250
3	250	200	100	100
4	250	150	250	100

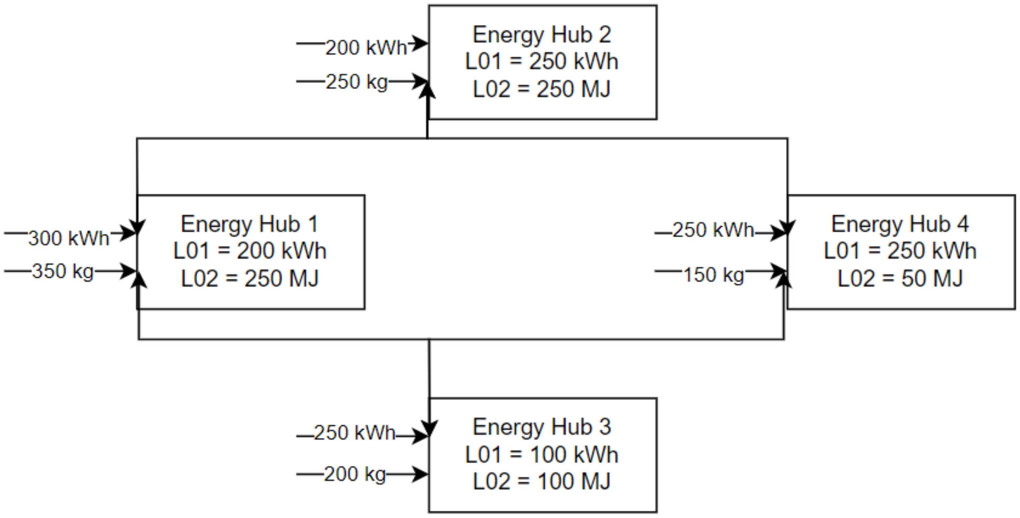

Figure 9: Visualization of the Basic Run showing the supply of energy and internal load for each hub. The energy hubs have interconnecting links which gives them the opportunity to buy and sell from other hubs before converting the energy.

6.2.2 Results of Basic Run

Optimal values to buy and sell Energy Hub 1 intends to sell about 50 units of the first input source and buy 100 units of the second input source. These results imply that Energy Hub 1 has an abundance of the first and second input sources. As a result, it intends to sell excess energy from both energy input sources to other energy hubs.

Energy Hub 2 wants to acquire 100 units of the first input source and 50 units of the second input source. This means that Energy Hub 2 has a shortage of energy for both input sources. It intends to buy the extra energy needed for both energy types from other energy hubs.

Energy Hub 3 intends to sell about 25 units of the first input source while purchasing 75 units of the second input source. This implies that Energy Hub 3 has a scarcity of the second input source but an abundance of the first input source. It intends to sell the extra energy from the first input source and purchase the energy necessary from the second input source.

Finally, Energy Hub 4 intends to sell both 25 units of the first and second input source. This suggests that Energy Hub 4 has an excess of both types of energy inputs. It is willing to sell the excess energy for both energy types to other hubs in the local market.

The resulting amounts to be bought or sold (variable q) within the local energy market are presented in table 2. Negative values imply that energy hubs will buy these amounts of energy from other local energy hubs whereas positive values indicate a desire to sell these amounts to other energy hubs within the local network.

Table 2: Optimal amounts for both energy types to be bought or sold from/to other energy hubs

Hub Number	Amount of ET 1 to buy(-) or sell(+)	Amount of ET 2 to buy(-) or sell(+)
1	50	100
2	-100	-50
3	25	-75
4	50	25

Corresponding Prices The dual variable λ represents the price within this project. It is updated every time step and finally converges to the optimal price. The price of Energy Type 1, which is electricity, is roughly -50. This price shows that there is an excess or surplus of input electricity. The negative number represents the increased cost or disutility of having a surplus of electricity. Energy hubs that have an excess of electricity are eager to sell it at a premium price to encourage other hubs to buy it and assist in balancing the system. Energy hubs having excess electricity energy strive to sell it at this price to encourage other energy hubs to buy the excess and contribute to a more balanced energy distribution.

The price of Energy Type 2, which is hydrogen, is roughly -75. This negative pricing suggests a large excess of inputted hydrogen. Hubs having a hydrogen excess are eager to sell it at this specific price in order for other energy hubs to buy their hydrogen, which lowers the burden of the individual energy hubs. This negative price encourages other hubs to buy it which does balance the system.

Evaluation of the Constraints The optimization process aims to minimize costs and disutility while ensuring an efficient allocation of energy resources based on the specific demand and availability within the network of energy hubs. For the system to be balanced, the total of the values for each energy type (type 1 and type 2) overall energy hubs should be zero. This constraint guarantees that the total energy supplied corresponds to the total energy requested by the network. If the values for q for each energy type are summed, the net result is zero, which ensures the energy input is equal to the energy output (after conversion). Furthermore, for each energy hub, the amount q reflecting the purchase or sale of energy cannot exceed the available energy supply p. This restriction guarantees that energy transactions do not exceed the limits of accessible energy sources. In none of the cases, q has a larger value than p, which means this constraint is satisfied as well.

6.3 Case Study

In this case study, the complexity of a local energy hub network is examined. It uses real-time data from solar electricity and hydrogen supply to evaluate the effectiveness of a cutting-edge algorithm created for smooth optimization and trading. In the smart grid infrastructure, energy hubs are key in facilitating a two-way flow of energy between decentralized producers and consumers. In this instance, the combination of solar electricity and hydrogen supply is the main focus, offering an interesting chance to effectively utilize renewable energy sources. Applying real data to the sophisticated algorithm created will improve local energy use and streamline buying and selling activities in the wholesale energy market. The case study will examine the results of using the algorithm to actual data, highlighting its advantages and disadvantages. In the end, this case study hopes to add to the continuing conversation about sustainable energy management.

6.3.1 Real-life data

The real-life data for electricity comes from a shopping center with ten shops all possessing solar panels [55]. Data on both the produced amount of electricity and the targeted amount are used within this project. Moreover, the transformer, which reduces the voltage of the incoming electricity to make it usable within the hub itself, has an efficiency of 70-90% [47]. Therefore, a randomized efficiency is used in the algorithm such that the conversion matrix of each hub is different. The input data on hydrogen come from a case study on the electrical processes for electrolysis and fuel cells [56]. The hydrogen input is based on an average size tank and hydrogen is stored under the pressure of 700 bar, which is 700 times normal atmospheric pressure. Then, hydrogen has a density of 42 kg/m^3 instead of 0.090 kg/m3 under normal pressure and temperature conditions [57]. At 700 bar, 12-15 kg of hydrogen can be stored in an average tank of 300-400 Liters. It is coded that each hub has between 12-15 kilograms of hydrogen in a tank. Lastly, the conversion of hydrogen to heat is represented by a factor of 100-120 MJ [49]. For the discomfort function Q_i, a random value between 1-10 is assigned to each hub. A higher numerical value signifies that a hub is less content with disutility compared to a hub assigned a lower value. Within this study, the architectural structure of an energy hub as in figure 5 is considered including the CHP and transformer as well as the dependency of electricity on hydrogen.

6.4 Results Case Study

Within this case study, the gathered data is applied to the created algorithm to evaluate how the algorithm responds and performs when confronted with realistic operational conditions. The inputs of the model, based on real-life data, are shown in table 3.

Table 3: Amount of electricity (kWh) and hydrogen (kg) as input for each hub

Hub Number	Input p_e (kWh)	Input p_h (kg)
1	33	12
2	66	14
3	45	15
4	65	12
5	13	12
6	45	15
7	49	12
8	35	12
9	45	15
10	66	14

6.4.1 Optimal Dispatch

Within this case study, two conditions are simulated. A situation in which there is an excessive input, which means the overall amount of energy present within the local network is larger than the overall internal loads. The second situation visualized a state in which the overall input into the energy hubs within the local network is not large enough to satisfy the overall internal demand of all the energy hubs. The input of both situations will the same, however, the internal demand will differ, influencing the distribution of energy and their prices.

Excessive input Within this run, ten different hubs are assumed, all with varying inputs and internal loads simulated in such a way that the overall input of energy is larger than the overall internal loads of the energy hubs within the local network. As a result of the employed algorithm, the quantified quantities of energy designated for sale or purchase by individual energy hubs to attain the global optimum have been determined and presented in figure 10 and table 5. The second column lists the amounts of electricity that need to be bought or sold, and the third column lists the hydrogen amounts. Noteworthy, negative numerical entries denote a deficit, indicating that there is a shortage and hubs need to buy the given amount. The constraint on the height of this value implies that the sum of input p_i and the extra amount q_i bought cannot exceed the load l_0^i by 30%. Conversely, positive values denote a surplus, indicating that the designated amounts need to be sold to other hubs. The latter values are therefore the only values that cannot exceed the corresponding input value p_i.

Table 4: Internal loads for electricity and thermal energy

Hub	Nominal Internal Load $l^i_{e,0}$ (kWh)	Nominal Internal Load $l^i_{th,0}$ (GJ)
1	34	1.5
2	15	0.8
3	34	1.5
4	31	1.1
5	15	0.5
6	23	0.7
7	19	0.9
8	13	1.2
9	15	1.1
10	15	1.5

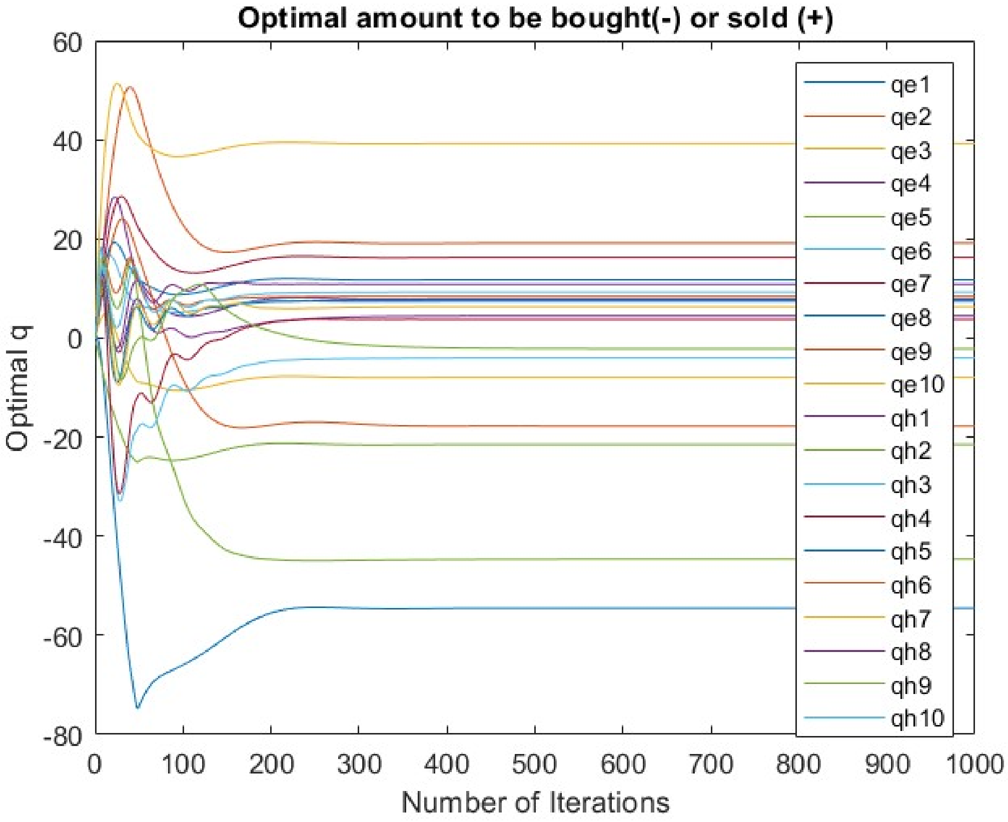

Figure 10: Amount of electricity and hydrogen to buy from or sell to other hubs in the local network

Table 5: Amounts of Electricity and Hydrogen sold in between energy hubs within a local market

Hub Number	Amount of electricity to buy(-) or sell(+)	Amount of hydrogen to buy(-) or sell(+)
1	-38.8571	4.0848
2	35.2722	2.5034
3	-26.4733	-24.6839
4	2.1673	-10.4518
5	-27.9296	7.5634
6	-10.0604	2.7568
7	19.8176	5.3377
8	7.8424	1.5984
9	1.6816	3.9419
10	36.5392	7.3493

Based on the results, shown in table 5, it can be seen that four energy hubs buy all electricity whereas the other six sell relatively smaller amounts to these hubs. When looking at the input for these four hubs, this buying behavior makes sense as the inputted amounts of electricity are significantly lower when comparing them with the electricity input of the other energy hubs in the local network. Although the input of hydrogen for these hubs lies relatively close to the inputs of other hubs, the second and third hubs still need to buy large amounts of hydrogen to fulfill their internal thermal load.

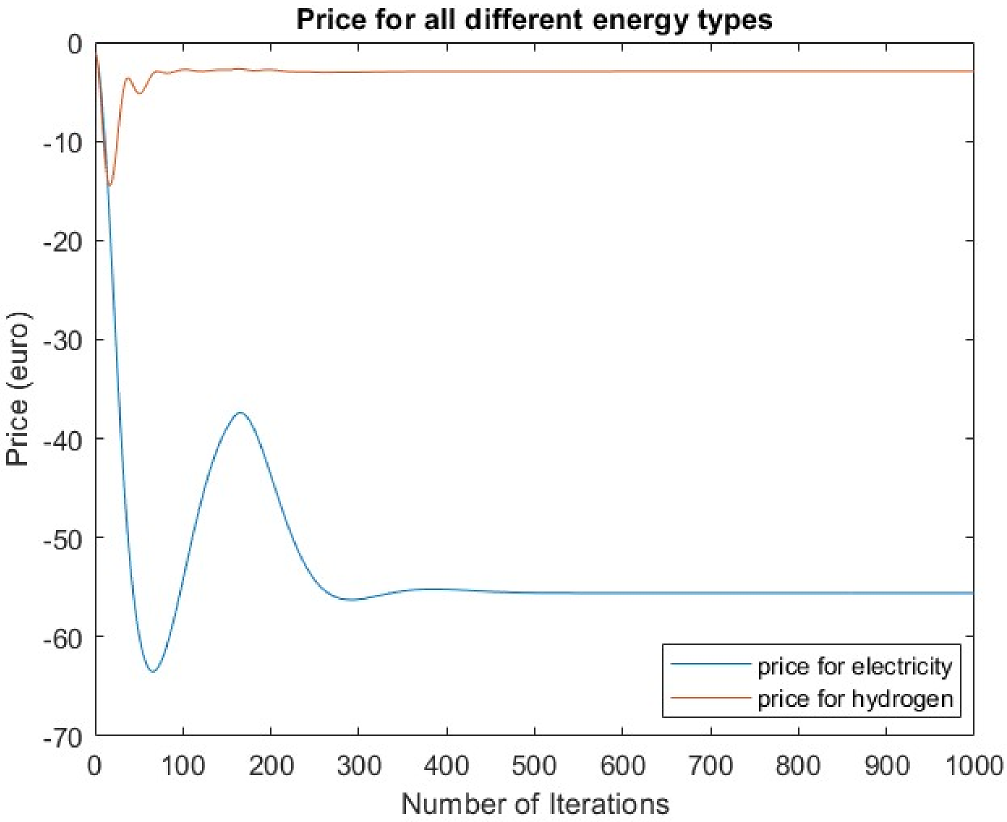

Figure 11: Corresponding prices of electricity and hydrogen

Table 6: Local Market Prices for both inputs

Type of Energy	Price (euros)
Electricity	-55.59
Hydrogen	-2.93

Resulting of the excessive input of energy within this simulation, it is expected that the resulting prices are negative. When looking at figure 11 and table 6, it can be concluded that the resulting prices converge to both negative values. This matches the initially expected result of negative pricing resulting from an overall larger input compared to the overall internal load of energy hubs within the local network. It makes sense that the price of hydrogen is relatively lower than the price of electricity because the excess of electricity is larger within this example.

Table 7: Final Internal Loads of each energy hub i

Hub Number	Final Load l_e^i (kWh)	Final Load l_{th}^i (GJ)
1	23.27222	1.821542
2	18.90905	1.031583
3	40.75024	0.637405
4	39.93503	1.426979
5	18.195824	0.649355
6	28.11666	0.904831
7	24.01759	1.169753
8	15.18809	1.490272
9	18.46546	1.420078
10	18.30951	0.641168

The constraint $q_i \leq p_i$ is satisfied for all i by directly comparing the corresponding elements of q and p. As energy hubs cannot sell more than their input, positive values of q, which indicate a sale to other hubs, should all be lower than the corresponding input p. Within this situation, this condition is satisfied. Moreover, it can be checked if the sum of all q_i values is equal to $\pm$ 0 by computing the sum. Since numerical precision might lead to a very small non-zero value, the sum could be compared with a small tolerance value close to zero. For this situation, the sum of the q values for each energy type equals a number close to zero. Lastly, none of the final internal loads, shown in table 7, exceeds $\bar{l}_i$, which equals 130% of the nominal electricity and thermal load of each energy hub i.

Shortage of input Within this run, the same ten hubs are assumed, however, this time with higher internal demand, creating a shortage of energy within the local market. The quantifiable amounts of energy that each energy hub will sell or buy in order to reach the global optimum have been determined and are shown in figure 12 and table 9. Again, the amounts of electricity that need to be purchased or sold are listed in the second column, and the amounts of hydrogen are listed in the third column. Negative number entries signify a deficit, suggesting a shortfall and the necessity for hubs to purchase the specified quantity. The restriction concerning the magnitude of this quantity indicates that the total of the input value p_i and the quantity q_i purchased must not surpass the capacity l_0^i by more than 30%. Positive numbers, on the other hand, signify a surplus, which means the prescribed amounts must be sold to other hubs. This value cannot surpass the input amount as it is not possible to sell more energy than the input of that energy type.

Table 8: Internal loads for electricity and thermal energy

Hub	Nominal Internal Load $l^i_{e,0}$ (kWh)	Nominal Internal Load $l^i_{th,0}$ (GJ)
1	34	0.5
2	45	0.6
3	25	1.5
4	51	1.1
5	15	1.3
6	23	1.2
7	29	0.9
8	33	1.2
9	15	1.1
10	15	0.5

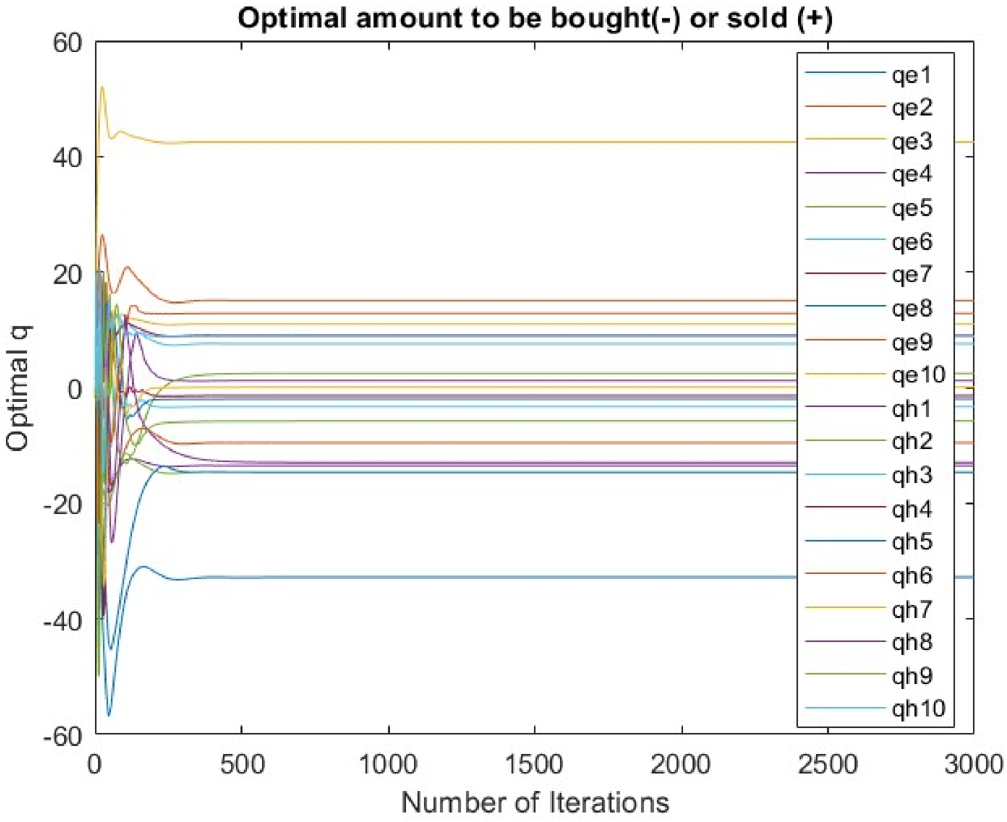

Figure 12: Amount of electricity and hydrogen to buy from or sell to other hubs in the local network

Table 9: Amounts of Electricity and Hydrogen sold in between energy hubs within a local market

Hub Number	Amount of electricity to buy(-) or sell(+)	Amount of hydrogen to buy(-) or sell(+)
1	-32.8195	1.205
2	-9.5346	2.4583
3	11.0093	-3.2801
4	-13.4847	-1.3968
5	-14.6159	-1.9734
6	7.5811	12.8182
7	9.0109	0.0576
8	-14.639	-12.9616
9	15.0509	-5.7998
10	42.4414	8.8726

As the electricity input for the first hub is still relatively low (the same in the previous simulation where there was an excess of energy), it still wants to acquire a significant amount of electricity as well as hydrogen. The same holds for the fifth and eighth hubs, whose input was also much lower compared to the other hubs. Noteworthy, the tenth hub has a significant excess of input electricity as it desires to sell a significant amount of it to the other hubs in the local market. For the other energy hubs, the amounts to be bought or sold for both electricity and hydrogen are all quite balanced, indicating slight shortages and excess of energy for these individual hubs optimized by selling and buying small amounts with other energy hubs.

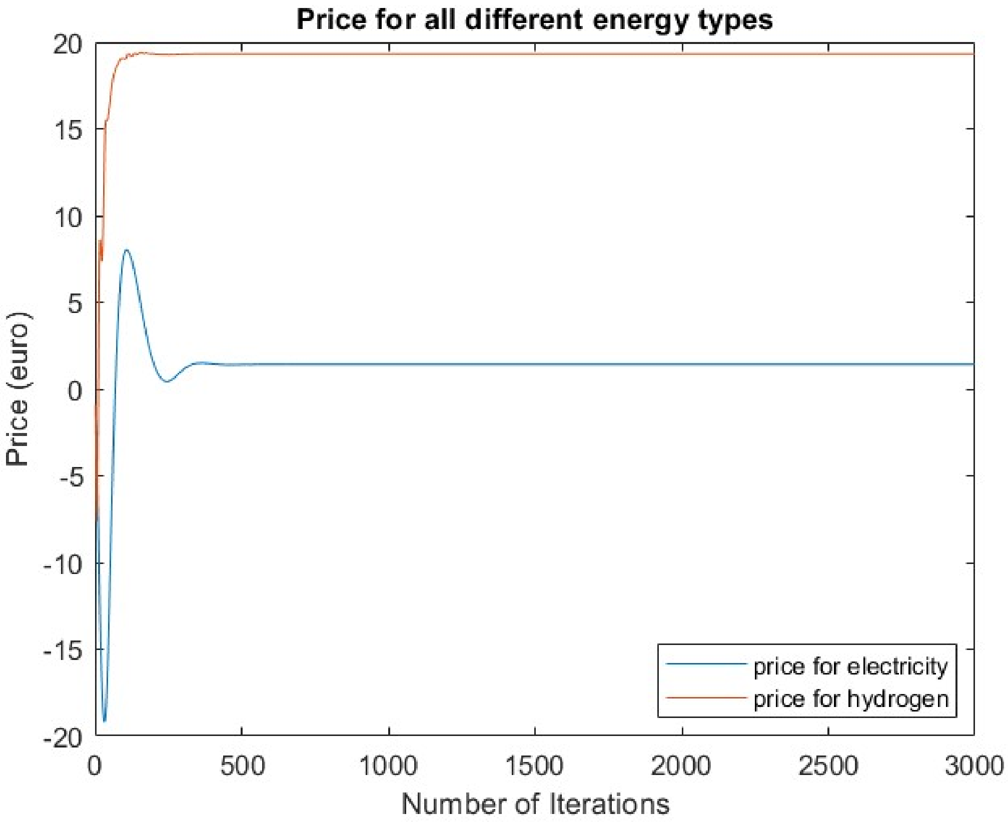

Figure 13: Corresponding prices of electricity and hydrogen

Table 10: Local Market Prices for both inputs

Type of Energy	Price (euros)
Electricity	1.42
Hydrogen	19.32

Based on the amounts bought from and sold to other hubs, the resulting prices are shown in table 10. It is anticipated that the resulting prices will be positive due to the significant energy deficit that is shown in this simulation. It is clear from looking at figure 13 and table 10 that the prices do, in fact, converge towards positive values. This result is consistent with the initial expectation of positive pricing, which came from the significant energy input shortage relative to the total internal load of energy hubs within the local network.

Table 11: Final Internal Loads of each energy hub i

Hub Number	Final Load l_e^i (kWh)	Final Load l_{th}^i (GJ)
1	41.80908	0.597946
2	52.51494	0.7743839
3	31.70745	1.93463
4	58.84558	1.406739
5	9.144156	1.620379
6	28.90073	1.558679
7	37.02114	1.164641
8	42.94207	1.542261
9	19.47924	1.425996
10	19.47054	0.649086

By directly comparing the equivalent elements of q and p, it is possible to satisfy the constraint $q_i \leq p_i$ for every i. Positive values of q, which represent sales to other hubs, should all be lower than the matching input p since energy hubs are unable to sell more than their input. This circumstance meets this requirement. Furthermore, by computing the sum, it is possible to determine if the total value of q_i is less than zero. The total might be compared with a modest tolerance value near zero since numerical accuracy may result in a very small non-zero number. In this case, the total of the q values for each form of energy is extremely near to zero. Finally, no final internal load, shown in table 11, surpasses $barl_i$, which is equivalent to 130% of the nominal electrical and thermal load of each energy hub i.

7 Conclusion

This report provides an introduction to the ideas behind creating a local energy hub network in which a local trading market is introduced. There is a greater need to develop an efficient energy infrastructure because of the rising global energy demand brought on by population growth and industrialization, as well as the limited supply of fossil fuels, congested transmission networks, and the desire to use more environmentally friendly and sustainable energy sources. Thanks to the development of technologies like effective multi-generation systems, the advantages of integrated energy infrastructure, such as electricity, natural gas, and district heating networks, are becoming more well-known. By adding a local network with energy hubs, these systems are created. The generation, conversion, storage, and consumption of MES take place in an energy hub. An energy hub represents an interface between different energy infrastructures and/or loads as it connects several energy carriers by way of its multiple input and output ports. Each of the energy hubs that make up the local energy market has its own projected inputs, energy converter types, and internal loads. This setup gives energy hubs the ability to purchase or sell any shortage or excess energy from/to other energy hubs in the local market. The energy hubs' loads can vary, so their energy inputs and needs are required to be managed and enhanced over time. The overall generation from all energy sources must be balanced with the total load demand at any given time in order to ensure a dependable and consistent power system. When there is a mismatch between the distribution of resources, opportunities, and results and the ideal or desired condition that would promote the highest overall societal welfare, a high disutility is the result within the local network. The purpose of this project is to come up with a collaborative resolution that maximizes the overall advantages for the local network while balancing the objectives of various energy hubs. Therefore, an algorithm is developed.

The algorithm's design encourages hubs to collectively match their individual ambitions. At the same time, hubs negotiate by expressing their preferences and limitations. Consequently, convergence towards solutions that optimize both individual and collective objectives and interests is the effective result. There are some constraints taken into account. Firstly, a constraint that ensures that the overall amount of energy given matches the entire amount of energy that the network has requested is introduced. Energy intake and output (after conversion) are identical if the amounts bought and sold for each energy type are added together; the net result is zero. Additionally, the amount of energy an energy hub sells cannot be greater than the input energy of that specific energy hub. Finally, the capacity limit is introduced which states that the internal load of an energy hub cannot exceed more than 20% of its ideal hub demand.

Real-life data is applied to the model in order to validate the application of the algorithm. Based on these real-life inputs and internal loads, the optimal values that need to be bought or sold within the local market are determined for each hub. As the internal loads can vary, the algorithm has been run until all amounts have converged to their optimal value. The results of the energy excess simulation support the finding that prices generally decrease due to the increased overall energy intake relative to the total internal load of the energy hubs within the local network, resulting in a negative price. This convergence is in line with the initial expected result. Similarly, the simulation of an energy shortage supports the trend of positive prices. This result is explained by the severe shortage in energy input when compared to the combined internal loads of the energy hubs in the local network, which is consistent with the initial assumption of positive pricing.

8 Discussion

This report offers a thorough investigation of local market dispatch optimization for energy hubs. Although the study shows great gains and new insights into how to distribute energy among energy hubs within a local market, there are still certain areas that need more research. The model's potential to scale to handle larger and more complicated energy networks, as well as its flexibility to new technologies and shifting market dynamics, both call for further study.

An example of this further study could be implementing a connection with the wholesale market. The connected energy hubs can transact directly in the purchase and sale of electricity or hydrogen. In order to balance the supply and demand within the network, if one hub has extra energy while another is in need of it, they can trade energy. This implies that in the case there is an energy shortage or excess present in the local market, hubs can buy or sell their shortcomings/abundance to the wholesale market. Within the existing local market, the duty of managing energy surpluses and deficits is shared collaboratively across all hubs in order to reach an optimal total load. If the wholesale market would be integrated, hubs would have the option of selling extra energy to this market when there is a surplus across all hubs (or buying when there is a shortage). As a result, the energy system becomes more adaptable to changes in energy output and demand.

The current model could also be expanded by adding connections with multiple local markets or increasing the number of individual energy hubs within a local market. Further research could also be realized by adding other architectural structures of the energy hub or even different types of structures for different hubs within the same local network. This would increase the complexity of the algorithm such that it could be possibly adapted to reconstruct real-life situations even better.

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