Consumer preference for green electricity

Integration Project

Industrial Engineering and Management
9509 words

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I. Abstract

The transition of energy production towards renewable and green sources has brought numerous benefits to society, such as helping the environment and reducing greenhouse gas emissions. However, it has also presented some challenges. Due to the unpredictability and intermittent nature of most renewable sources, we face electricity price volatility. This, combined with the current deregulated markets, has led to multiple instances of price spikes. For example, on April 4, 2022, the French wholesale market experienced a price spike, reaching up to 4000 euros per MWh for some hours. Incidents like these highlight the need for introducing regulations to current markets to facilitate a smoother transition to green energy. In this project, we design a control system for the agents participating in a green electricity market. Within this control system, agents collaborate through a market operator to maintain the electricity price at a pre-agreed socially acceptable level, which is affordable for all agents. When green electricity resources are scarce and demand is high, the controller intervenes by reducing the agents' demand to keep the electricity price affordable for everyone without disclosing their private information to the market operator. This controller is implemented in MATLAB, and simulations are performed to identify how the control system impacts the market price and demand of consumers. Furthermore, a sensitivity and impact analysis of the controller is conducted, measuring its performance under different scenarios. Lastly, an impact analysis is performed, providing insights into the short and long-term effects and assessing how the controller will influence consumers, the green electricity market, and society as a whole.

This research was made possible with the support of the Dutch Topsector Energy-subsidy of the Ministry of Economic Affairs and Climate.

CONTENTS

II. INTRODUCTION

Over recent years, the traditional electricity market has undergone significant changes concerning electricity supply, demand, and market prices [1]. At the base of this change stands the increasing worldwide necessity for non-polluting renewable energy sources, generating green electricity. Renewables are intermittent and heavily weather-dependant energy sources, which pose significant challenges for the electricity market [1]. Intermittent supply of green electricity are challenging, as they can reduce grid stability, which can decrease the reliability of the market and can disrupt the planning of the Independent system operator (ISO).

This ISO is appointed, which has the task of ensuring that the demand for electricity of consumers is reliably, efficiently, and continuously met [2]. The ISO achieves this by forecasting the demand profile of consumers for the upcoming 24 hours [2]. Following this, the ISO meets this demand by allocating most cost-effective electricity, regardless whether the electricity was produced through renewables (green electricity) or fossil fuels (gray electricity). This cost-effective allocation happens, as large proportion of electricity consumers prefer energy labels which suit their budget the best, rather than adopting green electricity at a premium price [3].

In the electricity market, consumers try to maximise their utility (the highest level of satisfaction) given their budget constraints [2]. In contrast, electricity suppliers try to maximise their profit, resulting in the establishment of a competitive equilibrium and market price for electricity [2][4]. However, this competitive equilibrium frequently results in prices that exceed socially acceptable thresholds of consumers, especially in the green electricity market where consumer may be required to pay a premium price [3]. Consumer will not accept this price and will opt out of the green electricity market reverting to conventional, non renewable energy sources that are more cost-effective [3] [4].

From this context it becomes apparent that the market price of the competitive equilibrium in the green electricity market is not always fair. Thus, in order to make the market price socially acceptable at all times, this research will contribute a control system design, that is able to reduce market price by changing consumer preference and demand. This controller will be implemented into MATLAB, and simulations will be performed to identify how the control system impacts the market price and demand of consumers. Furthermore, a sensitivity and impact analysis of the controller will be performed, measuring the performance of the controller under different scenario. lastly a impact analysis will be performed med, which will give insights in the short-and long-term, assessing how the controller will influence consumers, the green electricity market, and society as a whole.

III. Preliminaries

The traditional electricity market consists of multiple agents, that are either consuming or supplying electricity [4][3]. Due to multiple agents acting in the system, the electricity market can be seen as a multi-agent system (MAS) [3]. Such systems involve collecting data, allocating resources, and coordinating control between multiple agents and sub-systems [3]. In a MAS, each agent is conceived as an intelligent unit with its own rational decisions, objectives, and preferences [3]. These agents are therefore able to change market dynamics through their decisions. In context of the electricity market, agents can make decisions regarding the quantity of demand and supply, resulting in different demand and supply curves each having a different market outcomes.

A key challenge in the electricity market and MAS operation is efficiently allocating resources ensuring that supply and demand are balanced for efficient and secure operation [3]. This allocation problem, in context of the electricity market, involves local electricity demand, internal, and external supply of electricity, all interconnected through a network allowing for the transmission of this resource between agents [3]. Furthermore, agents in the electricity market decide on the resource that is consumed in order to maximise their individual payoffs, considering the utility of the consumption, as well as the income from the transmission of electricity [3].

The term for the market outcome where individual agents try to maximise their individual payoffs is called, competitive equilibrium [4]. The definition of competitive equilibrium is as follows: "Competitive equilibrium is a condition in which profit-maximising producers and utility-maximising consumers in competitive markets with freely determined prices arrive at an equilibrium price" [5]. This definition implies that for a competitive equilibrium, perfect competition in the market is assumed, meaning that all agents, consumer or supplier, are price takers and are unable to influence market price [4]. In order to form a competitive equilibrium, it is necessary that supply always matches the demand [3][2]. The competitive equilibrium serves many purposes as it describes how markets might settle on price, explaining how supply and demand can be balanced without a central planner and is used as an analytical tool to identify the efficiency of the market [5].

In order to characterise the competitive equilibrium, a market with only renewable resources and consisting of n number of agents will be considered. Each agent i has a local resource a_i and decides to use x_i units for his own consumption and satisfaction [3]. The utility function of agent i for consuming x_i is given by $f_i(x_i)$. Agents in the system are able to transmission local resources between agents through an interconnected network, meaning that agent i would incur a $x_i - a_i$ of surplus ($a_i > x_i$) or shortcoming ($a_i < x_i$) [3]. The transmission of local resources between agents is priced at λ, meaning that agents will generate (a_i - x_i)λ in income or expenditure. Considering the network resource allocation profile $x = (x_1 ... x_n)^T$, the following definition can be given for the competitive equilibrium:

Definition 1. Price-allocation (λ^*, x^*) is a competitive equilibrium if the following conditions hold true [3].

1) Each agent tries to maximise their payoff at x_i^*
2) Total demand and supply are balanced within the network

If these two conditions hold, then the competitive equilibrium will be a solution to the following optimisation problem [3]:

$$\max_{x_i} \quad f_i(x_i) + \lambda(a_i - x_i) \tag{1}$$

$$\text{s.t} \quad \sum_{i=1}^{N} x_i^* = \sum_{i=1}^{N} a_i$$

IV. Problem context

From earlier context it has become clear, that agents in the electricity market try to maximise their utility of electricity given their budget[3]. This maximisation of utility indicates, that consumers are inclined to prioritize short-term economic considerations by seeking the most cost-effective electricity [3]. Consequently, rather than paying a premium price for green electricity, consumers prefer adopting cheaper conventional electricity, as the premium price for green electricity is often not socially acceptable for consumers.

When the price surpasses the socially acceptable threshold, agent defection may occur as consumers switch to an more cost-effective alternative product (substitute) [6] [3]. This defection means that agents might consider withdrawing from the green electricity market and revert to adopting conventional, and non-renewable electricity an alternative. Thus, agents leaving the green electricity market can result in an upswing in the utilisation of conventional and polluting energy sources, enhancing the emission of greenhouse gases (GHG). The increase of GHG emission negatively contributes to climate change by affecting the surrounding environment, including raise of sea levels, increased desertification, and decrease in soil fertility [7].

Moreover, agents leaving the system can disrupt the market, as supply may not be equal to the demand. Additionally, agents leaving the system can decrease the competitiveness of the market, potentially resulting in even higher equilibrium prices [4]. Consequently, even more consumers might potentially leave the market. These impacts highlights the need to consider fairness and sustainability hen deciding on prices in electricity markets [3] [4].

This fairness can be achieved by lowering the equilibrium price of green electricity by influencing consumer demand. However, this change in demand requires consumers to adjust their preferences accordingly [3]. Nonetheless, consumers do not know how to adjust their preference accordingly to ensure that the market price becomes socially acceptable [3]. In order to adjust demand accordingly, the implementation of a control system is required. This system should influence consumer preferences to ensure that demand is reduced such that the market price is fair and socially acceptable at any given point in time. Nonetheless, the problem is that possible design options for such a control system have not been explored yet. Therefore, it is not known whether it is possible to change consumer preference in such a way that market price is lowered to socially acceptable prices. Furthermore, the short-term and long-term risks of such a controller on consumers, the market, and society have not explored yet.

V. WHY-WHAT MODEL

With the problem context above, a Why-What model has been created. This tool is used to help answer the questions *"Why do we want to solve this problem?"* and *"What is stopping us from solving the problem?"*. The constructed Why-what model can be seen below in Figure 1 and identifies the larger and smaller problems of the competitive equilibrium in the green electricity market.

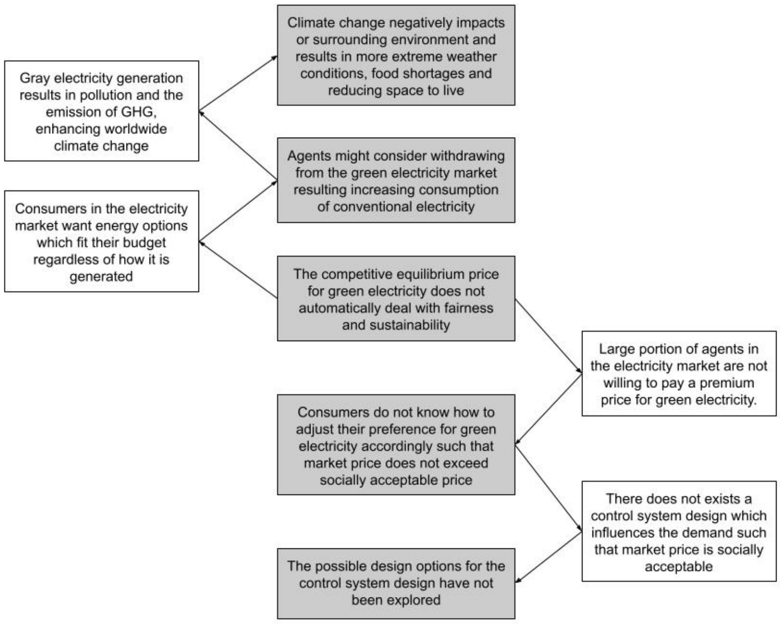


Fig. 1: The Why-What model visualises the main problem of the research while also identifying the smaller and broader problems involved

VI. SYSTEM ANALYSIS

In this section of the paper, the system will be identified, explained and analysed. This system analysis is used in research to provides a structured overview of system, as well as helps in identifying how the system can be improved.

A. *System description & social welfare problem*

As this research is focused on designing a control system capable of ensuring that the price of green electricity becomes fair and socially acceptable, it is essential to include a system analysis of the green electricity market. The system considered is therefore a green electricity market where agent can share their resources though an interconnected grid. This system, depicted in Figure 2, consists of n number of agents each having a local resource (a_i), and a desired demand x_i [3]. This local resource a_i, in context of system, represents green electricity produced by renewable energy sources (i.e., solar energy from solar panels).

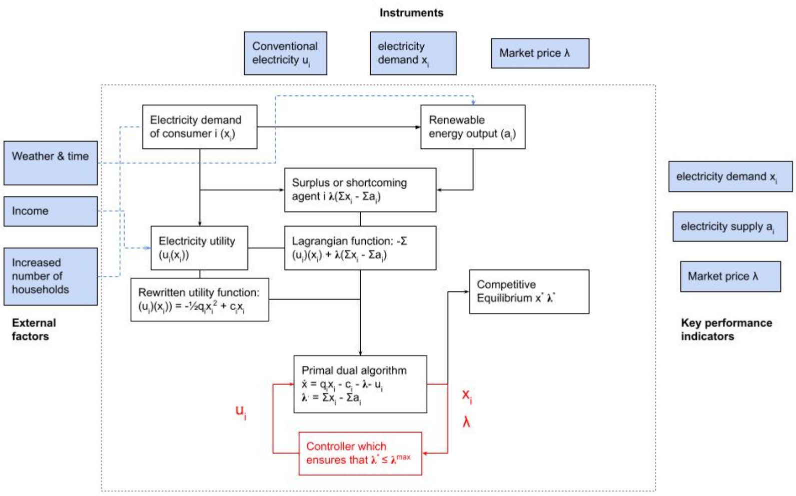


Fig. 2: The system model of the current electricity factors including external factors, instruments and Key Performance Indicators (KPI).

Each agent i in the system has its own individual desired demand for electricity at a given point in time (x_i) [3]. These agents, each has a utility function representing the level of satisfaction obtained by the agent as a function of the agents electricity consumption at a given point in time [8]. The utility function in this system is represented as a concave linear-quadratic utility function [8]. This utility function can be seen in equation (2) below.

$$u_i(x_i) = -\frac{1}{2}q_ix_i^2 - c_ix_i \tag{2}$$

Where $q_i > 0$ and $c_i < 0$. The value q_i of each agent is predetermined and c_i represents the desired consumer consumption level of agent i [8]. The consumers' desired consumption level refers how much agents value the quantity of goods or services that agents in the system wish to consume, taking into account market price, preferences, and budget constraints [8]. Therefore, if agents have a higher desired consumption level these agents will consume more units of electricity given the market price. A few examples of utility functions can be seen in figure 3.

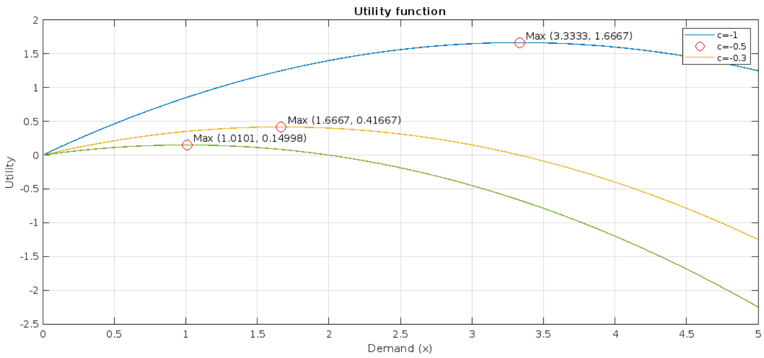


Fig. 3: Examples of utility functions for q_i = 0.3

Within the assumed electricity system, differences in demand and supply can occur, resulting in either a surplus ($a_i > x_i$) or a shortcoming ($x_i > a_i$) [3]. To match the supply and demand, agents can balance these surpluses and shortages by trading electricity resources through an interconnected electricity network via a pricing mechanism [3].

Thus, it becomes clear that agents interact with each other to find a social optimum. To do so, agents solve the following social welfare problem :

$$\min_{x_i} \quad \sum_{i=1}^{N} \frac{1}{2} q_i x_i^2 + c_i x_i \tag{3}$$

$$\text{s.t} \quad \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} a_i = a$$

Important to note is that this social welfare problem has been rewritten as a minimisation problem for convenience and clarity. This conversion from maximisation to a minimisation problem was achieved in following the property that *max* f(x) = *min* -f(x) [3]. Therefore, this convergence implies that, even though the optimisation problem is written as a minimisation problem, consumers still try to maximise their utility function, given the market price, preference, and budget constraints.

Within the social welfare optimisation problem, no production costs for electricity is included, as renewables are able to generate electricity without any production costs. Moreover, the optimisation problem includes the maximisation of the utility functions for all agents in the system, so that the sum of demand ($\sum_{i=1}^{N} x_i$) is equal to the total generated green electricity off all agents in the system a [3].

Definition 1: This maximisation of the utility functions, indicates that solving the optimisation will result in a competitive equilibrium (x^*, λ^*), as agents try to maximise their utility given that the suppliers maximise their profit [3].

With the social welfare problem defined an algorithm, capable of converging to optimal agent demand and price for electricity, can be defined. In order to develop this algorithm, its required to define the

Lagrangian function of the social welfare problem, which is achieved by incorporating the constraint of an optimisation problem within its objective function [9]. Thus, the following Lagrangian function was defined:

$$L = \sum_{i=1}^{N} (\frac{1}{2} q_i x_i^2 + c_i x_i) + \lambda \left(\sum_i x_i - \sum_i a_i \right) \tag{4}$$

Within this function, the desired demand x_i is defined as the primal variable and the market price λ is defined as the dual variable [3]. Primal variables are the decision variables from the objective function, while the dual variable emerges from the constraints of the optimisation problem. With this Lagrangian function and the identification of the primal and dual variable, the primal-dual dynamics were formulated by taking the partial derivatives of the Lagrangian function. This primal-dual algorithm looks as follows:

$$\begin{aligned} \dot{x} &= \frac{\partial L}{\partial x} = -q_i x_i - c_i - \lambda \qquad \forall i \in \{1, \dots, N\} \\ \dot{\lambda} &= \frac{\partial L}{\partial \lambda} = \sum_{i=1}^{N} x_i - a \end{aligned} \tag{5}$$

This algorithm will make sure that the outcome of the social welfare problem 3 will converge to the optimal solution (x^*, λ^*) . However, the market price λ^* at this optimal solution is not always fair and socially acceptable, as the market price can exceed the socially acceptable price λ_{max} of the agents [3]. Consequently, if the optimal solution's market price λ^* exceeds the socially acceptable price λ_{max}, agents might consider opting out of the system, therefore reverting to adopting conventional electricity [3] [8]. However, if the market price is socially unacceptable, it is assumed that the agents in the system are willing to change their utility functions and desired demand x_i such that the market price λ^* remains lower or equal to socially acceptable price λ_{max} [3]. For the utility function, it is assumed that the values of q_i remain constant for each agent. Therefore, a socially acceptable price is only feasible if agents change their desired consumer consumption level c_i [8].

Thus, in order to achieve the market outcome, where λ^* is lower or equal than the maximum price λ_{max}, a controller with input u will be designed. This controller will be able to alter the desired consumption level c_i of the agents, so that the desired demand of the agents changes so that the market price become socially acceptable. [3]. This input u, in the context of the green electricity market will represent conventional electricity in kWh. This input u implies that some agents in the system will change part of their demand to conventional electricity, so that the price can become socially acceptable.

However, changing this demand involves adopting gray electricity rather than green electricity. Thus, it becomes clear that it is crucial for the controller to minimise the input, to make market price socially. This is pivotal, as agents would otherwise increase the emission of greenhouse gases, further aggravating climate change.

VII. Problem Statement

With the problem context and the system analysis elaborated, a problem statement was constructed to clearly summarize the problem.

"The competitive equilibrium in market dynamics, can result in prices that exceed socially acceptable thresholds, leading to affordability concerns and unequal access to green energy sources. This disparity between equilibrium pricing and socially acceptable thresholds poses a significant challenge within the electricity market, requiring the implementation of a controller, which ensures that market prices remain within socially acceptable boundaries."

VIII. Research Objective

Using the theory of SMART objectives , combined with the problem statement, a corresponding research objective has been defined [10].

"The objective of this research is to design a control system in MATLAB for the social welfare problem, ensuring that the competitive equilibrium price will be socially acceptable. Additionally, the short-and long-term impacts of this control system on consumers, the market and society will be evaluated under different scenarios.

IX. Research Questions

In order to effectively carry out the research, a main research question has been formulated. This main research questions is as follows [11]:

"What are potential optimisation strategies for a control system to maintain the competitive equilibrium price at or below the socially acceptable price within the green electricity market, requiring minimal intervention, and what are the potential short-and long-term impacts of this control system on consumers, market dynamics, and societal well-being, considering different scenarios and sensitivities?"

The main question has been divided into three smaller sub-questions that will help to answer the main research question, as well as guide the research towards the desired research design and deliverable.

1) *What are potential design options for a control system so that the market price at competitive equilibrium is always socially acceptable with minimum intervention?*
2) *What impacts will the designed control system have on the demand and price of green electricity, compared to the same scenario without the controller?*
3) *What are the short-term and long-term impacts of the control system on consumer behavior, market dynamics, and societal well-being, considering various potential scenarios?*

X. Social welfare problem in MATLAB

In order to model the complex dynamics of the social welfare problem in MATLAB, it is necessary to rewrite the primal-dual algorithm in (5) in vector form. This vector form is as follows:

$$\begin{aligned} \dot{x} &= -Qx - c - \mathbf{1}\lambda \\ \dot{\lambda} &= \mathbf{1}^T x - a \end{aligned} \tag{6}$$

With Q being a diagonal $n \times n$ matrix with the q_i values of the agents on its diagonal, $\mathbf{1}$ is a $n \times 1$ vector of ones, and $\mathbf{1}^T$ is the transpose this ones vector. c is represented as a $n \times 1$ vector of the agents desired consumption level (c_i), and finally, a is a scalar representing the total generated green electricity of each agent in the system. The primal variable in this system is x, representing the demand profile of all the agents within the system and is expressed as a $n \times 1$ vector consisting of values x_i [3]. Lastly, the dual variable λ represents the market price for green electricity and is a scalar of size 1×1.

With the primal dual algorithm defined, it becomes necessary to rewrite the primal-algorithm dual algorithm in (6) in a state-space representation. This state-space representation has the following definition: "A state pace representation is a mathematical model of a physical system expressed as a function of input, output and state variables related by first-order differential equations" [12]. This representation provides a compact and systematic representation of the systems dynamics, which helps in further designing the control system [12].

By rewriting the primal-dual algorithm in (6), the following state-space representation was constructed in equation 7:

$$\dot{y} = \begin{bmatrix} -Q & -\mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} y + \begin{bmatrix} -c \\ -a \end{bmatrix} \tag{7}$$

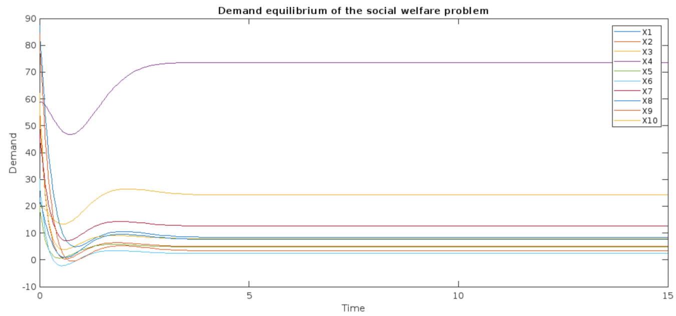
With y being defined as $\begin{pmatrix} x \\ \lambda \end{pmatrix}$, in which x is the demand profile of the agents in the system and λ is the market price for green electricity.

The state space representation above was then implemented within the "socialwelfare" function in MATLAB (See appendix (A)). This state space is solved by using the 'ode45' function in MATLAB, which is able to solve nonstiff differential equations [13]. This mathematical function describes how a system changes over time based on the rates of change of its variables (in this case, x and λ) [13]. In this research, the ode45 is used to model how primal-dual algorithm of the social welfare problem(5), convergences to its optimal agent demand x and market price λ^*.

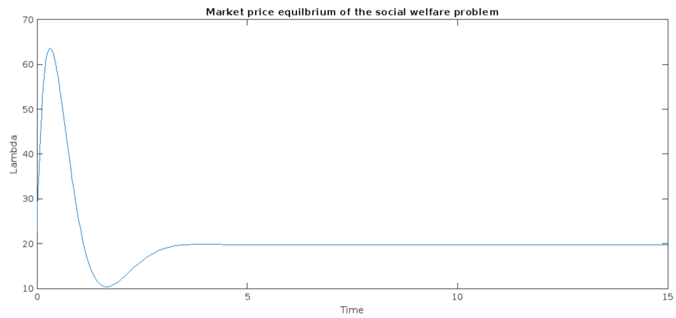
A. *Social welfare problem simulation*

With the 'Socialwelfare' function and 'ode45' implemented in MATLAB, it is now possible to simulate the social welfare problem. However, before simulating, it is necessary to define important parameters. The number of agents, participating in the green electricity market, was set to be equal to 10. For these 10 agents, random values for q_i and c_i were assigned to determine matrices Q and c, respectively. Its important to note that the values of q_i are diagonalised using the 'diag' function in MATLAB [13]. Random values (dummy data) were used for q_i and c_i since obtaining real-life data on consumers utility function parameters proved to difficult. The same principle of randomness is used for defining the initial conditions $y_0 = \begin{pmatrix} x_0^T & \lambda_0 \end{pmatrix}^T$. These random values or dummy data do not contain any real information, but rather serves as a placeholder in order to simulate the behavior of a system without needing meaningful data [14].

The final parameter that needs to be defined, is the total generated electricity a. For this work, it is assumed that each agent in the systems on average owns 3 solar panels. Therefore, assuming that each solar panel generates an average of 5kWh of electricity, the total generated electricity will be equal to $10 \times 3 \times 5 = 150$kWh if there are 10 agents in the system.
With the parameters defined, the social welfare problem was simulated over a time span of 15 seconds. This simulation was plotted in MATLAB, resulting in Figures 4a and 4b.



(a) Demand equilibrium of the social welfare problem



(b) Market price equilibrium of the social welfare problem

Fig. 4: Social welfare problem simulation (See appendix E)

In figure 4a and 4b, it can be seen that the social welfare problem quickly converges to its optimal solutions, representing the optimal demand and market price. Important to note is that the controller only determines the optimal demand and market price for one point in time. Looking at figure 4a, it can be observed that agent 4 has the highest demand for green electricity at 73 kWh. This demand represents around 50% of the total generated green electricity within the system. Other agents in the system have significantly lower demand, as most agents want to consume less than 20 kWh. This difference in consumption can be explained considering the utility function and demand profile of each agent. The demand of each agent is dependant on their values c_i and q_i. From the simulation it can be identified that, agents with low values of q_i and high values of c_i tend to have the highest demand. Thus, it can be concluded that, a decrease in q_i increases demand, and an increase in c_i also results in a higher demand. Normally, evaluating the agents initial demand values would be important. However, with the same values of q_i and c_i, the algorithm (5) always converges to the same optimal demand and market price.

When looking at the market price in figure 4b, it can be seen that, considering the demand profiles of the consumers, the equilibrium price settles around 20€/kWh. In this system, there is no controller capable of changing demand and market price. Consequently, this simulation results in a relatively high market price. This market price might not fall within socially acceptable thresholds, which is up to the agents to decide.

Lets assume the scenario where agents collectively decide on a socially acceptable price of 10€/kWh. Agents will not accept the current electricity price and might chooses to exit the green electricity market [3]. Leaving the system could lead in generated electricity going to waste, reduced competition and even higher prices for green electricity. While the market price in figure 4b might exceed the assumed socially acceptable price of 10€/kWh, this does not mean that every market outcome will exceeds this price, which is further confirmed by the graph in figure 5

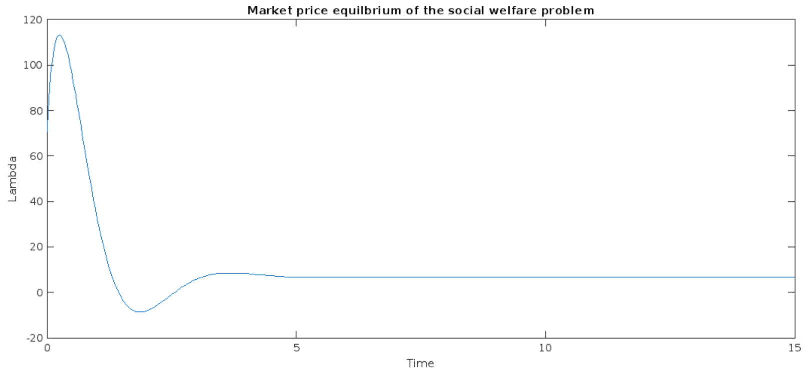


Fig. 5: The equilibrium price does not exceed socially acceptable thresholds

In this simulation, with completely different values for c_i and q_i, the market price is significantly lower at around 4 €/kWh. This indicates that in the case that the market price is lower than the socially acceptable price at 10€/kWh, agents will accept the market price and adopt green electricity for their end use without leaving the green electricity market.

XI. Control system design

From simulations in the the last, it has become clear that the market price of the social welfare problem is indeed not always fair and acceptable. Agents in the system might then decide to exit the green electricity market and seek conventional electricity as an alternative [3][6]. Furthermore, agents exiting the system might result in less competition and even higher market prices, leading into a potential cascade effect with more agents exiting the system [15].

Thus, a control system influencing consumer preference and desired demand in order to lower the market price is required. This control system will influence demand through changing the consumer desired consumption level c_i through the input u. Thus, for the primal-dual algorithm of the social welfare problem (5), an input u is added, resulting in the following adjusted algorithm:

$$\begin{aligned} \dot{x} &= -Qx - c - \mathbf{1}\lambda - u \\ \dot{\lambda} &= \mathbf{1}^T x - a \end{aligned} \tag{8}$$

The goal of ensuring that the market price is within socially acceptable thresholds is thus achieved by increasing input u to change agent desired demand. However, in the context of the green electricity market, this input u is represented by conventional electricity. The use of conventional electricity results in pollution and greenhouse gas emission, which worsens climate change. Therefore, the task of the control system is not only to ensure that the market price becomes socially acceptable but also minimise input u.

Summing up both problems, the controller will have to meet the following two constraints.:

1) $\lambda^* \leq \lambda_{max}$
2) Input u needs to minimised

With the two conditions for the controller clearly defined, it is thus possible to formulate another optimisation problem. This problem involves minimising the input u and ensuring that the market price λ^* is lower equal to the maximum price λ_{max} at steady state. Additionally, the optimisation problem is further constrained by the steady state of the primal-dual algorithm (6). Thus, in order to determine the optimal point for $y = \begin{bmatrix} x^T & \lambda \end{bmatrix}^T$ at steady state the following optimisation problem was formulated:

$$\begin{aligned} \min_{u,x,\lambda} \quad & \frac{1}{2}\|u\|^2 \\ \text{subject to} \quad & \lambda \leq \lambda_{\max}, \\ & Qx + (c+u) + \lambda \mathbf{1} = 0, \\ & \mathbf{1}^T x = a. \end{aligned} \tag{9}$$

The optimisation problem above is strictly convex, because of the quadratic objective function $\frac{1}{2}||u||^2$. This strict convexity indicates that the optimisation problem will have a unique solution ([16]). This convexity of the problem ensures the predictability of the objective function, making suitable for optimisation. Furthermore, the presence of a unique solution also indicates that the objective function will be minimised given the constraints, resulting in one optimal solution rather than multiple optimal solutions [16]. These properties help simplify the optimisation, as algorithms can reliably converge to the unique solution.

A. Lagrangian function of the control system

With the optimisation problem above, it is possible to define the Lagrangian function of the system . This Lagrangian function is obtained by incorporating the constraints within the objective function, which has resulted in the following function [9]:

$$L = \frac{1}{2}\|u\|^2 + \mu(\lambda - \lambda_{\max}) + \pi^\top (Qx + (c+u) + \lambda \mathbf{1}) + \nu \mathbf{1}^\top x \tag{10}$$

where μ, π, and ν being the dual-variables, which are used to penalises deviations of the defined constraints.

B. Karush-Kuhn-Tucker conditions

With the Lagrangian function defined, it is possible to form the necessary and adequate KKT conditions ([17]). Karush-Kuhn-Tucker conditions (KKT) are a set of mathematical conditions used to characterise the solutions to constrained optimisation problems [18][17]. These conditions are an extension of the Lagrange multiplier method in order to handle both equality and inequality constraints [19][20]. It is worth noting that for the social welfare problem without the controller, these KKT conditions were not defined, as this system only contained one straightforward equality constraint and no inequality constraints.

The KKT conditions generally consists of four different conditions: Stationary conditions, Primal feasibility, Dual feasibility, and Complementary slackness [20]. These four conditions collectively make up the KKT conditions and are fundamental for determining whether a given solution is optimal in an optimisation problem containing equality and inequality constraints [19][21]. Thus, KKT conditions provide a relationships between the objective function, constraints, and dual variables that must be satisfied at an optimal point.

1) Stationary conditions: The first KKT condition is the stationary condition, which involves the concept that at an optimal solution the gradient of the objective function is equal to zero [20]. Thus, for the stationary condition it pivotal that the sum of the changes in the objective function and constraints are equal to zero [21]. This condition is used to ensure that there is no feasible direction that could improve the objective function [20]. This is important as it identifies were potential minimums or maximums establish.

In summary, the stationary condition of the KKT conditions pinpoints potential optimal solutions by ensuring that the objective function is not changing in the direction of decision variables given the constraints.

2) Primal feasibility: The primal feasibility condition ensures that the proposed solution sticks to the equality and inequality constraints and is a statement that ensures that at the optimal condition the constraints are not violated. [20]. Therefore, this conditions ensures that the solution lies within the established feasible area of the constraints. [20][21]. In summary, the primal feasibility ensures that both equality and inequality constraints are satisfied, which is important as the feasibility of the solution is a fundamental requirement for optimally.

3) Dual feasibility: The dual feasibility condition introduces the requirement that the dual variables (Lagrange multipliers) associated with the inequality constraints must be non-negative [21]. This non-negativity is required because negative values for these Lagrange multipliers would result in results making no sense. In summary, the dual feasibility condition prevents the solution from making no sense by ensuring that the Lagrange multipliers will have no negative values.

4) Complementary slackness: The final condition, complementary slackness, states that the inequality constraint of the optimisation problem is either inactive or active, with the associated dual variable being positive [21] [20]. A constraint is considered inactive when the solution lies within the feasible region and is considered active when the solution touches the the boundary of the feasible region [21]. This condition ensures that if the solution is not within the feasible region, the inequality constraints will activate so that the solution will be guided to a solution within the feasible region [20].

C. KKT conditions for control system

With the role of the KKT conditions understood, they can be defined for the langrangian function in equation (10). Defining the KKT conditions is achieved by taking the partial derivatives of the Lagrangian function and setting them equal to zero, which has resulted in the following KKT conditions for the optimisation problem 9 [21]:

$$\begin{aligned}
\frac{\partial L}{\partial u} &= u^* + \pi^* = 0, \\
\frac{\partial L}{\partial x} &= Q\pi^* + \nu^* = 0, \\
\frac{\partial L}{\partial \lambda} &= \mu^* + \mathbf{1}^\top \pi^* = 0, \\
\frac{\partial L}{\partial \pi} &= Qx^* + (c + u^*) + \lambda^* \mathbf{1} = 0, \\
\frac{\partial L}{\partial \nu} &= \mathbf{1}^\top x^* = 0, \\
\\
0 \leq \mu^* &\perp \lambda^* - \lambda_{\max} \leq 0.
\end{aligned} \tag{11}$$

Within this system, the five partial derivatives represent the stationary conditions of the optimisation problem [21]. Additionally, the last row of the system represents the complementary slackness ($0 \leq \mu^* \perp$

$\lambda^* - \lambda_{\max} \leq 0$). The first part of the condition clarifies that μ^* needs to be greater or equal to zero (non-negative) while the second condition $\lambda^* - \lambda_{\max} \leq 0$ indicates that the maximum price always needs to be higher than the market price. These two conditions are orthogonal, indicating an indecent or uncorrelated relationship between them. This relationship of the conditions of μ^* and $\lambda^* - \lambda_{max}$ also indicates that the product of the conditions is equal to zero [20].

The partial derivative of variable u can be rewritten as $u^* = -\pi^*$, and by substituting this into the partial derivative of x the following equation for u^* can be obtained:

$$u^* = -\pi^* = Q^{-1}\mathbf{1}\nu^* \tag{12}$$

Note that given ν^*, it is possible to find u^*. This equation, paired with $u^* = -\pi^*$, is convenient as the KKT-system can be simplified using substitution. This simplified KKT system looks as follows:

$$\begin{aligned} &Qx^* + c + Q^{-1}\mathbf{1}\nu^* + \lambda^*\mathbf{1} = 0, \\ &\mathbf{1}^\top x^* = 0, \\ &0 \leq \nu^* \perp \lambda^* - \lambda_{\max} \leq 0. \end{aligned} \tag{13}$$

D. Connection to the dual program

The goal of restricting the dual variables is strongly connected to the dual problem of an optimisation problem, where the dual variable is considered a decision variable [22]. This connection means that the dual variable is treated as an unknown determined during the optimisation process.

Consider a linear inequality constraint $\lambda^* \leq \lambda_{max}$ that needs to be imposed on the dual variable λ. This inequality constraint will be imposed on the dual variable λ of the dual problem in equation (3). Then, by introducing slack variables, this dual problem can be converted into a primal problem. This converted primal problem looks as follows [22]:

$$\begin{aligned} \min_{x,s} \quad & \frac{1}{2}x^T Qx + c^T x + \lambda_{max} s \\ \text{subject to} \quad & \mathbf{1}^T x - s = a \\ & s \geq 0 \end{aligned} \tag{14}$$

where s is a scalar such that the inequality constraint $\lambda \leq \lambda_{max}$ can be written as a equality constraint.

As the primal problem above involves an inequality constraint, it is once again required to define KKT conditions. These KKT conditions for the primal problem look as follows:

$$\begin{aligned} &Q\bar{x} + c + \mathbf{1}\bar{\lambda} = 0 \\ &\mathbf{1}^T\bar{x} - \bar{s} = a \\ &0 \leq \bar{s} \perp \bar{\lambda} - \lambda_{\max} \leq 0. \end{aligned} \tag{15}$$

With primal problem (14) formulated, the question becomes interesting whether there exists a connection between problem (9) and (14). In other words, how are the optimal solutions of (9) and (14) related to each other? To answer this, the optimality conditions of (13) and (15) need to be compared to each other. It's important to note that $\lambda^* = \bar{\lambda}$, as both parameters indicate the price for green electricity. Thus, it can be stated that: $\lambda^* = \bar{\lambda} \leq \lambda_{max}$. Now, lets assume that $\nu^* = \bar{s} = 0$, then $x^* = \bar{x}$ and $u^* = 0$ are obtained. With these conditions, it is possible to create a map that relates (9) and (14) to each other. This map looks as follows:

$$\begin{bmatrix} Q & 0 \\ \mathbf{1}^T & -1 \end{bmatrix} \begin{bmatrix} \bar{x} \\ \bar{s} \end{bmatrix} = \begin{bmatrix} Q & Q^{-1}\mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \nu \end{bmatrix} \Rightarrow \begin{bmatrix} \bar{x} \\ \bar{s} \end{bmatrix} = M \begin{bmatrix} x^* \\ \nu^* \end{bmatrix} \tag{16}$$

With the map defined, the matrix M could be determined by multiplying the inverse of $\begin{bmatrix} Q & 0 \\ \mathbf{1}^T & -1 \end{bmatrix}$ with the matrix $\begin{bmatrix} Q & Q^{-1}\mathbf{1} \\ \mathbf{1}^T & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \nu^* \end{bmatrix}$. Thus, matrix M was defined as:

$$\begin{bmatrix} I & Q^{-2}\mathbf{1} \\ \mathbf{0} & \mathbf{1}^T Q^{-2}\mathbf{1} \end{bmatrix} \tag{17}$$

Here, I is the identity matrix of size $n \times n$, and $\mathbf{0}$ is a $1 \times n$ matrix of zeros.

E. Designed control system

This section, will expand further upon the last section by introducing the control system. The control system is obtained by using (16) as the change of variables formula in the optimisation problem (14) together with (12) [23]. The result is the definition of the following optimisation problem for the controller:

$$\begin{aligned} \min_{x,u,\nu} \quad & \frac{1}{2}x^\top Qx + (c+u)^\top x + \frac{1}{2}u^\top Q^{-1}u + (c+\lambda_{max}\mathbf{1})^T Q^{-1}u \\ \text{s.t.} \quad & \mathbf{1}^\top x = a, \\ & Qu = \mathbf{1}\nu, \nu \geq 0. \end{aligned} \tag{18}$$

The objective function of the minimisation problem will be minimised given the two constraints in (18), enabling the controller algorithm to converge to the optimal minimum. This optimal minimum is reached while ensuring that the constraints of market price needing to be socially acceptable ($\lambda \leq \lambda_{max}$) and minimising input u are satisfied.

Looking at (18), it becomes clear that there are no inequality constraints in the minimisation problem. Thus, it becomes possible to define the Lagrangian functions of the controller. This Lagrangian function is defined by incorporating the equality constraints in (18) into the objective function of the optimisation problem [9]. Consequently, the following Lagrange function could be defined:

$$\begin{aligned} L = & \frac{1}{2}x^T Qx + (c+u)^T x + \frac{1}{2}u^T Q^{-1}u + (c+\lambda_{\max}\mathbf{1})^T Q^{-1}u \\ & + \lambda\mathbf{1}^T x + \pi^T(Qu - \mathbf{1}\nu) - \mu^T \nu \end{aligned} \tag{19}$$

This Lagrangian function can then be solved by taking the partial derivative of the primal and dual variables x, λ, u, π, ν and μ. These partial derivatives have resulted in the following algorithm for the controller, allowing the algorithm to converge towards a solution a market outcome were the market price is lower or equal to the socially acceptable price:

$$\begin{aligned} \dot{x} &= -Qx - c - \mathbf{1}\lambda - u \\ \dot{\lambda} &= \mathbf{1}^T x - a \\ \dot{u} &= -Q^{-1}u - x - Q^{-1}(c+\lambda_{\max}\mathbf{1}) - Q\pi \\ \dot{\pi} &= Qu - \mathbf{1}\nu \\ \dot{\nu} &= \mathbf{1}^T\pi + \mu \\ \dot{\mu} &= [-\nu]^+_\mu \end{aligned} \tag{20}$$

Here, λ, π and μ are dual variables. The term $[-\nu]^+_\mu$ will be further explained by rewriting it as follows:

$$[-\nu]^+_\mu = \begin{cases} -\nu & \text{if } \mu > 0 \\ \max\{0, -\nu\} & \text{if } \mu = 0 \end{cases} \tag{21}$$

From the expression above, it becomes clear that if $\mu > 0$, then $\dot{\mu}$ will be equal to $-\nu$. Moreover, if the value for μ is equal to zero, then the following conditions holds: $\max(0, -\nu)$. This conditions indicates that if the value of $-\nu$ is either positive or zero, then the term will take the value zero. However, if the value of $-\nu$ is negative, then the term will take the value $-\nu$. In order to better understand the expression $\max(0, -\nu)$, it can be rewritten as follows:

$$\max\{0, -v\} \begin{cases} 0 & \text{if } -v \leq 0 \\ -v & \text{if } -v > 0 \end{cases} \tag{22}$$

From these mathematical expressions, it becomes clear that there are three possible modes, namely:

1) $\mu > 0$
2) $\mu = 0$ and $-\nu \leq 0$
3) $\mu = 0$ and $-\nu > 0$

From the mathematical expression (21) and (22), it shows that mode 1 and 3 are then the same. Thus, for the simulations there will only be two different systems. The system of mode 1 & 3 will have $\dot{\mu}$ being equal to $-\nu$, while system 2 will have $\dot{\mu}$ being equal to zero. These two systems will look as follows:

System 1

$$\begin{aligned}
\dot{x} &= -Qx - c - \mathbf{1}\lambda - u \\
\dot{\lambda} &= \mathbf{1}^T x - a \\
\dot{u} &= -Q^{-1}u - x - Q^{-1}(c + \lambda_{\max}\mathbf{1}) - Q\pi \\
\dot{\pi} &= Qu - \mathbf{1}\nu \\
\dot{\nu} &= \mathbf{1}^T \pi + \mu \\
\dot{\mu} &= -\nu
\end{aligned} \tag{23}$$

System 2

$$\begin{aligned}
\dot{x} &= -Qx - c - \mathbf{1}\lambda - u \\
\dot{\lambda} &= \mathbf{1}^T x - a \\
\dot{u} &= -Q^{-1}u - x - Q^{-1}(c + \lambda_{\max}\mathbf{1}) - Q\pi \\
\dot{\pi} &= Qu - \mathbf{1}\nu \\
\dot{\nu} &= \mathbf{1}^T \pi + \mu \\
\dot{\mu} &= 0
\end{aligned} \tag{24}$$

.

F. Control system in MATLAB

With the two systems defined, it is now possible to start implementing the controller algorithm into MATLAB. However, for the implementation, it is important to define the state-space representations of the two systems.

The state spaces of these two systems are defined as follows:

System 1 for mode 1 & 3

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \\ \dot{u} \\ \dot{\pi} \\ \dot{\nu} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -Q & -\mathbf{1} & -I & 0 & 0 & 0 \\ \mathbf{1}^T & 0 & 0 & 0 & 0 & 0 \\ -I & 0 & -Q^{-1} & -Q & 0 & 0 \\ 0 & 0 & Q & 0 & -\mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1}^T & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \\ u \\ \pi \\ \nu \\ \mu \end{bmatrix} + \begin{bmatrix} -c \\ -a \\ -Q^{-1}(c+\lambda_{max}\mathbf{1}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{25}$$

System 2 for mode 2

$$\begin{bmatrix} \dot{x} \\ \dot{\lambda} \\ \dot{u} \\ \dot{\pi} \\ \dot{\nu} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -Q & -\mathbf{1} & -I & 0 & 0 & 0 \\ \mathbf{1}^T & 0 & 0 & 0 & 0 & 0 \\ -I & 0 & -Q^{-1} & -Q & 0 & 0 \\ 0 & 0 & Q & 0 & -\mathbf{1} & 0 \\ 0 & 0 & 0 & \mathbf{1}^T & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \lambda \\ u \\ \pi \\ \nu \\ \mu \end{bmatrix} + \begin{bmatrix} -c \\ -a \\ -Q^{-1}(c+\lambda_{max}\mathbf{1}) \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{26}$$

Where x, u, and π are $n \times 1$ matrices, while λ, ν and μ are 1×1 matrices. These matrices imply that matrix

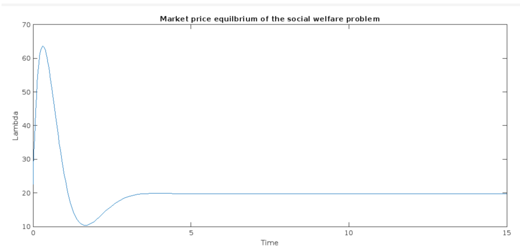
y will consist of 1 column and $3n+3$ rows, which is based on the number of agents in the system. The matrices for Q, I and **1** are the same as in the social welfare problem. The current system, as represented in the state-spaces, are not consistent with the dimensions. Therefore, multiple zero matrices were introduced so that both row and column are of size $3n+3$.

With the state-space representations of both systems presented, it becomes important to adjust the MATLAB function so that when $-\nu \leq 0$, system 2 will be simulated and when $-\nu > 0$ system 1 will be simulated. This was achieved by introducing a "ifelse" function into the MATLAB code. Thus if $\mu > 0$ then the "ode45" will integrate system 1 following: $dy = A_1 * y + B_1$ (see appendix C). Otherwise if $\mu = 0$ then either system 1 or 2 has to be simulated based on the value of ν. If $-\nu > 0$ then again system 1 will be simulated, however if $\nu \leq 0$ then system 2 will be simulated using: $dy = A_2 * y + b_2$ (see appendix C).

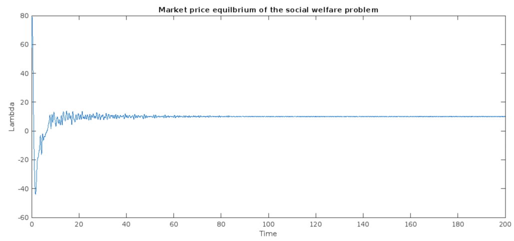
XII. Control system simulation

For the first simulation of the control system, the same random values were assigned for q_i, c_i and y_0. Furthermore, the maximum price was set to be equal to 10€ per kWh and the total generated green electricity was again set to 150 kWh. Important to note is that because of economic intuition only simulations were used where the demand of each agent is larger than zero ($x_i > 0$). This intuition was used as it is not possible for agents to have a negative amount of electricity demand. With the parameters defined, it was possible to simulate the control system over a time-span of 200 seconds.

This simulation has resulted in a price graph which can be seen in figure 6b below.



(a) Market price without control system

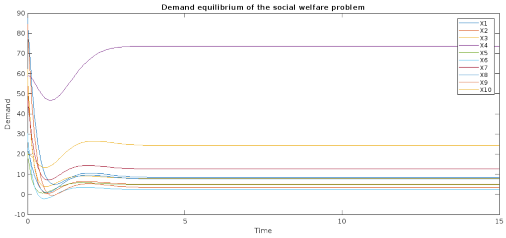


(b) Market price with the designed control system (See appendix F)

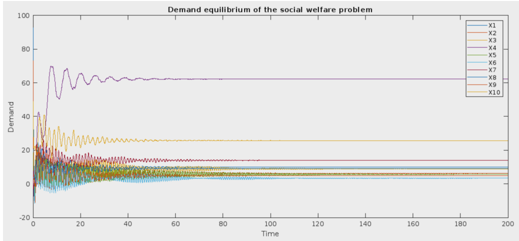
Fig. 6: Market price with and without controller

In figure 6a and 6b the difference between the social welfare problem with and without the controller can be seen. When looking at the graphs, it can be concluded that the system with the controller takes longer to reach equilibrium. This difference can be explained, as the newly developed control system algorithm has a lot more inputs and outputs, which all contribute to the longer convergence until the optimal demand and market price are determined. Furthermore, in figure it can be seen that the market price exceeds the maximum price in figure 6a , while in the controller simulation in 6b , the maximum price is capped at 10 €/kWh. Thus, the control system ensures that the market price will be lower or equal to the socially acceptable price λ_{max}. The developed algorithm accomplishes this by changing the agent desired consumption level so that agents with a high demand, switch to conventional electricity u. However, as supply needs to be equal to demand, agents with low demand will increase their demand, which leads to the leveling of desired consumer consumption level and demand. Important to note is that a high desired consumer consumption level, with respect to other agents, leads to high market prices. Therefore, changing the demand allocation will influences market dynamics so that market price will become lower and socially acceptable.

In this case where the consumer preference is to consume significant amounts of green electricity, occur until settling at a price of 10 €/kWh. This occurrence can be explained by looking at the dynamics of the controller and graph. At $t = 10s$ the market price exceeds the maximum price of 10 €/kWh, which implies that at this point the input u of the controller influences the desired consumption level c_i of the agents such that their demand changes. Due to their decrease of desired consumption level, agents with excessive demand, will revert part of their demand to conventional electricity. Therefore, other agents with low demand can increase their self-consumption, resulting in the change of market dynamics so that the price will decrease. The market price will then drop below the socially acceptable price, giving the agents the incentive to again increase their demand, which will result in a increase in market price. This oscillation goes on until the equilibrium price is set at 10€ per kWh as no agent wants to increase or decrease their demand at this equilibrium price.



(a) Agent demand without control system



(b) Agent demand with the designed control system (See appendix F)

Fig. 7: Agent demand with and without control system

In figure 7a and 7b the demand of the agents with and without the control system can be seen. In the control system simulation it can be seen that agent 3 and 4 reduce their consumption from 73 kWh and 25 kWh to 62 kWh and 24 kWh respectively. This difference implies that agent 3 and 4 will consume 11 and 1 kWh units of conventional electricity respectively. This decrease in green electricity, implies that other agents in the system are required to increase their demand. Therefore, agents with a low demand have increased their own demand, which can concluded by comparing the demand of agent 7 and 8 in figure 7a and 7b. These agent both have seen an increase in consumption of around 4 kWh and 2 kWh. This change in demand, will change the market dynamics so that the market price becomes socially acceptable.

XIII. Scenario simulations & Sensitivity analysis

In this section of the paper, comprehensive simulations will be performed, using the designed control system implemented within MATLAB. These simulations are performed to evaluate how different scenarios affect the outcome of the control system. The main objective is thus to analyse and compare the outcomes of these simulations, highlighting the impacts that these scenarios will have on the controllers outcomes and performance.

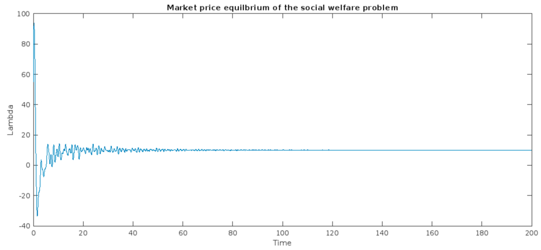
The simulations will revolve around altering important parameters whilst keeping other variables constant. By only changing one parameter it is possible to identify what effects it will have on the outcome and performance of the controller. Through this process it is possible to gain insights into the control systems performance, stability and sensitivity to change.

A. Case 1: Total generated green electricity

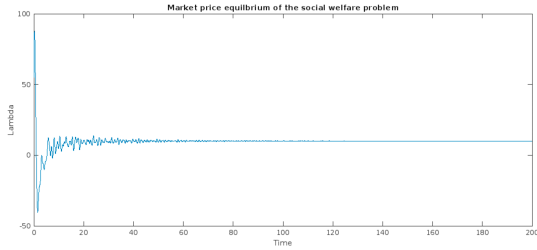
The first scenario analysis will involve a quantity analysis, which means that during these simulations only the quantity of green electricity generated (a) will be adjusted. For the other important parameters like maximum price and consumers desired consumption level dummy data is used (Different from dummy data in section XII. Throughout the simulations this dummy data will be held constant. This approach enables the comparison between the different simulations, which will aid in drawing a conclusion regarding the influence the total generated green electricity will have on demand and market dynamics.

For these simulations it was decided to perform three different simulations with the total generated green electricity a taking the values of 125 kWh, 150 kWh and 175 kWh. A difference of 25 kWh was chosen, as these changes are large enough to identify the changes in price and demand clearly.

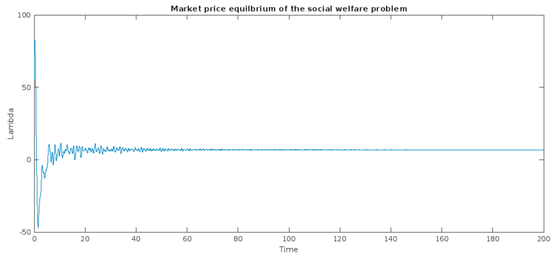
These simulations have resulted in the following market price equilibrium graphs:



(a) a = 125 kWh



(b) a = 150 kWh



(c) a = 175 kWh

Fig. 8: Market price at different green electricity supply values (a) (See appendix G)

In figure 8 three simulations using different values for *a* can be seen. The simulation with 150 kWh of green electricity is used as the reference point, as this standard value was also used in earlier simulations. The market prices were summarised in the following table:

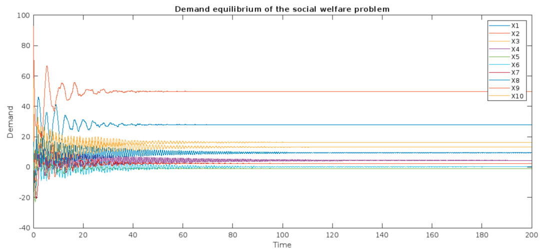
Table 1: Market price at different values of *a*

a = 125 kWh	a = 150 kWh	a = 175 kWh
10 €/kWh	10 €/kWh	6,67 €/kWh

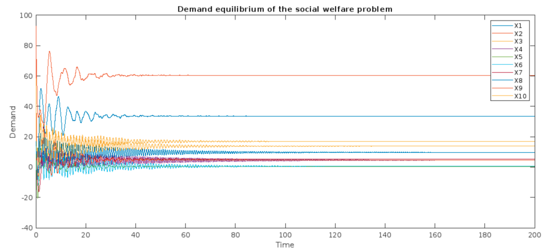
From the table above and figure 8a it can be concluded that reducing the total generated electricity does not affect the maximum price. Without the controller reducing the supply would, have resulted in a increased market price, as supply would be lowered. However, the controller is designed to cap the market price at 10 €/kWh, which implies that it reduce demand such that price becomes socially acceptable. In contrast to lower supply, increasing the electricity supply results in a decrease in market price, as can be seen in figure 8c. In this figure the market price has been decreased to 6,67 €/kWh. This decrease can be explained by the fact that there is more electricity demand and supply, which increases the competitiveness of the market, resulting in a decrease in market price [15]. In this case the controller is not required to adjust agent demand, as the price will already be within socially acceptable thresholds.

However, changing the total green electricity supply does not only affect the market price, but also the demand, as can be seen in figure 9.

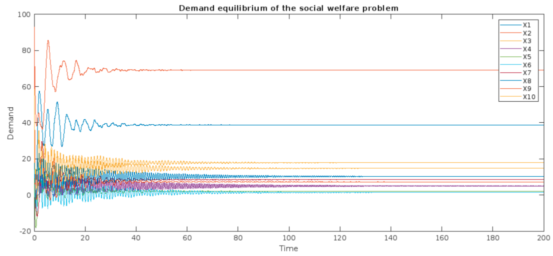
From these three figures in 9 it can be concluded that increasing the total green electricity supply will also increase the demand of the agents. This increase in demand occurs because supply has to match the demand, otherwise generated electricity will go to waste. Furthermore, reducing the total green electricity supply will result in agents changing their demand, so that the market price will not exceed socially



(a) Demand for a = 125 kWh



(b) Demand for a = 150 kWh



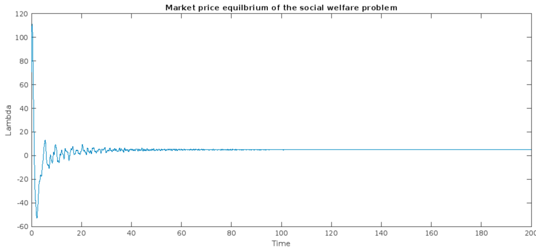
(c) Demand for a = 175 kWh

Fig. 9: Agent demand at different green electricity supply values (a) (See appendix G

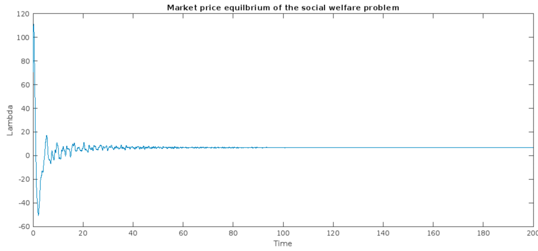
acceptable thresholds. In figure 9a however, the demand of some agents becomes negative. Following economic intuition This is not possible as agents can not consume a negative amount of electricity. This is a limitation of the controller, which can maybe be resolved by adjusting the controller so that if the demand of an agent becomes zero, the agent will leave the system.

B. *Case 2: Maximum price (λ_{max})*

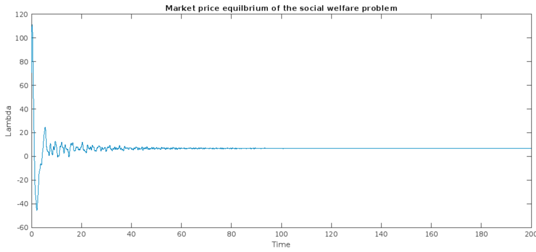
In the second scenario, the maximum price will be changed while keeping the other parameters constant. This case will give insights on how the maximum price will influence agent demand and market price. For this simulation the total generated green electricity will be kept constant at 150 kWh. A total of three simulations were performed with λ_{max} taking the values 5€/kWh, 10€/kWh and 20€/kWh. This has resulted in the following graphs:



(a) λ_{max} = 5€/kWh



(b) λ_{max} = 10€/kWh



(c) λ_{max} = 20€/kWh

Fig. 10: Market price at different maximum prices

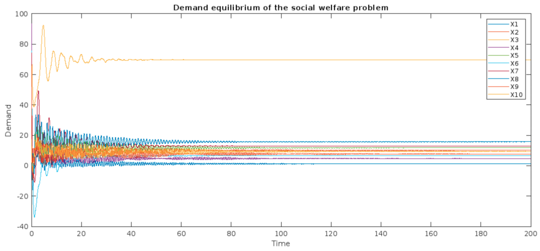
Again three different simulations have been performed, where each simulation has its own maximum price. The market prices were summarised in the following table:

Table 2: Market price at different maximum prices λ_{max}

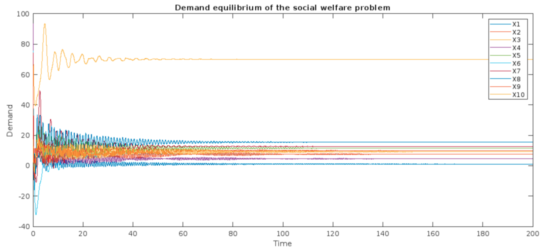
λ_{max} = 5 €/kWh	λ_{max} = 10 €/kWh	λ_{max} = 20 €/kWh
10 €/kWh	6,67 €/kWh	6,67 €/kWh

From the table above and figure 10b it can be concluded that the market price lies below the maximum price of 10 €/kWh at 6,67 €/kWh. Increasing the maximum price to 20€/kWh has no effect on the price, as the price will remain in at 6,67 €/kWh. The market price stays within socially acceptable thresholds, which means that the controller is inactive as demand is not required change. However, in the scenario where the maximum price is set to 5€/kWh, the current price of 6,67 €/kWh is not socially acceptable by the agents. Therefore, the controller regulates the demand of the consumers so that the price caps at the maximum price, as can be seen in figure 10a.

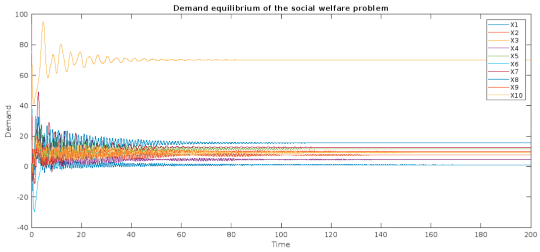
The price in figure 10b and 10c are both below the maximum price at 6,67 €/kWh. This means that the agents will have the same demand in both scenarios, which also becomes evident from figure 11b and 11c below. However, in figure 11a, with maximum price 5€/kWh, it becomes apparent that the demand of the agents is changed. This change in demand is required as otherwise the price would exceed socially acceptable at 6,67€/kWh.



(a) λ_{max} = 5€/kWh



(b) λ_{max} = 10€/kWh



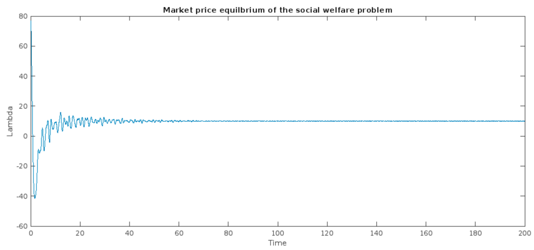
(c) λ_{max} = 20€/kWh

Fig. 11: Agent demand at different maximum prices

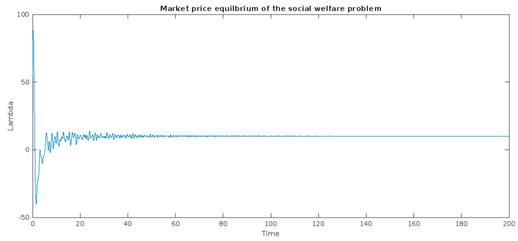
C. Case 3: Number of agents (n)

The third scenario which will be simulated, is the change in number of agents. This scenario will assess how agents entering or leaving would affect the market price and demand for green electricity. Three simulations will be performed with 8, 10 and 12 agents operating in the system. Furthermore, the total green electricity supply in the system will be based on the number of agents. One agent on average generates 15 kWh of electricity, which means that the total green electricity supply will be 120, 150 and 180 kWh respectively.

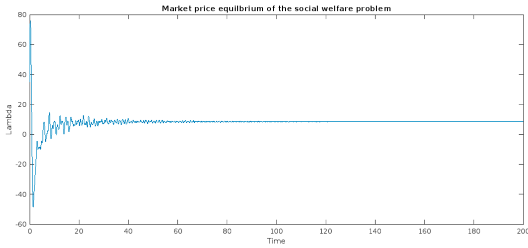
Simulating these three scenarios has resulted in the following market price graphs:



(a) n = 8 agents



(b) n = 10



(c) n = 12

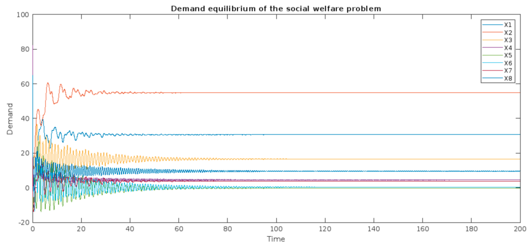
Fig. 12: Demand at different number of agents prices

In figure 12 the market price for different number of agents can be seen. These market prices are summarised in the following table:

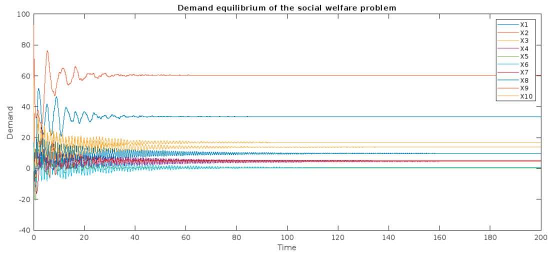
Table 3: Market price for different number of agents *n*

n = 8	n = 10	n = 12
10 €/kWh	10 €/kWh	8,56 €/kWh

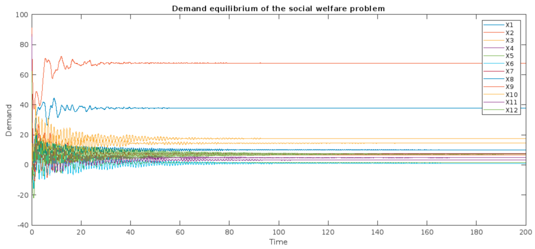
For 8 and 10 agents the maximum price caps at 10€/kWh, which means that regarding the market price little difference occur. However, with 12 agents, the market price drops to around 8,56€/kWh. This decrease in market price can be explained by using the competitiveness of the market. An increased number of agents will increase demand and supply making the market more competitive. Agents will have more options to choose from as there are is more supply, resulting in a downwards pressure on the price [15].



(a) n = 8 agents



(b) n = 10



(c) n = 12

Fig. 13: Demand at different number of agents prices (see appendix G)

Looking at figure 13 it can be seen that each scenario has a different green electricity allocation. When comparing figure 13b with 13a it becomes clear, that when agents leave the system that the reduction in green electricity affects agents with a high demand the most. This is shown in figure 13b and 13a, as agent 2 demand is decreased from 60 kWh to 55 kWh. The reverse is true for the scenario when agents enter the market, as now the demand of agent 2 increases from 60 kWh to 68 kWh. Thus it can be concluded that agents leaving or entering the system, will affect agents with high demand the most.

XIV. Impact analysis

By utilising the outlined control system in section XI and the scenario simulations in section XIII, an impact analysis can be performed. This impact analysis will evaluate the effects and consequences of the implementation of the control system on consumers, the market, and society. This will allow for insights on whether the implementation of such a control system can become feasible in the future. The analysis will be performed by first identifying what short-term and long-term impacts the controller will have on the system.

A. *short-term impacts*

1) *Affordability and protection*
The controller was specifically designed to ensure that electricity prices are always within socially acceptable thresholds. This indicates that implementing the controller within the green electricity market will ensure protection of consumers, as green electricity will always be within affordable thresholds [24]. Ensuring the affordability of green electricity implies that a large proportion of the population will have access to green electricity, which, in turn, stimulates the fairness and decreases difference in quality of life [24]. Furthermore sudden spikes in electricity price are prevented, resulting in more reliable prices for green electricity [25]. From more reliable prices it can therefore

be concluded that the market becomes more stable.

2) *Consumer confidence*

Additionally, with more reliable prices the confidence of consumers in the electricity market will increase [26]. This confidence among consumers often results in increased spending and investment, which in turn stimulates the economic growth of the electricity market [26]. Stimulating the growth of the green electricity market is important, as more green electricity consumption results in less pollution and green house gases emission, improving worldwide climate change.

3) *Constrained flexibility*

The introduced control system is able to regulate demand so that the green electricity price becomes socially acceptable. This regulation implies that when price is above socially acceptable thresholds, consumer preference will be influenced in such a way that the demand for green electricity will be reduced. This influence can result in consumers feeling constrained in their ability to adjust their consumption, based on their needs and revert to consuming conventional electricity. Demand flexibility in the electricity market is crucial, as it can adopt to abrupt and sudden changes in the supply-demand balance [27]. Thus, a lack of flexibility can result in dissatisfaction among consumers, which are keen on having more control over their electricity consumption.

4) *Market stability*

Another meaningful impact of the control system on the electricity market that was identified is market stability [25]. Market stability that there are little changes in market price over a period of time. This ensures, that the prices become more predictable allowing consumers to better predict and plan expenses. Most consumers prefer reliable prices and are risk averse, which therefore can increases the number of consumer participating in the market [25].

5) *Market dynamics*

The regulation of the controller heavily influences market dynamics. Due to pricing signals it is possible for the controller to incetivise consumers to change their demand, so that prices become socially acceptable. These pricing signals can optimise green electricity and renewable energy sources, as it can improve load balancing, as the controller ensures that supply is equal to demand. Load balancing is crucial for renewable energy sources, as these sources are intermittent and weather dependant.

B. ***long-term impacts***

1) *Investment in renewable energy*

As the market and market price become more reliable and stable long-term, investment in renewable energy infrastructure will be encouraged because these investment will become less risky [28]. However, if the capping prices are to low, the controller might discourage investors, as investors might anticipate limited returns from their investments. Thus, a fine line is required to be found, were socially acceptable prices are high enough for encouraging investment in renewable energy projects.

2) *Consumption patterns*

The controller can also stimulate consumers to have more consistent electricity consumption pattern. Developing these long term consumption patterns, is crucial in a society intermittent green energy sources for generation of electricity become more important. These consumption patterns include, balancing demand of green electricity during peak and off-peak demand, so that the grid is balanced throughout the day.

3) *Environmental Impact*

As the confidence in the market increases more agents will enter the market either as an investor or consumer [26]. This growth of the green electricity market will result in reduced reliance on conventional energy sources for electricity generation. Furthermore the increased use of green electricity will have positive effects on our surrounding environments, as greenhouse gas emission will be reduced. This reduction means that worldwide climate change will improve, thus contributing to worldwide sustainability goals.

From the short and long-term impact analysis it has becomes clear that the controller can have positive influences on the consumers, market and society. The implementation of the controller ensures affordable green electricity, promoting social fairness. It prevents sudden spikes, improving consumer confidence and stimulating economic growth, through increase investment. However the control may limit consumer consumption flexibility, which can result in consumers opposing the implementation of such a control system. However in the long run, the controller might be beneficial as it can encourage investments paired with market growth, creates consumption patterns, which positively affects the environment by improving the the transmission from conventional to renewable energy sources.

XV. Conclusion

In this work, a control system for capping the market price in green electricity market is designed, the green electricity market often deals with high electricity prices which may exceed the consumer's affordable and acceptable thresholds, resulting in consumers reverting to conventional, non-renewable energy sources. This designed controller is able to ensure socially acceptable price by changing the demand of the agents. This socially acceptable price is achieved by reallocating electricity from agents with high demand to agents with lower demand. This reallocation ensures a more equitable allocation of agent demand, leading to a fairer and lower market price. Furthermore, a sensitivity analysis was performed on the control system to assess the performance of the controller under different scenarios. From this analysis, it has become clear how changing the total green electricity, maximum price and number of agents can affect the demand and price outcomes of the controller.
Finally, the short and long-term impacts of the controller on consumers, the market and society were evaluated. It has been concluded that the controller can improve the green electricity market and protect consumers from paying prices that are to high prices. Furthermore, the implementation improves market stability, which may lead to increased investment and market growth. Therefore, further research on the implementation of this control system becomes interesting, considering the potential positive impacts on the market and green electricity consumption.

XVI. Discussion

The final section of this research will identify limitations of the controller, while also identify how future research could improve upon this control system.

One of the most important discussion points is that the system assumed only consists of green electricity supply and demand. In the real-life such an electricity market does not exist, as the grid is balanced by both green and gray electricity. Therefore, to further expand the controller, demand and supply for gray electricity can be included in the algorithm, which would result in a more accurate representation of the electricity market. Additionally, this research makes use of dummy data which acts as a placeholder for real data. Therefore, using this type of data can result in a lack of real world representation. Thus, it is not known whether the implementation of this control system in the real electricity market would work in the same way as presented in this research. Therefore, it is important to obtain real-life data for the utility functions of consumers, in order to better understand if this control system could be used for real-life practices.

The current controller simulates the market outcome at a single point in time, implying that the utility functions remain static over a given time span. This approach may not fully capture the changing supply and consumer preferences throughout the day. Thus, the controller can be improved by enabling a simulation over a given time period, which would give insights on how the controller changes demand and market price throughout the day.

From the sensitivity analysis, it had become clear that the designed control system, in case of to little green electricity in the system, will reduce the demand of some agents to a negative value. This outcome violates economic intuition and results in the simulations becoming incorrect. Therefore, future research could improve the control system by introducing a condition that demand can not become negative and that if agent demand is zero that they will exit the market. This improvement will then be able to simulate the scenario where total electricity is low, correctly. Furthermore, the controller currently takes some time to reach the equilibrium of the system. However, tuning the controller for faster convergence is outside the scope of this research and can be studied in later research.

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Appendix

A. Appendix A: Social welfare function

```
function dy = Social(t,y,Q,c,a,lmax,n)

O = ones(n,1);


A = [-Q -O ; O' zeros(1,1)];
B = [-c; -a];

dy = A * y + B;

end
```

B. Appendix B: Script for social welfare simulation

```
% Clear command window and workspace
clear
clc
close all

% Define parameters
n=10; %number of states
Q=10*diag(rand(n,1));

c=-100*rand(n,1);
lmax=20; %lambda_max
a = 250; % Scalar

% Set up parameters and initial conditions
tspan = [0, 15];  % Time span for integration
y0 = 100*rand(3*n+3,1);     % Initial conditions for x1, x2, and lambda

%Solve ODE45 function

[t,y] = ode45(@(t,y) SocialWelfare(t, y, Q, c, a, lmax, n),tspan,y0);


% Plot the first 10 state variables in one plot
figure;
plot(t,y(:, 1:11));
xlabel('Time');
ylabel('State Variables 1-10');
title('First 10 State Variables over Time');
legend('X1', 'X2', 'X3', 'X4', 'X5', ...
       'X6','X7', 'X8', 'X9', 'X10', 'Lambda');
```

C. Appendix C: function for social welfare simulation with controller

```
function dy = SocialWelfare(t, y, Q, c, a, lmax, n)

B = ones(n, 1); %Matrix of ones
I = eye(n); %Identity matrix of size n

% Define the matrices used for mode 1 and 2
A1 = [-Q -B -I zeros(n) zeros(n, 1) zeros(n, 1); B' zeros(1, 1) zeros(1,
     n) zeros(1,n) 0 0; -I zeros(n, 1) -inv(Q) -Q zeros(n, 1) zeros(n,
    1); zeros(n, n) zeros(n,1) Q zeros(n,n) -B zeros(n, 1); zeros(1, n)
    zeros(1, 1) zeros(1,n) B' 0 1; zeros(1, n) zeros(1, 1) zeros(1, n)
    zeros(1,n) -1 0];
B1 = [-c; -a; -inv(Q)*(c + lmax*B); zeros(n, 1); 0; 0];

% Define the matrices used for mode 3
A2 = [-Q -B -I zeros(n) zeros(n, 1) zeros(n, 1); B' zeros(1, 1) zeros(1,
     n) zeros(1,n) 0 0; -I zeros(n, 1) -inv(Q) -Q zeros(n, 1) zeros(n,
    1); zeros(n, n) zeros(n,1) Q zeros(n,n) -B zeros(n, 1); zeros(1, n)
    zeros(1, 1) zeros(1,n) B' 0 1; zeros(1, n) zeros(1, 1) zeros(1, n)
    zeros(1,n) 0 0];
B2 = [-c; -a; -inv(Q)*(c + lmax*B); zeros(n, 1); 0; 0];


if y(end, 1) > 0  % y(end, 1) is mu
    % mode1: you have system 1
    dy = A1 * y + B1;
elseif y(end-1, 1) <= 0 % y(end-1, 1) is nu
    % mode2: you have system 1
    dy = A1 * y + B1;
elseif y(end-1, 1) > 0
    % mode3: you have system 2
    dy = A2 * y + B2;
end
end
```

D. Appendix D: Script for social welfare simulation with controller

```
%clear command window and workspace
clear
clc
close all

% Define parameters
n=10; %number of agents
Q=10*diag(rand(n,1)); %Diagonal matrix with demand profile
c=-100*rand(n,1); % Desired consumption level of each agent
```

```
lmax= 10; % Price that may not be exceeded. If price exceeds then the
    market price is not socially acceptable
a = 150; % Scalar

% Define time span and initial conditions
tspan = [0, 200];  % Time span for integration
y0 = 100*rand(3*n+3,1);     % Initial conditions for x1, x2, and lambda

%Solve ODE45 function
[t,y] = ode45(@(t,y) SocialWelfare(t, y, Q, c, a, lmax, n),tspan,y0);

%Plot how the demand of agents converges to the optimal solution over
    time
figure;
plot(t,y(:, 1:10));
xlabel('Time');
ylabel('Demand');
title('Demand equilibrium of the social welfare problem');
legend('X1', 'X2', 'X3', 'X4', 'X5', 'X6','X7', 'X8', 'X9', 'X10');

%Plot how the price converges to the optimal solution over time
figure;
plot(t,y(:,11));
xlabel('Time');
ylabel('Lambda');
title('Market price equilbrium of the social welfare problem')
```

E. Appendix E: Social welfare problem simulations

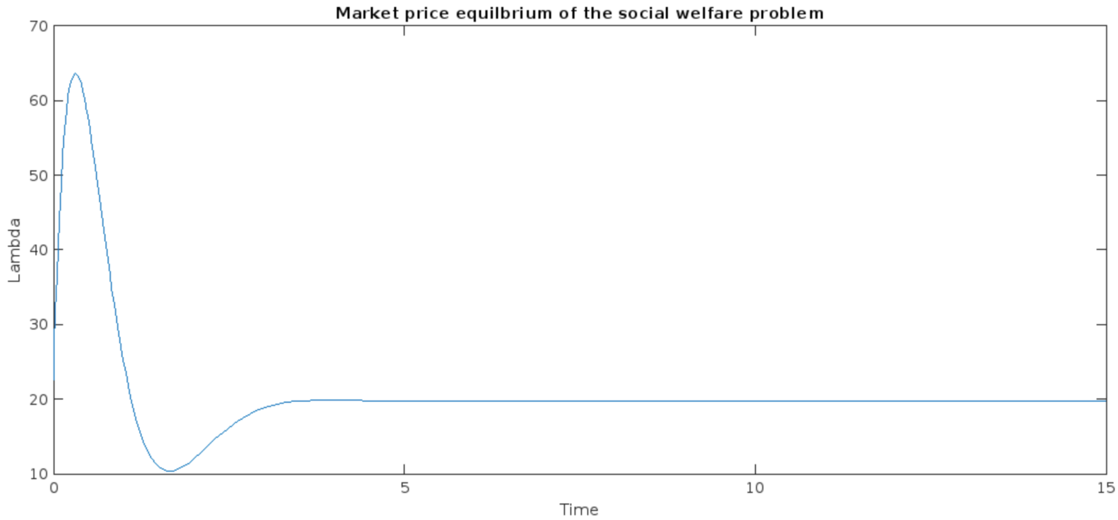

Fig. 14: The equilibrium price of the social welfare problem

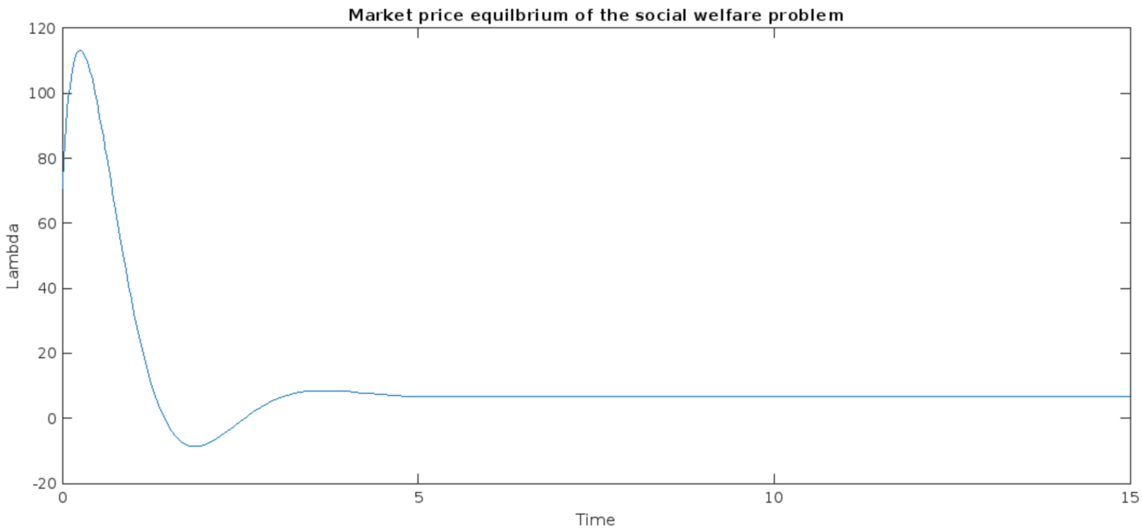

Fig. 15: The equilibrium price does not exceed socially acceptable thresholds

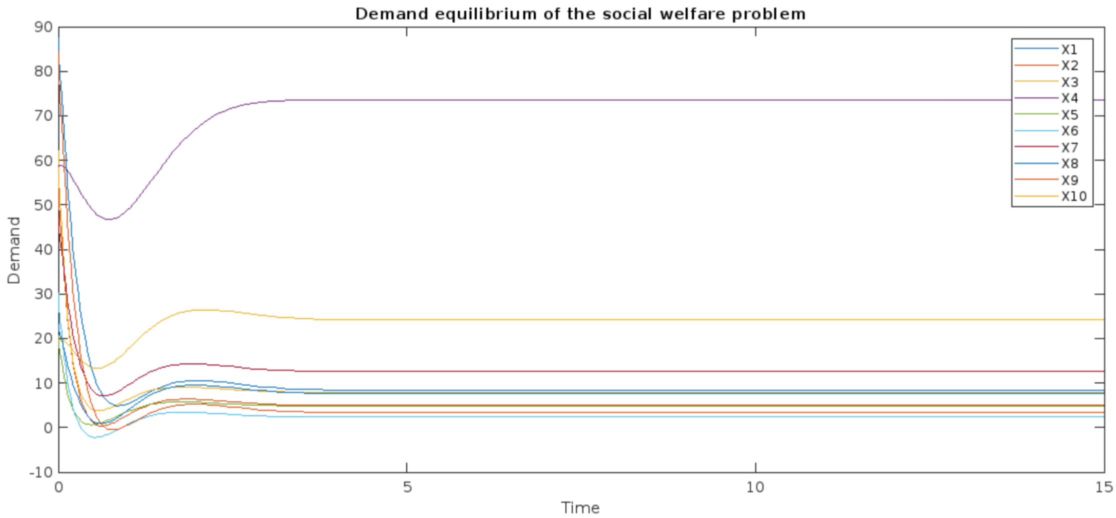

Fig. 16: Demand equilibrium of the social welfare problem

F. Appendix F: Control system simulations

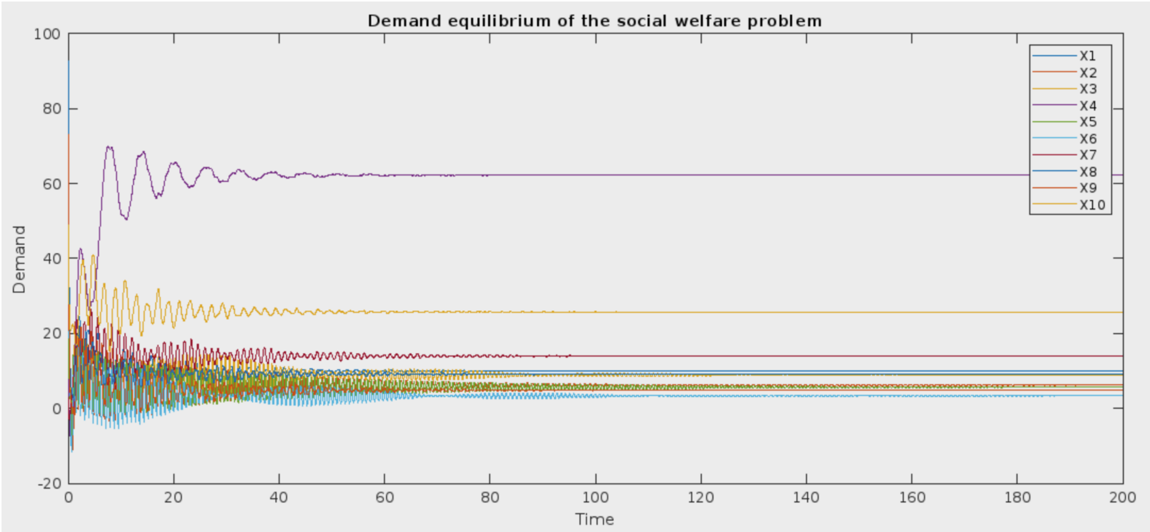

Fig. 17: Demand equilibrium of the social welfare problem with the designed control system

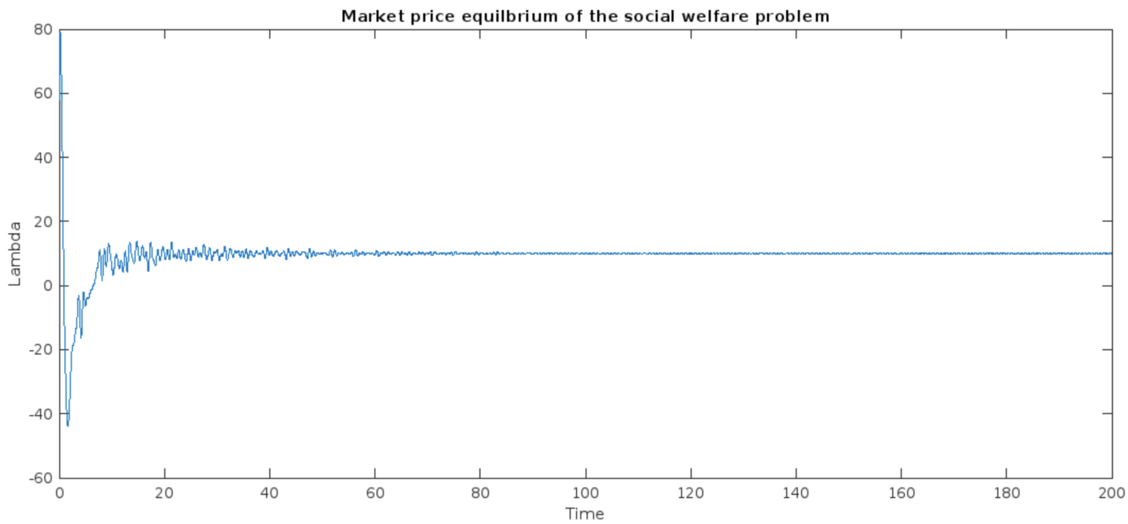

Fig. 18: Market price equilibrium of the social welfare problem with the designed control system

G. Appendix G: Sensitivity analysis

Case 1: Total generated green electricity

Fig. 19: Market price for a = 125 kWh

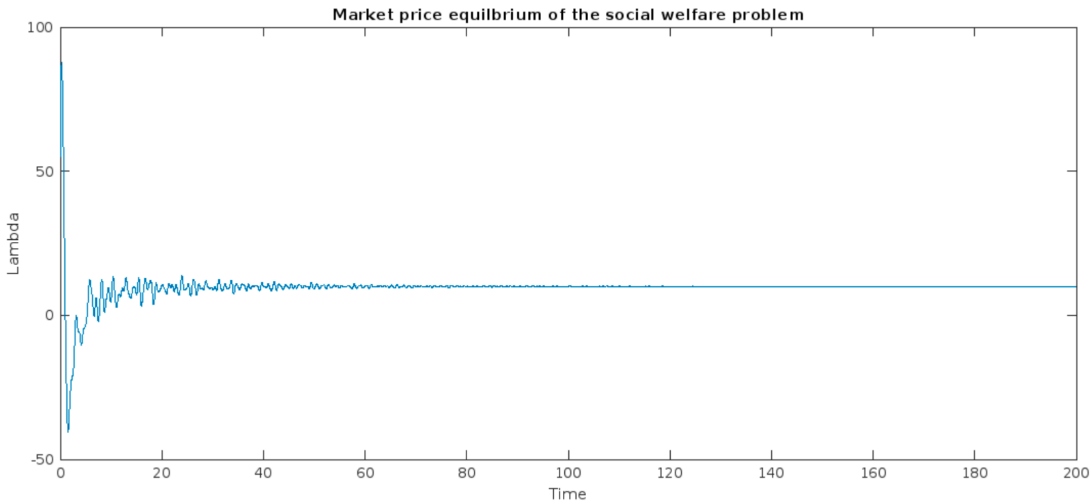

Fig. 20: Market price for a = 150 kWh

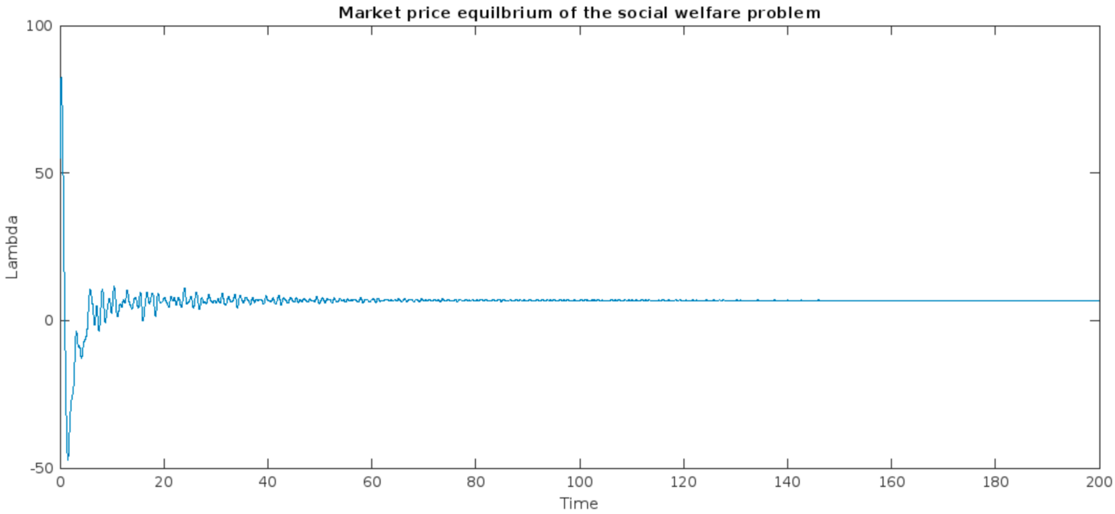

Fig. 21: Market price for a = 175 kWh

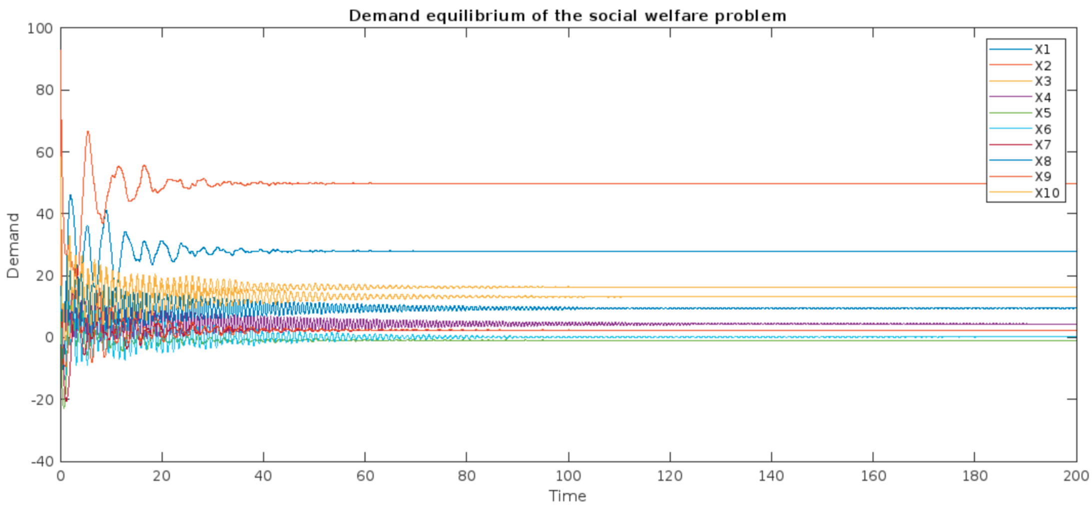

Fig. 22: Agent demand for a = 125 kWh

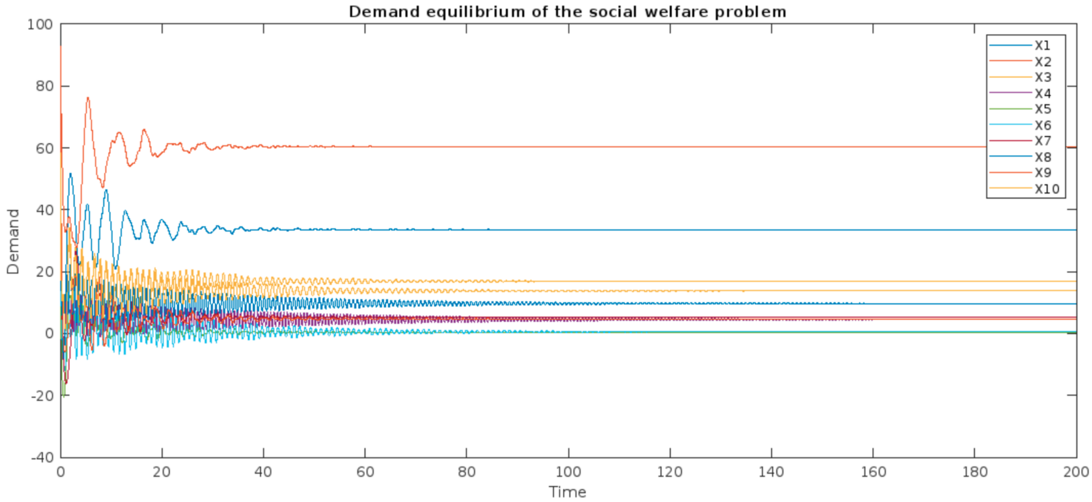

Fig. 23: Agent demand for a = 150 kWh

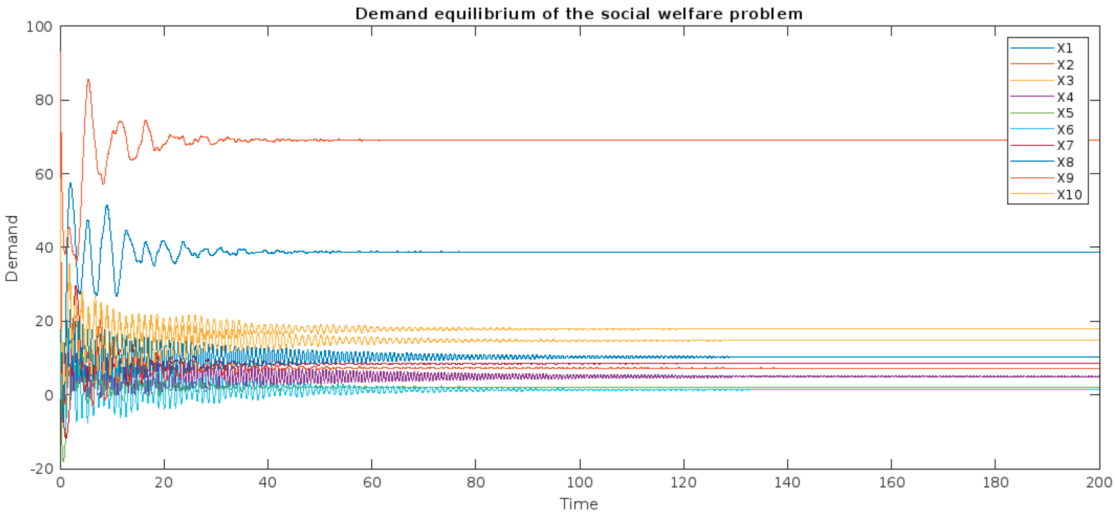

Fig. 24: Agent demand for a = 175 kWh

Case 2: Maximum price

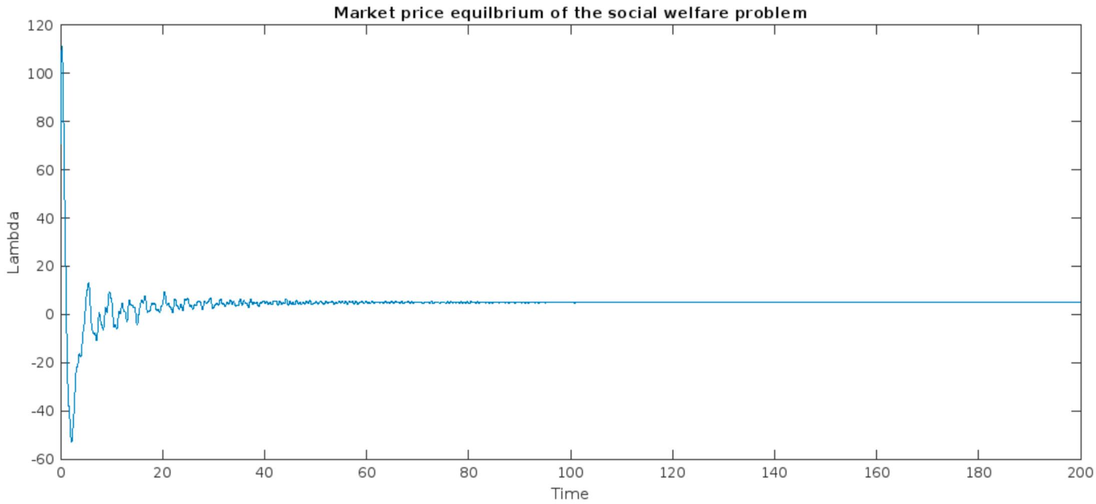

Fig. 25: Market price for λ_{max} = 5 €/kWh

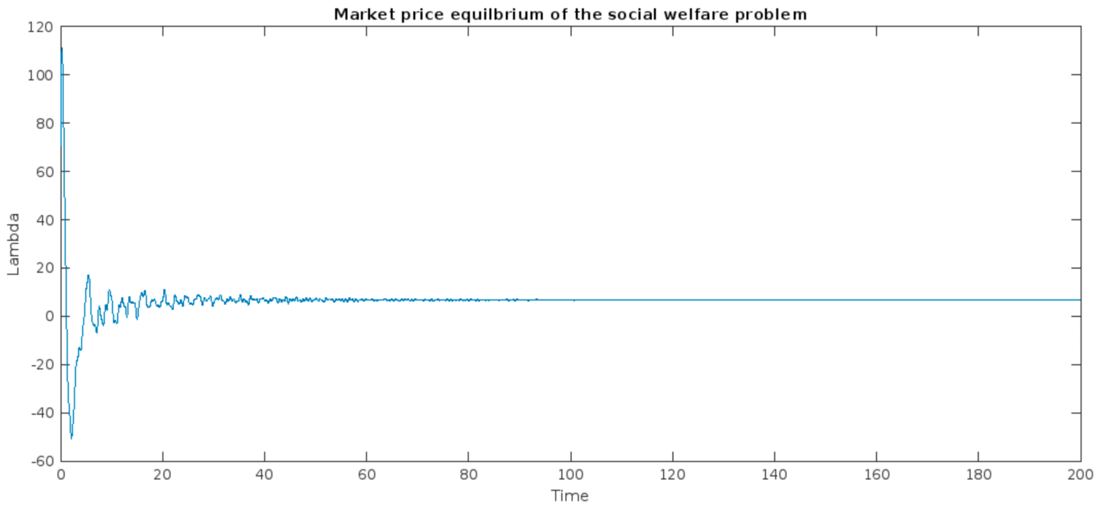

Fig. 26: Market price for λ_{max} = 10 €/kWh

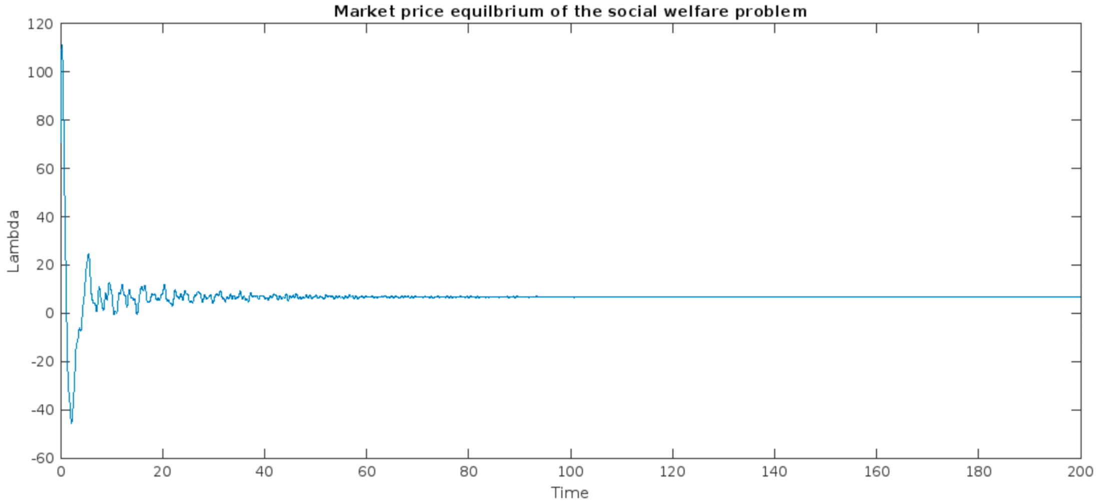

Fig. 27: Market price for λ_{max} = 20 €/kWh

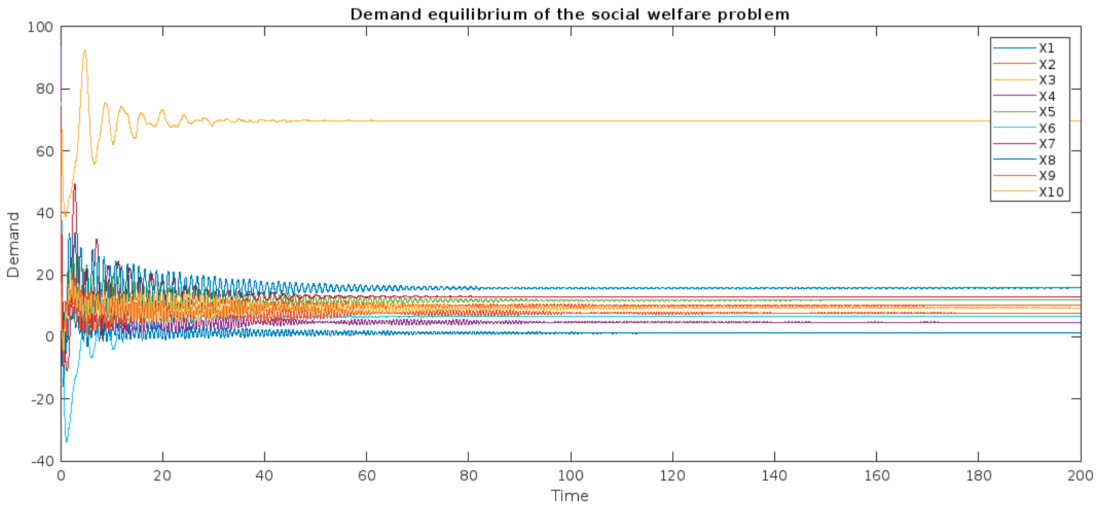

Fig. 28: Agent demand for λ_{max} = 5 €/kWh

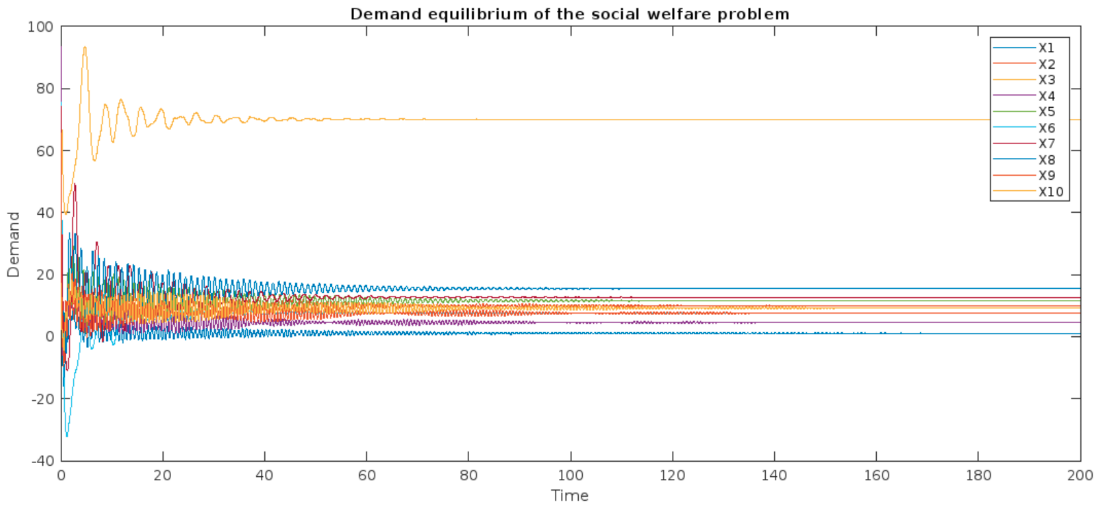

Fig. 29: Agent demand for λ_{max} = 10 €/kWh

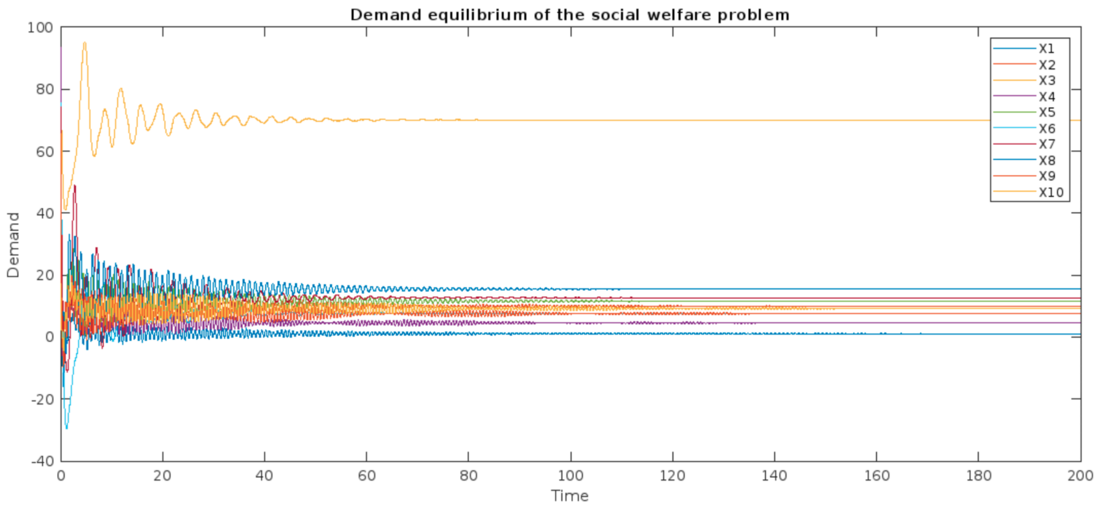

Fig. 30: Agent demand for λ_{max} = 20 €/kWh

Case 3: Number of agents

Fig. 31: Market price for n = 8

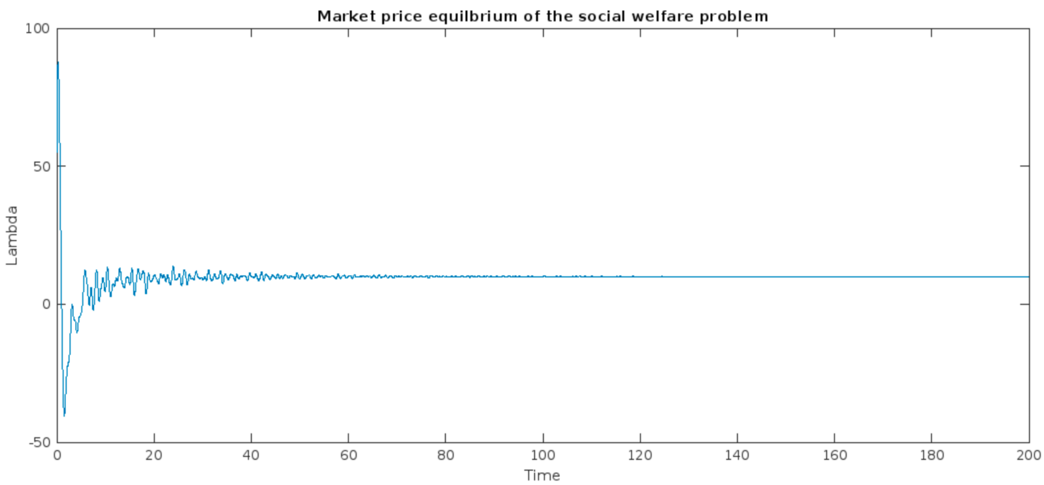

Fig. 32: Market price for n = 10

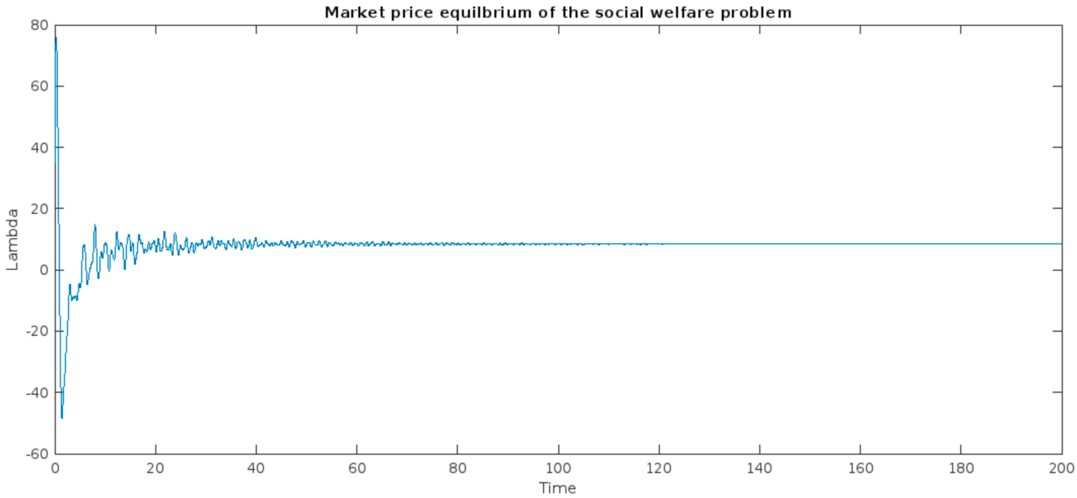

Fig. 33: Market price for n = 12

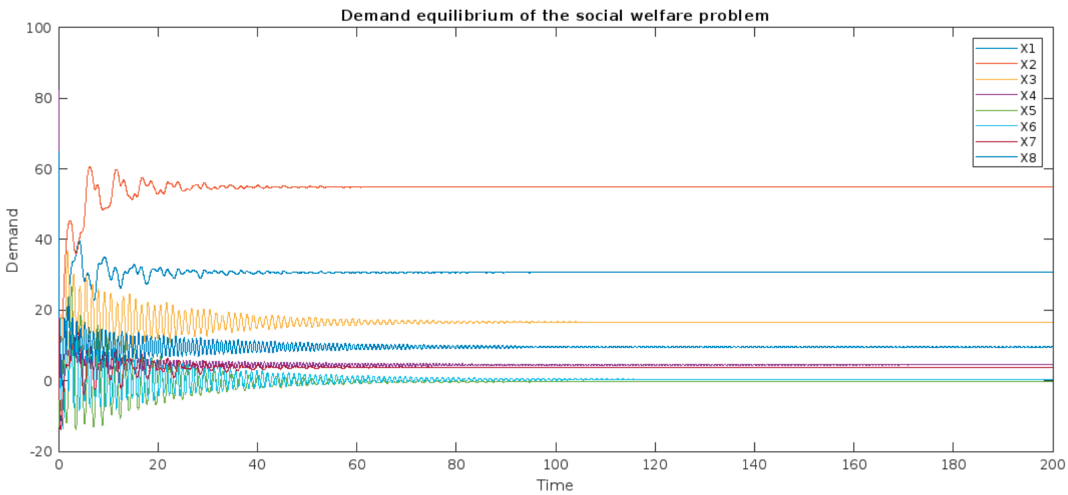

Fig. 34: Agent demand for n = 8

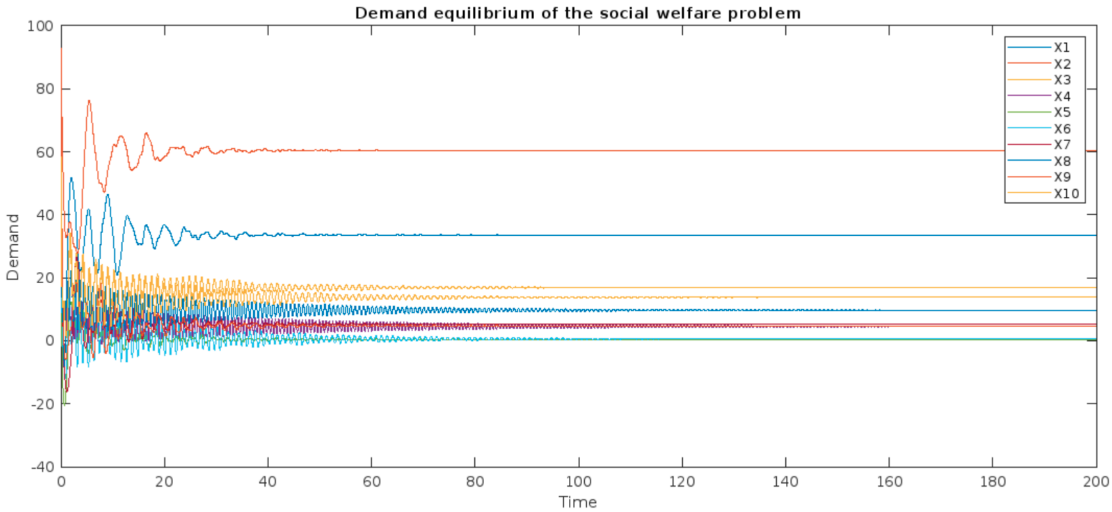

Fig. 35: Agent demand for n = 10

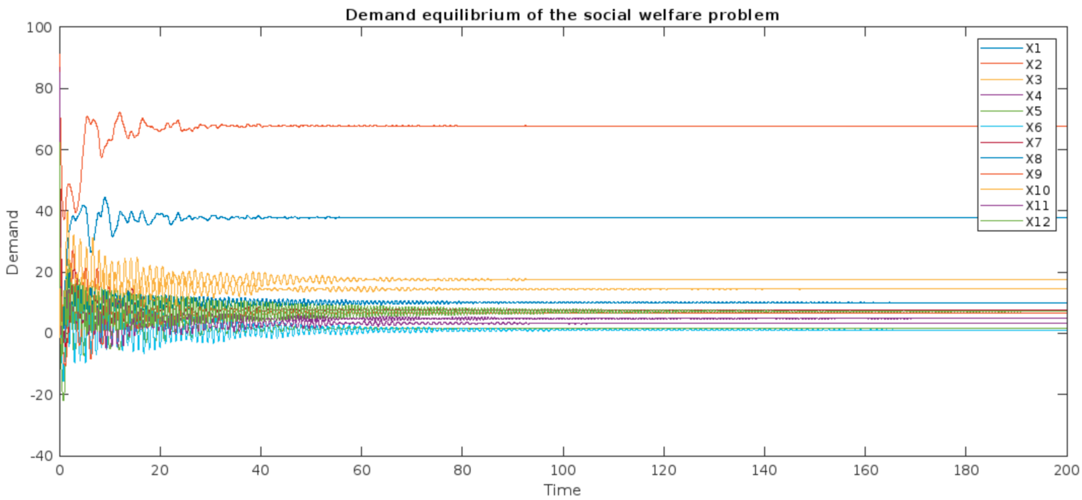

Fig. 36: Agent demand for n = 12