The Role of Real-time Information in Traffic Networks

Bachelor Integration Project

Industrial Engineering and Management

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Abstract

This research investigates the impact of Dynamic Toll pricing on traffic patterns and network performance using a Dynamic Traffic and Routing model. The study includes two stages, examining scenarios with both a constant and peak-based traffic inflow. The importance of this research lies in addressing the need for reducing traffic congestion and therefore reducing the average travel time experienced by drivers. Existing literature indicates the fact that drivers tend to minimise individual travel times, moving the network into a sub-optimal equilibrium, where average travel times are not minimised.

The research aims to identify the response of drivers to the introduction of Dynamic Toll and its impact on routing ratios, traffic density, and average travel times. Key findings present shifts in traffic distribution, with an increased volume of vehicles opting for the more distant route after the Dynamic Toll implementation. Resulting in an impact on the average travel times with a 5.39% reduction during peak hours, representing increased network performance.

The importance of these findings is the fact of providing insights into the effectiveness of Dynamic Toll pricing in optimising traffic distribution and reducing average travel times. Ensuring this leads to less congested roads and a reduction in environmental and economic impact. Furthermore, these findings are important to traffic operators, to understand the behaviour of drivers on implemented Dynamic Toll prices assigned to specific links in a network, leading to an improved overall network performance and efficiency.

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Chapter 1

Introduction

In the world of extensive transportation networks, ensuring an efficient flow of traffic on roads is incredibly important. Congestion control plays a vital role in making traffic networks more efficient. Besides the role of limiting traffic jams, it helps reducing travel times and environmental impacts. Real-time traffic information adds a deeper layer in giving immediate updates on traffic conditions, helping traffic operators create policies to further optimise traffic flows. However, it has turned out that the traffic distribution will reach the Wardrop Equilibrium, in which every individual driver selects the shortest route, leading to a sub-optimal equilibrium. To reach the social optimum, having the lowest average travel time further research should be executed. This research is executed for the SMS Cyber-physical Systems Group and this paper explores how congestion could be reduced by policies based on real-time traffic information to improve overall traffic flow and the responsiveness of the traffic network. These insights are generated by a traffic simulation model and analysing traffic flow in a network.

The paper's structure entails an initial exploration of the experienced problem and an examination of the theoretical background that functions as the foundation of this research. Subsequently, the research and design provide additional insight into the direction of this research. Given the use of a simulation model in this research, a thorough discussion of all the elements and their relation are discussed and extended with the integration of Dynamic Toll prices. This model serves as the source for the generated results and insights. These findings are used to formulate a conclusion, and based on these insights provide insights for the future research perspective.

Chapter 2

Problem analysis

Traffic congestion has a significant impact on both the economy and the environment. Traffic jams increase fuel consumption by vehicles, emitting additional greenhouse gases in the environment and negatively impacting the health of citizens. Despite this environmental impact, traffic jams result in a considerable loss of valuable productive working hours. The average Dutch citizen 37 hours, stuck in traffic jams on a yearly base [1]. Resulting in an impact on the Dutch economy of 15 billion euros in 2019. To effectively reduce traffic congestion, traffic operators could implement policies, based on real-time traffic data, to positively influence the distribution of traffic.

Navigation systems offer different route options to drivers, enabling them to select the fastest route and switch paths to reduce travel times based on real-time traffic information in case of congestion. These navigation systems are currently widely available and greatly adopted by 98% of drivers in the Netherlands [2] offering alternative routes to drivers to reduce travel times in case of congestion and drivers prefer to use navigation systems with real-time traffic information with a high level of detail. Positively influencing the level of drivers willing to alter their selected routes to reduce travel time [3] and further improves the overall traffic flow. However, according to Bianchin's research [4], these navigation systems often lack robustness. The provided information tends to oscillate the traffic distribution because navigation systems offer drivers available options with the shortest travel time [5]. Despite redistributing traffic among different roads, individually selecting the fastest route, based on information from navigation systems, may lead to an uncoordinated society in which only the social optimum is reached but not the desired societal optimum. At this desired optimum, the average overall travel time is reduced to a minimum [6], even though a fraction of the drivers may experience additional travel time and contribute to improving the overall performance of the network.

To reach the social optimum, or second Wardrop Equilibrium, coordination is required and not all drivers select their route based on minimising their individual travel time [7]. To accomplish this, the traffic operator's role becomes significant. The traffic operators can implement policies to influence drivers' responses, enhancing efficient traffic distribution. However, to distribute traffic efficiently, traffic operators should understand the potential switching behaviour of drivers under various conditions and create policies. Therefore, to overcome this gap, the **SMS Cyber-Physical Systems Group** has taken an interest in analytically investigating how drivers respond to real-time traffic information and what the behavioural effects are of communicating created policies to drivers. Because traffic operators are responsible for managing traffic flow, these insights prove valuable for entities such as Rijkswaterstaat in the Netherlands, for instance.

By creating a model, that has the ability to simulate the flow of traffic in a network and updates continuously traffic distribution, increased insights into this switching behaviour between selected routes can be obtained. By adding real-life behaviour from traffic operators, such as Rijkswaterstaat [2] in the Netherlands, additional insight is obtained on the drivers' response, which is highly valuable to implement effective policies that will distribute traffic in an effective manner to reduce traffic congestion and enhance traffic flow.

Based on research of He dynamical toll prices have a positive influence on the distribution of traffic at intersections, by more effectively distributing traffic among the roads and better using the maximum capacity of the road, the networks' throughput rate could be further improved while reducing the overall travel time [8]. Understanding the drivers' response to an introduced toll facility within a network, enables the effective implementation of the toll system, contributing to improved traffic flow and reducing traffic congestion.

Chapter 3

Theoretical Background

After identifying the specific problem in the previous part, section 2, it is necessary to delve into additional foundational theories to comprehend and explore the ongoing research to generate numerical insights into the introduction of a toll facility. An analysis of the fundamental aspects of traffic network and traffic distribution is conducted to understand the underlying principles of the flow of traffic within a network. This analysis serves as the foundation for executing a traffic simulation and subsequently analysing the outcomes, based on the insights of Bianchin et al. [4]. To assess the effectiveness of implementing the toll facility, the research should include a section directly associated with minimising the travel time experienced by drivers travelling through a network.

3.1 Minimising cost in flow problems

According to Hillier [9], to minimise travel time or costs in a flow problem, specific conditions must be met. Since in this research traffic is considered as a flow, the following conditions are applicable. The network must be **direct** and **connected**, highlighting the importance of linking nodes and assigning allowed directions for each link. At least one node within the network must serve as a **supply node**, from which traffic enters the network, and there should be at least one **demand node** where traffic exits. All remaining nodes function as **transshipment nodes**, directing traffic flow from the supply node towards the demand node. These transshipment nodes could both function to accommodate converging or diverging traffic. Flow on a given link is permitted only in the **direction indicated by the arrows**. Furthermore, the links must have **sufficient capacity** to handle the flow entering from the supply node. The cost to traverse each link is **proportional to the flow amount**, with costs specified for the flow surpassing the link. Ultimately, the objective of this type of problem is to **minimise the total costs** of transferring vehicles through the network to accommodate the entire flow generated by the supply node.

3.2 Networks

To meet the conditions of minimising cost in flow problems, for this research, the selected network is based on a **directed** and **connected network**, characterised by a supply node, 4 transshipment nodes and a final demand node. The directed graph in its full representation is denoted as G(V, L). V represents the nodes or intersections in the traffic network with n elements and is given by V = [v_1, v_2, v_3, v_4]. These transshipment nodes function as points where traffic converges or diverges along various links. The set of links or edges is represented by L and given by L = [l_1, l_2, l_3, l_4, l_5, l_6, l_7]. Transferring the network from the **supply node** to the **demand node** is allowed by multiple paths. The set of paths between the supply node and the demand node is represented by P. Figure 3.1 illustrates the traffic network, where traffic flows from the supply node and links 1 to node v_1. The traffic is distributed among the possible paths or links. Additionally, n denotes the number of links in the entire system. Because the system consists of 7 links, n equals 7.

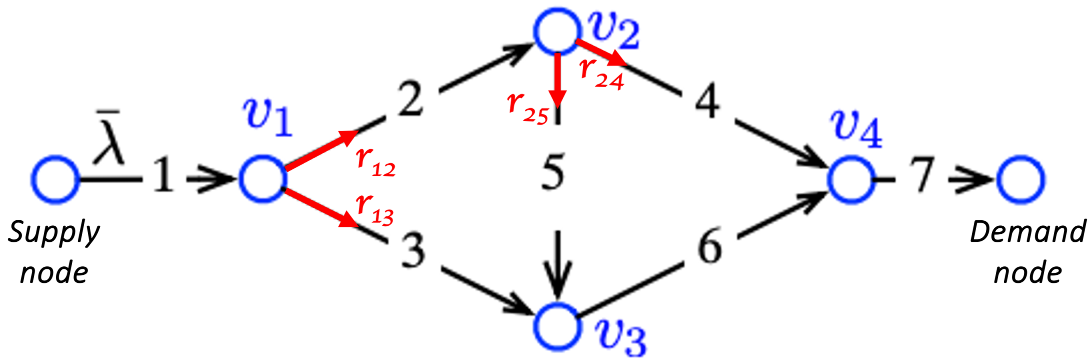

Figure 3.1: *Proposed road map for the traffic simulation using hypothetical input data. Traffic is meant to flow from one starting point to one destination and the driver can select at nodes* v_1 *and* v_2 *two different options, represented by r.*

The three possible paths (p) for traffic to travel from the origin to the destination are based on nodes and given by P = [p_1, p_2, p_3]. Each distinct path originating from the supply node and exiting at the demand node, involving the intersections of nodes, is labelled as follows:

- $p_1 = v_{supply} \rightarrow v_1 \rightarrow v_2 \rightarrow v_4 \rightarrow v_{demand}$
- $p_2 = v_{supply} \rightarrow v_1 \rightarrow v_2 \rightarrow v_3 \rightarrow v_4 \rightarrow v_{demand}$
- $p_3 = v_{supply} \rightarrow v_1 \rightarrow v_3 \rightarrow v_4 \rightarrow v_{demand}$

Given the specified paths for traffic movement from the supply node to the demand node, certain node connections are exclusively facilitated by specific paths. These include the connections between nodes v_2 and v_4 (represented by link 4),

v_2 and v_3 (represented by link 5), and v_1 and v_3 (represented by link 3). Traffic traversing one of these links has therefore chosen one of the three available paths, making them valuable indicators for understanding the traffic volumes associated with each of the three paths. Particularly, link 5 is expected to have lower usage compared to links 3 and 4. This is attributed to the fact that opting for this link implies selecting route option 2, which has the longest travel distance.

3.3 Traffic and Routing Dynamics model

Enabling to simulate traffic flow, entailing the behaviour of drivers, a dynamic model should be created to simulate traffic flow.

The traffic flow within the network is given by the coupled interconnection between the Traffic and Routing dynamics model. Using a Dynamic Traffic and Routing model is crucial for simulating traffic flow in the network. It considers the interconnection between Traffic and Routing dynamics, allowing for real-time adjustments and efficient route planning to adapt to changing conditions and demands effectively. The exact model, that is being used for this research is based on the insights of Bianchin [4] and is presented in figure 3.2.

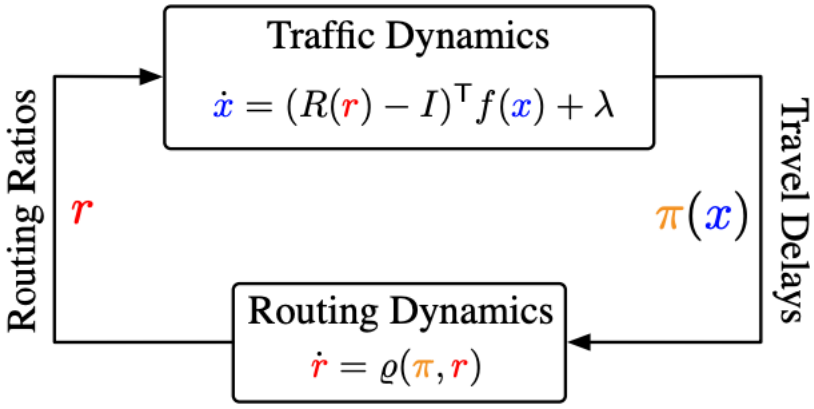

Figure 3.2: Coupled interconnection of the Traffic and Routing Dynamics model, functioning as a feedback system

This model consists of a feedback loop, that continuously updates the routing ratios r, based on minimising travel delays $\pi(x)$, that is experienced at the links of the network and functions as the fundamental basis. The routing ratio r is the fraction of traffic that will select a given link upon exiting a node. This routing ratio r influences ultimately the change in traffic density at each link. When traffic density changes, this consequently influences the travel delays π experienced at each link or road. Drivers get updated on current travel times via navigation systems. If travel delays are increasing, drivers tend to switch their selected routes and therefore the fraction of the routing ratios are changing.

Due to the importance of this model, a more extensive explanation of all the formulas (including variables) is provided in Section 5. This is done to enhance understanding of the complexity of the model.

3.4 Wardrop Equilibrium

In traffic networks, drivers tend to choose their routes based on minimising travel time, leading to the occurrence of the First Wardrop Equilibrium or social equilibrium where the average travel time is not minimised. The additional costs are defined as the Price of Anarchy [10]. This concept belongs to game theory and quantifies the reduction of performance due to the selfish behaviour of drivers in a network. The ratio between the total cost in social and societal equilibrium is defined as the Price of Anarchy. The higher the Price of Anarchy, the greater the inefficiency due to the lack of coordination among drivers transferring the network.

To reduce the Price of Anarchy, a part of the drivers can select alternative, longer routes, in terms of distance and travel time, contributing to the Second Wardrop Equilibrium or social optimum. According to Wardrop's second principle, some drivers must select longer routes to reduce traffic traffic density and delays [7]. This helps to reduce the overall average travel time in the network. However, drivers typically won't voluntarily choose more distant and time-consuming routes unless there is, for example, a financial incentive encouraging them to do so [8]. This financial incentive could be obtained by the implementation of toll prices on given roads. In situations with high traffic density, prices will increase and some drivers may select the more distant route, reducing the average travel time of all drivers. A further elaboration on Toll pricing will be in the next section 3.5.

Apart from this need to redistribute traffic through an alternative route to decrease the average travel time, certain drivers may opt for the longer route in advance. This is because the potential for higher driving speeds, resulting from fewer vehicles on that route, attracts some drivers even though travelling for a longer distance. As a result, there is always a part of the drivers already choosing the more distant route [11].

3.5 Dynamic Toll pricing

To influence the distribution of traffic among different roads, toll prices can be an incentive to let additional drivers switch their selected route to a different route, which is less occupied at that moment. Toll prices can help to force a part of the drivers to select a more time-consuming route. Additionally, by communicating the toll fare en route, the driver will respond to this changing toll price and impact the distribution of traffic [12]. This enhances the traffic efficiency of the network, reducing average travel times. In this research, the aim is to simulate the traffic distribution at networks where traffic will be distributed among two links in two situations at node v_1 and v_2. To influence this distribution, toll prices based on the current traffic could impact the distribution of these routing options. The introduction of Dynamic Toll Pricing to a network has a positive influence on assigning traffic to specific routes and therefore improving the distribution of traffic more optimally as mentioned earlier. Drivers opting for a more distant route will benefit from a lower toll price while experiencing a longer travel time. By distributing traffic more effectively among the roads and better using the maximum capacity of the road, the networks' throughput rate could be further improved while reducing the average overall travel time, through reducing congestion effectively. Based on the research of He [8], implementing dynamics toll prices increases the throughput of the roads by 20 per cent during peak hours and traffic density will be spread over a longer time frame because some drivers will postpone their trip.

Ensuring the effectiveness of the introduction of Dynamic Toll Prices, the actual price will be communicated to drivers by Digital Traffic Information Signs displayed on the roads. Ensuring an optimal transfer of information to drivers.

3.6 Price elasticity

To successfully implement toll prices, it is crucial to understand the responsiveness of drivers to changes in toll pricing. Price elasticity provides insight into how much the inflow of traffic (driver demand) of travelling via a road will respond to a change in toll prices. Therefore, the price elasticity is calculated as follows [13]:

$$ETP = \frac{\%\Delta driversdemand}{\%\Delta tollprice} \tag{3.1}$$

In Burris's research, which delved into the influence of price elasticity on traffic flow, it was discovered that the price elasticity of toll prices in the short term falls within the range of 0.076 to 0.15 [14]. This indicates that the behavioural response of drivers to a change in toll prices is minimal, therefore an increase in toll prices is required to redirect a portion of drivers to an alternative route. More recent research further supports the low price elasticity value, indicating the typical range of demand-price elasticity in transportation systems has a range between 0.10 and 0.20 [15]. Consequently, it can be inferred that the price elasticity is relatively inelastic, signifying that traffic does not promptly adjust to changes in toll prices. This is because toll prices represent only a portion of the overall costs associated with travelling by car from point A to point B, besides fuel and maintenance costs. Based on these insights, it could be said that the toll prices should be very inelastic as the value is far below the value 1. Therefore, to have a significant impact on the distribution of traffic and influence the distribution of traffic to persuade a part of the traffic to an alternative route, high changes in toll prices are required.

Chapter 4

Research & design

4.1 Research goal

Based on the insights gained from the Problem analysis 2 and the Theoretical Background 3, the following research goals are formulated. The research goal is to numerically analyse the behavioural effect of introduced Dynamic Toll Prices based on real-time traffic information to drivers and analyse how drivers respond to Dynamic Toll Pricing based on real-time information.

The insight of this analysis will help to understand how drivers respond to communicated real-time traffic information and based on this effective strategies for reducing overall traffic congestion within a traffic network and enhancing traffic efficiency in terms of the average travel time could be created. By reducing the average travel time and increasing the throughput rate of the network, the Societal Optimum or Second Wardrop Equilibrium could be reached or moving more towards this optimum. The proposed network, depicted in figure 3.1, is being analysed. Traffic is flowing in via the supply node, and crossing the network via the 3 possible paths to the demand node. For each path, no traffic can exit or enter the network and traffic is only able to flow in the direction of the arrows.

4.2 Research questions

Based on the Research goal, the following main research has been formulated: **What effect does the communication of real-time traffic-based Dynamic Toll Prices to drivers have on the travel times they encounter while travelling through traffic networks?**

To answer this main research question, three sub-research questions are created:

1. *What elements must be included in the traffic model to accurately simulate traffic across various routes from a single starting point and destination?*
2. *What observable changes in switching behaviour occur when real-time traffic conditions change, and how do these changes impact average travel time?*
3. *What strategy can a system operator implement to effectively communicate real-time traffic information to control congestion and enhance traffic efficiency within the network?*

4.3 Simulation

For the traffic flow simulation, the versatile tool MATLAB is being used. Through mathematical modelling and solving differential equations, important parameters such as traffic delays and traffic density on the roads could be calculated. Furthermore, the results and impacts of different policies could be made visible. The focus will be mainly on the Key Performance Indicators, which are discussed in section 4.6. An ODE function could help to optimise the created model to find a more optimal traffic distribution.

4.4 Boundaries

This research will investigate the effect of toll pricing on the distribution of traffic. To reduce complexity, the toll price will have a fixed ratio between the two links. In this case, the analysed node will be v_2, where drivers can select two options, r_{24} and r_{25}. Traffic selecting route 4 will experience a shorter road in terms of distance, while link 5 will take an additional travel distance to reach the final link 7. In this initial phase, the toll price will have a distribution of 2:1, meaning travelling via link 4 will have a toll price twice as high as the toll price compared to link 5. Leading to an additional financial benefit for drivers who selected link 5, to partially compensate for the longer experienced travel time. The toll price is effectively communicated to drivers through Digital Information Signs located near the roads. Thus, it is assumed that this information effectively reaches all drivers and makes decisions accordingly.

As attention is focused on node v_2 and toll prices are exclusively introduced to links 4 and 5, solving the overarching network problem is impacted by a more localised context, due to the implementation of the toll facility to only 1 of the 4 transshipment nodes. This shift includes the focus on how these changes will ultimately influence the performance of the network.

Moreover, introducing a Toll Facility ensures drivers will not postpone their trips. This implies that the traffic inflow remains equivalent in scenarios both with and without an extra toll fee applied to the two roads.

4.5 Design

The proposed design included the development of a model that can simulate traffic flow within a network, created by MATLAB. The model incorporates variables such as routing ratios (r), traffic density, and travel delay or travel time, all graphically represented by plots. Through this comprehensive simulation, the behaviour of drivers is accurately depicted, enabling to examination of the potential impact on the KPIs when dynamic toll prices are introduced to specific links within the network. The ultimate goal is to enhance network throughput and minimise average travel time. The dynamics of the model are in-depth explained in section 5.

4.6 Key Performance Indicators

To examine and evaluate the impact of the newly implemented Dynamic Toll Pricing system, it is crucial to establish specific Key Performance Indicators (KPIs) that facilitate a quantifiable comparison of results between two scenarios: one without and one with the dynamic toll price. The selected Key Performance Indicators are:

1. **Routing ratios:**
   The routing rations give a clear indication of the fraction of traffic selecting a specific link, upon exiting the previous link. This indicator gives insight in the fraction of traffic that changes their behaviour after introducing Dynamic Toll prices. This KPI is **dimensionless**, given that it is expressed as a fraction.

2. **Traffic density:**
   This indicator provides insight into the throughput of vehicles. An increase in traffic density represents a higher volume of traffic selecting a specific link. This is valuable, to understand the absolute impact on vehicle numbers after the introduction of Dynamic Toll to the links. The unit for this indicator is **vehicles/hour**.

3. **Average travel time:**
   The average travel time focuses on quantifying the average time drivers spend crossing the network from the supply node to the demand node. A lower average travel time suggests a reduction in congestion and increased traffic efficiency. The comparison is made between scenarios with and without the Dynamic Toll Pricing system. The unit for this KPI is **minutes**.

Chapter 5

Model setup

The previous chapters are fundamental for the model that will be created for the traffic simulation. An extensive explanation of all the parts of the model is given in this chapter to numerically analyse the behavioural effect of real-time communicated traffic information, with the implementation of Dynamic Toll Prices. It is important to note that the Dynamic Toll Price implemented by the introduction of a Toll Facility is an addition and manipulation of the Traffic and Routing model.

5.1 Traffic and Routing Dynamics model

The coupled Traffic and Routing Dynamics model, mentioned in section 3.3, is the foundation to accurately simulate traffic flow within the select network and investigate the drivers' responses. The created model, including the formulas, is based on the principles of the Cell Transmission model of Bianchin [4].

All the different parts and the relations between these and the formulas behind them are extensively explained in the following parts of this section.

5.1.1 Traffic Dynamics

For each designated link l, there will be an inflow and outflow with traffic density (x), denoted by $f_l^{in}(x)$ and $f_l^{out}(x)$ respectively, and is measured in unit *vehicles/hour*. Over time, traffic density will change, due to fluctuations of the routing ratios and traffic inflow at the supply node. The change in traffic density of link l is calculated by the difference between the inflow and outflow of the specified link:

$$\dot{x}_l = f_l^{in}(x) - f_l^{out}(x) \tag{5.1}$$

This principle change in traffic density for each link is combined and adjusted to fit in a matrix form where the change in traffic density $\dot{x}$, is calculated by the following formula:

$$\dot{x} = (R(r) - I)^T (f(x))) + \lambda \tag{5.2}$$

The matrix R(r) contains the network connectivity information between the links. The vector's routing ratios, denoted by r, signify the proportion of traffic assigned to a particular link because there is more than one link for the drivers to select upon arriving at the transshipment node. These routing parameters are specified into a matrix-valued map denoted as R^{nxn}. The R(r) matrix provides insights into the connection between the links and the routing ratios r_{lm}, representing the routing ratio or the fraction of traffic flow that selects link m upon exiting link l at node v. The fraction of the flow of traffic is further specified by the fact that the sum of routing ratios, exiting from a node always equals to one. Meaning that $r_{lm} + r_{l\hat{m}} = 1$ is at all times valid. For converging traffic from multiple links at one node, all the traffic will continue via this shared next link, because traffic is only allowed to travel in the direction of the arrow, therefore no fraction or routing ratio is required.

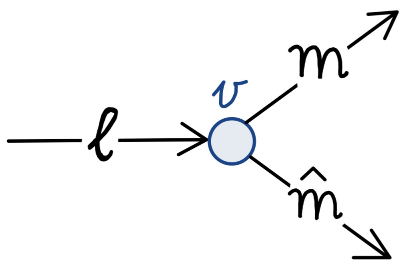

Figure 5.1: Distribution of traffic among multiple route options

Considering the network in figure 3.2, the entire network connectivity matrix is given by:

$$R(r) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ r_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ r_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & r_{25} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \tag{5.3}$$

At each specific link, the outflow of traffic is denoted by the following formula:

$$f(x) = [f_1^{out}(x_1), ..., f_n^{out}(x_n)]^T \tag{5.4}$$

The inflow of traffic is given by λ. The first value of this array is $\bar{\lambda}$, representing the inflow of traffic at the supply node. All this traffic will flow via the three described paths through the network to reach the final destination or demand node, which is reached via link 7.

$$\lambda = [\bar{\lambda}, ..., 0]^T \tag{5.5}$$

Based on the network in figure 3.1, which is being used for this research, there is only one supply node. However, when additional supply nodes are included in the network, the array λ serves as the supply of traffic at given supply nodes.

5.1.2 Travel Delays

The additional time it takes for a vehicle to travel along a specific link in the network due to congestion is represented by $\pi_l(x)$. The calculation of $\pi_l(x)$ is important to determine the change in routing ratios, denoted by $\dot{r}$. $\pi_l(x)$ is a function of the perceived travel costs τ_l and the traffic delay in the next link, or if there are multiple routes exiting from a node, the minimum delay is selected. This aligns with the observation that drivers opt for the shortest available routes, contributing to the principle of the First Wardrop Equilibrium, as discussed in Section 3.4. Because the delay of the next link or links is required, this approach requires back-propagation. The combined formula is denoted as:

$$\pi_l := \tau_l(x_l) + min(\pi_m, \pi_{\hat{m}}) \tag{5.6}$$

The specific delay functions for each link are denoted below. The delay for links 1 and 2 included the selection of the minimum value for the delay, in the possible links that are connected. Based on the selected network that will be analysed, the following formulas are created:

$$\pi_7 := \tau_7(x_7) \tag{5.7}$$

$$\pi_6 := \tau_6(x_6) + \pi_7 \tag{5.8}$$

$$\pi_5 := \tau_5(x_5) + \pi_6 \tag{5.9}$$

$$\pi_4 := \tau_4(x_4) + \pi_7 \tag{5.10}$$

$$\pi_3 := \tau_3(x_3) + \pi_6 \tag{5.11}$$

$$\pi_2 := \tau_2(x_2) + min(\pi_4, \pi_5) \tag{5.12}$$

$$\pi_1 := \tau_1(x_1) + min(\pi_2, \pi_3) \tag{5.13}$$

In these equations, τ_l represents the associated travel costs to transverse a specified link and is influenced by a_l and b_l. Contributing to the conditions of *minimising travel costs in flow problems* in section 3.1, that travel costs of transferring a link must be proportional the the flow amount or traffic density in this case. The relationship is described as follows:

$$\tau_l = a_l(x_l) + b_l \tag{5.14}$$

5.1.3 Routing Dynamics

This part of the model captures the required change of the routing ratios to distribute the traffic more efficiently to reduce travel times from the individual perspective of the driver. This is determined by the travel delay π and the current routing ratio r and their alternative routing options.

$$\dot{r}_{lm} = r_{lm}((r_{lm}\pi_m + r_{l\hat{m}}\pi_{\hat{m}}) - \pi_m) \tag{5.15}$$

The change in the routing ratio of a specific link is calculated by the input of the delay in the selected route and the alternative route that can be reached by the communal node, that traffic has travelled through.

In the situation for the network that will be investigated, figure3.1, the formulas for the change in routing ratios are described as follows:

$$\dot{r}_{12} = r_{12}((r_{12}\pi_2 + r_{13}\pi_3) - \pi_2) \tag{5.16}$$

$$\dot{r}_{13} = r_{13}((r_{12}\pi_2 + r_{13}\pi_3) - \pi_3) \tag{5.17}$$

$$\dot{r}_{24} = r_{24}((r_{24}\pi_4 + r_{25}\pi_5) - \pi_4) \tag{5.18}$$

$$\dot{r}_{25} = r_{25}((r_{24}\pi_4 + r_{25}\pi_5) - \pi_5) \tag{5.19}$$

5.1.4 Routing Ratios

The change in routing ratios, denoted by $\dot{r}$, is incorporated into the existing routing ratio during each calculation cycle. This iterative process ensures continuous updates of the routing ratios, allowing to create a simulation of the distribution of traffic across the network, based on minimising travel times for the individual driver, reaching the social optimum. The calculated change in routing ratios, $\dot{r}$, can be either negative or positive and is accumulated with the routing ratio r from the previous iteration, which is also utilised in the computation of the change in routing ratios, $\dot{r}$. This iterative mechanism continues throughout the entire simulation duration, contributing to the continuous refinement of the traffic distribution within the network.

5.1.5 Capacity constraint

According to constraints of minimising cost in flow problems, the capacity of the network should be sufficient to accommodate all the traffic moving from the supply node, across the network, to the demand node.

The capacity constraint C of the links denotes the maximum flow that can be accommodated through a given link l in the network. This constraint is a fundamental aspect of modelling traffic as it gives insight into the limitations of the infrastructure of the links. The capacity constraint involves factors mainly external factors such as the condition of the road, traffic regulations and the capability of the infrastructure to accommodate a certain volume of vehicles. The capacity constraint sets a limit on the number of vehicles that can traverse a particular link. Due to the selected time frame during peak hours, initial traffic is available on the links of the network and the traffic inflow $\bar{\lambda}$ at the supply node will never exceed the maximum capacity of the network, ensuring the traffic density x to be the left side of the graph in figure 5.2, below the maximum capacity.

To clarify, the model lacks a strict road capacity condition. Instead, the capacity constraint is partially connected to the delay function. As the number of vehicles rises, travel costs, including travel time, result in increased delays. Although the capacity constraint affects delay, restricting additional traffic flow, the network can accommodate all traffic given the chosen inflow at the supply node as mentioned earlier. Consequently, there is no need for a specific hard-coded capacity constraint value within the model to simulate driver behaviour.

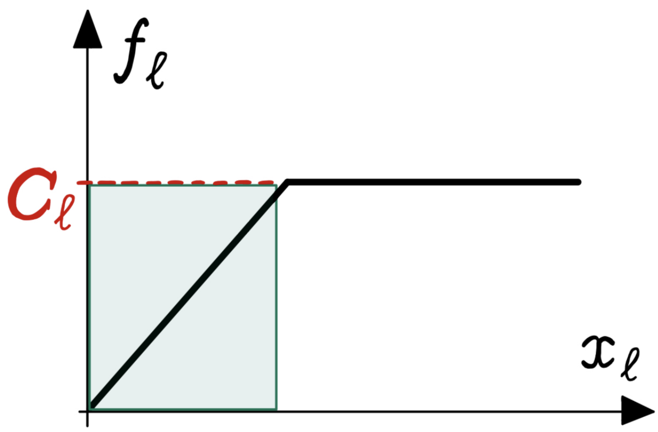

Figure 5.2: Visualization of capacity constraint in phase 1. The area denoted in green represents the maximum traffic density x

5.1.6 Choice of affine travel costs

The choice of affine travel costs involves the selection of a_l and b_l associated with each link l in the network and directly influences the associated travel costs to transverse a link, see formula 5.14. The term a_l represents the coefficient linked to the linear component, explaining the impact of the travel costs on variations in traffic density on link l. On the other hand, b_l represents the foundation of the travel delay or cost on link l in the absence of traffic.

The specific values chosen for a_l and b_l are crucial as they directly influence the behaviour of the dynamical network system. The coefficients are part of the link cost function, contributing to providing input for the travel delay calculations for this network, see formula 5.6. The selection of the affine travel costs is visible in table 5.1. For this initial phase, there is chosen for uniformity between each link except for link 5. All links will experience the same link costs increase by traffic density, $a = 1$. However, link 5 will not experience any initial delay, to stimulate a part of the traffic selecting this route. Additionally, links 1 and 7 do not have any initial delay component, because all traffic is required to travel via this link.

Table 5.1: *Impact of travel delay represented by a_l and the initial travel delay or costs by b_l.*

link(l)	**1**	**2**	**3**	**4**	**5**	**6**	**7**
a_l	1	1	1	1	1	1	1
b_l	0	0	0	0	0	0	0

5.2 Introduction Toll facility

In this section, Dynamic Toll Prices are added to the existing Traffic and Routing Dynamics model, discussed in the previous section 5.1. The influence of Dynamic Toll Pricing has explained in section 3.5 and this section explains the actual implementation of Dynamic Toll Pricing based on congestion level at given links. Therefore it is important to note that the implementation of the Dynamic Toll Price is an addition and editing of the Routing and Dynamics model. The implementation may influence the division of traffic at the nodes. In this case, the analysis will focus on node v_2, where traffic will arrive via link 2 and the drivers can select either link 4 or 5. In this phase of the research, a toll price will be added to the associated travel costs to cross a specified link.

Due to the inelastic behaviour found in section 3.6 when toll prices are added to roads, a significant increase in the price is required. As a result, in this study, the toll price for crossing link 4 will be set at twice the amount of link 5. Derived from the findings in section 3.6, the chosen elasticity for the toll ratio is set at 0.2. The adjusted formula for travel costs associated with transferring these two specified links is as follows:

$$\tau_4 = a_4(x_4) + toll_4 \tag{5.20}$$

$$\tau_5 = a_5(x_5) + toll_5 \tag{5.21}$$

Taking into consideration that the price elasticity for additional toll price is set to 0.2, the toll, for the links indicated by formulas 5.22 and 5.23. It is important to notice that the toll is related to the number of vehicles crossing the specified link, denoted by x_l.

$$toll_4 = elasticity * tollratio_4 * (x_4) = 1 * (x_4) \tag{5.22}$$

$$toll_5 = elasticity * tollratio_5 * (x_5) = 0.5 * (x_5) \tag{5.23}$$

5.3 Traffic inflow $\bar{\lambda}$

Equation 5.5 gained insight into the initial traffic inflow at link 1. The first value of this array is $\bar{\lambda}$ and includes the inflow at link 1. This traffic inflow may change and can oscillate over time, influencing the entire network.

5.3.1 Phase 1 - Constant inflow

This section specifies the initial response of the system to a continuous traffic inflow offered by the supply node. In this phase the traffic inflow $\bar{\lambda}$ is set at 3000 vehicles per hour. The primary emphasis is on analysing the behaviour of the seven links within the network. The traffic density observed at each link provides insights into the impact of the introduced toll facility. Furthermore, the examination continues with the routing ratios and their response within the time range of 0 to 30. These insights help to validate the working of the model.

5.3.2 Stage 2 - Peak hour based inflow

The supply node will provide the network with a specific traffic inflow. The inflow for this simulation will be dependent on the inflow during peak hours. Here the time frame of 6:30 am until 9:30 am is being analysed, based on these insights, an inflow $\bar{\lambda}$ at the supply node is created. According to Jenelius [16], the portion of traffic demand during peak hours for each time interval of 15 minutes is given by figure 5.3. Resulting in maximum traffic demand at around 8 am.

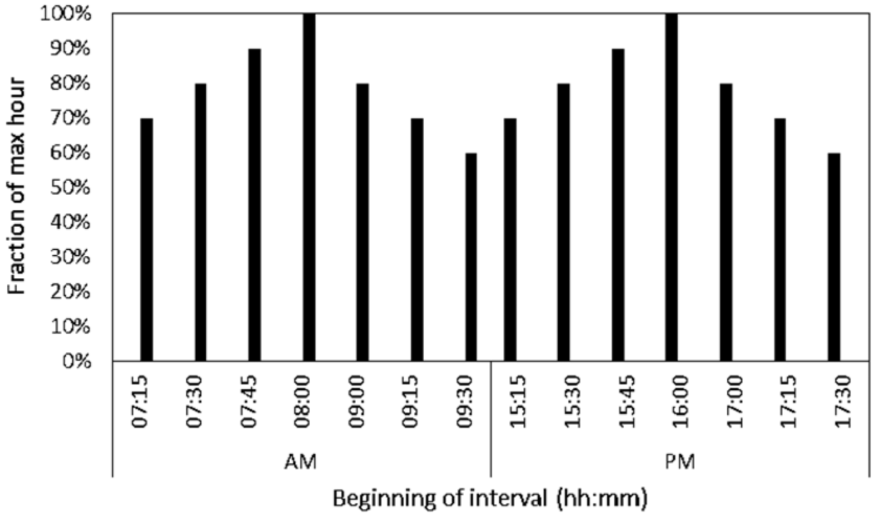

Figure 5.3: Fraction of traffic during peak hours in vehicles per hour[16].

Based on this, a typical pattern will be generated to mirror real-life scenarios. The maximum traffic inflow at the busiest time, 8 am, is set to 4000 cars/hour. The remaining time intervals are approximated according to the values of figure 5.3.

The allocation of traffic fractions per time interval is maintained as static in the configuration used in figure 5.3, with predictable variations in increments or decrements of 10 or 20 per cent. However, actual traffic inflow shows more fluctuations and therefore an adjustment is made to the selected inflow for this network.

Consequently, the inflow pattern generated included a maximum inflow of 4000 cars per hour at around 8 am. Additionally, the selected time frame for the research is extended to cover the period from 6:30 am to 9:30 am, resulting in a simulation of traffic over a timeframe of 3 hours. This adjustment creates a more accurate representation of real-world traffic conditions and increases the robustness of the generated findings. The traffic inflow $\bar{\lambda}$ has been made visible in figure 5.4

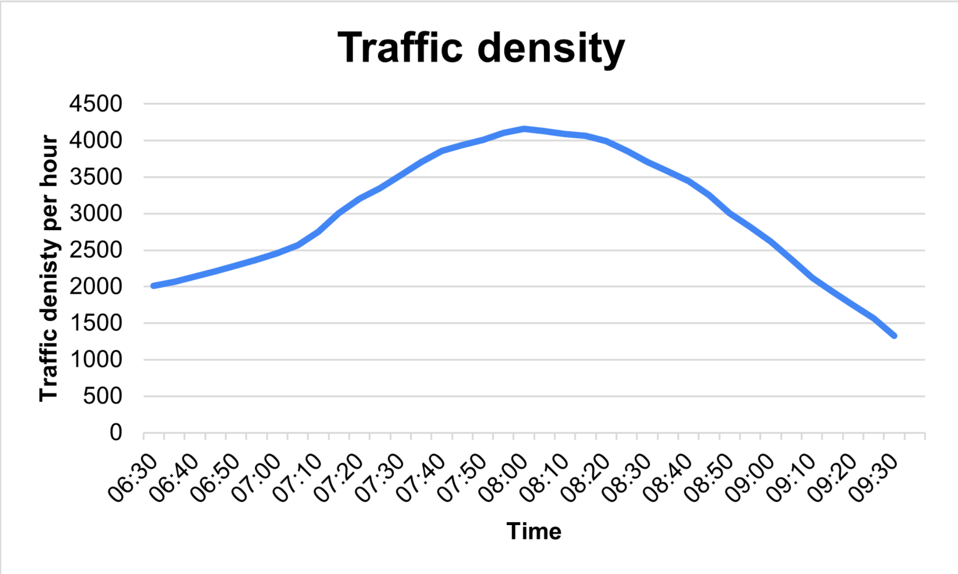

Figure 5.4: The traffic inflow, supplied by the supply node, cars per hour

The traffic inflow equation presented in figure 5.4 is as follows:

$$\bar{\lambda} = 2250 + 1900 * sin(0.04 * \pi * t) + 50 * cos(0.5 * \pi * t) \tag{5.24}$$

After the traffic is analysed, the impact of a Dynamic Toll Price is introduced within the network. The inflow will be equal for both situations, and Key Performance Indicators can be compared provided in section 4.6.

5.4 Minimising travel costs optimisation tool

For the assignment of the introduced toll price, dependent on the traffic density, an optimisation tool is required. As mentioned in section 3.1, the aim is to minimise travel costs, and therefore the introduced tool seeks the lowest aggregate travel time experienced by the vehicles. This is performed in numerous iterations with different assigned toll values. The investigated toll ratio is selected to be between 0 and 10. It is pertinent to note that the optimisation tool operates with a precision of 0.1, therefore, 100 iterations are executed to find the minimal average travel time for this given initial conditions and traffic inflow. This optimisation tool provides the toll ratio assigned to link 4. For the research, the distribution between the toll for links 4 and 5 stays 2:1 at all times. Based on this tool, the minimum value is found and could be introduced to the model. The impacts of the reduction in travel time will be analysed and discussed in section 6.

5.5 Data visualisation

The Key Performance Indicators (KPIs) outlined in section 4.6 provide significant and valuable indicators for analysing and drawing conclusions on the impact of implemented Dynamic Toll prices on a specific network. However, it is equally crucial to observe the overall response of drivers within the network. This involves presenting traffic density per road, allowing for a clear overview of changes in the two scenarios and understanding the shifts of traffic within the network. The traffic density is given by the number of cars per hour and the time frame.

Furthermore, the four routing ratios illustrate the fraction of traffic assigned to each link after this traffic has approached nodes v_1 and v_2. Observing the actual response of the system and validations ensures that the sum of routing ratios exiting one node is always equal to one. With the implementation of toll prices, there might be a noticeable change in the distribution of routing ratios, meaning that a larger part of the traffic is opting for different routes. This visualisation improves the understanding of the dynamics and response of the system and validates the effectiveness of the Dynamic Toll Pricing system in influencing driver behaviour and route distribution. The routing ratios are presented by the fraction at the given time frame.

Finally, the average travel time offers insight into the duration it takes for drivers to traverse the network from the supply node to the demand node. This aligns with the objective of minimising travel costs and moving the network towards a societal optimum or Second Wardrop Equilibrium, in which the average travel time is minimised. The examination of average travel time, over time, conditions with and without a Dynamic Toll and a uniform inflow, facilitates a clear and comprehensive understanding of the response and impact of the introduced dynamic toll, which can be both numerically and relatively compared.

Chapter 6

Results

This section is split into two separate parts. The initial part, labelled as stage 1, provides insights into the system response when Dynamic Toll Prices are implemented with a constant traffic inflow. The subsequent part, stage 2, employs peak hour-based traffic inflow, reflecting traffic patterns between 6:30 am and 9:30 am during the morning peak. This approach challenges the model to respond to high traffic numbers and fluctuations in the traffic supply.

6.1 Stage 1 - Constant inflow

This section includes the examination of the system's response under a constant traffic inflow originating from the supply node, denoted as $\bar{\lambda}$. The inflow remains constant at 3000 vehicles per hour. An analysis of all seven links in the system, together with their respective impacts on the traffic density at each link and the four routing ratios, is conducted. The objective is to investigate the changes in the proportion and the amount of traffic selecting specific links. Additionally, the average travel time is subject to be minimised or reduced in the given situation.

The initial part of this section involves the analysis of the system's response without the introduction of any additional toll. Subsequently, a toll facility is introduced at node v_2 for links 4 and 5, with a fixed distribution of 2:1. This implies that traffic traversing link 4 will incur a toll fare twice as high as that for traffic on link 5. The introduction of this toll is aimed at reducing the average travel time across the network.

6.1.1 Initial conditions

The routing ratios represent a fraction of traffic that will select the next link upon arriving at the shared node. It is important to note that this fraction does not include several vehicles and is dimensionless. The initial values of the routing ratios are given in table 6.1. The initial traffic density experienced at each link is indicated in table 6.2.

Table 6.1: *Representation of the routing rations including the initial values assigned to them.*

Routing ratio	**Value**
r_{12}	*2/3*
r_{13}	*1/3*
r_{24}	*2/3*
r_{25}	*1/3*

Table 6.2: *Representation of the initial traffic density at each link.*

Link	**Initial Traffic Density (veh/h)**
1	*3000*
2	*0*
3	*0*
4	*0*
5	*0*
6	*0*
7	*0*

6.1.2 Initial System Response

The graphical representation in Figure 6.1 illustrates the response of traffic density observed for individual network links within the designated time frame spanning from 0 to 90 units of time. The supply node serves as the point of origin for traffic inflow into the network, subsequently facilitating its distribution across diverse links of the network.

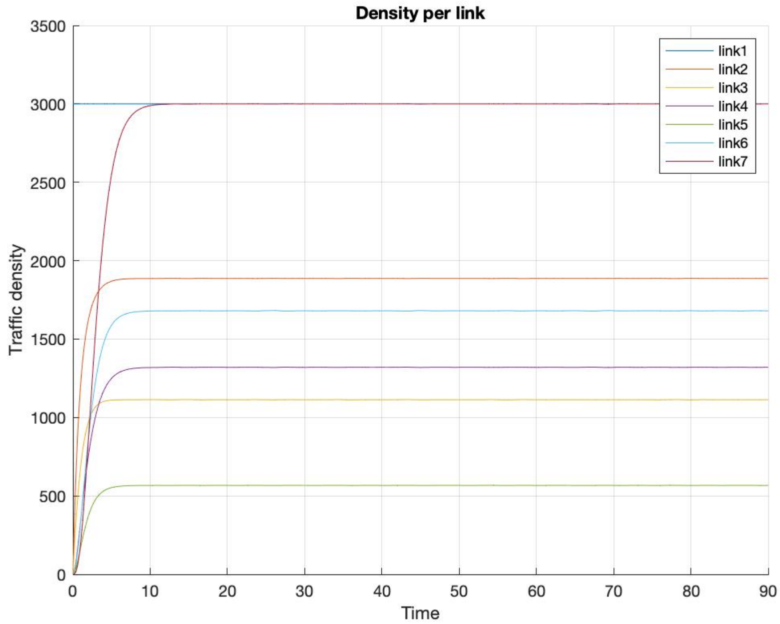

Figure 6.1: Representation of the traffic density over time for all links of the network without Dynamic Toll

Following the inflow of traffic originating from the supply node, the traffic flow undergoes a process of distribution across the various roadways within the network, ultimately seeking an equilibrium characterised by a constant volume of vehicles crossing each specified link. The graphical representation provides evident insights, revealing that the minimal traffic selects link 5. The equilibrium point for this link is identified to be exceeding 500 vehicles per hour. This is considerably lower compared to the traffic on links 3 and 4, which have higher traffic densities. As a result of reaching a stable traffic density, the corresponding values will be examined and compared with the traffic density equilibrium following the implementation of the toll facility in table 6.3.

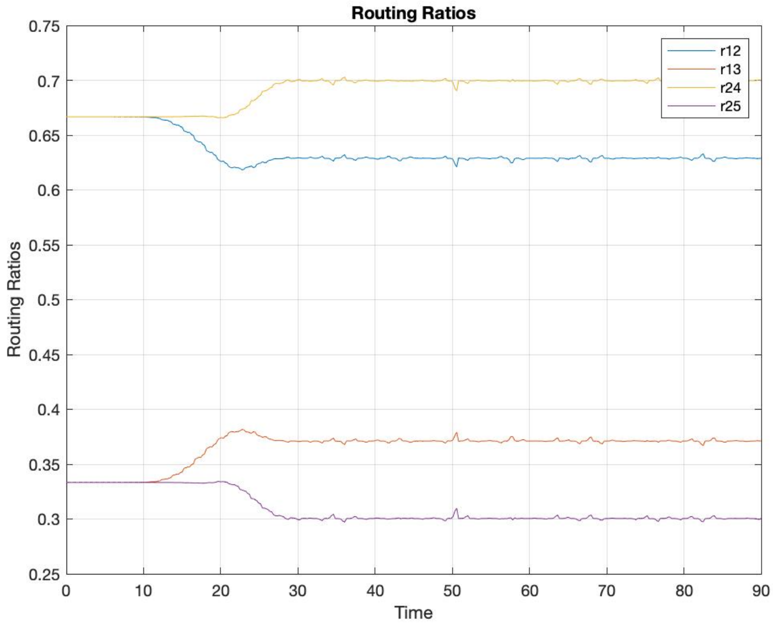

Figure 6.2: Response of the 4 routing ratios over time, without Dynamic Toll

The findings are supported by the routing ratios, indicating that only 30% of the traffic reaching node v_2 opts for link 5, a figure below the initially assigned value of 1/3 or 33.3%. The functionality observed from the routing ratios can be attributed to the relationship between the two connected routing ratios, wherein the sum always equals 1. This is expressed by the equations $r_{12} + r_{13} = 1$ and $r_{24} + r_{25} = 1$, which is constantly met.

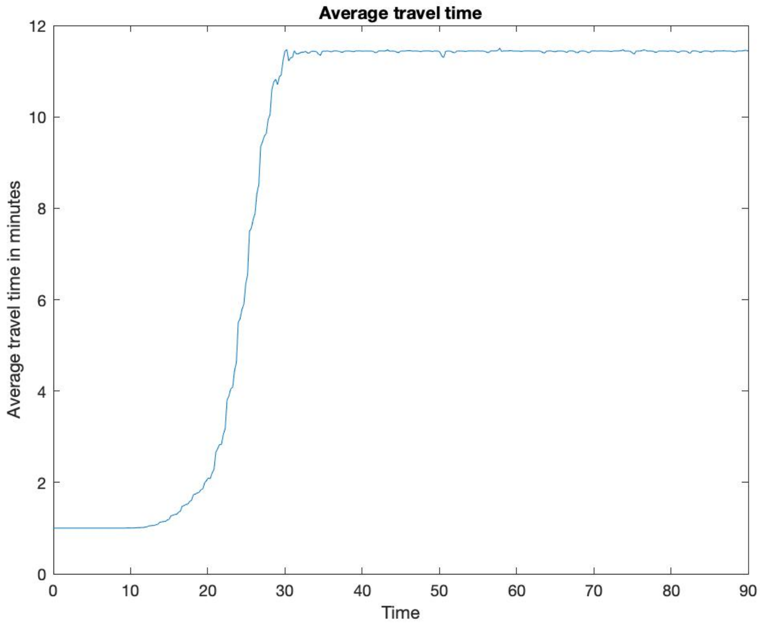

Figure 6.3: Average travel experienced by drivers under a constant inflow of traffic and without a toll facility.

Figure 6.3, shows the average travel time experienced by the vehicles, travelling from the supply node to the demand node. As initial traffic is assigned to link 1, and the remaining links are not utilised, the initial vehicles crossing the network experience a significantly low travel time. However, as traffic from the supply node continues to the other network links, the average travel time gradually increases, approaching an equilibrium state with a value of 11.4 minutes for traversing the network from the supply node to the final demand node.

6.1.3 Optimisation stage 1

To determine an appropriate toll ratio for links 4 and 5, the proportional toll component of link 4 is computed. The objective of this analysis is to minimise the aggregate of average travel time across all links. This analysis provides insight into the optimal value to be assigned to link 4, aiming to minimise average travel time.

After executing the optimisation, with the code given in section A, of the average aggregated travel time, it becomes evident that the optimal dynamic toll price for link 4 should be set at 6.0 to minimise the aggregate average travel time in this specific simulation with the given initial conditions, see section C.1. An important note is that the optimisation tool has a precision of 0.1 and therefore, the values should be divided by a fraction of 10. It is crucial to emphasise that the dynamic toll ratio for link 5 consistently remains half of that for link 4, resulting in a value of 3.0. These findings will be further elaborated in the next subsections, addressing the results and corresponding effects of an introduced toll facility.

6.1.4 System response - Toll facility

Based on the findings of the optimisation toll, the assigned toll ratio of links 4 and 5 should be 6.0 and 3.0 respectively, to minimise the average aggregated travel time. Firstly, the traffic density of all the links will be presented and the routing ratios. Because the traffic density will reach an equilibrium, value, these will be compared to the between the two situations with and without the added Dynamic Toll Prices to the network.

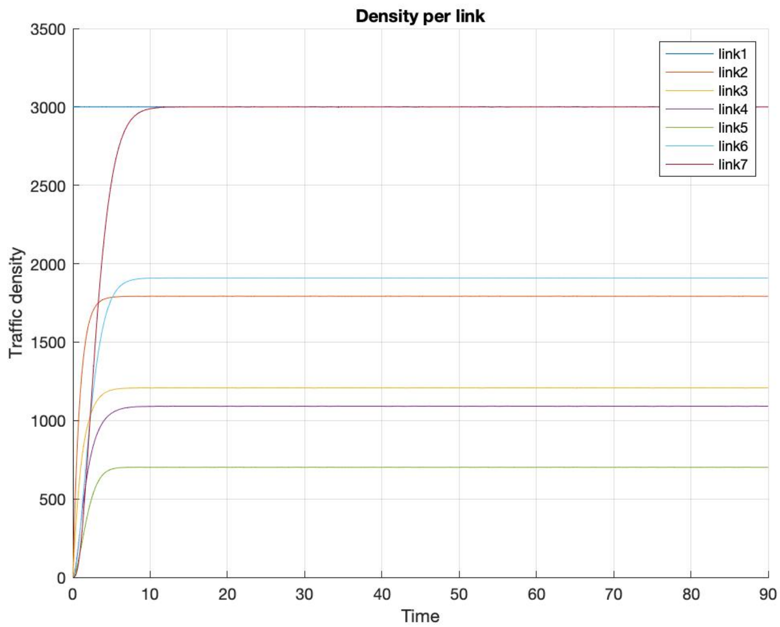

Figure 6.4: Traffic density with implemented Dynamic Toll for link 4 and 5

Figure 6.4, shows the traffic density at all links of the network after the introduction of the toll facility at node v_2 with Dynamic Toll prices for link 4 and 5. At first sight, also in this situation, the traffic density will reach anequilibrium.

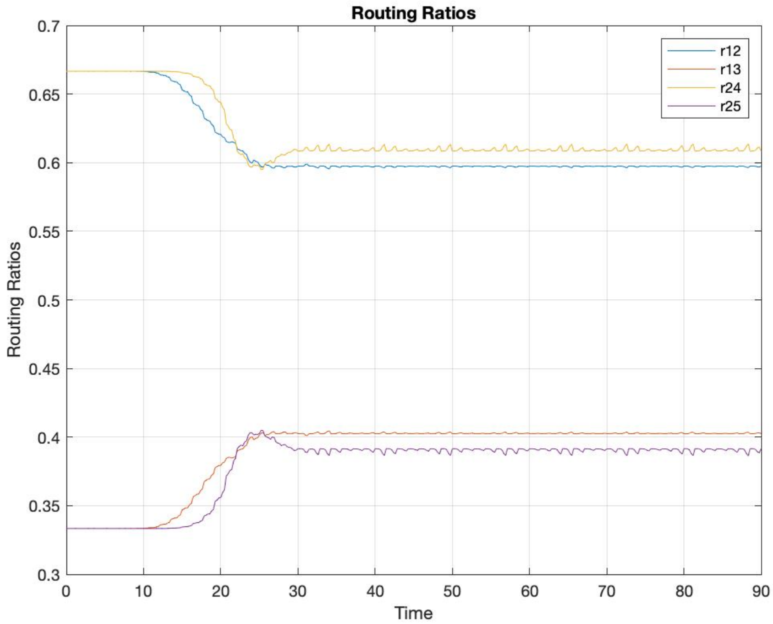

Figure 6.5: Routing ratios with implemented Dynamic Toll for link 4 and 5

The evaluation of traffic density at time = 80, when the system at a steady-state, is conducted for both scenarios with and without tolls. This approach facilitates a clear assessment of the links where traffic has decreased or increased. Such insight is important to understand the impact of the introduced toll facility. Specifically, links 3, 4, and 5 correspond to the traffic density of the three distinct paths (2, 1, and 3 respectively). These findings reveal that a greater proportion of traffic now utilises links 3 and 5, while link 4 experiences reduced traffic. Consequently, link 3 became the most preferred route for the drivers in absolute numbers.

Table 6.3: *Traffic density each link in equilibrium phase at time=80, with the corresponding percentual change.*

Link	**No toll**	**Toll**	**Difference**
1	3000	3000	*0%*
2	1887	1792	*- 5.03%*
3	1113	1208	*+ 8.54%*
4	1320	1090	*- 17.42%*
5	567	702	*+ 23.81%*
6	1681	1910	*+ 13.62%*
7	3000	3000	*0%*

Figure 6.6, the average travel experienced by the drivers travelling through the network is presented. The blue line represents the average travel time, without toll and the red line gives the average travel time with the included Dynamic Toll price. The travel time has reduced after the introduction of the Dynamic Toll Price. The exactly assigned values for links 4 and 5, are 6.0 and 3.0 respectively and gathered by the optimisation tool.

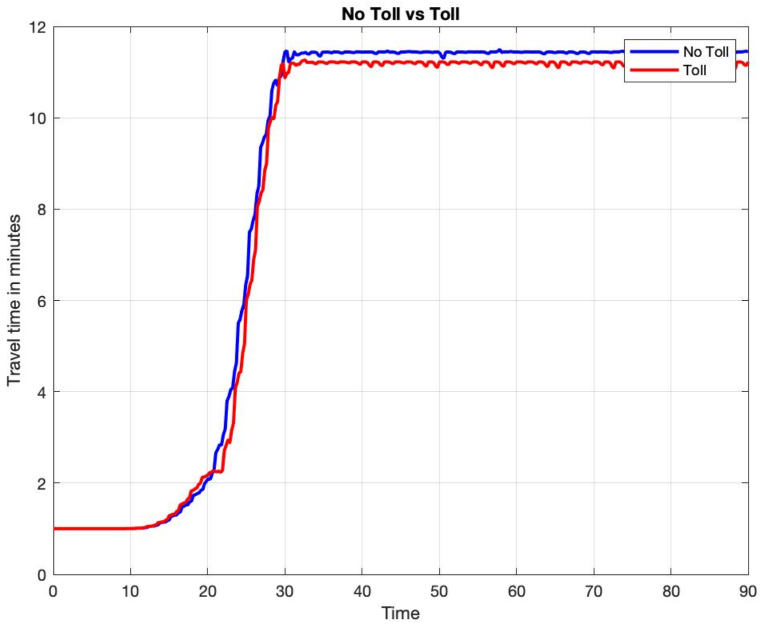

Figure 6.6: Travel time with the corresponding travel times experienced by the drivers during the given time frame.

Nonetheless, the lines provide valuable indications of the effects of the implementation of the toll facility, a reduction in travel time has been observed. However, the percentage changes in travel time will provide a relative reduction of the average travel time. These values will be presented in table 6.4. This comparative analysis encompasses the average travel time across the entirety of the simulation time frame for both situations.

Table 6.4: *Average travel time compared for the situation without and with the introduced toll facility.*

	No Toll	**Toll**	**Percentual difference**
Average travel time (minutes)	8.61	8.39	**- 2.56%**

The implementation of toll prices results in a 2.56% reduction in the average travel time for drivers. It is crucial to highlight that these values represent averages. This reduction is attributed to a 23.81% increase in drivers opting for link 5, despite it being the longer route. However, this choice contributes to an overall decrease in average travel time, playing a role in moving towards the societal equilibrium with a constant inflow of traffic.

6.2 Stage 2 - Peak hour-based inflow

This section presents the model's response to the generated traffic inflow, as illustrated in Figure 5.4, representing the average traffic density denoted as $\bar{\lambda}$. The maximum traffic inflow peaks at approximately 4000 vehicles per hour, this is below the system's maximum capacity. The simulation time spans from 6:30 to 9:30 am.

First, the initial results are discussed without the implementation of the Dynamic Toll. Subsequently, the same optimisation tool is used to determine the optimal toll ratio assignment to the links. Due to variations in inflow and a reduction in inflow after the absolute peak, this value may deviate from the previously calculated optimal ratio and is therefore recalculated accordingly. Additionally, in this phase, the assigned value to toll 4 is again twice as high as that assigned to toll 5.

6.2.1 Initial conditions

The initial values of the routing ratios have the following values, made visible in table 6.5 and the initial traffic density specified for each link is presented in table 6.6.

The 7 links of the system and their response on the inflow of traffic at link 1 have been made visible in figure 6.7. This system is based on the model described in section 5. Additionally the routing ratios are depicted for r_{12}, r_{13}, r_{24} and r_{25} and change over time and their respective behaviour is visible in figure 6.8.

Table 6.5: *Representation of the routing ratios including the initial values assigned to them.*

Routing ratio	**Value**
r_{12}	*5/9*
r_{13}	*4/9*
r_{24}	*5/9*
r_{25}	*4/9*

Table 6.6: *Representation of the initial traffic density at each link.*

Link	**Initial Traffic Density (veh/h)**
1	*1000*
2	*1000*
3	*1000*
4	*1000*
5	*1000*
6	*1000*
7	*1000*

6.2.2 Initial System Response

Figure 6.7 displays the absolute change in traffic density over the selected time frame from 6:30 am to 9:30 am, represented along the axis between 6.5 and 9.5. Initially, the traffic density begins at 1000 vehicles per hour, marking the beginning of the simulations. Subsequently, the traffic redistributes among the possible links and paths, and the peak-based generated inflow originates from the supply node, traversing through the system. The absolute peak occurs just before 8:00 am. It is important to node that all traffic enters the network via link 1 and follows three paths to link 7, converging to reach the final demand node. The travel time from link 1 to link 7 is clearly evident. Considering the importance of traffic using link 5, the least selected route due to its longest travel distance, the maximum traffic selecting this link (or path 2) is approximately 850 vehicles per hour. Additionally, following the absolute peak of the peak hours, just before 8:00 am, the traffic inflow diminishes, leading to a reduction in traffic density at all links of the network. The analysis of the path-specific links, namely links 3, 4 and 5, will be elaborated and compared later on in section 6.2.4.

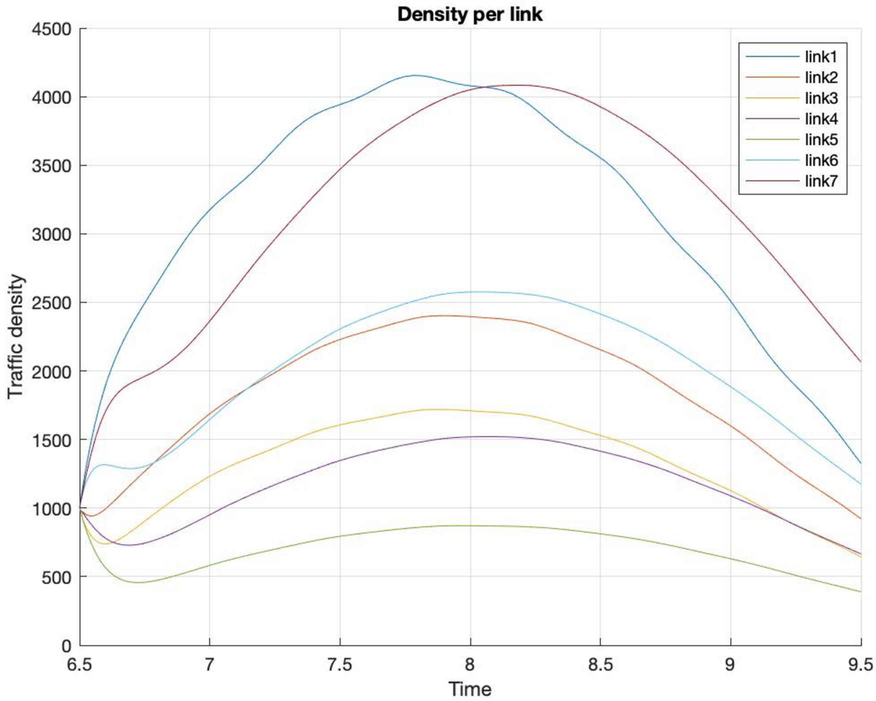

Figure 6.7: Traffic density presented of all links of the network, without the introduction of the Toll Facility.

The routing ratios shows a rapid response to changes in traffic density within the system, due to the fact that traffic changes their paths. Given that all links initially shared an equal traffic density of 1000 vehicles per hour, the system reacts to changing traffic inflow. The initial conditions of routing ratios changes rapidly, with r_{12} and r_{24} increasing to at around 0.9, while r_{13} and r_{24} decrease to around 0.1. However, shortly after this abrupt change, the routing ratios quickly stabilise and moving to a new equilibrium, undergoing only minor adjustments towards a more steady state. This indicates that the traffic needs to react to redistribute the initially assigned traffic within the network. Subsequently, traffic distribution became more streamlined. For validation of the model and the response on a fluctuating inflow, the sum of the two interconnected routing ratios is always 1.

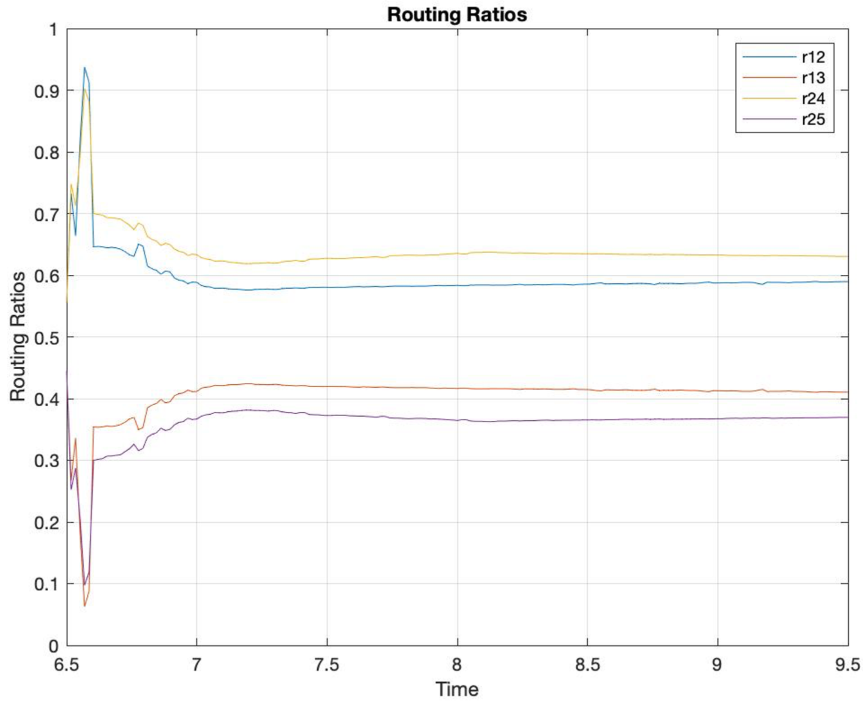

Figure 6.8: Presentation of the four routing ratio changing over time, without the included Dynamic Toll.

The fast response of traffic redistribution, represented by the routing ratios, leads to a rapid increase in travel time, increasing nearly instantaneously to 11 minutes. The initial benefit of reduced traffic density on the roads during the first time span of the simulation is quickly negated by drivers' behaviour to minimise their individual travel times, without contributing to achieving the societal optimum characterised by the lowest average travel time for all drivers.

However, after this initial increase of average travel time, they slightly decrease, only to rise again upon reaching the absolute peak of the peak hours.

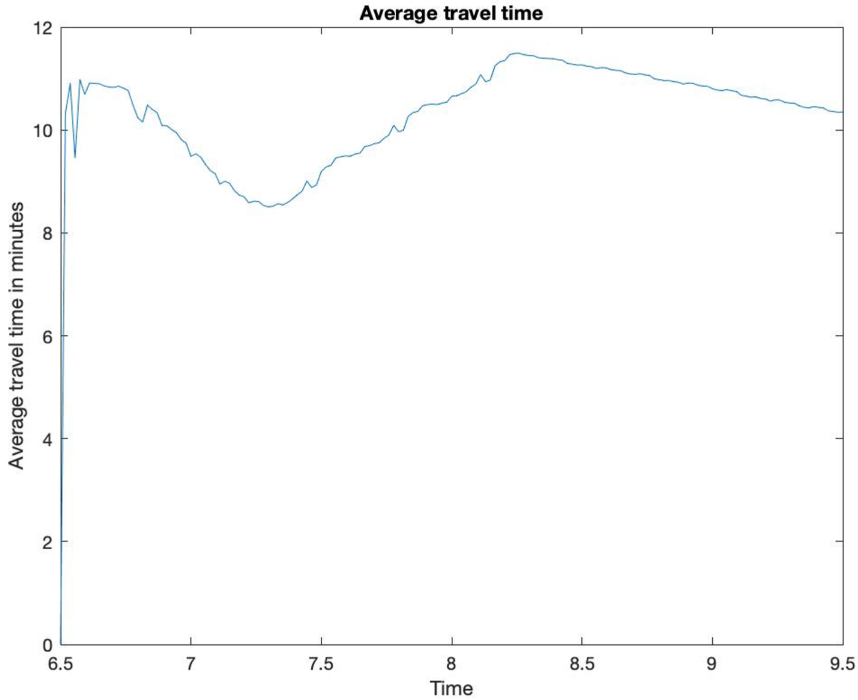

Figure 6.9: Average travel time experienced by drivers without the implemented Dynamic Toll.

6.2.3 Optimisation stage 2

After consulting the optimisation, found by the code in section A, it becomes evident that the optimal dynamic toll price, based on the peak-based traffic inflow, for link 4 should be set at 6.5 to minimise the sum of average travel time in this, see section C.2. An important note is that the optimisation tool has a precision of 0.1 and therefore, the values should be divided by a fraction of 10. It is crucial to emphasise that the dynamic toll price for link 5 consistently remains half of that for link 4, resulting in a value of 3.25. Both toll values are dependent on traffic density experienced at each link. These findings will be further elaborated in the next subsections, addressing the results and corresponding effects of an introduced toll facility.

6.2.4 System response - Toll facility

Based on the findings of the optimisation toll, the assigned toll ratio of links 4 and 5 should be 6.5 and 3.25 respectively, to minimise the average travel time.

Firstly, the traffic density of all the links will be presented and thereafter the routing ratios. To understand what the switching behaviour of drivers is under the new implemented toll facility, the traffic density of link 3, 4 and 5 are presented in 3 separate graphs, including traffic density with and without the Dynamic Toll prices.

Figure 6.10 presents the absolute response of traffic density at each link. The patterns of links 1 and 7 closely mirror the traffic response in the situation without the toll facility presented in the previous section. However, a noticeable difference is observed, with a higher volume of traffic, nearly 1000 vehicles per hour, selecting for link 5. This is significantly higher, than the situation without an introduced toll, where the maximum traffic density at link 5 only reached about 850 vehicles/hour. To precisely understand the shifts between the three distinct paths, represented by links 3, 4, and 5, the specific changes in traffic density will be discussed later in this section.

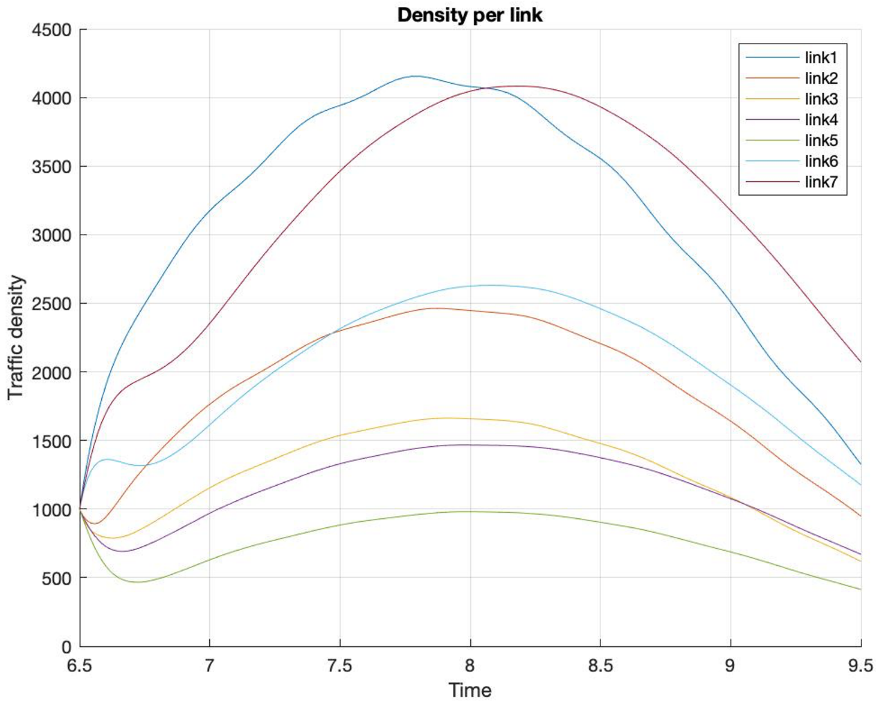

Figure 6.10: Representation of traffic density in vehicles per hour for all 7 links of the network, after the implementation of the toll facility.

Based on Figure 6.11, the changes in the routing ratios are presented. It is evident that r_{12} and r_{24} increase, while r_{13} and r_{24} decrease. However, this response is notably less rapid and reactive compared to the change in routing ratios observed in the scenario without the implemented Dynamic Toll, as shown in Figure 6.8. Nonetheless, it is important to note that routing ratios only represent the fraction of traffic arriving from the previous link and selects the given next link. Therefore, the traffic density provides a more absolute measure of the change in traffic, indicating the number of vehicles switching their paths.

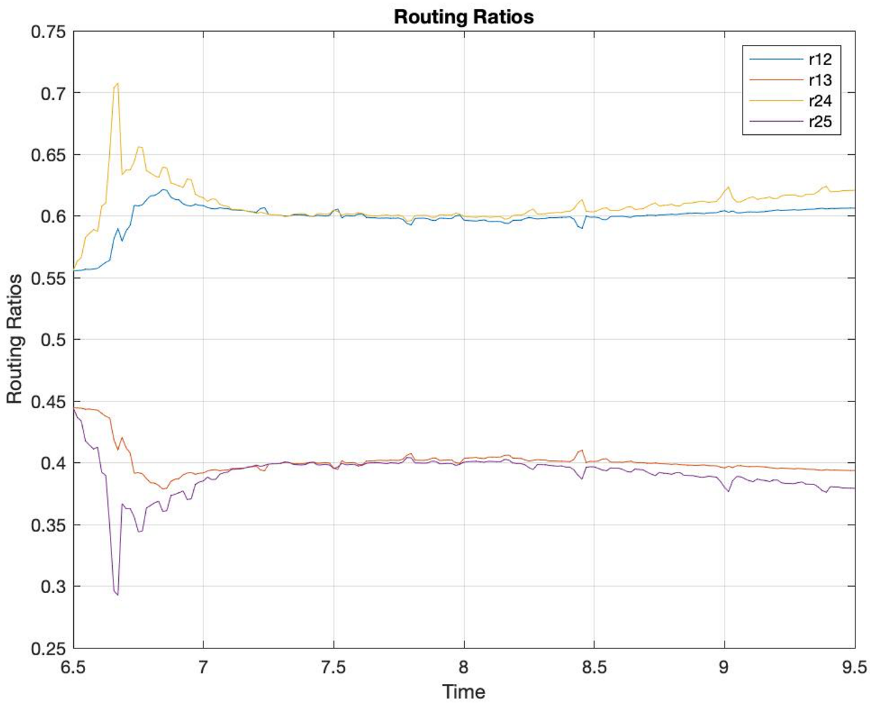

Figure 6.11: Routing ratio after introduction of the toll facility

Given that links 3, 4, and 5 exclusively represent traffic assigned to one of the three available paths used for reaching the final demand node, the numerical response of these links are significant to analyse and provides valuable insights to understand the drivers' response on the introduced toll facility. The used data is visible in both section D and E.

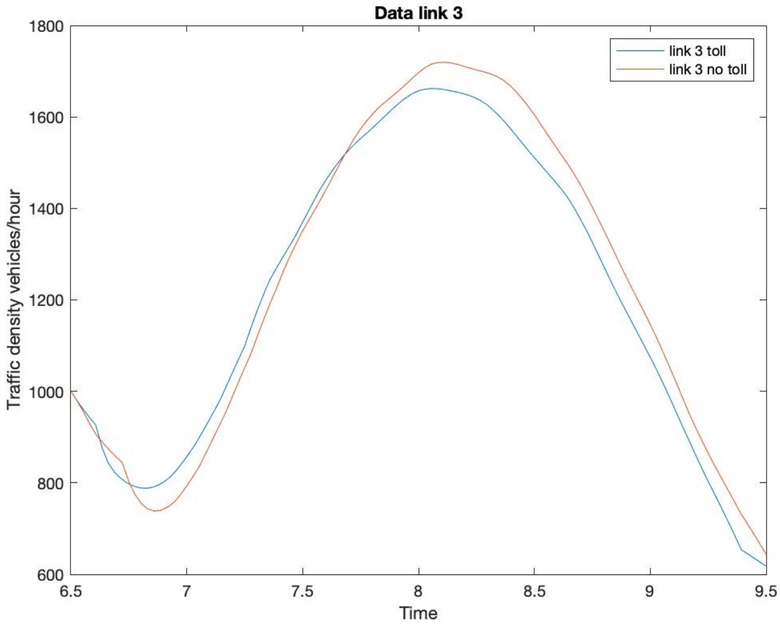

Figure 6.12: Traffic density link 3 response after introduction Dynamic Toll.

Figures 6.12 and 6.13 depict the traffic density for link 3 and 4 respectively, both before and after the introduction of the toll. It is evident that the traffic for both these links decreases after the implementation of the toll facility at node v_2. Consequently, the anticipated outcome would be an increased traffic density observed at link 5.

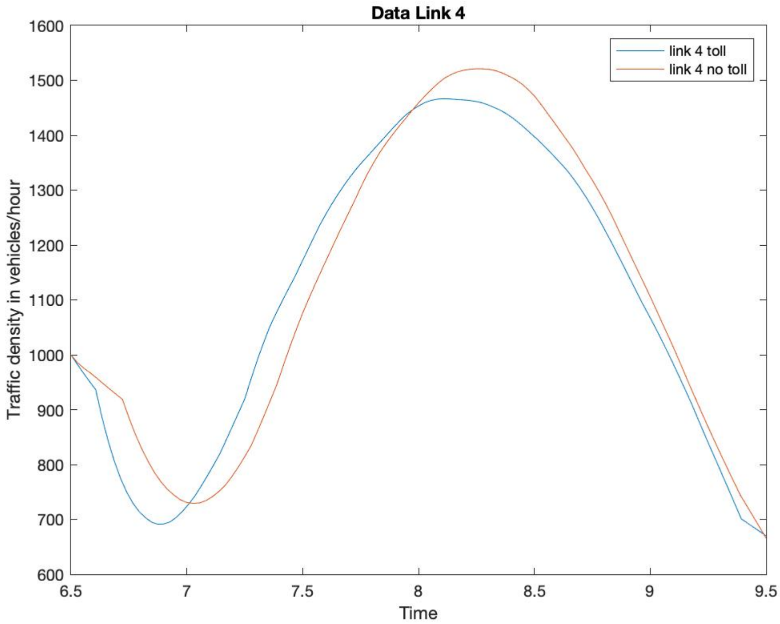

Figure 6.13: Traffic density link 4 response after introduction Dynamic Toll.

Figure 6.14, shows that it indeed true that additional drivers opts for the longer travel route, and therefore travel via link 5. The increase of traffic is significant and peaks just after 8:00 am to almost 1000 vehicles per hour, increased from approximately 850 vehicles per hour. This redistribution of traffic is expected to have a positive influence on reducing the average travel experienced by the drivers.

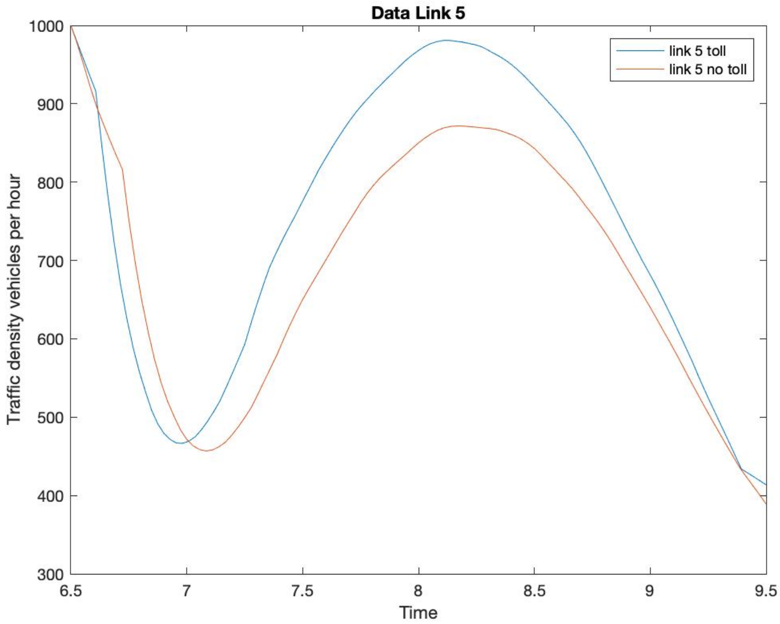

Figure 6.14: Traffic density link 5 response after introduction Dynamic Toll.

Following the implementation of the Toll Facility at node v_2, there has been a reduction in average travel time. The average delays with and without the implemented Dynamic Toll prices are visible in figure 6.15.

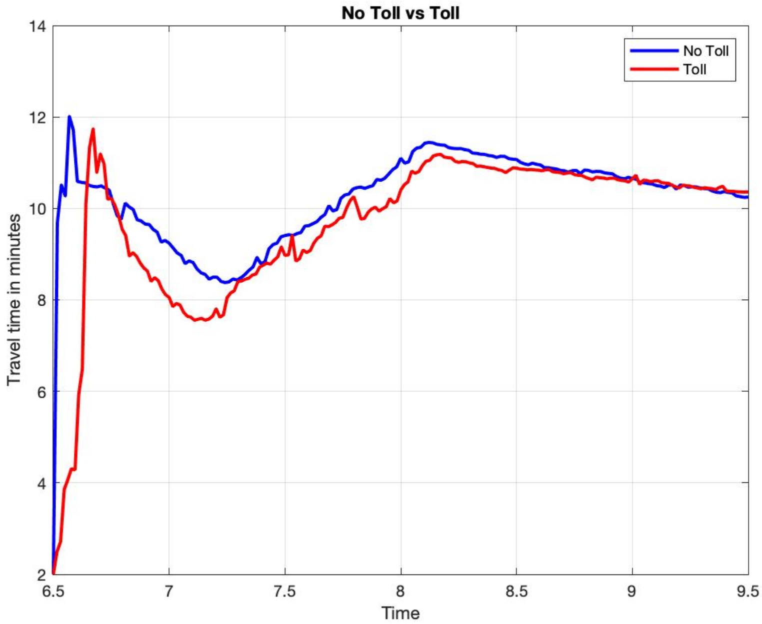

Figure 6.15: Comparison average travel times between toll and without introduced toll

Given the less reactive response of drivers to the toll introduction, the traffic distribution did not change that fast and achieved a more optimal state. Consequently, the increase in average travel time occurred considerably later after the toll's introduction. Despite that fact that travel times also peak, with the Dynamic Toll Price, the average travel time consistently remains significantly lower than the average travel time without toll and is only slightly higher at the end of the simulation time. To gain a deeper understanding of the numerical impact on average travel time reduction during peak hours, a comparison of average travel times is conducted. Although the scenario with the introduced toll prices shows slightly higher average travel times in the final segment of the simulation, the results indicate a reduction of the average travel time of **5.39%** during the selected time frame, signifying an enhancement in network performance in terms of reducing average travel time.

Table 6.7: *Average travel time compared for the situation without and with the introduced toll facility.*

	No Toll	**Toll**	**Percentual difference**
Average travel time (minutes)	10.21	9.66	**- 5.39%**

Chapter 7

Conclusion

The Dynamic Traffic and Routing model presents drivers' behaviour as they aim to minimise individual travel times when navigating through the network. Essential inputs for this model include the traffic density on each link, directly influencing the travel time for traversing a specific link. The effort of minimising individual travel times by drivers results in a social equilibrium, where the average travel time is not minimised and the outcome sub-optimal. This occurrence is related to the observation that, in both scenarios, the least number of drivers opt for selecting link 5, characterised by a longer travel distance. Under these circumstances, no driver experiences a reduction in travel time through individual switching selected paths.

To disrupt this equilibrium, a toll facility was introduced at node v_2, imposing a Dynamic Toll on both link 4 and link 5. Notably, the toll assigned to link 4 was twice as high as that assigned to link 5. These Dynamic Toll prices are proportional to the traffic density and therefore increase if the link is subjected to higher traffic densities. To assess the impact of the implemented Dynamic Toll on driver behaviour, the response was evaluated against key performance indicators, including routing ratios, traffic density, and average travel time. The optimisation tool identified the most optimal toll ratios assigned for links 4 and 5, minimising the average travel time for the given traffic inflow and selected time frame. Based on these insights and tools, the research was conducted in two phases, one involving a constant inflow of traffic and the other generated on a peak-based inflow, provided to the network through the supply node.

The analysis indicates that the implementation of Dynamic Toll pricing, particularly with generated toll ratios for links 4 and 5, significantly shifts traffic from links 3 and 4 to link 5. The optimisation tool plays a pivotal role in generating the toll ratios, based on the given inflow and given constraints. The outcomes of the simulations present a significant shift in traffic distribution, with additional vehicles opting for the longer route in favour of lower toll costs on link 5. The Dynamic Toll Pricing strategy, grounded in the number of vehicles, effectively minimizes average travel times, ensuring a more optimal traffic distribution across the network's links. This dynamic approach not only improves the overall network performance but also results in a reduction of 5.39 % in average travel times during peak-hour simulations. Consequently, the network is shifted closer to the societal optimum, given the localised nature of the problem to implement the toll facility only to node v_2, thereby making a substantial contribution to the improvement of the network's efficiency and reducing congestion to further reduce economic loss and negative environmental impact.

Chapter 8

Discussion & Future research

The findings of this research can be a starting point for implementing Dynamic Toll Pricing in transportation networks. The research shows that these Dynamic Tolls can make drivers choose different routes and spread traffic more evenly to reduce the average travel time. For future studies, some adjustments can be made to reduce average travel times experienced by drivers get closer to the societal optimum and minimise the average travel time.

However, the limitations and assumptions of this research should be considered in future research to generate a more useful simulation which is closer related to real-life behaviour. There is space available to adjust the model in such a way that it matches real situations better. In this research, it was assumed that all traffic from the starting point would travel through the entire network to reach the same destination, the demand node and the Dynamic Toll Price were only introduced to two links of the model, namely links 4 and 5.

Another important side note is the greater extent of drivers' behaviour. Some drivers might change their plans if toll prices go up, using other transportation modes, sharing rides with others or postponing their trips. Adding these real-life factors to the model makes it more realistic and helps to understand how drivers react to Dynamic Toll prices.

Furthermore, gaining insight into the socially and politically acceptable toll fare for crossing a specific link is essential. This aspect varies depending on the country, considering different governmental policies. Therefore, the system and models used should be adjusted in such a manner that they better reflect specific behaviours and situations in the selected country or even area.

The research established a proportional relation between toll prices and traffic density on each link. However, a more useful approach may involve implementing tolls only during specific peak hours or only if traffic densities are increasing to impact the fraction of traffic selecting specific routes, aiming to assess the reduction of traffic during these times. In other words, the ability to activate or deactivate the Dynamic Toll Prices. Therefore, a useful addition to the model could be to include the optimisation tool to find the optimal toll ratio for each specific link and change this accordingly, resulting in a better adaptation of Dynamic Toll fares to the traffic density.

If the simulation tools are adjusted correctly, traffic many traffic issues could be solved and impacts on the network performance could be minimised. This Dynamic Toll approach could be helpful in those situations.

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Appendix A

Code optimisation tool

```
clear all;
clearvars; clc; close all;

global r12 r13 r24 r25 pi6 pi5 pi4 pi3 pi2 pi1 pi7 Rplot j Delayplot ...
    h time_per_car traveltime toll_global

% Enter initial value of routing ratios
r12 =    ; %Initial value routing ratio r12
r13 =    ; %Initial value routing ratio r13
r24 =    ; %Initial value routing ratio r24
r25 =    ; %Initial value routing ratio r25
pi7 = 0; pi6 = 0; pi5 = 0; pi4 = 0; pi3 = 0; pi2 = 0; pi1 = 0;
Rplot = zeros(0, 0, 0);

m = 1;n = 7;j = 0;h = 0;s = 0;
initial_activator = true;

% Enter initial traffic on links:
initial = zeros(1, 7);
initial(1) = ; %Initial Traffic density link 1
initial(2) = ; %Initial Traffic density link 2
initial(3) = ; %Initial Traffic density link 3
initial(4) = ; %Initial Traffic density link 4
initial(5) = ; %Initial Traffic density link 5
initial(6) = ; %Initial Traffic density link 6
initial(7) = ; %Initial Traffic density link 7

% Optimisation tool
precision = 0.1;
range4 = [0 10];
xlen = range4(1):precision:range4(2);
for i=1:length(xlen)
    toll(1,:) = xlen; % Toll 4
    toll(2,:) = 0.5.*xlen; % Toll 5 - Toll ratio is 2:1,
    %therefore is Dynamic Toll 5 half of Toll 4
end


for i=1:length(toll)
    traveltime=0;
    time_per_car=0;
    toll_global=toll(:,i) ;
% ODE function:
time =       ; % Enter selected simulation time
timespan = [0, time];
[t, s] = ode23(@DRM2, [0 time], initial);
traveltime_vec_sum(i)=sum(traveltime);
end
```

```
figure(1)
plot(traveltime_vec_sum)
xlabel('Toll ratio link 4');
    ylabel('Aggreagred average travel costs');
    xlabel('Value toll ratio link 4')
    title('Summation average travel costs all links');
    grid on;
% Plots:
% Plot Traffic Density at each link

function [dx] = DRM2(t, x)
global r12 r13 r24 r25 pi6 pi5 pi4 pi3 pi2 pi1 pi7 Rplot j Delayplot ...
    h time_per_car traveltime toll_global

toll4= toll_global(1,1);
toll5 = toll_global(2,1);
a = [1 1 1 1 1 1 1];
b = [0 0 0 0 0 0 0];
tollratio = [0 0 0 toll4 toll5 0 0];

% Price elasticity
elasticity = 0.2;

for i = 1:7
   % Dynamic toll pricing incorporating elasticity
    tau(i) = a(i) * x(i)+ b(i) + tollratio(i)*elasticity * x(i);
end

tau_norm = normalize(tau, 'range');

pi7 = tau_norm(7);
pi6 = tau_norm(6) + pi7;
pi5 = tau_norm(5) + pi6;
pi4 = tau_norm(4) + pi7;
pi3 = tau_norm(3) + pi6;
pi2 = tau_norm(2) + min(pi4, pi5);
pi1 = tau_norm(1) + min(pi2, pi3);

Delay = [pi1, pi2, pi3, pi4, pi5, pi6, pi7];
traveltime = [pi1 + tau_norm(1) pi2 + tau_norm(2) pi3 + tau_norm(3) pi4 ...
    + tau_norm(4) pi5 + tau_norm(5) pi6 + tau_norm(6) pi7 + tau_norm(7)];
h = h + 1;
Delayplot(:, :, h) = Delay;
Traveltimeplot(:, :, h) = traveltime;

r12_dot = r12 * ((r12 * pi2 + r13 * pi3) - pi2);
r13_dot = r13 * ((r12 * pi2 + r13 * pi3) - pi3);
r24_dot = r24 * ((r24 * pi4 + r25 * pi5) - pi4);
r25_dot = r25 * ((r24 * pi4 + r25 * pi5) - pi5);
```

```
R = [-1 0 0 0 0 0 0; r12 + r12_dot -1 0 0 0 0 0;...
    r13 + r13_dot 0 -1 0 0 0 0; 0 r24 + r24_dot 0 -1 0 0 0; ...
    0 r25 + r25_dot 0 0 -1 0 0; 0 0 1 0 1 -1 0; ...
    0 0 0 1 0 1 -1];

j = j + 1;
Rplot(:, :, j) = R;

% Enter initial selected traffic inflow, represented by lambda_bar
lambda_bar =     ;
lambda = [lambda_bar; 0; 0; 0; 0; 0; 0];

dx(1:7, 1) = R * [x(1); x(2); x(3); x(4); x(5); x(6); x(7)] + lambda;

dx(1:7, 1);

time_per_car(j) = (pi1 + pi2 + pi3 + pi4 + pi5 + pi6 + pi7);
end
```

Appendix B

Code Dynamic Traffic simulation

```
clear all;
clearvars; clc; close all;

global r12 r13 r24 r25 pi6 pi5 pi4 pi3 pi2 pi1 pi7 Rplot j ...
    Delayplot h time_per_car traveltime
% Enter initial routing ratios
r12 =    ;
r13 =    ;
r24 =    ;
r25 =    ;

pi = zeros(1,7);
Rplot = zeros(0, 0, 0);
traveltime=zeros(1,7);

m = 1; n = 7; j = 0; h = 0; s = 0;

% Enter initial traffic on links:
initial = zeros(1, 7);
initial(1,1) =      ; % Initial traffic density link 1
initial(1,2) =      ; % Initial traffic density link 2
initial(1,3) =      ; % Initial traffic density link 3
initial(1,4) =      ; % Initial traffic density link 4
initial(1,5) =      ; % Initial traffic density link 5
initial(1,6) =      ; % Initial traffic density link 6
initial(1,7) =      ; % Initial traffic density link 7



% ODE function:
time =      ; % Enter simulation time
[t, s] = ode45(@DRM2, [0 time], initial);
```

```
% Plotting of traffic density all 7 links
figure(1)
hold on
for i = 1:size(s, 2)
    plot(t, s(:, i), 'DisplayName', ['link' num2str(i)])
end
xlabel('Time')
ylabel('Traffic density');
legend('show');
title('Density per link');
grid on;

% Plotting of all 4routing ratios
figure(2)
[m, n, k] = size(Rplot);
for q = 1:k
    r12_plot(q) = Rplot(2,1,q);
    r13_plot(q) = Rplot(3,1,q);
    r24_plot(q) = Rplot(4,2,q);
    r25_plot(q) = Rplot(5,2,q);
end


tmax = max(t);
tmin = min(t);
step = (tmax-tmin)/(length(Rplot)-1);

plot_vec_rr = tmin:step:tmax;

plot(plot_vec_rr, r12_plot, 'DisplayName', 'r12');

hold on
plot(plot_vec_rr,r13_plot, 'DisplayName', 'r13');
plot(plot_vec_rr,r24_plot, 'DisplayName', 'r24');
plot(plot_vec_rr,r25_plot, 'DisplayName', 'r25');
xlabel('Time');
ylabel('Routing Ratios');
legend;
title('Routing Ratios');
grid on;
```

```
[m, n, k] = size(Delayplot);
for q = 1:k
     pi1_plot(q)=Delayplot(1,1,q);
     pi2_plot(q)=Delayplot(1,2,q);
     pi3_plot(q)=Delayplot(1,3,q);
     pi4_plot(q)=Delayplot(1,4,q);
     pi5_plot(q)=Delayplot(1,5,q);
     pi6_plot(q)=Delayplot(1,6,q);
     pi7_plot(q)=Delayplot(1,7,q);
end

figure(3)
time_per_car;
step = (max(t)-min(t))/(length(time_per_car)-1);
plot_vec = tmin:step:tmax;
plot(plot_vec, time_per_car)
hold on
plot(plot_vec, time_per_car_toll)
xlabel('Time')
    ylabel('Average travel time')
    title('Time per car')
    grid on
```

```
function [dx] = DRM2(t, x)

global r12 r13 r24 r25 pi6 pi5 pi4 pi3 pi2 pi1 pi7 Rplot j ...
    Delayplot h time_per_car traveltime

h = h + 1;

toll4 =     ; % Dynamic toll ratio for link 4
toll5 =     ; % Dynamic toll ratio for link 5


a = [1 1 1 1 1 1 1];
tollratio = [0 0 0 toll4 toll5 0 0];
elasticity = 0.2;

% Tau calculation
for i = 1:7
    tau(i) = a(i)*x(i) + tollratio(i)*elasticity*x(i);
end

% Pi calculation
tau_norm = normalize(tau, 'range');
pi7 = tau_norm(7);
pi6 = tau_norm(6) + pi7;
pi5 = tau_norm(5) + pi6;
pi4 = tau_norm(4) + pi7;
pi3 = tau_norm(3) + pi6;
pi2 = tau_norm(2) + min(pi4, pi5);
pi1 = tau_norm(1) + min(pi2, pi3);

%travel time calculation
Delay = [pi1, pi2, pi3, pi4, pi5, pi6, pi7];
traveltime = [pi1 + tau_norm(1) pi2 + tau_norm(2) pi3 + tau_norm(3) pi4...
    + tau_norm(4) pi5 + tau_norm(5) pi6 + tau_norm(6) pi7 + tau_norm(7)];
h = h+1;
Delayplot(:, :, h) = Delay;
Traveltimeplot(:, :, h) = traveltime;


r12_dot = r12 * ((r12 * pi2 + r13 * pi3) - pi2);
r13_dot = r13 * ((r12 * pi2 + r13 * pi3) - pi3);
r24_dot = r24 * ((r24 * pi4 + r25 * pi5) - pi4);
r25_dot = r25 * ((r24 * pi4 + r25 * pi5) - pi5);
```

```
R = [-1 0 0 0 0 0 0; r12 + r12_dot -1 0 0 0 0 0; r13 + ...
    r13_dot 0 -1 0 0 0 0; 0 r24 + r24_dot 0 -1 0 0 0; 0 r25 + ...
    r25_dot 0 0 -1 0 0; 0 0 1 0 1 -1 0; 0 0 0 1 0 1 -1];

j = j + 1;
Rplot(:, :, j) = R;

lambda_bar =    ; % Traffic inflow at the supply node
lambda = [lambda_bar; 0; 0; 0; 0; 0; 0];



dx(1:7, 1) = R * [x(1); x(2); x(3); x(4); x(5); x(6); x(7)] + lambda;
time_per_car(j) = (pi1 + pi2 + pi3 + pi4 + pi5 + pi6 + pi7);
end
```

Appendix C

Optimisation tool results

C.1 Optimisation tool stage 1

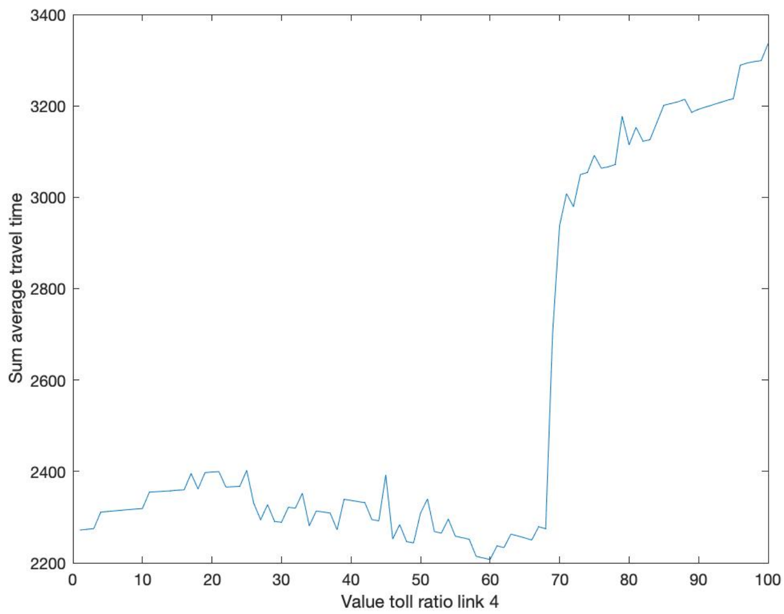

Figure C.1: Caption

C.2 Optimisation tool - stage 2

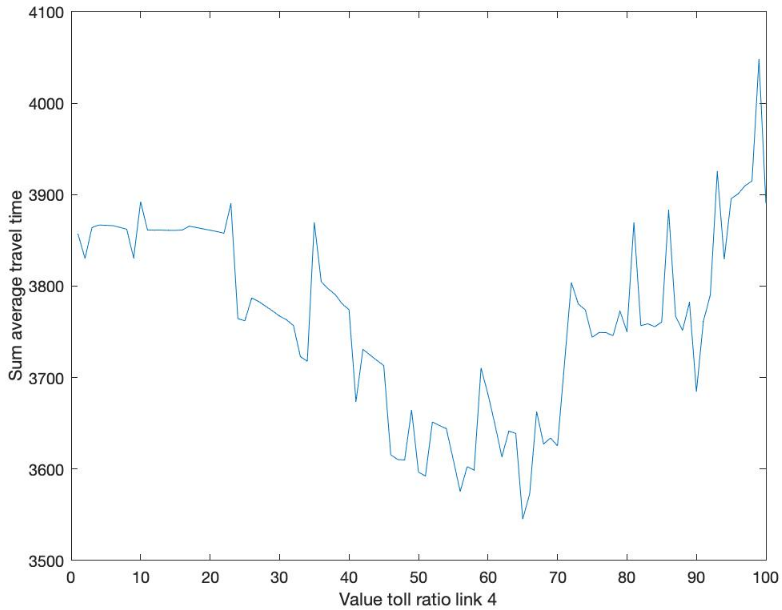

Figure C.2: Caption

Appendix D

Traffic density link 3, 4 & 5 without toll

No Toll	**Link 3**	**Link 4**	**Link 5**
	**10^3*	**10^3*	**10^3*
1	1.0000	1.0000	1.0000
2	0.9782	0.9858	0.9761
3	0.9536	0.9756	0.9490
4	0.9269	0.9670	0.9215
5	0.9034	0.9575	0.8962
6	0.8869	0.9476	0.8751
7	0.8714	0.9378	0.8547
8	0.8569	0.9282	0.8351
9	0.8435	0.9188	0.8161
10	0.8025	0.8856	0.7522
11	0.7732	0.8555	0.6969
12	0.7537	0.8287	0.6493
13	0.7425	0.8055	0.6089
14	0.7383	0.7850	0.5736
15	0.7401	0.7682	0.5444
16	0.7467	0.7547	0.5206
17	0.7571	0.7444	0.5015
18	0.7736	0.7358	0.4840
19	0.7930	0.7309	0.4716
20	0.8145	0.7291	0.4633
21	0.8372	0.7301	0.4586
22	0.8657	0.7344	0.4567
23	0.8946	0.7416	0.4582
24	0.9236	0.7511	0.4621
25	0.9524	0.7625	0.4681
26	0.9859	0.7780	0.4771
27	1.0190	0.7953	0.4876
28	1.0515	0.8143	0.4994
29	1.0834	0.8344	0.5120
30	1.1220	0.8611	0.5285

31	1.1591	0.8889	0.5454
32	1.1944	0.9173	0.5625
33	1.2275	0.9461	0.5793
34	1.2625	0.9806	0.5987
35	1.2942	1.0140	0.6171
36	1.3236	1.0456	0.6341
37	1.3509	1.0759	0.6501
38	1.3754	1.1034	0.6645
39	1.3999	1.1299	0.6784
40	1.4248	1.1558	0.6923
41	1.4504	1.1809	0.7061
42	1.4772	1.2061	0.7201
43	1.5037	1.2310	0.7340
44	1.5292	1.2552	0.7474
45	1.5527	1.2790	0.7603
46	1.5740	1.3045	0.7736
47	1.5923	1.3283	0.7855
48	1.6086	1.3494	0.7959
49	1.6228	1.3689	0.8052
50	1.6346	1.3859	0.8132
51	1.6463	1.4016	0.8207
52	1.6583	1.4165	0.8281
53	1.6708	1.4306	0.8353
54	1.6843	1.4452	0.8429
55	1.6968	1.4590	0.8501
56	1.7077	1.4715	0.8564
57	1.7155	1.4833	0.8618
58	1.7190	1.4944	0.8663
59	1.7195	1.5037	0.8694
60	1.7182	1.5103	0.8711
61	1.7150	1.5152	0.8717
62	1.7109	1.5184	0.8715
63	1.7067	1.5201	0.8708
64	1.7027	1.5211	0.8701
65	1.6991	1.5209	0.8692
66	1.6950	1.5199	0.8684
67	1.6895	1.5177	0.8673
68	1.6820	1.5134	0.8649

69	1.6712	1.5078	0.8620
70	1.6576	1.5018	0.8589
71	1.6414	1.4938	0.8547
72	1.6237	1.4836	0.8493
73	1.6048	1.4717	0.8430
74	1.5834	1.4565	0.8348
75	1.5623	1.4404	0.8261
76	1.5414	1.4245	0.8176
77	1.5208	1.4080	0.8086
78	1.5008	1.3919	0.7999
79	1.4792	1.3751	0.7906
80	1.4557	1.3564	0.7802
81	1.4293	1.3366	0.7691
82	1.4028	1.3181	0.7588
83	1.3747	1.2980	0.7476
84	1.3457	1.2766	0.7356
85	1.3166	1.2540	0.7230
86	1.2867	1.2292	0.7092
87	1.2578	1.2042	0.6952
88	1.2295	1.1796	0.6815
89	1.2019	1.1549	0.6677
90	1.1744	1.1304	0.6539
91	1.1462	1.1058	0.6400
92	1.1167	1.0802	0.6255
93	1.0854	1.0542	0.6107
94	1.0534	1.0289	0.5963
95	1.0202	1.0026	0.5814
96	0.9865	0.9754	0.5660
97	0.9529	0.9474	0.5501
98	0.9224	0.9208	0.5350
99	0.8930	0.8942	0.5198
100	0.8648	0.8681	0.5051
101	0.8378	0.8425	0.4905
102	0.8119	0.8172	0.4761
103	0.7864	0.7926	0.4620
104	0.7608	0.7682	0.4480
105	0.7345	0.7440	0.4342
106	0.7126	0.7247	0.4231

107	0.6901	0.7052	0.4119
108	0.6670	0.6855	0.4006
109	0.6436	0.66561	0.3891

Appendix E

Traffic density link 3, 4 & 5 with toll

Toll	**Link 3**	**Link 4**	**Link 5**
	**10^3*	**10^3*	**10^3*
1	1.0000	1.0000	1.0000
2	0.9794	0.9831	0.9786
3	0.9603	0.9669	0.9575
4	0.9427	0.9513	0.9366
5	0.9266	0.9363	0.9161
6	0.8774	0.8861	0.8450
7	0.8437	0.8429	0.7798
8	0.8234	0.8060	0.7208
9	0.8102	0.7746	0.6689
10	0.8000	0.7495	0.6263
11	0.7933	0.7294	0.5897
12	0.7894	0.7142	0.5586
13	0.7881	0.7032	0.5329
14	0.7897	0.6949	0.5089
15	0.7946	0.6913	0.4913
16	0.8022	0.6917	0.4791
17	0.8127	0.6957	0.4714
18	0.8276	0.7039	0.4670
19	0.8447	0.7150	0.4666
20	0.8635	0.7283	0.4694
21	0.8833	0.7431	0.4748
22	0.9066	0.7610	0.4837
23	0.9306	0.7798	0.4945
24	0.9552	0.7995	0.5070
25	0.9801	0.8197	0.5205
26	1.0101	0.8443	0.5378
27	1.0402	0.8692	0.5558
28	1.0700	0.8943	0.5742
29	1.0992	0.9194	0.5928
30	1.1389	0.9553	0.6194
31	1.1761	0.9896	0.6451
32	1.2107	1.0206	0.6688

33	1.2426	1.0502	0.6911
34	1.2670	1.0743	0.7086
35	1.2903	1.0971	0.7250
36	1.3130	1.1191	0.7405
37	1.3355	1.1404	0.7551
38	1.3612	1.1640	0.7707
39	1.3872	1.1875	0.7860
40	1.4131	1.2110	0.8013
41	1.4386	1.2344	0.8167
42	1.4602	1.2547	0.8301
43	1.4805	1.2741	0.8434
44	1.4992	1.2920	0.8560
45	1.5161	1.3087	0.8680
46	1.5311	1.3245	0.8795
47	1.5446	1.3389	0.8900
48	1.5570	1.3521	0.8995
49	1.5686	1.3643	0.9081
50	1.5814	1.3771	0.9168
51	1.5942	1.3896	0.9248
52	1.6071	1.4022	0.9328
53	1.6197	1.4145	0.9405
54	1.6317	1.4265	0.9483
55	1.6423	1.4375	0.9558
56	1.6509	1.4460	0.9623
57	1.6572	1.4533	0.9681
58	1.6606	1.4593	0.9731
59	1.6619	1.4634	0.9769
60	1.6617	1.4657	0.9794
61	1.6602	1.4666	0.9808
62	1.6577	1.4662	0.9808
63	1.6548	1.4652	0.9799
64	1.6516	1.4646	0.9788
65	1.6478	1.4636	0.9772
66	1.6426	1.4620	0.9756
67	1.6357	1.4598	0.9733
68	1.6270	1.4562	0.9694
69	1.6152	1.4514	0.9648
70	1.6022	1.4465	0.9607

71	1.5874	1.4403	0.9557
72	1.5713	1.4327	0.9499
73	1.5546	1.4240	0.9431
74	1.5378	1.4143	0.9356
75	1.5214	1.4041	0.9274
76	1.5055	1.3939	0.9188
77	1.4899	1.3831	0.9102
78	1.4743	1.3715	0.9016
79	1.4585	1.3597	0.8929
80	1.4422	1.3476	0.8838
81	1.4246	1.3349	0.8744
82	1.4026	1.3201	0.8638
83	1.3786	1.3042	0.8521
84	1.3531	1.2869	0.8388
85	1.3259	1.2683	0.8246
86	1.2975	1.2485	0.8097
87	1.2689	1.2277	0.7941
88	1.2404	1.2064	0.7784
89	1.2129	1.1846	0.7626
90	1.1866	1.1623	0.7468
91	1.1608	1.1400	0.7311
92	1.1351	1.1176	0.7157
93	1.1089	1.0951	0.7004
94	1.0834	1.0741	0.6862
95	1.0568	1.0528	0.6717
96	1.0290	1.0306	0.6563
97	1.0000	1.0077	0.6404
98	0.9700	0.9842	0.6240
99	0.9396	0.9600	0.6070
100	0.9094	0.9354	0.5899
101	0.8799	0.9105	0.5727
102	0.8501	0.8836	0.5541
103	0.8216	0.8570	0.5361
104	0.7941	0.8309	0.5188
105	0.7673	0.8054	0.5021
106	0.7397	0.7793	0.4852
107	0.7118	0.7534	0.4684
108	0.6833	0.7272	0.4512

109	0.6537	0.7009	0.4339
110	0.6448	0.6932	0.4289
111	0.6358	0.6855	0.4239
112	0.6268	0.6778	0.4188
113	0.6177	0.6701	0.4136