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A Scalable Hybrid Model for the Parameterization of Material Requirements Planning under Uncertainty

Thesis Report

Master Research Project

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Abstract

Material Requirements Planning (MRP) is a core element in manufacturing management, capable of managing complex production systems and extensive Bill of Materials (BOM) structures. A significant challenge within MRP is determining the optimal planning parameters for systems characterized by uncertainty. This research deploys a scalable hybrid model to identify these optimal planning parameters amidst uncertain conditions, to minimize overall costs. The proposed model simulates a make-to-order production environment that has demand and lead time uncertainty. This scalable model is defined by the input it receives from BOM and routing information. Stochastic behaviour is applied to processing times, replenishment lead times, customer-required lead time and customer order size. Then a genetic algorithm is utilized to find the planning parameters to minimize the sum of inventory and backorder costs. The hybrid model is subjected to different scenarios regarding BOM size and complexity. The results indicate that the hybrid model successfully identifies optimal planning parameters across a range of BOM sizes. Demonstrating adaptability to diverse demand pattern scenarios accompanied by uncertain lead times underscores the model's broad applicability. This adaptability highlights the method's potential for generic use in optimizing manufacturing planning parameters.

Acknowledgements

I want to thank my supervisor Alexander Hübl for his support and guidance during the project. It was a great opportunity to explore the field of production scheduling and hybrid modelling from which I learned a lot. Furthermore, I would like to thank my friends Sem and Masseh who helped me a great deal by performing many simulation runs.

List of Abbreviations

BOM	Bill of Materials
CI	Confidence Interval
CU	Cost Unit
DES	Discrete Event Simulation
FIFO	First In First Out
FOQ	Fixed Order Quantity
GA	Genetic Algorithm
KPI	Key Performance Indicator
LLC	Low-Level Code
MPS	Master Production Schedule
MRP	Material Requirements Planning
PDF	Probability Distribution Function
PLT	Planned Lead Time
SL	Service Level
SS	Safety Stock

1 Introduction

1.1 Background and context

In today's ever-evolving manufacturing and supply environment, the signals for generating supply orders within our supply chains have grown increasingly out of alignment with real-time demand, due to the increased complexity of global manufacturing and supply landscapes. The MRP (Material Requirements Planning) plays a critical and relevant role in the efficiency of modern supply chains [1]. MRP has been a core component in nearly every computerized approach to manufacturing management, such as Enterprise Resources Planning (ERP), and Supply Chain Management [2]. The global market size in 2021 was 5,314 million USD and is projected to reach 11,139 million USD by 2031 [3], which signifies the importance of MRP in industries.

The primary objective of MRP is to satisfy the material needs arising from external demand by scheduling jobs and purchase orders while determining the appropriate order quantities for a wide array of items. This includes the final products destined for sale, components essential for final product assembly, and raw materials procured for manufacturing. Furthermore, it must ensure the fulfilment of order due dates by determining the production timing. The interconnection between the end items and their constituent parts, often referred to as lower-level items, is described by the Bill of Materials (BOM).

MRP can be used for complex production systems and large BOM structures, hence its broad applicability [2]. The MRP planning parameters that need to be optimized are the lot sizing policy, the planned lead time and the safety stock which are defined for each material in the production system. This necessitates a comprehensive array of planning parameters for effective MRP execution [4]. One of the principal challenges confronting MRP systems lies in identifying the optimal set of three planning parameters per item, resulting in many parameters. This task is both critical and complex due to the dynamic nature of production environments and the multiple factors influencing demand and supply.

1.2 Problem statement

Analytical models can be used for optimizing MRP parameters, however, they tend to oversimplify production systems by making a set of assumptions, often overlooking the primary sources of uncertainty and complexity within the production system. The interaction between materials, production orders and resources in a production planning process can often not be captured by closed-form models [5]. As the complexity of most real-world systems is too great to be evaluated analytically, the systems must be analysed through simulation [6]. One of the most frequently employed methods for analysing manufacturing systems is discrete event simulation (DES) [7]. Simulation models allow the incorporation of stochasticity within the production system being evaluated. This inclusion of randomness enables the integration of uncertainty into the analysis, yielding a more realistic depiction of the system's behaviour. The primary challenge encountered is that the complexity of optimizing MRP parameters exceeds what can be practically addressed through simulation alone. Most real-world scheduling problems such as MRP are NP-hard problems as the solution space grows exponentially, rendering it impossible to solve many real-sized problems optimally using exact methods. The exhaustive evaluation of all potential solutions across the parameter space would demand an infeasible amount of time. To mitigate this issue, various techniques can be applied to effectively explore the search space, such as metaheuristics [8].

Traditionally, in the field of operational research, simulation and optimization were regarded as separate approaches, however, advancements in computational power have paved the way for methodologies that integrate both techniques [9]. Simulation-based optimization involves identifying specific settings of input parameters in a stochastic simulation that minimizes an objective function typically derived from the simulation's output [10]. After establishing the simulation model, meta-heuristic optimization methods can be applied to find near-optimal production planning parameters. An important concern of simulation-based optimization is the runtime consumption, as evaluating a solution candidate might demand several seconds or even minutes [11]. The solution space for planning parameters grows exponentially, potentially rendering the computational runtime impractical for real-world applications.

1.3 Research objective

This research aims to parameterize an MRP system under conditions of uncertainty with the goal of improving an objective function, which could relate to costs or specific Key Performance Indicators (KPIs). As far as available knowledge permits, the existing research regarding simulation-optimization methods to determine optimal parameters for MRP systems in the presence of uncertainty is limited. Most recent studies [4] [5] [12] are often restricted to a specific real-world scenario. Therefore, a generalized approach is required, along with well-defined benchmarks for evaluation. The proposed hybrid model is envisioned as a generalized framework for optimizing MRP parameters, serving as a foundation for subsequent research and development. Furthermore, it is essential to assess the model with scenarios featuring BOM structures that vary in both size and complexity. This results in the following research objective:

> *Apply a scalable hybrid model for the parameterization of an MRP system in the presence of uncertainty, intending to minimize the overall costs incurred.*

In addition to achieving the objective, the study tries to answer the following research questions:

Q1: How can a hybrid model be applied to optimize parameters of material requirement planning under uncertainty?

Q2: What is the influence of different scenarios regarding BOM structure size and complexity on the hybrid model?

Q3: How can a hybrid model framework be established for a generalized approach to setting the optimal MRP parameters?

1.4 Research scope

In this research, the term "hybrid model" refers to the methodology of integrating a DES model with an external optimizer. The scalability of the model is ensured by allowing BOM structures of various sizes and complexity with minimal adjustments required to the model. The types of uncertainty that the model addresses are customer demand variability and lead time fluctuations.

A significant limitation arises due to the lack of access to real-world data, which impedes the validation process, particularly for large and complex BOM structures. Instead, the research uses simulation-based validation against predetermined scenarios to analyze and validate the model's effectiveness. Validating and fine-tuning the model for larger instances lies beyond the current study's scope.

1.5 Structure of the report

The subsequent sections of this report are structured as follows: Section 2 provides a comprehensive review of the theoretical base and related literature in the field. Section 3 delineates the methodological approach adopted for the creation of a hybrid model designed to optimize MRP parameters. In Section 4, the practical execution of the model is elucidated, detailing both the configuration of the model and the setup for various experiments. Section 5 presents both the validation process and the outcomes of the experiments conducted. Lastly, Section 6 engages in a thorough discussion of the findings, and the study's conclusions are summarized in Section 7.

2 Literature review

In this section, a literature review is provided that emphasizes the exploration of various methodologies for optimizing MRP systems. First, the fundamentals of MRP systems are explained, followed by the influence of uncertainty in MRP. Analytical models have been developed to analyse and optimize MRP parameters under uncertainty. These models lead to proven solutions which can give insight into the behaviour of the system, however, assumptions must be made to simplify the model. To solve this issue simulation can be used to evaluate the MRP parameters while considering the system's complexity and inherent uncertainty. When the production system has complex BOM structures, resulting in many MRP parameters, the solution space becomes too large to be evaluated by simulation only. Therefore, simulation-based optimization with meta-heuristics can be applied to find near-optimal values within a reasonable amount of time. In this study, a genetic algorithm (GA) is selected as a heuristic.

2.1 Fundamentals of MRP systems

The MRP system addresses the medium-term planning problem by calculating production and procurement orders through a straightforward algorithm that was developed decades ago [13]. In MRP the demand for end items triggers demand for its lower-level items. While all demand for end items is characterized as independent demand, the majority of demand for lower-level items is considered dependent demand. MRP relies on BOM data and on information related to independent demand (customer orders and forecasts), which originates from the master production schedule (MPS) [2]. MRP starts with the end items at level 0 and then iterates over all the items, level by level. This process is illustrated in figure 1. From the figure, the configuration of the BOM can be derived. For instance, Item C serves as a lower-level component for Items B and Y. Given that Item B is positioned at Level 1, Item C must accordingly be situated at Level 2. Consequently, to maintain the hierarchical integrity of the BOM, Item C is also assigned to Level 2 in relation to Item Y. This hierarchical structuring is crucial for the execution of MRP calculations. During MRP, four basic steps are performed for each item: netting, lot sizing, time phasing and BOM explosion.

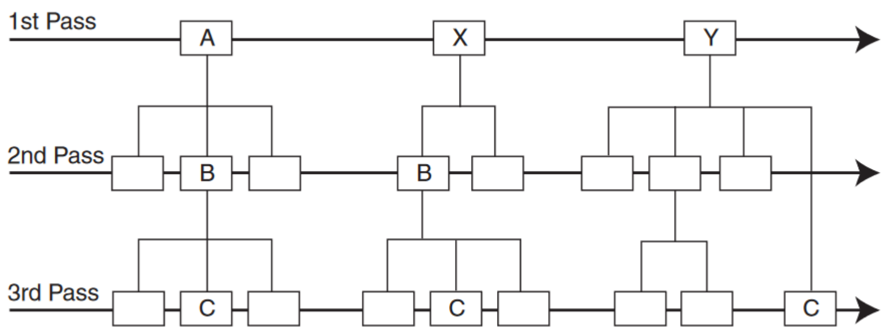

Figure 1: Level-by-level processing, where item A, X and Y are the end items [1].

In the first step *netting*, the net requirements are determined by deducting the on-hand inventory and any scheduled receipts from the gross requirements. As stated before, the gross requirements for end items at the highest level (level 0) originate from the MPS, while those for the lower-level items are generated from prior MRP iterations. The result after deducting is the projected on-hand inventory, and if it is less than zero it signifies a material requirement, i.e. the net requirements. During *lot* *sizing* the net requirements are divided into suitable lot sizes, determined by the lot size policy, to create job orders. *Time phasing* offsets the due dates of the jobs according to the planned lead times to determine the order release times. Finally, *BOM explosion* utilizes the lot sizes, the planned releases and the BOM to calculate the gross requirements for any necessary components at the subsequent level(s). The steps are repeated until all levels are processed. As MRP operates on deterministic principles, measures should be taken to address uncertainty and randomness. Production environments are typically subject to variations caused by factors like stochastic processing times, customer order interarrival times, customer order size, customer-required lead times and other unexpected events [2]. To account for this uncertainty, safety stock can be used to mitigate these challenges, which is a good choice to minimize the effects of uncertainty in customer demand [14].

Two significant weak points associated with MRP systems include capacity infeasibility and long, fixed lead times [15]. The core principle of MRP revolves around a production line with a predetermined lead time that remains constant regardless of the workload. This implies an assumption that the production line will consistently have adequate capacity, essentially assuming an infinite capacity for all production lines. However, this assumption can pose challenges when production reaches or approaches full capacity, resulting in infeasible plans that require rescheduling. Moreover, long lead times inherently result in increased inventory levels. Nonetheless, given that the penalty for delayed orders often surpasses those associated with excess inventory, longer planned lead times are favoured in practice. This tendency is further aggravated by the MRP system's reliance on fixed lead times, despite the reality that actual manufacturing durations fluctuate constantly. In an attempt to mitigate this discrepancy, planned lead times are often resorted to lengthy estimates, aiming to buffer against the variability and uncertainties in production times. Lastly, nervousness in MRP systems is the phenomenon where minor adjustments in the master production schedule can result in substantial changes in planned order releases, potentially culminating in unforeseen events with strange effects [2].

2.2 Uncertainty in MRP systems

The primary uncertainties encountered in MRP systems are related to demand and lead time uncertainty [16]. Addressing these uncertainties typically involves the implementation of planned lead times, safety stock and the adoption of suitable lot-sizing rules. Nevertheless, the financial implications of maintaining safety stocks can be substantial, presenting a significant consideration for organizations striving to balance operational resilience with cost efficiency. Dolgui and Prodhon [17] have conducted a comprehensive review focused on the parameterization of MRP systems under uncertainty. Their survey outlines various models documented in the literature addressing either stochastic demand or lead time variations. The authors' findings indicate a substantial volume of research focused on demand uncertainty, contrasting with a more limited exploration of lead time uncertainties. Notably, investigations that concurrently address uncertainties in both demand and lead times are identified as particularly scarce, attributed to the complexity of managing these dual uncertainties simultaneously. The majority of existing research tends to concentrate on the effects of individual MRP parameters, underscoring a gap in the literature regarding comprehensive approaches that tackle the combined challenges of uncertain demand and lead times.

Analytical methodologies addressing uncertainties in MRP systems have been explored through various contributions. Grubbström and Tang [18] present a comprehensive analysis utilizing input-output examination and Laplace transforms to assess MRP systems. Milne et al. [19] have developed an analytical method to identify optimal planned lead times, employing a mixed integer program (MIP) that formulates order releases in a manner analogous to traditional MRP logic. Buzacott et al. [14] delve into the effects of implementing safety stock and safety lead times within MRP frameworks, employing a stochastic model for a single-stage system to evaluate production planning outcomes. Furthermore, Mula et al. [20] introduce a mathematical programming model characterized by fuzzy constraints and coefficients aimed at optimizing production planning amidst uncertainty within industrial settings. This model facilitates the determination of the MPS and MRP for each product as well as raw materials, highlighting the strategic importance of addressing uncertainty in production environments. To evaluate and address the challenges that arise from demand and lead time uncertainty in MRP systems, the application of scheduling heuristics and simulation modelling emerges as the most relevant experimental methods [16].

2.3 Simulation studies for MRP systems

Real-world systems often exhibit complexities that disregard analytical evaluation. This is also the case when dealing with uncertainty in MRP systems. In this context, simulation stands out as a potent instrument for model assessment and data collection, aimed at estimating the model's desired attributes. Simulation is widely used in operations research and management science [6]. A primary benefit of conducting simulation studies is their capacity to incorporate uncertainty within the model, thereby facilitating the derivation of outcomes that more accurately mirror real-world scenarios. Within the literature, numerous studies have leveraged simulation as a methodological tool for addressing uncertainties in MRP systems.

A simulation study was applied by Molinder [21] to study an MRP system affected by stochastic demand and lead times. The MRP parameters are optimized using simulated annealing, and the effects of safety stocks and lead times are compared. The study's findings suggest that in scenarios characterized by high demand variability coupled with low variability in lead times, maintaining safety stocks emerges as the most effective strategy. Conversely, in situations where both demand and lead times exhibit high variability, extending safety lead times proves to be more effective. Enns [22] studied the effects of lot size and planned lead time in an MRP system by applying a simulation study. The findings indicate that the optimal choice of lot size is relatively independent of the settings for planned lead times. This suggests that the initial step in optimizing MRP processes should involve identifying an appropriate lot size. Subsequently, adjustments to planned lead times can be made to align to achieve the intended level of performance in customer delivery. A simulation study by van Kampen et al. [23], studied the effects of safety stock and safety lead time in a multi-product system with variable supply and unreliable demand information. Safety stock is praised for its ability to improve responsiveness to demand fluctuations, while safety lead times enhance the system's flexibility. The analysis demonstrates that, when dealing with supply variability, extending safety lead times proves to be a more effective strategy than accumulating safety stock. Conversely, in the presence of demand uncertainties, maintaining safety stock is typically more advantageous as it provides a direct mechanism to address sudden increases in customer orders. In situations characterized by uncertainties in both supply and demand, the study suggests that safety lead times offer greater benefits compared to maintaining an equivalent level of safety stock. Altendorfer et al. [24] studied the impact of an MRP system that encounters stochastic demand. To determine the overall costs, which include capacity, backorder, and inventory costs, the multi-item and multi-stage MRP system is simulated.

The insights gathered from simulation studies examining MRP systems in the presence of uncertainty highlight a strategic preference for increasing safety stock as a method to cope with high demand fluctuations. Conversely, when dealing with uncertainties in both demand and lead times, extending safety lead times is a preferred method.

The literature reviewed primarily concentrates on analyzing MRP parameters in systems dealing with uncertainty. Furthermore, there is a need for a more generalized approach concerning simulation frameworks in manufacturing contexts. In the paper of Negahban and Smith [7] a comprehensive review of discrete event simulation in manufacturing is provided. For a generic simulation model, Hübl et al. [25] provide a framework. The authors developed a flexible DES, where the model is generated based on BOM and routing information. The flexible model includes MRP and stochastic settings can be applied to analyze complex production systems. This scalable simulation model is utilized by Felberbauer et al. [26] to study MRP and Kanban systems.

The observations derived from the above findings predominantly concentrate on analysing MRP systems under uncertainty. While these insights are invaluable, they fall short of offering a practical framework for optimizing MRP parameters across diverse scenarios. To fulfil the objective of this research, it is essential to venture beyond the existing analysis and explore alternative methodologies that can provide a more dynamic and scenario-specific optimization of MRP parameters. This exploration must continue to leverage the benefits of simulation, utilizing its strengths in replicating complex systems and uncertainties, while integrating additional techniques that allow for the adaptive and strategic tuning of MRP settings to enhance system performance under varying operational conditions.

2.4 Simulation-based optimization and scheduling heuristics

Simulation optimization seeks the optimal configuration of input parameters for a stochastic simulation model, aiming to minimize a specific objective derived from the simulation's outputs. This process is particularly crucial, given that many advanced simulations can demand significant resources, including time, money, or other resources. Therefore, optimizing the number of simulations conducted during the search for these optimal parameters becomes paramount. In scenarios where the parameter space is extensive or potentially infinite, the optimization technique must incorporate a mechanism for systematic exploration. In this context, random search methods are particularly beneficial due to their proficiency in navigating complex, high-dimensional spaces. The inherent stochasticity of random search methods enables them to evade local optima more efficiently. A typical example of a random search strategy is the Genetic Algorithm (GA). The GA is widely recognized for its simplicity of implementation and its broad adoption across various software platforms. Its effectiveness in a diverse array of optimization problems underscores its versatility and power [10].

Simulation-based optimization in production environments is an area of research that has been explored through various studies. First, a review of the main approaches for simulation optimization is given in the work of Fu et al. [27] and Amaran et al. [10]. Sobottka et al. [28] developed a hybrid simulation-based model to increase energy efficiency in production environments. The research showed that a tuned genetic algorithm as the optimization method provided the best results. Jodlbauer and Huber [29] present a simulation-based optimization to parameterize various production planning methods, such as MRP. For the optimization, a standard evolutionary algorithm is applied to the simulation model to find the optimal parameters. Similar to MRP, demand-driven material requirements planning (DDMRP) depends on many parameters. A recent study [30] presents a genetic algorithm and a simulation algorithm that calculates the objective functions to parameterize the DDMRP. Gansterer et al. [31] applied simulation-based optimization of the three MRP parameters in a make-to-order stochastic environment. Various heuristics are studied, of which the Variable Neighborhood Search (VNS) gave the best results in this study. Authors [4] [12] used simulation-based optimization for optimizing the planning parameters of MRP, where simheuristic algorithms are proposed to minimize overall costs. This hybrid approach combines operational optimization rooted in metaheuristics with simulation as an iterative process. Karder et al. [32] investigate simulation-based optimization of a real-world MRP system. Surrogate models approximate the objective function values of the stochastic production environment. The work of Werth et al. [5] describes the simulation-based optimization of the MRP parameters with an NSGA-II algorithm on a single real-world use case. The computational cost was reduced by the use of a surrogate model.

A comprehensive review of production scheduling heuristics with genetic programming can be found in Nguyen et al. [33]. In Sukkerd et al. [34], a hybrid genetic algorithm for solving an MRP system with finite capacity constraints in a flexible flow shop is proposed. The hybrid algorithm consists of a GA and tabu search.

Over recent decades, the efficacy of heuristic optimization techniques across a diverse array of problem domains has created numerous optimization paradigms, many of which draw inspiration from natural processes. Examples include evolutionary algorithms, simulated annealing, and ant colony optimization. The development and practical application of these heuristic optimization algorithms in scientific research and industry necessitate sophisticated, adaptable, and user-friendly software systems. Heuristic optimisation software systems' design and architectural development present considerable challenges for developers. These challenges stem from the vast array of algorithms, the wide spectrum of problems to be addressed, and the diverse requirements of distinct user groups [35]. An available framework for heuristic optimization is HeuristicLab which serves as a framework specifically designed for heuristic optimization, offering a comprehensive suite of tools for the development, testing, analysis, and optimization of complex problems [36].

At its core, a heuristic optimization process can be described as an iterative sequence in which an algorithm modifies one or more solutions to a problem in a specific manner to enhance their quality, while also generating results that communicate the optimization progress to stakeholders. Within the HeuristicLab framework, this process is encapsulated by four fundamental modules: Algorithm, Problem, Solution, and Result. These modules serve as the primary actors in the optimization process and are implemented as base classes within the framework. HeuristicLab is structured to facilitate the development of a wide variety of optimization algorithms and problems. This is achieved by allowing for the extension of these four base classes, where developers can inherit from them and flesh out their abstract methods to tailor the framework to specific optimization needs. HeursticLab offers functionality that can be expanded through the integration of algorithm and problem plugins, enriching its capability to address a broad spectrum of optimization challenges. To bridge the gap between the foundational framework and the end-user, HeuristicLab includes a layer that transcends the base classes to offer a graphical user interface front-end. This interface is pivotal for facilitating all user interactions, presenting the results generated by the algorithms, and managing administrative tasks such as the installation and removal of plugins or the modification of global settings [37].

Utilizing simulation-based optimization techniques to parameterize MRP systems shows promise in improving production planning. Among these techniques, the GA stands out as a widely acknowledged heuristic for solving production scheduling challenges, showcasing its ability to address complex optimization problems effectively. However, its application within an MRP system under uncertainty that is optimized through a simulation-based framework is relatively unexplored. This gap offers an opportunity to investigate the GA's effectiveness in optimizing MRP parameters and its potential for generalizability.

2.5 Genetic Algorithms

Evolutionary learning encompasses a variety of algorithms to tackle optimization problems. Inspired by natural evolution and population dynamics, evolutionary learning serves as a computational intelligence approach, aimed at improving individuals across successive generations, underpinning its efficacy in solving complex problems [38]. Most evolutionary algorithms fall under the category of adaptive heuristic search algorithms, with GAs being the most well-known example. GAs resemble the principles of natural selection, wherein only the fittest individuals are capable of surviving, reproducing, and passing on their traits to the succeeding generation, thereby adapting to environmental changes [39].

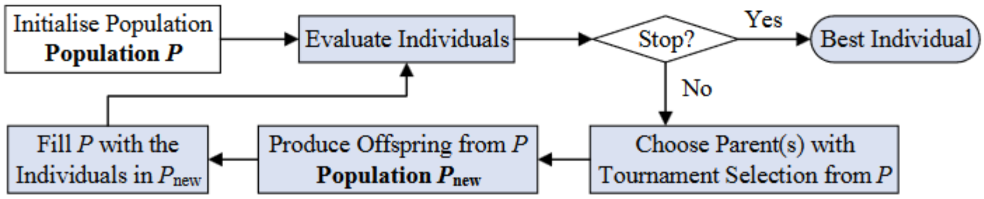

Figure 2: Flowchart of a typical evolutionary algorithm using a tournament selector [38].

Genetic algorithms stand apart from many other metaheuristics due to their utilization of three fundamental concepts: (1) employing a population of solutions to direct the search process, (2) incorporating crossover operators to merge two or more solutions, thereby generating potentially improved alternatives, and (3) actively controlling diversity to maintain exploration throughout the optimization process [40]. In figure 2, the flowchart for an evolutionary algorithm, specifically a GA, is presented. The GA starts by randomly generating a population with individuals representing solutions to the problem. Each individual's fitness is evaluated, typically representing the objective function relevant to the optimization problem under consideration. Individuals of higher quality are more likely to be selected as parents to produce offspring. Subsequently, genetic operators, including crossover, mutation, and reproduction, are employed on the chosen parents to produce offspring. Crossover involves replacing certain genes in one parent with corresponding genes from the other parent. Another common operator is mutation, where a subset of genes is randomly selected, and the value of those genes is altered [41]. This iterative process persists until a termination criterion, such as reaching a specified number of iterations, is fulfilled. Subsequently, the best individual is chosen as the output of the GA [38].

The primary considerations of tuning the GA revolve around the size of the population and the method of individual selection. Population size typically involves a trade-off between efficiency and effectiveness. A population that is too small may not adequately explore the search space, while an excessively large population could impede method efficiency, rendering timely solutions unattainable. Maintaining population diversity for as long as possible is crucial for achieving good performance. Selection tends to diminish diversity, and certain methods can lead to a rapid reduction in diversity. This challenge can be addressed by employing larger populations, increasing mutation rates, or implementing other techniques specifically designed to preserve diversity [41].

3 Methodology

In addressing the research objectives and questions, this section delineates the methodologies employed, structured into several key areas of focus. Initially, the framework underpinning the hybrid model is examined. Following this foundational overview, the functionality of the simulation model is explored in depth, explaining the mechanisms through which the model replicates a make-to-order production environment under conditions of demand and lead-time uncertainty. After the simulation model, attention shifts to the external optimizer responsible for identifying optimal planning parameters. This discussion encompasses a comprehensive problem description alongside an explication of the quality function. The quality function plays a pivotal role in the GA by evaluating the fitness of solutions, thereby guiding the optimization process toward the most cost-effective planning parameters. Concluding the methodological overview, the validation strategy is elaborated upon.

3.1 Hybrid model framework

To enhance the optimization of MRP planning parameters, a hybrid model has been proposed. This hybrid simulation-optimization approach offers a significant advantage in terms of accuracy and detail over traditional mathematical models, making it considerably more effective in dealing with uncertainty. The incorporation of simulation allows for a more realistic representation of the production environment, capturing the complexities and variabilities inherent in real-world systems.

The hierarchical structure of the developed hybrid model is characterized as Optimization with Simulation-based Iterations (OSI) [9]. This model is distinguished by conducting at least one simulation run for every iteration within an optimization cycle. Specifically, the simulation model represents the MRP system, with an evaluative function designed to assess the solution's performance. Given the large solution spaces associated with MRP systems, the application of metaheuristics becomes necessary. These high-level frameworks employ a variety of methods and heuristics to navigate the search space efficiently [8].

The framework for the hybrid model is illustrated in figure 3. This framework incorporates a user interface that has the parameters for the optimization algorithm, along with the BOM and routing information and stochastic settings as input. The output of this system is the most optimal solution found by the optimizer, characterized as the best quality. Within this hybrid model, the optimizer dispatches a set of planning parameters to the simulation, which undergoes evaluation through multiple replications. Subsequently, the simulation model sends a quality metric which represents the overall costs back to the optimizer, which in turn generates a new set of parameters for evaluation. This iterative process aims at converging towards near-optimal solutions while minimizing computational time. In the model, the optimizer sends the best quality back to the user interface whenever the specified criterion is fulfilled. Within this framework, the criterion is the predefined number of simulation iterations.

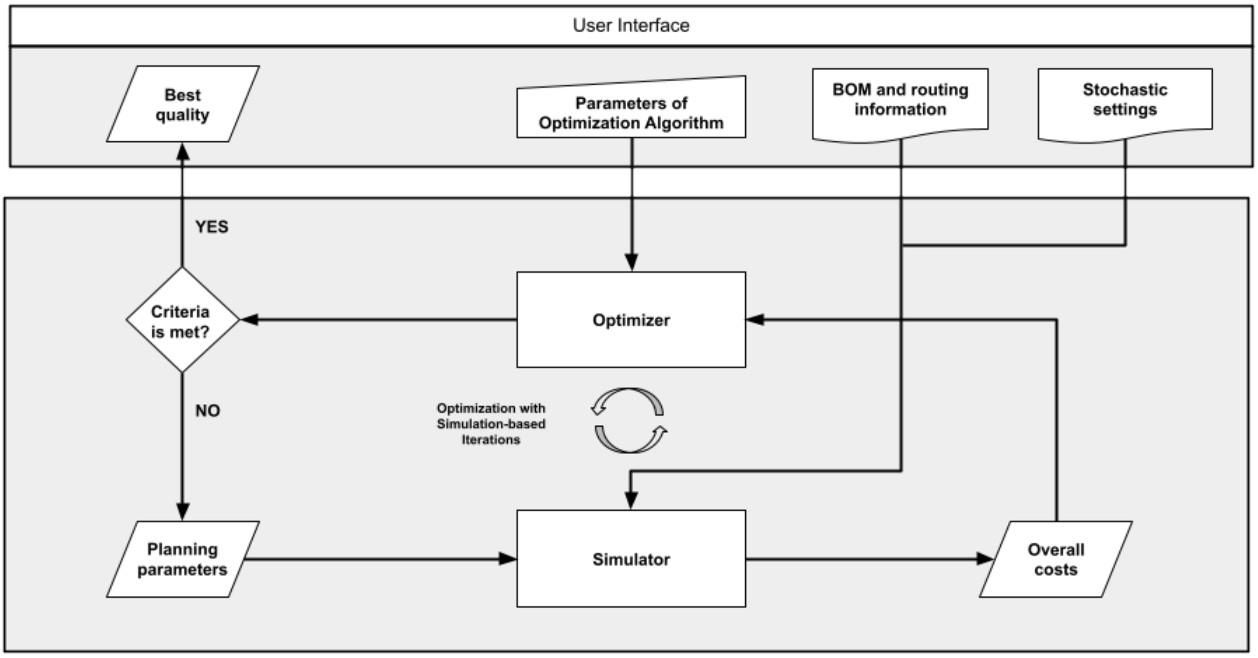

Figure 3: The hybrid model framework

3.2 Simulation model

The MRP system is simulated within a DES environment using AnyLogic PLE version 8.8, similar to the model found in the work of Hübl et al. [25]. This simulation model is designed to simulate a production system characterized by make-to-order manufacturing and the handling of multiple items. The scalable model is adaptable based on the input it receives. The primary input data for the model is derived from a spreadsheet in Excel, containing information regarding the BOM and routing details. This Excel table is imported into the AnyLogic database at the start of each simulation experiment. The imported table encompasses essential data related to both parent items and child items, including relevant information such as processing times, required quantities, and machine routing details. A more comprehensive explanation of the data structure and its generation process is provided in Section 3.2.1.

Figure 4 illustrates the simulation model's main components, comprising five key elements. The first component, 'Production Planning', is where customer orders are received and stored in a match that serves as the input for production planning. The method for production planning is MRP, which generates production orders based on these inputs. Subsequently, these production orders are forwarded to the 'Material Availability Check'. In this phase, raw materials are directly dispatched to 'Procurement'. Concurrently, following the material availability assessment, the production orders are transferred to the 'Machine Group' for processing. Additionally, within the 'Production Planning' stage, a customer order is duplicated to 'Customer Order Completion'. Here, finished goods are retrieved from inventory to fulfil the orders, and any resulting backorders are recorded and managed.

The simulation model incorporates stochasticity in customer demand and lead times. Customer demand has uncertainty in order size, customer required lead time and customer order interarrival time. The uncertainty in lead times is described by the processing times of the machines and replenishment lead times for procurement. The stochastic settings are explained in more detail in Chapter 4 for the various scenarios.

The simulation model is based on the following assumptions:

1. The MRP system has no capacity constraints and unlimited inventory capacity.
2. MRP is updated daily.
3. The lotsizing policy used is the fixed order quantity (FOQ).
4. Inventory of each item is treated equally, i.e. the holding costs are equal for each item.
5. Initial on-hand inventory of each item is equal to the safety stock.
6. Machines are sequence independent.
7. Customer order size and customer due date are discrete and uniformly distributed.
8. The customer order interarrival time is exponential or lognormal distributed.
9. Processing and replenishment times follow either an exponential or triangular probability distribution. Replenishment time is independent of the order size.
10. All the logic in the simulation model uses FIFO as a dispatching rule.

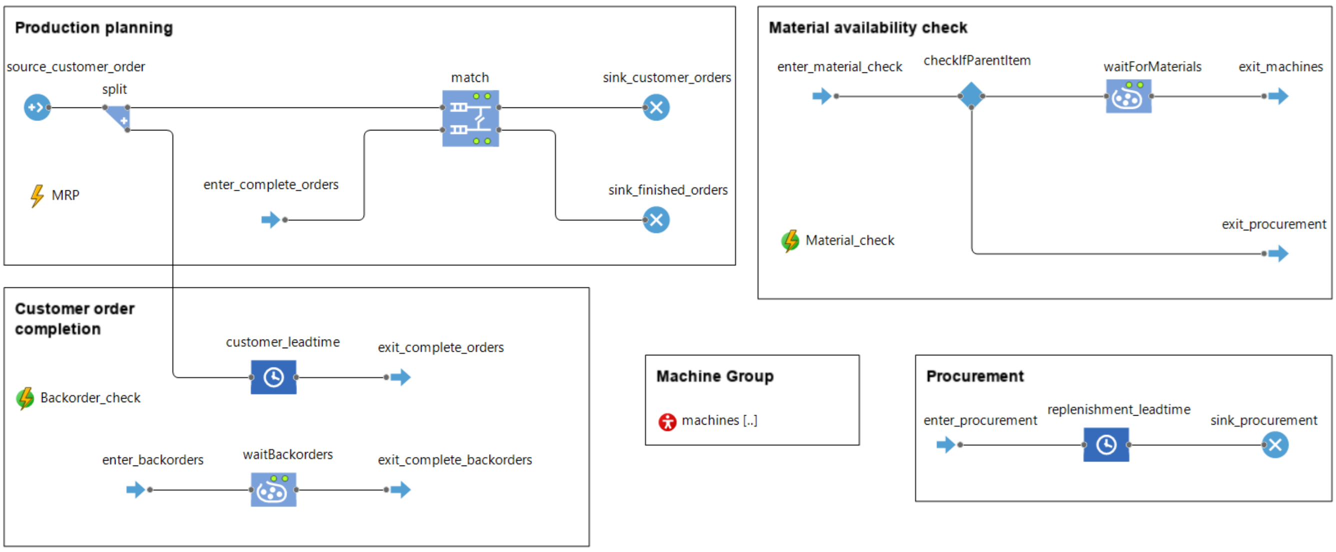

Figure 4: Top-level overview of the simulation model in AnyLogic.

3.2.1 Data generation

The main inputs for the simulation model are customer orders and the Excel file that contains the BOM and routing information which is imported to the AnyLogic database at the start. The components of the file structure are:

- **LLC:** the low-level code of an item, where end items have an LLC of 0 and raw materials have the highest LLC.
- **ParentItem:** consists of one or more childitems.
- **ChildItem:** are required to produce a parent item.
- **Quantity:** required quantity of a child item to produce a parent item.
- **MachineID:** indicates at which machine the parent item is produced.

- **ProcessingTime:** the mean processing time of a produced item.
- **ReplenishmentTime:** the mean replenishment time of a purchased item.

The database's architecture as described in Hübl et al. [25] is structured around three primary data types: ParentItem, ChildItem, and MachineID. These data types are critical in defining the relationships and processes within the database. An illustrative example of a database for a multi-item BOM is presented in table 1. This example is constructed based on the BOM structure depicted in figure 5, providing a practical context for understanding the database's configuration and application in MRP scenarios.

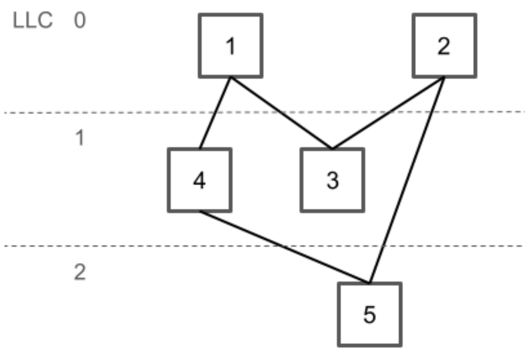

Figure 5: Multi item BOM structure example

ParentItem	ChildItem	MachineID
1	3	M1
1	4	M1
2	3	M2
2	5	M2
3	0	
4	5	M3
5	0	

Table 1: BOM and routing table

The table begins with the end items, each assigned an LLC of 0. For instance, item 1, which has two child items, namely item 3 and item 4, appears in two separate rows within the table. This reflects its dual relationship with these child items. The machine routing for each item is derived from the MachineID; in the case of item 1, it is produced at machine 1. Furthermore, items 3 and 5 are identified as raw materials, indicated by setting their ChildItem value to 0. The simulation model is designed to be universally applicable across various BOM structures, provided that the assignment of low-level codes is correct.

Customer orders within the simulation model are generated in the source source_customer_order according to an exponential or lognormal distributed interarrival time. The selection of the product type, corresponding to the final item in the BOM, is executed randomly with a uniform distribution. Subsequently, the customer's required due date and the size of the order are determined using a discrete uniform distribution. This method introduces an element of randomness while maintaining a uniform probability across all outcomes. These customer orders, along with the BOM and routing information are the input data for the simulation model. This data enables the simulation to represent and process the dynamics of customer demand and production planning within the modelled environment.

3.2.2 Production planning

Customer orders are generated in source_customer_order and remain in the match block until the fulfilment of the order is achieved. The orders contain information on the customer's required due date and the order size. A split is employed to duplicate the customer order, directing the copy towards 'Customer order completion'. This process ensures that a parallel track is maintained for the completion of customer orders alongside the main production scheduling and execution processes. The MRP event conducts daily reviews of the pending orders within the match and, based on these reviews, generates corresponding production orders. These production orders are then forwarded to the 'Material availability check' section, where they proceed to enter_material_check. Upon completion of the customer orders, wherein final items are retrieved from inventory, the completed customer orders are routed back into the match via enter_complete_orders. Within the match, the product type, order size, and customer due date of both the original customer order and the completed order are compared. A match signifies that the order has been completed, allowing both the original and completed orders to be dispatched to a sink, effectively concluding the order fulfilment process. Simultaneously, within the match, KPIs such as service level, tardiness, and customer order lead time are calculated. This approach enables the continuous monitoring and improvement of KPIs within the MRP system.

The MRP event is the core mechanism for executing the MRP process, incorporating all necessary logic for its operation. This logic is based on the examples provided in the book of Hopp and Spearman [2], which can in turn be used to validate the outcome of the simulation model. Several key assumptions are made in the application of MRP logic within this model.

Primarily, the MRP system updates daily, with the planning horizon segmented into daily time buckets. This segmentation into time buckets facilitates the translation of continuous simulation time into discrete intervals, which are then utilized for MRP calculations. Given the daily update frequency, production orders are released only when their dates are due (MRP time bucket is equal to t=0). Consequently, scheduled receipts are generated immediately upon the release of production orders. This approach diverges from traditional models that might allocate scheduled receipts to specific future time buckets. Instead, it allows for a more dynamic and responsive MRP system, capable of adapting to the real-time flow of production orders and the immediate scheduling of necessary receipts. It is important to recite that the MRP system described operates without being subject to capacity constraints. This absence of capacity limitations facilitates the dynamic adjustment of planned releases.

Finally, the model adopts a fixed-order quantity (FOQ) policy for lot sizing, which acts as a predetermined planning parameter for each item. This policy reduces the complexity associated with varying order sizes, but can still be enhanced by the optimization. A more detailed description of the logic within the MRP event can be found in Appendix A.

3.2.3 Customer order completion

The process flow for handling customer orders involves directing the duplicate of the customer order to a delay, named customer_leadtime, where the order is delayed until the due date is met. Once the customer lead time elapses, the order progresses to the exit_complete_orders stage, where an inventory check is conducted. If the inventory is adequate to fulfil the order, the order is rerouted back to the match via enter_complete_orders, and the inventory levels are adjusted to reflect the fulfilment of the order. In scenarios where the inventory is insufficient to meet the demands of the customer order, the order is diverted to enter_backorders, where it is then held in waitBackorders. Upon any update that replenishes inventory of the end items, the backorders are re-evaluated. This mechanism ensures that unfulfilled orders are tracked and prioritized for fulfilment as soon as inventory becomes available. Orders that can now be fulfilled are sent back to the match through enter_complete_orders, at which point the inventory is again updated to reflect the new status.

3.2.4 Material availability check

Following the completion of the MRP run, product orders proceed to the enter_material_check. At the select output checkIfParentItem, it is determined whether a product order relates to a parent item or constitutes raw material. Raw materials bypass further processing steps and are directly forwarded to procurement. Parent items, which necessitate the assembly of child items for production, are routed to waitForMaterials. Here, a material check process is initiated to assess the availability of the required child items. If these child items are readily available, the product order advances to the machine group, and the inventory levels of the involved child items are correspondingly adjusted to reflect their allocation to production. At the exit_machines, the specific MachineID associated with the product order is identified and the product order is transferred to the appropriate machine for manufacturing. Conversely, if any child items are found to be unavailable during the initial check, the product order remains in waitForMaterials to ensure that product orders are not prematurely advanced to production without the necessary components. Each time inventory is replenished, a reevaluation of material availability for all product orders within waitForMaterials is performed.

3.2.5 Machine group & procurement

The final stages of the simulation model are the machine group and procurement. The procurement stage is modelled as a delay named replenishment_leadtime. This delay is dynamically adjusted for each product order based on its specific replenishment lead time. The replenishment lead time is exponential or triangular distributed. The machine group within the simulation is depicted as an agent population, providing a flexible framework to simulate the operations of multiple machines within a production facility. A single line representing one machine in the machine group is depicted in figure 6. Product orders enter this stage at enter_machine and are placed in a queue, awaiting processing. Due to the capacity constraint of one order per machine, product orders must queue until a machine becomes available. The processing time for each order is calculated based on the order size and the specific machine's capabilities, introducing a realistic delay that mirrors actual production times. Each item's processing time is calculated with a probability distribution, either exponential or triangular.

Figure 6: Single line in the machine group.

Upon the completion of a product order, it is transitioned to a sink, signifying the end of the procurement or manufacturing process for that order. In this final stage, inventory levels are updated to reflect the addition of new items. Additionally, the completion of orders triggers the material availability and backorder check, ensuring that backorders are addressed as efficiently as possible.

3.3 External optimizer

The proposed approach involves discrete event simulation with an external optimization algorithm to determine the optimal parameters for MRP. For the external optimizer the open-source optimizer HeuristicLab [42] is used which is a framework for heuristic and evolutionary algorithms. This introduces an approach designed to integrate a robust meta-heuristic optimization framework, with the simulator serving as an evaluator for potential solutions. HeuristicLab allows for integration with AnyLogic and offers users access to a large number of algorithms and problem sets. The main goal of this integration is to empower users to identify and customize the most suitable optimization method for their requirements [11]. The primary challenges regarding the external optimizer are algorithm selection, parameterization of the algorithm, runtime consumption, robustness, and the stability of generated solutions.

3.3.1 Problem description and notation

The performance of the MRP parameters is assessed based on the quality metric derived from the simulation model's evaluation. This quality metric, serving as the output of the simulation run, represents the overall costs associated with the process. Table 2 details the decision variables and parameters of the model.

Sets of indices	
N	Number of periods in the simulation run $(n = 1, ..., N)$
T	Number of periods in the planning horizon $(t = 1, ..., T)$
J	Number of final products in the BOM $(j = 1, ..., J)$
I	Number of items $(i = 1, ..., I)$
M	Number of machines $(m = 1, ..., M)$
Decision variables	
$P_{i,t}$	Quantity to produce of the item i on period t
ss_i	Safety stock of the item i
L_i	Planned lead time of the item i
Cost coefficients	
h_i	Holding cost per unit of the item i
g_j	Backorder cost per unit of the product j
Model variables	
$d_{i,t}$	Gross requirements of the item i on period t
$SR_{i,0}$	Scheduled receipts of the item i on period 0
$INV_{i,0}$	Inventory of the item i on period 0 (initial on-hand inventory)
$INV_{i,t}$	Inventory of the item i at the end of period t
$B_{j,t}$	Backorders of the product j at the end of period t

Table 2: Decision variables and parameters of the MRP model

The simulation time is set to a number of periods N. The MRP is conducted based on a rolling horizon planning where planning horizon T, represents the duration over which calculations for MRP are performed. The MRP time bucket is denoted by t, and in this model, it is set to days. Central to this framework are three decision variables critical to optimizing MRP that represent the planning parameters for each item: the lot sizing policy, planned lead time, and safety stock levels. The variable $P_{i,t}$ embodies the lot sizing policy, dictating the specific production quantities. The model variables, shaped by both the decision variables and the stochastic elements within the simulation model, play a pivotal role. The output derived from these variables, following a simulation run, determines the quality value, effectively measuring the efficacy of the MRP parameters under scrutiny.

3.3.2 Quality function

In the context of the multi-item manufacturing model, the quality metric employed for evaluation encompasses both inventory costs and backorder costs. This metric is used for assessing the performance of the planning parameters. The quality of each simulation run is determined by aggregating the total backorders times the backorder costs and the mean inventory level times the holding costs. The sum of the two represents the overall costs incurred. It is important to note that penalties for backorders are exclusively applied to end items.

The decision to utilize a mean inventory level for the quality function calculation, under the assumption that holding costs are identical for each item, significantly simplifies the model's complexity. This simplification not only streamlines the computational process but also enhances the model's scalability. The calculation of the mean inventory is executed through the usage of a statistical element within AnyLogic. Given that the holding costs are uniform across all items, the statistical element undergoes an update each time the inventory levels for a specific item are adjusted, which also logs the time. This approach affords a more precise depiction of inventory levels over the temporal scope of the analysis.

The simplicity of the quality function, nevertheless, offers a realistic approach to the optimization process by compelling the optimizer to balance the penalties associated with failing to meet customer due dates against the implications of maintaining inventory levels.

3.3.3 Setup in HeuristicLab

HeuristicLab is configured as a single-objective external evaluation problem, employing integer vector encoding to represent each distinct planning parameter. These parameters are assigned bounds that delineate their range of possible values. The selected algorithm is the genetic algorithm, although it is possible to select a variety of different algorithms to address the same problem. For the algorithm's effective operation, several parameters require configuration, including the crossover method, the number of elites, mutation probability, the specific mutator to be used, population size, and the selection mechanism. The number of iterations is configured in the simulation model.

3.4 Validation strategy

Before deploying the hybrid model for experiments, it is imperative to validate both the simulation model and the external optimizer. Validation of the simulation model can be initiated through the application of simplistic structures, exemplified by figure 7. The model is then configured with deterministic inputs, facilitating the verification of outputs via manual calculations, presented in tabular format. An illustration of such calculations is offered by Hopp and Spearman [2]. Because the model is comprised of five distinct components, each can be individually tested to ascertain its functionality. Upon the successful validation of these components, the integrity of the entire model structure can be assessed.

The validation of the external optimizer for smaller instances can be executed through a parameter variation experiment within AnyLogic. This experiment involves testing all potential solutions within a specified parameter range to identify the most optimal one for the scenario. Consequently, the external optimizer is expected to identify a solution that is either optimal or near-optimal. To ensure reproducibility, the simulation model uses identical seed values for both the parameter variation experiment and the runs of the external optimizer. In this experiment, the primary objective is to demonstrate the optimizer's ability to identify solutions that are (near) optimal. Therefore, the precise accuracy of the solution is not the primary concern, meaning there's no need for extensive warm-up periods or replications for this analysis. The utilization of identical seed values ensures that the outcomes derived from both the parameter variation experiment and the optimizer, for a given set of parameters, are consistent. This methodological choice underscores a focus on evaluating the effectiveness of the optimizer in navigating the solution space rather than on the absolute accuracy of the solutions it generates.

Due to the unavailability of real-world data, validating the external optimizer for larger and more complex instances falls outside the scope of this research. The complexity and size of the solution spaces associated with these instances render them impossible to assess through the parameter variation experiment.

4 Model Implementation

This section delves into the implementation and configuration of the hybrid model, beginning with an overview of the initial setup used for validation. After this, the various scenarios subjected to evaluation are explored in detail.

4.1 Initial setup for validation

The initial setup is designed to analyze and validate the hybrid model. The absence of real-world data underscores the importance of this preliminary setup as a means of model validation. A successful outcome in this initial phase is essential, as it not only confirms the model's validity but also lays the groundwork for further experiments involving more complex BOM structures.

In this setup, a BOM comprising three items and spanning three levels is utilized, as depicted in figure 7. In this configuration, the items are arranged in series, single quantities of the child items are required with item 3 functioning as the raw material. The size of the solution space remains within manageable limits, yet the structure possesses sufficient complexity to test and validate the optimizer's capability.

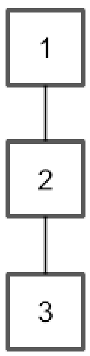

Figure 7: 3 item BOM used for model validation. Item 1 is produced at machine 1 and item 2 at machine 2.

Description	Distribution	Value	Unit
Simulation time		100	days
Warm-up phase		-	days
Replications		10	-
Customer order arrival rate	exponential	7	per day
Order size	uniform	[10, 20]	pieces
Customer required lead time	uniform	[6, 13]	days
Holding costs		10	CU
Backorder costs		100	CU
Processing time item 1	exponential	5	min
Processing time item 2	exponential	4	min
Replenishment lead time item 3	exponential	8	hours

Table 3: Simulation model settings for the initial setup used for the validation experiments.

4.1.1 Probability distributions in the simulation model

The simulation model introduces stochastic settings to account for uncertainty within the MRP system being analyzed. This model captures uncertainty primarily through variations in customer demand, alongside process and replenishment lead times. Specifically, customer demand is characterized by three elements: interarrival time, order size, and the lead time required by the customer. Both the size of the order and the lead time required by the customer follow a discrete and uniform distribution, indicating that each possible outcome is equally likely. The arrival rate is denoted by the rate λ orders per day, correlating to an exponential distribution of interarrival times with a mean of $1/\lambda$.

The processing and replenishment times within the system are likewise modelled using an exponential distribution. These distributions introduce a higher variability in the system. Notably, the processing times vary following the size of each order. These times are calculated for each item within an order individually, and the aggregate of these durations yields the total processing time for the entire order. Conversely, the replenishment lead times do not vary with order size and are directly calculated based on their mean value.

Beyond the usage of exponentially distributed interarrival times, the model incorporates a lognormal distribution to introduce an additional layer of variability. The lognormal distribution is characterized by a shape parameter, σ, which modulates the distribution's variability. This distribution is specifically integrated to facilitate sensitivity analysis, enabling the examination of the model's responsiveness to changes in the underlying distributional assumptions. The mean M of the lognormal distribution can be calculated using [6]:

$$M = e^{\mu + \frac{\sigma^2}{2}} \tag{1}$$

Rewriting the equation into μ facilitates the computation of the lognormal distribution's probability density function (PDF). Figure 8 illustrates this distribution across a spectrum of σ values, demonstrating how varying σ can modulate customer demand. A σ value of 0.1, indicative of low variability, results in a more condensed range of interarrival times, thereby rendering customer demand more predictable. Conversely, higher σ values introduce a dual scenario: potential surges in customer demand as well as intervals of minimal demand. This variability illustrates the lognormal distribution's capacity to model a wide range of demand patterns, from stable to unpredictable.

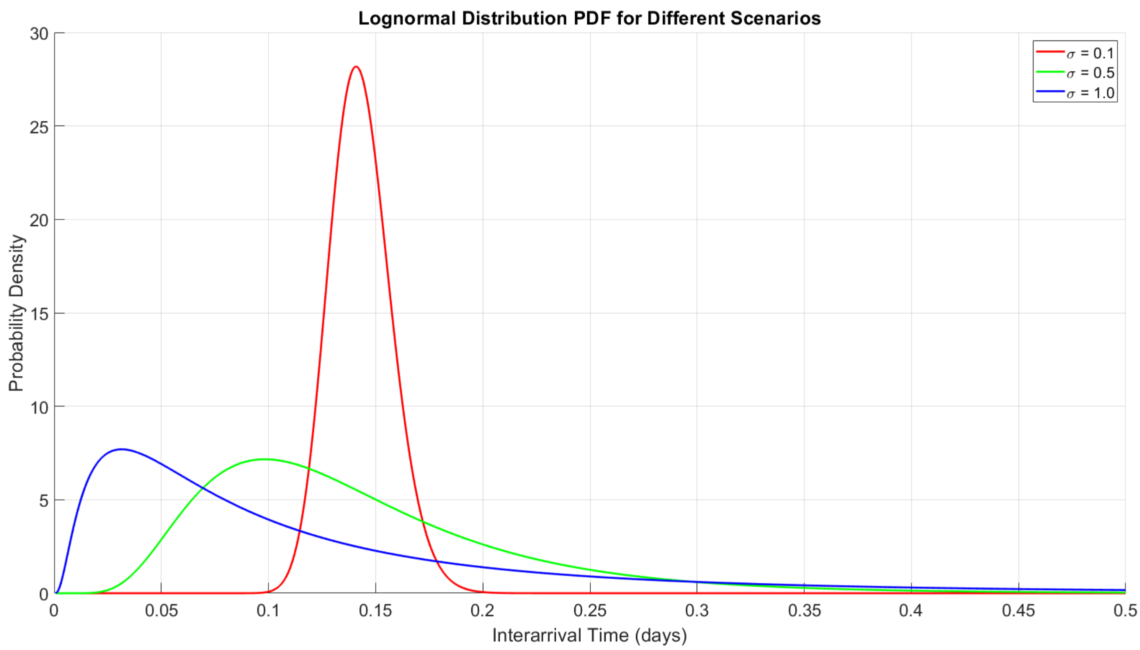

Figure 8: Lognormal probability density function for different σ values for the interarrival time of customer orders. The lognormal mean used in this plot is $\frac{1}{7}$.

For the validation experiment, the seed value is set equal to the current replication, to ensure reproducibility. The use of multiple replications, each with a different randomly selected seed value, serves as a mechanism to ensure the reliability of the results.

4.1.2 Simulation model configuration

The settings of the simulation model are outlined in table 3. In the validation experiments, the absence of a warm-up phase is a deliberate choice aimed at minimizing computational time. Given that fixed seed values are utilized, the optimizer yields the same results as in the parameter variation experiment for a given set of parameters. As a result, the accuracy of the solution becomes less critical for these validation purposes. For the experiments, an exponential distribution is used for the interarrival, processing and replenishment times.

Upon analyzing the cost structure, it becomes apparent that backorder costs significantly surpass holding costs. This discrepancy leads to the logical inference that the model is likely configured to maintain elevated inventory levels as a preventive measure against backorders. Furthermore, the model incorporates an assumption that replenishment lead times, which follow an exponential distribution, are independent of the order size. Consequently, it is anticipated that both the lead time and safety stock levels for item 3 will be maintained at minimal levels.

4.1.3 Pre-experiments with the external optimizer

The configuration of the external optimizer in HeuristicLab is established through the adjustment of seven key parameters within the algorithm: iterations, crossover, elites, mutation probability, mutator, population size, and selector. The settings for these parameters are delineated in table 4. The number of iterations and the size of the population collectively dictate the number of generations produced within the experiment. The range for the parameters aligns with those detailed in table 5. Unlike the parameter variation experiment, the external optimizer operates without employing a predefined step size. This design choice reflects the optimizer's capability to efficiently navigate through the larger solution space.

The parameters for the GA, as delineated in the provided table, were determined through an empirical process of pre-experiments. Given the distinctive nature of each optimization challenge, it becomes difficult to prescribe a universally applicable configuration for the GA. In practice, the refinement of the algorithm's settings is accomplished through a process of experimentation and observation. After these experiments, the present configuration emerged as the most effective in yielding results characterized by low-quality output. Nonetheless, it is important to underscore the possibility that this configuration may not represent the most optimal setup for the GA, especially in terms of efficiency.

Algorithm parameter	**Setting**
Total iterations	2000
Crossover	Single Point Crossover
Elites	4
Mutation probability	5%
Mutator	Uniform One Position Manipulator
Population size	150
Selector	Tournament Selector (group size 6)

Table 4: Parameters of the GA setup in HeuristicLab

4.1.4 Parameter variation experiment for model validation

The objective of the parameter variation experiment is to explore the solution space and find an optimal parameter that is used as a benchmark to validate the external optimizer. The setup for the parameter variation experiment is delineated in table 5.

The settings for the planning parameters are equal for each item, resulting in 9 planning parameters. A specific step size was applied to efficiently reduce the size of the solution space. The FOQ is defined to follow either a lot-for-lot policy or to exceed the minimum order size of 20 units. Similarly, the safety stock level is configured to be either zero or surpass the minimum order size, thereby accommodating variations in inventory requirements.

Parameter	**min**	**max**	**step**
planned lead time	1	3	1
FOQ	1	30	29
safety stock	0	30	30

Table 5: Parameter variation settings

The experimental design incorporates 10 replications, with the seed value for each replication being set to match the corresponding replication number, thereby ensuring the reproducibility of the results. This approach guarantees that each run of the experiment can be precisely replicated, providing a consistent basis for analysis. Following the completion of these replications, the results are transferred to CSV for further analysis and interpretation.

To analyze the results of the parameter variation experiment, the focus is placed on the average quality of each iteration, which represents the overall costs incurred. This measure is analogous to the output quality derived from the optimizer. The iteration exhibiting the lowest overall costs is subsequently designated as the benchmark for the validation process. Given the context of this validation phase, where the accuracy of the solution is not the primary concern, no additional statistical analysis is deemed necessary. This approach simplifies the validation process by concentrating on whether the optimizer can identify solutions that are consistent with or improve upon this benchmark.

4.2 Simple scenario

The first scenario under evaluation is depicted in figure 5, illustrating a BOM comprising five items distributed across three levels. Detailed information regarding the BOM and its associated routing can be located in Appendix B. The ”simple” BOM structure introduces a layer of complexity beyond the initial setup by featuring two end items that share child items. The inclusion of additional items considerably expands the solution space, making this scenario an effective testing ground for assessing the capability of the hybrid model. The primary objective in this context is to identify an optimal solution and to analyze the corresponding KPIs, including service level, tardiness and utilization. This approach allows for the validation of the discovered solution based on the KPIs, without the need for real-world data. Through this methodology, the efficacy of the hybrid model in navigating and optimizing BOM structures can be evaluated, providing insights into its potential applicability and performance in realistic settings.

Pre-experiments and model configuration

In response to the absence of real-world data, preliminary experiments were conducted to refine the simulation scenario, aiming to approximate a more realistic manufacturing environment. The primary goal of these pre-experiments was to achieve a target machine utilization of at least 85%. Operating at higher utilization levels poses increased challenges for the model in identifying optimal parameters while maintaining a high service level, due to reduced flexibility in handling fluctuations in demand and production constraints. To achieve the desired utilization, the customer arrival rate and the processing and replenishment lead times were adjusted. Furthermore, the model underwent testing over an extended period to assess its long-term stability. This phase was to observe the behaviour of inventory levels as they evolved, a process that inherently requires a period of adjustment before reaching equilibrium. It was noted that the mean inventory level reached a stable state approximately 200 days into the simulation. Based on this observation, a warm-up phase of 200 days was implemented to allow the system to stabilize, followed by a total simulation period of 300 days. This approach, incorporating a warm-up phase, ensures that the simulation reflects more reliable outcomes.

The configuration of the simulation model, derived from the pre-experimental phase, is detailed in table 7. A key deviation from the initial setup is the extension of the minimum customer-required lead time. Additionally, the interval for order sizes has been expanded, and the arrival rate of customer orders has been raised to 18 orders per day. Details on processing and replenishment times are provided in Appendix B. Both follow an exponential distribution. The GA setup employed for this experiment remains consistent with the parameters outlined in table 4. For this particular scenario, the solution space has been enlarged, as delineated in table 6. This expansion is designed to explore the effects of broader parameter ranges on the model's ability to find optimal solutions, while also testing the flexibility and adaptability of the GA under conditions of increased complexity.

Parameter	**min**	**max**
planned lead time	1	7
FOQ	1	100
safety stock	0	100

Table 6: Parameter interval used for the scenarios

Description	**Distribution**	**Value**	**Unit**
Simulation time	-	300	days
Warm-up phase	-	200	days
Replications	-	50	-
Customer order arrival rate	exponential / lognormal	18	per day
Order size	uniform	[10, 40]	pieces
Customer required lead time	uniform	[10, 14]	days
Holding costs	-	10	CU
Backorder costs	-	100	CU
Processing time	exponential	-	min
Replenishment lead	exponential	-	hours

Table 7: Simulation model settings for the simple scenario.

Experimental design

The first experimental setup consists of four scenarios aimed at examining the model under the variability in customer demand. This examination is facilitated by applying an exponential distribution and a lognormal distribution (with σ values of 0.1, 0.5, and 1.0) to the interarrival times of customer orders. The exponential distribution is characterized by higher variability, whereas the lognormal distribution is adjusted for low, medium, and high variability settings. For each distribution, the arrival rate is maintained at 18 orders per day. By fixing the arrival rate, the model ensures that the average volume of customer orders remains consistent. This approach allows the model to be assessed for varying demand patterns, as detailed in Section 4.1.1.

The second experiment, conducted with the simple scenario framework, is designed to evaluate the effect on the model of varying the cost function. By altering the quality metric through adjustments in cost coefficients, the experiment aims to assess the model's adaptability. Specifically, the holding costs are adjusted to 100 CU and backorder costs to 10 CU, presenting a new set of financial incentives for inventory management and order fulfilment strategies. In this setup, the probability distribution of the customer order interarrival time is defined as exponential. The outcomes of this experiment are then compared with those obtained from the previous experiment.

4.3 Complex scenario

In the next scenario, the complexity of the BOM structure is increased. The ”complex” BOM under examination, illustrated in figure 9 exhibits a hierarchical structure comprising six levels and twelve final products. The BOM table with routing information can be found in Appendix B. The complexity of production planning is significantly increased when twelve items share common child items. This scenario introduces intricate challenges in managing inventory levels, scheduling production runs, and ensuring the timely fulfilment of all end products. A notable characteristic of this BOM arrangement is the shared utilization of machinery across various items, indicating an overlap in the manufacturing processes for different components within the system. The BOM structure in this scenario necessitates the optimization of 84 planning parameters, presenting a comprehensive test of the hybrid model's performance and robustness. This extensive parameter set underscores the complexity of the task at hand, requiring the model to navigate a highly dimensional solution space effectively. If the hybrid model can optimize the parameters of this BOM within a reasonable time it provides strong argumentation for its capability to handle real-world MRP-related problems.

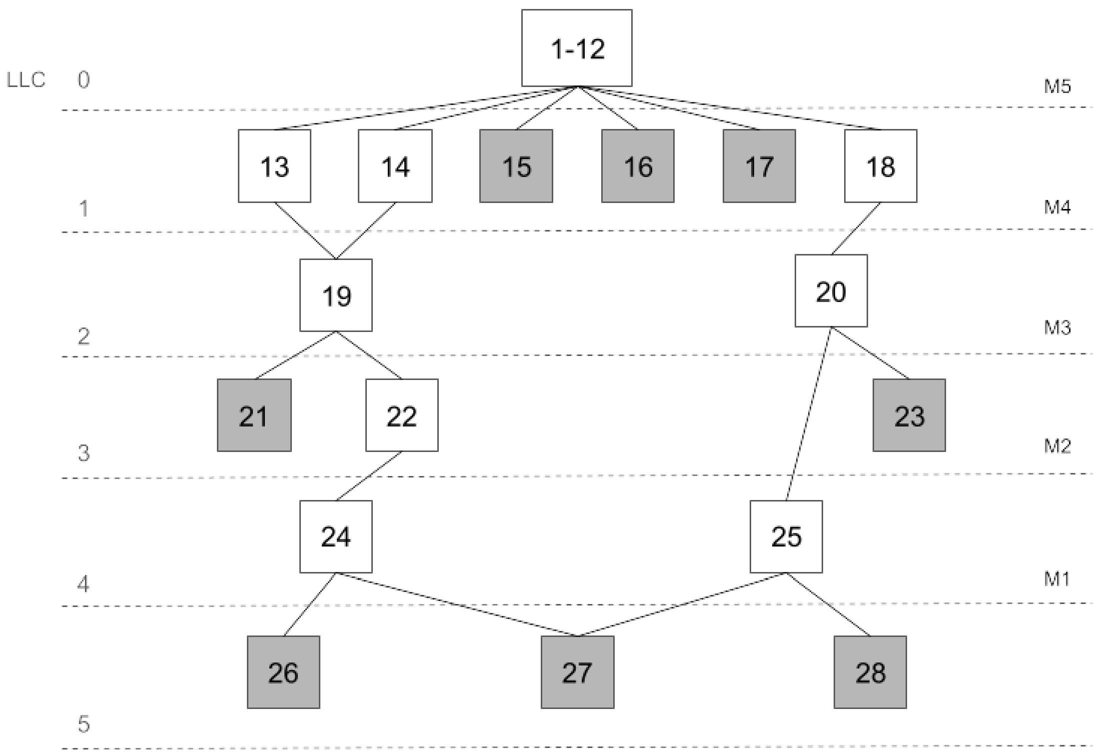

Figure 9: BOM structure with 6 levels and 12 end items [43]. The shaded items are raw materials and unshaded items are (semi-)finished products. In this instance, multiple items are produced on the same machine.

Pre-experiments and model configuration

In preparation for the experiments and similar to the approach taken with the simple BOM structure, preliminary tests were conducted to adjust the machine utilization to at least 85%. Analysis of the warm-up phase indicated that the system reached a stable state around 200 days. Consequently, for the experimental phase, the total duration of the simulation was set at 300 days. These preliminary findings also revealed that the complexity inherent to the BOM significantly increases the system's sensitivity to instability. To mitigate this in the scenario experiments, a triangular distribution has been applied to both processing and replenishment lead times. This distribution allows for the specification of minimum and maximum values, effectively preventing the occurrence of extreme values that could disproportionately impact the system's stability. Appendix B provides the min, max and mode that shape the triangular distribution of both the processing and replenishment times. Additionally, customer lead times have been lengthened to accommodate the increased number of items and levels within the BOM. The settings for the simulation model, informed by these pre-experimental adjustments, are outlined in table 8.

Description	**Distribution**	**Value**	**Unit**
Simulation time	-	300	days
Warm-up phase	-	200	days
Replications	-	50	-
Customer order arrival rate	exponential / lognormal	4	per day
Order size	uniform	[10, 40]	pieces
Customer required lead time	uniform	[14, 18]	days
Holding costs	-	10	CU
Backorder costs	-	100	CU
Processing time	triangular	-	min
Replenishment lead time	triangular	-	hours

Table 8: Simulation model settings for the complex scenario.

The preliminary experiments conducted for the complex BOM demonstrated that an expanded solution space benefits from increased population size, as this enhances the diversity of potential solutions. To guarantee an adequate number of generations for thorough exploration, the total number of iterations has been raised to 15000, as specified in table 9. The parameter range for the GA remains consistent with the descriptions provided in table 6.

Algorithm parameter	**Setting**
Total iterations	15000
Crossover	Single Point Crossover
Elites	4
Mutation probability	5%
Mutator	Uniform One Position Manipulator
Population size	1000
Selector	Tournament Selector (group size 6)

Table 9: Parameters of the GA setup for the complex BOM

Experimental design

The experiment with the complex BOM is also focussed on the variability in customer demand. Both exponential and lognormal distributed interarrival times are used in four different scenarios. The arrival rate determined in the pre-experiments is set to 4 orders per day. Then the corresponding values for the lognormal distribution with σ of 0.1, 0.5 and 1.0 can be determined to maintain a consistent arrival rate. This is used to see the effects of low, medium and high variability on the outcome of the model.

The main objective of this experiment is to evaluate how to model reacts to increased complexity. This can give insight into how it reacts on a much larger solution space while also assessing the runtime.

4.4 Planned Analysis

All results from the experiment are transferred into a CSV file, facilitating the evaluation of KPIs such as mean tardiness, service level, mean inventory, number of backorders, and overall costs, the latter serving as the quality function for assessment. Tardiness quantifies the delay of an order's completion beyond its anticipated due date, while service level indicates the percentage of customer demands fulfilled within a designated timeframe. Incorporating these KPIs into the analysis enhances the robustness of the optimizer's results. This evaluation approach allows for an understanding of the trade-offs involved, acknowledging that achieving the lowest overall costs might coincide with a compromise in service level, which could be considered undesirable. Thus, by analyzing these metrics, it becomes possible to balance cost efficiency with operational performance, ensuring that optimizations do not adversely impact customer satisfaction or service quality.

To accommodate the stochastic behaviour in the model, multiple replications of each experiment are conducted. Following these experiments, the average value for each iteration is calculated. Subsequently, a confidence interval is applied to assess the accuracy of the experiment's outcomes. The objective extends beyond merely minimizing costs; it also includes reducing the width of the confidence interval. A narrower confidence interval indicates a higher degree of consistency among the outcomes of random replications with a specified set of planning parameters, thereby enhancing the reliability of the model's predictions.

5 Validation and Experimental Results

In this chapter, the experimental results are presented, beginning with an analysis of the validation strategy employed and the results obtained from this process. Following the validation phase, a comparative analysis of the outcomes from two distinct scenarios is conducted. Subsequently, the focus shifts to examining how the model performs under different conditions by comparing the results of two scenarios. This comparative analysis aims to highlight the model's adaptability, efficiency, and effectiveness in navigating varying conditions, thereby providing insights into its applicability and potential limitations. It is important to note that the architecture of this hybrid model precludes the possibility of conducting parallel evaluations, a limitation that inherently constrains the processing speed of the model.

5.1 Validation of the optimizer using parameter variation experiment

For the validation of the optimizer, an initial parameter variation experiment consisting of 10 replications was conducted. The experiment identified an optimal solution for items 1, 2 and 3, characterized by planned lead times of [2,1,1], FOQ of [1,1,1], and safety stock levels of [30,30,0], yielding overall costs of 455.82. This result confirmed the pre-experimental rationale. Specifically, item 3, featuring an order size-independent replenishment lead time, necessitates neither safety stock due to its shorter delivery time nor an extended lead time, which in turn would result in a higher inventory level. The adoption of a lot-for-lot policy across all items aligns with the absence of setup costs and the presence of inventory-related expenses, thereby justifying this approach as cost-effective under the given circumstances.

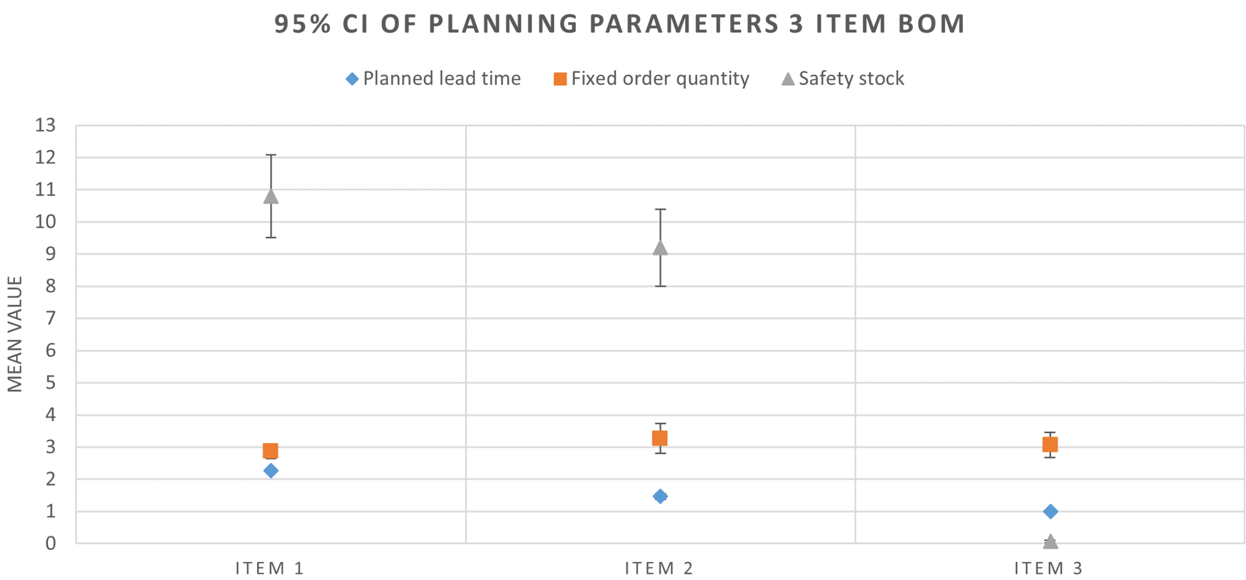

Figure 10: 95% confidence interval of the optimal planning parameters found by the external optimizer. The value on the vertical axis is represented by the average value of the 15 replications.

Due to the stochastic nature of genetic algorithms, fifteen experiments were conducted using the external optimizer to ensure reliability in the outcomes. The parameters identified through this process are presented in figure 10, depicted within a 95% confidence interval (CI). This graphical representation reveals a close alignment between the found parameters and the solutions derived from parameter variation experiments, indicating a robust adherence to expected outcomes. Specifically, the planned lead times demonstrate a noteworthy consistency with the optimal solutions identified in the parameter variation experiment. For instance, the planned lead time for item 1 fluctuates between 2 and 3 days, yet the mean value closely approximates 2 days. Furthermore, the FOQ values exhibit low variability and remain minimal across different runs. Notably, the established values for the planned lead time and safety stock for item 3 are set at 1 and 0, respectively, aligning with the rationale previously discussed. The safety stock values for items 1 and 2 demonstrate a broader confidence interval. The minimum order size is 10 items, which correlates to the values found of for the safety stock of items 1 and 2. The safety stock levels for items 1 and 2 are increased to account for the uncertainty in processing times, which are also order-dependent. Furthermore, the financial implications of maintaining additional inventory are less significant than the potential consequences of encountering backorders.

The optimizer's performance yielded an average overall cost of 468.53, with a CI of ±2.19. Notably, the quality output observed within the same parameter interval is slightly above that achieved through the parameter variation analysis. However, the solution space given to the external optimizer is much larger as no step sizes are included. Therefore, it can be concluded that the external optimizer possesses the capability to identify near-optimal values. This conclusion is further supported by the observation that expanding the solution space—namely, the range of parameter intervals—enables the optimizer to explore larger solution spaces.

Additional experiments demonstrated that enlarging the solution space enabled the optimizer to identify solutions of even lower quality outcomes. This capability is not replicable in parameter variation experiments, where the expansion of the solution space results in a scale too vast for effective exploration.

5.2 Results from scenario: Simple BOM structure

The first experimental results, focusing on a simple BOM structure, are encapsulated in table 10, which presents a comparison across four distinct probability distributions for customer order interarrival times. From the results, the following findings have been identified.

Finding 1 A notable observation from these experiments is the uniformity of machine utilization across all scenarios, attributable to the consistency in arrival rates that ensures a comparable volume of customer orders in each case. This consistency can be attributed to the make-to-order nature of the system being analyzed. In a make-to-order environment, production is initiated only in response to specific customer orders, which is constant across the scenarios.

Finding 2 The most optimal outcome, in terms of minimizing overall costs, is observed when employing a lognormal distribution with a σ value of 0.1. This is primarily due to the reduced variability in interarrival times, fostering a steady inflow of orders that allows the system to maintain lower inventory levels as a buffer against demand fluctuations. Conversely, as variability intensifies, the model compensates by aggregating inventory reserves to mitigate the increased unpredictability in order arrivals.

Finding 3 A comparison between lognormal distributions with σ values of 0.1 and 0.5 reveals similar inventory levels, yet the scenario with higher σ demonstrates a greater average overall cost and wider confidence intervals. This discrepancy is indicative of the challenges posed by heightened variability in customer orders, which, despite similar inventory strategies, leads to increased cost implications due to the broader dispersion of order arrival patterns. The rise in costs can be related to the increased number of backorders.

					Utilization		
Distribution	**Overall costs**	**SL**	**Backorders**	**Mean inventory**	M1	M2	M3
Exponential	7041±138	0.999	2.2±1.2	682	0.937	0.853	0.936
Lognormal σ=0.1	5071±160	0.998	3.6±1.8	471	0.938	0.858	0.937
Lognormal σ=0.5	5381±363	0.996	7.0±3.8	469	0.933	0.857	0.93
Lognormal σ=1.0	7322±320	0.997	5.36±3.3	679	0.936	0.855	0.935

Table 10: Results experiment with varying customer demand settings for the simple BOM structure. All the distributions use an arrival rate of 18 orders per day. The values represent the average of the 50 replications whereas the ± represents the 95% confidence interval.

The results highlight the model's adaptive response to demand uncertainty while maintaining a high service level. Furthermore, the model has identified optimal planning parameters, as detailed in table 11. The following observations regarding the planning parameters are identified.

Finding 4 A straightforward observation from the results is that the model compensates for demand variability by extending planned lead times. Items 1 and 4 which have to be produced demonstrate an increase in planned lead times for high variability scenarios. The planned lead time of item 2 remains consistent across all scenarios. In a scenario with lognormal $\sigma = 0.1$, the planned lead time of item 3 is the highest. In addition, the FOQ and safety stock are also comparatively high. This can be attributed to the increased demand for item 3, which is a child item for both the end items.

Finding 5 Concluding order quantity and safety stock adjustments is more complex for two primary reasons: the relatively low holding costs and the simplicity of the lot-sizing policy in use. The low holding costs imply that minor modifications in order quantity and safety stock levels have a marginal impact on overall costs, making changes to these parameters less significant compared to adjustments in planned lead times. However, the model seems to balance the safety stock and planned lead time. For instance, when comparing scenarios with high variability for item 1, the extension of planned lead times from 4 to 6 days acts as a primary mechanism for mitigating variability, enabling the system to maintain operational resilience without necessitating substantial increases in safety stock levels.

Finding 6 Conversely, for item 2, where planned lead times remain unchanged, the analysis reveals a different adaptive strategy. Given that item 2's composition is solely derived from raw materials, which are characterized by relatively shorter, order size-independent lead times, the model does not extend the planned lead time. Instead, it opts to increase safety stock as a defensive measure against variability. This approach demonstrates the model's ability to distinguish between items based on their unique characteristics. It employs tailored strategies to optimize inventory levels and strengthen system resilience in response to uncertainties in both demand and supply.

The second experiment with the simple BOM concerns changing the cost function of which the results can be found in table 12.

	Planned lead time					Fixed order quantity					Safety stock				
Distribution	PLT_1	PLT_2	PLT_3	PLT_4	PLT_5	FOQ_1	FOQ_2	FOQ_3	FOQ_4	FOQ_5	SS_1	SS_2	SS_3	SS_4	SS_5
Exp.	6	3	2	7	3	49	35	25	11	53	4	79	14	4	50
Log. σ=0.1	4	3	3	4	1	27	23	55	27	62	3	48	77	65	40
Log. σ=0.5	4	3	2	4	1	79	35	29	91	98	30	11	12	35	68
Log. σ=1.0	6	3	2	7	3	37	35	78	49	33	24	79	14	14	19

Table 11: Optimal planning parameters identified by the model.

Finding 1 When setting the holding costs a factor of 10 higher than the backorder costs the overall costs increase significantly as well as a drop in service level. This is because the system must fulfil orders and as a result, there will always be inventory in the system. Instead of maintaining a high inventory level to satisfy customer orders, the model is trying to minimize inventory as the backorder penalty is much lower. Because the orders must be fulfilled it is still important to minimize backorders as well. This can be related to the service level which is still 65%.

Finding 2 When comparing the planning parameters this results in shorter lead times for the high holding costs scenarios which results in lower inventory over time. In general, the order quantities are also lower which results in less overcapacity being produced. The safety stock levels are somewhat higher to account for the decrease in planned lead times. This is underscored for items 1 and 4 which show the largest decrease in planned lead time.

Costs function	Overall costs	SL	Backorders	Mean inventory
h=10, b=100	7014 ±138	0.999	2.2 ±1.2	682
h=100, b=10	24506 ±491	0.644	635.7 ±48.4	181

	Planned lead time					Fixed order quantity					Safety stock				
Costs function	PLT_1	PLT_2	PLT_3	PLT_4	PLT_5	FOQ_1	FOQ_2	FOQ_3	FOQ_4	FOQ_5	SS_1	SS_2	SS_3	SS_4	SS_5
h=10, b=100	6	3	2	7	3	49	35	25	11	53	4	79	14	4	50
h=100, b=10	2	2	1	3	2	13	11	16	26	1	52	9	2	17	34

Table 12: Results of two different scenarios regarding the holding costs h and backorder costs b cost coefficients. Both scenarios use an exponential interarrival time with an arrival rate of 18 orders per day. The values for the KPIs represent the average of the 50 replications whereas the ± represents the 95% confidence interval.

5.3 Results from scenario: Complex BOM structure

The experimental results from the complex BOM are documented in table 13, mirroring the findings observed with the simple BOM experiments.

Finding 1 Uniformity across the machine utilization of the different scenarios is observed similar to the results found in the simple BOM structure. This stability across different scenarios can be attributed to the constant rate of order arrivals and the make-to-order nature of the system, similar to the discussion of the results of the simple BOM.

Finding 2 Applying a lognormal distribution with a standard deviation σ of 0.1 yields the lowest overall costs, in contrast to a σ of 1.0, which incurs the highest costs. This outcome stems from the reduced variability associated with a σ of 0.1, as opposed to the significantly increased variability seen with a σ of 1.0. Consequently, the scenario with a higher σ necessitates the maintenance of larger inventory levels, experiences a greater number of backorders and increased tardiness. This is due to the need to buffer against the unpredictability introduced by the higher variability, which directly impacts the effectiveness of the system.

Finding 3 An analysis of the KPIs indicates that achieving a near-perfect service level is significantly more challenging in scenarios involving a complex BOM. A notable 9% difference in service level is observed between scenarios with a σ of 0.1 and 1.0, a variance not seen in the experiments with a simple BOM. This challenge is closely associated with the make-to-order system and the capacity of the machines. Specifically, both machine 1 and machine 4 are highlighted for their high utilization rates, exceeding 90%, which positions them as potential bottlenecks within the system. In instances of sudden order surges, these machines are at risk of reaching full capacity, leading to delays in order fulfilment. Additionally, the fundamental characteristics of a make-to-order system, which restricts the preemptive stockpiling of products. The model's strategy for mitigating uncertainty involves elevating inventory levels through adjustments to the planning parameters. Nonetheless, excessively high settings for these parameters can lead to elevated inventory costs, especially during intervals of diminished customer demand. Such adjustments, while aimed at preserving service levels, may inadvertently contribute to an increase in overall costs.

Distribution	Overall costs	SL	Backorders	Mean tardiness	Mean inventory	Utilization				
						M1	M2	M3	M4	M5
Exp.	7132±583	0.942	23.6±6.2	1.46±0.42	477±7.6	0.908	0.896	0.883	0.940	0.881
Log. σ=0.1	3957±53	0.997	1.4±0.5	0.29±0.08	382±3.4	0.911	0.899	0.887	0.946	0.887
Log. σ=0.5	4182±63	0.997	1.5±0.8	0.25±0.07	403±5.9	0.911	0.900	0.887	0.945	0.886
Log. σ=1.0	7990±1099	0.907	36.7±11.7	1.85±0.53	432±10.8	0.907	0.894	0.884	0.941	0.881

Table 13: Results experiment with varying customer demand settings for the complex BOM structure. All the distributions use an arrival rate of 4 orders per day. The values represent the average of the 50 replications whereas the ± represents the 95% confidence interval.

The optimization of 84 planning parameters for a complex BOM introduces a significant challenge in deriving clear insights from a broad array of parameters. The optimal planning parameters identified by the model are detailed in Appendix C. To facilitate analysis, the focus is narrowed to the planning parameters of the twelve end items, comparing their values across different scenarios of customer demand variability. Through this comparative approach, several key findings emerge, shedding light on how variations in demand impact the optimization of planning parameters.

Finding 4 Upon comparing the parameters between scenarios with σ of 0.1 and 0.5 for end items 1-12, several patterns emerge regarding the adaptation of planning parameters to manage increased demand variability. Specifically, in the scenario with $\sigma = 0.5$, the planned lead times for 7 out of the 12 items are observed to be higher than those in the $\sigma = 0.1$ scenario. Within this subset, four items exhibit a reduction in FOQ and/or safety stock in comparison to the $\sigma = 0.1$ distribution. Conversely, three items show a decrease in planned lead time relative to the $\sigma = 0.1$ setting, with all three witnessing an increase in FOQ and/or safety stock to mitigate the higher variability. For the two items where planned lead times remain unchanged, the scenario with $\sigma = 0.5$ demonstrates an increase in either FOQ and safety stock parameters. These trends indicate that in response to greater customer demand variability, over half of the items assessed have elevated planned lead times. Furthermore, when planned lead times are equivalent to or shorter than those in the $\sigma = 0.1$ scenario, there is a compensatory increase in FOQ and/or safety stock.

Finding 5 Comparing the planning parameters between scenarios with σ of 0.5 and 1.0 for end items 1-12 reveals how strategies adjust in response to further increased customer demand variability. In the scenario with $\sigma = 1.0$, 5 out of 12 items have longer planned lead times than those in the $\sigma = 0.5$ setting. Among these 5 items, 4 exhibit a reduction in FOQ and/or safety stock parameters. Conversely, there are 5 items for which the planned lead times in scenario $\sigma = 1.0$ are shorter than in $\sigma = 0.5$. Of these, 4 items compensate with increased FOQ and/or safety stock parameters compared to $\sigma = 0.5$. The remaining two items maintain consistent planned lead times across both scenarios, with one item in the $\sigma = 1.0$ scenario showing an increase in FOQ and safety stock. This analysis illustrates a pattern of adaptation to heightened levels of customer demand variability. A significant portion of items exhibit increased planned lead times in the scenario with higher variability ($\sigma = 1.0$). For items where lead times are reduced or remain unchanged, adjustments are made through increased FOQ and/or safety stock, aiming to buffer against unpredictability.

Similar patterns are observed in the comparison between σ of 0.1 and 1.0. However, distinguishing the planning parameters, particularly planned lead times, between the customer demand scenarios is more challenging for the complex BOM compared to the simple BOM structure. This difficulty largely stems from the fact that numerous items within the complex BOM are produced using the same machinery. For instance, 12 end items are manufactured on machine 5, all of which exhibit similar demand patterns but only slight variations in processing times. Consequently, uniformly extending the planned lead times for all twelve items results in a clustering of production orders within the same period. A potentially more effective approach could involve sequencing the production orders to ensure a more distributed workload over time. Such an approach may account for the observed phenomena where planned lead times in scenarios with higher customer demand variability do not consistently show an extension compared to scenarios with lower variability.

5.4 Runtime benchmark

An additional benchmarking experiment was undertaken for the complex BOM structure, utilizing a more powerful desktop computer for the evaluation. The benchmark was conducted on a PC equipped with an AMD Ryzen 9 5950X 16-core CPU and 64GB of RAM. To test the runtime the same setup as described in table 8 is used. The runtime for experimenting with data transfer to CSV of the complex BOM scenario with 15000 iterations and 50 replications was 785 minutes.

6 Discussion

6.1 Results

The scalable hybrid model presented demonstrates promising potential in optimizing MRP parameters. Unlike similar studies that often confine their models to specific contexts, the findings from this research illustrate the model's versatility in adapting to a variety of scenarios with minimal modifications necessary. In addition, there is a lack of similar studies that consider MRP systems under uncertainty. This simulation model is adept at incorporating stochastic elements, providing a robust platform for result analysis. Simultaneously, the external optimizer utilizing a genetic algorithm has proven effective in navigating the search space to identify optimal solutions efficiently. This framework integrates simulation and optimization as a generalized approach to refine MRP planning parameters effectively.

The hybrid model has been applied to study two scenarios, with a focus primarily on the outcomes related to the complex BOM structure. Preliminary experiments revealed that the BOM structure within the hybrid model exhibited sensitivity to variations in customer demand. The experiments showed that increasing the variability in customer demand led to a nearly 10% decrease in service level. Although an increase in overall costs might be anticipated, such a decline in service level indicates the model's limitations in maximizing customer satisfaction under conditions of heightened demand variability, which may be attributed to high machine utilization. This outcome further highlights the challenges associated with production planning for complex BOM structures, particularly in efforts to balance cost minimization with other objectives. Incorporating a multi-objective problem into the optimization process could potentially enhance the model's performance by providing it with a broader spectrum of information. However, this addition is likely to also increase the complexity of the model. Integrating multiple objectives would necessitate a more sophisticated approach to decision-making within the model, enabling it to weigh various factors, such as cost, service level, and production efficiency, against one another in a more nuanced manner.

Upon examining the KPIs from the experimental results, they seem to meet the necessary criteria. Machine utilization rates are sustained above 85%, and a high service level is attained. Because of the long runtime of the model, it was challenging to set all machine utilization during pre-experiments to approximately 85%, especially for the complex scenario. Due to time constraints, values above 85% were accepted, but it should be noted that the increased utilization might have caused bottlenecks in the system. Furthermore, it is observed that with an increase in customer demand variability, overall costs rise due to the need to maintain higher inventory levels. However, evaluating the overall cost efficiency presents a challenge, as conducting a parameter variation experiment to fully assess the quality of the findings is not feasible. To accurately assess the model's effectiveness and cost implications, it would be essential to apply the model in a real-world use case scenario. This application would provide a concrete context within which to evaluate the model's performance, allowing for a more thorough assessment of its capacity to balance costs, service levels, and utilization rates in a practical setting.

The runtime for solving the complex BOM structure required a substantial duration of 785 minutes, which may initially seem infeasible for the practical application of the model. However, it is crucial to acknowledge that this extended runtime resulted from the limitation of sequential processing. Both AnyLogic and HeuristicLab, the platforms used for simulation and optimization respectively, are capable of parallel processing. Given the hardware utilized in the experiment was equipped with

32 threads, enabling parallel processing could theoretically reduce the runtime to approximately 25 minutes. Furthermore, there is potential for further optimization of the optimizer itself for enhanced performance, as the focus was mainly on achieving optimal solutions. With the application of parallel processing and additional performance optimizations, solving complex BOM structures within a reasonable timeframe appears to be a feasible objective.

6.2 Theoretical and practical implications

The research contributes to the existing body of knowledge by offering a conceptual framework that can be further developed and refined. This research highlights the potential benefits of employing the applied hierarchical structure within simulation-based optimization. The results obtained suggest that this approach can effectively address complex optimization problems such as MRP parameterization by cohesively integrating simulation and optimization processes. In addition, the outcomes of this research can be used as a benchmark for future studies.

The complexity of MRP systems and the unpredictable nature of production environments make it challenging to identify optimal parameters. As a result, planning strategies often require frequent adjustments which is a difficult and time-consuming assignment. The model offers a tool for approximating optimal conditions within the MRP system. By using this model, businesses can gain insights into the best configurations for MRP parameters like inventory levels, reorder points, and lot sizes, considering various operational scenarios and demand patterns. This initial estimation provides a starting point for refining and fine-tuning MRP settings to minimize costs while maximizing service levels and operational efficiency. Further enhancing the model could lead to its integration into existing MRP software platforms, enabling continuous optimization processes. This integration would automate parameter adjustments based on real-time data, production environment changes, supply chain dynamics, or demand forecasts. Such dynamic adjustments would significantly enhance the responsiveness and agility of MRP systems, helping businesses maintain optimal performance despite market fluctuations and uncertainties. The potential integration of this model into daily MRP operations highlights its ability to improve traditional planning methods by offering a more adaptive, data-driven approach to resource planning and management.

6.3 Limitations and future work

The study's main limitation is the lack of real-world data, as the experiments are confined to predefined scenarios. Validation relies solely on parameter variation experiments within the model, which hinders the detection of anomalies in the results. To enhance the research's credibility, applying the model to a real case study would be beneficial. Moreover, the model's tested processing is limited to relatively small BOM structures, raising concerns about runtime when scaling up to entire organizational systems. Additionally, the system involves simplifications, and incorporating capacity constraints, a common challenge in real-world production environments, would significantly enhance the practical usability of the MRP system.

In the current framework, the utilization of the computer CPU's maximum performance is not achievable due to its reliance on sequential processing, which leads to extended model runtimes. By integrating parallel evaluations, the computational time could be significantly reduced, given that contemporary CPUs are equipped with multiple threads. This enhancement would enable simultaneous processing of tasks, leveraging the full potential of modern multi-core processors to improve efficiency and significantly reduce the time required for model computations.

Considering these limitations, future research could benefit from integrating more complexity into the hybrid model, particularly by incorporating capacity constraints. The implication of dynamic lead times could be an interesting approach to solving the issue. Furthermore, the influence of different lotsizing policies could be studied. Incorporating multi-objective optimization into the hybrid model presents an appealing advancement, especially since real-life production environments invariably need to balance multiple objectives. This approach would allow the model to simultaneously consider various factors such as cost minimization, service level optimization, and resource utilization efficiency. Fine-tuning the genetic algorithm could enhance the external optimizer's efficiency and as a result, minimize the runtime. Exploring the effects of various heuristic algorithms could offer deeper insights into how these methods affect the optimization process's efficiency and effectiveness. Such investigations would not only advance the understanding of the model's adaptability to different optimization strategies but also potentially reveal more effective approaches to navigating complex solution spaces. Additionally, conducting a case study would be essential for improving the model's outcomes.

7 Conclusion

This study examines an MRP system characterized by uncertain demand and lead times, aiming to identify the optimal MRP planning parameters for each item to minimize costs. The proposed method is a hybrid model, integrating simulation and optimization in a generic framework. A scalable simulation model, developed in AnyLogic, assesses the MRP planning parameters within a stochastic make-to-order production environment. Subsequently, a genetic algorithm, implemented in HeuristicLab, is employed to determine the optimal parameters based on the metrics derived from the simulation model output. This approach facilitates a comprehensive analysis of the MRP system, enabling the identification of cost-effective planning parameters under conditions of uncertainty.

The hybrid model shows promising results for the parameterization of MRP systems under uncertainty. Furthermore, the results of the experiments showed that the model can adjust to different BOM structures in size and complexity. A simple structure of 5 items and a more complex structure of 28 items have been evaluated. The model was able to find optimal parameters for different scenarios regarding customer demand variability. The results of the model can serve as a benchmark for further research.

This research contributes to a field where existing studies on MRP systems under uncertainty are scarce. Unlike the majority of similar research, which tends to concentrate on specific real-world cases, the findings of this study offer a more generalized approach.

The present study has several limitations that need to be addressed. Firstly, it employs a simple lot sizing policy without taking capacity constraints into account. Additionally, the model is confined to single-objective optimization, limiting its ability to simultaneously address multiple operational goals that are often crucial in practice. Another significant constraint is the architecture of the hybrid model, which relies on sequential processing, leading to prolonged runtimes. Moreover, the study lacks the validation for large BOM structures due to the unavailability of real-world data.

Future research should include more complex lot sizing and multi-objective optimization. It would be interesting to see if it is possible to include capacity constraints during the optimization. Furthermore, different optimization algorithms could be explored. Finally, the model should be tested and validated with the use of real-world data.

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Appendices

A MRP event

The MRP event commences by initializing the variables: gross requirements, net requirements, planned receipts, planned releases, and projected inventory. At the onset of each run, these variables are reset to 0. The calculation of gross requirements for end items, which are assigned an LLC of 0, is the initial step. This process involves iterating over all customer orders stored in the match block, during which the due date specified by the customer is converted into an MRP time bucket corresponding to the daily update frequency of the MRP system. Upon updating all the gross requirements, the application of the MRP logic is set into motion.

The MRP logic processes all items by a level-by-level approach starting at LLC 0. Then each item at the LLC is iterated over the planning horizon t starting at 0. The MRP procedure based on Hopp and Spearman [2] with some assumptions is as follows:

Netting In the first step the net demand is calculated. The algorithm starts with calculating the projected on-hand inventory at $t = 0$, which gives:

$$I_t = I_{OH} - D_t + S$$

where I_t is the projected on-hand inventory on period t, I_{OH} is on-hand inventory, D_t are the gross requirements on period t and S are the scheduled receipts. The on-hand inventory and the scheduled receipts are only taken into account at $t = 0$. Then for $t > 0$, the projected on-hand inventory is:

$$I_t = I_{t-1} - D_t + N_{t-1}$$

where N_{t-1} are the net requirements of the previous period. The net requirements of the previous period have to be included to update the projected on-hand inventory correctly. Because at $t = 0$ the on-hand inventory and the scheduled receipts are added to the projected inventory, they do not have to be included in the next calculations. In the netting step, the safety stock is taken into account. Whenever the projected on-hand inventory is lower than the safety stock, the net requirements are computed as:

$$N_t = SS - I_t$$

where SS is the safety stock planning parameter that is set for each item.

Lot sizing When there is net demand, a lot sizing policy has to be applied. In this model, a fixed order quantity (FOQ) is applied, which is a planning parameter set for each item. The planned receipt is then equal to or a multiplication of the FOQ.

Time phasing The planned releases are computed by subtracting the planned lead time from the time of the planned receipts. The planned lead time is a planning parameter set for each item.

BOM explosion In the final stage of the MRP run the gross requirements of the item's child items are updated. For example, if part A requires two units of part B, then the gross requirements of part B are computed by doubling the planned release of part A. If all items of the LLC are processed the MRP logic is repeated for the next LLC.

Upon completing the MRP procedure is performed for all items, product orders are created and released. As mentioned in Section 3.2.2, only the planned releases at $t = 0$ are released into production. These planned releases at $t = 0$ are converted into product orders, which are then assigned with all necessary attributes related to the order.

B BOM tables

B.1 Simple scenario

Level	ParentItem	ChildItem	Quantity	Machine	ProcessingTime	ReplenishmentTime
0	1	3	1	1	6	0
0	1	4	1	1	6	0
0	2	3	1	2	5.5	0
0	2	5	1	2	5.5	0
1	3	0	0	0	0	10
1	4	5	1	3	6	0
2	5	0	0	0	0	10

Figure 11: BOM table for the easy scenario

B.2 Complex scenario

Level	ParentItem	ChildItem	Quantity	Machine	ProcessingTime_mode	ProcessingTime_min	ProcessingTime_max	ReplenishmentTime_mode	ReplenishmentTime_min	ReplenishmentTime_max
0	1	13	1	5	10.5	8	20	0	0	0
0	1	14	1	5	10.5	8	20	0	0	0
0	1	15	1	5	10.5	8	20	0	0	0
0	1	16	1	5	10.5	8	20	0	0	0
0	1	17	1	5	10.5	8	20	0	0	0
0	1	18	1	5	10.5	8	20	0	0	0
0	2	13	1	5	10	8	20	0	0	0
0	2	14	1	5	10	8	20	0	0	0
0	2	15	1	5	10	8	20	0	0	0
0	2	16	1	5	10	8	20	0	0	0
0	2	17	1	5	10	8	20	0	0	0
0	2	18	1	5	10	8	20	0	0	0
0	3	13	1	5	11	8	20	0	0	0
0	3	14	1	5	11	8	20	0	0	0
0	3	15	1	5	11	8	20	0	0	0
0	3	16	1	5	11	8	20	0	0	0
0	3	17	1	5	11	8	20	0	0	0
0	3	18	1	5	11	8	20	0	0	0
0	4	13	1	5	10.5	8	20	0	0	0
0	4	14	1	5	10.5	8	20	0	0	0
0	4	15	1	5	10.5	8	20	0	0	0
0	4	16	1	5	10.5	8	20	0	0	0
0	4	17	1	5	10.5	8	20	0	0	0
0	4	18	1	5	10.5	8	20	0	0	0
0	5	13	1	5	10	8	20	0	0	0
0	5	14	1	5	10	8	20	0	0	0
0	5	15	1	5	10	8	20	0	0	0
0	5	16	1	5	10	8	20	0	0	0
0	5	17	1	5	10	8	20	0	0	0
0	5	18	1	5	10	8	20	0	0	0
0	6	13	1	5	11.5	8	20	0	0	0

0	6	14	1	5	11.5	8	20	0	0	0
0	6	15	1	5	11.5	8	20	0	0	0
0	6	16	1	5	11.5	8	20	0	0	0
0	6	17	1	5	11.5	8	20	0	0	0
0	6	18	1	5	11.5	8	20	0	0	0
0	7	13	1	5	10	8	20	0	0	0
0	7	14	1	5	10	8	20	0	0	0
0	7	15	1	5	10	8	20	0	0	0
0	7	16	1	5	10	8	20	0	0	0
0	7	17	1	5	10	8	20	0	0	0
0	7	18	1	5	10	8	20	0	0	0
0	8	13	1	5	10.5	8	20	0	0	0
0	8	14	1	5	10.5	8	20	0	0	0
0	8	15	1	5	10.5	8	20	0	0	0
0	8	16	1	5	10.5	8	20	0	0	0
0	8	17	1	5	10.5	8	20	0	0	0
0	8	18	1	5	10.5	8	20	0	0	0
0	9	13	1	5	10	8	20	0	0	0
0	9	14	1	5	10	8	20	0	0	0
0	9	15	1	5	10	8	20	0	0	0
0	9	16	1	5	10	8	20	0	0	0
0	9	17	1	5	10	8	20	0	0	0
0	9	18	1	5	10	8	20	0	0	0
0	10	13	1	5	11	8	20	0	0	0
0	10	14	1	5	11	8	20	0	0	0
0	10	15	1	5	11	8	20	0	0	0
0	10	16	1	5	11	8	20	0	0	0
0	10	17	1	5	11	8	20	0	0	0
0	10	18	1	5	11	8	20	0	0	0
0	11	13	1	5	10.5	8	20	0	0	0
0	11	14	1	5	10.5	8	20	0	0	0
0	11	15	1	5	10.5	8	20	0	0	0
0	11	16	1	5	10.5	8	20	0	0	0

0	11	17	1	5	10.5	8	20	0	0	0
0	11	18	1	5	10.5	8	20	0	0	0
0	12	13	1	5	10	8	20	0	0	0
0	12	14	1	5	10	8	20	0	0	0
0	12	15	1	5	10	8	20	0	0	0
0	12	16	1	5	10	8	20	0	0	0
0	12	17	1	5	10	8	20	0	0	0
0	12	18	1	5	10	8	20	0	0	0
1	13	19	1	4	3.5	2	8	0	0	0
1	14	19	1	4	3.5	2	8	0	0	0
1	15	0	0	0	0	0	0	9	8	36
1	16	0	0	0	0	0	0	9	8	36
1	17	0	0	0	0	0	0	9	8	36
1	18	20	1	4	3	2	9	0	0	0
2	19	21	1	3	4	2	7	0	0	0
2	19	22	1	3	4	2	7	0	0	0
2	20	23	1	3	3.5	2	7	0	0	0
2	20	25	1	3	3.5	2	7	0	0	0
3	21	0	0	0	0	0	0	9	8	36
3	22	24	1	2	5.5	2	12	0	0	0
3	23	0	0	0	0	0	0	10	8	36
4	24	26	1	1	3	2	8	0	0	0
4	24	27	1	1	3	2	8	0	0	0
4	25	27	1	1	3.5	2	8	0	0	0
4	25	28	1	1	3.5	2	8	0	0	0
5	26	0	0	0	0	0	0	12	8	36
5	27	0	0	0	0	0	0	11	8	36
5	28	0	0	0	0	0	0	11	8	36

C Optimal planning parameters complex BOM

	Planned lead time per item																											
Distribution	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Exp.	3	1	1	5	4	1	4	1	1	2	3	6	1	1	4	4	7	4	4	5	5	3	3	5	3	3	4	3
Log. 0.1	4	7	7	1	3	4	1	4	3	5	3	4	1	2	3	2	2	4	3	3	3	3	5	5	5	5	7	2
Log. 0.5	4	2	6	4	5	5	2	5	4	7	2	4	1	2	5	1	2	5	3	2	5	3	2	3	4	4	2	5
Log. 1.0	7	7	4	4	2	6	1	5	1	1	6	5	1	2	7	1	4	2	2	6	3	4	6	5	2	3	5	4
	Fixed order quantity per item																											
Distribution	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Exp.	8	97	49	14	36	53	6	21	10	29	16	32	7	59	100	8	5	46	24	98	25	49	58	73	6	86	67	30
Log. 0.1	9	31	38	17	16	18	62	8	1	27	13	31	27	36	18	2	93	3	7	19	5	62	35	83	62	63	55	92
Log. 0.5	32	54	21	31	20	25	25	18	32	24	19	38	6	63	48	32	25	3	69	21	92	78	83	23	30	20	55	19
Log. 1.0	43	16	25	11	9	53	13	35	20	20	15	5	84	35	54	25	55	70	99	60	98	50	8	33	89	24	22	15
	Safety stock per item																											
Distribution	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Exp.	100	98	29	83	83	80	43	96	61	51	72	100	48	99	0	90	91	43	87	69	5	95	12	15	95	16	81	94
Log. 0.1	100	51	84	100	31	47	95	42	67	24	50	45	25	84	44	12	60	32	46	46	97	97	0	75	12	94	88	44
Log. 0.5	21	81	94	93	38	76	84	45	53	17	83	89	23	58	12	25	63	98	100	12	54	81	45	100	41	34	33	38
Log. 1.0	55	39	70	65	54	66	71	78	73	89	65	62	50	82	88	24	69	100	88	97	76	61	86	44	62	97	32	56

Table 14: Optimal planning parameters identified by the model for the complex BOM.