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Exploring Experimental Modal Analysis of Beam Structures Through Data-Driven Modelling

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Groningen, 21/06/2024

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Abstract

This research aims to explore the accuracy and uses of data-driven modelling in experimental modal analysis. An Auto-regressive with Exogenous Excitation (ARX) model will be used to find the modal parameters, respectively the Natural Frequency, Damping Ratios, and Mode Shape. The results from the model will be compared to the modal parameters found using Analytical Equations. Initial results find that the ARX-model is able to find the natural frequencies and the mode shapes with high accuracy. However, finding the damping ratios remains a difficult task due to noise affecting the amplitude. These findings show that data-driven modelling has real-world applications in the Structural Health Monitoring of civil structures, as continuous monitoring of said structures along with the use of the ARX-model will act as proactive maintenance in the form of early damage identification.

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List of Abbreviations

1. EMA: Experimental Modal Analysis
2. ARX: Auto-regressive with exogenous excitation
3. D&V group: Dynamics and Vibrations group
4. FFT: Fast Fourier Transform
5. FRF: Frequency Response Function
6. SHM: Structural Health Monitoring

1 Introduction

When it comes to designing buildings, airplanes, bridges and other structures, it is important for the engineers to know that their structures will not collapse due to external forces or vibrations. One way of ensuring the structural integrity is by using experimental modal analysis (EMA). EMA is a method that determines the modal parameters such as natural frequency, damping ratio, and mode shape of a structure through experiments [1] [2] [3]. The mode shape reflects the shape of the structure at a particular natural frequency. An example of such experiments is by conducting an impact test with a hammer [4] [5]. This method requires a lot of calculations and sufficient knowledge of signal processing and data analysis, which requires skilled personnel [2] [6]. However, with the use of an ARX (auto-regressive with exogenous excitation) model these calculations can be conducted in a much quicker manner without the need for understanding the physics behind the vibrations [7] [8] [9] [10]. ARX-models are data-driven models and auto-regressive means that the system takes the outputs of previous iterations to calculate the current output [11]. Although the use of ARX-models for calculating the modal parameters is not uncommon and are fairly accurate, the robustness of the resulting parameters has not been studied. Therefore, this project, under the guidance of the Dynamics & Vibrations (D&V) group, aims to study and systematically discuss the robustness of the extracted parameters, by conducting an experimental modal analysis on a cantilever beam structure with the use of hammer impact testing and data-driven modelling (ARX) to compute the modal parameters. These parameters will be compared to the results of the same parameters from the analytical model in MATLAB. The results of this research hope to provide researchers and engineers a better understanding of ARX-modelling within modal analysis and show its accuracy which in turn will aid with their calculations and in the discovering of the physical dynamics of a wide variety of structures. Therefore ensuring the structural health of bridges, planes, and many more.

2 Experimental Setup

2.1 Data Acquisition System

Before the EMA could be conducted, the data acquisition system had to be designed. For this, LabVIEW was used as this is a fairly easy to use program with a simple but effective dashboard that showcases the acquiring of data. For the actual acquiring of data, the NI-DAQ was used, as this is compatible with LabVIEW. Figure 1 shows the resulting dashboard in LabVIEW.

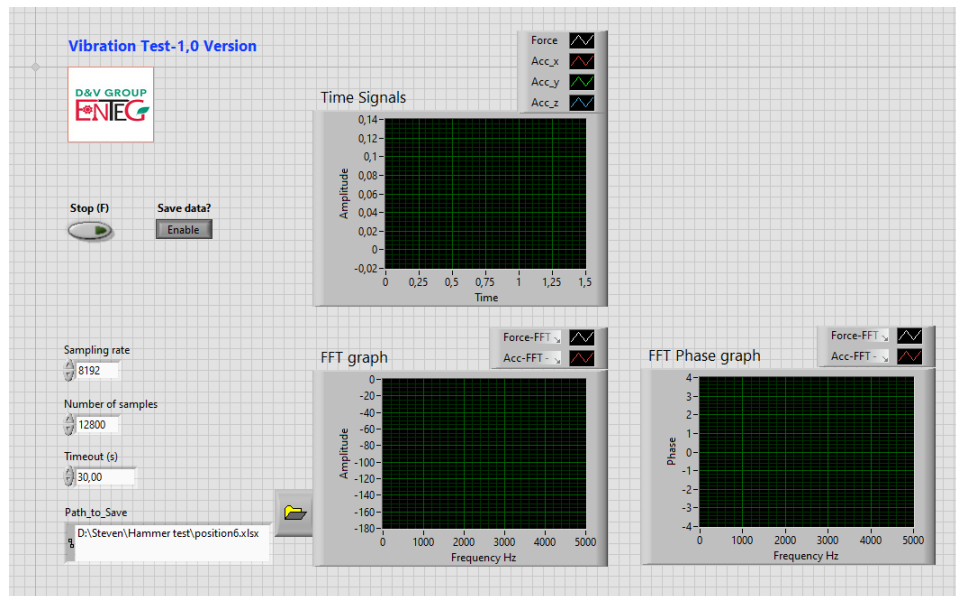


Figure 1: Dashboard for the Data-Acquisition System in LabVIEW, showing the Time Signals, Fast Fourier Transform graph and the Fast Fourier Transform Phase graph. Sampling parameters and Path to Save is also shown on the Dashboard.

In the dashboard several graphs are showcased. The top graph shows the time signals during the experiment, in particular the force from the hammer and the acceleration of the beamed measured by the accelerometer. Below that is the Fast Fourier Transform (FFT) graph which converts the time signal to the frequency domain and amplitude, while to the right of that is the FFT Phase graph which does the same but converts the phase instead of the amplitude. On the left side of the dashboard, the sampling rate, number

of samples, timeout, and the Path to Save can be manually set. In this case the sampling rate is set to 2000 Hz, the number of samples is set at 12800, which means that the buffer is 12800 samples, and the Timeout setting is set to 30 seconds, meaning that the system will wait 30 seconds to write all the samples.

Figure 2 shows the "behind the scenes" block diagram in LabVIEW. The most important block is that of the DAQ Assistant, as this allows for the recording of the data.

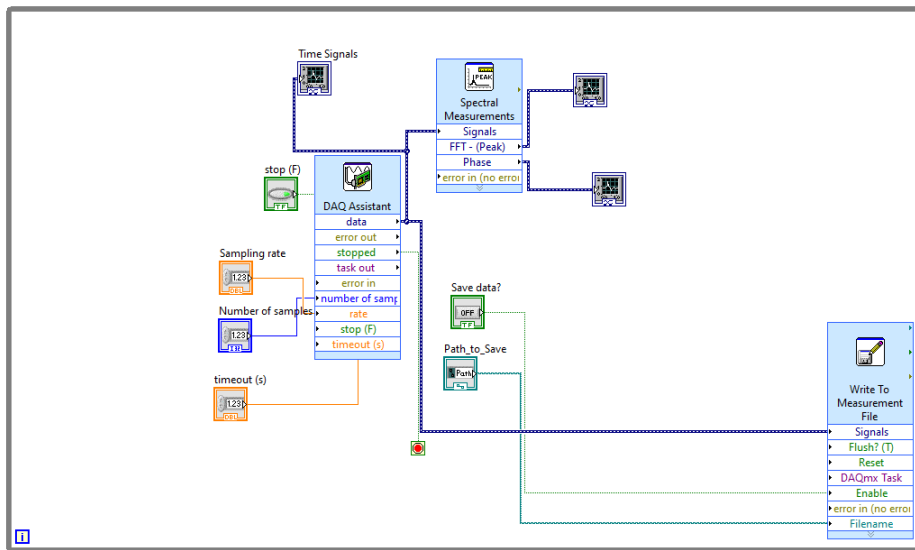


Figure 2: Block Diagram for the Dashboard of the Data-Acquisition System in LabVIEW.

2.2 Lab Setup

Figure 3 shows a schematic overview of the lab setup, with figure 4 showing the top-view of the cantilever beam upon which the hammer impact testing was conducted. The hammer can be seen in figure 5. The hammer is connected to the NI-DAQ in order to read the impact forces. The accelerometer stayed in place while the impact location changed for each reading. The positioning of the impacts can be seen in table 1. For each impact test it was essential that the beam was only impacted once in order to create the

best response. If it was found that there was a double or triple knock, the file would be deleted and the impact test was repeated.

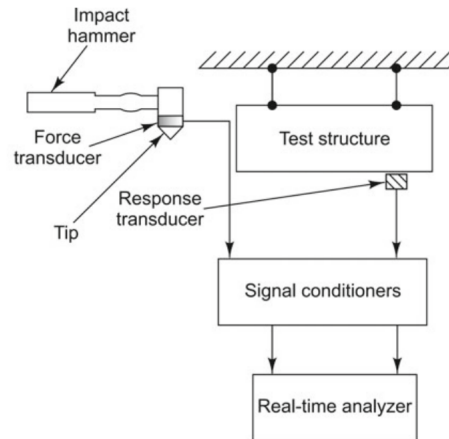


Figure 3: Schematic Overview of the Lab Setup.

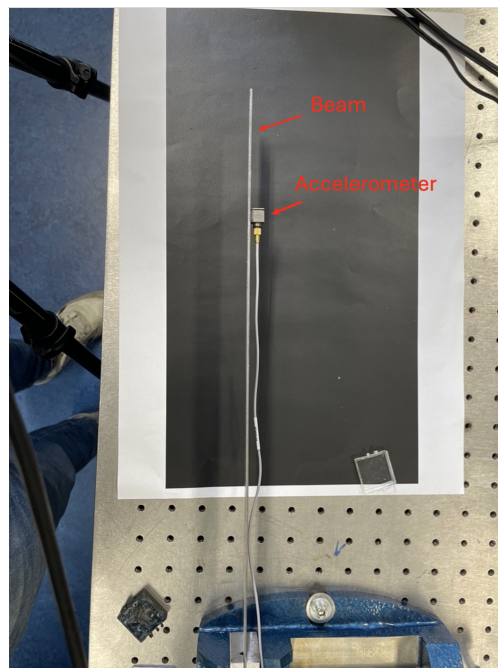


Figure 4: Top-view of the experimental set-up in the D&V lab. Here the Test Beam and the Accelerometer are visible.



Figure 5: Hammer used for impact-testing.

The parameters of the beam are: 420mm x 30mm x 2mm. The locations of the impacts are shown in the following table:

| Position | Distance to free end (cm) |
|----------|---------------------------|
| 1 | 9 |
| 2 | 13 |
| 3 | 20 |
| 4 | 25 |
| 5 | 34 |
| 6 | 39 |

Table 1: Overview of the positions of the hammer-impact on the beam.

3 Theory on ARX-modelling

The General ARX-model form can be written as the following:

$$y(t) + A_1 y(t-1) + \dots + A_{na} y(t-na) = B_0 u(t) + B_1 u(t-1) + \dots + B_{nb} u(t-nb) + e(t) \quad (1)$$

From this, the polynomial form can be written as:

$$A(q)y(t) = B(q)u(t) + e(t) \quad (2)$$

Where $A(q)$ and $B(q)$ can be written as:

$$A(q) = I + A_1q^{-1} + A_2q^{-2} + \dots + A_{na}q^{-n} \quad (3)$$

$$B(q) = B_0 + B_1q^{-1} + B_2q^{-2} + \dots + B_{nb}q^{-n} \quad (4)$$

Equation 2 is the ARX-model, where $A(q)y(t)$ refers to the Autoregressive part and $B(q)u(t)$ the exogenous input. $y(t)$ is the output of the model and $u(t)$ is the input while $e(t)$ is the error. $A(q)$ and $B(q)$ are polynomials in the delay operator q^{-1} and n_a, n_b are the model order of $A(q)$ and $B(q)$, which are set to 40 [11]. The error is minimized using the Bayesian Information Criterion (BIC) in MATLAB.

The ARX-model then makes a regressive connection between the input and output, $u(t)$ and $y(t)$. Using the least squares method, the optimal modal parameters matrices for A and B are estimated. From there, the state matrix can be determined in the following form:

$$A = \begin{bmatrix} -A_1 & -A_2 & -A_3 & \dots & -A_{na} \\ I & 0 & 0 & \dots & 0 \\ 0 & I & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & I & 0 \end{bmatrix} \quad (5)$$

The same goes for B .

The next step is to calculate the eigenvalues (λ) from this matrix. The natural frequency and damping ratios can then be calculated with the following equations:

$$f_i = \frac{\sqrt{Re^2(\lambda_i) + Im^2(\lambda_i)}}{2\pi} \quad (6)$$

$$\xi_i = -\frac{Re(\lambda_i)}{2\pi f_i} \quad (7)$$

With i being the mode index 1-6.

For the mode shapes, the eigenvector of matrix A of all the input signals must be found. From there, the real parts must be taken and plotting the first 6 values per column will show the mode shape. As the mode shapes are properties of the material, they should result in plots similar in shape to these:

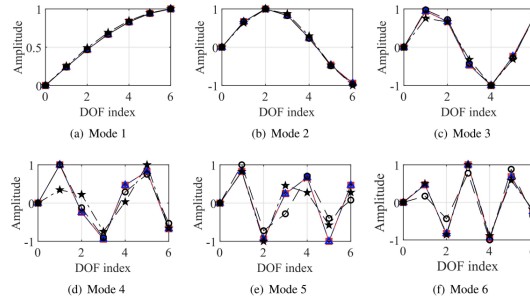


Figure 6: Examples of mode shapes 1-6, taken from [12].

4 Results from ARX-model

With the developed ARX-model, the following plots for the impact force, response signal, frequency domain, and the frequency response function were found.

4.1 Response signal & Impact Force

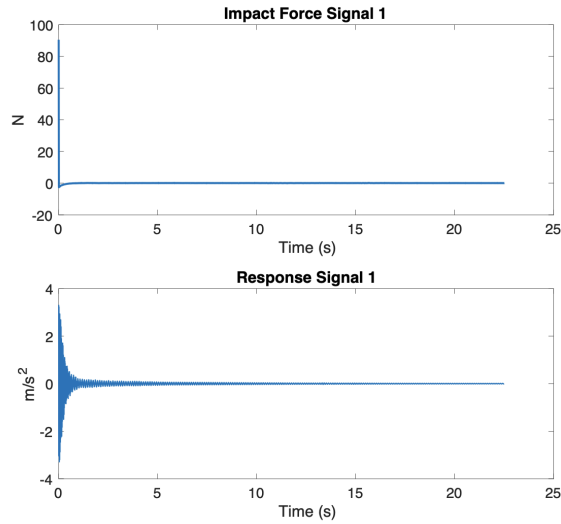


Figure 7: Response Signal of the Beam and Impact Force of the Hammer at Position 1.

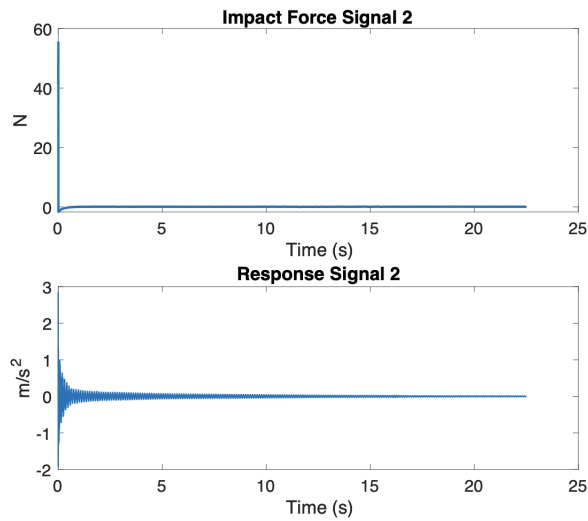


Figure 8: Response Signal of the Beam and Impact Force of the Hammer at Position 2.

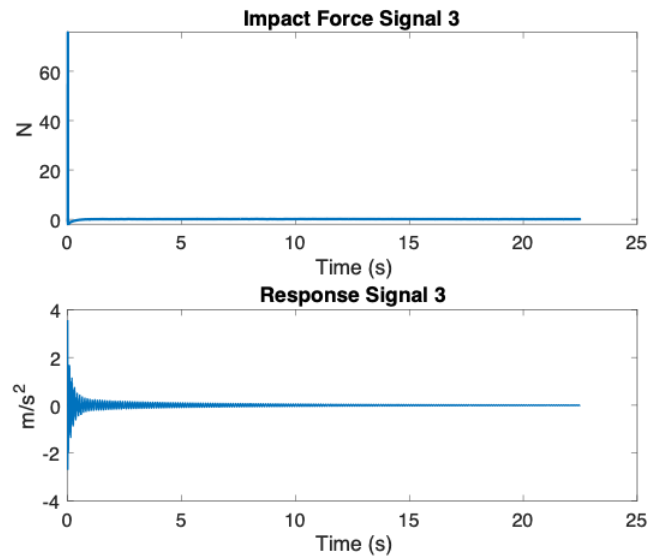


Figure 9: Response Signal of the Beam and Impact Force of the Hammer at Position 3.

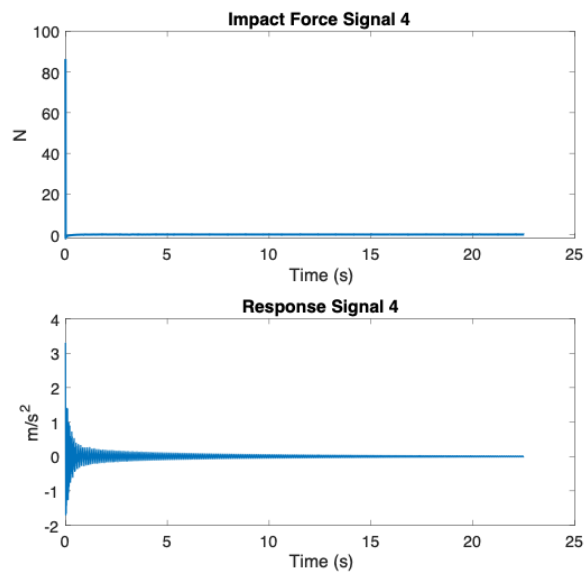


Figure 10: Response Signal of the Beam and Impact Force of the Hammer at Position 4.

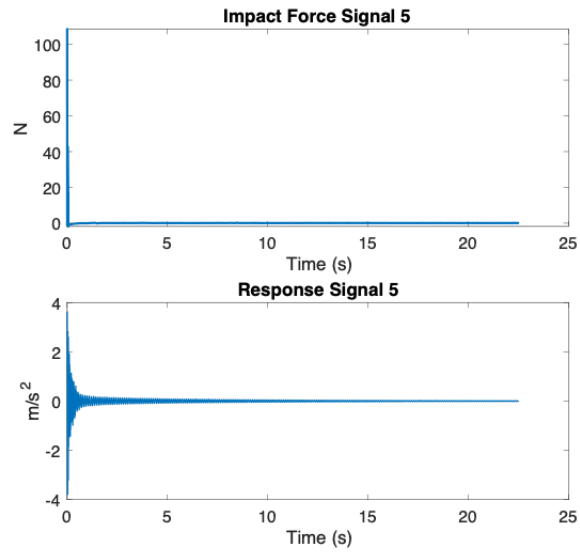


Figure 11: Response Signal of the Beam and Impact Force of the Hammer at Position 5.

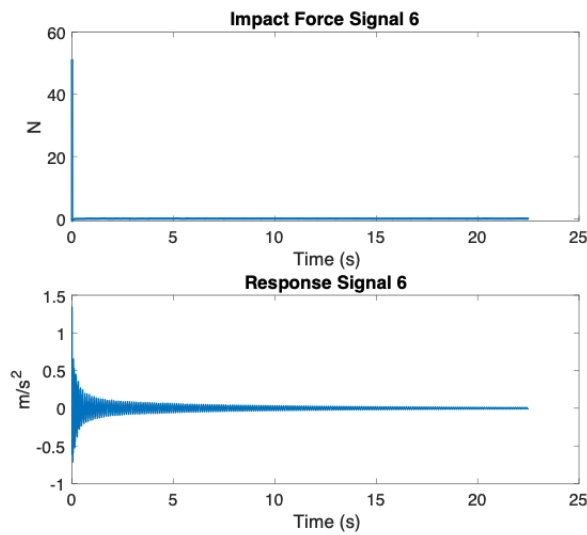


Figure 12: Response Signal of the Beam and Impact Force of the Hammer at Position 6.

4.2 Frequency Domain

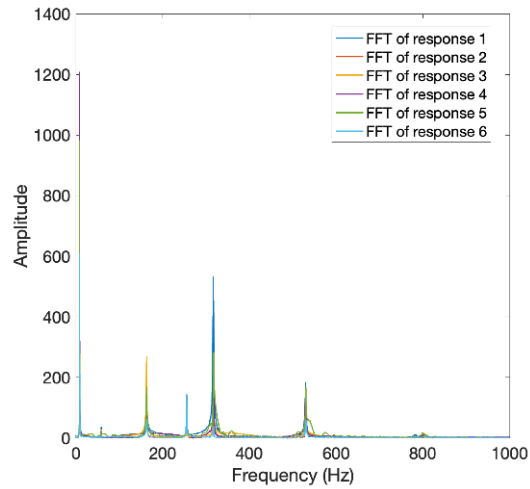


Figure 13: The FFT of all 6 Signals in the Frequency Domain. The peaks indicate the Natural Frequencies.

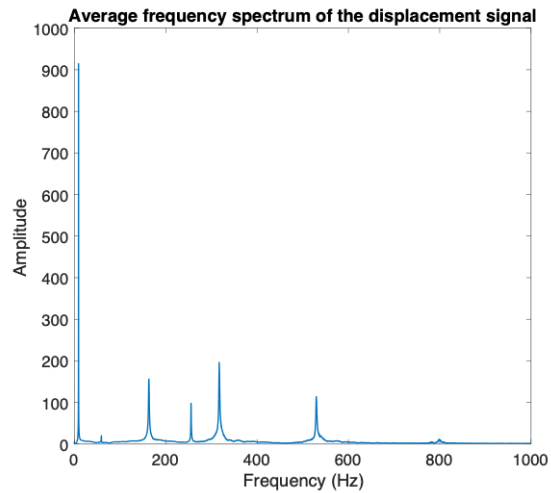


Figure 14: The Average FFT taken from the Signals of figure 13 in the Frequency Domain. The peaks indicate the Natural Frequencies.

4.3 Frequency Response Function

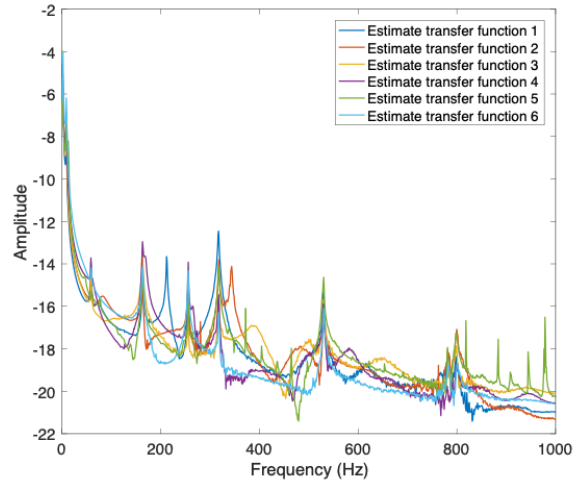


Figure 15: The Estimated Frequency Response Function in the form of the Transfer Function of all 6 Signals.

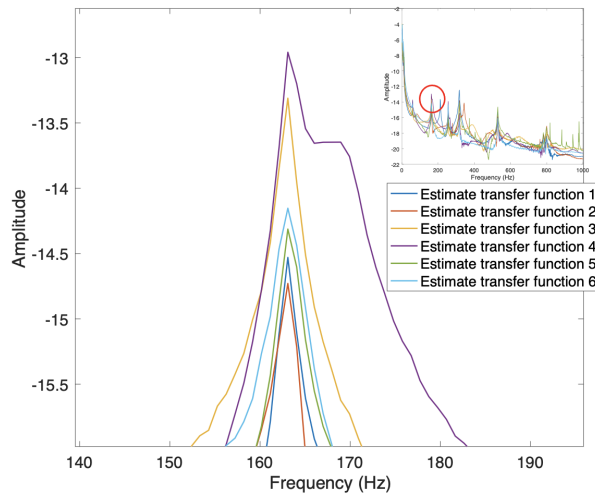


Figure 16: Zoomed In Section of Figure 15, showing all 6 Signals sharing a peak at the same Frequency.

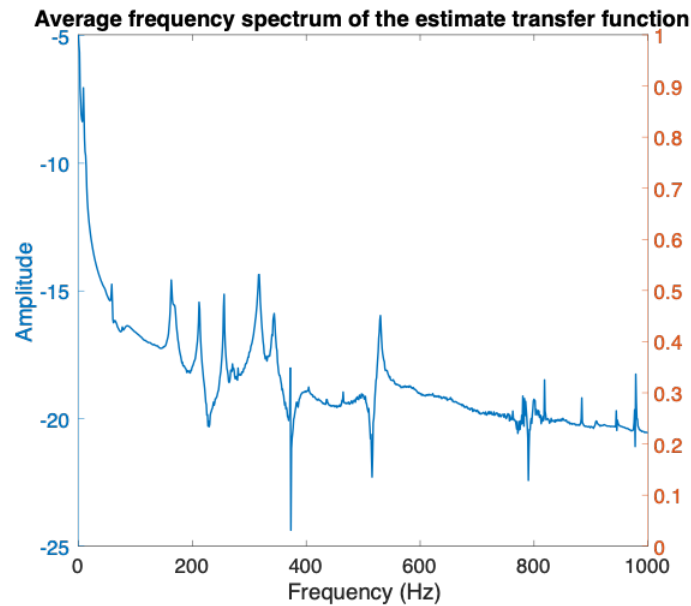


Figure 17: The Average Frequency Response Function of the 6 Signals from figure 15.

5 Modal Parameter Extraction

5.1 ARX-model

With the help of the ARX-model and the collected data, the modal parameters can be found in the plots by running the code. 6 natural frequencies were found, along with 2 damping ratios and 5 mode shapes.

5.1.1 Natural Frequencies

For the natural frequencies, the x-values of the peaks in figure 14 are needed, which are found with equation 6. This results in the following table:

| Natural Frequencies |
|---------------------|
| 9.0711 Hz |
| 60.2707 Hz |
| 164.5189 Hz |
| 255.5638 Hz |
| 317.8106 Hz |
| 529.8480 Hz |
| 798.8191 Hz |

Table 2: Natural Frequencies found using ARX-model.

5.1.2 Damping Ratios

The damping ratios are found by using equation 7. The ARX-model then plots the results in a figure. The actual damping ratios are then found by finding 4 results in a row in a line, as highlighted in the following figure:

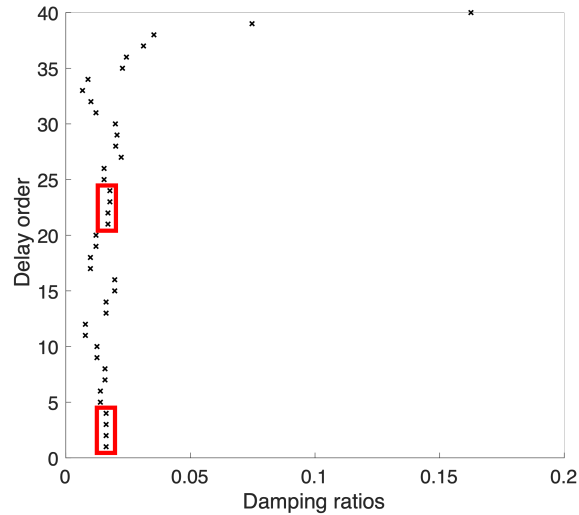


Figure 18: Damping Ratios found by the ARX-model. The red boxes indicate a Damping Ratio.

As can be seen, only 2 damping ratios were found, with one of them only narrowly being allowed to be classed as a damping ratio.

5.1.3 Mode Shapes

As mentioned before, for finding the mode shapes the real parts of the Eigenvector of the A matrix are needed. Then, by plotting the first 6 values per column, the mode shapes can be found. Using this method, the first 5 mode shapes can be found by going through all the columns. The results are shown below:

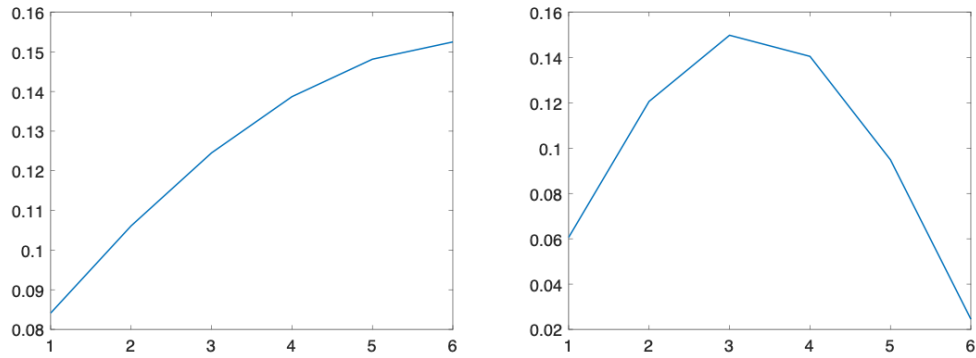


Figure 19: First and Second Mode Shapes found using the ARX-model.

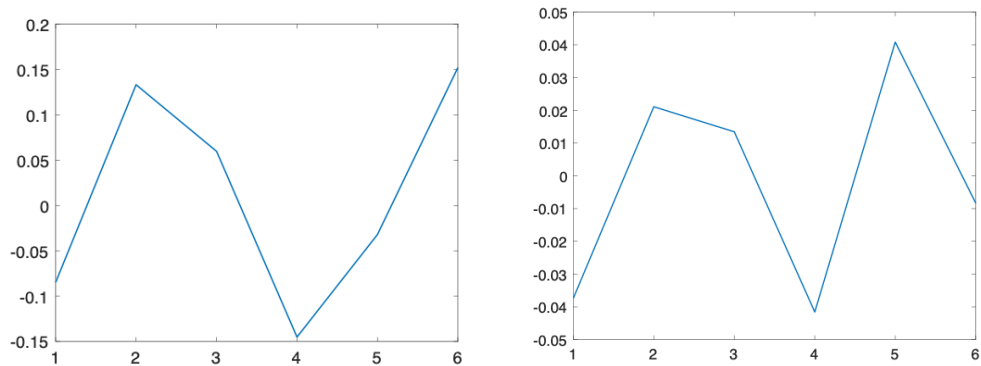


Figure 20: Third and Fourth Mode Shapes found using the ARX-model.

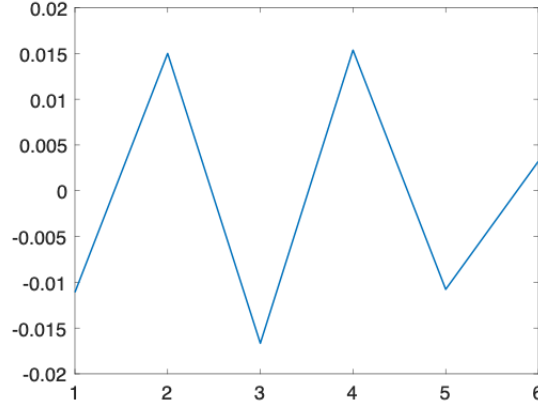


Figure 21: Fifth Mode Shape found using the ARX-model.

5.2 Analytical model

5.2.1 Natural Frequencies

For calculating the natural frequencies analytically, the following equations were used:

$$\omega_n = \left(\frac{2n-1}{2} \pi + e_n \right)^2 \frac{1}{l^2} \sqrt{\frac{EI}{\rho A}} \quad (8)$$

$$f_n = \frac{\omega_n}{2\pi} \quad (9)$$

When calculating the natural frequencies, the following parameters need to be considered.

| | |
|-------------------------------|-----------------------|
| Dimensions of Cantilever Beam | 420x30x2 mm |
| Material | Steel |
| Young's Modulus | $205 \cdot 10^9$ Pa |
| Poisson's ratio | 0.3 |
| Density | 7850 kg/m^3 |

Table 3: Parameters of the Cantilever Beam

Per Adavardkar and Admuthe, e_n in equation 8 are small and rapidly diminishing correction terms, namely 0.3042 for $n=1$, -0.018 for $n=2$, and

0.001 for $n=3$ [13]. Plugging in the equations and values into MATLAB yields the following results:

| Natural Frequencies |
|---------------------|
| 9.3584 Hz |
| 58.6626 Hz |
| 164.2455 Hz |
| 321.8391 Hz |
| 532.0198 Hz |
| 794.7456 Hz |

Table 4: Natural Frequencies calculated with Analytical Model

5.2.2 Mode Shapes

For calculating the mode shape the following equations were used:

$$Y_n(x) = -\sinh \beta_n x + \sin \beta_n x + \left[\frac{\sinh \beta_n l + \sin \beta_n l}{\cosh \beta_n l + \cos \beta_n l} \right] (\cosh \beta_n x - \cos \beta_n x) \quad (10)$$

$$\beta_n = \frac{\left(\frac{2n-1}{2}\pi + e_n\right)}{l} \quad (11)$$

By plugging these equations in MATLAB, the following plots for the first 5 mode shapes were found:

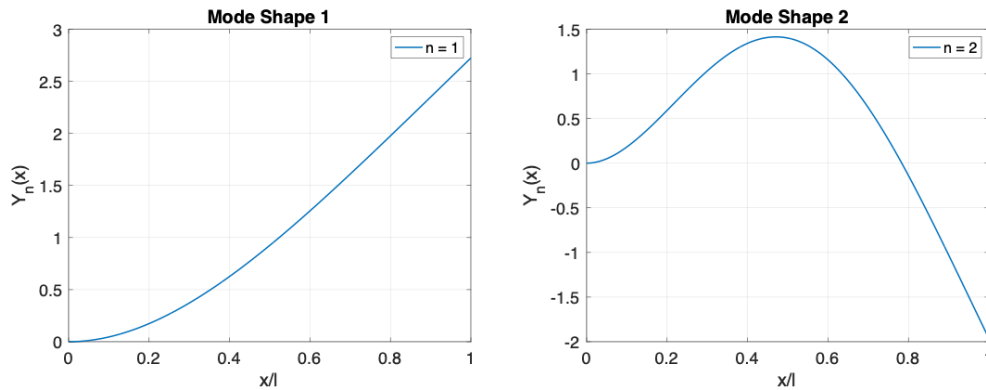


Figure 22: First and Second Mode Shapes from Analytical Equations.

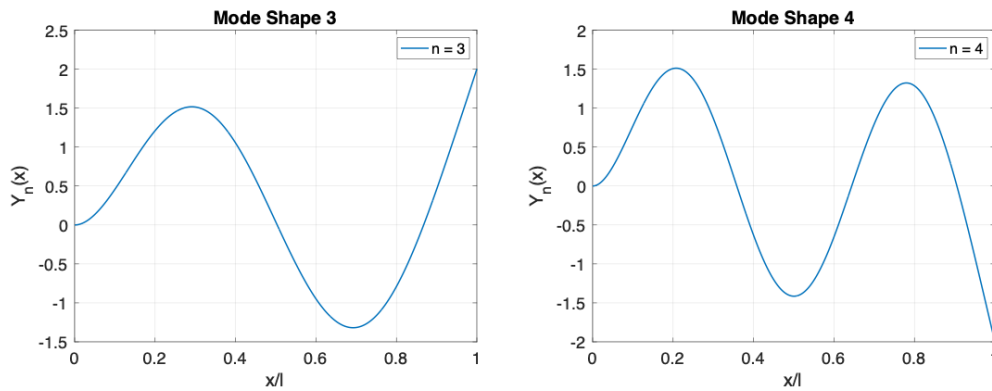


Figure 23: Third and Fourth Mode Shapes from Analytical Equations.

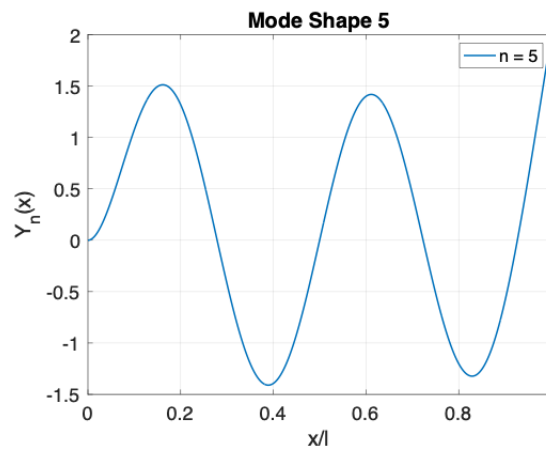


Figure 24: Fifth Mode Shape from Analytical Equations.

6 Discussion

6.1 Comparing the Results

With the results from both methods found, they can now be compared to each other, with as goal determining whether using the ARX-model is a suitable way to find the modal parameters via EMA.

Firstly, comparing the ARX-model mode shapes from figures 19, 20, and 21 with the analytically found mode shapes from figures 22, 23, and 24, it can be deduced that they are fairly similar. The data-driven mode shapes follow the same general path as the analytical mode shapes, but with fewer values, therefore not flowing as neatly as the analytical mode shapes.

For the natural frequencies, the error percentage is calculated to showcase the accuracy of the ARX-model. This is done using the following equation:

$$\text{Error percentage} = \left| \frac{\text{Analytical Value} - \text{ARX Value}}{\text{Analytical Value}} \right| * 100\% \quad (12)$$

Before the error percentages are calculated, it must be noted that one value from the ARX-model was not found by the analytical method, namely the value 255.511 Hz. This is most likely due to that value being the torsional natural frequency. This value will therefore not be taken into account for the analysis.

Following this, table 5 shows the final natural frequencies and their error percentage.

| ARX-Model Natural Frequency | Analytical Natural Frequency | Error Percentage |
|--------------------------------|---------------------------------|------------------|
| 9.0711 Hz | 9.3584 Hz | 3.07% |
| 60.2707 Hz | 58.6626 Hz | 2.74% |
| 164.5189 Hz | 164.2455 Hz | 0.17% |
| 317.8106 Hz | 321.8391 Hz | 1.25% |
| 529.8480 Hz | 532.0198 Hz | 0.41% |
| 798.8191 Hz | 794.7456 Hz | 0.51% |

Table 5: Natural Frequencies and their Error Percentage

Following the results from table 5, the average error is calculated to be 1.36%.

Furthermore, it can be noted based on figure 13 and the results from table 5 that the ARX-model is sufficiently good at finding the natural frequencies of the beam even from different impact locations, as figure 13 and figure 16 show that peaks are at the same frequency for all locations.

Lastly, only 2 damping ratio's were found, with one of them only barely meeting the criteria, therefore no comparison can be made. This is an existing issue for ARX-models as the noise can easily affect the amplitudes.

6.2 Applications

The main application of this research is in Structural Health Monitoring (SHM) which is currently a very topical area as the amount of articles regarding SHM using vibration based is increasing rapidly [14]. Combined with the fact that the focus of engineering communities is shifting towards automation, the use of the ARX-model promotes such automation [15]. The vibrations of, for example a bridge, can be easily measured using a single sensor at any accessible point in the structure [16]. By continuously monitoring the vibrations of the bridge and extracting the natural frequencies and mode shapes from the data using the ARX-model, potential issues such as damages can be identified and addressed before they become a bigger problem. When a structure becomes damaged, it causes a change in the physical properties of the structure. This in turn changes the modal properties, as the natural frequency will be affected if there is damage at any point in the structure [15] [17]. Furthermore, by continuously monitoring the structure and finding the modal properties, maintenance can be conducted in a proactive manner, negating the problem of taking the entire structure out of operation, thus reducing downtime and extending the time between major repairs or replacements. This all leads to a decrease in costs caused by maintenance [6] [14] [15].

As mentioned, the changing of the natural frequencies indicates that there is damage within the structure, but not where. However, with the use of some extra steps and methods, localisation of the damage(s) is possible with the natural frequencies [16]. Therefore using the ARX-model to find the modal properties is a useful tool, as skilled personnel in signal processing and data analysis is not needed as much, as the ARX-model does it for them.

Lastly, the ARX-model has applications in the identification of modal parameters in earthquake-resistant design. As these structures undergo seismic events, their dynamic characteristics change ever so slightly [18] [19]. By testing the structures in an simulated or controlled environment, the structural requirements for these major vibrations can be found and implemented in the design stage. The ARX-model can also be used after an earth-quake to find the modal parameters and see if any damage has occurred.

6.2.1 Damage Detection Example

In order to show that the ARX-model is capable at detecting damage within a structure, an experiment was added to the research. For this experiment, a beam with the same dimensions was taken and a notch of 15x8mm was made at 250mm from the free end, as can be seen in figure 25. Furthermore, the same experiment was conducted, with the hammer hitting the beam at the same locations as the previous test, as seen in table 1. The expectation is that the natural frequencies all shift left, as the beam is damaged.

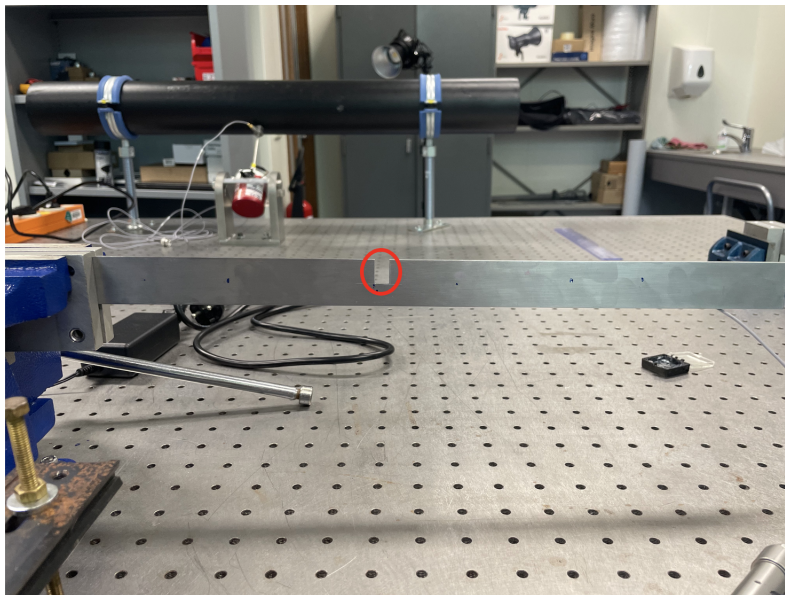


Figure 25: Sideview of the damaged beam upon which the same test was conducted.

For this experiment only the natural frequencies were analysed, as the beam is damaged, the change in natural frequency is highly visible. Only 5 natural frequencies were identified in this experiment. The ARX-model yielded the following plots for the frequency domain and the frequency response function:

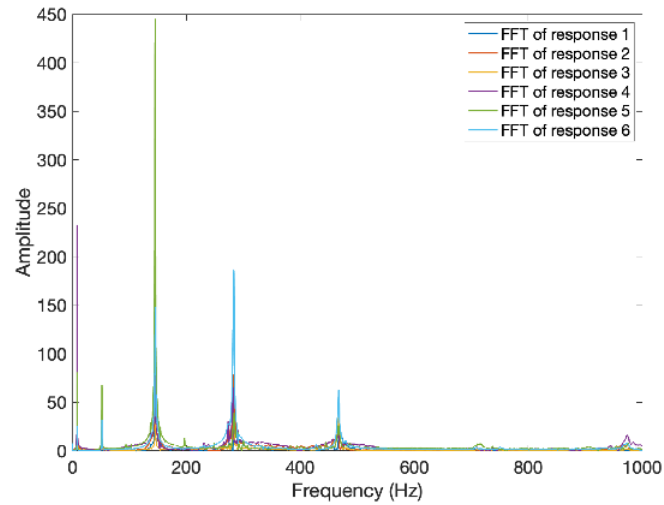


Figure 26: The FFT of the 6 Signals, measured on the Damaged Beam. The peaks indicate the Natural Frequencies.

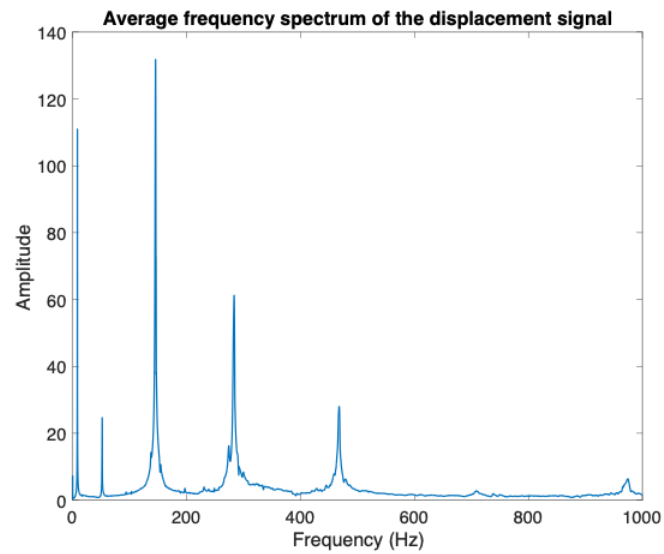


Figure 27: The Average FFT of the Damaged Beam, taken from the Signals in figure 26. The peaks indicate the Natural Frequencies.

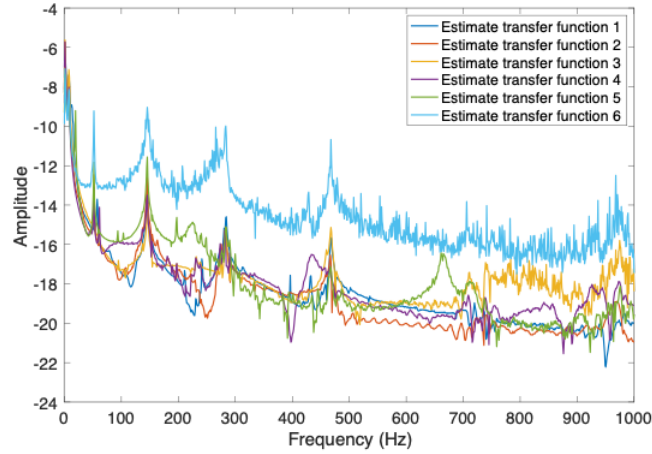


Figure 28: The Estimated Frequency Response Function in the form of the Transfer Function of all 6 Signals of the Damaged Beam.

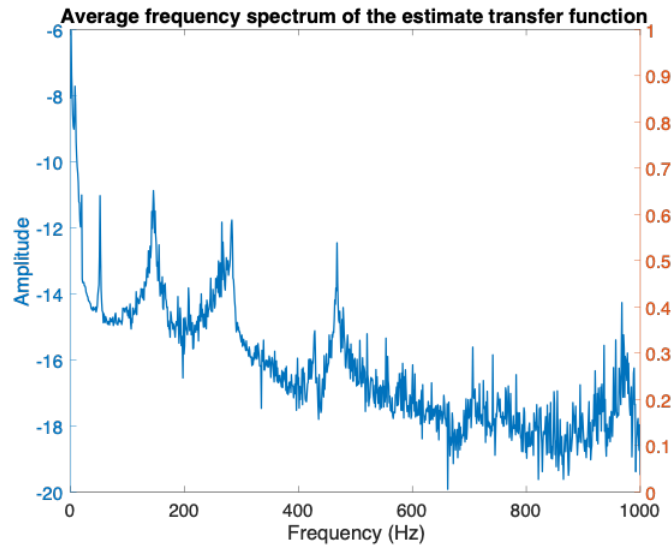


Figure 29: The Average Frequency Response Function of the 6 Signals of the Damaged Beam, from figure 28.

With the use of equation 6 and the plots above, the natural frequencies are found and listed in table 6, along with the original natural frequencies found by the ARX-model.

| ARX-Model Natural Frequency | Damaged Beam Natural Frequency |
|--|---|
| 9.067 Hz | 6.0339 Hz |
| 58.8 Hz | 56.2309 Hz |
| 163.111 Hz | 145.8346 Hz |
| 317.156 Hz | 286.9010 Hz |
| 529.778 Hz | 465.2930 Hz |
| 797.733 Hz | - |

Table 6: Overview of the Natural Frequencies found with the ARX-model.

As expected, the natural frequencies all shift to the left, towards 0.

7 Conclusion

Following the results from this research, it can be concluded that using the ARX-model is a suitable method for finding the Natural Frequencies and the Mode Shapes. The natural frequencies differ on average a mere 1.36% to the analytical natural frequencies and the mode shapes are similar in shape to each other. It is however not suitable for finding the damping ratios, as only 2 were identified.

With these results, it can be concluded that the ARX-model is a useful tool for Structural Health Monitoring, namely for identifying damage within a structure, as shown in the extra experiment, and, with some extra steps, localisation of the damage. Further research should be focused on extracting damping ratio's more accurately and on conducting this experiment but then as a Multiple Input Multiple Output (MIMO) system, in order to find its applications in that area.

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8 Appendix

8.1 MATLAB Code for ARX Model

```

1  clc
2  clear all
3  close all
4  %% Load data, six acceleration response signals, six
   external force signals
5  load('Acc6.mat');
6  load('Force6.mat');
7  Acc=Acc';
8  Force=Forces';
9  %% set parameters
10 fs=2000;% sampling frequency
11 dt=1/fs;% time vector
12 %% Delay setting for ARX modle
13 nf=40;% output
14 nb=40;% input
15 nk=0;
16 %% Stores all natural frequencies and damping ratios
17 ARX_f_all = [];% Stores the natural frequency
   obtained from 6 signals.
18 ARX_d_all = [];% Stores the damping ratios obtained
   from 6 signals.
19 %% ARX solution for each signal
20 for i = 1:6
21     %% Find index of maximum value in current row
22     [peak_value, peak_loc] = max(Acc(i,:));
23     N = length(Force); % Number of samples
24     %% define the length of vibration signals (ending
   point)
25     Len = 45000;
26     if peak_loc + Len - 1 > N
27         error('Selected length exceeds data length.'
28             );
29     end

```

```

29
30     %% Construct new signals
31     Force_cut = Force(i,peak_loc:peak_loc + Len - 1)
           ; % Update the force vector
32     Acc_cut = Acc(i,peak_loc:peak_loc + Len - 1); %
           Update the acceleration vector
33     Acc_all(i,:)=Acc_cut;
34     Force_cut_all(i,:)=Force_cut;
35     N_cut = length(Force_cut); % Update the number
           of sampling points
36     t_cut = 0 : 1/fs : (N_cut - 1)/fs; % Update the
           time vector
37     x=Force_cut';
38     y=Acc_cut';
39
40     %% visualize the new time signals
41     figure(i)
42     subplot(2,1,1)
43     plot(t_cut, Force_cut, 'LineWidth', 1.5)
44     xlabel ('Time (s)')
45     ylabel ('N')
46     title(sprintf('Impact Force Signal %d', i));
47     subplot(2,1,2)
48     plot(t_cut, Acc_cut, 'LineWidth', 1.5)
49     xlabel ('Time (s)')
50     ylabel ('m/s^2')
51     title(sprintf('Response Signal %d', i));
52     hold on
53     %% ARX model
54     z2=iddata(y,x,1/fs);
55     sys = arx(z2,[nf nb nk]);% ARX model
56     %% Construct the right_matrix
57     sub_vector = sys.A(2:end);
58     neg_sub_vector = -sub_vector;
59     right_matrix = zeros(nf, nf);
60     right_matrix(1, :) = neg_sub_vector';
61     for j = 2:nf
62         right_matrix(j, j-1) = 1;

```

```

63     end
64     %% Eigenvalue decomposition
65     [V, D] = eig(right_matrix);
66     D=diag(D);
67     ARX_f=abs(log(D) / dt) / (2 * pi); % natural
        frequencies
68     ARX_d=(-real(log(D) / dt) )/ (abs(log(D) / dt));
        % damping ratios
69     %% Order the natural frequencies and damping
        ratios from smallest to largest.
70     [sorted_f, sorted_indices] = sort(ARX_f);
71     [sorted_d, sorted_indices] = sort(ARX_d);
72     %% Extract the non-zero element columns from the
        ARX_d matrix
73     [row, col] = find(sorted_d); % Get row and
        column indexes for non-zero elements
74     values = ARX_d(sorted_d ~= 0); % Get the value
        of a non-zero element
75     nonZeroData = [values]; % Formation of new
        package groups
76     ARX_d_all=[ARX_d_all,nonZeroData];% Store the
        non-zero damping ratio obtained from 6
        signals
77     ARX_f_all=[ARX_f_all,sorted_f];
78     %% estimate transfer function
79     [H,f] = modalfrf(Force_cut,Acc_cut,fs,rectwin
        (2048),1024,Estimator="H2");
80     H_all(i,:)=H';% Store the transfer function for
        all signals
81 end
82 %% Find the average of the 6 natural frequencies and
        the damping ratios
83 averages_f = zeros(size(ARX_f));
84 averages_d = zeros(size(ARX_d));
85 averages_f= mean(ARX_f_all, 2);
86 averages_d= mean(ARX_d_all, 2);
87 %% visualize the Frequency domain analysis of new
        response signals

```

```

88 figure(7);
89 for i=1:6
90 frequency = 0:fs/length(t_cut):(fs/2-fs/length(
    t_cut));
91 FF=abs(fft(Acc_all(i,:)));
92 plot(frequency, FF(1:length(frequency)), 'LineWidth
    ', 1.5);
93 % plot(frequency, log(FF(1:length(frequency))), '
    LineWidth', 1.5);
94 xlabel('Frequency (Hz)');
95 ylabel('Amplitude');
96 hold on
97 % Add legend
98 legendStrings = arrayfun(@(k) sprintf('FFT of
    response %d', k), 1:6, 'UniformOutput', false);
99 legend(legendStrings);
100 end
101
102 figure(8);
103 averF = mean((abs(fft(Acc_all(1:6, :))))), 2);
104 plot(frequency, averF(1:length(frequency)), '
    LineWidth', 1.5);
105 xlabel('Frequency (Hz)');
106 ylabel('Amplitude');
107 title('Average frequency spectrum of the
    displacement signal');
108
109
110 figure(9);
111 for i=1:6
112 plot(f, log(abs(H_all(i,:))), 'LineWidth', 1.5)
113 xlabel('Frequency (Hz)');
114 ylabel('Amplitude');
115 hold on
116 % Add legend
117 legendStrings = arrayfun(@(k) sprintf('Estimate
    transfer function %d', k), 1:6, 'UniformOutput',
    false);

```

```

118 legend(legendStrings);
119 end
120
121 H_mean = mean(H_all, 1);
122
123 figure(10);
124 yyaxis left;
125 plot(f,log(abs(H_mean)), 'LineWidth', 1.5)
126 xlabel('Frequency (Hz)');
127 ylabel('Amplitude');
128 title('Average frequency spectrum of the estimate
        transfer function');
129 %hold on
130 %yyaxis right;
131 %scatter(averages_f, (1:numel(averages_f)), 'kx', '
        LineWidth', 1.5);
132 %ylabel(' Delay order');
133 % Set x-axis limits
134 %xlim([0 1000]);
135
136 figure(11);
137 scatter(averages_d, 1:numel(averages_d), 'kx', '
        LineWidth', 1.5);
138 xlabel('Damping ratios');
139 ylabel(' Delay order');
140
141 %% Mode shape
142 modeshape=real(V);

```

8.2 MATLAB Code for the Analytical Natural Frequencies

```

1 % Set the Parameters of the Beam
2 l = 0.420 ;
3 E = 205*10^9;
4 rho = 7850;
5 A = 0.03*0.002;

```

```
6 I = (0.03*0.002^3)/12;
7 n = 1;
8
9 e_n_values = [0.3042, -0.018, 0.001, 0, 0, 0];
10
11 % Loop over different n
12 for n=1:6
13
14 % Calculate e_n
15     if n <= length(e_n_values)
16         e_n = e_n_values(n);
17
18
19 % Calculate Natural Frequencies f_n
20     omega_n = (((2*n-1)/2)*pi + e_n)^2 * 1/(l^2) *
21         sqrt((E*I)/(rho*A));
22     f_n = omega_n/(2*pi)
23
24     n= n+1;
25 end
end
```

8.3 MATLAB Code for the Analytical Mode Shapes

```
1 l = 0.420;
2 n_values = 1:5;
3 x = linspace(0, 1, 1000);
4
5 e_n_values = [0.3042, -0.018, 0.001, 0, 0];
6
7 % Loop over different n values
8 for idx = 1:length(n_values)
9     n = n_values(idx);
10
11     % Calculate e_n
12     if n <= length(e_n_values)
13         e_n = e_n_values(n);
```

```
14     else
15         e_n = 0;
16     end
17
18     % Calculate beta_n
19     beta_n = ((2*n - 1)/2 * pi + e_n) / l;
20
21     % Calculate Y_n(x)
22     term1 = sinh(beta_n * x);
23     term2 = sin(beta_n * x);
24     term3 = (sinh(beta_n * l) + sin(beta_n * l)) / (
25         cosh(beta_n * l) + cos(beta_n * l));
26     term4 = (cosh(beta_n * x) - cos(beta_n * x));
27
28     Y_n = -term1 + term2 + term3 * term4;
29
30     % Plot the result
31     figure;
32     plot(x/l, Y_n, 'DisplayName', sprintf('n = %d',
33         n));
34     xlabel('x/l');
35     ylabel('Y_n(x)');
36     title(sprintf('Mode Shape %d', n));
37     legend show;
38     grid on;
39 end
```