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An overview of warm inflation

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Contents

Abstract

This paper provides an overview of warm inflation, an alternative to the standard cold inflationary model. Unlike cold inflation, warm inflation includes radiation during the inflationary period, maintaining a thermal bath, and eliminating the need for a separate reheating phase. The study examines the dynamics of warm inflation, focusing on the equations of motion of the inflaton field, energy scales, and the derivation of the primordial power spectrum, where thermal fluctuations dominate over quantum ones. The scalar dissipation function $G(Q)$ from the literature is presented, enhancing the accuracy of power spectrum calculations and facilitating comparisons with empirical data. Various inflation models are reviewed for their compatibility with warm inflation and observational constraints, aiming to align theoretical predictions with cosmological observations.

Acknowledgments

I would like to dedicate this work to my grandmother, who taught me to read, add, and subtract. She left secondary education to work the land after the Spanish Civil War, and her biggest dream was for her granddaughters to attend university.

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1 Introduction

What is the origin of the universe?

This is a timeless question, inherent to human nature. To answer it, a countless number of ideas have been proposed across the ages, always intertwining belief, philosophy and science [1]. Nowadays, one of the most compelling scientific answers is the Hot Big Bang theory [2], supported by numerous empirical observations from the cosmic microwave background (CMB) [3]. However, the theory still faces significant challenges that have long beset cosmologists, notably the horizon, flatness, and monopole problems. Addressing these issues within the Hot Big Bang framework is possible, but requires a great level of fine-tuning of the theory.

Inflation emerged as an alternative and more elegant solution, as it does not require fine-tuning [4]. Inflation posits an epoch of accelerated expansion before the Big Bang, driven by vacuum energy dominance, causing the scale factor to grow exponentially. The inflationary expansion would have allowed for the universe to be sufficiently flat, homogeneous and isotropic on the largest observable scales [5].

Initially, the application of particle physics to the early universe in this theory was revolutionary [6]. Not only does it address the shortcomings of the Hot Big Bang theory, it also introduces the concept of primordial fluctuations [7]. These fluctuations can manifest as perturbations in the metric, tensor perturbations, or density perturbations within the homogeneous background universe. In this paper, we will focus on the density perturbations, which are responsible for the formation of the large-scale structures observable today. A robust early universe model must include a source of these primordial perturbations, often explained by homogeneous scalar fields. In most inflation theories, a specific type of scalar field, denoted as the inflaton is the driver of the inflationary expansion [8].

The standard inflation scenario, known as cold inflation, involves a single scalar field slowly rolling down an almost flat potential, generating a quasi-de Sitter phase [1] through which the universe expands superluminally. Once the field reaches the potential minimum, vacuum energy converts to radiation, thereby reheating the universe [10]. An alternative to cold inflation is warm inflation, first introduced by Arjun Berera in 1995 [11]. Warm inflation includes radiation during the inflationary period. Due to the presence of radiation, the inflaton can interact with other fields and decay into radiation and matter. This process consistently supplies radiation, supporting a thermal bath during inflation, which will make the temperature of the universe not go to zero. The emerging radiation bath invokes a dissipation term in the equation of motion of the inflaton [12], revoking the need for a separate reheating phase at the end of inflation [13]. The presence of this thermal bath requires a different formalism than cold inflation with regards to the primordial perturbations, as fluctuations would be primarily thermal in origin.

[1] The de Sitter universe is a spacetime characterized by a positive constant four-curvature, maintaining uniformity and symmetry in spatial and temporal dimensions [9].

Warm inflation is a relatively new concept that still requires further investigation. This paper introduces the topic of warm inflation and examines how the evolution of the power spectrum differs in this paradigm compared to cold inflation. Chapter 2 explores the limitations of the Hot Big Bang theory and introduces the fundamentals of inflation, particularly how it addresses the horizon and flatness problems. Chapter 3 focuses on warm inflation, detailing its dynamics, energy scales, and the derivation of the primordial power spectrum, with an emphasis on the dominance of thermal fluctuations and the scenario in which the inflaton field couples with radiation, using the function $G(Q)$. Chapter 4 critically examines various models of warm inflation, evaluating their compatibility with observational data and theoretical constraints. Finally, the paper summarizes the theory and assesses the viability of warm inflation, highlighting its potential advantages over cold inflation and suggesting directions for future research.

2 An introduction to inflation

2.1 Big Bang shortcomings

As mentioned in the introduction, the Hot Big Bang theory is a widely accepted framework for understanding the origin of the universe. However, this theory presents certain challenges, which we will discuss in the following sections. To address these challenges, the theory of inflation was proposed. Although this might appear to be circular reasoning, as the resolution of these issues is a postdiction rather than a prediction of inflationary expansion, the theory of inflation does make testable predictions, such as the imprint of density perturbations in the cosmic microwave background. In this paper, we will explore the horizon and flatness problems associated with the Hot Big Bang theory. The monopole problem, however, is outside the scope of this discussion. For a detailed summary of the monopole problem, refer to Barbara Ryden's introductory book on cosmology [4].

2.1.1 The horizon problem

The first shortcoming of the Hot Big Bang model to be discussed is the horizon problem. The horizon, or particle horizon, is defined as the largest distance that a signal can travel from the time corresponding to the initial singularity (τ_i) to a later time (τ) [10]. It is given by the following equation:

$$\chi_p(\tau) = \tau - \tau_i = \int_{\tau_i}^{\tau} dt' = \int_{a_i}^{a} \frac{Ha^2}{a(t')} \sim a^{(1+3w)/2} - a_i^{(1+3w)/2}. \tag{1}$$

This equation gives an expression for the comoving [2] distance $\chi_p(\tau)$ at conformal time τ. H is the Hubble parameter, a is the scalar factor, and a_i is the initial scalar factor. w is a constant that signifies the type of matter that dominates in the universe.

As the universe expands, the horizon does as well, and equation 1 will be dominated by the contribution of the latest time when w is greater than $-\frac{1}{3}$. This constraint for w is further explained in section A of the appendix. This results in regions of space that were never in causal contact, coming into contact for the first time. If the universe was not initially homogeneous, these regions should look different from each other, according to the special theory of relativity. However, causally disconnected regions in the CMB are remarkably similar. How is this possible? This is known as the horizon problem. We will discuss in section 2.2.1 how inflation offers a plausible answer to this question.

2.1.2 The flatness problem

The second shortcoming of the Hot Big Bang theory to be discussed is the flatness problem. The curvature of the universe is not predicted by the theory of inflation. The Friedmann-Robertson-Walker metric (FRW), further explained in section A of the appendix is used in inflation and is a solution for three different geometric scenarios; an open, a closed and a flat universe. Therefore, it does not give a definite answer about the flatness of the universe. It is then better to study the flatness of the universe in terms of its energy content. To do that, we can look at the energy fraction $\Omega_k(a)$ in the following form:

$$\Omega_k(a) = -\frac{k}{a^2 H^2(a)}, \tag{2}$$

[2] A comoving coordinate system is one in which the frame we use as reference grows along with the expansion of the universe [14].

where k is the curvature parameter of the universe. When $w > -\frac{1}{3}$, the solution $\Omega_k = 0$ becomes an unstable point. Notably, Ω_k can reach at most ± 1; in such cases, w approaches $-\frac{1}{3}$ if $k < 0$, while for $k > 0$, the universe would collapse [10]. Interestingly, current observations show that Ω_k is smaller than 10^{-3} [3]. This means that the geometry of our universe now is relatively close to being flat. Such flatness would require very specific conditions for the early universe, so what causes this flatness?

2.2 Inflation formalism

Before solving the two problems of the Hot Big Bang theory previously introduced, we need to understand what is meant by inflation. In this section, an overview of the framework of inflation is offered, as well as a description of its mathematical formalism. This is mainly based on the following literature [4, 5, 8, 10, 15].

The first empirical evidence of the expansion of the universe was discovered by Edwin Hubble. He found that the redshift of galaxies was directly proportional to the their distance from Earth. This can be seen in Figure 1, created by Hubble himself in his paper from 1929 [16]. His measurements led him to conclude that galaxies farther away from Earth, were moving away at a rate faster than the ones closer to us. He stated that the universe must be expanding. This idea had been introduced by Lemaitre a few years earlier, who was quite ahead of his time, since his proposition was that if the universe is expanding now, it must have been smaller in the past [17].

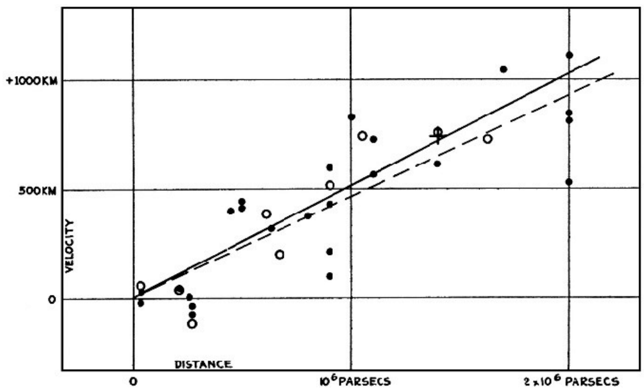

Figure 1: Velocity-distance relation among extra galactic nebulae Edwin Hubble 1929 [16].

It was not until much later, that this idea of inflation was applied to the early universe as a solution to the limitations of the Hot Big Bang theory. Even though the shortcomings mentioned in the previous section can be avoided by fine-tuning of the initial state of the universe, there is no reason why there could not be another solution to the problem. A far more elegant approach is inflation. The idea of an inflationary expansion in the early universe was first proposed in the 1960s [18, 6]. While several earlier papers had touched on this concept, it was Guth's work in 1981 that became the most cited. Guth's paper was the first to argue convincingly that inflation could resolve both the horizon and flatness problems [19].

The standard theory of inflation suggests an exponential expansion phase that occurred before the Hot Big Bang. In this phase, the universe's scale factor grew exponentially, while the Hubble radius decreased. More information about the Hubble radius can be found in section A of the appendix. During inflation, the universe's energy density was dominated by a vacuum energy similar to a cosmological constant. At the end of the inflationary period, before the Hot Big Bang, this energy transformed into radiation and matter through a reheating process, both elements occupying the universe today [4, 20].

2.2.1 Inflation as a solution to the horizon and flatness problems

For a clear and comprehensive explanation of how inflation addresses the horizon and flatness problems, some analogies can be presented. The horizon problem questions why two points that are non-causally connected in the cosmic microwave background have the same properties, i.e. the same temperature. To understand how inflation solves the problem, we can visualize an Olympic-sized swimming pool with two dyes of different colors dropped at opposite edges. The time it takes for the dyes to completely mix represents the horizon problem. Now, consider a shot glass with two dyes dropped at the edges. The dyes in the shot glass mix faster than in the pool. This analogy illustrates that for the vast size of the universe observed in the CMB, it would take an impossibly long time for distant points to reach the same temperature. However, if these points started in a much smaller region (like the shot glass) and then underwent rapid expansion, they could have the same properties.

The solution to the horizon problem is represented as well in Figure 2, where we see how two points denoted as P and Q are not in causal contact now, but they were in the past in some region of space that expanded exponentially. As they were connected in the past, they can have the same properties in the present.

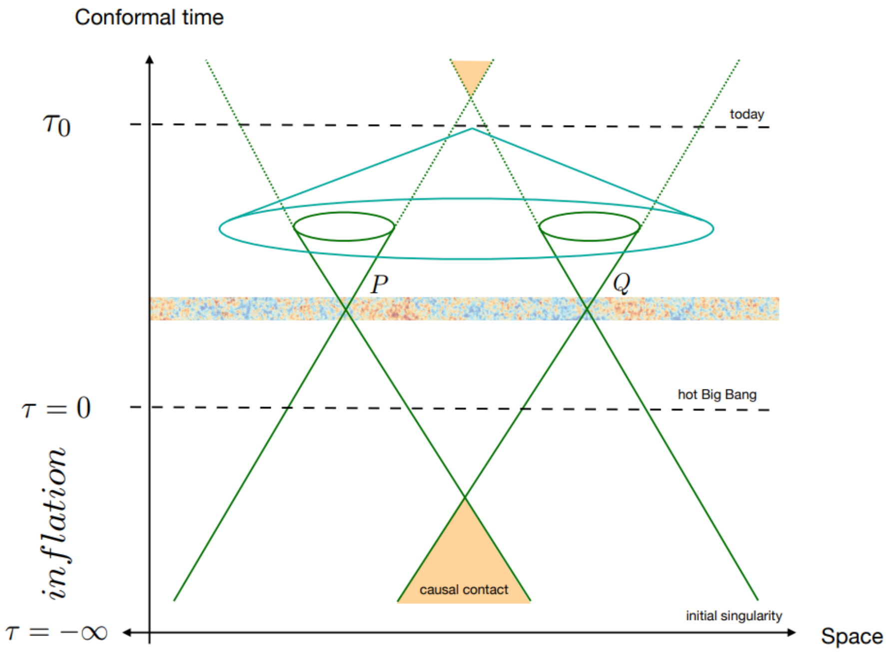

Figure 2: Conformal time versus space diagram representing the horizon problem. P and Q are causally disconnected in the CMB, but were causally connected in the past due to an inflationary period [21].

The solution to the flatness problem can be effectively illustrated by picturing a very small observer, often depicted as an ant, on an inflatable surface like a balloon. This can be seen in Figure 3.

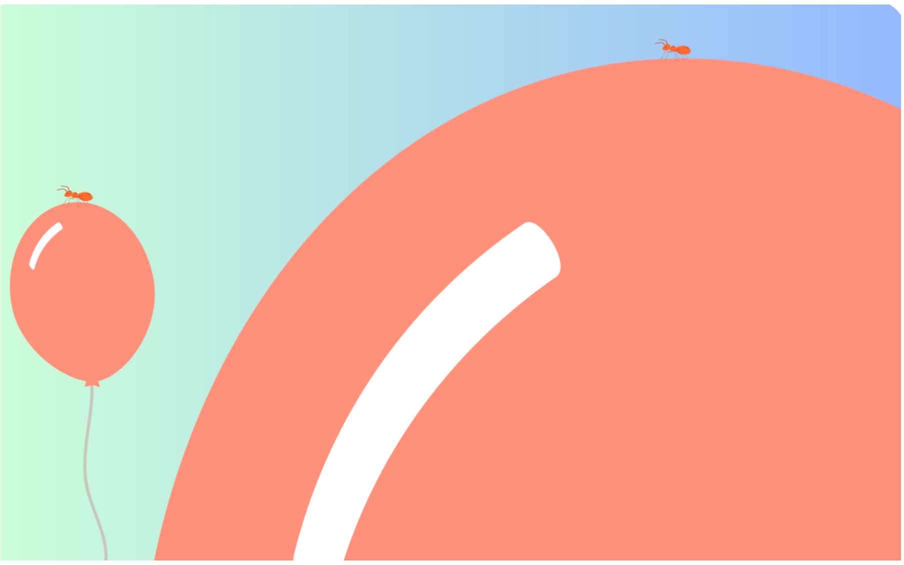

Figure 3: An ant on a balloon, illustration of the flatness problem.

Initially, the ant perceives the balloon's surface as curved. However, if the balloon inflates rapidly, the surface appears locally flat to the ant. This analogy can be extrapolated to our universe: with the balloon's inflation representing the expansion of the universe, with rapid inflation smoothing out initial curvature, making the universe appear flat on a large scale despite its initial state [15].

Inflation provides a solution to the shortcomings of the Hot Big Bang theory without needing fine-tuning and can be considered as a viable model for the early universe. In the following section, we will examine the standard inflation scenario.

2.2.2 The inflaton field

In particle physics, spin zero particles are modelled by scalar fields. These are conserved under coordinate transformations and are often associated with symmetry breaking mechanisms, such as the Higgs mechanism or the ones present in Grand Unified Theories [20] . These fields are thought of as being relevant to numerous physical processes, including inflation.

In most scenarios for inflation, a scalar field is chosen to drive this process of expansion, the inflaton field ϕ. The most common models for the evolution of the scalar field are denoted as "slow roll" models of inflation, because the scalar field slowly rolls down to the lowest value of the potential. If the field rolls slow enough, we can disregard the time derivative of the field in comparison with the value of the potential $V(\phi)$. At this point, the scalar field acts as a cosmological constant and the universe experiences a period of expansion [5]. This process can be seen in Figure 4.

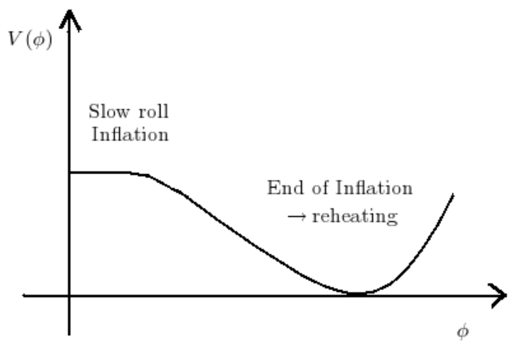

Figure 4: Potential for slow-roll inflation. [22]

For inflation to occur, there needs to exist a negative pressure to initiate the process. The scalar field is initially in a state denoted as a "false vacuum", which occurs as a result of large negative pressure. This is the reason why the potential energy $V(\phi)$ needs to dominate in the universe during inflation [19]. The scalar field stays in a potential plateau, and then rolls down a potential energy hill. Whenever the rolling speed of the field is much slower than the expansion of the universe, inflation occurs. The field approaches the potential minimum value (true vacuum) and when the hill is steeper again inflation ends and a process called reheating starts.

Due to the exponential expansion during inflation, radiation and matter become diluted, making any other sources of energy density negligible as they scale with a negative power of $a(t)$. This accelerated expansion results in an almost empty universe with no radiation, meaning the temperature during inflation is zero in the standard cold inflation model. At the end of the inflationary period, the field begins to oscillate around the minimum of its potential. These oscillations cause the field's kinetic energy to decay into radiation and matter through interactions with other fields. This phase, known as reheating, occurs only after the inflationary period has ended and is not considered part of it. [5].

In this paradigm, the inflaton field is isolated during inflation and does not interact with other fields. In the next section, another aspect of the standard inflation scenario will be discussed, primordial fluctuations.

2.2.3 Introduction to perturbations in the cold early universe

The universe is currently described as homogeneous and isotropic. However, when examining the Cosmic Microwave Background (CMB), anisotropies are observed [23]. These anisotropies pose a puzzling question that has led cosmologists to a compelling idea. The initial hypothesis suggested that the universe's initial conditions were not perfectly homogeneous and that gravitational clumping was responsible for structure formation. However, a period of inflation would smooth out these inhomogeneities, making this explanation unlikely. This raises the question: how did these anisotropies form?

The answer lies in the nature of inflation, which is driven by a scalar field governed by the principles of quantum mechanics. According to Heisenberg's uncertainty principle, we cannot precisely determine the value of the inflaton field and its conjugate momentum simultaneously [24]. As a result, there are small quantum fluctuations or perturbations in the field. To understand how these quantum fluctuations affect the inflaton field, we can divide the field into two parts, as shown in the following equation:

$$\phi = \bar{\phi}(t) + \delta\phi(t, \mathbf{x}). \tag{3}$$

The first term is the background field, which is not dependent on space, homogeneous and it obeys classical laws. The second term is the primordial density fluctuations. These can be quantized and included in the equations of motion of the field. To understand their behaviour we will look at their power spectrum in section 3.3.3. First, we will discuss these perturbations and what their effects are on the universe.

These density fluctuations lead to the small anisotropies in the CMB [4]. These anisotropies are the temperature fluctuations observed in the CMB and are thought to be responsible for formation of structures, such as galaxies in the universe [25]. This is due to the quantum nature of the field. Quantum properties cause inflation to end at slightly different times in different points of space, so there will be regions denser than others at the end of inflation. Small fluctuations get augmented during inflation and increase in size by the gravitational attraction after inflation.

During inflation there exists as well other types of perturbations, such as metric perturbations. If we are to measure the polarization of the photons in the CMB we would find that there were fluctuations in the metric caused by gravitational waves generated during inflation. These are also called tensor perturbations. We will not focus on them in this paper, but it is to be noted that they are quite fascinating. For further information on tensor perturbations see [26]. The ratio of the amplitude of these tensor perturbations to the amplitude of scalar perturbations, which are another type of metric perturbations, is denoted as the tensor-to-scalar ratio r and it is relevant for observational constraints in inflation models [27].

2.3 Inflation dynamics

2.3.1 Equations of motion of the inflaton field

To understand the dynamics of the inflaton field, we define a Lagrangian density of the form:

$$\mathcal{L} = \mathcal{L}_s + \mathcal{L}_R + \mathcal{L}_I, \tag{4}$$

where $\mathcal{L}_s$ is the inflaton system Lagrangian, $\mathcal{L}_R$ is the Lagrangian for radiation fields, and $\mathcal{L}_I$ is the Lagrangian describing the interaction between the inflaton field and other fields. The simplest Lagrangian is defined as $\mathcal{L} = T - V$, with T being the kinetic energy contribution and V the potential. In the cold inflation scenario, there is no radiation present during inflation so the inflaton is an isolated field, as we discussed previously. To understand its dynamics we can model it with the Lagrangian density equation for a real scalar field, given by:

$$\mathcal{L}_s = \frac{1}{2}\ddot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2 - V'(\phi). \tag{5}$$

The first term accounts for the kinetic energy, the second for the gradient energy, and the third for the potential energy of the scalar field [6]. To derive the equations of motion of the field, we will use the action of a real scalar field:

$$S = \int d^4x \sqrt{-g}\left(-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V'(\phi)\right). \tag{6}$$

As we are treating a homogeneous and isotropic universe, we will use the Friedmann-Robertson-Walker metric described in section A of the Appendix. Hence, the metric will be given by $\sqrt{-g} \equiv \sqrt{-|(g_{\mu\nu})|} = a^3$. Note that $\frac{1}{2}\partial_\mu\phi\partial^\mu\phi = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2$, and that $V'(\phi) = \frac{\partial V(\phi)}{\partial\phi}$ [10]. By applying the previous conditions on the real scalar field, we can vary its action to find:

$$\begin{aligned}
\delta S &= \int d^4x a(t)^3 \left(-g^{\mu\nu}\partial_\mu\phi\partial_\nu\delta\phi - V'(\phi)\delta\phi\right) \\
&= \int d^4x \left[\partial_\nu\left(a(t)^3 g^{\mu\nu}\partial_\mu\phi\right) - a(t)^3 V'(\phi)\right]\delta\phi \\
&= \int d^4x \left[-\partial_t\left(a(t)^3\dot{\phi}\right) + \partial_i\left(a(t)\Upsilon^{ij}\partial_j\phi\right) - a(t)^3 V'(\phi)\right]\delta\phi \\
&= \int d^4x \left[-3\dot{a}(t)a(t)^2\dot{\phi} - a(t)^3\ddot{\phi} + a(t)\nabla^2\phi - a(t)^3 V'(\phi)\right]\delta\phi \\
&= \int d^4x \left(-a(t)^3\right)\left[\ddot{\phi} + 3\frac{\dot{a}(t)}{a(t)}\dot{\phi} - \frac{\nabla^2\phi}{a(t)^2} + V'(\phi)\right]\delta\phi.
\end{aligned} \tag{7}$$

From the previous expression, one can obtain the equation of motion for the inflaton field background:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0. \tag{8}$$

The Hubble parameter H is the rate of expansion of the universe $\left(\frac{\dot{a}}{a}\right)$. In this paper the term Hubble scale is used for this value as well. For more information see section A in the appendix.

Equation 8 is the equation of motion for the inflaton in the scenario that there are no dissipation effects and that radiation is not emitted during inflation. It is the first approximation in which the density perturbations are not included in the equation, and therefore, we do not have a noise term from the quantum density perturbations in cold inflation.

2.3.2 Reheating

During inflation, the inflaton field gradually moves toward the minimum value of its potential. Upon reaching this minimum, the inflaton oscillates around it. In this process, the kinetic energy of the inflaton decays into radiation and matter, marking the end of inflation and the beginning of the reheating period [28]. Subsequently, the temperature of the universe is no longer zero, and the inflaton field can couple to other fields. To account for the dissipation effects of this process an additional term needs to be added to the equations of motion of the inflaton [5],

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + V'(\phi) = 0. \tag{9}$$

The additional term Υ is the dissipation term, which accounts for the interaction of ϕ with other fields. For inflation to happen, we need $H >> \Upsilon$. At the end of inflation H decreases, so Υ is the relevant parameter for the reheating process.

The cold inflation model proposes that if the inflaton interacts with certain Standard Model particles, these interactions will establish the initial conditions needed for all Standard Model particles to reach thermal equilibrium. This scenario sets the stage for the occurrence of the Hot Big Bang [29].

The reheating temperature is bounded by the energy density at the end of inflation but is not necessarily the same. The exact expression for the temperature is dependent on the chosen model [30].

3 Warm inflation

The previously discussed theory and formalism correspond to the standard view on inflation. However, this is not the only model that can drive the inflationary process. The idea that we wish to discuss is warm inflation. The main difference between warm and cold inflation, lies, as the names suggest, in the temperature during the inflationary process.

There are numerous cosmological models for inflation, yet this paper introduces a new one. The obvious question arises: why should we consider this model? The warm inflation model originated from an idea by Berera in 1995, inspired by a talk on cosmological inflation [11]. Berera proposed that since almost all dynamical systems involve a dissipation term, the inflaton field during the inflationary period should be no different. This dissipation term is negligible in the cold inflation scenario because it is assumed that the process is too fast for any microphysical processes to take place, resulting in a zero temperature state [31]. In this scenario, radiation would be neglected, and the inflaton would not interact with other fields. However, is possible that there is enough time for microphysical processes to occur and for radiation and dissipative terms to be significant [12, 29]. Consequently, it is possible to have a scenario in which the scalar field is not isolated but interacts with other fields during the inflation period. These interactions would result in particle production and radiation during inflation, thereby eliminating the need for a separate reheating phase [32].

This model is drastically different from the cold inflation paradigm and leads to a very interesting conclusion: the density perturbations in the early universe could be of a classical nature, instead of a quantum nature due to their thermal nature [26]. As microphysical processes are permitted in this new paradigm, there would be a suitable dynamical range during the inflation period for these processes to generate statistical states such as thermalization, making the thermal perturbations of classical nature predominant in the universe [12].

In this chapter, we discuss the formalism of warm inflation, its dynamics (3.1), its conditions (3.2), and its density perturbations (3.3). The density perturbations have a special focus to provide an accessible but thorough interpretation of the power spectrum and its derivation in the warm inflation scenario. This is extremely relevant, as understanding it will be essential to contrast this theory with observational data [6].

3.1 Dynamics of warm inflation

In this section, a derivation of the dynamics of the inflaton field in the warm scenario is shown along with an explanation of the implications of this new idea.

The Lagrangian considered for most inflation scenarios is of the form of equation 4, which contains a term accounting for the interaction of the scalar field with other fields $\mathcal{L}_I$ and for the interaction of the inflaton field with radiation fields $\mathcal{L}_R$. In the cold inflation scenario, both of these terms were neglected and did not contribute to the equation of the Lagrangian density of the inflaton during inflation, since the inflaton field was isolated. However, in the warm inflation scenario, these terms are not neglected. The Lagrangian density describing the field has a dissipation term that arises from the interaction between the inflaton field and other fields.

The equation of motion of the inflaton field in the warm inflation scenario is therefore a Langevin type equation [3] of the form:

$$\ddot{\phi} + 3H\dot{\phi} + \Upsilon\dot{\phi} + V'(\phi) = \zeta. \tag{10}$$

This is the same equation as the inflaton equation in the reheating period, equation 9, but it includes a term, that accounts for the density perturbations and their noise in the right hand side. We will elaborate further on the noise term in section 3.3. This is no coincidence, as in the reheating period the inflaton is interacting with other fields and radiation is present. This also shows that for the warm inflation scenario, a separate reheating period is not needed, since matter and radiation are being produced during inflation.

During the inflationary period in warm inflation, the inflaton field is also conditioned by slow roll motion, the slow roll parameters of warm inflation are described in section B of the appendix. As radiation is present, the value of the energy densities in the warm inflation scenario is different than in the standard cold one. For the warm inflation paradigm, the general relativity cosmological energy conservation equation[4], gives the derivative of the radiation energy density ρ_r, as:

$$\dot{\rho}_r = -4H\rho_r + \Upsilon\dot{\phi}^2. \tag{11}$$

We can see that the first term on the right hand side is a sink term that depletes radiation energy density, but the second term sources energy density. This means that radiation energy cannot be neglected because it will not go to zero. In warm inflation models, it is assumed that the radiation energy density is finite when inflation starts [36]. This does not happen in the standard cold inflation model, where radiation and matter only become present in the reheating process. At long enough times, the radiation present in our universe will be independent of the primordial conditions because it will only depend on the rate at which it is being produced by the source term. It is worth highlighting that the vacuum energy is still the dominant energy in the universe in warm inflation. This agrees with general relativity and the initial idea for inflation [19].

In the warm inflation paradigm there is no reheating period, but inflation still needs to end. The end of the inflationary period in this model occurs when the radiation energy density exceeds the vacuum energy density [11]. This transition signifies the universe moving from an inflationary phase to a radiation-dominated phase, leading to a smooth exit from inflation without the need for a reheating period.

[3] A Langevin type equation is a stochastic differential equation describing the evolution of some degrees of freedom dependent on time. These equations have frictional and random forces. For a further analysis on Langevin equations, see [33] and for more information about stochastic processes see [34].

[4] The general relativity cosmological energy equation is given by $\frac{\partial \mathscr{T}^{\mu\nu}}{\partial x^\nu} = 0$, where $\mathscr{T}^{\mu\nu} \equiv \sqrt{-g}T^{\mu\nu}$ defines the total energy, momentum, and stress. For an exhaustive explanation, and the definition of the parameters in these equations, see [35].

3.2 Energy scales in inflation

For a comprehensive conceptual understanding of warm inflation, it is useful to examine the relationships between the different energy scales. In inflation models we often encounter the following energy scales: the vacuum energy E_v, the radiation energy E_r, the Hubble scale H, the mass of the inflaton field m_ϕ, and in the case of warm inflation, the dissipative coefficient Υ. In the context of cold inflation, the vacuum energy is significantly larger than the radiation energy, the Hubble scale is larger than the mass of the inflaton, which in turn surpasses the radiation energy. Additionally, the Hubble scale greatly exceeds the dissipative coefficient, thereby making dissipative effects are negligible in this context.

In the context of warm inflation, the vacuum energy exceeds the radiation energy, creating a negative pressure that initiates inflation. However, the radiation energy will surpass the mass of the inflaton field, making a zero temperature state impossible. In warm inflation, there are two regimes: one where $\Upsilon > 3H$ and another where $\Upsilon \leq 3H$. These are referred to as the strong, and weak dissipative regime respectively. The dominating term controls the damping evolution of the field during inflation [32, 37]. Table 1 summarizes the relationships between different energy scales in the cold and warm inflationary scenarios.

Table 1: Comparison of Energy Scales in Cold and Warm Inflation

Cold Inflation	**Warm Inflation**
$E_v >> E_r$	$E_v > E_r$
$H > m_\phi$	$H < m_\phi$ (Strong)
$m_\phi > E_r$	$m_\phi < E_r$
$H >> \Upsilon$	$\Upsilon > H$ (Strong)
	$\Upsilon \leq H$ (Weak)

One of the most important outcomes of the warm inflation theory is the possibility of an inflaton mass greater than the Hubble parameter in the strong dissipative regime (coloured in blue in table 1). This is a great difference in comparison with the standard inflation model. In the latter, the inflaton mass is very small and strictly less than the Hubble scale [38]. This affects "The η problem" in cosmology, and the swampland conditions in string theory [39]. Gravity guides the mass of the inflaton to be close to the value of H, but for inflation to occur, they cannot be equal [39, 40]. In the case of warm inflation, this problem disappears, since the mass can be even greater than the Hubble scale [37]. This is one of the main advantages of the use of the warm inflation framework.

There exists another significant energy scale that warrants discussion, namely, the quantum gravity scale. In cosmology, the influence of quantum gravity becomes important at energy levels approaching the Planck scale [23]. During cold inflation, the amplitude of the inflaton field is damped solely by the Hubble parameter, $3H$, as seen in equation 8. It is greater than the Planck scale, which makes it be in the quantum gravity scale regime. In contrast, during warm inflation, the amplitude of the inflaton field is further reduced due to dissipation effects by $3H + \Upsilon$. Thermal fluctuations impose a limit on the perturbation amplitude in the field, as they surpass quantum effects in magnitude [37].

Additionally, during the strong regime of warm inflation, in which $\Upsilon > 3H$, the dissipation further reduces the amplitude of the inflaton field. This damping effect in the strong regime is governed by the factor $3H + \Upsilon$, as depicted in equation 10. This makes the amplitude of the inflaton field be below the Planck scale. Consequently, the inflaton field amplitudes differ between the two scenarios, with the amplitude in warm inflation being below the Planck scale, thereby mitigating potential quantum gravity effects [6]. Avoiding the complexities of quantum gravity effects facilitates a clearer and more comprehensive understanding of the theory, given that the field of quantum gravity is still under development and remains largely unexplored.

We have observed that the relationships between different energy scales of inflation vary between the two models in certain cases. The primary advantages of the warm inflation model over the cold inflation model includes an inflaton mass greater than the Hubble scale in the strong regime, and an inflaton amplitude below the Planck mass, thus evading the influence of quantum gravity.

3.3 Density perturbations in warm inflation

In the warm inflation scenario, the temperature T of the thermal bath present during inflation is greater than the mass of the inflaton and particularly larger than the Hubble parameter in the strong regime. In this case, thermal fluctuations of the scalar field will be dominant in comparison with the quantum fluctuations [12]. This contrasts the cold inflation scenario, where quantum fluctuations are dominant and thermal fluctuations are neglected.

3.3.1 Derivation of the primordial power spectrum

To understand the evolution of the density perturbations, we will derive its power spectrum. It is essential to be able to derive this quantity in most of the theories of inflation, since it also allows us to compute the tensor-to-scalar ratio r, and the spectral tilt n_s [5]. These two quantities are crutial to compare models to experimental data [41]. And it is expected that due to the influence of thermal fluctuations on the scalar perturbations, the tensor-to-scalar ratio value will be significantly lower in the warm inflation scenario, in comparison with the standard one [42]. This is one of the most important differences between the cold and warm scenarios of inflation, and a difference that can be experimentally tested. The spectral tilt value is model dependent within both scenarios [42].

The power spectrum is defined as the statistical average of the distribution of power among the frequency components that make up a signal. This is achieved through Fourier analysis, as any physical signal can be decomposed into discrete frequencies over a continuous range [43]. In this section, we will derive the primordial power spectrum of density perturbations, focusing on the thermal noise present in the warm inflation scenario. We will follow a similar approach to that used by Ramos and Silva [44]. In the mentioned literature, the power spectrum is derived for inflation models covering regimes from cold inflation to warm inflation. The results of the paper were also consistent with CMB data from the Wilkinson Microwave Anisotropy Probe (WMAP) [45]. Our focus, as previously mentioned, is on deriving the contributions to the power spectrum from thermal perturbations.

[5]The spectral tilt, denoted as n_s, is a crucial parameter in cosmology that characterizes the distribution of primordial density fluctuations generated during the inflationary period. It measures the deviation from a perfectly scale-invariant spectrum, where $n_s = 1$, which would indicate no tilt.

In section 3.1, the equation of motion for the inflaton field in the warm inflation regime was derived, and it is given by equation 10, which is a Langevin-type equation. The equation includes a term that describes the dissipation of energy from the field to the radiation bath and a term accounting for the backreaction of the thermal perturbations in the radiation bath on the inflaton field. This latter term is a stochastic noise term, which can be of thermal or quantum nature depending on whether the warm or cold regime is being studied, respectively.

There is a relevant contribution from the radiation bath to the power spectrum and as previously stated, a domination of thermal fluctuations $\delta\varphi_t$ over the quantum inflaton field fluctuations $\delta\varphi_q$ when $T > H$, since $\delta\varphi_q \sim H$ [12]. We have also introduced that there exists two regimes of warm inflation, the strong dissipative regime when $\Upsilon > 3H$ and the weak dissipative regime when $\Upsilon \leq 3H$ [6]. In the strong dissipative regime the dissipation caused by the radiation production dominates over the friction of the metric expansion in the equations of motion. The strong dissipative regime offers a more interesting theoretical framework, this is also why we encounter more literature on the dynamics of the inflaton field within this regime [6].

To study the density perturbations in warm inflation we will take equation 10 and perform a coarse-graining [6] on the inflaton field following a similar procedure as previous literature has used for the perturbations in cold inflation [46]. This procedure is denoted as the stochastic inflation approach and it consists on reducing the number of degrees of freedom of the complex system by splitting the field in a long wavelength part and a short wavelength part.

$$\Phi(x,t) \to \Phi_{>}(x,t) + \Phi_{<}(x,t). \tag{12}$$

The short wavelength term represents the field modes with a wavelength smaller than the horizon and describes quantum vacuum fluctuations, high momentum modes of ϕ with $k \gtrsim \dot{k}_h \approx aH$, where k is the comoving coordinate wave-vector, K_h is its value at the horizon and $k \equiv |k|$. The separation of modes is also done through a window filter function often constructed as a Gaussian function in the literature [47, 48].

The long wavelength modes are decomposed according to [44] as:

$$\phi_{<}(x,t) \equiv \phi_q(x,t) = \int \frac{d^3k}{(2\pi)^{3/2}} W(k,t) \left[\phi_k(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \hat{a}_k + \phi_k^*(t) e^{i\mathbf{k}\cdot\mathbf{x}} \hat{a}_k^\dagger \right]. \tag{13}$$

The functions $\phi_k(t)$ are the field modes in momentum space, $\hat{a}_k^\dagger$ and $\hat{a}_k$ are the creation and annihilation operators respectively, and $W(k,t)$ is the window filter function.

[6] Coarse-graining is a method used in complex mathematical and physical systems to simplify its study by dividing them into smaller parts, easier to evaluate, for more information on the use of this method see the paper by Ramos and Silva [44].

The most common microscopic mechanism for generating radiation in warm inflation involves the transfer of energy from the inflaton field to various other fields through field interactions. In a quasi-equilibrium thermal state where the radiation is rapidly thermalized, allowing the state to be maintained, and by applying the Friedmann-Robertson-Walker metric, equation 10 can be written as:

$$\frac{\partial^2\Phi}{\partial t^2}+(3H+\Upsilon)\frac{\partial\Phi}{\partial t}-\frac{1}{a^2}\nabla^2\Phi+\frac{\partial V_{eff,r}(\Phi)}{\partial\Phi}=\xi_T. \tag{14}$$

$V_{eff,r}$ is the renormalized effective potential for the inflaton field. The term in the right hand side is the noise component that describes thermal fluctuations in the local approximation for the equation of motion in warm inflation. The connection of this noise term with the dissipation coefficient Υ previously mentioned is given by a Markovian fluctuation dissipation relation [7], given by the following equation in Ramos and Silva paper [44]:

$$\langle\xi_T(x,t)\xi_T(x',t')\rangle=2\Upsilon Ta^{-3}\delta(x-x')\delta(t-t'). \tag{15}$$

The average is done over a statistical ensemble. The dissipation term is found to be dependent on the amplitude of the inflaton, the temperature T and the mass of the fields coupled to the inflaton. In this paper we will study the model presented by Gabriele Montefalcone on his 2024 paper because of its relevance as the only known paper presenting a model able to give a numerical solution to the power spectrum of the thermal density perturbations accounting for coupling effects between the inflaton and the radiation field [50].

$$\Upsilon(\phi,T)=C\Upsilon M^{1-c+m}\frac{T^c}{\phi^m}. \tag{16}$$

C_Υ is a dimensionless constant, which is model dependent, M is a mass scale for the model, and c and m are integers corresponding to the coupling of the inflaton with other fields. We will expand further on the work done by Montefalcone in section 3.3.3, where we will also see how the power spectrum changes if the coupling between the inflaton field and the radiation field denoted as c is not zero. The strength of the dissipative term is parametrized by the following equation [50]:

$$Q=\frac{\Upsilon}{3H}. \tag{17}$$

To obtain the total power spectrum for the perturbations we will follow the stochastic approach introduced by Starobinsky as cited before in the warm inflation scenario. We will describe the inflaton field in equation 14 split into the quantum fluctuation part for sub-horizon modes as $\phi_q(\mathbf{x},t)$ and in a super horizon term behaving as the classical inflaton mode, which is divided into the homogeneous inflaton field $\phi(t)$ and the fluctuations $\delta\varphi(x,t)$, so:

$$\Phi(x,t)=\phi(t)+\delta\varphi(x,t)+\phi_q(x,t). \tag{18}$$

If we are to substitute this equation in equation 14 and expand on the classical and quantum perturbations to first order, we can obtain the equations for the background field and the classical fluctuations. The equation for the latter $\delta\varphi(x,t)$ is given by:

$$\left(\frac{\partial^2}{\partial t^2}+[3H+\Upsilon(\phi)]\frac{\partial}{\partial t}-\frac{1}{a^2}\nabla^2+\Upsilon(\phi)\dot{\phi}+V_{\phi\phi}(\phi)\right)\delta\varphi=\tilde{\xi}_q+\xi_T. \tag{19}$$

[7] For more information on Markovian perturbations and the Fluctuation Dissipation Theorem in statistical physics see [49].

$V_{\phi\phi}(\phi)$ represents the average value of the potential in the long-wavelength regime with respect to the de Sitter vacuum. For more information on its derivation, refer to [44]. $\tilde{\xi}_q$ is the quantum noise term that accounts for the quantum modes in the inflaton field, as understood in the standard scenario of cold inflation, and ξ_T is the thermal noise term described previously. Since this paper works with the local approximation of the Langevin equation, these terms are understood as Markovian stochastic noises [8].

The dynamics can be better understood by writing equation 19 in momentum space. For that we can use the variable $z \equiv k/aH$. If we also substitute the values of the slow roll coefficients $(\varepsilon, \eta, \beta)$ appended to this paper in section B and obtained in [54], we will get:

$$\delta\varphi''(k,z) - \frac{1}{z}(3Q+2)\delta\varphi'(k,z) + \left(\frac{1+3}{z^2}\right)\eta - \frac{\beta Q}{1+Q}\delta\varphi(k,z) = \frac{1}{H^2 z^2}\left(\xi_T(k,z) + \tilde{\xi}_q(k,z)\right) \quad (20)$$

The primes indicate derivatives with respect to the variable z. The general solution for the equation is obtained through a Green function, and can be expressed as:

$$\delta\varphi(k,z) = \int_z^\infty dz'\, G(z,z') \frac{(z')^{1-2\nu}}{z'^2 H^2}\left(\tilde{\xi}_q(z') + \xi_T(z')\right). \quad (21)$$

At the same time, and for the sake of simplicity, the Green function can be presented in terms of Bessel functions:

$$G(z,z') = \frac{\pi}{2} z'^{\nu}[J_\alpha(z)Y_\alpha(z') - J_\alpha(z')Y_\alpha(z)], \quad (22)$$

with $z' > z$, $\nu = 3(1+Q)/2$ and $\alpha = \sqrt{\nu^2 + \frac{3\beta Q}{1+Q} - 3\eta}$.

Finally, the following equation is derived, where thermal and quantum noise terms are uncorrelated [44]. This enables a separate study of thermal and quantum noises.

$$\begin{aligned} \langle \delta\varphi(k,z)\delta\varphi(k',z)\rangle = {} & \frac{1}{H^4}\int_z^\infty dz_2 \int_z^\infty dz_1 G(z,z_1)G(z,z_2)\frac{(z_1)^{1-2\nu}}{{z_1}^2}\frac{(z_2)^{1-2\nu}}{{z_2}^2}\langle \tilde{\xi}_q(k,z_1)\tilde{\xi}_q(k',z_2)\rangle \\ & + \frac{1}{H^4}\int_z^\infty dz_2 \int_z^\infty dz_1 G(z,z_1)G(z,z_2)\frac{(z_1)^{1-2\nu}}{{z_1}^2}\frac{(z_2)^{1-2\nu}}{{z_2}^2}\langle \tilde{\xi}_T(k,z_1)\tilde{\xi}_T(k',z_2)\rangle. \end{aligned} \quad (23)$$

It is possible then to evaluate the thermal noise contribution to the density perturbations in equation 18. Since we are focusing on the study of the warm inflation dynamics, we will not derive the quantum noise component for the power spectrum. However, interested readers can refer to chapter IV, section A in the paper by Ramos and Silva [44].

[8] A Markov process occurs when an stochastic process has a finite number of possible stage outcomes and the outcome of any stage depends only on the outcome of the previous stage. Further information on Markov processes can be found in the following literature [51]. For more information on the non-Markovian case, refer to [52]. Additionally, the work by Jacob Barandes on stochastic processes and causal locality provides a different understanding of the theory and a possible new approach to the quantum paradigm [53].

3.3.2 Thermal noise component for the power spectrum

The definition of the power spectrum of the inflaton in terms of the two-point correlation function is described in the book by Liddle and Lyth [25] as:

$$P_{\delta\varphi} = \frac{2\pi^2}{k^3}\int \frac{d^3k'}{(2\pi)^3}\langle \delta\varphi(k,z)\delta\varphi(k',z)\rangle \tag{24}$$

To derive the term of the spectrum associated with thermal noise, equation 15 can be expressed in terms of momentum space and using the z variable. This gives:

$$\langle \tilde{\xi}_T(k,z_1), \tilde{\xi}_T(k',z_2)\rangle = \frac{2\Upsilon T}{k^2k'}H^4 z_1^3 z_2 \delta(z_1-z_2)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}'). \tag{25}$$

Substituting the previous equation in the power spectrum from equation 24, the thermal contribution to the power spectrum is given by:

$$\begin{aligned} P_{\delta\varphi}^{(\mathrm{th})} &\equiv \frac{k^3}{2\pi^2H^4}\int \frac{d^3k'}{(2\pi)^3}\int_z^\infty dz_2 \int_z^\infty dz_1 G(z,z_1)G(z,z_2)\frac{(z_1)^{1-2\nu}}{z_1^2}\frac{(z_2)^{1-2\nu}}{z_2^2}\langle \tilde{\xi}_T(k,z_1)\tilde{\xi}_T(k',z_2)\rangle \\ &= \frac{\Upsilon T}{\pi^2}\int_z^\infty dz' z'^{2-4\nu} G(z,z')^2. \end{aligned} \tag{26}$$

The argument z in the equation can be approximated to zero in the late stages of inflation, which are the observable ones. Using the gamma function $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}\,dt$ and the asymptotic form of the Bessel functions given by $Y_\alpha(z) \simeq -\frac{\Gamma(\alpha)}{2}\sqrt{\frac{\alpha z}{\pi}}$ following the derivation in [44], we get that if we neglect terms proportional to the slow-roll coefficients such that $\alpha \simeq \nu = 3(1+Q)/2$ and consider a spatially flat gauge, the power spectrum can be approximated as:

$$\begin{aligned} P_{\delta\phi}(z)^{th} &\simeq \frac{z^{2\nu-2\alpha}}{16\pi^2}\frac{[2\nu\Gamma(\nu)]^2\Gamma(\nu-1)\Upsilon T}{\Gamma(2\nu-1/2)\Gamma(\nu-1/2)} \\ &= z^{2\eta-2\beta Q/(1+Q)}\frac{HT3Q8^Q\left[\Gamma\left(\frac{3}{2}+\frac{3Q}{2}\right)\right]^3}{\pi^2(1+3Q)\Gamma\left(\frac{5}{2}+3Q\right)\Gamma\left(1+\frac{3Q}{2}\right)}. \end{aligned} \tag{27}$$

By incorporating the quantum noise derived by Ramos and Silva with the thermal noise, the total power spectrum is given by:

$$P_{\delta\varphi}(z) = \frac{HT}{4\pi^2}\left[\frac{3Q}{2\sqrt{\pi}}2^{2\alpha}z^{2\nu-2\alpha}\frac{\Gamma(\alpha)^2\Gamma(\nu-1)\Gamma(\alpha-\nu+3/2)}{\Gamma(\nu-1/2)\Gamma(\alpha+\nu-1/2)} + \frac{H}{T}\coth\left(\frac{zH}{2T}\right)z^{2\eta}\right]. \tag{28}$$

The power spectrum described in the previous equation takes into account the thermal and quantum noises and describes the distribution of the primordial perturbations into different frequencies at horizon crossing, i.e, when the wavelength of the mode is equal to the horizon size, $z \equiv \frac{k}{aH} = 1$ [44].

3.3.3 Evolution of the perturbations

However, the power spectrum is not constant on scales well outside the horizon [25]. This raises the question: how can observations from the present epoch be compared to conditions during inflation?

Deriving a transfer function is then crucial to determine the initial density perturbations, and represent them in the present epoch. It is noteworthy that this is possible because different regions of the universe that are not locally connected can be treated as independent, unperturbed evolutions on scales far outside the horizon [25]. These different regions are considered identical up to the synchronization of their clocks. Rather than examining the density of species in the universe, this study will focus on their density contrast, defined by $\delta \equiv \delta\rho/\rho$. At the initial epoch, each density contrast is related to the total density contrast δ_k, due to the radiation dominated period. This relation is defined as the adiabatic condition and a set of density contrasts satisfying it is known as an adiabatic density perturbation [10].

To derive the transfer function, it is essential to introduce a new perturbation, denoted as the curvature perturbation $\mathbf{R}(x,t)$. The crucial characteristic of this new parameter is its constancy over time [25]. It maintains consistent values from the initial to the present epoch and is related to the inflaton perturbation. If we select the FRW metric with a flat gauge, it is given by $\mathbf{R} = H\delta\phi/\dot{\phi}$ Under this metric, the comoving curvature [9] and the amplitude of the curvature perturbation are equal according to the derivations in [50] to:

$$\Delta_R^2 = \frac{H^2}{\dot{\phi}^2} P_{\delta\varphi} = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2 1 + 2n_* + \frac{T}{H}\frac{2\sqrt{3}\pi Q}{\sqrt{3+4\pi Q}}, \tag{29}$$

where n_* is the inflaton statistical distribution caused by the presence of the radiation bath. This is generally assumed to be the equilibrium Bose-Einstein distribution, $n_* = [\exp(H/T)) - 1]^{-1}$.

It is evident that to reach this point multiple approximations and assumptions were made. That is why we denote this result as an analytical estimate. The principal constraints considered were that the temperature power of the dissipation rate is zero ($c = 0$) in equation 16, and that the metric perturbations mentioned in the theory (2.2.3) are neglected, and not addressed. Other restrictions were that: (1) H and T are constant with respect to the z parameter to put them outside of the integral in equation 26; (2) the η slow-roll coefficient described in appendix B is ignored so $\alpha \approx \nu$.

According to Montefalcone in his paper [50], the only relevant assumption that can change the final result substantially is the deviation from a temperature power zero in the dissipation rate. In the case that $c \neq 0$, the scalar power spectrum will be significantly affected, particularly in the strong dissipation regime in warm inflation.

This complication drove the research of Montefalcone to introduce a function addressing the dissipation strength $G(Q)$, which multiplies the already derived analytical expression. This factor $G(Q)$ can only be solved numerically. To this end, Montefalcone developed an open-source code explained in the aforementioned paper [50]. In the following section we will gain some insight in the functioning of the code and the numerical method to find the dissipation strength function. This is particularly relevant since it is the first paper known to the author, that has explored this area. A calculation in a similar direction but working with Fock space appears in a paper by Ballesteros from 2023 [55].

[9] For more information on the comoving curvature parameter see [25].

3.3.4 Numerical determination of the scalar dissipation function $G(Q)$

In recent literature on warm inflation, the power spectrum is given as an expression similar to equation 24. However, this expression is only an analytical estimate. To include the coupling between the inflaton and radiation perturbations, where the power in the dissipation rate $c \neq 0$, numerical calculations are required. In order to do so, the power spectrum is multiplied by a function describing the dissipation strength $G(Q)$. The function $G(Q)$ is determined by dividing the numerically computed value by its analytical counterpart:

$$G(Q) \equiv \frac{\Delta^2_{R,numerical}|c \neq 0}{\Delta^2_{R,\mathrm{analytic}}}. \tag{30}$$

Understanding the differences in observations between warm and cold inflation is crucial. A precise numerical fit for $G(Q)$ for various dissipation rates and potentials is essential for calculating observable parameters, such as the spectral tilt n_s or the tensor-to-scalar ratio r.

The detailed derivation and explanation of these concepts are beyond the scope of this paper, however, a brief introduction to the tensor-to-scalar ratio will be provided for context. The tensor-to-scalar ratio is the ratio of the amplitude of tensor perturbations (metric perturbations, i.e., gravitational waves) to the amplitude of scalar perturbations. This variable is crucial for distinguishing between the cold and warm inflation regimes. According to the literature [6, 41, 56], the curvature power spectrum is significantly affected by the dissipation term in warm inflation, which dramatically alters the tensor-to-scalar ratio compared to cold inflation. This is the motivation for a numerical derivation of the dissipation function. The code for the numerical calculation of $G(Q)$ in Montefalcone's paper [50] can be found here.

The code is thoroughly discussed in Montefalcone's paper, which includes a review of the code, a summary of its functions, and an explanation of its working mechanism. For further details, please refer to Montefalcone's paper.

Here, a review and summary of the paper are provided to give an overview of the code's functionality and future implementation. First, it is important to consider that obtaining the dissipation function requires other parameters, which depend on the inflaton model used. The body of the code is written in the Python script `WI_Solver_utils.py`. It is divided into four modules: (1) `inflaton_Model`, (2) `Background`, (3) `Perturbations`, and (4) `Scalar_Dissipation_function`.

1. `inflaton_Model`

   In this module, the model chosen for the inflaton field is defined. This is not related to the type of field we are to treat, since this one will always be a scalar field, but rather the type of potential of the field. In this block, all the required parameters dependent on the potential are defined. In the literature, numerous potentials have been presented [57, 58, 59, 60, 61], some of which will also be discussed in section 4. In this code, the author proposes four potentials for the definition of the model. These are the Monomial potential, the Hilltop-like potential, the potential associated with Natural inflation and the β-exponential potential.

We will focus on the Monomial and Hilltop potential for our discussion due to its simplicity and popular use for the discussion of the dynamics of the scalar field. The Monomial potential is given by:

$$V(\phi) = \frac{\lambda}{n!}\phi^n, \tag{31}$$

λ depicts the coupling of the inflaton field with itself when $n = 4$, being n the steepness of the potential. This inflaton model does not agree with the new observational data offered by BAO and BICEP2/Keck Array data for the cold inflation scenario. However, this model agrees with the observations for a good amount of parameters in the warm inflation scenario [55].

The Hilltop potential is given by:

$$V(\phi) = V_0 \left[1 - \left(\frac{\phi}{\phi_f}\right)^{2n}\right]^2. \tag{32}$$

In this case $n \geq 1$ and $\phi << \phi_f$ initially, because ϕ_f is assumed to be large enough for the inflation to end before arriving to the inflection point of the potential. This potential is suitable for an effective field theory and it was found to agree with the data in warm and cold inflation regimes [62].

2. `Background`

This module is responsible for solving the background evolution based on the number of e-folds, N_e. Initial conditions are set so that inflation ends when the initial number of e-folds equals the number of e-folds at the end of inflation, which is zero. According to observations N_e between horizon crossing and inflation is 60 [63]. For more information on the calculation of the background evolution see [50] .

The most important aspect of Montefalcone's approach for solving the background dynamics is the assumption of a constant dissipation strength Q during the expansion. This implies that the background dynamics are no longer dependent on the temperature or the power-law exponents of the inflaton field's dissipation rate Υ (specifically, c and m). This assumption limits the code's use for studying background evolution in warm inflation. However, it can still be applied to interpret dissipation effects given a specific background model. Future work focusing on the background dynamics would greatly enhance the code, enabling precise solutions for the background evolution in warm inflation. Montefalcone mentions in his paper that he plans to address this issue in the future.

3. `Perturbations`

Taking the background evolution computed in the previous module for a specific inflaton potential, and the values for c and m, the module solves numerically the perturbations that are obtained from equation 23. The `Perturbations` module gives the scalar power spectrum at horizon crossing, as well as the standard deviation, which is dependent on Q.

4. `Scalar Dissipation function` $G(Q)$ is computed by taking the previous modules results as well as the appropriate values for c, m and the parameters dependent on the inflaton potential choice. This is done according to equation 30. This module is in charge of calculating the function that fits best $G(Q)$ using the method of least squares. In the literature, the most common fitting functions are given by [61, 64]

$$G_{\text{pol}}(Q) = \begin{cases} 1 + AQ^{\alpha} + BQ^{\beta}, & \text{if } c > 0 \\ (1 + AQ^{\alpha})(1 + BQ^{\beta})^{-\gamma}, & \text{if } c < 0. \end{cases} \tag{33}$$

However, the polynomial function does not provide a satisfactory result for positive values of c, i.e, it does not fit the data in Montefalcone's paper well when the dissipation rate is proportional to the temperature. That is something to keep in mind when referring to the paper. They resolved the issue by using a logarithmic fitting function instead of a polynomial one, which perfectly fits their data. The differences between the fitting function and the computed values for c can be observed in Figure 5. The logarithmic fit is given by:

$$G_{\log}(Q) = 10 \sum_{n=1}^{4} a_n x^n, \quad x \equiv \log_{10}(1+Q). \tag{34}$$

The WarmSPy program by Montefalcone was executed for the Monomial potential, and the fits for the dissipation function $G(Q)$ were plotted using these resources.

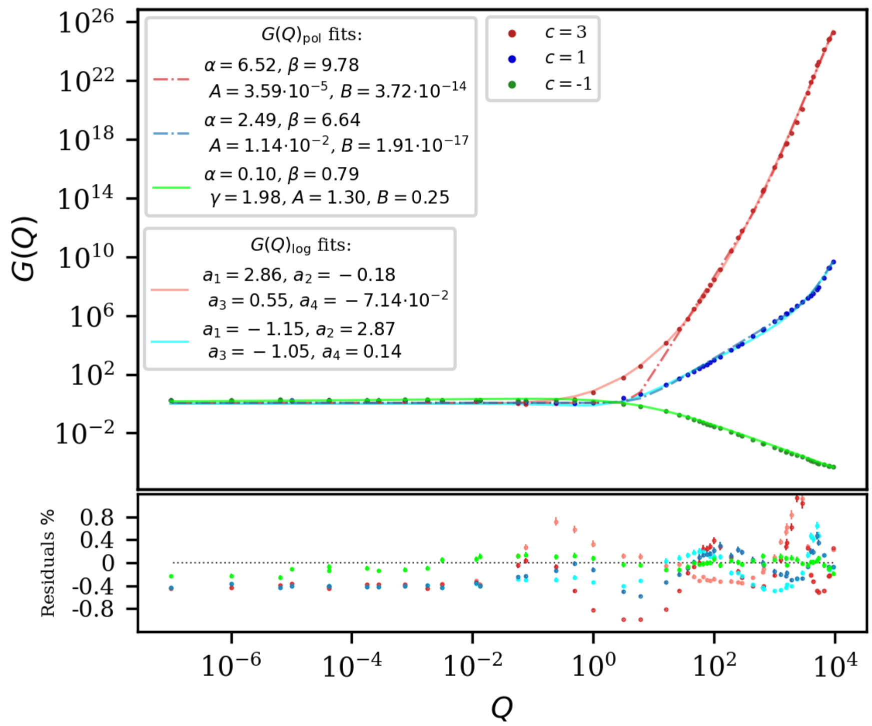

Figure 5: Solutions computed numerically for the function $G(Q)$ with a Monomial potential depending on the temperature power dependence of the dissipation rate. Values of $c = 3, 1, -1$ were chosen to be coherent with Montefalcone's results. Lower frame: the fractional residuals between the computed result for $G(Q)$ and the different fit choices are displayed.

Figure 5 provides an illustrative analysis of the behavior of the dissipation function for the Monomial potential scenario across different values of c, along with an evaluation of the quality of the fits. These results can guide future research in appropriately selecting fits for dissipation functions in the warm inflation scenario.

The results were obtained under the following limitations: no inflaton field dependence in the dissipation rate ($m = 0$), 60 e-folds after horizon crossing, and neglecting metric perturbations. According to Montefalcone's study, these assumptions do not affect the validity of the results, as the fits for the scalar dissipation function $G(Q)$ exhibit broader applicability, extending beyond the constraints of the mentioned assumptions [50].

The polynomial fitting function better represents a negative temperature dependence of the inflaton field with the radiation bath, whereas, the logarithmic fitting function, is more suitable for a positive temperature dependence. This discrepancy arises because the magnitude of $G(Q)$ cannot be fully captured by simple polynomial fitting functions. Additionally, the accuracy of the polynomial fit is highly dependent on the range of Q values to which it is applied. For precise calculations, a narrow range of Q is preferred.

Obtaining the function $G(Q)$ is crucial for calculating the power spectrum of primordial density perturbations in the warm inflation scenario. This function accounts for the effects of thermal fluctuations, which distinguish warm inflation from the cold inflation scenario. While the specific calculation of the power spectrum depends on the chosen warm inflation model, the method is universally applicable to all warm inflation cases. An accurate calculation of the power spectrum, enabled by determining $G(Q)$, is essential for testing warm inflation models against current and future CMB data. This calculation allows for the determination of experimental constraints, such as the tensor-to-scalar ratio r and the spectral tilt n_s.

4 Selecting warm inflation models

As outlined in section 3.3.4, the investigation of warm inflation is dependent on the choice of potential for the inflaton field. Previously, the Monomial and Hilltop potentials were analyzed, with the Monomial potential utilized to derive the dissipation function $G(Q)$. In this section, we will explore more recent potentials introduced in literature, evaluating their suitability for describing inflaton dynamics within the warm inflation framework. This discussion follows Arjun Berera's proposition in his 2023 paper [6].

In cosmology, the greatest difficulty is to prove theories against observations, as these observations are extremely difficult to achieve. The empirical information available to test theoretical cosmological theories is very limited. Whilst this poses a challenge to theorists, it is also a motivation to do a thorough evaluation of the different models available. In the context of inflation, the cosmological data results in an upper bound on the tensor-to-scalar ratio, on the amplitude of the scalar perturbations, and on the spectral tilt. The goal is to have a model that adequately fits these parameters; while some other conditions are met, such as the temperature at the end of inflation and the specific theoretical constraints of the model presented.

According to Berera [6], a constructive way of evaluating different potential models is to understand how speculative they are. Berera considers models that include theories and ideas significantly different from the Standard Model to be very speculative. He asserts that while the Standard Model is insufficient to describe all cosmological theories, it can serve as the foundation for constructing a robust theory of inflation, particularly, for warm inflation. Berera distinguishes between two speculative features in cosmological models, fundamental and technical. The table that he introduces defining the features of the fundamental and the technical category is shown below:

Table 2: Range of speculative features in cosmological models

Category	Features
Fundamental [F]	Quantum gravity, additional spacetime dimensions above four, modifications to gravity beyond general relativity, sub-Hubble mass scalar fields and supersymmetry/other new spacetime symmetries or adjustments to them
Technical [T]	Effective field theory methods with cutoff scale below m_p, symmetries included in the model not of the type in the Standard Model and excluding new spacetime symmetries, symmetries included in the model, extra fields added beyond the Standard Model and not attributed to any symmetry and model building beyond the Standard Model

The fundamental category involves theories with unknown unknowns and the technical category presents theories with known unknowns as Berera describes. All the elements in the technical category have been tested to some extent, making it possible to use them to create a robust theory that could potentially be tested as well. However, if a theory relies on the fundamental category, making empirical progress is challenging. This is not to suggest that some models are superior to others, but to highlight the challenges associated with models in theoretical cosmology. In his paper, Berera analyses models from a purely theoretical aspect to analyse their "degree of speculation". This discussion is valuable not only to see what models are more easily testable, but also for reviewing the constraints and paradigms of each model presented. Three of the models presented by Berera will be reviewed: the D-Brane inflation model. The R^2 Starobinsky model and the Warm Little inflaton model. The latter was also proposed by Arjun Berera et al. in their article from 2016 [61].

1. D-brane inflation model

   This is a model within the string theory paradigm. In it, there exists an interaction energy between two parallel brane and anti-branes[10], which are solutions arising from string theory. Their potential energy would drive inflation. The potential for the inflaton would have the form:

$$V_{D-\text{brane}}(\phi) = M^4 \left(1 - \frac{\alpha}{\phi^4}\right). \tag{35}$$

   This model has as fundamental speculations: quantum gravity, higher dimensions, inflaton mass under the Hubble scale, and supersymmetry. In the technical category, it contains symmetries outside of the Standard Model and model building beyond the Standard Model. This gives the model a very high count of fundamental speculations. For more information on this model, see [66].

2. R^2 Starobinsky model

   This model proposes a modification in the curvature by adding $R^2/(6M)^2$ to the action of the field. R is the Ricci scalar and $M > m_p$. This is essentially a model with modified gravity that poses a potential of the form:

$$V_{R^2}(\phi) = \Lambda^4 \left(1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{m_p}\right)\right)^2. \tag{36}$$

   This model has as fundamental speculations: quantum gravity, modifications in general relativity, and a mass of the inflaton field below the Hubble scale. As technical challenges, it includes symmetries outside of the Standard Model and model building also outside of the Standard Model. Further information about this model can be found in the following literature [67].

[10] A brane is an object extended in one or more dimensions in space. These arise in string theory. For a further analysis the following literature is recommended [65].

3. Warm Little inflaton model

   In this model two complex Higgs fields are present. The expectation value of the vacuum for both fields is nonzero, giving rise to two bosons. The relative phase of the two fields gives a singlet, the inflaton. The potential for the inflaton in this case is a monomial potential given by:

$$V_{\text{warm little}}(\phi) = \frac{\lambda}{4!}\phi^4 \text{ , or } \frac{1}{2}m^2\phi^2. \tag{37}$$

   This model does not contain fundamental speculations, but has technical challenges, such as effective field theory methods, fields beyond the Standard Model not attributed to any symmetries, and model building outside the Standard Model. Futher information about the model can be found in [61].

Among the models presented, the Little inflaton model stands out as particularly interesting according to Berera's discussion. This is because it does not depend on altering fundamental theories. While further study is needed to fully understand its behavior in both the weak and strong regimes of warm inflation, its lack of fundamental speculation makes it a promising candidate for comparison with CMB data.

5 Discussion and Conclusion

In this paper, we introduced the topic of warm inflation, a theoretical framework for inflation that differs from the standard cold inflation model. We examined the differences between the warm and cold inflation scenarios, particularly focusing on the radiation and dissipation effects in the equations of motion of the inflaton field and the differences between the primordial density fluctuations in the two scenarios. One of the most interesting outcomes of warm inflation is the impact of the dissipation term in the inflaton field equation. In warm inflation, this term introduces friction due to interactions of the inflaton field with other fields, resulting in a scenario where radiation and matter are continuously produced during inflation, eliminating the need for a separate reheating phase. This continuous production of radiation prevents the radiation energy density from being neglected, in contrast with the cold inflation scenario. Therefore, there exists a thermal bath maintained by the continuous production of radiation. This thermal bath also influences the perturbations in the early universe.

The power spectrum for the density perturbations was derived, taking into account the thermal noise present in warm inflation. This was crucial, as the literature on warm inflation argues that thermal fluctuations dominate over quantum fluctuations when the temperature is higher than the Hubble parameter, in the strong dissipative regime of warm inflation. This dominance of thermal fluctuations leads to a different contribution to the power spectrum than in the cold inflation case. The power spectrum can be calculated analytically without considering the contribution of the coupling between the inflaton field and the radiation. However, if this coupling is present, the power spectrum can only be calculated numerically. In this paper, we presented a summary of the numerical calculation of the power spectrum using the code introduced by Montefalcone in his 2024 paper [50]. To successfully derive the power spectrum accounting for thermal fluctuations and the coupling between the inflaton field and the radiation, the function G(Q) introduced by Montefalcone was presented.

The calculation of this power spectrum can also be used to determine the tensor-to-scalar ratio and the spectral tilt. In the warm inflation scenario, due to the presence of thermal perturbations, the tensor-to-scalar ratio is expected to be significantly smaller than in the cold inflation case. The spectral tilt depends on which model of warm or cold inflation is considered and serves to disregard models that do not agree with the experimental data from the CMB.

This paper confirms that the warm inflation model can address several issues present in the cold inflation paradigm. For instance, the presence of a thermal bath allows for a inflaton mass even greater than the Hubble parameter. This remedies the η-problem is cosmological theories, and swampland conditions associated with the cold inflation scenario in string theory. Additionally, the amplitude of the inflaton field remains below the Planck scale in warm inflation, reducing potential quantum gravity effects that could complicate the study of inflation.

Finally, some models for warm inflation were explored, providing an overview of the current research landscape on warm inflation. These models were evaluated according to the criteria presented by Berera in [6], which can serve as a guideline for which warm inflation models to choose in the future.

This paper aimed to demonstrate the viability of warm inflation as an alternative to the standard cold inflation models and to provide an introduction to the subject. Future research should focus on developing models that accurately represent dissipative processes during inflation, both in thermal and non-thermal equilibrium, and rigorously compare these models with experimental data. That is why identifying data capable of falsifying specific warm inflation models should be a priority. Additionally, it is crucial to investigate the potential effects of the coupling of the inflaton field with other fields, as this could provide significant insights into particle production in the early universe. An extensive examination of inflaton decay channels and interactions with other fields is essential. Furthermore, exploring the impact of thermal perturbations on other cosmological parameters and the implications of warm inflation for various areas of cosmological research is crucial. Warm inflation has the potential to offer solutions to several cosmological puzzles, including dark energy, baryogenesis, and black hole formation.

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Appendix

A Friedmann-Robertson-Walker Metric and important parameters

In this section an introduction to Friedman-Robertson-Walker (FRW) cosmology is presented, along with some relevant parameters.

FRW Cosmology

In this paper we are working with a homogeneous and isotropic universe described by the Friedmann-Robertson-Walker metric. This is given by:

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\right] \tag{38}$$

Where ds^2 represents the spacetime interval, c is the speed of light in a vacuum, $a(t)$ is the scale factor, which describes how the size of the universe changes as a function of time t, r is the comoving radial coordinate in this equation, k is the curvature parameter. The angles are given by performing the calculations in spherical coordinates and are also comoving with the expansion of the universe.

Friedmann equations

The expansion rate of the universe is determined by its content, which is characterized by the energy density ρ and pressure p. By substituting the Friedmann-Robertson-Walker (FRW) metric into Einstein's field equations, and neglecting the cosmological constant, we can derive the Friedmann equations for a perfect fluid, which describe the dynamics of the universe's expansion. In this section we follow derivations from [5].

The first Friedmann equation is expressed as:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}, \tag{39}$$

where $H = \frac{\dot{a}}{a}$ denotes the Hubble parameter, reflecting the rate of expansion. Here, G is the gravitational constant, k is the curvature parameter, and c is the speed of light. This equation connects the expansion rate with the energy density and spatial curvature of the universe.

The second equation or acceleration equation, is given by:

$$\dot{H} + H^2 = \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right). \tag{40}$$

This equation describes how the acceleration of the universe's expansion is influenced by its energy density and pressure.

For the universe to be accelerating or at least not decelerating too rapidly, the term inside the parentheses must not be too negative. If $\omega \leq -\frac{1}{3}$, the pressure term $\frac{3p}{c^2}$ becomes sufficiently negative to make the acceleration negative, leading to a decelerating universe. For a positive expansion rate, $\omega > -\frac{1}{3}$ ensures that the combined effect of density and pressure does not decelerate the universe excessively.

The constraint on w also ensures that in the early universe, matter and radiation dominated the energy content. For non-relativistic matter, $w \approx 0$ and for radiation (relativistic matter), $w = \frac{1}{3}$. Hence, both cases satisfy $w > -\frac{1}{3}$. Dark energy on the other hand, would have $w \approx -1$ [5].

From these equations, we can derive the continuity equation, which governs the time evolution of the energy density:

$$\frac{d\rho}{dt} = -3H\left(\rho + \frac{p}{c^2}\right). \tag{41}$$

Solving this yields:

$$\rho \propto a^{-3(1+\omega)}, \tag{42}$$

where $\omega = \frac{p}{\rho c^2}$ is the equation of state parameter.

Utilizing the first Friedmann equation, the comoving Hubble radius, which is explained later, is derived as:

$$\frac{1}{Ha} = \frac{1}{\dot{a}} = \left(H_0^2 a^{-(1+3\omega)} - \frac{kc^2}{a^2}\right)^{-\frac{1}{2}}, \tag{43}$$

where H_0 represents the Hubble constant. In the case of a flat universe ($k = 0$), this simplifies to:

$$\frac{1}{aH} = \frac{1}{H_0} a^{\frac{1}{2}(1+3\omega)}. \tag{44}$$

Hubble Parameter

The Hubble parameter is defined as:

$$H = \frac{\dot{a}}{a}, \tag{45}$$

where $\dot{a}$ is the time derivative of the scale factor a. It measures the rate of expansion of the universe.

B Slow roll parameters for Warm Inflation

The warm inflation slow roll parameters have to also account for the presence of the dissipation term Υ. According to Hall, Moss and Berera in their 2003 paper [54], the slow roll parameters for warm inflation are given by:

$$\varepsilon = \frac{1}{16\pi G}\left(\frac{V_\phi}{V}\right)^2 \tag{46}$$

$$\eta = \frac{1}{8\pi G}\left(\frac{V_{\phi\phi}}{V}\right) \tag{47}$$

$$\beta = \frac{1}{8\pi G}\left(\frac{\Upsilon_\phi V_\phi}{\Upsilon V}\right) \tag{48}$$

The slow roll approximation is valid when all of the slow-roll parameters are smaller than $1 + Q$, Q denoted the strength of the dissipative parameter Υ and is given in equation 17. In cold inflation, they have to be smaller than 1.