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Social Balance: The dynamics of weak and strong links on social network balance

Bachelor's Project Thesis

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Abstract: This project investigates the dynamics and influence of weak and strong ties in social networks according to social balance theory. Social balance theory is a field of study in which interpersonal relationships are quantified as positive or negative, such that statements can be made about the degree of internal strife in a social network. By abstracting real life social networks into a graph based model, the dynamics of changing weak or strong social connections and their influence on the balance of the entire network are studied. Analysis of the data shows that the influence that changing either weak or strong social links has on the network balance is dependent on the size of the network and the initial balance that the network has, but that strong links have a much more significant influence.

1 Introduction

Social balance theory or structural balance theory is a field concerned with the dynamics of relationships in social networks. The theory of balance in social networks has many uses in differing fields of science such as sociology, computer science, and psychology, and therefore has been studied quite extensively since its inception by Heider (1946). In this paper, he hypothesized that in a social setting, the relations between the actors are either balanced (for example each actor has a positive opinion towards all other actors) or there exists tension in the interaction which, if possible, will lead the actors to change their opinion to achieve a balanced state. Later, Cartwright and Harary (1956) extended the idea and made it into a more formal definition by stating it in the mathematical theory of linear graphs, abstracting the concept slightly but allowing for more rigorous methods of research by now being able to apply graph theory to the field of social sciences. Since nearly all we do in life is in the context of some form of a social network, be it online or in the real world, the dynamics of how these networks and the actors in them adapt their behaviours towards each other is of great interest. For studying the dynamics of social networks, it is impractical to use human test subjects and interview them about their feelings towards each other, not in the least because relationships between people are complex and multifaceted, and also because getting an adequate sample size to produce scientifically sound data is extremely difficult when you have to collect data by asking people to fill in surveys. Once the internet and concurrently internet forums appeared, however, research into the field became more straightforward since it was now possible to perform experiments with scientifically appropriate sample sizes. After all, there now existed a mountain of data on human-to-human interactions, with recorded interactions from which positive or negative sentiments can be analysed. From these data-sets, ideas that had been merely theoretical in the field of structural balance could be tested on real-world data. Belaza et al. (2019) found when analysing data on alliances in an online game as well as data from the Cold War political landscape, that not only did the established theories hold up, but they could even be furthered by, for example, realizing the importance of non-active connections in social networks.

Since the mass adoption of the internet is a recent phenomenon along with its possibilities of collecting big sentiment analysed data-sets, most of the fundamental research in the field has been and is done by representing social networks as an un-directed graph of nodes and edges (Wasserman et al. (1994); Watts and Strogatz (1998); Albert and Barabási (2002); Newman (2003); Fowler and Christakis (2010); McPherson et al. (1992); Borgatti et al. (2009); Hanaki et al. (2007); Lazer et al. (2009); Carrington et al. (2005). The edges in these networks are then often denoted by + or - to indicate positive or negative relations between the nodes((Wasserman et al., 1994); (Easley and Kleinberg, 2010); (Katai and Iwai, 1978)). The concept of balance within these networks can be described by the age-old adage: "The friend of my friend is my friend, the friend of my enemy is my enemy and the enemy of my enemy is my friend". In more practical terms, this means that when looking at such networks, one can divide them up into triads of nodes. In these triads, there is balance when the multiplication of the signs of the edges is positive. So +++ and -+- are balanced but - - - and -++ are not. Under generalized balance, which is more lenient in its balance definition, a - - - triad is also considered balanced (Van de Rijt, 2014) and other

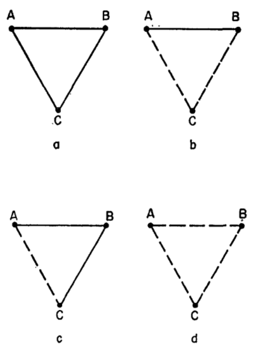

Figure 1.1: Example of triads, where a solid line represents a positive connection and a dotted line a negative connection. ***a*** **and** ***b*** **are balanced, but** ***c*** **and** ***d*** **are not.**

models consider a triad in which one node has no connection to the others to be "pressure-free" and such a triad can thus be counted as neutral (Hoek et al., 2022). These "pressure-free" or neutral triads are only possible in network that is not fully connected. This idea of these balance dynamics finds its origin in the real world in the fact that people like to be in social situations that are free of conflict. Since it is, in general, not pleasant to be in a situation where two of your friends do not like each other, thereby causing conflict.

In a fully connected network (in which every node knows and has a relationship with every other node) finding out whether the network as a whole is balanced becomes trivially easy. There are different definitions of the balance of a network, some research considers a network balanced only when all of the triads in the network are balanced (Easley and Kleinberg, 2010), while others have a looser definition of balance, based on the ratio of balanced triads divided by the total amount of triads in the network (Situngkir and Khanafiah, 2004). For the process of balancing an unbalanced network there are many different methods, all of which eventually lead to a balanced outcome, but differ in the specific methods by which they perform balancing. For example: A node could balance one of its triads by changing one of the signs of its edges based on a probability value, or it could change one of its edges based on knowledge it has about the other nodes in the network. The end result will be the same in both cases, but the dynamics of the procedure will differ. Wang and Thorngate (2003) found that when given an arbitrary fully connected network, two options exist pertaining to the possible balance of such a network. The balancing rules they used worked as follows: The function will select one of the edges of a triad at random and change its sign to balance the triad. If the triad had a neutral connection, it would either change it to a positive or negative relation to balance the triad. If the triad was balanced or had two or three neutral connections, it would leave the triad untouched. When applying these balancing rules on such a network, all nodes in the network either become friends with each other or can be divided into two distinct groups of nodes. These groups of nodes are all friends of each other and enemies of all the nodes in the other group. They also proved that in a fully connected network no more than two such subgroups can exist. If such a network contains no unsigned edges between nodes (where there is an edge between two nodes, but it is neither positive nor negative), it is predetermined to which subgroup each of the nodes will eventually belong after the balancing procedure. If there are unsigned edges between the nodes, however, the balancing rules with which the network is balanced becomes a factor in determining to which subgroup the nodes will belong. Although a signed graph is naturally not a good representation of the grey-valued relationships that can be had in the real world, it forms a good basis for more research on more complex types of networks that better represent the real world. It also seems interesting that this property of balanced graphs overlaps with the tendency of people in groups to either all like each other or divide themselves into two vehemently opposed sides who stand directly against each other.

In incomplete graphs, the story changes a little. In such graphs, not all nodes are aware of each other and consequently do not have an opinion on every other node in the network. When trying to figure out if such a network is balanced, more options become available than all nodes being friends or being divided into two camps. When looking at incomplete networks, it becomes much more interesting to make claims about the *amount* of balance in a network and what this entails. Davis (1967) found that when a network contains no cycle with exactly one negative relation, the graph becomes clusterable. In his definition of a cluster, a node has only positive connections within the cluster, and all the negative connections are considered to be outside the cluster. This means that in such a graph there are two, or often more clusters of agents where each node lies at the center of a dense network of positive relations, but the graph as a whole might be very disconnected, containing a multitude of node pairs that have neither direct nor indirect connections to each other. In clusterable graphs, the balance of an entire network, therefore, could be significantly different when viewed as a whole as opposed to subdividing the network into smaller balanced clusters. A better measure for balance, in this case, might be localized balance (Situngkir and Khanafiah, 2004), where a balance value is calculated for each pair of nodes by taking the number of balanced triads it is a part of and dividing by the total number of triads it participates in.

In the process of balancing a network, by shifting the balance of one triad another triad may become imbalanced, this means when all the nodes are allowed to reassess their relationships a snowball effect can occur in the entire network by creating a single local change in a triad. In a paper on the dynamics of balancing social networks by Antal et al. (2006), two different methods are discussed concerning such a balance shift. *Local triad dynamics* are when a random unbalanced triad is made balanced, without considering the effect of this balancing act on the rest of the network. The creation of balance in this triad might result in the unbalancing of multiple other triads. A single random opinion change can thus have a big impact on the balance of the network as a whole. With these dynamics, if the probability of changing a negative edge in an unbalanced triad with only one negative edge to a positive edge is lower than 50 percent (which one could consider an "asocial" society), they show that the system will not reach a balanced state. Increase this probability above 50 percent, however, and eventually, balance will be reached. When looking at *constrained triad dynamics*, a random unbalanced triad is also made balanced, but now only if the total amount of balanced triads increases (a more "social" approach). If the amount of balanced triads stays equal, a random link in the triad is adjusted based on a 50 percent probability. Since the number of imbalanced triads can never increase, these dynamics create a fully balanced network.

In real-world social networks, there is a difference between weak and strong ties. Dunbar and Spoors (1995) found for example that amongst the British population on average a person had 10-15 people they would contact frequently (multiple times per month, in said paper named a "sympathy group" who all know one another well), while also having a group of around 100 people who they would ask for a favour, but rarely contact (a few times per year, considered "acquaintances"). Based on this research in this paper strong and weak ties are defined as follows: Strong ties are relationships such as exist in close-knit friend groups, where many of the people you know also know each other and weak ties on the other hand are acquaintances that are known to you, but not or barely known by the rest of your friend group. Sandstrom and Dunn (2014) found in a study on Cambridge University students that people on average have more interactions with weak ties than strong ties with respectively 11.40 to 6.70 interactions per day. Through these interactions information is dispersed that can allow one to reevaluate opinions and relationships to others. Gravenotter (1983) discusses that weak ties between individuals are like bridges between close knit groups of friends. Here he also discusses the theory that weak ties allow for more cognitive flexibility by manner of being subjected to information that might not have reached a person via their strong ties and close knit friend groups. Zhao et al. (2010) found in their research on the importance of weak ties in online information diffusion that weak ties are essential for the wider spread of information in a network, but are significantly less used than strong ties for exchanging information.

When looking at the previous research done in the field of social balance theory, there seems to be a hole in the literature concerning the influence of weak and strong ties on social balance theory. What is the importance of weak and strong ties respectively on the balance of the entire network? And if, as the literature suggests, strong ties are more influential than weak ties, is there a certain number of weak ties that have the same influence as one strong tie? If this is the case, why and what might be the specific properties of the network that cause this? Based on the above mentioned research this paper tries to investigate the role that weak ties play in the balance properties of a network. When a strong tie is changed in a close knit friend group, many of the interconnected relationships of that specific tie are also impacted, and go through a process of re-evaluation. To investigate the impact this change has on the rest of the network, a balancing process of the whole network is started from this initial seed of change. In this paper a weak tie is constituted as an edge that is not a part of many triads. By changing such an edge the assumption is made that the nodes it is connected to re-evaluate one of their own triads, based on the hypothetical new information that the change in relation gave them. From there the balancing process of the whole network is started just as with the changing of a strong tie.
The main origin of this question is a combination of reading the literature on the subject and combining that with observations of real-world social events. In the real world, a shift in relationship in a close group has more influence than changes in relationship with acquaintances, based on my own observations. Therefore this paper investigates which, if any, properties of a network have influence of the importance of strong and weak ties. My hypothesis, therefore is as follows: The changing of a strong tie in a signed social network has significantly more influence on the balance of the network than changing multiple weak ties. To study this question, I have built a model in Python, which can create random signed social networks and perform tests on the dynamics of these networks to test my hypothesis.

2 Methods

To test my hypothesis, I have built a model in Python, making use of the network analysis package NetworkX. Instead of a fully connected graph, I have chosen to run the simulations on a small-world network, since research (Barrat and Weigt (2000), Crossley (2008)) indicates this is a fairly accurate (although slightly abstracted) representation of real-world social networks, since the properties of these graphs show similarities to the organization of real-world social groups. To generate the graphs the Watts-Strogatz method is used, which generates random graphs with small-world properties. The built-in function in Networkx generates these graphs with a specified amount of nodes and degree of connectivity. The degree of connectivity specifies to how many other nodes a node is connected to. The graphs generated by this method are inherently not fully connected based on the degree of connectivity, which fits the type of networks I want to research, namely representations of real world social networks.
After the graph is generated the edges will be randomly assigned to be either positive or negative, based on a percentage value that governs the amount of positive edges in relation to negative edges. For simplicity's sake there are no edges in the graph with a neutral value, since these are considered as no connection under most interpretations of strict as well as generalized balance. An example of such a graph can be seen in figure 2.1.

The following properties of the generated network are then recorded: Total balance of the graph with a strict balance criterium, the average degree of connectivity and percentage of positive edges in the graph. The balance of the network is calculated by dividing the number of balanced triads over the total number of triads. A triad is considered balanced by the criteria earlier mentioned, namely having 1 or 3 positive connections in a triad. Seeing if a triad is balanced can be easily calculated in the

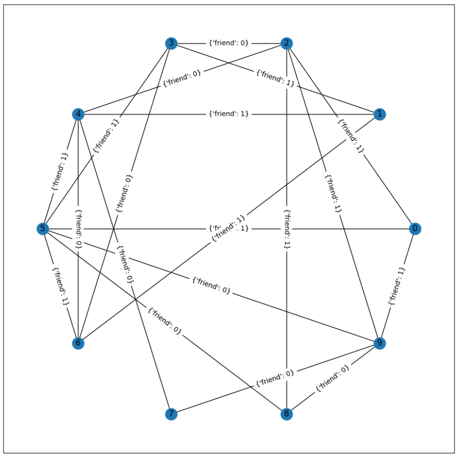

Figure 2.1: Example of a network. An edge labeled friend:0 means a negative relation and an edge labeled friend:1 denotes a positive relation

program by checking the added integer values of all the edges in the triads, and counting the results 1 and 3 as balanced while disregarding the rest.

Ordered lists are then created of the most important nodes based on the following methods:

- Node degree, the amount of edges a node has
- Node clustering coefficient, the fraction of possible triads a node can have that exist
- Node eigenvector centrality, a measure of influence that a node has in the network

The node degree is a representation of the number of people someone knows, the clustering coefficient represents a person with a dense social network, and the eigenvector centrality represents someone who knows a lot of people that in turn also know a lot of people. In lay-mans terms if a node has many connections to other well connected nodes, the eigenvector centrality of this node will be high. It is calculated by the following formula:

$$x_v = \frac{1}{\lambda} \sum_{t \in M(v)} x_t = \frac{1}{\lambda} \sum_{t \in V} a_{v,t} x_t \tag{2.1}$$

where x_v is the eigenvector centrality of node v, λ is the eigenvalue of the adjacency matrix of the graph, $M(v)$ is the set of neighbouring nodes of v, x_t the eigenvector centrality of node t, V the set of all nodes and $a_{v,t}$ the adjacency matrix of the graph. The clustering coefficient of node u is calculated as follows:

$$c_u = \frac{2T(u)}{deg(u)(deg(u)-1)} \tag{2.2}$$

where *T(u)* is the number of triads through node u and *deg(u)* is the degree of u.
To record the balance of the network the following formula is used:

$$b(G) = \frac{t_+(G)}{t(G)} \tag{2.3}$$

where *b(G)* is the balance value of the network, $t_+(G)$ is the number of balanced triads in the network and *t(G)* is the total number of triads in the network.

After creating these lists, the node with the greatest value from each method is selected and used as the starting point for the balancing procedure. Only the best node from each lists is selected, since exploratory testing indicated that starting the balancing procedure with lesser nodes showed similar trends in the data but with overall lower values and less pronounced effects. On each of the selected nodes, the following procedures are performed:

The strongest tie of that node will be selected, which in this simulation is the edge of the node that is part of the largest number of triads. In the case where there are multiple edges in the model with the same number of triads connected to them, one of these is selected at random. This edge will have its value flipped from positive to negative or vice versa. Flipping this edge represents a change in opinion of the strongest tie in the network and is used as a starting place for the balancing procedure of the network that follows. In this way, the first change to the network differs from the rest of the balancing procedure, in that it does not actively try to improve the balance in the network but merely changes its opinion. After this flip the change in balance will be recorded as well as the number of triads impacted by this change. The absolute impact on the balance of the network is calculated (since the change can positively or negatively impact balance) by taking the absolute difference in balance and dividing it by the mean of the initial and new balance values. This is done to later calculate the number of weak ties that have to be changed to have the same impact on the balance of the network. After this initial step, all of the nodes impacted by the change through their triads are added to a list. The nodes in this list are then allowed to improve one of their triads, by changing one of their edges from negative to positive. The other node connected to this changed edge is then allowed to do the same. This changed edge is then added to a list to prevent it from being changed again and creating an unending loop. In this manner, a node is allowed to be selected multiple times if the procedure circles back to it through other edges and can improve another one of its triads. The process functions in a breadth-first search manner, spreading through the network in parallel and adjusting edges that do not necessarily have a direct connection to each other, instead of following one trail of nodes until it can reach no more This procedure continues until no more nodes are reachable via this method, or if all reachable nodes have improved all of their possible triads.

The final balance of the network will be recorded, as well as the number of edges changed and the number of triads impacted

When the balancing procedure has been completed for the strongest tie, another balancing procedure will take place on an exact copy of the original graph for the weak ties. A list is made of the weakest ties of the node, in this simulation the edges that are part of the smallest number of triads. Then the weakest of these edges is selected and will have its value flipped. Thereafter the absolute impact this flip has on the balance of the network is calculated and stored. From there the next weakest edge is flipped and its absolute impact is added to the impact of the previous edge. This continues until the total impact of changing multiple weak edges is the same or greater as the impact of changing the strong edge in the same network. During each step the change in balance and the number of impacted triads is recorded. A list is also made of all the nodes impacted by the change of the edge, and from the list the same balancing procedure is performed as with the changing of a strong edge. Once the balancing procedure is completed, the final balance, number of edges changed and number of triads impacted is recorded.

These procedures are repeated 10 times on graphs generated with the same values for node count, initial positive edge percentage and connectivity degree. The experiment will be repeated for all node counts between 20 and 90 in steps of 10 to simulate different network sizes and for each node count with initial positive edge percentages between 30 and 80 in steps of 10 to simulate different initial amounts of friendliness in the network, which is associated with balance through the fact that having a greater number of positive edges in the network increases the chance of a balanced triad being generated at random, and a connectivity degree of between $n/2$ and $n-5$ in steps of 5, where n is the number of nodes. The connectivity degree starts at half of the node count because exploratory testing showed that a connectivity degree lower than that caused there to be too few triads to perform balancing in a meaningful way.

When all the data is collected the average for all the values recorded in the 10 runs with the same starting conditions is calculated. These values are than normalized on the node count of the network. The difference in balancing between weak and strong ties is analysed. The averages are then compared with the values of the other starting conditions to figure out if the initial positive edge percentage and the connectivity degree have a significant effect on the balancing of the network.

3 Results

Calculating the average number of weak edges that have to be changed to equal the same impact on balance as one strong edge gives a result of 2.73 weak edges per strong edge with a standard deviation of 5.08. This number is similar over all different initial settings changed.

When looking at the final balance value for all node selection types in Figure 3.1, we can see that the final balance value for the strong tie method is significantly higher than the value for the weak tie. This would indicate that changing a single strong tie has a much greater effect on the final balance value than changing multiple weak ties. When digging deeper in the data, however, we find that to reach these final balance values, the method for the strong tie changes on average 60.5% more edges. In order to determine the effect that each of these changes has on the final balance value, we need to control for this difference in the number of edge changes made. Since the effect that each edge has on the balance of the network is in part determined by both the initial positive edge percentage and the connectivity degree, we plot the effect that a single changed edge has (on average, after the balancing procedure has completed) against these "Friend value" (initial positive edge percentage) and "K-value" (connectivity degree), respectively, as can be seen in Figures 3.2 and 3.3.

From these figures, we can see that when controlling for the difference in the number of edge changes made, the strong tie still has a greater effect on the balance of the network than the weak tie. Of note is that the difference between the strong and weak ties is relatively even when comparing to the friend value, but diminishes when comparing against the connectivity degree. Since a lower connectivity degree causes the network to have fewer triads (by virtue of having fewer connections between the nodes), the effect of changing a strong tie becomes more pronounced in comparison to changing a weak tie in a sparser network.

Performing a linear regression test on these findings to see if they are significant produced the following results: Testing to see if the initial positive edge percentage significantly predicted a change in balance for the change in a strong and weak tie showed that it significantly predicted change in balance with values of strong and weak ties respectively of ($\beta = 0.003, 0.002$; p <.000) and ($R^2 = 0.08, 0.15$; t = 7.87, 10.35; p <.000), where β represents the correlation between the change in balance and the initial positive edge percentage and R^2 is the coefficient of determination, measuring the strength of the linear relationship between the two variables. A p-value of <.005 indicates that the result is statistically significant.
For predicting the change in balance value for strong and weak ties the connectivity degree also showed a significant, albeit smaller result with value for strong and weak ties respectively of ($\beta = -7.583\text{e-}5, -4.420\text{e-}5$; p <.000) and ($R^2 = 0.437, 0.359$; t = -22.06, -18.75; p <.000)

Looking at the results by the different methods to determine the importance of a node, we can investigate if there is a difference in the results based on the measure of importance. Since the data for all the different importance methods follows the same trends seen in Figures 3.2 and 3.3 when plotting against the friend or K value, we plot the average impact that a single edge change has on balance for all the types in Figures 3.4. In this plot, we can see there is a slight increase in impact values when selecting the initial most important node based on the node degree. Although the difference is small (about 0.0006 per edge), considering that on larger networks there are northwards of 2000 edge changes, these effects add up and might have a significant impact on the final balance value. To see if there is a significant difference in the effect a single edge change has on the final balance of the network based on the different importance criteria for the nodes used, an ANOVA analysis was done on these results. This analysis showed that the different importance methods have no significant effect on the impact that a single edge has on the final balance of the network, with a p-value of 0.363 indicating there is no significant relation between these variables.

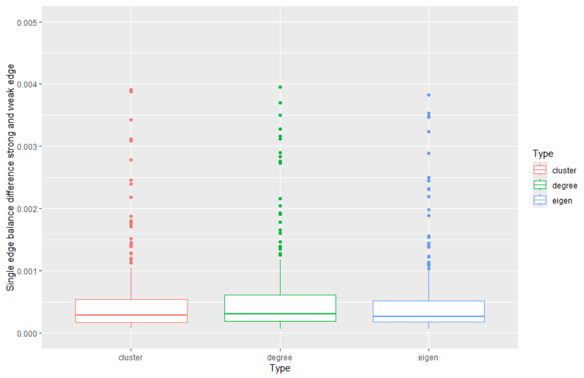

Figure 3.4: Boxplot of the average impact of a single edge change on balance per type

4 Conclusion and discussion

From these results, multiple things can be concluded. First and foremost, the data shows that the changing of strong ties has significantly more influence on the balance of a network than changing multiple weak ties. This is the case after the first

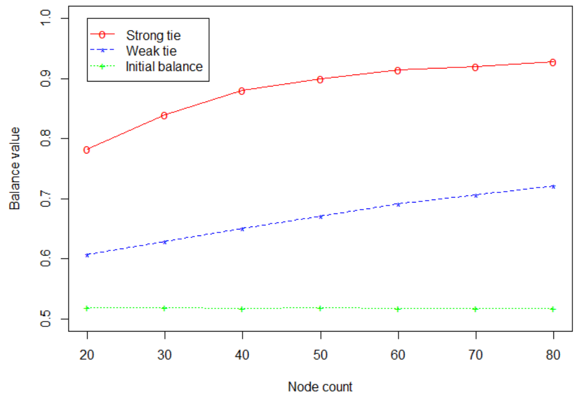

Figure 3.1: Final balance value for all node selection types

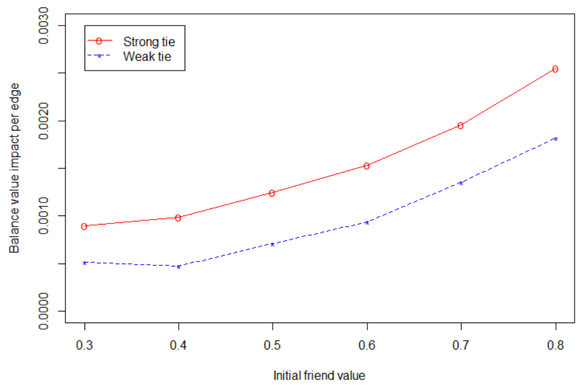

Figure 3.2: Balance effect of a single edge by friend value

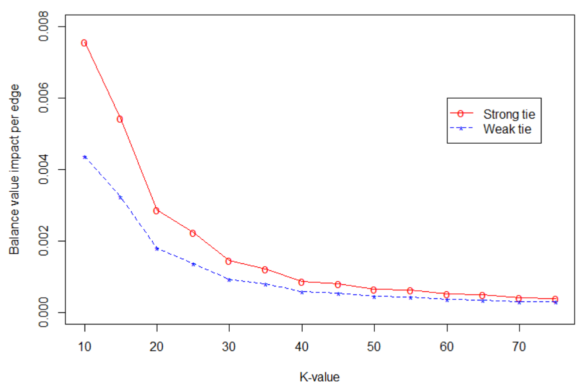

Figure 3.3: Balance effect of a single edge by K-value

change as well as when the balancing of the network is propagated for a couple of iterations. The data also shows that while there are differences in the dynamics of the change in balance based on the different types of selection criteria of what constitutes an important node, these differences are not significant. The initial percentage of positive edges and the degree of connectivity of the network do have a significant effect on the balance of the network, where a higher initial percentage of positive edges results in a higher final balance value and increases the impact each individual edge has, while a higher degree of connectivity decreases the effect that changing a single edge has on the network.

To further investigate the dynamics of strong and weak ties on the balance of a network, multiple changes can be made to this research. For example: instead of allowing the nodes only to improve their relationships also allow them to worsen the balance of the network, in order to see if this changes the dynamics between strong and weak ties when further balancing the network. Although there is no direct real-world parallel for this type of behaviour, it is theoretically interesting to see for example what amount of "bad faith" actors would be needed to prevent the balancing of a network entirely. A change to the network can be made as to where each node has a certain "belief" value (to represent for example the same favorite sports team, or similar tastes in music), which is taken into account when deciding changes in its relationship to others. Methods could also be added to allow for the forming and breaking of new relations between the nodes, which more realistically represent human interaction. The experiment could also be performed on a different type of network, instead of a small-world network, which might produce entirely different results.

Another avenue for further research could be creating an agent-based model of a social network and investigating what the impact of different types of agents is on the dynamics of a network. For example what could the effect of a single bad actor as opposed to a single good actor be on the balance of a network. What would the effect of allowing agents to make or break connections based on their internal feelings or external stimuli be? Could there be some emergent properties discovered from such a simulation, that reflect the dynamics of human-to-human interaction and group forming? Would it be possible to devise a method for quickly balancing an entire network by careful pruning of certain connections between nodes, thereby "enforcing" harmony in a group. In the same vain could a method exist, that if applied to a group would descend the entire group into disarray in the fastest time possible by systematically forming or breaking connections between in-groups to cause the most imbalance?

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