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University of Groningen
Faculty of Science and Engineering
NL-eEDM Collaboration

Bachelor's Thesis

*To fulfill the requirements for the degree of
Bachelor of Science in Physics*

Upgrading and Characterizing the Electronics of a Traveling-Wave Stark Decelerator

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May 27, 2025

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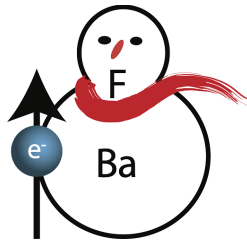
Victor Albert Bakker

Abstract

The electron electric dipole moment (eEDM) is a powerful probe for physics beyond the Standard Model, with implications for understanding the observed matter-antimatter asymmetry in the universe. The NL-eEDM collaboration aims to improve the sensitivity of the eEDM measurement by using cold beams of heavy polar molecules. A critical component of the setup is the traveling-wave Stark decelerator, which slows the molecular beam to increase interaction time and molecule count in the measurement region, which are key factors in reducing statistical uncertainty. Optimal operation of the decelerator requires the generation of (near) ideal waveforms, which is currently challenged by limitations in the supporting electronics and by parasitic effects such as capacitive coupling.

This thesis focuses on upgrading and characterizing the electronics that drive the decelerator, with the goal of enabling waveform generation that closely approximates the theoretical ideal. This involves ensuring that the electronics can support the required peak-to-peak voltage of 20 kV, and verifying that the components across different channels exhibit consistent behavior. Transformers and amplifiers were characterized individually and in coupled configurations via Bode plot analysis. It was found that, while the amplifiers behave consistently across channels, transformers exhibited two distinct behavioral groups, attributed to mechanical faults. Repairing such faults successfully restored desirable transformer characteristics. To increase the maximum voltage through the channels, the Amplifiers were reconfigured from 'stereo mode' to 'bridge mode'. In combination with a capacitor replacement, this resulted in a maximum peak-to-peak voltage of 26.5 kV, hence comfortably reaching the required value for ideal waveform generation. While capacitive coupling showed only minor influence on gain and phase in Bode plots, practical evidence suggests it significantly distorts waveforms, indicating that the current measurement approach under-represents its full effect.

Overall, the findings demonstrate that the electronic components - if properly selected, repaired, and upgraded - can support the requirements for ideal waveform generation in the NL-eEDM experiment.



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1 Introduction

1.1 The NL-eEDM experiment

The electron electric dipole moment (eEDM, d_e) is a powerful probe for physics beyond the Standard Model (SM). The SM allows for an exceedingly small eEDM in the order of $10^{-35} e \cdot \text{cm}$ [1], current experimental efforts have pushed the upper limit down to $|d_e| < 4.1 \cdot 10^{-30} e \cdot \text{cm}$ at 90% confidence level [2]. A non-zero eEDM would imply simultaneous violation of parity (P) and time-reversal (T) symmetry, and hence CP violation via the CPT theorem [3]. Such a discovery would contribute to the explanation for the observed matter-antimatter asymmetry in the universe, which the SM alone cannot account for.

The purpose of the NL-eEDM collaboration [4] is to improve the precision of the eEDM measurement using an intense beam of slow, cold molecules. The use of electrically neutral, heavy polar molecules such as BaF (barium monofluoride) and BaOH (barium monohydroxide) have been proposed due to their favorable properties such as having a large internal electric field and being amenable to laser manipulation. To ensure high precision, the molecules must spend a significant amount of time in the measurement region of the setup. This is accomplished by longitudinally decelerating the molecular beam, for which the traveling-wave Stark decelerator plays a central role.

1.2 General setup

Before discussing the details of the decelerator it is important to understand the NL-eEDM experiment as a whole to get an idea of what the role of the decelerator is within the entire project. A schematic overview of the proposed experimental setup is given in Figure 1.

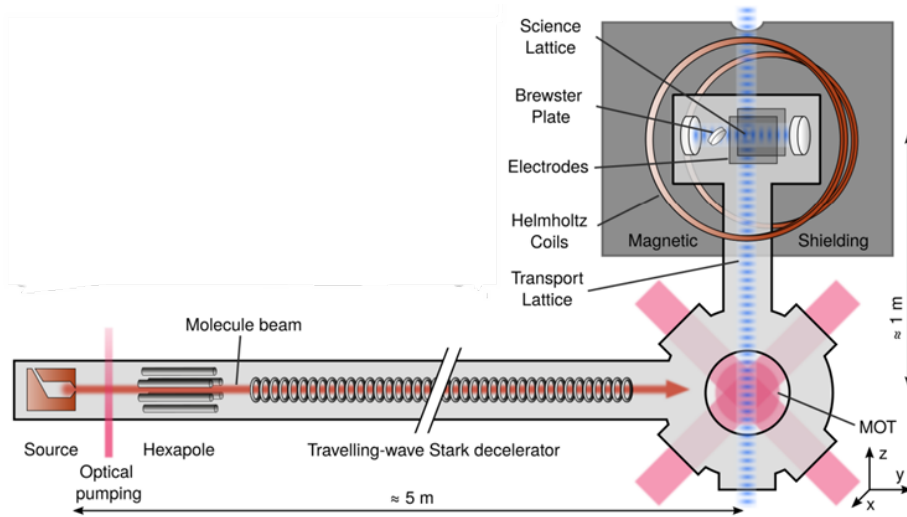


Figure 1: *Schematic overview of the proposed experimental approach for the NL-eEDM experiment.* The molecular beam is generated in a cryogenic source, after which it is focused using a hexapole lens. The beam is then decelerated with a traveling-wave Stark decelerator and subsequently captured and further cooled in a Magneto-Optical Trap (MOT). Once cooled, the molecules are transported to the science lattice (measurement region). Here, their spin precession is analyzed, which is used in the determination of the eEDM [5].

The cryogenic **source** is where the desired molecules are created. Using the buffer gas method, the molecules are cooled to cryogenic temperatures, mainly reducing the longitudinal kinetic energy, which is useful for high-precision measurements [6]. The source generates a pulsed beam, resulting in short bursts of molecules. **Optical pumping** is used to prepare the internal quantum states of the molecules. This ensures population of specific rotational and vibrational levels that are low-field seeking and can be efficiently manipulated by the Stark decelerator. The molecules leaving the source are spread out in all directions. Therefore, a guide in the form of a **hexapole** lens is introduced to focus the molecules in a narrow beam, resulting in less transverse spreading, which reduces the loss of the molecules.

The **traveling-wave Stark decelerator** slows the molecules down even more longitudinally, from 200 ms^{-1} to 30 ms^{-1} , by using high voltage to generate electric fields. This part of the setup is the main focus of this paper; more details of this will be given in Section 2.1.

The **magneto-optical trap (MOT)** is a stage where the molecules are spatially confined and further cooled using a combination of laser cooling and magnetic field gradients, converting a decelerated molecular beam into a tightly confined, ultra-cold sample suitable for the final spin-precession and measurement stages. Then the molecules are transferred to the **measurement region** using an optical transport lattice. In this highly controlled region, a homogeneous electric field and uniform magnetic field are applied, allowing for proper eEDM measurements [5].

1.3 eEDM measurement

The electron electric dipole moment d_e can currently not be measured directly. Instead, its value is constrained by the statistical uncertainty σ_d , which can be calculated using Equation 1 [7],

$$\sigma_d = \frac{\hbar}{E \cdot A \cdot P(E) \cdot \varepsilon \cdot T \cdot \sqrt{n}}. \quad (1)$$

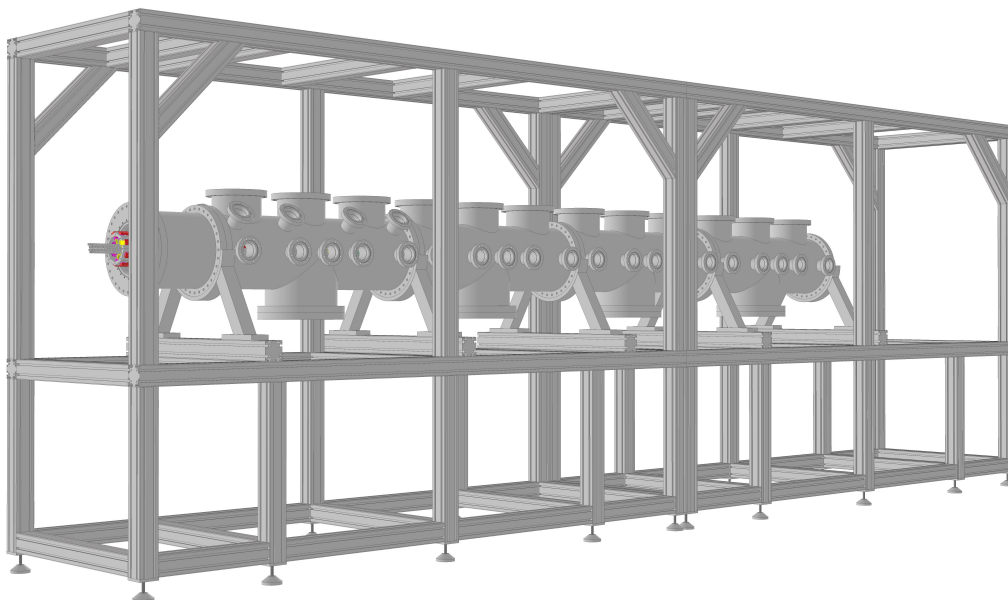
In this equation, E is the applied electric field strength in the measurement region, $\hbar$ is the reduced Planck constant. A is the molecular enhancement factor, a dimensionless constant. $P(E)$ is the polarization factor of the molecule, it determines how well the molecule is polarized by the external electric field. ε is the total detection and measurement efficiency, it determines how many of the molecules that enter the measurement region are actually detected and contribute to the signal.

T is the interaction time spent by molecules in the electric field of the measurement region during spin precession, n is the total number of molecules that enter the measurement region. These two quantities are effected greatly by the traveling-wave Stark decelerator. By decelerating the molecules to lower velocities, the time they spend in the measurement region increases, leading to a larger interaction time T . Improved trapping and guiding efficiency in the decelerator prevents molecule loss, resulting in a larger number of molecules reaching the measurement region, thereby keeping n as high as possible. As shown by Equation 1, larger values of T and n reduce the statistical uncertainty σ_d , highlighting the crucial role of the decelerator in maximizing the sensitivity of the experiment.

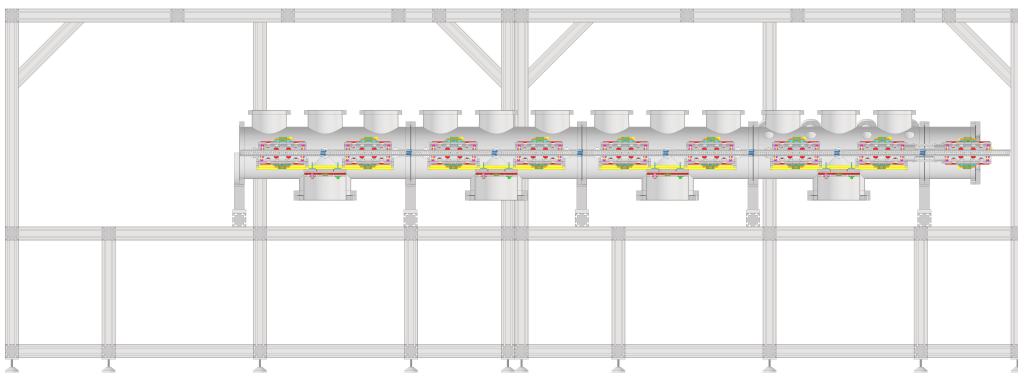
2 Deceleration of Molecules

2.1 Traveling-wave Stark decelerator

The traveling-wave Stark decelerator is a cylindrically shaped vacuum chamber with nine modules inside, where the electric field is generated. It operates under ultrahigh vacuum conditions, with pressures in the range of 10^{-8} mbar. This low pressure minimizes the probability that the molecules collide with residual gas particles while traveling through the structure, which could scatter them. Visualizations are shown in Figure 2.



(a) Isometric view of the decelerator. In this figure the left side is the back of the decelerator and the right side is the front.



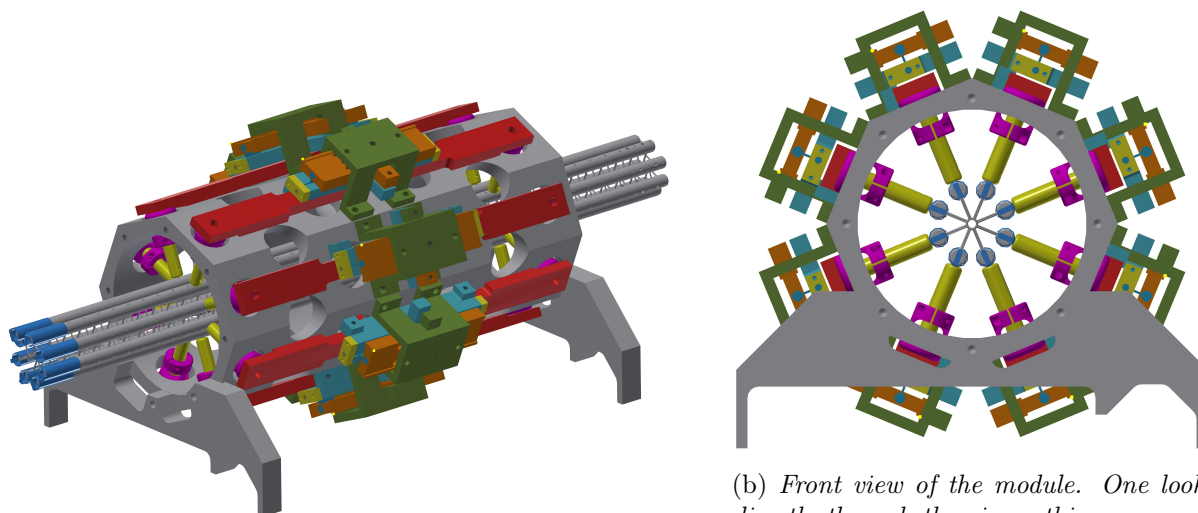
(b) Cross-sectional front view of the decelerator. This clearly visualizes the nine modules inside the vacuum chamber. In this figure the left side is the front of the decelerator and the right side is the back.

Figure 2: **Decelerator visualizations.** Two different views of the module are shown: an isometric view (a) and a cross-sectional front view (b). The length of the decelerator in this figure is about 4.5 meters. More perspectives of the decelerator can be seen in Figures A3 and A4. Figure courtesy of Leo Huisman.

Ultimately when the whole setup is assembled, the front of the decelerator will be connected to the hexapole lens, and the back will be connected to the measurement region. The present decelerator is 3 meters long instead of 4.5 meters. The reason for this is that three of the nine modules could handle lower voltage than the other six, hence they were taken out for maintenance. When repaired, these three modules could be put back into the decelerator to increase the decelerating potential. The decelerator was designed and built by the NL-eEDM Collaboration at the University of Groningen.

2.1.1 Modules

The modules contain eight stainless steel electrode rods mounted in an octagonal pattern, these rods are connected to a sequence of tantalum ring electrodes. By applying oscillating, phase-shifted voltages to the ring electrodes, the decelerator creates a sequence of traveling electric potential wells ('traveling-wave') - effectively moving traps - in which the molecules can be captured and gradually slowed down. Polar molecules experience a force due to the Stark effect - the interaction between their electric dipole moment and the inhomogeneous electric field - and become trapped in these wells. By gradually reducing the speed of the traveling waves, the trapped molecules are decelerated without leaving the minima of the potential wells, allowing for a continuous deceleration of the molecular beam. More information about the Stark effect and the electric field in the decelerator can be found in Sections 2.2 and 2.3. Each module is designed to ensure proper alignment of the rings so that the molecule beam can pass through a well-defined central channel. The rings can be precisely aligned through several adjustment screws on the modules. Figure 3 shows several visualizations of the the module.



(a) Isometric view of the module. The length of a module is 50.4 cm.

(b) Front view of the module. One looks directly through the rings; this appears as the tiny hole in the center. The diameter of the rings is 4 mm.

Figure 3: **Module visualizations.** Two different views of the module are shown: an isometric view (a) and a front view (b). The separate parts of the module have been given a variety of colors, but in real life this is not the case. A photo of the module can be seen in Figure A5, as well as a side view in Figure A6. Figure courtesy of Leo Huisman.

2.2 Stark curves

The Stark effect refers to the shifting and splitting of molecular energy levels due to the presence of an external electric field [8]. In polar molecules like BaF and BaOH - that have been proposed for the NL-eEDM experiment - this effect arises because the molecule's permanent dipole interacts with the external field. The stark curves for BaF and BaOH are shown in Figure 4.

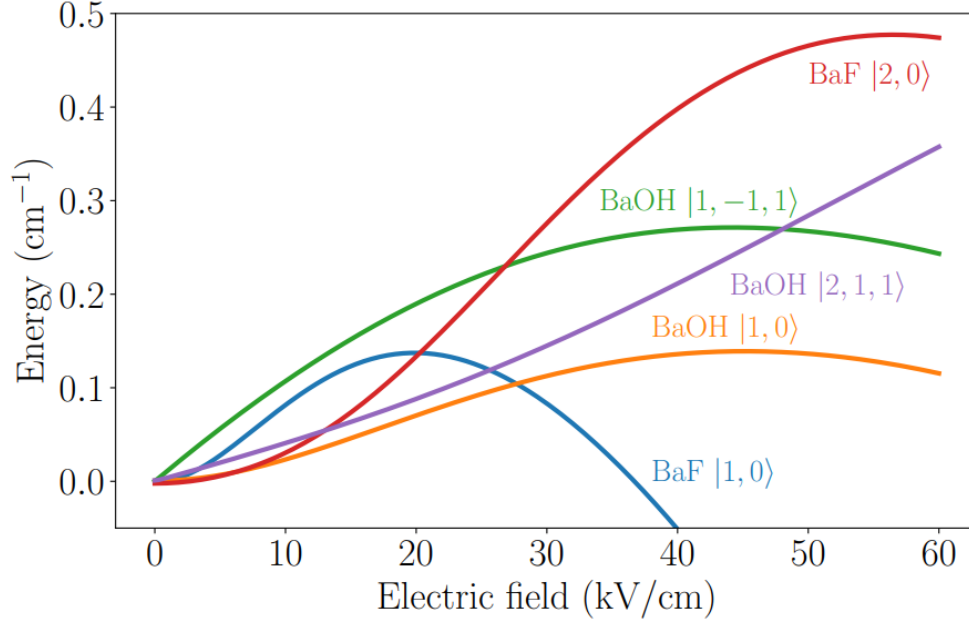


Figure 4: ***Stark shifts of BaF and BaOH for different rotational and vibrational levels.*** For BaF and BaOH, the states represent $|N, M\rangle$ and $|N, -K, M\rangle$ respectively. Here, N is the rotational angular momentum quantum number, M is the projection of N onto the space-fixed (laboratory frame) quantization axis, and K is the vibrational angular momentum quantum number from bending motion. An offset is added so that all curves pass through zero [9].

Prior to the peak of the Stark curve, the molecule resides in a low-field-seeking state, where its energy increases with electric field strength. This regime is favorable for trapping, as the molecule experiences a restoring force toward the minimum of the electric field, typically located at the center of the trap. Beyond the peak, however, the molecule transitions into a high-field-seeking state, in which it is repelled from the trap center and is therefore more likely to escape. The peak of the Stark curve thus marks a critical turning point, where the derivative of the energy with respect to the field vanishes, indicating that the molecule is in equilibrium and experiences no net force.

Although the trap depth, defined as the maximum potential energy difference confining the molecule, is greatest near this peak, effective trapping requires remaining on the low-field-seeking side. Accordingly, the applied electric field (and hence the peak-to-peak amplitude of the waveform) must be sufficiently high to provide a deep potential well, yet remain below the threshold where the molecule becomes high-field-seeking. For the $N = 2$ rotational state of BaF, this optimal operating point lies just below the peak of the Stark curve and corresponds to a peak-to-peak voltage of approximately 20 kV, which forms the basis for the chosen waveform amplitude in the experimental design.

2.3 Electric field in the decelerator

The eight waveforms create an electric field in the traveling-wave Stark decelerator that is used to decelerate and trap molecules, a plot of this is shown in Figure 5.

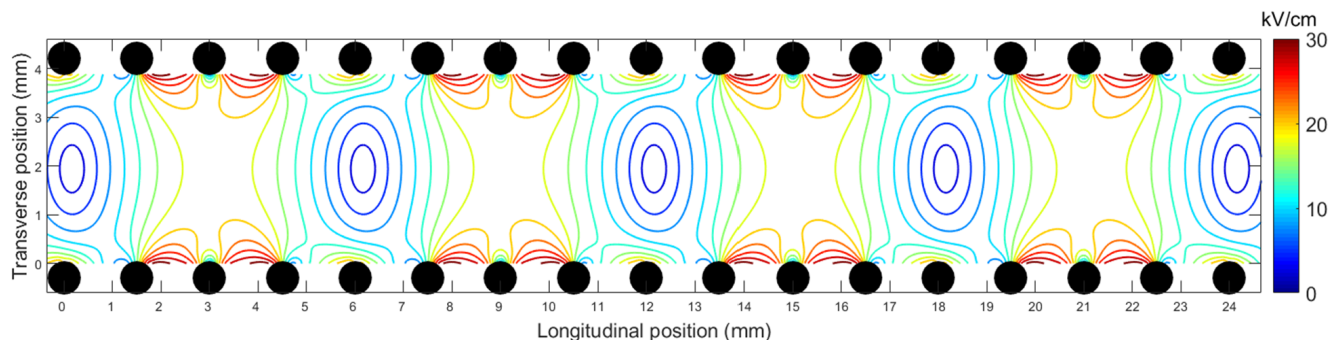


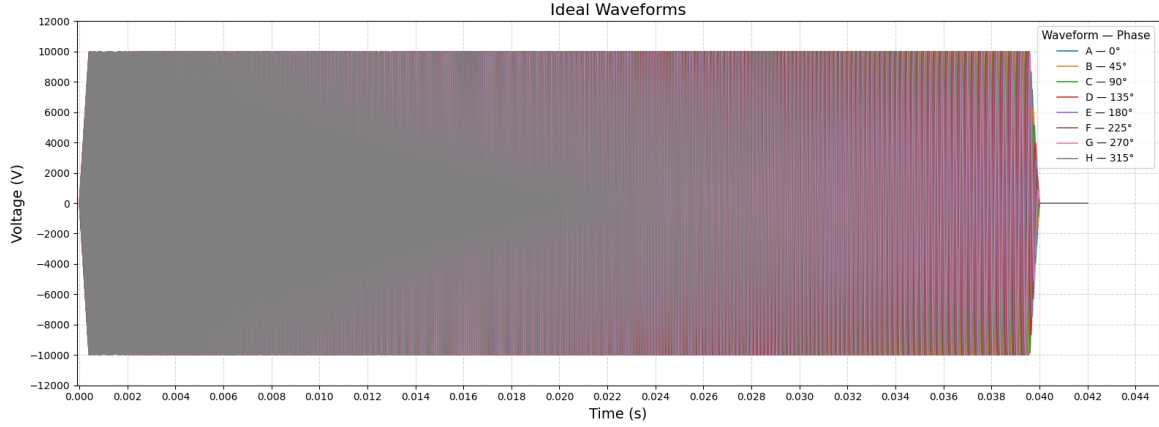
Figure 5: *Electric field within the traveling-wave Stark decelerator.* The potential wells are depicted by the blue areas, where the magnitude of the electric field is the smallest. The black dots represent the tantalum ring electrodes. Figure courtesy of Steven Hoekstra.

The colored contours depict the magnitude of the electric field in units of kV/cm . Series of potential wells are created along the longitudinal axis, providing the low-field-seeking traps necessary for molecular trapping and deceleration. The minima of the field magnitude, corresponding to the centers of these traps, showing a moving array of three-dimensional potential wells [10]. This structure allows for the controlled manipulation of molecular beams.

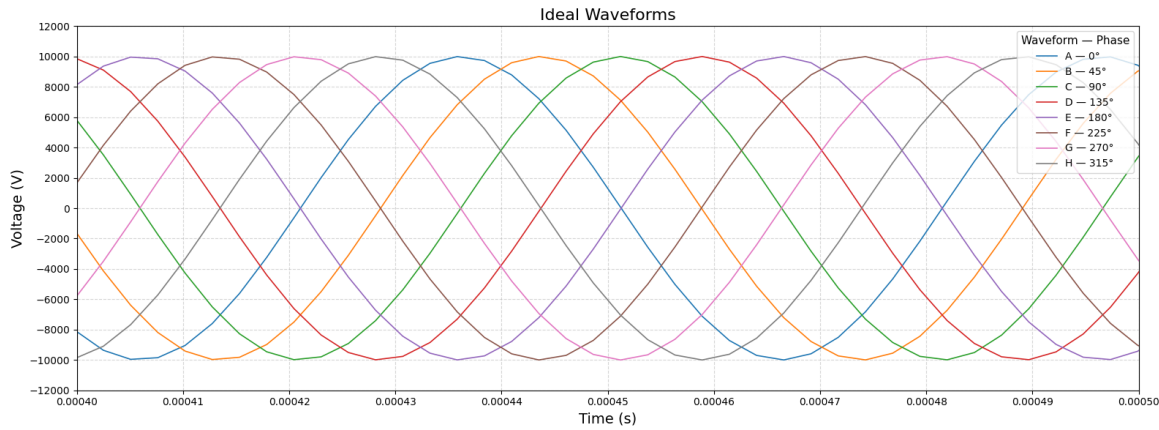
2.4 Ideal waveforms

As previously discussed in Section 2.1, the decelerator has eight stainless steel rods, each connected to a series of tantalum ring electrodes. These rods serve as eight distinct channels, with each channel receiving a dedicated waveform. The electric field within the decelerator is generated by the coordinated application of these eight waveforms. To achieve efficient molecular deceleration while minimizing losses, the waveforms must adhere to specific criteria, ensuring stable trapping and controlled deceleration of the molecular beam. The ideal waveforms have a **peak-to-peak amplitude of 20 kV** to effectively trap and decelerate the molecules. A pulse duration of 40 ms with a repetition rate of 10 Hz is used, over this duration a frequency sweep from 16.7 kHz to 2.5 kHz is applied. This corresponds to the longitudinal velocity of the molecule beam of 200 m/s when leaving the source to 30 m/s when leaving the decelerator.

A **flat envelope** throughout the frequency sweep is required, as variations in amplitude would result in instability of the trapping potential and lead to molecular loss. The envelope makes use of a ramp up and ramp down of the amplitude at the start and end of the pulse, this way the changes in amplitude magnitude are smooth. Finally, the decelerator requires a **phase difference between adjacent waveforms of 45°** . This ensures the formation of a continuous traveling wave along the decelerator, maintaining smooth and uniform movement of the electric potential wells along the decelerator. This allows the molecules to stay trapped in the same moving potential well throughout the entire deceleration process. These criteria define the *ideal waveforms*, a simulated version of which is shown in Figure 6.



(a) The ideal waveforms over the full 40 ms pulse duration. All eight phase-shifted channels are shown, resulting in a densely populated plot. For improved clarity, a version displaying only a single waveform over the full pulse is provided in figure A2.



(b) The Ideal waveforms along only a small fraction of the pulse duration to clearly show the shape of all eight waveforms and their position relative to each other.

Figure 6: **Visualization of the theoretical ideal waveforms.** Two different views of the plot are shown: Along the entire pulse length (a) and a zoomed-in version along a fraction of the time axis (b). The eight waveforms are 45° out of phase relative to their adjacent waveforms and have a peak-to-peak ‘flat’ amplitude of 20 kV. These waveforms would generate the ideal circumstances (ideal electric field) for deceleration the molecule beam in the NL-eEDM experiment.

2.5 Capacitive coupling

Getting close to the ideal waveforms in practice is more challenging than one might expect. Due to capacitive coupling between channels, the waveforms influence one another; any adjustment made to one channel typically affects the remaining seven. Prior to this project, the waveforms were manually tuned using analog filters on the amplifier outputs. While achieving a satisfactory shape for a single waveform might only take a few minutes, ensuring that all eight waveforms meet the ideal criteria simultaneously can take weeks due to the interdependence introduced by capacitive effects. When changes are made to the setup, one needs to repeat the entire tuning process. This approach is therefore unsustainable in the long term. A visualization of capacitive coupling is shown in Figure 7.

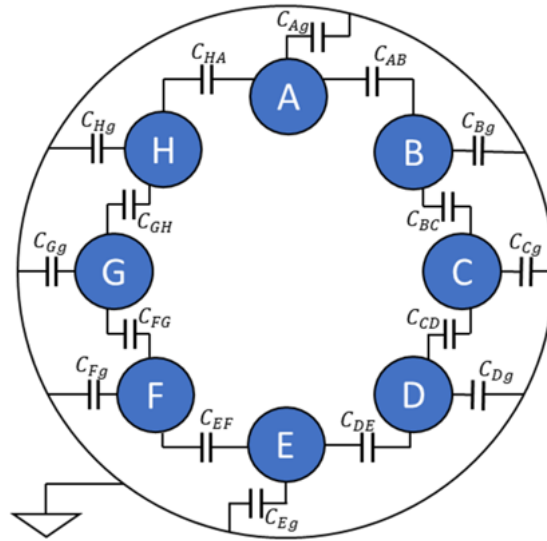


Figure 7: ***Schematic representation of capacitive coupling between the eight waveform channels.*** Capacitive coupling refers to the unintended interaction between adjacent electrical circuits due to parasitic capacitance. In this setup, each of the eight waveform channels (labeled A-H) is connected to its neighbors via coupling capacitors $C_{XY} \approx 160 \text{ pF}$, and to ground via stray capacitance $C_{Xg} = 57 \text{ pF}$ [11]. These capacitive links cause voltage changes in one channel to induce cross-talk in the others, making it challenging to generate and maintain eight ideal waveforms simultaneously. Figure courtesy of Lucas van Sloten.

The desired solution for this is implementing a feedback system in the control software that continuously monitors the waveform shapes and updates them automatically when significant deviations from the ideal waveforms are detected. More about this feedback system can be read in Section 3.2.

3 Decelerator Setup

3.1 Upgraded decelerator electronics

The decelerator system is driven by a chain of electronic components in which a PC-controlled waveform generator produces low-power signals, which are subsequently amplified and transformed into high-voltage waveforms using amplifiers and transformers. These high-voltage signals are then used to generate the electric fields inside the Stark decelerator. During this project, the setup was upgraded to achieve higher output voltages by configuring the amplifiers in bridge mode instead of stereo mode. In bridge mode, the inputs and outputs of both internal amplifier channels are combined to drive a single waveform with increased voltage. Since each amplifier can now support only one waveform, the number of amplifiers had to be doubled from four to eight to accommodate all eight channels. The upgraded setup is shown in Figure 8. The previous setup, right before these upgrades, is shown in Figure A1.

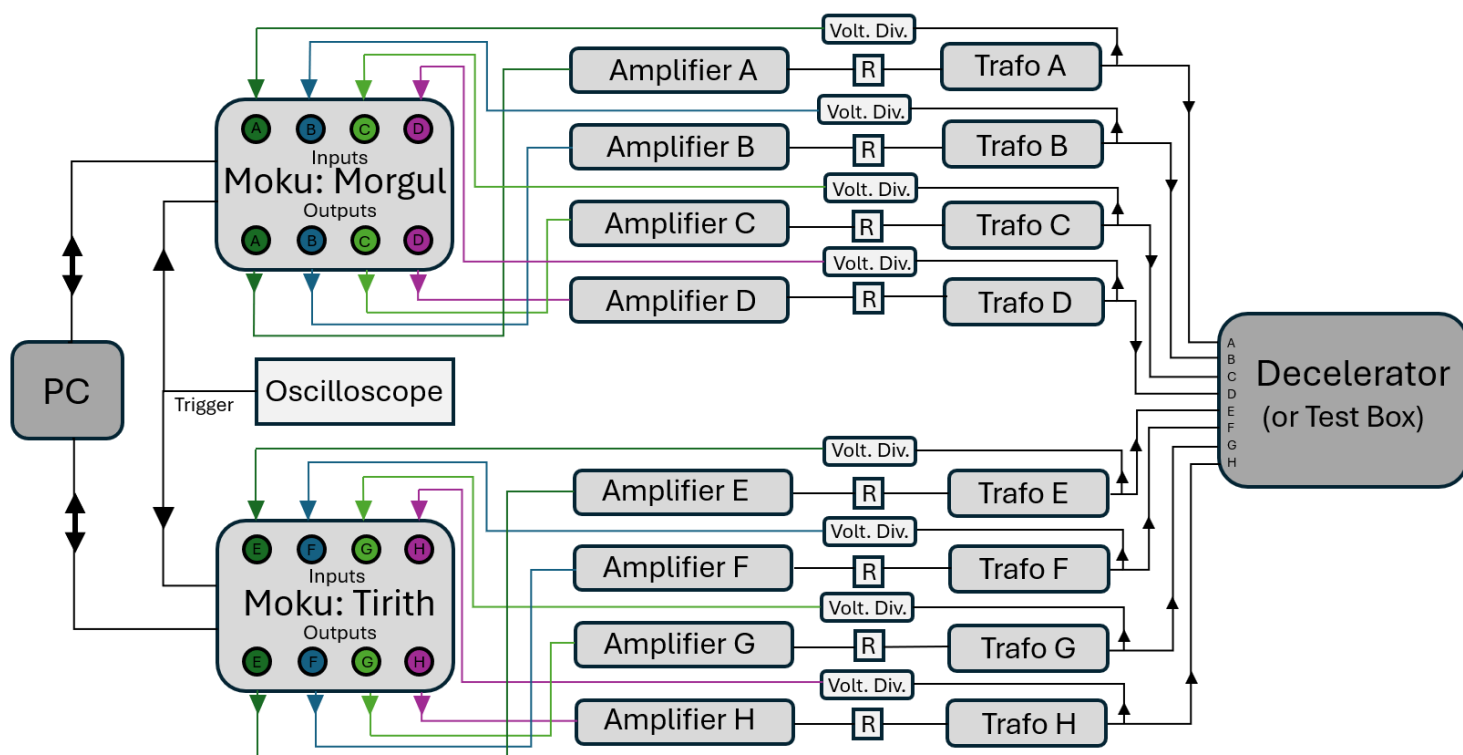


Figure 8: *Schematic overview of the upgraded decelerator setup with eight amplifiers in bridge mode.* The PC communicates with both Moku:Pro devices, named Morgul and Tirith, via a python script. Eight waveforms are generated by the Moku's, which are then amplified and transformed by its corresponding amplifier and transformer. The waveforms are then sent into the decelerator, generating a carefully controlled electric field that can be used to slow down molecules. The resistors, R , have a resistance of 2.2Ω . The voltage dividers (volt. div.) reduce the voltage by a factor of about 1224 in order for the signal to be safe for the Moku's to receive back. The oscilloscope is used to trigger both Moku's in order to synchronize them. The test box is used instead of the decelerator during test measurements, mimicking its capacitive behavior.

3.1.1 Specifics of decelerator setup components

The components shown Figure 8 will now be explained in more detail. A **Moku:Pro**, created by Liquid Instruments [12], combines multiple laboratory instruments into a single reconfigurable device. In this experiment, it is used as an arbitrary waveform generator and oscilloscope. Two of these devices can generate eight waveforms in total. To trigger and consequently synchronize both Moku's, another external **oscilloscope** is used. Specifically in this experiment, the KEYSIGHT InfiniiVision DSO-X 3024T is used [13]. It can also be used to plot the waveforms in order to verify what is observed in the plots generated on the PC, as well as saving datasets during measurements.

The **amplifier** is an electronic device that increases the amplitude of a signal without significantly altering its shape. In this experiment, they are mainly used to amplify the voltage and current ($\sim 10^0\text{V}$ and $\sim 10^{-3}\text{A}$) of the output waveform signal generated by the Moku:Pro. The signal can then be amplified to about $\sim 10^1\text{V}$ and $\sim 10^1\text{A}$. Especially a significant increase in current can be noted, about four orders of magnitude more. The specific amplifier used in this experiment is the Behringer NX3000D [14]. This is a class D amplifier, of which more explanation is given in Section 4.4. It has filter knobs - adjusting the frequency, gain, and quality - that can be used selectively correct and flatten the waveforms. The frequency sets the center point along the spectrum where the adjustment is applied. The gain modifies the amplitude at that specific frequency. The quality factor (Q) controls the width of the adjustment: a high Q results in a narrow, precise correction, while a low Q produces a broader adjustment over a wider frequency range.

The main function of the **transformer** is to transform the received signal from the amplifiers from high current to high voltage, to about $\sim 10^4\text{V}$. The working principle and equivalent circuit are explained in Section 4.3. An equivalent circuit of the transformer is given in Figure 12 and a visualization can be seen in Figure 10. To safely monitor the waveform, a **voltage divider** is used. It reduces the output voltage of the transformers by a factor of about 1224 before it is received by the Moku:pro input. An equivalent circuit of the voltage divider is shown in Figure 9 and a visualization can be seen in Figure 10.

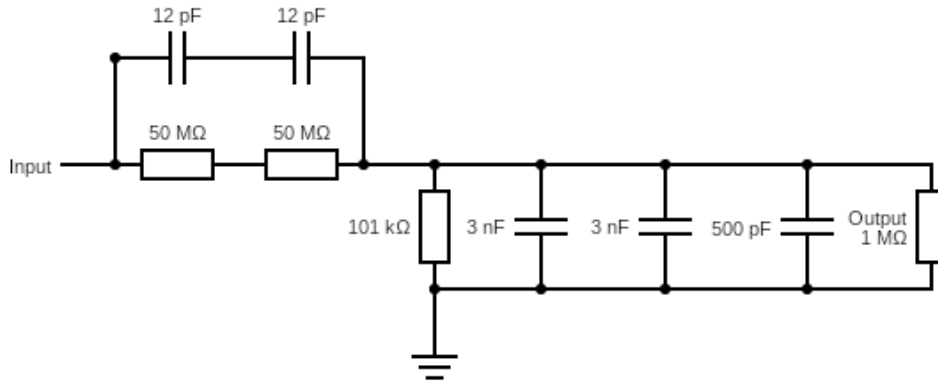


Figure 9: **Equivalent circuit of the voltage divider.** The voltage divider is placed in between the output of the transformer and the input of the Moku:Pro, as can be seen in Figure 8. The peak-to-peak voltage limit of the voltage dividers is about 8.4 kV, which makes them unsuitable for the ideal peak-to-peak voltage of 20 kV. Therefore a new voltage divider design has been created, this is visualized in Figure A9.

The **resistors** - R - used in the decelerator setup shown in Figure A1 are required at the output of the amplifiers to maintain a stable signal, and have a resistance of $2.2\ \Omega$. Its power limit is around $1500\ W$. A visualization of the resistors can be seen in Figure 10.

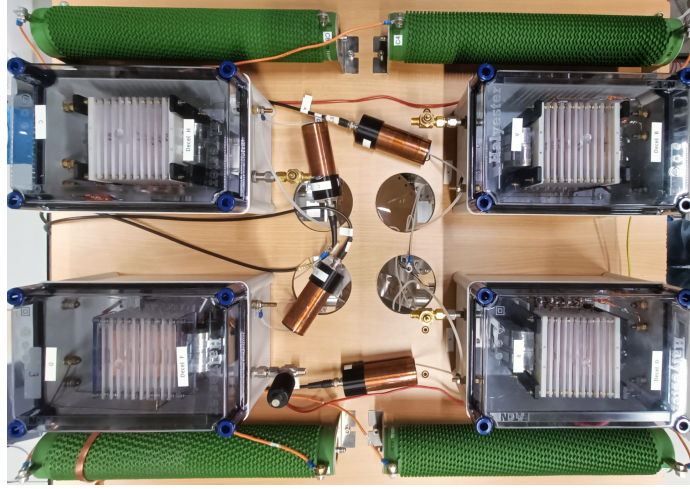


Figure 10: *Photo of the transformers, voltage dividers and resistors in the lab. The green cylinders are the resistors, the brown cylinders are the voltage dividers and the see-through rectangular boxes are the transformers.*

To safely test waveform behavior without exposing the actual Stark decelerator to potentially damaging high voltages, a **test box** was developed. This device replicates the capacitive loading and coupling characteristics of the decelerator electrodes - as explained in Figure 7 - allowing experimental waveforms to be realistically tested and optimized. A schematic of the equivalent circuit is shown in Figure 11.

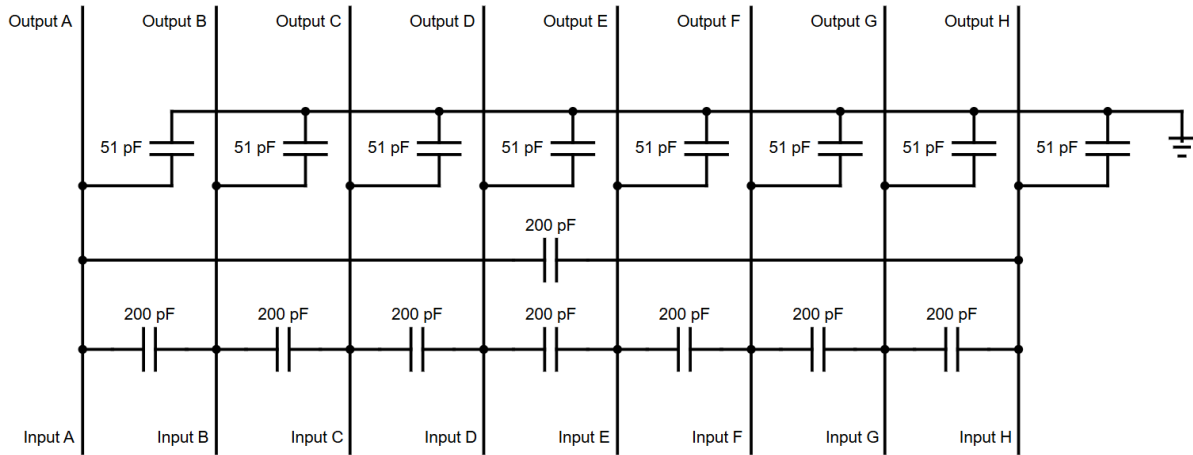


Figure 11: *Equivalent circuit of the decelerator test box. Each of the eight waveform channels (A-H) are connected to ground via a $51\ pF$ capacitor, representing the stray capacitance of an individual electrode to ground. Additionally, adjacent channels are interconnected through $200\ pF$ capacitors to simulate capacitive coupling between neighboring electrodes in the actual decelerator. Figure courtesy of Lucas van Sloten.*

3.2 Upgraded control software: Feedback system

The waveform generation and analysis will be carried out using a dedicated Python script developed for the NL-eEDM decelerator setup, visualized in Figure 8 as the PC. In contrast to the previous script, which only generated waveforms without any form of feedback or automated optimization, the current implementation will incorporate an automated feedback system designed to iteratively optimize waveform quality. Pre-distortion is applied, after which the feedback system iteratively refines the waveform to achieve the desired output shape. These feedback routines will continuously correct for deviations between the ideal and measured waveforms in real time. As discussed in section 2.5, the absence of real-time feedback in the earlier method made it highly limited and time-consuming to obtain suitable waveforms.

The input waveforms will be generated and transmitted through the hardware in the order described in Figure 8. The resulting output waveforms are expected to initially deviate from the ideal shape. To correct for this, the output will be processed through a sequence of feedback loops in the following order: safety feedback $\Rightarrow$ envelope feedback $\Rightarrow$ amplitude feedback $\Rightarrow$ phase feedback. Each of these systems will compare the measured waveform properties to pre-defined criteria and apply targeted adjustments to the input waveform. This iterative procedure is intended to enable autonomous convergence toward the ideal waveform profile, within the decelerator (or test box) environment.

Safety feedback

A safety check routine will be executed at the beginning of each measurement iteration. This routine is responsible for verifying that the generated waveforms fall within hardware-safe operational boundaries. It calculates the maximum voltage difference between adjacent waveforms to ensure that the voltage difference between adjacent channels does not exceed the maximum of 8 kV. If this safety criterion is violated, the program automatically engages a scaling mechanism. The routine identifies the largest voltage difference between any two adjacent channels and computes a global scaling factor based on this worst-case value relative to the maximum allowable voltage. This factor is then uniformly applied to all waveforms, proportionally reducing their amplitudes to bring the entire set within safe limits.

Envelope feedback

The envelope feedback algorithm will assess the flatness of the measured output waveform envelope. The comparison will be restricted to the region between the end of the ramp-up and the start of the ramp-down, as this is the relevant portion for the optimization. For each waveform, a fourth-order (possibly higher order on the future) polynomial will be fitted to the envelope segment to obtain a continuous representation. The percentage deviation from flatness is then quantified. This is shown by equation 2,

$$\Delta_n = \frac{\max(\text{env}_n) - \min(\text{env}_n)}{\max(\text{env}_n)} \cdot 100\%. \quad (2)$$

In this equation, env_n is the fitted envelope segment of waveform n . If Δ_n exceeds a predefined threshold, the corresponding input envelope will be rescaled. By repeating this process iteratively, the envelope feedback routine will guide the system toward waveforms with minimal amplitude ripple in their flat-top sections, improving the uniformity of the decelerating electric field.

Amplitude Feedback

The goal of the amplitude feedback is to ensure that the peak amplitudes of all eight waveform channels are of equal magnitude. This is essential to preserve the symmetry and coherence of the traveling wave. In each iteration, the routine will determine the maximum output voltages $V_{\text{max},n}$ for each channel n , one channel will be changed per iteration. A scaling factor β_n will then be computed such that all channels are scaled relative to the maximum allowed amplitude, which is denoted by $V_{\text{max, allowed}}$. The relation for this is given by equation 3,

$$\beta_n = \frac{V_{\text{max, allowed}}}{V_{\text{max},n}}. \quad (3)$$

These scaling factors will be applied multiplicatively to the corresponding input waveforms. As a result, the channels can be scaled up or down to match the desired value.

Phase Feedback

The phase feedback will monitor and correct phase misalignment between the eight output waveforms and their ideal counterparts. As explained in section 2.4, the ideal waveforms are separated by a phase difference of 45° relative to adjacent channels. To evaluate the phase alignment, waveform A will first be used as a temporal reference by shifting it such that it crosses zero at the start of the pulse. The other waveforms will then be aligned with respect to this reference. These temporal offsets are translated into phase differences and used to correct the input waveforms by shifting them in time, thereby correcting relative phase difference across all output waveforms. In contrast to the other feedback systems described above, the phase feedback does not work properly yet in practice and is therefore subject to change.

Link to electronics

The feedback system itself is not the primary focus of this thesis and will therefore not be discussed in more detail. Instead, this thesis aims to upgrade and characterize the electronics of the decelerator setup shown in Figure 8, in order to support the feedback system in generating waveforms that closely approximate the ideal form. More details on the precise motivation and aim of this thesis are provided in Section 5.

4 Theoretical context

4.1 Waveform generation

The waveforms for the NL-eEDM experiment are generated by pulsed sine-waves over a frequency sweep that smoothly transitions from high to low frequency over the pulse duration. The analytical form of the waveform is given by Equation 4;

$$s(t) = \sin[2\pi (f_d \cdot t_{\text{shifted}} + (f_i + 2f_d \cdot \Delta t) \cdot t_{\text{shifted}})]. \quad (4)$$

In this equation, Δt is associated with the phase difference between channels. It is calculated by $t_{\text{shifted}} = t - \Delta t - \frac{n}{F} - s_t$, where F is the repetition rate (typically 10 Hz). n is the number of pulses produced, this is set to one as the Moku:pro will generate one pulse every iteration. t_{shifted} is a global time delay applied to all waveforms, t is the time variable and s_t are the delays. f_i is the initial frequency at the start of the pulse, typically 16.7 kHz. f_f is the final frequency, typically 2.5 kHz. f_d is half the rate of the frequency chirp, given by $f_d = \frac{f_f - f_i}{2P}$, where P is the pulse length.

At higher frequencies, the phase difference introduced by a fixed time delay becomes more significant, whereas at lower frequencies its effect is reduced. This behavior is crucial for maintaining synchronized trapping conditions throughout the deceleration process.

4.2 Peak-to-peak voltage limit in the decelerator

To prevent damage to the decelerator, a safety limit is imposed on the maximum allowed voltage difference between adjacent channels. This limit is 8 kV in amplitude. Based on this constraint, the maximum permissible amplitude for the decelerator waveforms, denoted as A_{max} , can be determined. From the ideal waveform structure shown in Figure 6, the instantaneous voltage difference between two adjacent channels, $V_{\text{diff}}(t)$, can be expressed analytically as given in Equation 5;

$$V_{\text{diff}}(t) = A \cdot \sin(t) - A \cdot \sin(t + \frac{\pi}{4}). \quad (5)$$

In this equation, A denotes the amplitude of the generated waveforms, and the term $\frac{\pi}{4}$ corresponds to a phase shift of 45°, expressed in radians. The time points t at which $V_{\text{diff}}(t)$ reaches a maximum can be determined by differentiating and setting the derivative equal to zero;

$$A \cdot \cos(t) - A \cdot \cos(t + \frac{\pi}{4}) = 0,$$

$$\cos(t) = \cos(t + \frac{\pi}{4}),$$

$$t_{\text{max}} = -\frac{\pi}{8} + n\pi, \quad n \in \mathbb{Z}.$$

To determine the maximum allowable amplitude in the decelerator channels, A_{max} , Equation 5 is evaluated at the time $t_{\text{max}}^{n=1}$, which corresponds to the first maximum of $V_{\text{diff}}(t)$. At this time,

the voltage difference between adjacent rings reaches its maximum value, which is constrained to 8 kV, as previously stated. One then finds;

$$t_{max}^{n=1} = \frac{7\pi}{8},$$

$$V_{diff}(t_{max}^{n=1}) = A_{max} \cdot \sin(t_{max}^{n=1}) - A_{max} \cdot \sin(t_{max}^{n=1} + \frac{\pi}{4}),$$

$$A_{max} = \frac{V_{diff}(t_{max}^{n=1})}{\sin(t_{max}^{n=1}) - \sin(t_{max}^{n=1} + \frac{\pi}{4})} = \frac{8 \text{ kV}}{\sqrt{2} - \sqrt{2}} = \boxed{10.45 \text{ kV}}. \quad (6)$$

The maximum permissible amplitude in the decelerator channels is found to be 10.45 kV, corresponding to a peak-to-peak voltage safety limit of 20.9 kV. This result indicates that, in principle, the ideal peak-to-peak voltage of 20.0 kV can be safely reached in the decelerator channels.

4.3 Transformers

The working principle of a transformer is based on electromagnetic induction. When an alternating current (AC) flows through the primary coil, it generates a time-varying magnetic field within a shared ferromagnetic core. This changing magnetic flux induces a voltage across the secondary coil via mutual inductance. In this experiment, transformers are used to convert the high-current output from an amplifier into the high-voltage signal required to drive the decelerator electrodes. The circuit includes resistive (R), inductive (L), and capacitive (C) components, forming a coupled second-order RLC circuit. An equivalent circuit of the transformer system is shown in Figure 12.

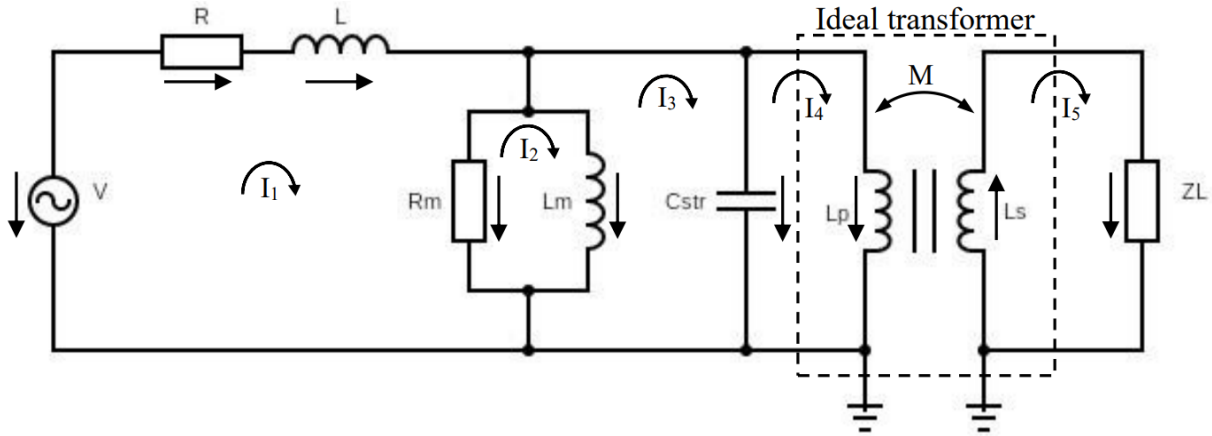


Figure 12: **Equivalent circuit for the transformers.** In this figure, V is the input AC voltage, and $I_{1,2,3,4,5}$ are the currents flowing through the five mesh loops. L , C , and R represent inductance, capacitance, and resistance, respectively. The amplifier injects a high-current, 'low'-voltage signal into node V . The primary winding L_P generates a time-varying magnetic field, which induces a voltage across the secondary winding L_S via mutual inductance M . The resulting high-voltage output drives current I_5 through the load impedance Z_L , representing one of the decelerator's electrode rods. Figure courtesy of Lucas van Sloten.

4.3.1 Damping of second order RCL circuit

The damping ratio ζ of a second-order RLC-like resonant system, such as a transformer with parasitic inductance and capacitance, is given by Equation 7 [15];

$$\zeta = \frac{R}{2} \sqrt{\frac{C}{L}}. \quad (7)$$

In this equation, R is the resistance (in Ω), L is the inductance (in H), C is the capacitance (in F), ζ is the dimensionless damping ratio. For $\zeta < 1$, the system is underdamped (oscillatory), and for $\zeta > 1$, the system is overdamped. The system is critically damped when $\zeta = 1$, which corresponds to a critical resistance R_{crit} . R_{crit} can be calculated by rewriting Equation 7, and is given by Equation 8;

$$R_{crit} = 2\sqrt{\frac{L}{C}}. \quad (8)$$

4.3.2 Magnetic Coupling

Magnetic coupling in transformers refers to the ability of two inductive coils to transfer energy via mutual inductance. When alternating current flows through the primary winding, it generates a time-varying magnetic field, which induces a voltage in the secondary winding. The strength of this magnetic interaction is quantified by the coupling coefficient k , which can be determined with Equation 9 [16];

$$k = \frac{M}{\sqrt{L_1 L_2}}. \quad (9)$$

In this equation, M is the mutual inductance and L_1 , L_2 are the self-inductances of the primary and secondary windings, respectively. Perfect coupling, $k = 1$ is ideal but never achievable in practice due to factors such as imperfect core material or winding misalignment. In this case, the magnetic flux generated by the primary winding is almost entirely linked to the secondary winding, ensuring efficient energy transfer and predictable performance. Conversely, loosely coupled transformers, $k \ll 1$, exhibit a considerable amount of leakage flux that does not contribute to energy transfer, leading to reduced efficiency and altered impedance characteristics.

The degree of coupling affects how the transformer responds to changes in load conditions, such as the addition of a resistor. Tightly coupled transformers are more sensitive to such changes, displaying noticeable variations in output voltage and current. This sensitivity arises because the strong magnetic linkage ensures that any change in the secondary circuit directly influences the primary circuit. In contrast, loosely coupled transformers show minimal response to load changes due to the weaker magnetic linkage.

4.3.3 Sealing

To ensure safe and stable operation under high-voltage conditions, the transformers can be enclosed in vacuum-sealed chambers. These enclosures are designed to be gas-tight (unlike the standard plastic housings), as they are filled with sulfur hexafluoride (SF_6), an insulating gas with significantly higher dielectric strength than air. Visualizations of this can be seen in Figure 13.

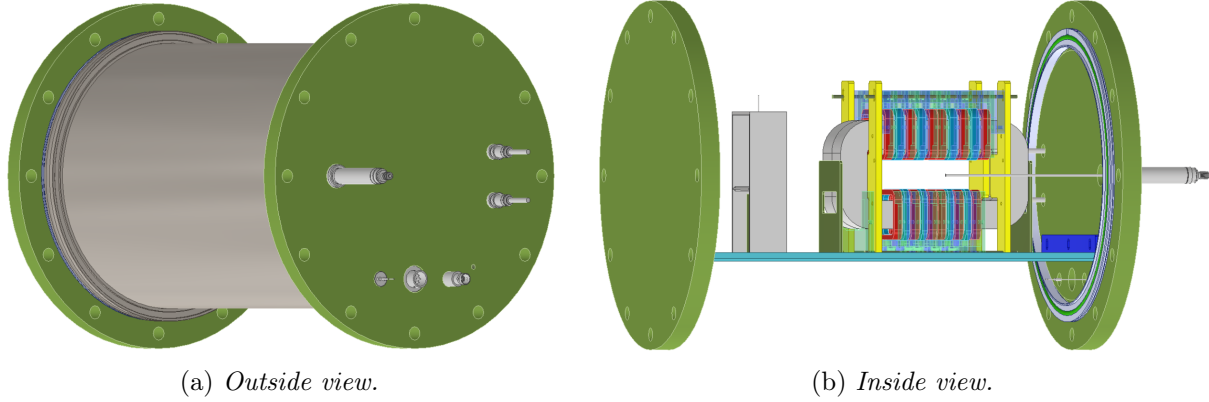


Figure 13: **Visualizations of a sealed transformer.** At present, this is only the case for transformer X. The same procedure is expected to be applied to the other transformers as well. Figure courtesy of Leo Huisman.

4.4 Amplifiers

Amplifiers are electronic devices designed to increase the power, voltage, or current of a signal. In the NL-eEDM experiment, Class D amplifiers are used due to their high efficiency, fast switching behavior. Unlike linear amplifiers, Class D amplifiers operate by switching output transistors on and off rapidly. Figure 14 shows an example schematic of a Class D amplifier.

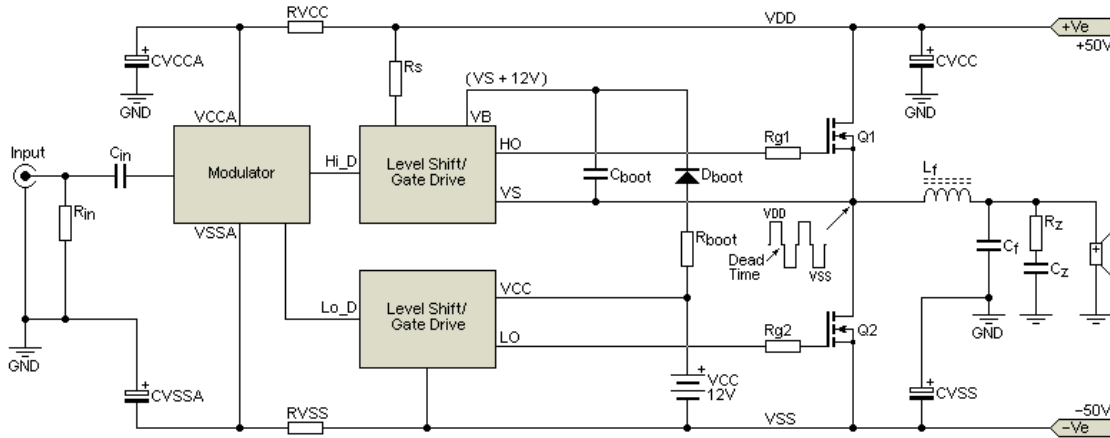


Figure 14: **Example circuit of a Class D amplifier.** This is the same amplifier class as the Behringer NX3000D used in the NL-eEDM experiment. Figure courtesy of Leo Huisman.

During the rapid transitions, brief but substantial current surges are required. The power supply alone cannot respond quickly enough to these demands. To compensate, decoupling capacitors C_{VCC} and C_{VSS} are placed close to the output stage to locally supply this current. Their performance is critically affected by their internal resistance, known as the Equivalent Series Resistance (ESR). A capacitor with lower ESR can deliver higher peak currents during switching events, improving output voltage stability and reducing power loss. Variations in capacitor quality—such as replacing a degraded unit with a lower-ESR capacitor—can therefore lead to observable changes in amplifier behavior, including increased output amplitude and reduced distortion.

4.5 Bode plots

A Bode plot is a logarithmic graphical representation of a system's frequency response. It consists of two plots: one showing the gain (in decibels) and the other the phase shift (in degrees), both as functions of frequency. Bode plots provide insight into the system's stability, bandwidth, and dynamic behavior across a wide frequency range. In this thesis, they are used to characterize the gain and phase response of different transformers and amplifiers. The gain and phase difference can be calculated using equation 10 and 11, respectively.

$$Gain \text{ (dB)} = 20 \log_{10} \left(\frac{V_{\text{output}}}{V_{\text{input}}} \right) \quad (10)$$

$$\phi(f) = \phi_0 - \phi_{\text{span}} \cdot \arctan \left(\left(\frac{f}{f_c} \right)^n \right) \cdot \left(\frac{180}{\pi} \right) \quad (11)$$

In equation 11, ϕ_0 is the phase offset at low frequencies, ϕ_{span} is the total phase shift across the frequency range, f is the frequency, f_c is the cutoff frequency where the phase starts to change significantly, and n is a steepness parameter that determines how rapidly the transition occurs. The factor $\frac{180}{\pi}$ converts the arc-tangent from radians to degrees.

5 Motivation, research questions & hypothesis

As explained in Section 1.1, a precise measurement of the eEDM is of fundamental importance to modern physics, as its discovery would provide clear evidence of physics beyond the Standard Model. Improving the sensitivity of such measurements requires higher T and n in Equation 1, which is further explained in Section 1.3. In the NL-eEDM experiment, this is achieved by generating waveforms in a traveling-wave Stark decelerator that approximate the ideal waveforms, which are described Section 2.4.

Achieving near-ideal waveforms in the traveling-wave Stark decelerator requires not only an effective feedback system but also compatible and uniformly performing electronics. This thesis focuses on upgrading and characterizing the electronics of the setup, rather than the feedback algorithm itself. A key upgrade involves configuring the amplifiers in bridge mode to enable higher voltage outputs, with the aim of reaching the target ideal peak-to-peak voltage of 20 kV. Figure 6 illustrates that, apart from a 45° phase offset between adjacent channels, all ideal waveforms are identical. To accurately and efficiently realize this in practice with the feedback system, the electronics in all eight channels must exhibit similar behavior. Deviations between components will hinder the feedback system's ability to converge to the ideal waveforms. Therefore, this thesis includes a detailed characterization of the transformers and amplifiers, to identify and possibly minimize discrepancies between channels.

The overall aim of this thesis is to upgrade and characterize the electronics of the traveling-wave Stark decelerator to create optimal conditions for ideal waveform generation. This leads to the following research questions with corresponding hypotheses:

- **RQ:** *How do frequency-dependent gain and phase responses of the transformers and amplifiers differ, and how can these differences be minimized to ensure consistent waveform generation in the traveling-wave Stark decelerator?*

Hypothesis: Since the amplifiers are commercially manufactured and the same model, their frequency responses are expected to be nearly identical, with only minor deviations due to production tolerances. The transformers are custom-built and thus more susceptible to variation; nonetheless, they were designed to be functionally identical, so significant differences in behavior are not expected. If major deviations in behavior are observed in certain components, one could dismantle, inspect and possibly repair these components. Theoretical background on amplifiers and transformers is provided in Sections 4.4 and 4.3, respectively. The theory behind their frequency response characteristics, represented in Bode plots, is given in Section 4.5.

- **RQ:** *What peak-to-peak voltages can be reached in the channels of the traveling-wave Stark decelerator after setting the amplifiers to bridge mode, and how could the voltage output be increased even further?*

Hypothesis: Setting the amplifiers in 'bridge mode' instead of 'stereo mode' is expected to significantly increase the maximum achievable output voltage per channel to a level that makes it possible to reach the ideal peak-to-peak voltage of 20 kV in each channel. This was explained in Section 3. The peak-to-peak voltage per channel is expected to approximately double - from around 10 kV to 20 kV - by combining both internal amplifier channels into one.

- **RQ:** *What is the effect of capacitive coupling between channels on the gain and phase difference behavior of the transformers?*

Hypothesis: Section 2.5 showed that capacitive coupling has a significant influence on the channels, which makes it challenging to approximate the ideal waveforms. Therefore it is expected that the effect of capacitive coupling on the frequency response of the channels will also be significant.

- **RQ:** *What is the safe upper limit for the peak-to-peak voltage in the channels of the traveling-wave Stark decelerator, and how does this compare to the ideal peak-to-peak voltage of 20 kV?*

Hypothesis: In the setup shown in Figure 8, the transformers, decelerator and the voltage dividers should be able to safely withstand the ideal peak-to-peak voltage of 20 kV. In Section 4.2 it was determined that the voltage limit for the decelerator is around 20.9 kV, which is well above the ideal voltage. The transformers are expected to be able to withstand the ideal voltage after being sealed, which is further explained Section 4.3.3. As explained in Section 3.1.1, the current voltage dividers are not able to withstand the ideal voltage. Therefore a new design will be proposed, with as theoretical peak-to-peak voltage limit of 25.2 kV. It is expected that the channels will safely be able to withstand the ideal peak-to-peak voltage of 20 kV, as long as the transformers are sealed and the new voltage dividers will be implemented.

6 Experimental Methods

In order to evaluate and possibly improve the performance of the waveform generation in the decelerator, the behavior of the electronics in the setup will be researched. By identifying units that under-perform or deviate from expected behavior, suitable combinations of components can be selected, and hardware-specific adjustments can be implemented. Establishing a uniform and high-performing electronics environment across all eight channels is crucial for achieving higher voltages and for facilitating the generation of consistent, near-ideal waveforms. A general setup used for the measurements is shown in Figure 15.

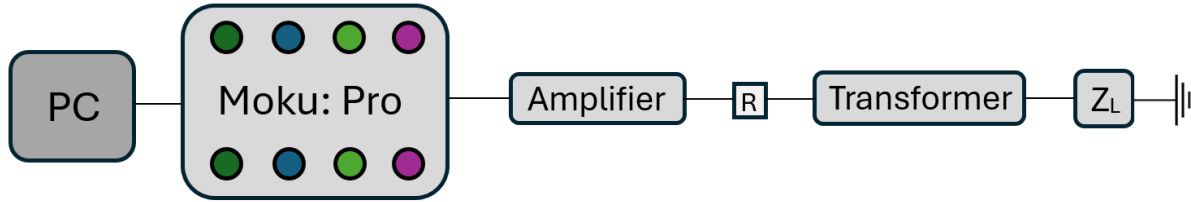


Figure 15: *Schematic overview of the general setup used for the measurements involving individual testing transformers or amplifiers.* This setup is used to measure one of the eight channels at the time. The load Z_L has a capacitance of 200 pF and is used to simulate the decelerator environment for one channel. The resistor R has a resistance of 2.2 Ω .

6.1 Transformer testing

This experiment aims to characterize the performance of ten transformers (labeled A through H, X, and Y) and identify potential differences between them. All transformers are intended to be constructed identically and are therefore expected to exhibit comparable behavior, with the exception of transformer X, which is a shielded transformer. More details on shielding can be found in Section 4.3.3. A frequency sweep from 20 kHz to 1 kHz will be used while maintaining the amplifier gain knob on a constant setting. During this sweep, the input voltage, input current, and output voltage will be recorded. All amplifiers will operate in bridge mode without the use of filters. Data acquisition will be performed using an oscilloscope with four input channels. The input voltage, output voltage, and input current of the transformer will be measured using a low-voltage probe, a shielded high-voltage probe, and a current probe, respectively. The fourth oscilloscope channel will record the corresponding signal from the voltage divider to cross-check the high-voltage probe readings.

Measurement data will be stored digitally for post-processing. For each transformer, the time series of input and output voltages will be used to generate Bode plots by curve-fitting. These plots will show the gain (in decibels) and phase shift (in degrees) as functions of frequency, derived using Equations 10 and 11. Frequency will be extracted from the time axis of the measurements. Two experimental configurations will be studied: (1) individual channel measurements, to isolate the behavior of each transformer, and (2) simultaneous operation of all eight active channels, to investigate the influence of inter-channel capacitive coupling. For Case 1, the setup shown in Figure 15 will be used with the probes at the input and output of the transformer. The same amplifier will be used for these measurement, and the transformer will be varied.

For Case 2, eight transformers (A, B, C, D, E, F, G, H) will be connected simultaneously to the test box, so for this measurement Figure 15 does not directly apply. The setup for this experiment

is given in Figure 8, with the probes added at the input and output of the transformers. During measurements, only the position of the probes will be varied, as the input and output voltages need to be measured at each transformer. Theoretical background on transformer behavior and Bode plots is provided in Sections 4.3 and 4.5.

6.2 Amplifier testing

To characterize the performance of the eight amplifiers used in the NL-eEDM setup, each unit was individually connected to the same shielded reference transformer (transformer X) under identical conditions. The setup shown in Figure 15 will be used with the probes at the input and output of the amplifier. The same transformer will be used for these measurements, and the amplifier will be varied. For each amplifier, the maximum achievable peak-to-peak output voltage and the corresponding output current were determined by gradually increasing the gain knob until the amplifier's internal protection mechanism triggered a shutdown. The input signal was fixed at a peak-to-peak amplitude of 2 V, and a frequency sweep from 16.7 kHz to 2.5 kHz was applied. Prior to significant gain adjustments, manual filtering was applied to flatten the output waveform amplitude as much as possible. All channels were tested individually, without capacitive coupling effects from other active channels.

To further analyze amplifier behavior, Bode plots were constructed for each amplifier using transformer X and data collected via a four-channel oscilloscope. The same setup was used as in the previous amplifier measurement, visualized in Figure 15. The input voltage to the amplifier, taken from the output of the Moku:Pro, was recorded directly via coaxial connection and adjusted by a factor of two in post-processing to account for bridge mode operation. The output voltage before the series resistor (the amplifier output) was measured using a low-voltage probe, as was the voltage after the resistor. The fourth oscilloscope input monitored the high-voltage output of the transformer, although this signal was not central to the analysis. Bode plots were generated to show the gain and phase behavior of the amplifiers as a function of frequency. For all eight amplifiers, separate Bode plots were generated using the input voltage and either the output voltage before or after the resistor, enabling assessment of both intrinsic amplifier behavior and load effects.

7 Results & Discussion

7.1 Transformers

7.1.1 Gain and phase difference behavior when tested separately

Following the methodology described in Section 6.1, Bode plots were generated for each transformer to characterize the gain and phase difference between input and output voltages as a function of frequency. Figure 16 shows the results in the absence of the resistor R .

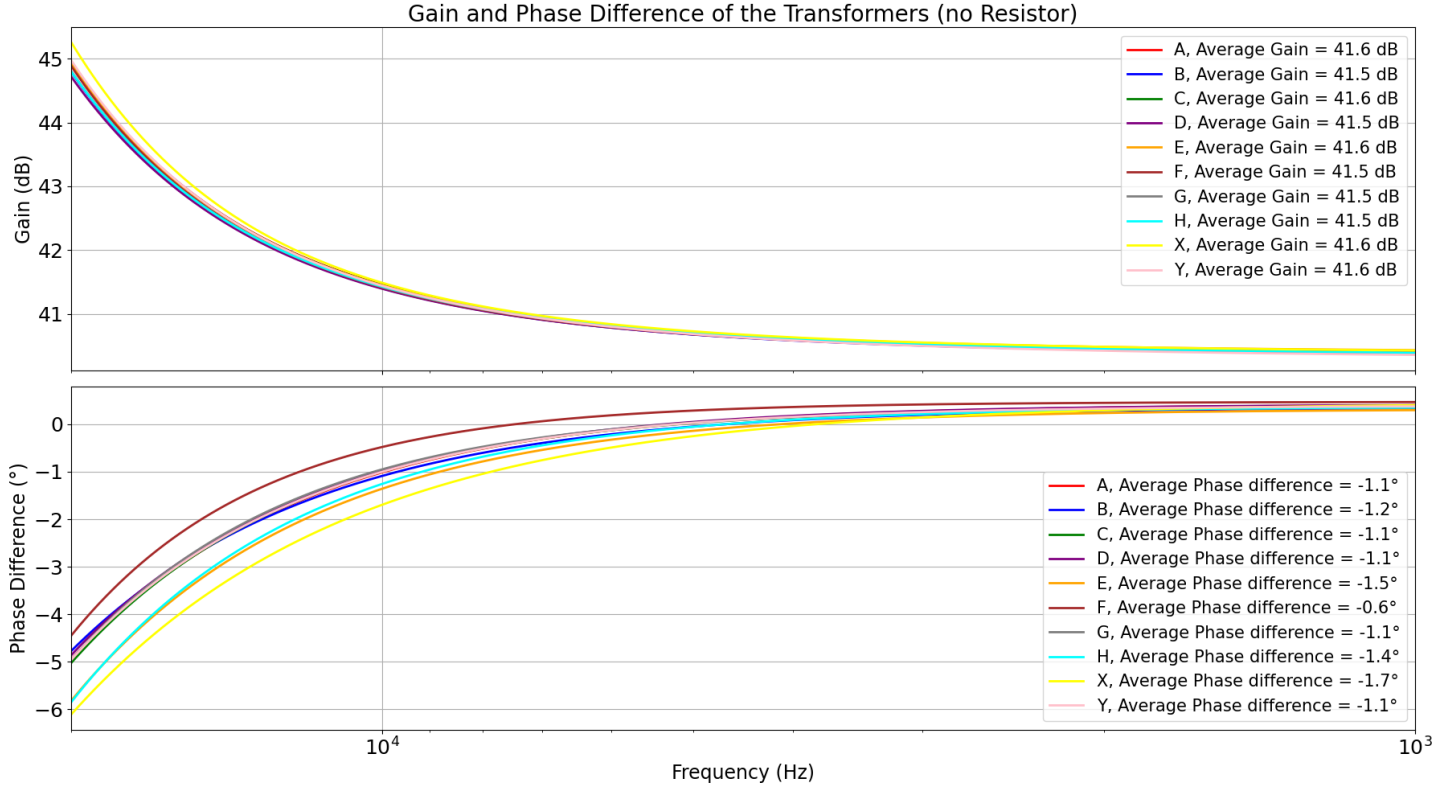
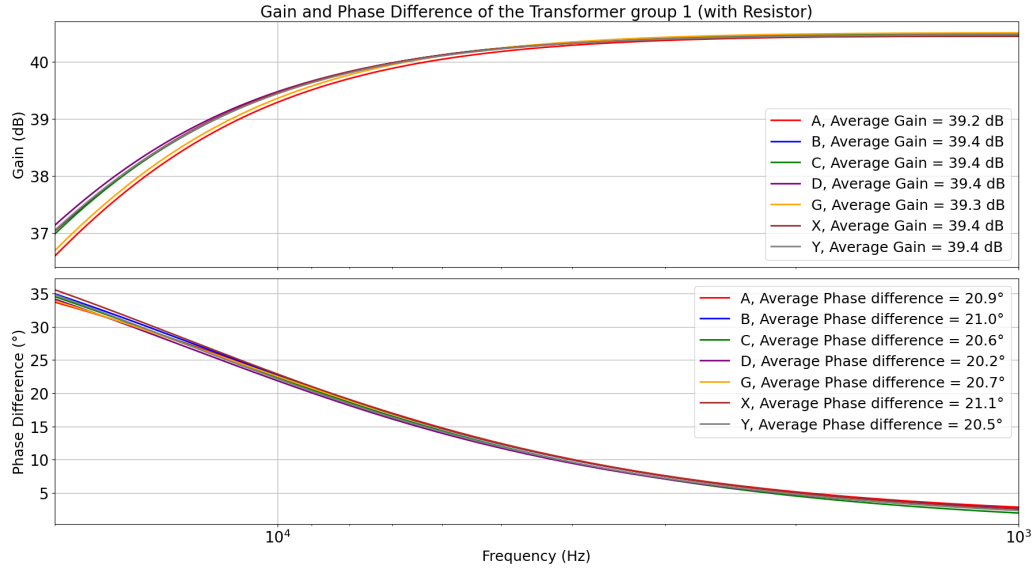


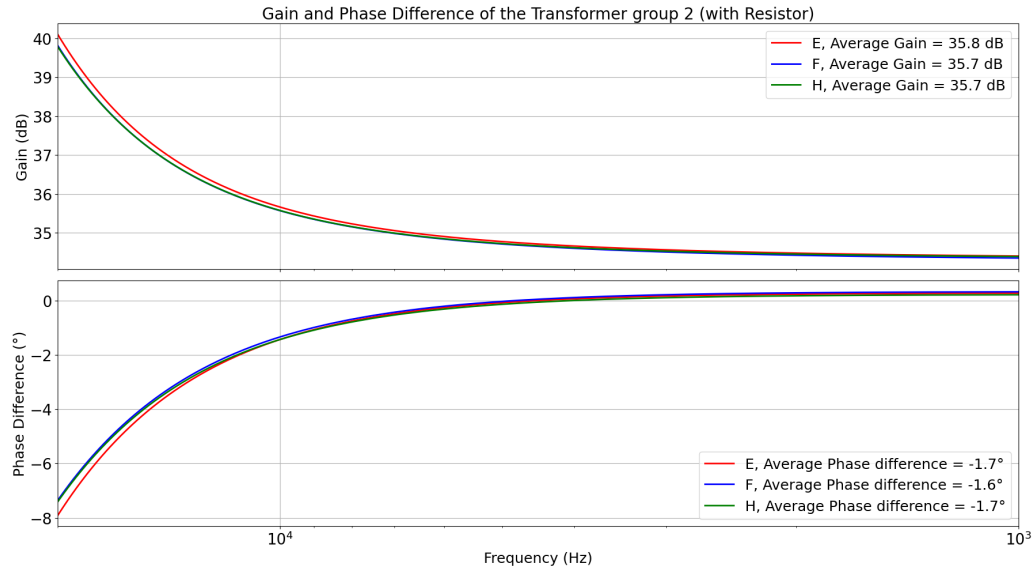
Figure 16: *Bode plot of the transformers when no resistor is connected in the circuit. A frequency sweep from 20 kHz to 1 kHz was used for data acquisition and plotting. For the computation of the average gain and phase difference shown in the legend, only the data in the range of 16.7 kHz to 2.5 kHz was considered, as this corresponds to the frequency band relevant for future eEDM measurements. The transformers were tested individually, meaning that capacitive coupling had no influence on these results. A load of 200 pF was used to simulate the the decelerator. No resistor was placed between the amplifiers and transformers in this configuration.*

It can be observed that all transformers exhibit nearly identical frequency responses in both gain and phase difference when no resistor is present. The gain characterizes the ratio between input and output voltages. With Equation 10 it can be determined that an average gain of 41.5 dB and 41.6 dB correspond to a voltage gain ($\frac{V_{out}}{V_{in}}$) of 119.4 and 122.1, respectively. The higher this ratio, the easier it becomes to reach the ideal peak-to-peak amplitude of 20 kV, hence a high gain is desired for the transformers. The phase difference between the input and output

voltages reflects the temporal phase shift introduced by the transformer. It can be observed that for the output voltage slightly lags behind the input voltages - especially for higher frequencies - which is common behavior for transformers. In order to make it easier to achieve the ideal phase difference of 45° between channels, it is desired that the phase difference between the input and output of the transformers is low, and of about equal magnitude for all transformers. When no resistor is used, the transformers show very similar behavior. But as one can recall from Figure 8, the resistors will be present during actual eEDM measurements. Therefore the behavior of the transformers when the resistors are connected in the circuit needs to be studied as well. Resulting Bode plots of this scenario can be seen in Figure 17.



(a) Group 1: transformers A, B, C, D, G, X, and Y.



(b) Group 2: transformers E, F, and H.

Figure 17: *Bode plot of the transformers with the resistor in the circuit. Apart from the resistor R of $2.2\ \Omega$ now being present, the methodology is the same as before.*

Figure 17 shows that the presence of the resistor has a significant impact on most of the transformers. These transformers are from now categorized in 'Group 1' and correspond to transformers A, B, C, D, G, X, and Y. The behavior of Group 1 is shown in Figure 17a. The behavior of transformers E, F, and H does not seem to depend much on the presence of the resistor, these transformers are from now on categorized as 'Group 2'. The behavior of Group 2 is shown in Figure 17b. The gain and phase difference behavior of Group 1 inverts in the presence of a resistor; with decreasing frequencies, increasing gain and decreasing phase values are found, while that was opposite in the absence of R . To investigate this behavior more, a transformer from both Group 1 and 2 will be tested over variable resistance, the resulting Bode plots are shown in Figures 18 and 19.

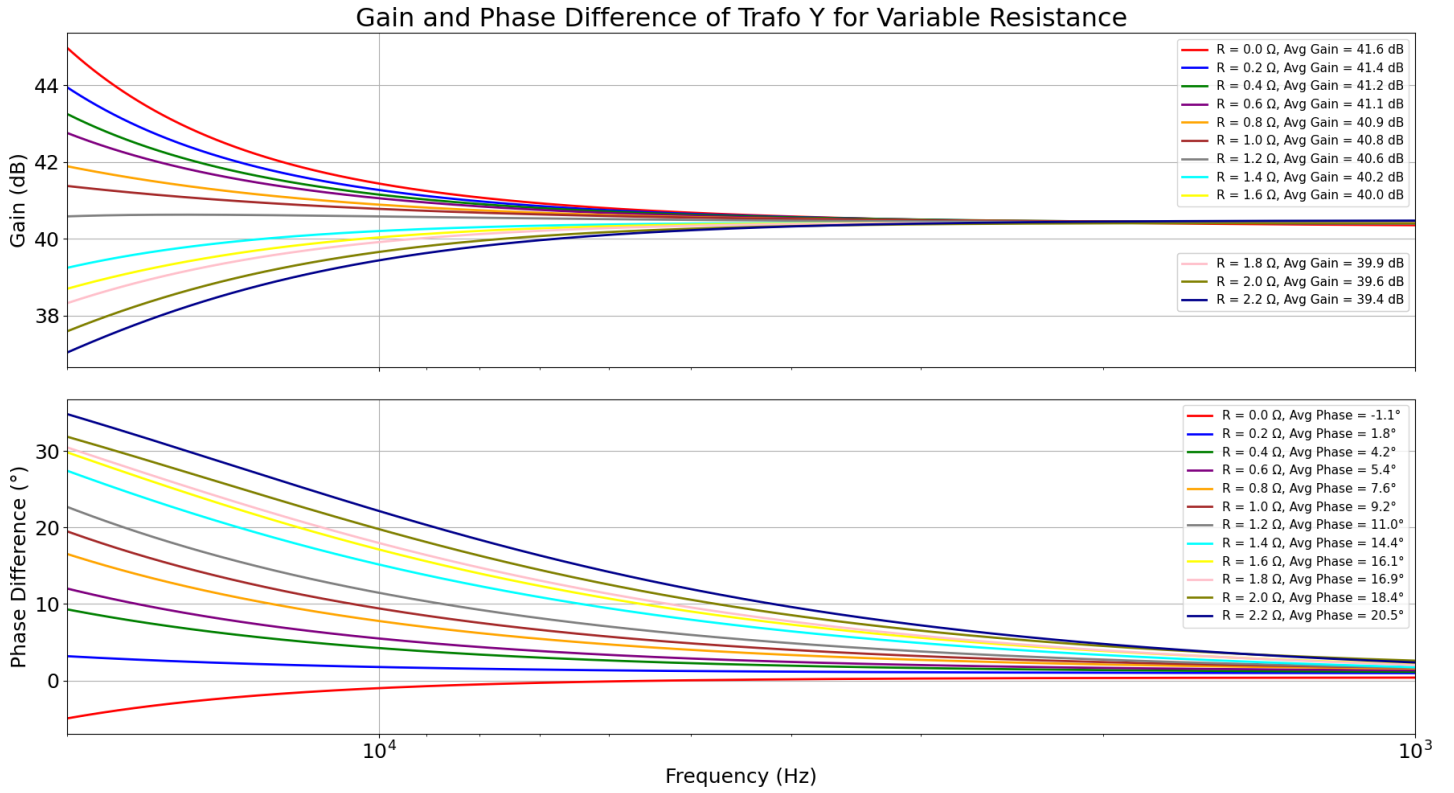


Figure 18: *Bode plot of transformers Y over variable resistance.* Resistance values ranging from 0.0Ω to 2.2Ω were used, with steps of 0.2Ω . For the boundary values - 0.0Ω and 2.2Ω - the same data of transformer Y shown in Figure 16 (no resistor) and Figure 17 were used, respectively. Apart from that the methodology is the same as before.

Figure 18 shows the dependence of the gain and phase difference of transformer Y over variable resistance. This Figure functions as an example of how transformers of Group 1 behave over variable resistance. It follows a predictable pattern; the higher the value of R , the more it behaves like Figure 17 ($R = 2.2 \Omega$) & the lower the value of R , the more it behaves like Figure 16 (no resistor). The average gain values range between 39.4 dB and 41.6 dB. This corresponds to $93.3 \leq \frac{V_{out}}{V_{in}} \leq 120.2$, which means the ratio between input and output voltage decreases with 28.8% over the increasing R . The average phase difference increases by 21.6° over increasing R . The pattern shown is likely explained by damping of second-order RLC circuit, of which the

theory was explained in Section 4.3.1. Where the curve appears flat, the value of R is likely close to R_{crit} of the system, where damping ratio $\zeta \approx 1$. When $R > R_{crit}$ the system is overdamped, and when $R < R_{crit}$ the system is under-damped.

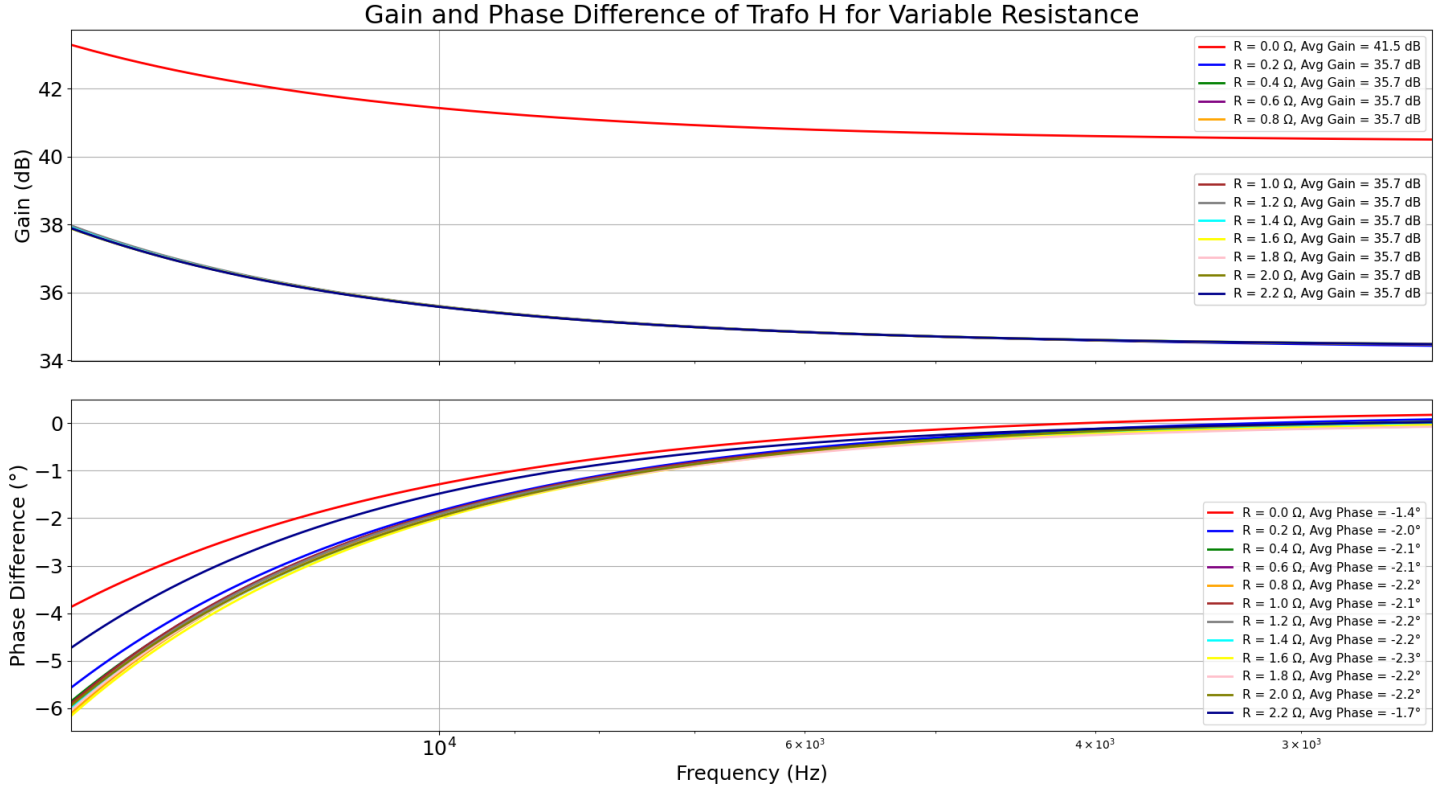


Figure 19: *Bode plot of transformers H over variable resistance.* Resistance values ranging from 0.0Ω to 2.2Ω were used, with steps of 0.2Ω . For the boundary values - 0.0Ω and 2.2Ω - the same data of transformer H shown in Figure 16 (no resistor) and Figure 17 were used, respectively. Apart from that the methodology is the same as before.

Figure 19 shows the dependence of the gain and phase difference of transformer H over variable resistance. This figure functions as an example of how transformers of Group 2 behave over variable resistance. It can be seen for transformer H, the dependence on R of both the gain and the phase difference is rather insignificant. This weak response of the performances of transformer H on changes in the setup is likely explained by loose winding and weak magnetic coupling compared to Group 1, of which the meaning was explained in Section 4.3.2. A discrepancy is shown in the gain plot at $R = 0.0 \Omega$, where no resistor is used. One can observe a significantly higher gain for only this value of R , which is likely caused by instabilities in the system when no resistor is connected at all.

To investigate the cause of differences in behavior of both groups of transformers, transformer E was disassembled and inspected. It turned out that multiple coils were loose and almost touching one another, which caused sparking between different sets of coils. A visualization of this is shown in Figure A8 in Appendix Section A. After this was repaired, the same measurements were repeated. The previous data collected of transformer E (the same data shown in Figure

17b) and data after repair were plotted in the same figure to visualize the possible changes in behavior. The old and new datasets will be called E_{broken} and $E_{repaired}$, respectively. Both datasets were acquired with the resistor $2.2\ \Omega$ present in the circuit. The results are shown in Figure 20.

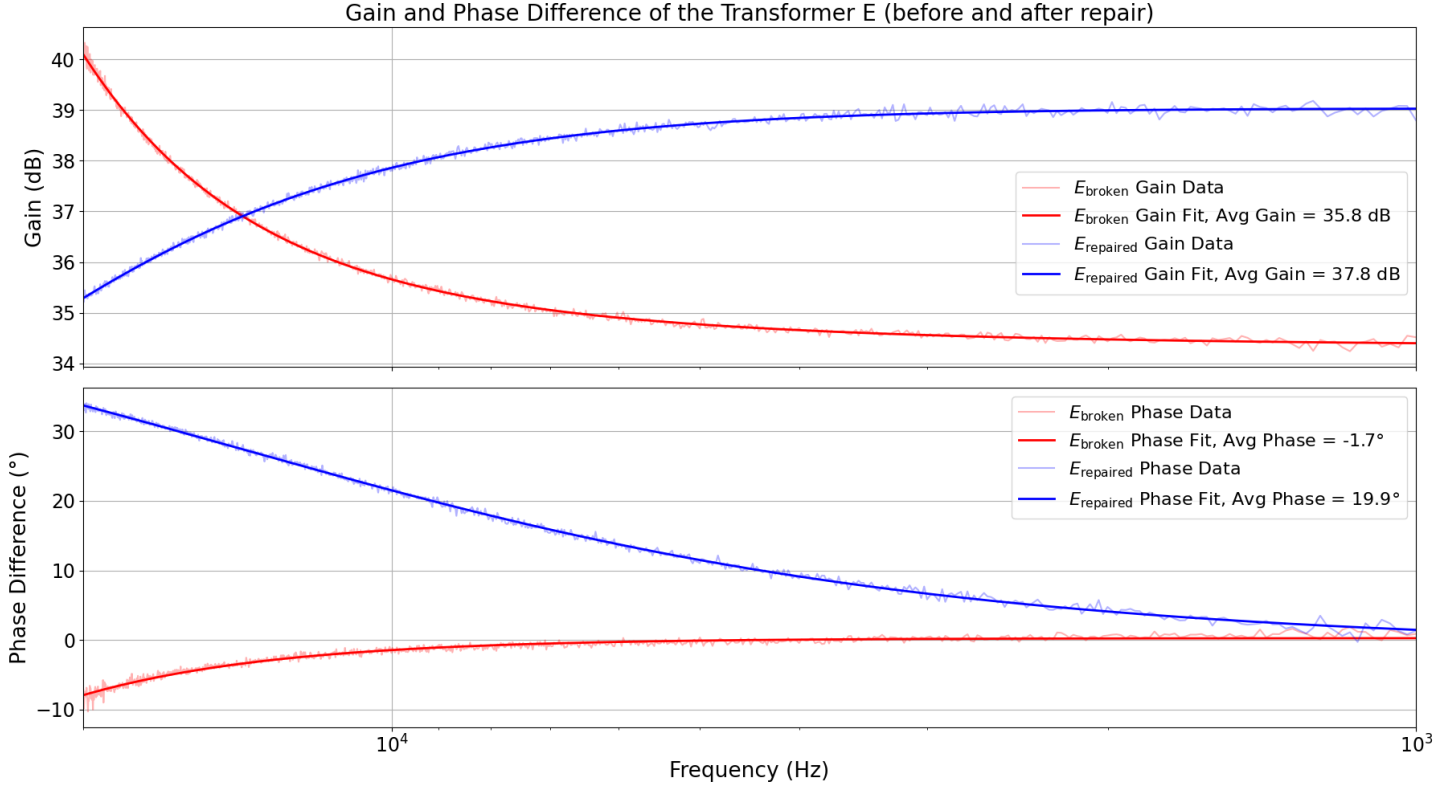


Figure 20: *Bode plots of transformers E before and after repair.* A constant resistance of $2.2\ \Omega$ was used. E_{broken} shows the data collected of transformer E before it was repaired, and $E_{repaired}$ shows the data collected of transformer E after repair. The lines of best fit and the data itself is shown, as explained by the legend. Apart from that the methodology stays the same as before.

One can directly see that the behavior of transformer E changed significantly after repair. It showed Group 2 behavior before. But the $E_{repaired}$ plot shows that after repair, transformer E started behaving like Group 1 transformers; increasing gain and decreasing phase difference as the frequency decreases. So when transformer E was functioning properly, it displayed Group 1 behavior. This suggests that a properly functioning transformer should behave like Group 1, which could mean that the remaining transformers from Group 2 (F, H) also have sparking windings. To verify this, transformer F and H should be disassembled and inspected as well, like was done with transformer E.

For the NL-eEDM experiment, the transformers must operate in a manner that facilitates the generation of near-ideal waveforms. Consequently, transformers exhibiting high gain, low and consistent phase difference, and uniform frequency response characteristics are preferred. Table 1 summarizes the most important differences between transformers of Group 1 and Group 2.

Property	Group 1 (A, B, C, D, E _{repaired} , G, X, Y)	Group 2 (E _{broken} , F, H)
Gain shape	Gain increases as frequency decreases	Gain decreases as frequency decreases
Average gain	Higher; Gain _{avg, 1} = 39.2 ± 0.2 dB	Lower; Gain _{avg, 2} = 35.7 ± 0.0 dB
Phase difference shape	Phase difference decreases as frequency decreases	Phase difference increases as frequency decreases
Average phase difference	Larger magnitude and spread; $\phi(f)_{\text{avg, 1}} = 20.6 \pm 0.1^\circ$	Smaller magnitude and spread; $\phi(f)_{\text{avg, 2}} = -1.7 \pm 0.0^\circ$
Build quality	Functioning as expected; E exhibited Group 1 behavior after repair	Likely malfunctioning; F and H require further testing

Table 1: *Comparison of Group 1 and Group 2 transformer behavior.* Values for average gain and phase difference are based on measurements from Figures 17 and 20. All data reflect conditions with a 2.2Ω resistor in the circuit.

Table 1 shows that the transformers in Group 1 have a higher average gain in the relevant frequency range of 16.7 kHz to 2.5 kHz . By Equation 10, the ratio $\frac{V_{\text{out}}}{V_{\text{in}}}$ can be determined for both groups.

$$\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)_{\text{Avg, 1}} = \boxed{91.2 \pm 2.1} > \left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)_{\text{Avg, 2}} = \boxed{60.9 \pm 0.0}$$

This means that on average, the average voltage gain $\left(\frac{V_{\text{out}}}{V_{\text{in}}}\right)$ of the transformers of Group 1 is approximately $49.8 \pm 3.4 \%$ higher than that of the transformers of Group 2.

The average phase difference of Group 1 is about twelve times larger than the average phase difference of Group 2. While in general it is desirable for $\phi(f)$ to be as low as possible - suggesting that Group 2 is favorable in this regard - the most critical factor is uniformity. As long as all eight waveforms are shifted by similar amounts, the channels still experience nearly equal circumstances. Therefore, despite Group 2 being favorable in this regard, the phase differences of Group 1 are acceptable as the relative error is only 0.5%. This means that the phase differences of the transformers of Group 1 are of similar magnitude, which should therefore not introduce difficulties in the ideal waveform generation process.

Transformer E, originally classified in Group 2, began to behave like a Group 1 transformer after it was repaired. This indicates that the behavior of Group 1 likely reflects the correct operating conditions for the transformers used in the NL-eEDM experiment. Using transformers with loose or damaged windings that produce sparks is not a viable long-term solution and may compromise reliability.

In conclusion, Group 1 exhibits superior gain performance and build quality, while Group 2 shows only a slight advantage in phase difference. This makes Group 1 the preferred choice for the NL-eEDM experiment. Currently, eight transformers - A, B, C, D, E, G, X, and Y - exhibit Group 1 behavior, which is precisely the number required. These should therefore be selected for use.

To safely achieve higher voltages, all transformers must be sealed; at present, only transformer X meets this condition. Further details on transformer sealing are provided in Section 4.3.3. Transformers F and H should be disassembled and inspected to determine whether they can be repaired to exhibit Group 1 characteristics. If successful, follow-up Bode measurements - similar to those shown in Figure 20 - can be created. This would allow for the selection of the eight transformers most alike in gain, phase and shape characteristics out of the ten available transformers, enabling the most optimal configuration given the available components.

7.1.2 The effect of capacitive coupling

Investigating the effect of capacitive coupling is important, as during actual measurements all eight channels will be simultaneously active, making capacitive effects - as discussed earlier in Section 2.5 - relevant. Figure 21 shows the results of these measurements.

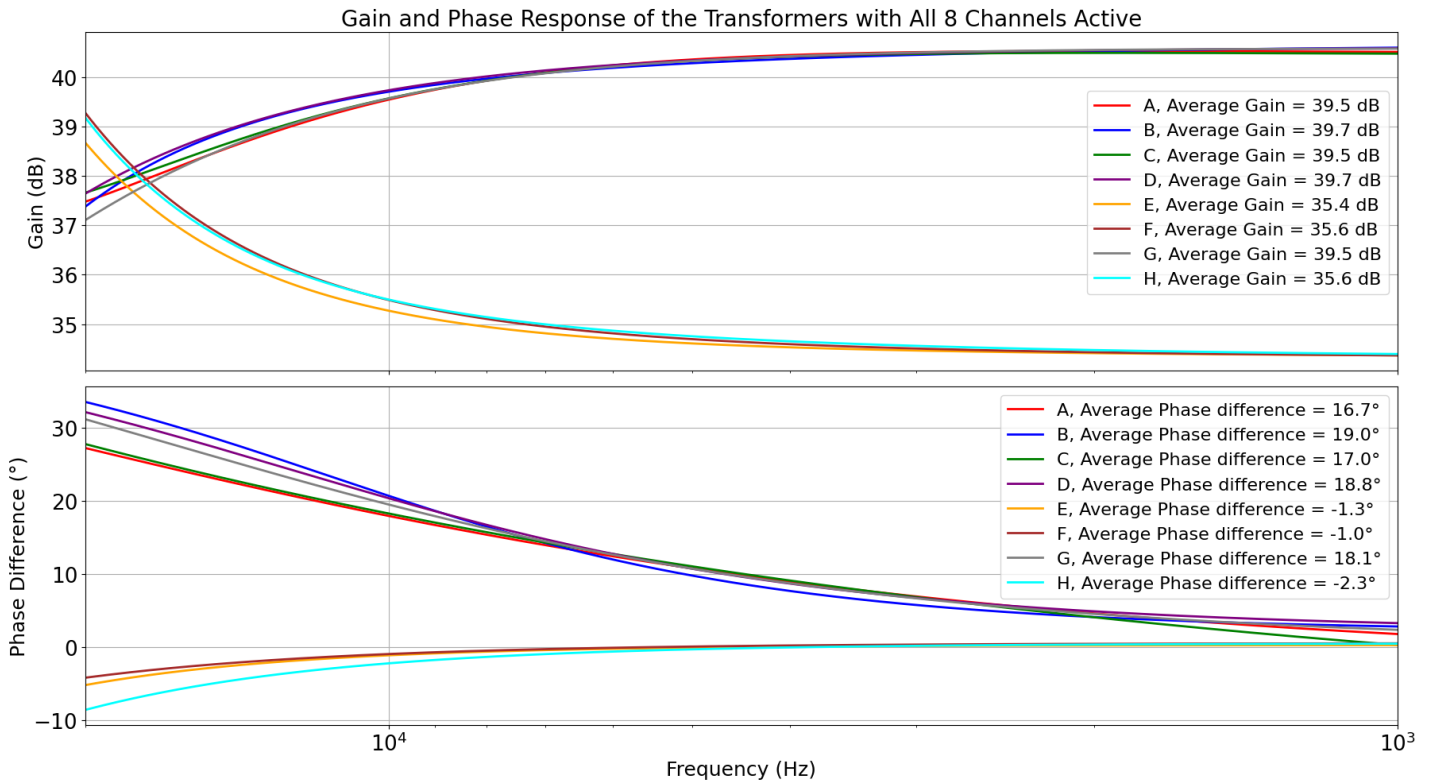


Figure 21: *Bode plots of transformers A, B, C, D, E, F, G, and H when all eight channels are simultaneously active. A constant resistance of $2.2\ \Omega$ was used for all channels. The data of transformer E used is the data before repair, hence explaining the 'Group 2' behavior. Apart from all channels now being simultaneously active, the methodology stays the same as before. Transformers X and Y were omitted from this measurement, as only eight channels can be simultaneously connected during operation, corresponding to the configuration used in the actual experiment.*

When comparing Figure 21 with the individual transformer testing shown in Figure 17, it can be seen that the effect of capacitive coupling is rather insignificant: both the shape and magnitude of the gain and phase difference curves remain largely unchanged. The differences are summarized in Table 2.

Condition	Average gain	Average phase difference
Group 1, no coupling	39.3 ± 0.0 dB	$20.7 \pm 0.1^\circ$
Group 1, with coupling	39.6 ± 0.0 dB	$17.9 \pm 0.5^\circ$
Group 2, no coupling	35.7 ± 0.0 dB	$-1.7 \pm 0.0^\circ$
Group 2, with coupling	35.5 ± 0.1 dB	$-1.5 \pm 0.4^\circ$

Table 2: *Comparison of Group 1 and Group 2 transformer behavior with and without capacitive coupling.* Values for average gain and phase difference are based on measurements from Figures 17 (no coupling) and 21 (with coupling). All data were obtained with a 2.2Ω resistor in the circuit. Group 1 includes transformers A, B, C, D, and G; Group 2 includes transformers E, F, and H.

Table 2 confirms that the effect of capacitive coupling on the gain and phase difference behavior of the transformers is minor. However, an increased spread in the phase difference is observed when capacitive coupling is present, indicating that the channels behave less uniformly. The relative errors for Group 1 and Group 2 are 2.8% and 26.7% respectively. This means that the relative spread in phase difference of especially Group 2 is rather large. While this could make it more challenging for the feedback system to achieve the ideal phase differences, it is not expected to be problematic as it was already concluded before that only Group 1 transformers should be used.

Although the measurements presented in this section suggest that capacitive coupling has a limited effect on gain and phase behavior, practical experience - discussed in Section 2.5 - indicates that capacitive effects significantly hinder the generation of ideal waveforms. This discrepancy may be partially attributed to the increased spread in phase differences observed when coupling is present, but it does not fully account for the pronounced distortions encountered in practice. It is therefore likely that the Bode plot methodology used here is not sufficiently capture the full impact of capacitive coupling. Capacitive leakage voltages cause by the capacitive coupling between channels might appear at both ends of the transformer, which causes it to cancel out in the Bode plot representation as the ratio $\frac{V_{out}}{V_{in}}$ is used. To better understand the influence of capacitive effects, a revised experimental approach could be proposed, specifically one that directly visualizes absolute distortions.

7.2 Amplifiers

7.2.1 Gain and phase difference behavior

The gain and phase difference behavior of the amplifiers was characterized by creating Bode plots, the results are shown in Figure 22. The shapes of the gain and phase difference curves as functions of frequency are similar across all amplifiers, indicating similar frequency responses. The phase differences converge to zero degrees as the frequency decreases, which is expected behavior. It can also be observed that amplifiers B and D exhibit a significantly higher gain compared to the others. This can be explained by the fact that these two amplifiers were previously repaired, possibly causing improved performance. Further details about the repairs will be discussed later.

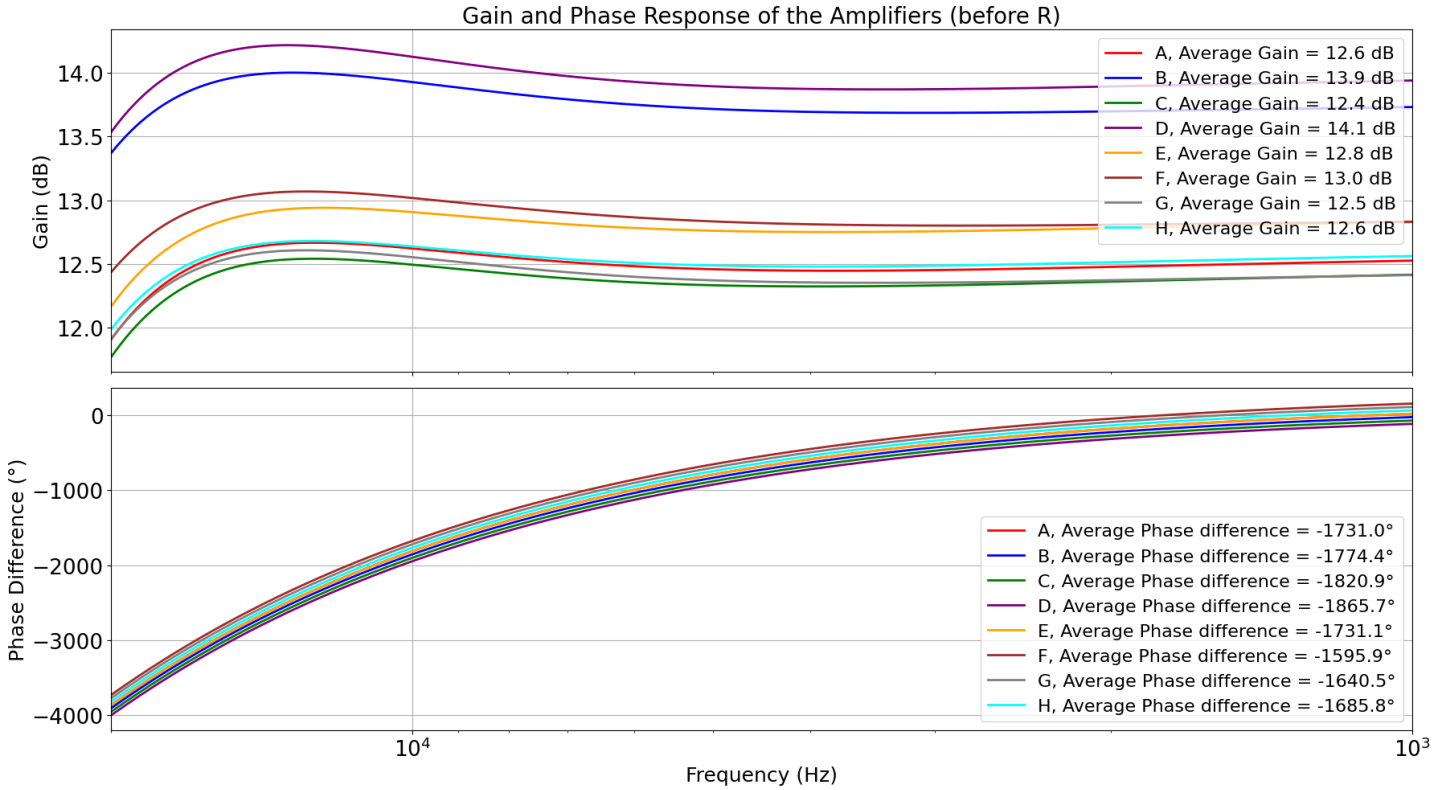


Figure 22: *Individual Bode plots of the amplifiers.* The output voltage of the amplifiers was measured before passing through the resistor. Bode plots with output voltage after the resistor were also created, this can be found in Figure A7. A constant resistance of 2.2Ω was used across all channels.

The average amplifier gain was found to be 13.0 ± 0.2 dB, which is significantly lower than that of the transformers. This is consistent with expectations, as the primary function of the amplifiers is to increase current rather than voltage. The average phase difference was measured to be $-1730.7 \pm 31.8^\circ$, much larger than that observed for the transformers. This is also in line with expectations, as the amplifiers are known to introduce a substantial time delay of approximately $610 \mu\text{s}$, a more elaborate table with amplifier delays can be found in Table A2.

The relative errors in the average gain and phase difference were calculated to be 1.5% and 1.8%, respectively, indicating that the spread in these quantities across different amplifiers is low. Combined with the similarity in the overall frequency response, it can be concluded that all amplifiers behave consistently, and no significant issues are expected.

7.2.2 Peak-to-peak voltage limit

The peak-to-peak output voltage limits of the channels were measured with the amplifiers in bridge mode, the results are shown in Table 3.

Amplifier	V_{out} (± 1.0 kV)
A	18.9
B	26.5
C	18.5
D	20.1
E	18.9
F	20.9
G	19.3
H	20.5

Table 3: **Peak-to-peak output voltage limits in kV of the channels with the amplifiers in bridge mode.** A more detailed version of this table is given in Table A1.

Examining the lower bounds of the uncertainty ranges, it is observed that only amplifier B can safely reach the ideal peak-to-peak voltage of 20 kV. For the second-best performing amplifier in this test, amplifier F, a peak-to-peak voltage of 20.9 ± 1.0 kV was measured, implying a lower bound of 19.9 kV. However, in practice, one would avoid operating near the lower bound to ensure stability and prevent damage during actual eEDM measurements. These tests were conducted over only a few seconds; sustained operation of the amplifiers near the voltage limits could still result in damage over extended periods of time. As such, the practical operating limit is below 20 kV, meaning that only amplifier B can comfortably and safely reach the ideal voltage.

The significantly better performance of amplifier B can be explained by a recent repair: after a capacitor failure during earlier measurements, the original capacitor (Firma LSG, 3300 μF , 100 V) was replaced by a Vishay 3300 μF , 100 V capacitor. The new capacitor has an internal equivalent series resistance (ESR) of approximately 64 m Ω at 100 Hz, which is likely lower than that of the original component. This reduction in ESR improves voltage stability under load and likely explains the significant improvement in output performance observed for amplifier B. ESR is explained more in Section 4.4.

It is recommended that the other amplifiers undergo a similar capacitor replacement in order to reliably achieve the ideal peak-to-peak voltage of 20 kV with sufficient safety margin.

8 Conclusion & Outlook

To draw clear conclusions to this thesis, the four research questions, as stated in Section 5, will now be explicitly answered.

How do frequency-dependent gain and phase responses of the transformers and amplifiers differ, and how can these differences be minimized to ensure consistent waveform generation in the traveling-wave Stark decelerator?

Section 7.1.1 showed that transformers A, B, C, D, E (post-repair), G, X, and Y displayed the highest gain and were similar in behavior. Therefore, using these eight transformers would be ideal at this time. Transformers F and H likely have damaged coils and should be inspected and possibly repaired, like was done with transformer E. If so, then the behavior can be tested and the best possible combination of eight out of the ten transformers can be selected. Section 7.2.1 showed that the amplifiers showed consistent gain and phase characteristics, with minor variations between units. This result is largely consistent with the hypothesis presented in Section 5. While significant deviations between transformers were not specifically anticipated, the possibility of such deviations arising from defective components was acknowledged. Therefore, the hypothesis is accepted.

What peak-to-peak voltages can be reached in the channels of the traveling-wave Stark decelerator after setting the amplifiers to bridge mode, and how could the voltage output be increased even further?

Section 7.2.2 showed that when operated in bridge mode, only channel B (with amplifier B) was able to reliably reach the ideal peak-to-peak voltage of 20 kV. The superior performance of amplifier B was attributed to a capacitor replacement that likely reduced the equivalent series resistance, thereby improving current handling and voltage stability during switching. To increase output voltage across all amplifiers, it is recommended that similar capacitor upgrades be applied to the remaining units (A, C, D, E, F, G, H). Alternatively, more powerful amplifiers such as the Behringer NX6000D could be considered, offering higher voltage output while retaining compatibility with the current setup. This result is largely consistent with the hypothesis presented in Section 5. Although not all channels can reach the ideal peak-to-peak voltage of 20 kV at this time, channel B has proven that it is possible with the Behringer NX3000D in bridge mode. Therefore, the hypothesis is accepted.

What is the effect of capacitive coupling between channels on the gain and phase difference behavior of the transformers?

Section 7.1.2 showed that, aside from an increased spread in phase differences across channels, capacitive coupling has only a minor effect on the frequency response of the transformers. However, this does not imply that capacitive coupling is negligible overall, as it has been demonstrated in practice to significantly hinder the generation of ideal waveforms (see Section 2.5). This discrepancy suggests that the current measurement method does not fully capture the influence of capacitive coupling, rendering the results somewhat inconclusive. Since the findings of Section 7.1.2 contradict the initial hypothesis presented in Section 5, the hypothesis is rejected.

What is the safe upper limit for the peak-to-peak voltage in the channels of the traveling-wave Stark decelerator, and how does this compare to the ideal peak-to-peak voltage of 20 kV ?

The components that must withstand the ideal peak-to-peak voltage of 20 kV include the transformers, voltage dividers, and the decelerator. Section 4.2 showed that the theoretical safe upper limit for the peak-to-peak voltage in the decelerator is around 20.9 kV , but so far this has not yet been tested properly in practice. In Section 7.2.2 it has been proven that shielded transformer X is able to withstand voltages up to at least 26.5 kV . Section 3.1.1 showed that the limit of the present voltage dividers is about 8.4 kV , but the new voltage divider design - visualized in Figure A9 - has a theoretical limit of 25.2 kV . This means that in the theory, the ideal peak-to-peak voltage of 20 kV can safely be reached in the channels of the decelerator as long as the transformers are shielded and the new voltage divider design is used. While this has been confirmed in practice for the transformers, the decelerator and new voltage dividers still require practical validation. These findings support the hypothesis presented in Section 5, which is therefore accepted.

Outlook

- As was done for the transformers and amplifiers, the remaining hardware components - such as voltage dividers and resistors - should also be systematically tested to identify and mitigate any anomalies that could impact overall system performance.
- Transformers F and H should be disassembled and inspected for mechanical defects, such as damaged or loose coils. If issues similar to those previously found in transformer E are identified, they may be repairable and could potentially be restored to proper functioning.
- Amplifiers A, C, D, E, F, G, and H should be disassembled and inspected, with particular attention to replacing internal capacitors. This approach has already proven effective in improving the performance of amplifier B.
- A new experimental method to investigate the effect of capacitive coupling on the channels should be developed, as creating Bode plots yielded an inconclusive result.
- The safe voltage limits of both the decelerator and the newly designed voltage dividers should be thoroughly validated through experimental testing, in order to determine whether the ideal peak-to-peak voltage of 20 kV can safely be reached in practice.

Acknowledgments

Firstly, I would like to express my gratitude to Steven Hoekstra for welcoming me into the team and for his supervision and feedback throughout the course of my thesis. I would also like to thank Lucas van Sloten for his valuable guidance and daily support in the lab throughout this project. His dedication and expertise played a crucial role in the progress of this work. The feedback system we initiated together with Marianne has since become his full-time project, and I am confident it will result in excellent outcomes upon its completion.

I am also grateful to Marianne Westerhof for the collaboration at the start of the project, when we worked together with Lucas on the feedback system and the decelerator electronics. Working as a team during the early stages contributed greatly to both the efficiency and enjoyment of the work. I would like to thank Leo Huisman for his technical assistance in the lab, and for providing the visualizations of the decelerator, modules, and more.

It has been an honor to be part of the NL-eEDM collaboration. I sincerely thank all members of the team for their support and for providing an inspiring research environment.

NL-eEDM collaboration

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A Appendix

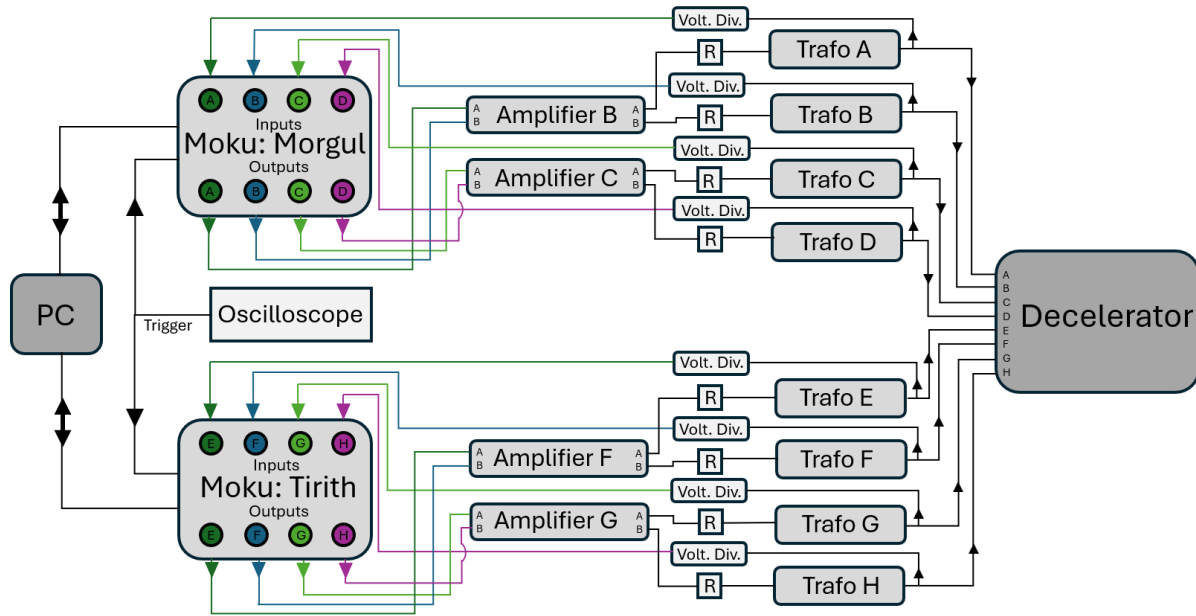


Figure A1: *Schematic overview of the old decelerator setup with four amplifiers. This figure describes what the decelerator setup was like at the start of the project described in this paper, when there were only four amplifiers without bridge mode.*

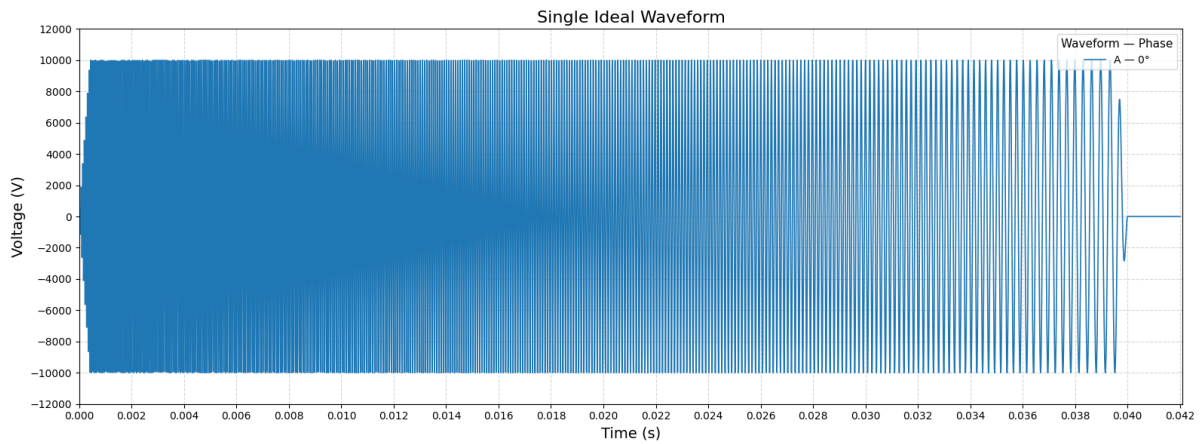
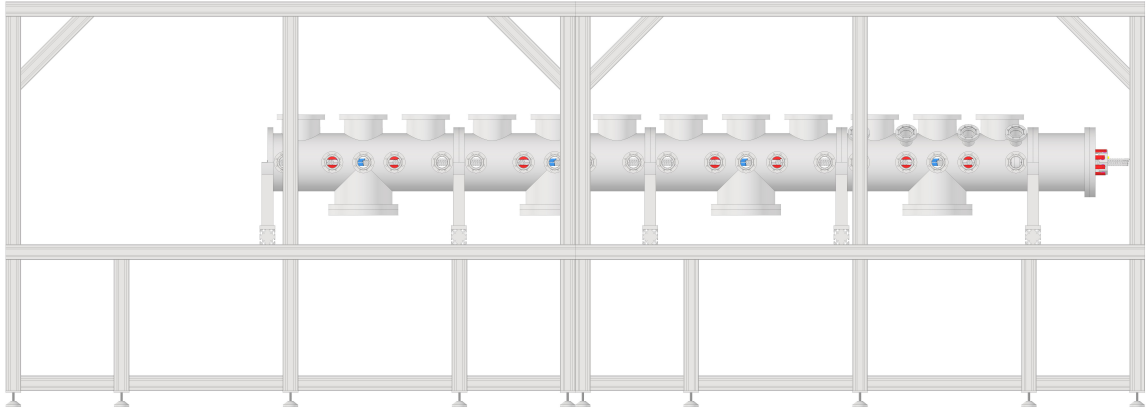
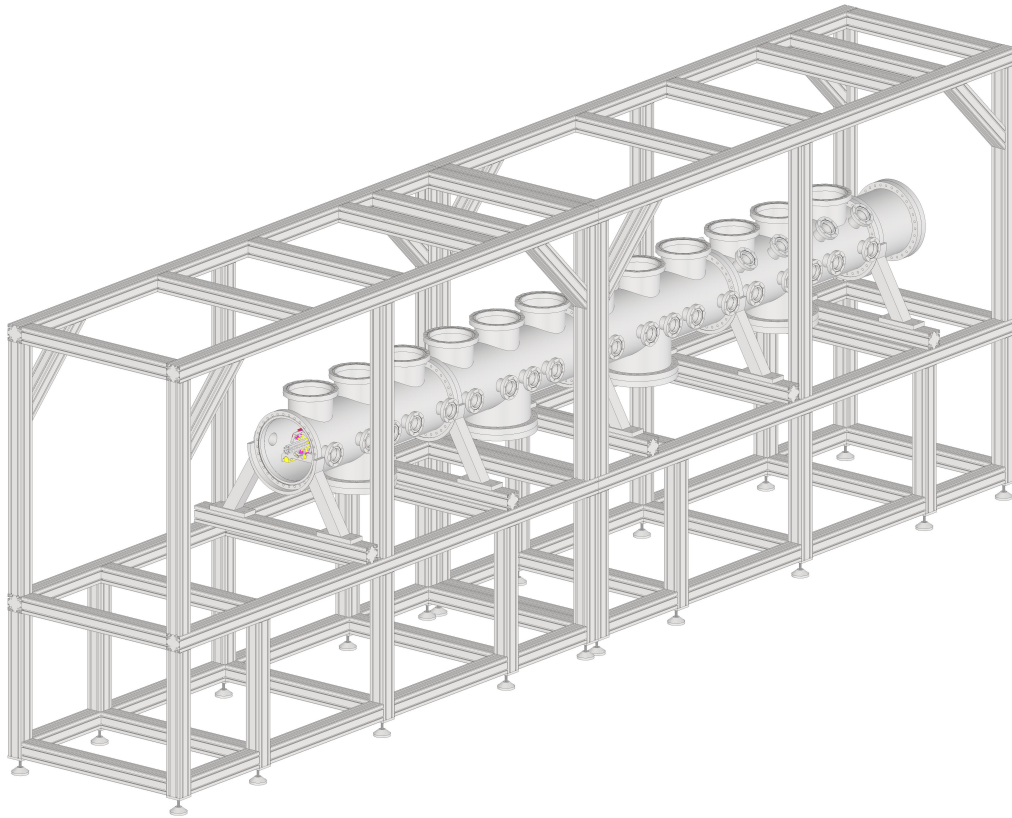


Figure A2: *Visualization of the ideal waveform. This plot functions as an addition to figure 6a to show the shape of a single waveform along the entire pulse length of 40ms.*



(a) Side view of the decelerator. In this figure the left side is the front of the decelerator and the right side is the back.



(b) Isometric view of the decelerator. In this figure the left side is the front of the decelerator and the right side is the back.

Figure A3: **More decelerator visualizations.** Two different views of the decelerator are shown: an isometric view (a) and a side view (b). Figure courtesy of ing. Leo Huisman.



Figure A4: ***Decelerator visualizations.*** Two different views of the decelerator are shown: a back view (a) and a front view (b). Figure courtesy of Leo Huisman.

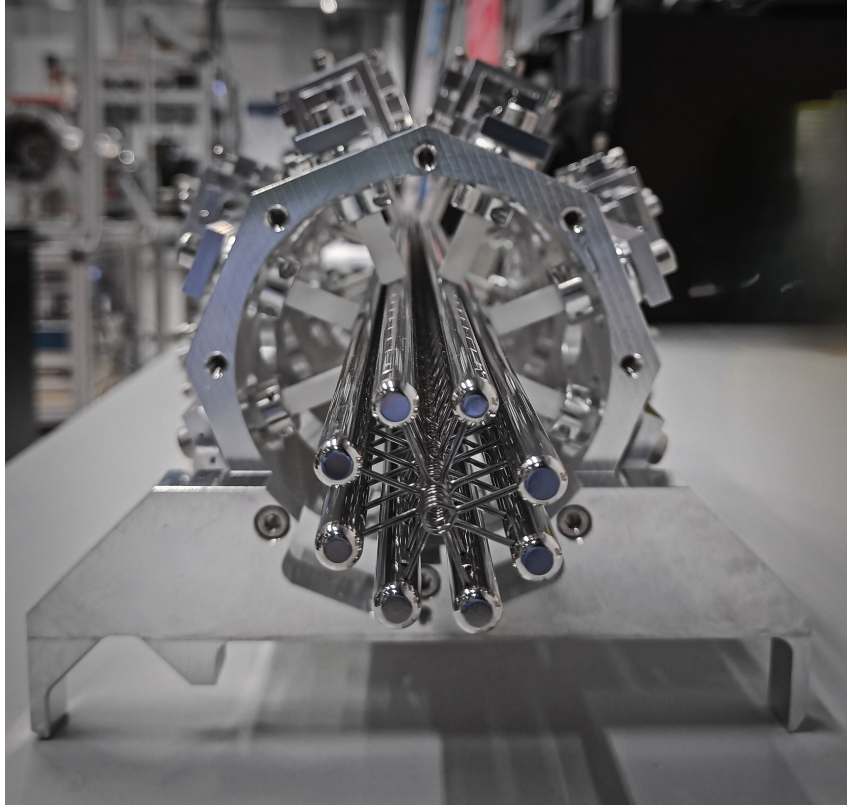


Figure A5: *Photo of one of the modules.* This gives a realistic idea of what the module looks like in terms of colors. This view also clearly shows the structure of the rings.

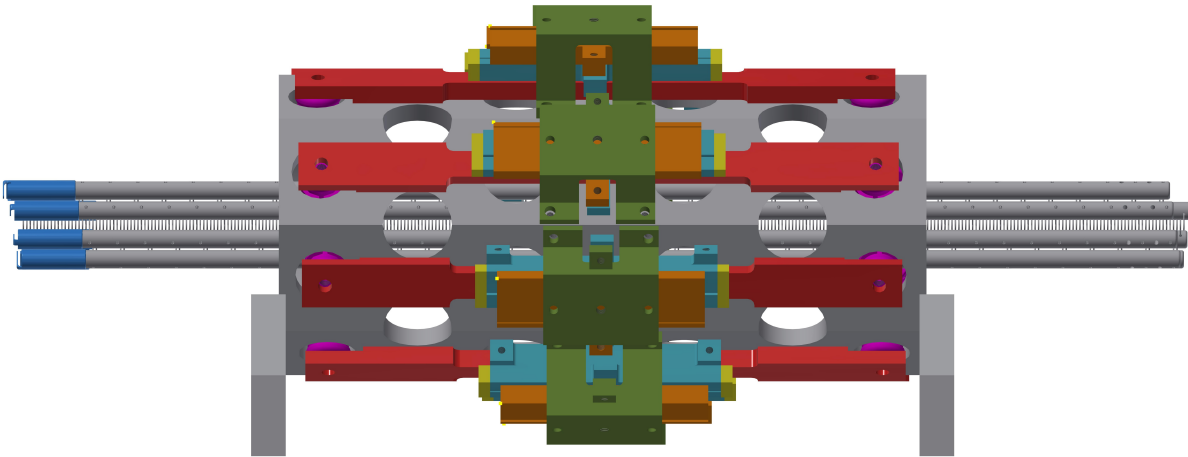


Figure A6: *Side view of the module.* Figure courtesy of ing. Leo Huisman.

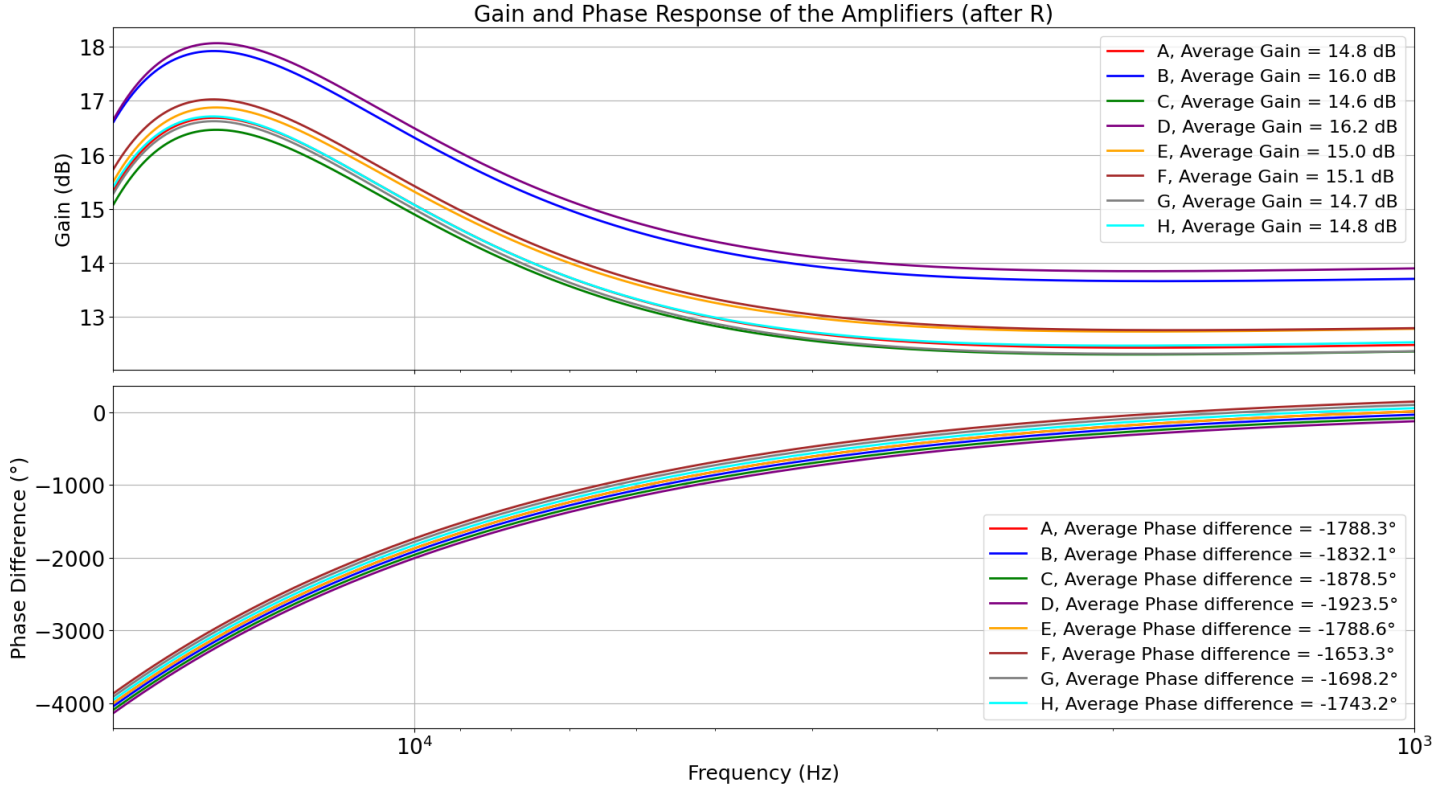


Figure A7: *Individual Bode plots of the amplifiers.* The output voltage of the amplifiers was measured after passing through the resistor. A constant resistance of 2.2Ω was used across all channels.

Amp	I_{out} (± 1 A)	$V_{out,amp}$ (± 1.0 kV)	$V_{out,trafo,before}$ (± 1.0 V)	$V_{out,trafo,after}$ (± 1.0 V)
A	64.0	18.9	74.0	116
B	88.0	26.5	129.0	163
C	64.0	18.5	72.8	113
D	68.0	20.1	81.0	123
E	64.0	18.9	74.8	115
F	70.0	20.9	85.0	128
G	64.0	19.3	74.8	117
H	68.0	20.5	82.0	125

Table A1: **Peak-to-peak output voltage limits in kV of the amplifiers.** shows a more detailed version of this table.

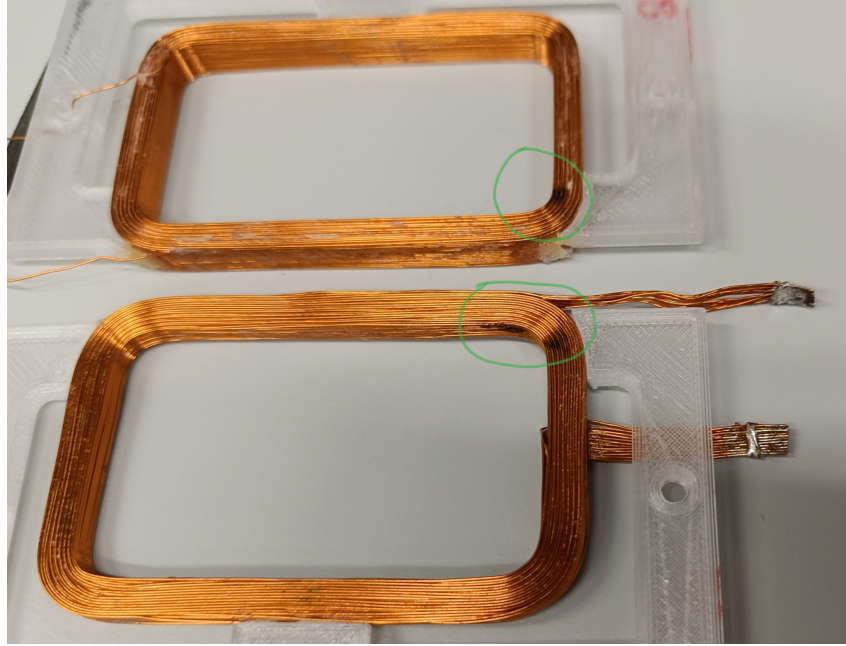


Figure A8: *Visualization of the sparking coils of transformer E. Black spots from burning as a result from the sparking can be observed, these spots are encircled in green.*

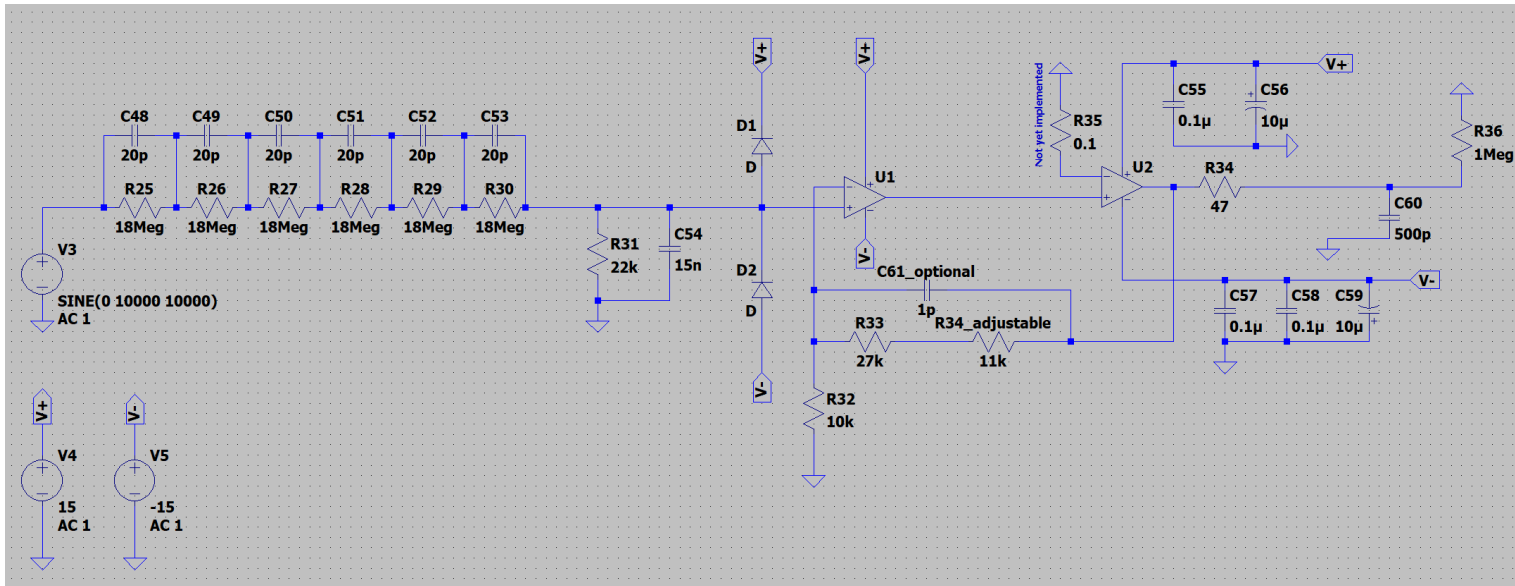


Figure A9: *Circuit for the newly designed voltage divider. This new design is expected to be able to withstand about three times higher maximum voltages than the old voltage divider design, given in Figure 9. This is because the new design has six C-R components (shown as C48-R25, C49-R26, etc), while the old design had only two of these components. If the new design uses the same type of resistors and capacitors, then theoretically the new design should be able to withstand three times higher voltages than the old design, which results in 25.2 kV. This is well above the ideal peak-to-peak voltage of 20 kV.*

Amplifier	Delay (μs)
A	610
B	608
C	609
D	612
E	610
F	610
G	610
H	612

Table A2: Measured amplifier delays. All delays are expressed in microseconds (μs).