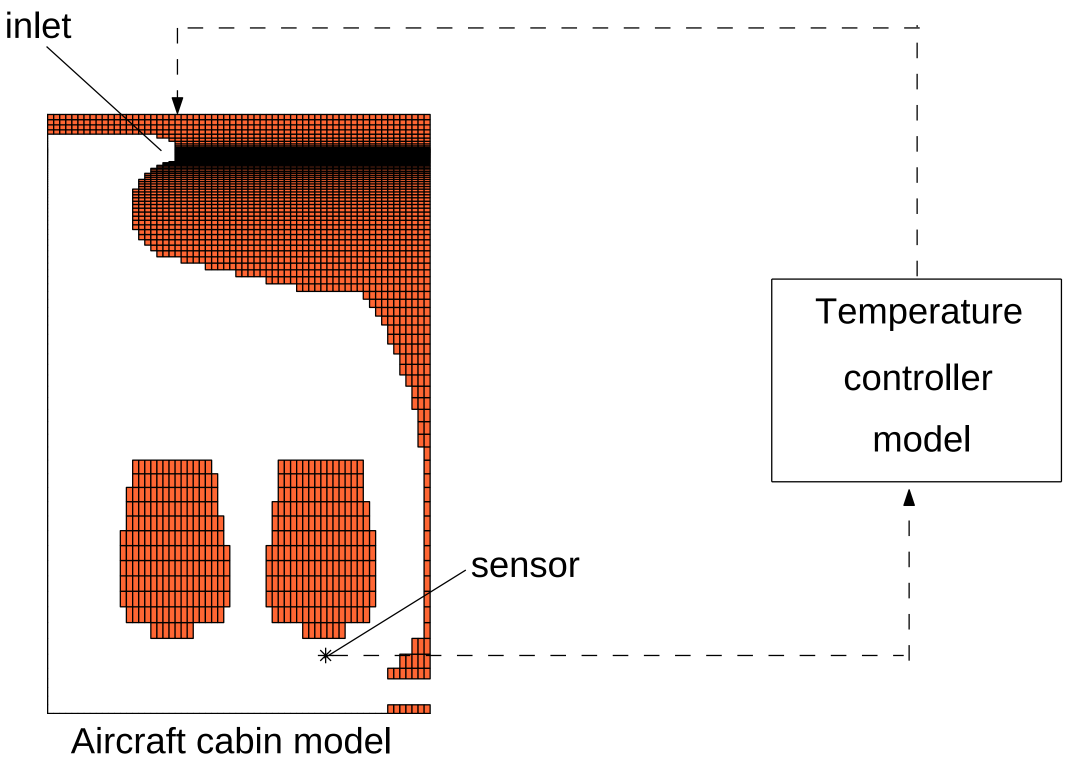
Coupling of computer models for aircraft cabin and temperature controller

Albert Kremer



Department of Mathematics

RuG

Master's thesis

Coupling of computer models for aircraft cabin and temperature controller

Albert Kremer

Supervisor:
Prof.dr. A.E.P. Veldman
Department of Mathematics
University of Groningen
P.O. Box 800
9700 AV Groningen

August 2002

Contents

Nomenclature

List of abbreviations

Abbreviation	Description
ASICA	Air management SImulation for aircraft CAbins
CFD	Computational Fluid Dynamics
HEAT	HEAT97-cv, CFD code for incompressible Navier-Stokes equations
NLR	National Aerospace Laboratory NLR
RuG	Rijksuniversiteit Groningen

List of symbols

Symbols	Description	Units
c_p	specific heat at constant pressure	$m^2\, s^{-2}\, K^{-1}$
d_{nh}	width of the inlets	m
d_{out}	width of the outlets	m
k_p	proportionality parameter of the PID-controller	-
k_i	integral parameter of the PID-controller	s^{-1}
k_d	derivative parameter of the PID-controller	s
L	length of the cabin	m
M_a	total mass of the air in the cabin	kg
$\dot{m}_i$	massflow through the nozzles	$kg\, s^{-1}$
$\dot{m}_o$	massflow through the outlets	$kg\, s^{-1}$
P_c	cabin pressure	Pa
R	gas constant	$J\, kg^{-1}\, K^{-1}$
t	time	s
T_c	cabin temperature	K
T_{nh}	inflow/nozzle temperature	K
ν	kinematic viscosity	$m^2\, s^{-1}$
ρ	density	$kg\, m^{-3}$

Chapter 1

Introduction

The temperature of the air in an aircraft cabin is important for a comfortable climate for passengers. An Environmental Control System (ECS) is used to control the temperature in the cabin. Both the air in the cabin and the temperature controller are modeled separately. To get a useful impression of the dynamics of the complete system, a coupling between the cabin model and the controller model is necessary.
For the aircraft cabin and the temperature controller different models exist. In Section 2 simple mathematical models for the cabin and the controller are presented. Numerical models are obtained by discretization of the simple models. These numerical models are shown in Section 3. The accuracy and the convergence of these models are investigated. The results of this investigation are presented in Section 4. Instead of the simple cabin model (Thermal Cabin Model), the CFD code HEAT97-cv (HEAT) is coupled to the controller model. In Section 5 the results of the coupling of the Thermal Cabin Model with the controller are compared to the results of the coupling of HEAT with the controller. An investigation of the accuracy of the coupling of HEAT with the controller is done in Section 6. Section 7 contains the conclusions and the recommendations.

Chapter 2

Simplified models for flow solver and controller

Two computer models are used to simulate the cabin temperature and the control system of an aircraft. The temperature in a cabin is simulated with a CFD code, HEAT. The control system of an aircraft cabin is simulated with a complicated controller model (ECS). Coupling of these complicated models is necessary to say anything about the dynamics of the whole system. To conclude anything about the coupling of the flow solver HEAT and the ECS model, two simplified models have been created. The coupling between these simplified models has been studied.

2.1 Thermal Cabin Model

Modeling the temperature of a cabin the law of conservation of heat is used. The heating of the air in the cabin equals the heat coming in minus the heat that flows out:

$$Q_{cabin} = Q_{inlet} - Q_{outlet} \tag{2.1}$$

The terms in the equation are defined as:

$Q_{cabin} = M_a c_p \frac{dT_c}{dt}$, rate of change of heat in the cabin,

$Q_{inlet} = \dot{m}_i c_p T_{nh}$, heat of the incoming air,

$Q_{outlet} = \dot{m}_o c_p T_c$, heat of the out flowing air.
This results in the equation of the Thermal Cabin Model:

$$M_a \frac{dT_c}{dt} = \dot{m}_i T_{nh} - \dot{m}_o T_c \tag{2.2}$$

The temperature calculated with this equation is independent of the place. So the geometry of the cabin has no influence on the temperature. An assumption is made to the outflow temperature. This temperature is chosen the same as the average temperature of the cabin. The fluid in the cabin is incompressible so that $\dot{m}_i = \dot{m}_o$.

2.2 PID-controller

The other part of the coupled system is a PID-controller. This controller calculates the output as follows:

$$y = k_p u + k_i \int_0^t u \, \mathrm{d}\tau + k_d \frac{\partial u}{\partial t} \tag{2.3}$$

The input u is a temperature difference, namely the reference temperature minus the temperature of the cabin: $T_{ref} - T_c$. The reference temperature (T_{ref}) can be taken zero. So the input of the controller is $-T_c$. The output of the controller plus the reference temperature is the inflow temperature of the cabin, T_{nh}. And so equation 2.3 results in:

$$T_{nh} = -k_p T_c - k_i \int_0^t T_c \, \mathrm{d}\tau - k_d \frac{dT_c}{dt} \tag{2.4}$$

2.3 Coupling of the Thermal Cabin Model and the PID-controller

An investigation of the coupling between these two models has been done. Combining Equations 2.2 and 2.4 results in the coupled model:

$$\frac{\dot{m}_o}{\dot{m}_i} T_c + \frac{M_a}{\dot{m}_i} \frac{dT_c}{dt} = -k_p T_c - k_i \int_0^t T_c \, \mathrm{d}\tau - k_d \frac{dT_c}{dt} \tag{2.5}$$

It is possible to solve this equation analytically. First equation 2.5 is differentiated:

$$\frac{\dot{m}_o}{\dot{m}_i} \frac{dT_c}{dt} + \frac{M_a}{\dot{m}_i} \frac{d^2 T_c}{dt^2} = -k_p \frac{dT_c}{dt} - k_i T_c - k_d \frac{d^2 T_c}{dt^2} \tag{2.6}$$

The solution of Equation 2.6 can be written as:

$$T_c(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \tag{2.7}$$

$$\lambda_{1,2} = \frac{-(\frac{\dot{m}_o}{\dot{m}_i} + k_p) \pm \sqrt{(\frac{\dot{m}_o}{\dot{m}_i} + k_p)^2 - 4(\frac{M_a}{\dot{m}_i} + k_d) k_i}}{2(\frac{M_a}{\dot{m}_i} + k_d)} \tag{2.8}$$

If $\Re(\lambda_{1,2}) < 0$, the physical solution is damped. Because $M_a, \dot{m}_o, \dot{m}_i > 0$, and, $k_p, k_i, k_d \geq 0$ the physical solution is damped always. The values of c_1 and c_2 are dependent on the initial condition as follows:

$$c_1 = \frac{((k_p - 1) + \frac{\dot{m}_o}{\dot{m}_i} + \frac{M_a}{\dot{m}_i} \lambda_2)(T_{nh}(0) - T_c(0))}{\frac{M_a}{\dot{m}_i}(\lambda_1 - \lambda_2)}$$

$$c_2 = -c_1 + T_c(0)$$

The solution of the inflow temperature ($T_{nh}(t)$) can be determined by Equation 2.2.

Chapter 3

Numerical model of the coupling of the Thermal Cabin Model and the PID-controller

This section describes some numerical models of the Thermal Cabin Model and the PID-controller. To discretize the equations given in Section 2, different methods are possible. Some of these methods are investigated and described in this section. The differences between the described models are small.
To discretize the model, the Euler method is used. This is a single step method:

$$\left(\frac{\partial \phi}{\partial t}\right)^n = \frac{\phi^n - \phi^{n-1}}{\Delta t}$$

Method A
The Thermal Cabin Model, given by Equation 2.2 is discretized as follows:

$$M_a \frac{T_c^n - T_c^{n-1}}{\Delta t} = \dot{m}_i T_{nh}^n - \dot{m}_o T_c^n \tag{3.1}$$

The mathematical equation of the PID-controller, Equation 2.4, contains an integral. This integral is calculated as a function of the cabin temperature and the previous value of the integral. Because the previous value of the integral is not the previous value of the inlet temperature (T_{nh}^{n-1}), a new variable is introduced: $I(t) = -\int_0^t k_i T_c \, \mathrm{d}\tau$. The time derivative is given by:

$$\frac{\partial I}{\partial t} = -k_i T_c \tag{3.2}$$

This equation is also discretized with the Euler method:

$$\frac{I^n - I^{n-1}}{\Delta t} = -k_i T_c^n \tag{3.3}$$

In the following applications $k_d = 0s$. So the total discretized equation of the PID-controller is:

$$T_{nh}^n = -k_p T_c^n + I^n \tag{3.4}$$

In this equation T_{nh}, T_c, and I are given at $t = t^n$. This equation is used to compute the inlet temperature afterwards, because T_{nh} is not a parameter of the iteration process. Equation 3.4 is unchanged in the different methods. The convergence of the results of the numerical model to the physical solution will be investigated. The convergence of I and T_c is sufficient to get convergence of T_{nh}. Also the accuracy of T_{nh} depends on the accuracy of I and T_c. So it is possible to eliminate the parameter T_{nh}. The convergence and the accuracy of the coupled system have been unchanged.
The found method is an implicit method. Combining Equations 3.1, 3.3 and 3.4 the following system is found:

$$\begin{pmatrix} 1 & k_i\Delta t \\ -\Delta t\frac{\dot{m}_i}{M_a} & 1 + k_p\Delta t\frac{\dot{m}_i}{M_a} + \Delta t\frac{\dot{m}_o}{M_a} \end{pmatrix} \begin{pmatrix} I \\ T_c \end{pmatrix}^n = \begin{pmatrix} I \\ T_c \end{pmatrix}^{n-1} \tag{3.5}$$

Solving I^n, T_c^n from I^{n-1}, T_c^{n-1} requires inverting the matrix in Equation 3.5. This inverted matrix is called the iteration matrix:

$$A = \begin{pmatrix} 1 & k_i\Delta t \\ -\Delta t\frac{\dot{m}_i}{M_a} & 1 + k_p\Delta t\frac{\dot{m}_i}{M_a} + \Delta t\frac{\dot{m}_o}{M_a} \end{pmatrix}^{-1}$$

The spectral radius of the iteration matrix A has to be less than one to get convergence **[5]**. The convergence and the accuracy of this system is described in the next section.

Method B

To solve the equations without inverting a matrix, a choice is made of the value at the new time step, t^n.
In the right hand sides of Equations 3.1 and 3.3, T_c^n is changed into T_c^{n-1} and T_{nh}^n is changed into T_{nh}^{n-1}. This yields the explicit expression:

$$M_a\frac{T_c^n - T_c^{n-1}}{\Delta t} = \dot{m}_i T_{nh}^{n-1} - \dot{m}_o T_c^{n-1} \tag{3.6}$$

In this case the value of T_{nh} is calculated as in Equation 3.4:

$$T_{nh}^{n-1} = -k_p T_c^{n-1} + I^{n-1} \tag{3.7}$$

$$\frac{I^n - I^{n-1}}{\Delta t} = -k_i T_c^{n-1} \tag{3.8}$$

The Equations 3.6 and 3.8 can be written as a system with an iteration matrix B:

$$\begin{pmatrix} I \\ T_c \end{pmatrix}^n = B \begin{pmatrix} I \\ T_c \end{pmatrix}^{n-1}$$

$$B = \begin{pmatrix} 1 & -k_i\Delta t \\ \Delta t\frac{\dot{m}_i}{M_a} & 1 - \Delta t\frac{\dot{m}_i}{M_a}k_p - \Delta t\frac{\dot{m}_o}{M_a} \end{pmatrix}$$

Method C

Another way to solve the system is a combination of method A en B. The cabin temperature is calculated as in method A (Equation 3.1). To calculate the value of the integral, Equation

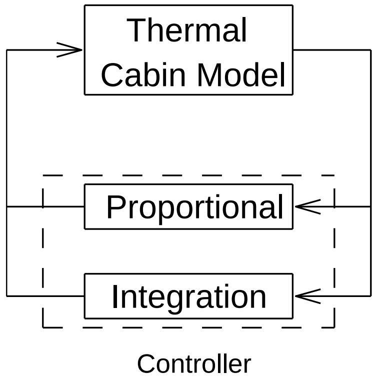


Figure 3.1: Schematic representation of the coupling in Simulink.

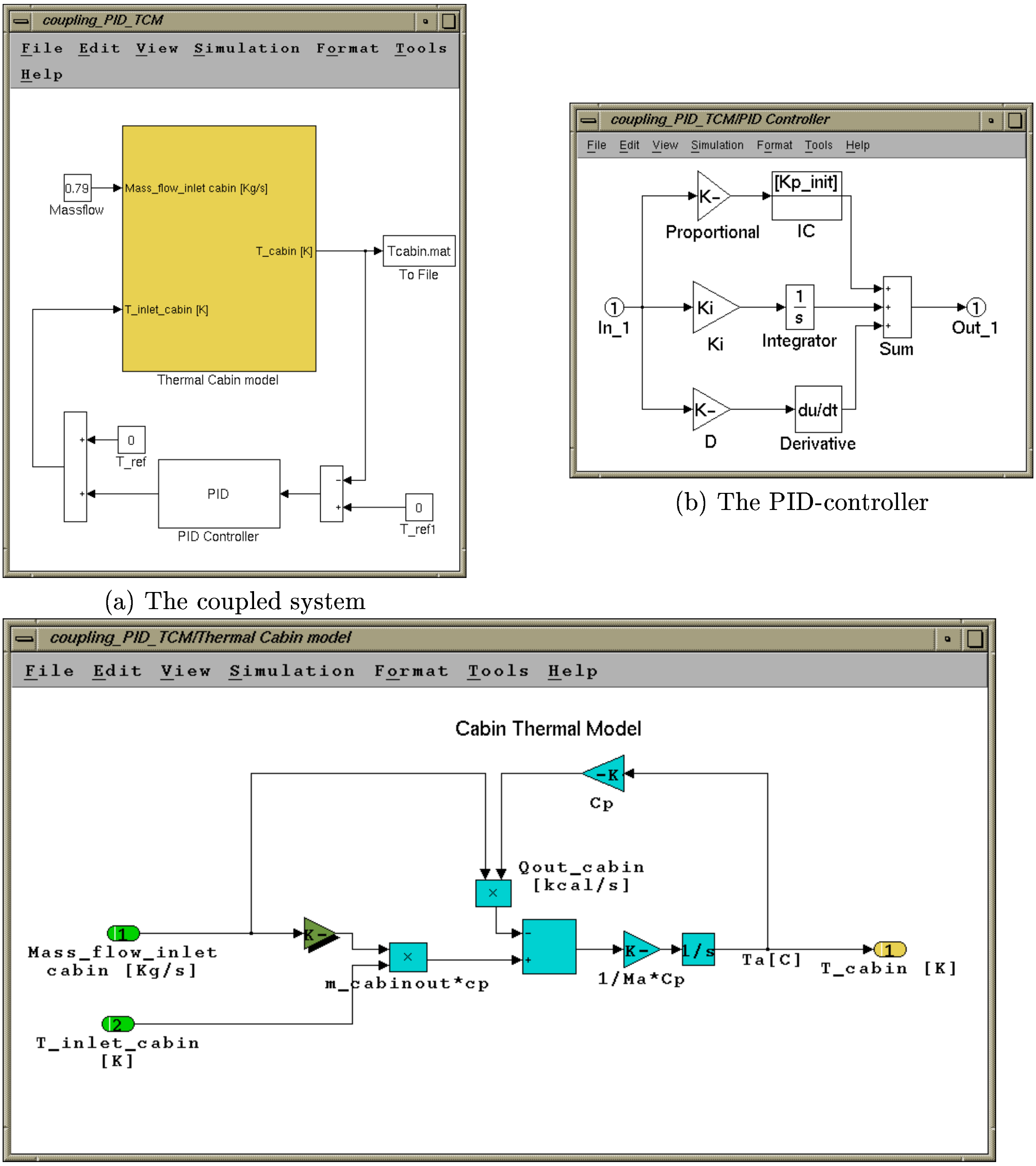
3.8 of method B is used. The iteration matrix is found similar to the other methods:

$$\begin{pmatrix} I \\ T_c \end{pmatrix}^n = C \begin{pmatrix} I \\ T_c \end{pmatrix}^{n-1},$$

where

$$C = \begin{pmatrix} 1 & -k_i\Delta t \\ \frac{\Delta t \frac{\dot{m}_i}{M_a}}{1+\Delta t \frac{\dot{m}_i}{M_a} k_p + \Delta t \frac{\dot{m}_o}{M_a}} & \frac{1-\Delta t^2 \frac{\dot{m}_i}{M_a} k_i}{1+\Delta t \frac{\dot{m}_i}{M_a} k_p + \Delta t \frac{\dot{m}_o}{M_a}} \end{pmatrix}.$$

The convergence of the system deteriorates by using the old values of the I and T_c. In Section 4 the convergence of the different Methods A, B and C is discussed. Also the results are compared with the results found by the Simulink model of the coupled system. If the coupled system is modeled in Simulink it contains three parts: the Thermal Cabin Model, the proportional part of the PID-controller and the integration part of the PID-controller. The coupling between these parts is represented schematically in Figure 3.1. The Simulink model of the coupled system is shown in Figure 3.2.



(a) The coupled system

(b) The PID-controller

(c) The Thermal Cabin Model

Figure 3.2: The coupled system and the components in Simulink.

Chapter 4

Results of the coupling of the Thermal Cabin Model and the PID-controller

In this section the convergence and the accuracy of the numerical methods are discussed. The following parameters of the PID-controller are used: $k_p = 0.5$, $k_i = 0.01s^{-1}$ and $k_d = 0.0s$. The system will converge for a Δt_{max} that depends on the iterative method. Also the accuracy of the system is dependent on the used method.
The convergence of method A is independent of the time step size. On the other hand using method B, the eigenvalues are greater than 1 if the time step is greater than $150s$. Hence, the system will diverge if the time step is chosen greater than 150 seconds. Using method C the maximum Δt is about 375 seconds. These results are shown in Figure 4.1. In Simulink the same model is build (Figure 3.2). The parameters of the PID-controller are again: $k_p = 0.5$, $k_i = 0.01s^{-1}$ and $k_d = 0.0s$. Some simulations are done with the Euler solver *ODE1*. Moreover, a fixed time step is used. This model gives the same convergence domain as method B. If $\Delta t > 150\,s$ the system will diverge. Some Simulink results are presented in Figure 4.2.
The accuracy of the different methods and the Simulink model are investigated with the same parameters: $k_p = 0.5$, $k_i = 0.01s^{-1}$ and $k_d = 0.0s$. The initial condition of the cabin temperature and the integral are given: $T_c(0) = -5$ and $I(0) = -5$. The total simulation time is 1000 seconds. In Table 4.1 an overview of the maximum norm of the errors of the solutions of the different methods is given.
This table shows for each method an error of $O(\Delta t)$. The used Euler method is $O(\Delta t)$. The accuracy of method C is the best. The results of Simulink are the same as the results of method B. This method has the smallest domain in which the solution converges. On the other hand, with the explicit method it is possible to solve the most coupled systems. This is necessary in Simulink.
For different values of the PID parameters the stability is investigated. The results are shown in Figure 4.3. Figure 4.3 (a)-(c) shows the maximum time step for method B. In Figure 4.3 (d)-(f) the maximum time step for method C is presented. The smaller k_i the greater the maximum time step. The k_p-value shows the opposite. The greater k_p the greater the convergence domain. The maximum time step of method C is always greater than the maximum time step of method B.

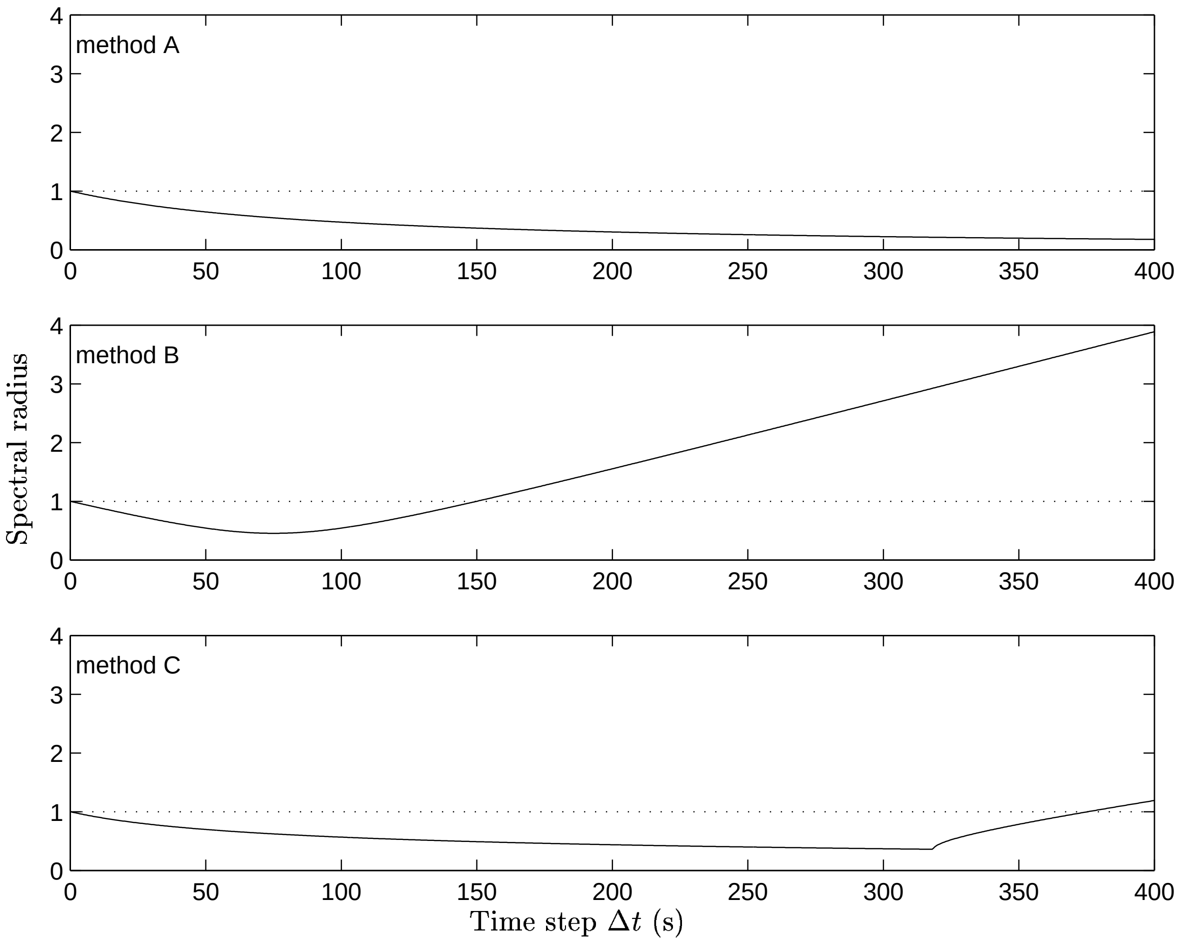


Figure 4.1: The convergence speed of the different methods. Method A: The new values of the temperature are used. Method B: The old values of the temperature are used. Method C: A combination of the old and the new values.

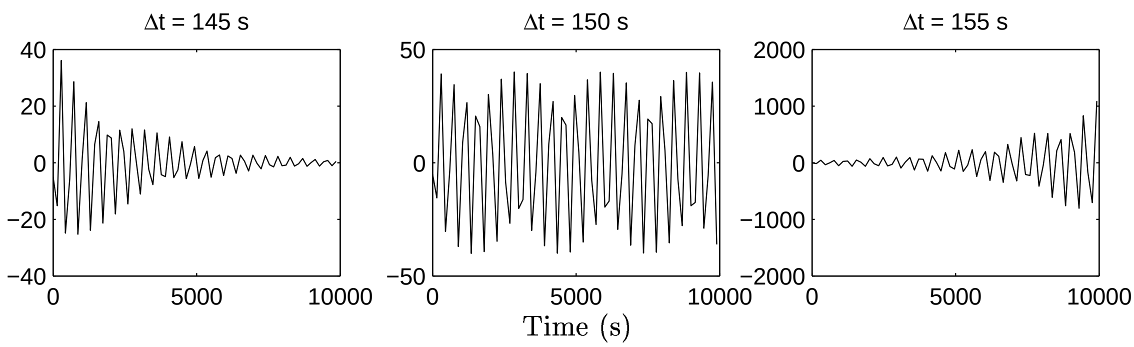


Figure 4.2: The coupled system in Simulink with different time steps.

Δt (s)	Simulink max norm K	Method A max norm K	Method B max norm K	Method C max norm K
4.0	0.0400	0.0385	0.0400	0.0313
2.0	0.0198	0.0194	0.0198	0.0156
1.0	0.0099	0.0098	0.0099	0.0078
0.5	0.0049	0.0049	0.0049	0.0039
0.25	0.0025	0.0024	0.0025	0.0020
0.125	0.0012	0.0012	0.0012	0.0009
0.0625	0.0006	0.0006	0.0006	0.0005

Table 4.1: Maximum norm of the error of the Simulink model and the numerical methods with respect to the analytical solution.

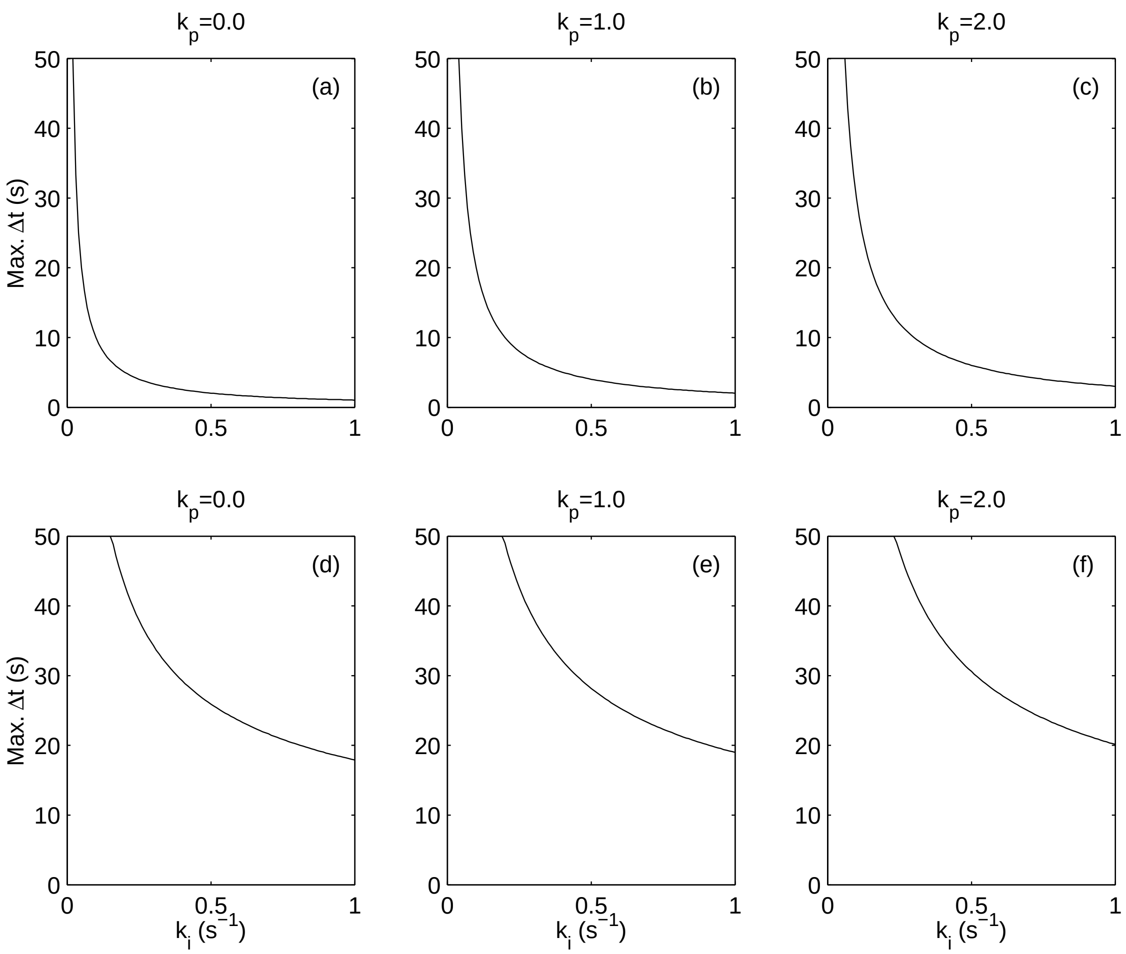


Figure 4.3: k_i versus maximum time step in which the system converges. Method B (a)-(c), method C (d)-(f).

Chapter 5

Comparison of HEAT with the Thermal Cabin Model

Besides the simple Thermal Cabin Model, a two-dimensional CFD code (HEAT) has been used to simulate the temperature in the aircraft cabin. The mathematical and the numerical model used in HEAT have been described in [**1**]. Two aspects are investigated in this section. A comparison of HEAT and the Thermal Cabin Model is given. Both models, HEAT and the Thermal Cabin Model, are coupled with a PID-controller. For the coupling of the Thermal Cabin Model with a PID-controller the analytical solution, described in Section 2, has been used. Secondly it is investigated if the results satisfy certain thermal comfort requirements. The Thermal Cabin Model can be used only to compute the average cabin temperature. The CFD code HEAT is also suitable to compute the position dependent temperature. The average temperature is given as the mean of nine measure points. The cabin geometry and the nine measure points (p1,p2,..,p9) are given in Figure 5.1. Some important parameters of this geometry are given in Table 5.1. The exact position of the measure points is given in [**3**]. All simulations are done with the same cabin parameters. An overview of the used parameters during these simulations is given in Table 5.2. With the exception of simulation *SSC-k1-pm3* the inflow velocity in HEAT is calculated with the following formulas:

$$T_{av} = \frac{\sum_{i=1}^{9} T_i}{9},\ \rho = \frac{P_c}{R\,T_{av}},\ v_{nh} = \frac{\dot{m}_i}{2\,L\,d_{nh}\,\rho}.$$

In this case it means, $v_{nh} \approx 4\,m\,s^{-1}$. T_i is the temperature at pi, with $i = 1, 2, .., 9$). For simulation *SSC-k1-pm3* the average temperature of all grid points has been used to calculated the inflow velocity. All simulations have been done in Simulink. The coupled system is shown in Figure 5.2. A description of the integration of the flow solver HEAT in Simulink is given

grid	64×100
floor condition	free slip
d_{nh}	$0.006\,m$
d_{out}	$0.08\,m$

Table 5.1: Parameters of the cabin geometry used during the simulations.

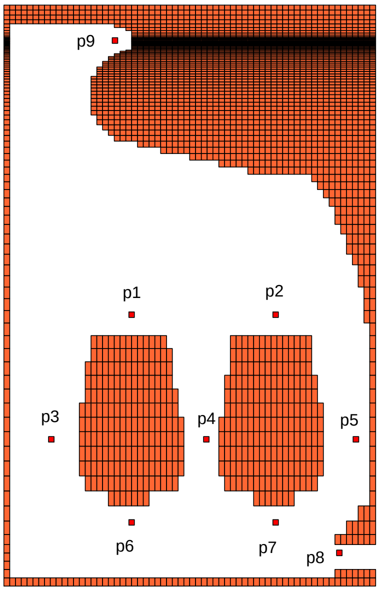


Figure 5.1: Position of the measure points (p1,p2,..,p9) in the cabin geometry.

$\dot{m}_i$	$0.79\,kg\,s^{-1}$
$\dot{m}_o$	$0.79\,kg\,s^{-1}$
M_a	$56\,kg$
P_c	$10^5\,Pa$
ν	$0.0007\,m^2\,s^{-1}$
$T_c(0)$	$18\,^{\circ}C = 291\,K$

Table 5.2: Overview of parameters during the coupled simulations.

in Appendix A. To solve the coupled system, the Euler method is used. This is a first order solver. A fixed time step $\Delta t = 0.375\,s$ is chosen.

It is important to notice the differences between HEAT and the Thermal Cabin Model. The Thermal Cabin Model calculates the average temperature of the cabin. This average temperature is used as an input for the PID-controller. HEAT calculates the temperature at each grid point. The temperature of a sensor, chosen at one point in the cabin is used as an input for the PID-controller. The difference between the average temperature and the sensor temperature is dependent on the position of the sensor. The outflow temperature in the Thermal Cabin Model is chosen the same as the average temperature.

An investigation of the influence of the position of the sensor for certain fixed controller parameters is given in Section 5.1. The average temperature of the cabin is calculated in different ways. These differences are described in Section 5.2. Also the influence of the parameters of the PID-controller with a sensor at p7 is studied. The results of the simulations for different controller parameters are presented in Section 5.3. In Section 5.3 optimal controller parameters are found for the case of the sensor positioned in p7. These controller parameters are also used to investigate the dependence on the sensor position. The results of this investigation are presented in Section 5.4.

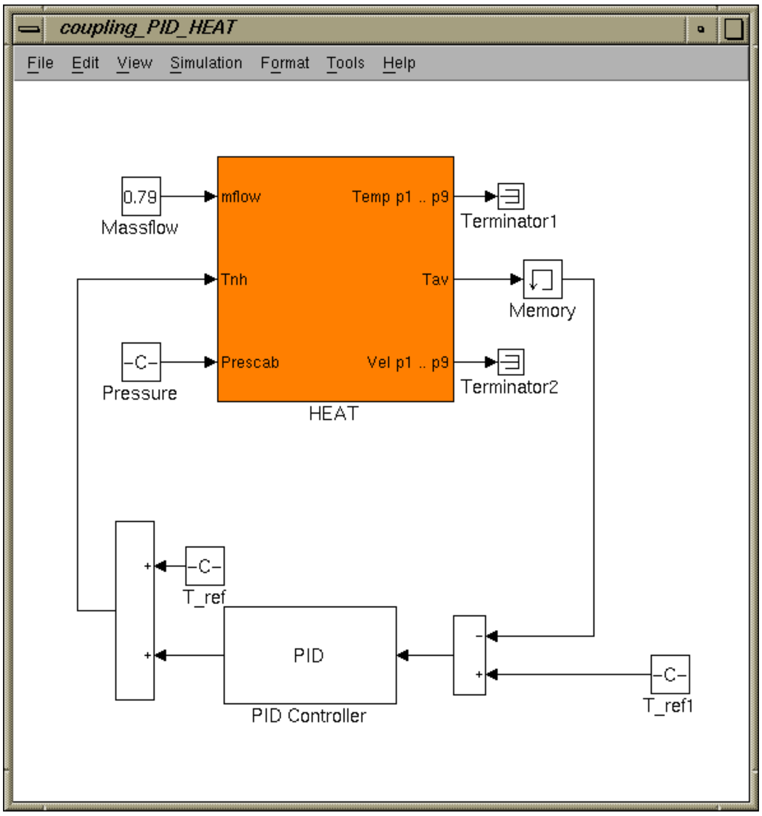


Figure 5.2: HEAT controlled by a PID-controller.

5.1 Results at different sensor positions with a certain set of PID parameters

In this section the coupled system is investigated with the sensor at different measure points. The influence of the position of the sensor is discussed.
In Table 5.3 an overview of the simulations discussed in this section is given. A comparison

Simulation	k_p	k_i	k_d	sensor position
SSC-k1-p1	1.0	0.05	0.0	p1
SSC-k1-p2	1.0	0.05	0.0	p2
SSC-k1-p3	1.0	0.05	0.0	p3
SSC-k1-p4	1.0	0.05	0.0	p4
SSC-k1-p5	1.0	0.05	0.0	p5
SSC-k1-p6	1.0	0.05	0.0	p6
SSC-k1-p7	1.0	0.05	0.0	p7
SSC-k1-p8	1.0	0.05	0.0	p8
SSC-k1-p9	1.0	0.05	0.0	p9

Table 5.3: Overview of simulations of the coupled system, with the sensor at different positions.

of the results for different sensor positions is done. The average temperature of HEAT is the average temperature of the nine measure points. This average temperature is compared to the temperature of the Thermal Cabin Model. These results are shown in Figure 5.3. The average temperature of HEAT is similar to the temperature of the Thermal Cabin Model if the sensor is placed at p1, p2, and p3. The corresponding simulations are respectively SSC-k1-p1, SSC-k1-p2, and SSC-k1-p3. The average temperature of the simulations with the sensor placed at pi, $i = 4, 5, .., 8$ shows an overshoot. This overshoot is greater than the overshoot shown by the Thermal Cabin Model. A possible explanation of the overshoot can be given by the delay of time which is not modeled by the Thermal Cabin Model.
Figure 5.4 shows the temperature at the sensors in the different simulations. The temperature calculated with the Thermal Cabin Model gives an impression of the sensor temperature calculated with HEAT if the sensor is placed at p1 or p2. If the sensor is placed lower in the cabin the overshoot is higher. A similarity between the height of the overshoot at p3, p4, and p5 is seen. The time in which the overshoot arises is dependent on the position of the sensor. If the sensor is placed at p3 the overshoot is seen first. At p4 and p5 the overshoot arises later. At p6, p7 and p8 the overshoot is the highest. The sensor at p9 reacts immediately on the controller. If the parameters of the PID-controller are optimized for the Thermal Cabin Model, these parameters are not useful for HEAT if the sensor is not positioned appropriately.
The results of simulation *SSC-k1-p1* at the other measure points (p2, p3,..,p9) have been investigated also. In this simulation the sensor is placed at p1. The temperatures at the other measure points are shown in Figure 5.5. The deviation of results of the Thermal Cabin Model with respect to the results calculated with HEAT is dependent on the position of the sensor. At p9 the temperature increases very soon. A high temperature is expected. The temperature at p3 shows a similar behavior.
The results of the Thermal Cabin Model are the same as the results of the HEAT if the sensor temperature and the temperature at the outlet (p8) are the same as the average temperature of the Thermal Cabin Model. With exception of $t = 0$ and $t \to \infty$ this situation will only

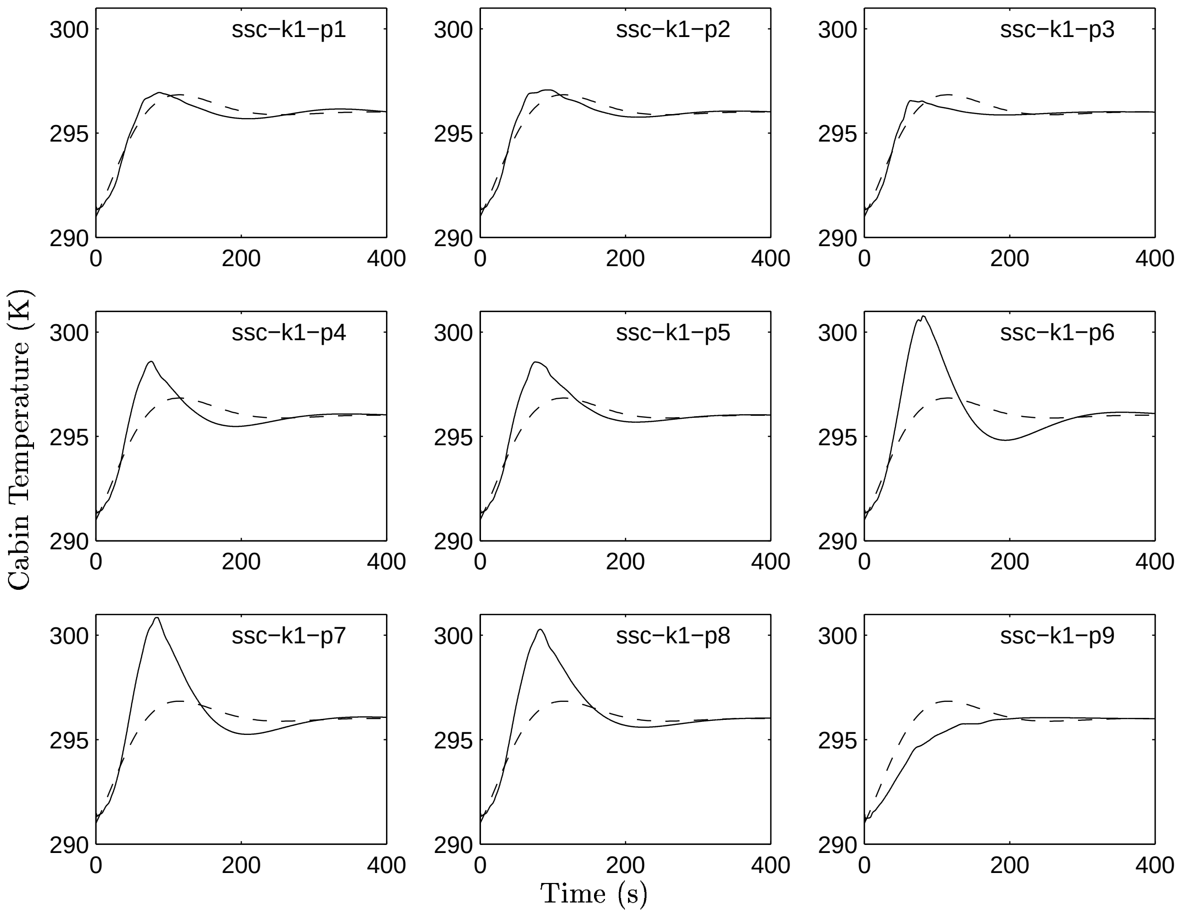


Figure 5.3: Cabin temperature of simulations *SSC-k1-pi*, $i = 1, 2, .., 9$. (– – Average temperature as computed by the Thermal Cabin Model, – Average temperature calculated by HEAT)

occur incidentally.

The average temperature of HEAT, is in the different simulations reasonably similar to the temperature at the sensor position. In Figure 5.6 these temperatures are shown. The average temperature of the cabin is similar to the temperature of the sensor, independent of the position at the sensor. So the sensor temperature always gives a reasonable impression of the average cabin temperature. The reaction time of the sensor temperature depends on the position of this sensor. If the controller is optimized (Section 5.3) for the sensor, the behavior of the average temperature of the cabin is optimal also.

5.2 More than one sensor

Some simulations are done with more than one sensor. An overview of these simulations is given in Table 5.4. In simulation *SSC-k1-pm1* the input of the controller is the average temperature of the nine measure points. Most of the nine sensors are placed near the seats in the cabin. The temperature in the upper half of the cabin is measured in p9 only (Figure

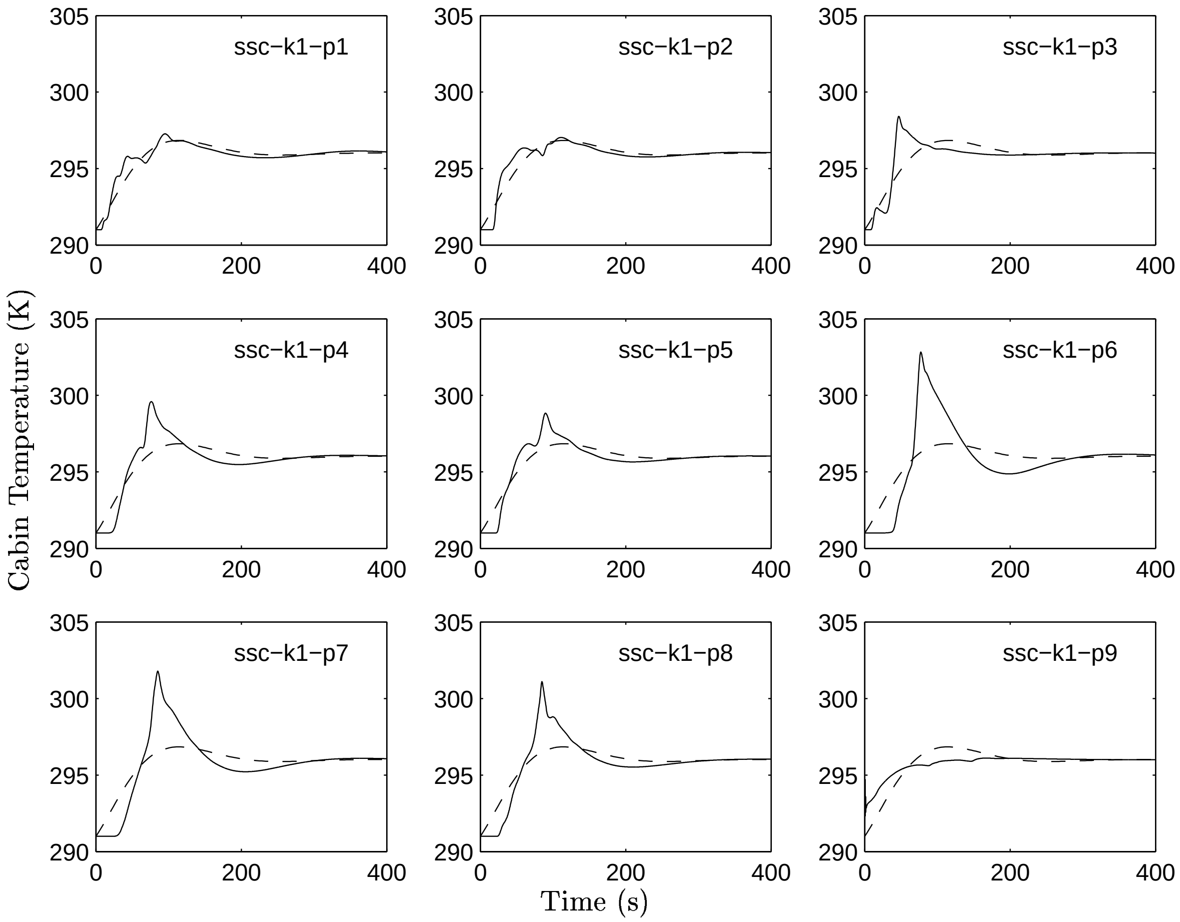


Figure 5.4: Temperature at the sensor pi computed by HEAT in simulations *SSC-k1-pi* (solid lines) compared to the cabin temperature as calculated by the Thermal Cabin Model (dashed lines)

Simulation	k_p	k_i	k_d	sensor position
SSC-k1-pm1	1.0	0.05	0.0	p1,p2,..,p9
SSC-k1-pm2	1.0	0.05	0.0	p1,p2,..,p7,p8,p9,p9
SSC-k1-pm3	1.0	0.05	0.0	T_{av}^*

Table 5.4: Overview of simulations of the coupled system, with more than one sensor.

5.1). A simulation is done with the average temperature of the measure points p1,..,p8 and 2 times p9, simulation *SSC-k1-pm2*. Because differences have been found between these two simulation, a simulation is also done with the average temperature of all grid points (T_{av}^*) in the aircraft cabin (simulation *SSC-k1-pm3*). The differences found are shown in Figure 5.7. All these simulations are different from the result calculated by the Thermal Cabin Model. In simulations *SSC-k1-pm1* and *SSC-k1-pm2* the temperature difference ($T_{ref} - T_c$) is greater than for simulation *SSC-k1-pm3* for $t < 50\,s$. This temperature difference ($T_{ref} - T_c$) is integrated by the PID-controller. And so the higher overshoot after $t = 50\,s$ is explained.

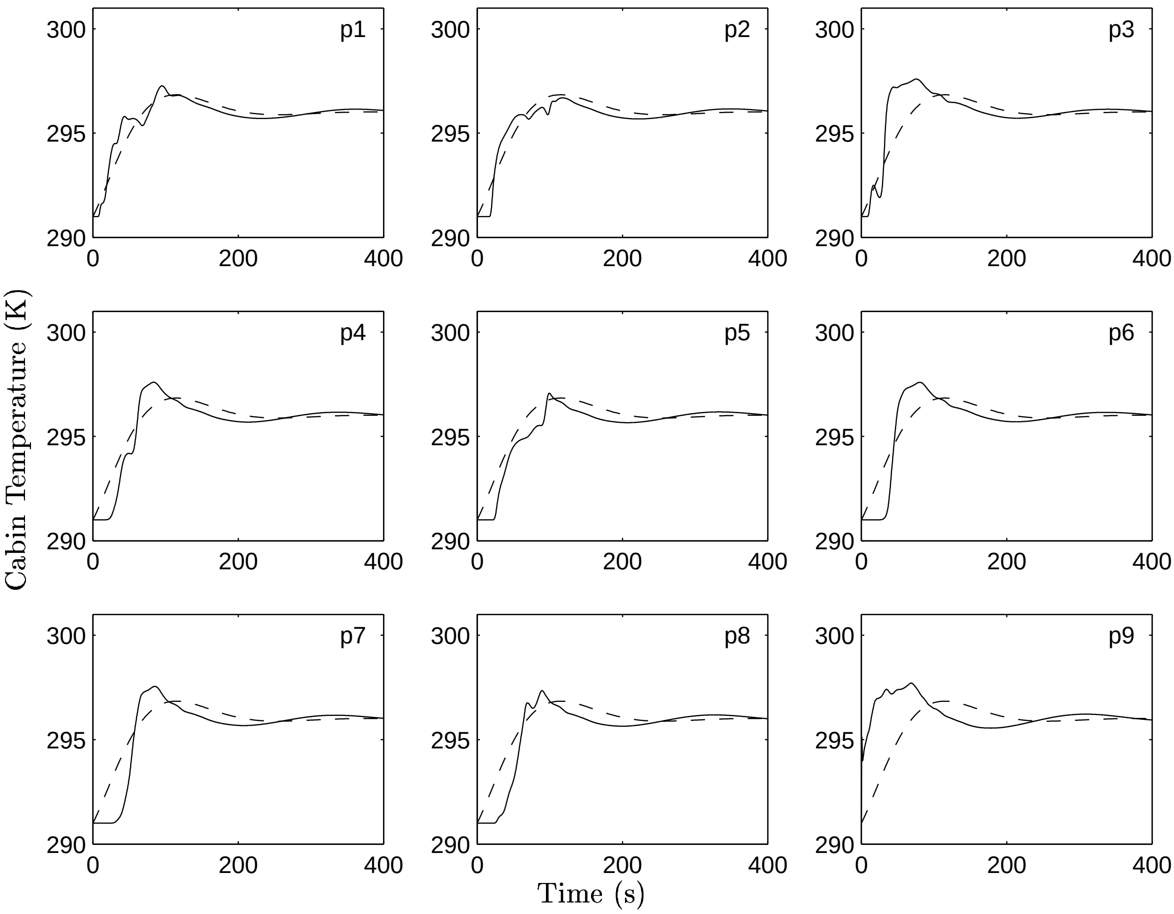


Figure 5.5: Temperature at the different measure points for simulation *SSC-k1-p1*. The sensor is placed at p1. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the nine measure points)

For the Thermal Cabin Model the temperature of the outflow air is chosen the same as the average temperature. Especially in the beginning of the simulation, an error is made by this assumption. For simulation *SSC-k1-pm3* and the Thermal Cabin model the slope at $t = 0$ is the same. This can be shown by the equation of the Thermal Cabin Model. We assume that the outlet temperature is the average temperature plus a deviation: $T_o = T_c + f(t)$. The equation of the Thermal Cabin Model can be written as:

$$\dot{m}_i T_i = M_a \frac{\partial T_c}{\partial t} + \dot{m}_o (T_c + f(t)) \tag{5.1}$$

A few properties of the function $f(t)$ are known: At $t = 0$, the temperature in the cabin is constant. So the average temperature equals the temperature at each point, especially the temperature at the outlet. So $f(0) = 0$. Moreover, if the solution is damped:

$$\lim_{t \to \infty} f(t) = 0.$$

Because $f(0) = 0$ the derivative of the average temperature of the Thermal Cabin Model equals the derivative of the average temperature of HEAT. This is shown in Figure 5.8. The

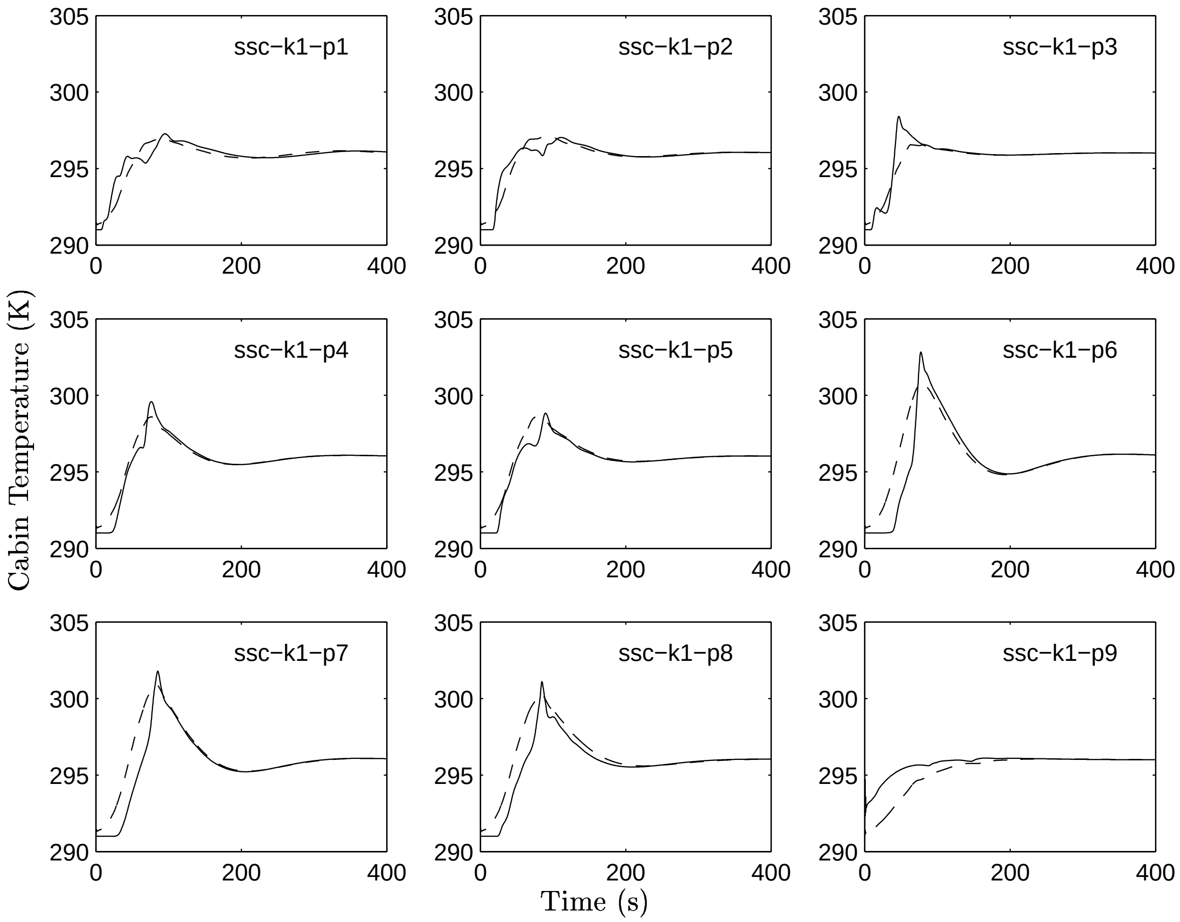


Figure 5.6: The average temperature of the cabin compared to the temperature at the sensor position in the simulations *SSC-k1-pi*, for $i = 1, 2, .., 9$. (– – Average temperature calculated by HEAT, – Temperature calculated by HEAT at the sensor positions)

difference between the outlet temperature and the average temperature depends on different parameters. So a simple adaptation of the Thermal Cabin Model is not possible. More study about the different parameters, which influence the inflow velocity is necessary. The width of the nozzle is not a parameter of the Thermal Cabin Model. A change in the width of the nozzle leads to another inflow velocity. As a consequence, the time before the temperature of the outlet rises can be changed. Because the temperature of the outlet affects the average temperature, the average temperature changes also.

The differences between simulation *SSC-k1-pm3* and the Thermal Cabin Model are shown in Figure 5.8. The temperature of the air at the outlet is not necessarily the same as the average temperature of the cabin. In the Thermal Cabin Model this is an assumption. In the beginning of the simulation the outlet temperature is lower than the average temperature. In the Thermal Cabin Model too much heat leaves the cabin, and so the average temperature of the Thermal Cabin Model is too low. The nozzle temperature, which reacts on the low average temperature is too high.

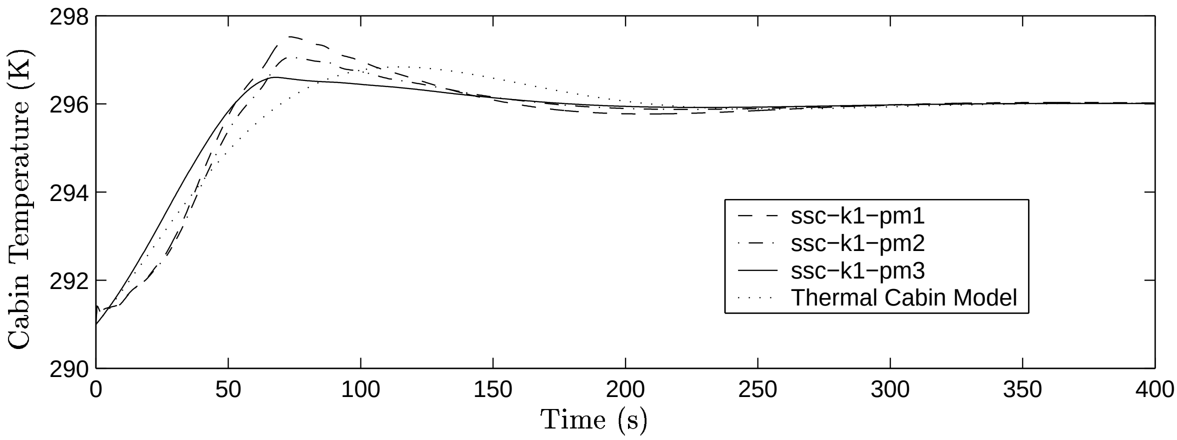


Figure 5.7: Average temperature of the cabin. Results of simulations *SSC-k1-pm1*, *SSC-k1-pm2* and *SSC-k1-pm3* compared to the results of the Thermal Cabin Model.

5.3 Results for different PID parameters at one sensor position

In this section an investigation of different parameters of the PID-controller is done. The sensor position is chosen at p7, because this is a point under the outboard seat. It is possible to place a sensor at this position without hindering the passengers. An overview of the simulations is given in Table 5.5.

Simulation	k_p	k_i	k_d	sensor position	$T_{nh}(0)$
SSC-k1-p7	1.0	0.05	0.0	p7	296
SSC-k2-p7	1.0	0.01	0.0	p7	296
SSC-k3-p7	1.0	0.03	0.0	p7	296
SSC-k4-p7	1.0	0.05	1.0	p7	296
SSC-k5-p7	2.0	0.01	0.0	p7	301
SSC-k6-p7	1.0	0.0	0.0	p7	301
SSC-k7-p7	0.0	0.01	0.0	p7	291
SSC-k8-p7	0.0	0.05	0.0	p7	291
SSC-k9-p7	0.0	0.015	0.0	p7	291
SSC-k10-p7	0.5	0.01	0.0	p7	293.5
SSC-k11-p7	1.0	0.02	0.0	p7	296
SSC-k12-p7	0.5	0.02	0.0	p7	293.5
SSC-k13-p7	0.5	0.018	0.0	p7	293.5

Table 5.5: Overview of simulations of the coupled system, with the sensor at p7 and different PID parameters.

The equation of the PID-controller for $k_i \neq 0\,s^{-1}$ at $t = 0$ can be written as: $T_{nh}(0) = k_p(T_{ref} - T_c(0)) + I(0)$, with I as in Section 3. The cabin temperature T_c and the initial value of the integral are chosen $291K$. If $k_i = 0\,s^{-1}$, $T_{nh}(0) = k_p(T_{ref} - T_c(0)) + T_{ref}$. So the initial value of the nozzle temperature is dependent on k_p and k_i. In the Figures 5.9, 5.10, 5.11, and B.1 the dotted lines mark the following comfort requirements:

- The rate of the cabin temperature should not exceed $3K/min$.

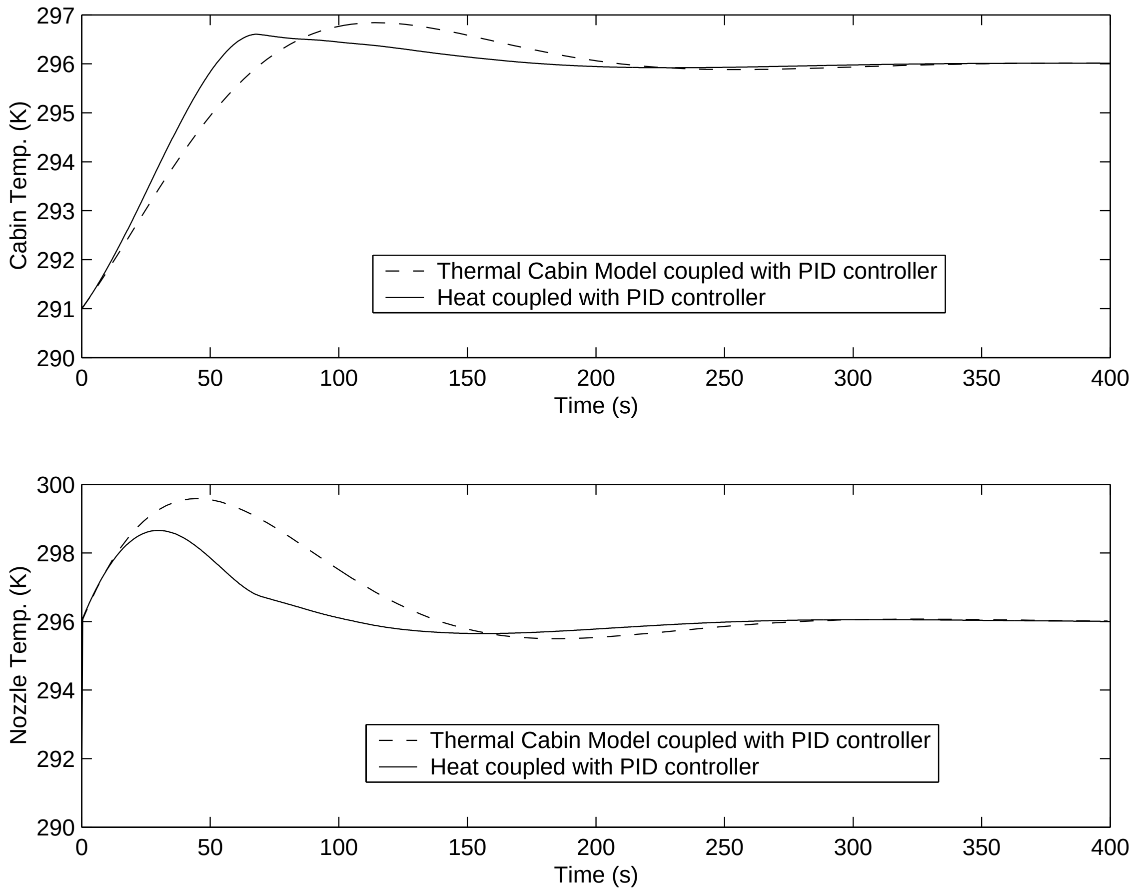


Figure 5.8: A comparison of HEAT and the Thermal Cabin Model. Both models are coupled with a PID-controller. Simulation *SSC-k1-pm3*.

- The temperature has to stabilize in a range (5%) around the reference temperature ($296K$). This range is calculated as follows: $0.05(T_{ref} - T_c(0)) = 0.05(296 - 291) = 0.25K$.

In simulations *SSC-ki-p7*, $i \in \{2, 5, 7, 10\}$, $k_i = 0.01s^{-1}$ and $k_d = 0.0s$. The influence of the variation of k_p is seen in these simulations. The results of these simulations are presented in Figure 5.9. The smaller k_p, the smaller the overshoot. And the smaller k_p, the greater the similarity with the Thermal Cabin Model. Simulations *SSC-k7-p7* and *SSC-k10-p7* show the most similarity with the Thermal Cabin Model. The initial condition of the inflow temperature are $291K$ and $293.5K$ respectively. The very small k_i parameter leads to a very slow heating of the cabin. Each sensor position shows the same behavior. And so in these cases, the average temperature is approximately equal to the outflow temperature. These cases satisfy the assumption of the Thermal Cabin Model.
Variation of k_i is seen in simulations *SSC-ki-p7* $i \in \{1, 2, 3, 11\}$. During these simulations $k_p = 1.0$ and $k_d = 0.0s$. The smaller k_i, the smaller the overshoot. And the smaller k_i, the greater the similarity with the Thermal Cabin Model. The results of these simulations are shown in Figure 5.10.

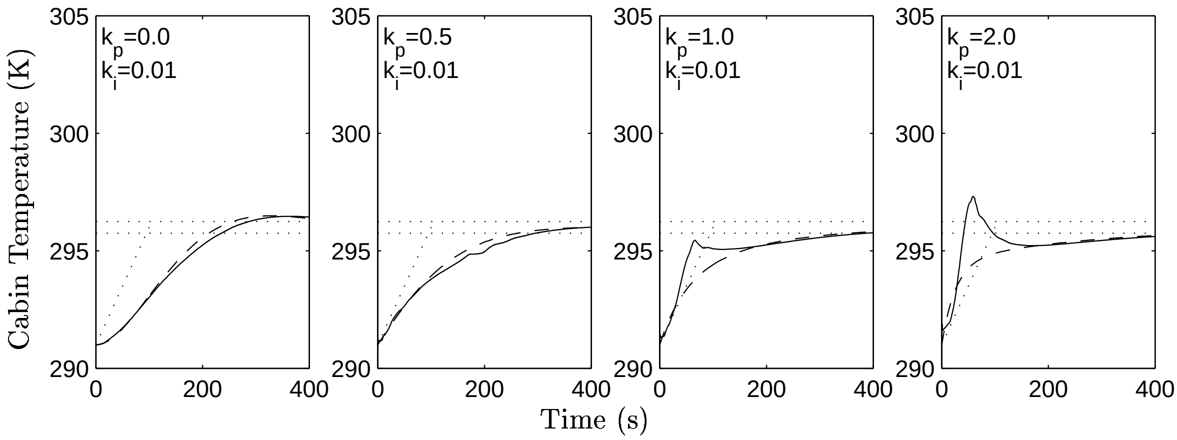


Figure 5.9: A variation in the parameter k_p. The temperature of the Thermal Cabin Model compared to the temperature at the sensor position. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the sensor position, $\cdots$ Requirements for an optimal behavior of the temperature)

A variation of k_d is seen in simulations *SSC-k1-p7* and *SSC-k4-p7*. In these simulations $k_p = 1.0$ and $k_i = 0.05s^{-1}$. This variation does not have an influence on the results. This is shown in Figure 5.11.
If all parameters are chosen zero, the inflow temperature is constant. An investigation of this situation is done in [**1**].
The average temperature in these simulations (*SSC-ki-p7*, $i = 1, .., 13$) is similar to the sensor temperature (Figure 5.12). If a desired time dependent behavior of the temperature is found at p7, the average temperature has the desired behavior also.
Three cases are distinguished:

- **The results of the Thermal Cabin Model are similar to the results of HEAT, and these results satisfy the requirements.**
  A special case of the parameters is necessary to get a similarity in the results of HEAT and the results of the Thermal Cabin Model. Simulations *SSC-k7-p7* and *SSC-k10-p7* give results similar to the coupling of the PID-controller with the Thermal Cabin Model. The temperature of simulation *SSC-k10-p7* satisfies the requirements the best. The PID parameters in this case are: $k_p = 0.5$, $k_i = 0.01s^{-1}$, and $k_d = 0.0s$

- **The results of the Thermal Cabin Model satisfy the temperature requirements.**
  In Appendix B the graphical results of the simulations with the sensor at p7 are given. For the simulation with the PID parameters: $k_p = 1.0$, $k_i = 0.02s^{-1}$, and $k_d = 0.0s$ the Thermal Cabin Model satisfies the temperature requirements the best.

- **The results of HEAT satisfy the temperature requirements.**
  The best behavior of the temperature calculated by HEAT is found with the parameters: $k_p = 0.5$, $k_i = 0.018s^{-1}$, and $k_d = 0.0s$. This can be seen in Appendix B also.

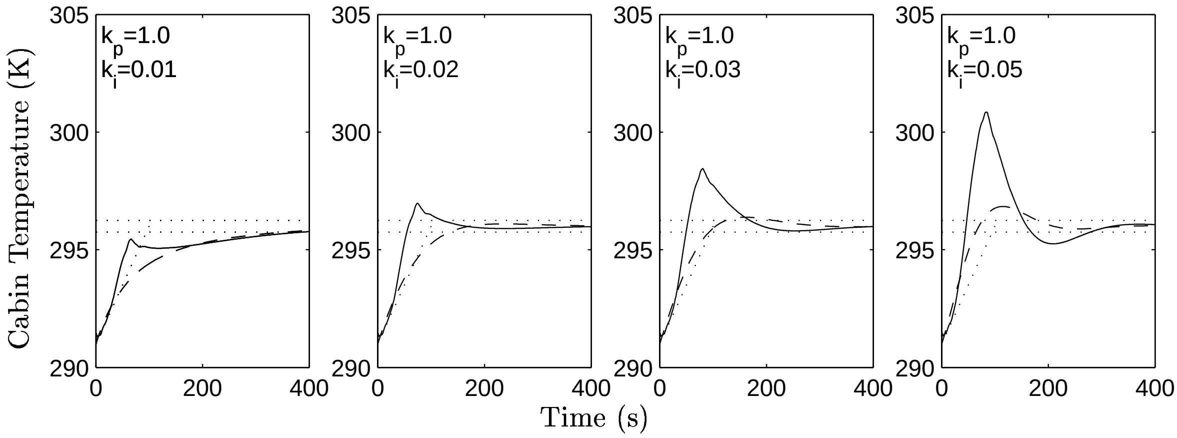


Figure 5.10: A variation in the parameter k_i. The temperature of the Thermal Cabin Model compared to the temperature at the sensor position. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the sensor position, ··· Requirements for an optimal behavior of the temperature)

5.4 Results at different sensor positions for optimal PID parameters

The results of the coupling of the Thermal Cabin Model with the PID-controller are similar to the results of the coupling of HEAT with the PID-controller if the following parameters are used: $k_p = 0.5$, $k_i = 0.01s^{-1}$ and $k_d = 0.0s$. With these parameters the influence of the position of the sensor is studied also. An overview of the simulations is given in Table 5.6. The results of these simulations are presented in Figure 5.13. The deviation of the results of

Simulation	k_p	k_i	k_d	sensor position
SSC-k10-p1	0.5	0.01	0.0	p1
SSC-k10-p2	0.5	0.01	0.0	p2
SSC-k10-p3	0.5	0.01	0.0	p3
SSC-k10-p4	0.5	0.01	0.0	p4
SSC-k10-p5	0.5	0.01	0.0	p5
SSC-k10-p6	0.5	0.01	0.0	p6
SSC-k10-p7	0.5	0.01	0.0	p7
SSC-k10-p8	0.5	0.01	0.0	p8
SSC-k10-p9	0.5	0.01	0.0	p9

Table 5.6: Overview of simulations of the coupled system, with the sensor at different positions.

HEAT with respect to the Thermal Cabin Model are small in these simulations. In Section 5.3 optimal PID parameters have been found for a sensor at p7. If the sensor is placed at the other sensor positions (p1,p2,..,p6,p8,p9) the sensor temperature is similar to the cabin temperature of the Thermal Cabin Model also.
Two simulations have been done with the sensor at p1. In Section 5.1 simulation *SSC-k1-p1* is presented, and in this section the results of simulation *SSC-k10-p1* are shown. In Table 5.7

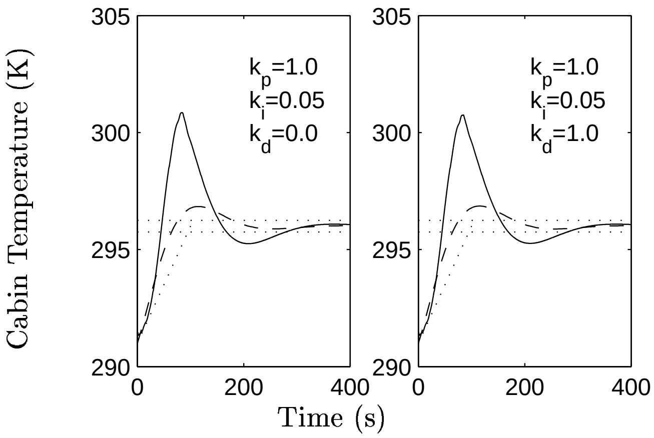


Figure 5.11: A variation in the parameter k_d. The temperature of the Thermal Cabin Model compared to the temperature at the sensor position. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the sensor position, $\cdots$ Requirements for an optimal behavior of the temperature)

the parameters of these two simulations are repeated. For both simulations the results of the Thermal Cabin Model are similar to the results of HEAT at p1. If the sensor is placed at p1, the results of the Thermal Cabin Model are not similar to the results of HEAT for arbitrarily chosen PID-controllers. Another choice of the PID parameters can lead to a greater difference between the results of HEAT and the results of the Thermal Cabin Model. Simulation *SSC-k14-p1* is a simulation with such parameters. This simulation is shown in Table 5.7 also.

Simulation	k_p	k_i	k_d	sensor position
SSC-k1-p1	1.0	0.05	0.0	p1
SSC-k10-p1	0.5	0.01	0.0	p1
SSC-k14-p1	2.0	0.2	0.0	p1

Table 5.7: Simulations of the coupled system.

The result of this simulation is presented in Figure 5.14.

5.5 Conclusions

Simulations with sensors at different places and simulations with different PID-controllers have been done. The results of the Thermal Cabin Model have been compared to the results of HEAT. An investigation of the dependence on the place of the sensor has been done with the PID parameters $k_p = 1.0$, $k_i = 0.05s^{-1}$, and $k_d = 0.0s$. The sensor has been placed at nine measure points. The similarity of the results of the Thermal Cabin Model to the results of HEAT is dependent on the place of the sensor.
To make the result of the Thermal Cabin Model similar to the result of HEAT, the parameters of the PID-controller are essential. With the sensor at p7 (a sensor below the right seat)

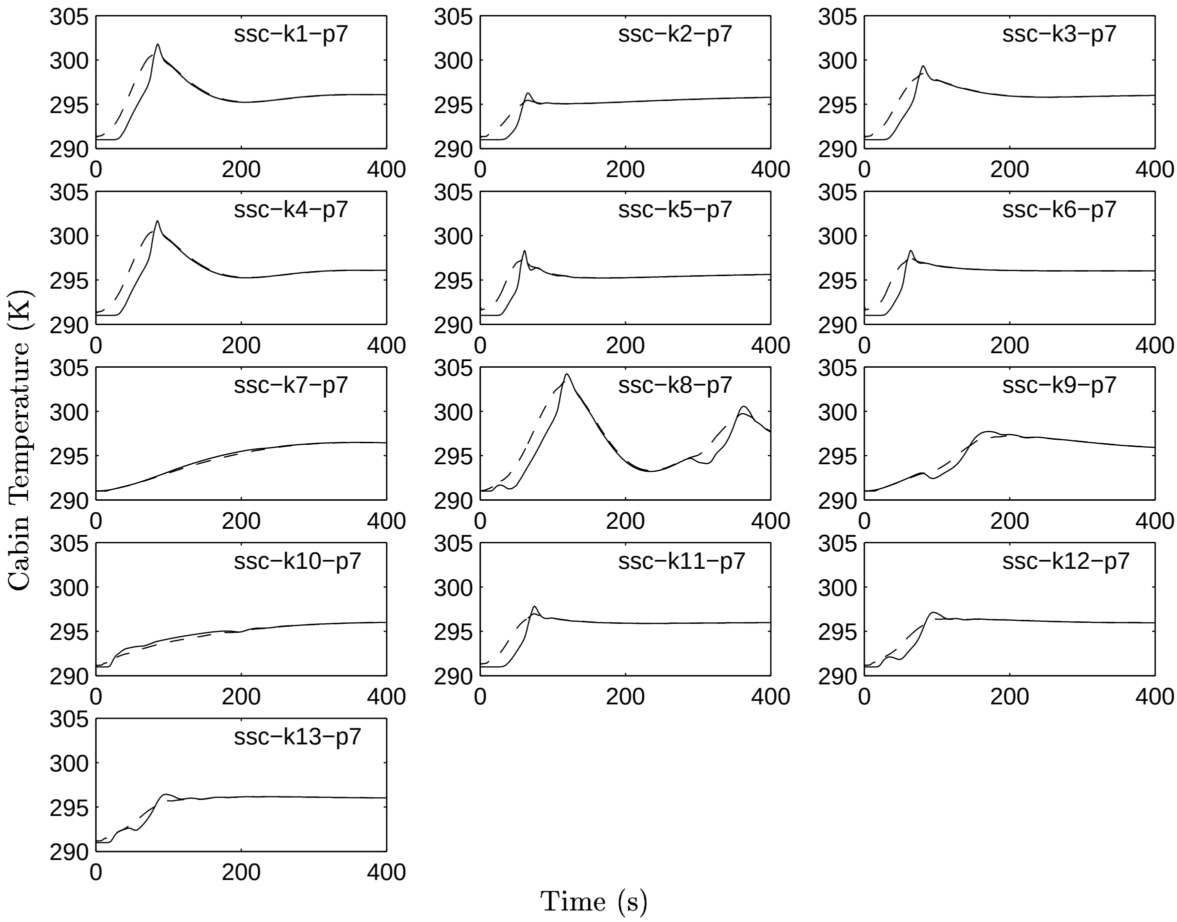


Figure 5.12: The average temperature of the cabin compared to the temperature at the sensor position in the simulations *SSC-ki-p7* $i = 1, 2, .., 13$. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the sensor position)

different parameters of the PID-controller are used. The influence of the different controller parameters has been investigated. The influence of k_d is not noticeable. This parameter is chosen zero. If the other parameters of the controller are chosen sufficiently small ($0 \leq k_p < 1$, and $0 \leq k_i < 1/\tau\, s^{-1}$, with $\tau = \frac{\dot{m}_i}{M_a}$ the time constant), the temperature of the cabin increases slowly. Only in these situations the results of the Thermal Cabin Model are similar to the results of HEAT. For the case that the sensor is placed at p7 the following ranges have been found: $0 \leq k_p < 0.5$, and $0 \leq k_i < 0.01s^{-1}$. Simulations with these parameters, and the sensor at the other measure points (p1,..,p6,p8,p9) gives results similar to the results of the Thermal Cabin Model. If the sensor is placed at p1, the limiting values of the PID parameters are higher. The exact values are not investigated.

In all simulations performed the average temperature of HEAT is similar to the temperature at the sensor position. In other words, in each case the average temperature is similar to the sensor temperature. If the sensor temperature is acceptable the average temperature is also acceptable.

With the coupled system an optimal temperature for the passengers is searched. With the

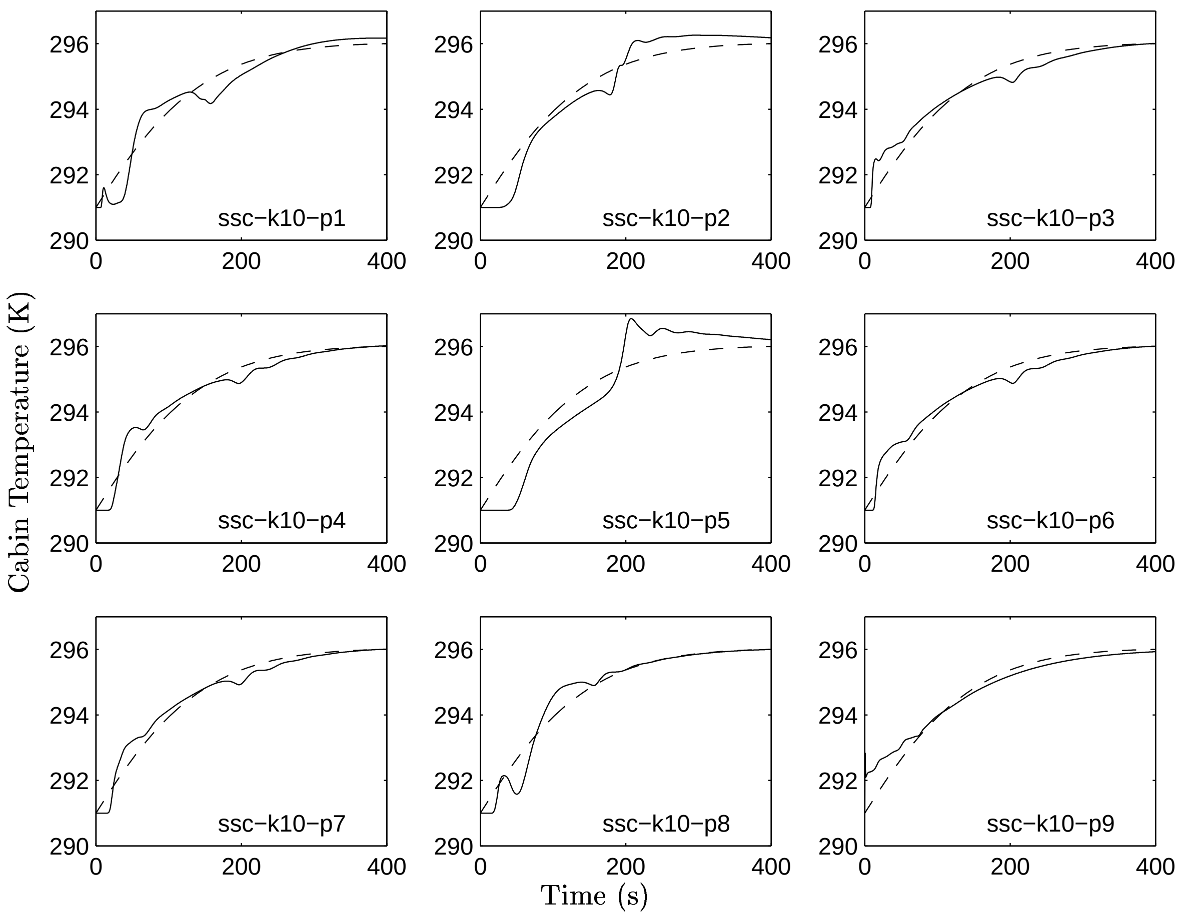


Figure 5.13: The results of the Thermal Cabin Model compared to results of HEAT at the sensor position. Simulations *SSC-k10-pi* $i = 1, 2, .., 9$. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT measured at the sensor position)

coupling of the Thermal Cabin Model and the PID-controller an optimal set of PID parameters can be found. In general these PID parameters are not the optimal parameters if HEAT is coupled to the PID-controller. The dead time is defined as the time before the sensor reacts on the controller output. This dead time plays an important role in the comparison of the Thermal Cabin Model with HEAT. The dead time is dependent on the position of the sensor and also on the inflow temperature. The higher the inflow temperature the longer it takes before the heated air reaches the sensor. This dead time is not modeled in the Thermal Cabin Model. Moreover, for the average temperature , the sensor temperature, and the outflow temperature in the Thermal Cabin Model the same temperature T_c is used. In HEAT these temperatures are not the same.

Only if the variations in the cabin temperature are small, the results of the Thermal Cabin Model are similar to results of HEAT. So the Thermal Cabin Model is not useful in all dynamical simulations. In the steady state solution however, the differences between the temperatures of HEAT and the temperatures of the Thermal Cabin Model are zero.

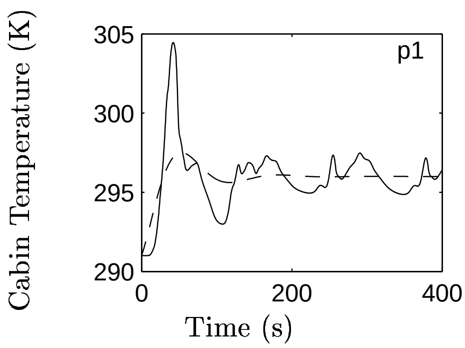


Figure 5.14: The results of the Thermal Cabin Model compared to results of HEAT at the sensor position. Simulation *SSC-k14-p1*. (– – Average temperature as computed by the Thermal Cabin Model, – Temperature calculated by HEAT at the sensor position p1)

Chapter 6

Numerical aspects of the coupling of HEAT and the PID-controller

In this section the accuracy of the coupling of the flow solver HEAT and the PID-controller is investigated. The influence of the coupling time step is discussed.
Simulation *SSC-k13-p7* has been repeated for different values of the coupling time step. For simulations with different time steps, the results are compared. The following results are used: the velocities at the nine measure points ($|u|_i, i = 1, 2, .., 9$), the temperatures at the nine measure points ($T_i, i = 1, 2, .., 9$), the average temperature of the nine measure points (T_{av}), and the nozzle temperature (T_{nh}). The parameters of the PID-controller are presented in Table 6.1. With these parameters HEAT satisfies the requirements the best (Section 5.3). The time step of Simulink is chosen fixed. The greatest time step used is $\Delta t = 4$ seconds. This is a great time step with respect to the time step used in HEAT. The average time step which HEAT has been used is $0.012\,s$. The accuracy has been determined with the formula:

$$q(\Delta t, result) = \frac{||result[4\Delta t] - result[2\Delta t]||_\infty}{||result[2\Delta t] - result[\Delta t]||_\infty} \tag{6.1}$$

In this equation $result[\Delta t]$ means a vector of results at fixed points in time, which are found with a coupling time step Δt:

$$result[\Delta t] \equiv (result(t_1), result(t_2), ..., result(t_N))^T$$

For example $result[\Delta t]$ can be $(|u|_1(t_1), |u|_1(t_2), ..., |u|_1(t_N))^T$. During the simulations a first order solver (Euler) has been used. An accuracy of $O(\Delta t)$ is expected. This results in $q(\Delta t, result) = 2$, for small Δt. For each Δt, the results have been measured at the same points in time: $t \in \{4s, 8s, 12s, ..., 148s\}$. For all results the quotient q (Equation 6.1) has been calculated. The average of all these quotients is presented in Table 6.2. In Table 6.2 the expected factor 2 is found.

k_p	k_i	k_d
0.5	0.018	0.0

Table 6.1: The parameters of the PID-controller used in the investigation of the accuracy

Δt	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
$q(\Delta t)$	1.31	1.44	1.64	1.73	1.86	1.91	2.08

Table 6.2: Results of the quotient q (Equation 6.1) for different time step sizes.

With the results of two simulations (Equation 6.2) the order of magnitude of the error can be approximated.

$$||result[2\Delta t] - result[\Delta t]||_\infty \tag{6.2}$$

For all used coupling time steps and results, this difference is presented in Appendix C. For example, if an accuracy of $0.1K$ is required at each measure point, and we assume that the grid is fine enough the coupling time step should be smaller than $2^{-4}s$.

Chapter 7

Conclusions and recommendations

Three methods of coupling have been described: An implicit method (A), an explicit method (B), and a combination of both (C). All methods are first-order accurate. The maximum time step by which the coupled system converges is dependent on the method used. Method B has a smaller maximum time step by which the system converges than method C. The convergence of method A is independent of the time step. If in Simulink the first order solver ODE1 is used, the results of Simulink and the results of the explicit method (B) are the same.
An investigation of the differences between two models of the aircraft cabin has been done. The Thermal Cabin Model is based on conservation of heat. This model calculates the average temperature of the cabin. With the CFD code HEAT, the position dependent information of the simulation has been calculated.
Independent of the sensor position and the parameters of the PID-controller, the average temperature of the nine measure points is similar to the sensor temperature.
The results of the Thermal Cabin Model are similar to the results of HEAT in some special cases only. An investigation of the influence of the PID parameters k_p and k_i has been done. The results of the Thermal Cabin Model are similar to the results of HEAT if the controller parameters are small. The maximum value of these controller parameters are dependent on the position of the sensor.
Only if the variations in the cabin temperature are small, the results of the Thermal Cabin Model are similar to results of HEAT. So the Thermal Cabin Model is not useful in all dynamical simulations.

Bibliography

[1] Kortleve, E.; *Numerical simulation of aircraft cabin ventilation*, NLR Memorandum IW-2001-022, The Netherlands, 2001

[2] Kremer, G.A.; *Coupling of dynamical systems: a literature survey.*, NLR Memorandum IW-2002-016, The Netherlands, 2002

[3] Kremer, G.A.; *Numerical simulation of aircraft cabin ventilation (II)*, NLR Memorandum IW-2002-008, The Netherlands, 2002

[4] User manual of Matlab/Simulink version 4.1. An online version of this manual can be found at the Internet address: `http://www.mathworks.com/`

[5] Veldman, A.E.P.; *Numerieke stromingsleer*, lecture notes, R*u*G, The Netherlands, 1994

Appendix A

HEAT as S-function

In Simulink, a part of Matlab, it is possible to have very complicated models. To combine different models in Simulink you only have to have Simulink blocks which you can connect to each other. We have a program called HEAT, which we want to integrate in a Simulink model. But the form of HEAT is not correct. HEAT is not a Simulink block. This appendix is divided in three parts. First some general properties of an S-function are described. Secondly, special adaptations to HEAT are presented. And finally an implementation of the S-function in a Simulink model is given.

A.1 Description of an S-function in Simulink

To use HEAT in Simulink you have to make an S-function of HEAT. There are some requirements for an S-function. An S-function does not have a main program, but contains subroutines only. An S-function is made of two files: `simulink.F` and another file called `'ownfile'.f`. Compiling these files together results in a mex-file. The file `simulink.F` and examples of `'ownfile'.f` can be found in: `<matlabroot>/simulink/src/`. The global structure of both files should be preserved, but additions to these files are possible.
During the simulation the subroutines in the file `simulink.F` are called by the Simulink model. The arguments Simulink passes to an S-function are:

- **t**, the current time
- **x**, the state vector
- **u**, the input vector
- **flag**, an integer value that indicates the task to be performed by the S-function.

The value of the flag prescribes which subroutine of `'ownfile'.f` is called in the file `simulink.F`. Possible values of the flag and their descriptions are given in Table A.1.

The Fortran file `'ownfile'.f` contains the following subroutines:

- SIZES(SIZE)
- OUTPUT(T, X, U, Y)

Flag	Description
0	Defines basic S-function block characteristics.
1	Calculates the derivatives of the continuous state variables.
2	Updates discrete states, sample times, and major time step requirements.
3	Calculates the outputs of the S-function.
4	Calculates the time of the next hit in absolute time.
9	Performs any necessary end of simulation tasks.

Table A.1: Overview of possible flags.

- INITCOND(X0)
- DERIVS(T, X, U, DX)
- DSTATES(T, X, U, XNEW)
- DOUTPUT(T, X, U, Y)
- TSAMPL(T, X, U, TS, OFFSET)
- SINGUL(T, X, U, SING)

Table A.2 gives an overview of the flags with its corresponding subroutines. That means, which subroutine of `'ownfile'.f` is called by `simulink.F` if the flag has a certain value.

Flag	corresponding subroutine
0	SIZES(SIZE), INITCOND(X0)
1	DERIVS
2	DSTATES
3	OUTPUT, DOUTPUT
4	TSAMPL
5	SINGUL
9	-

Table A.2: Overview of flags and corresponding subroutines.

It is noticed that the value five is not defined in Table A.1. This value is not defined in the Matlab User Guide [**4**]. The influence of this value is not studied yet. If the flag is nine no subroutine of `'ownfile'.f` is called. The subroutines in Table A.2 are necessary for the S-function because they are called in the file `simulink.F`. On the other hand it is possible to change the subroutines. It is possible to add your own subroutines in this Fortran file also. These subroutines are used in the computation of the S-function if they are called in the standard subroutines. We give a short description of the used subroutines: SIZES, OUTPUT, and INITCOND. The changes of these subroutines if we want to make an S-function of HEAT are described in the next section. The other subroutines are not used in the S-function of HEAT. It means calling these subroutines, the S-function has nothing to do.

SIZES

This routine returns a vector which determines model characteristics. This vector contains the sizes of the state vector and other parameters. More precisely:

SIZE(1): number of continuous states

SIZE(2): number of discrete states

SIZE(3): number of outputs
SIZE(4): number of inputs
SIZE(5): number of discontinuous roots in the system
SIZE(6): set to 1 if the system has direct feed through of its inputs, otherwise 0

OUTPUT
This is the routine which is called if Simulink takes a time step. If the Simulink model calls the Simulink block, the commands in the subroutine OUTPUT will be executed.
INITCOND
This subroutine is executed only at $t = 0$.

A.2 Adaptations to HEAT

To use HEAT as a Simulink S-function, we have to change HEAT:

- We split the PROGRAM HEAT in three parts: the initialization, the time loop, and the termination. An S-function contains only subroutines, and so these three parts have been placed in the subroutines INITCOND, HEATPROGRAM, and STOPHEAT respectively. These subroutines are called by the file `simulink.F`. The flag, presented in Table A.2 prescribes which subroutine is called. The flags corresponding to the mentioned subroutines are zero, three, and nine respectively. In this case the `'ownfile'.f` is called `heatsfunctionv6.f`.

- Some variables are doubly defined, both in the Simulink part and in the HEAT part. The following Simulink variables are used in HEAT also:
  X: a vector of the states of the S-function
  X0: a vector of the initial values of the state
  U: a vector of the inputs of the S-function
  Y: a vector of the outputs of the S-function
  T: a scalar which contains the current time
  In the Simulink part these variables have been changed in X_S, X0_S, U_S, Y_S, and T_S respectively. In HEAT, the variables are unchanged.

- The subroutine SIZES is defined as:

```
SUBROUTINE SIZES(SIZE)

INTEGER*4     SIZE(*)
INTEGER*4     NSIZES
PARAMETER     (NSIZES=6)

SIZE(1)=0  {number of continuous states}
SIZE(2)=0  {number of discrete states}
SIZE(3)=19 {number of outputs}

SIZE(4)=3  {number of inputs}
SIZE(5)=0  {number of discontinuous roots in the system}
SIZE(6)=1  {set to 1 if the system has direct feed through of its inputs,
            otherwise 0}
```

```
RETURN
END
```

The outputs are defined in the subroutine HEATPROGRAM. The first nine outputs are the absolute velocities at the nine measure points. The following nine outputs are the temperatures at these nine points. The last output is the average temperature of the nine measure points. The three defined inputs at SIZE(4) are respectively, the mass flow, the nozzle temperature and the cabin pressure. HEAT is a stateless program. So the S-function block is a direct feed through block. The value of the parameter SIZE(6) is set to 1. A possibility to give HEAT a state may be studied later.

- Before HEAT takes a time step, the time of HEAT is compared to the time of Simulink. HEAT only advances if T_S greater than T. The Simulink model does not have influence on the time step of HEAT.

- A Simulink block needs inputs (U_S) and outputs (Y_S). Firstly, those have been added in the time loop. If the time step of the Simulink model is smaller then the time step of HEAT, HEAT does not make a time step, so that the output would be undefined. This problem is solved by declaration of the outputs outside of the time loop, but inside the subroutine HEATPROGRAM. The structure of the subroutine HEATPROGRAM is given below:

READ THE INPUT

```
mflow_nh=U_S(1)
Tnh=U_S(2)
pres_cab=U_S(3)
     TnH=TnH-273
     rho=pres_cab/(287*(Tav+273))
     Vnh=mflow_nh/(2*14*Dnh*rho)
```

TAKE A TIME STEP

```
DO WHILE (T_S.GT.T)
   T = T + DelT
   {calculation of the flow}
ENDDO
```

CALCULATE THE OUTPUT

```
Tav=0.0
DO n=1,nrh
  i=pdi(n)
  j = pdj(n)
```

```
          UAV = .5*(U(i-1,j) + U(i,j))
          VAV = .5*(V(i,j-1) + V(i,j))
          SGU = Alpha*SIGN(1.0,UAV)
          SGV = Alpha*SIGN(1.0,VAV)
          Ugm = .5*(U(i-1,j) + U(i,j) + SGU*(U(i-1,j) - U(i,j)))
          Vgm = .5*(V(i,j-1) + V(i,j) + SGV*(V(i,j-1) - V(i,j)))
          norm = sqrt(Ugm**2 + Vgm**2)
          IF (Igdn.NE.0) THEN
            IF (norm .LT. 0.05) THEN
              PD=0.
            ELSE
             PD=(34.-Temp(i,j))*((norm-.05)**0.62)*(0.37*norm*sig+3.14)
            ENDIF
          ENDIF

          IF (Icabin.NE.0) THEN
            IF (PPDform.eq.1) THEN
              IF (norm .LT. 0.05) THEN
                PD=0.
              ELSE
               PD=(34.-Temp(i,j))*((norm-.05)**0.62)*(0.37*norm*sig+3.14)
              ENDIF
            ENDIF
            IF (PPDform.eq.2) THEN
              IF (norm .LT. 0.04) THEN
                PD=0.
              ELSEIF (Temp(i,j) .LE. 13.7) THEN
                PD=100.
              ELSE
                PD = 13800.*(((norm-.04)/(Temp(i,j)-13.7)+0.0293)**2 -
     >             0.000857)
              ENDIF
            ENDIF
          ENDIF
          Y_S(n)=norm
          Y_S(nrh+n)=Temp(i,j)+273
C         IF (PD.GT.100) PD = 100.
C        WRITE(unrh-1+n,'(e11.3,f9.3,2e11.4)') T,PD,norm,Temp(i,j)
          Tav=Tav+Temp(i,j)
        ENDDO
        Tav=Tav/nrh
        Y_S(2*nrh+1)=Tav+273
```

Independent of both time steps, Simulink model time step and HEAT time step, always the output of the S-function is defined. In Figure A.1 it is shown which information is used to solve the next time step.

A.3 Implementation in the Simulink model

To compile the created S-function we have to start Matlab. We give the command:
mex heatsfunctionv6.f simulink.F

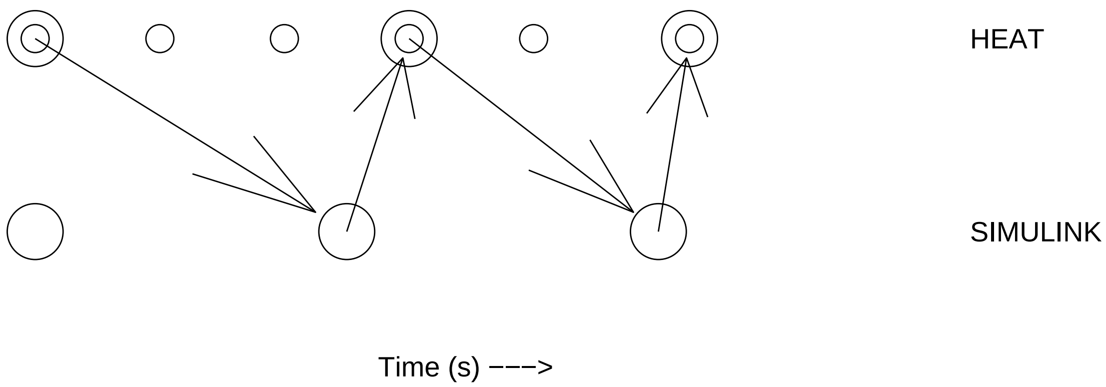


Figure A.1: This figure shows which time step of the other system is used as input. The great circles are the coupling time steps, the small circles define a time step of HEAT.

platform	extension
sol2, SunOS 5.x	`.mexsol`
hpux	`.mexhpux`
hp700	`.mexhp7`
ibm_rs	`.mexrs6`
sgi	`.mexsg`
alpha	`.mexaxp`
glnx86	`.mexglx`
Windows	`.dll`

Table A.3: Overview of the extensions of the mex-file at the different platforms

The compiler creates a file, `heatsfunctionv6`, with an extension dependent on the used platform. The possible extensions are shown in Table A.3.
The mex command calls a Fortran compiler. It is possible to choose another compiler. You have to copy the file `mexopts.sh` to a directory in your home called `<home>/.matlab/R12`. The file `mexopts.sh` is set in the directory: `<matlabroot>/bin/`. In this file the used compiler on each platform is presented. It is possible to change the compilers in this file.
Now we can use the mex-file as a Simulink block. An S-function block is standard in Simulink and showed in Figure A.2. Double clicking this block displays the block parameters as shown in Figure A.2. We rename this S-function block into heatsfunctionv6. To get the right interface a subsystem in Simulink has been made. This subsystem is shown in Figure A.3.
Figure A.3 shows the S-function heatsfunctionv6 with its inputs and outputs. Only one line in connected to the input and the output. It means that the input and the output of the S-function are defined as a vector. The data computed during the simulation is send to a file, `data.mat`. This file is a matlab file. Loading this file in Matlab gives a matrix with 23 rows. The first row of this matrix contains the time and the other rows the output data as given in the following order: on rows two to ten the absolute velocity of the nine measure points, on the rows 11 to 19 the temperature of the nine measure points, on row 20 the mean temperature of the nine measure points. On row 21 the nozzle temperature. On row 22 the cabin pressure and on the last row the mass flow. It is possible to use the uncoupled subsystem. After setting constants at the input ports and displays at the output ports you

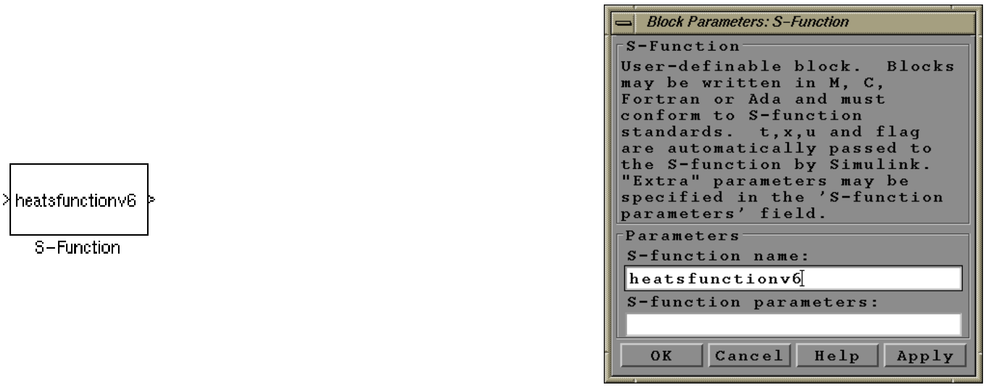


Figure A.2: S-function block with parameters.

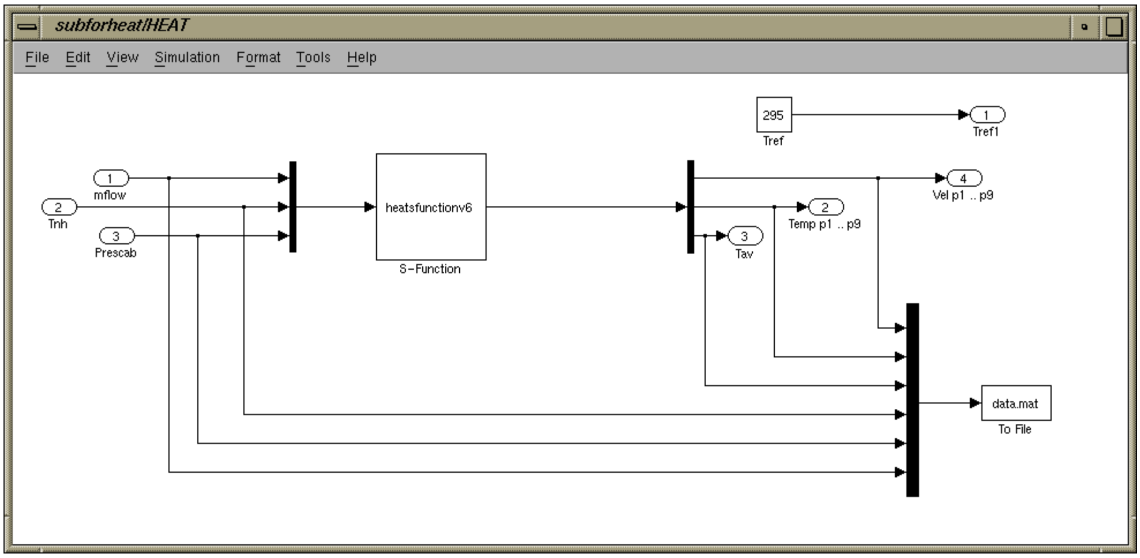


Figure A.3: Subsystem used in the complete ECS model

can run HEAT. The files you need in the current work directory to run HEAT are:

1. A Simulink model which contains the subsystem HEAT. This subsystem is shown in Figure A.3
2. The HEAT input files which are used always: `heat97-4.in` and `heat97.geo`
3. The executable which is made by the mex compiler: `heatsfunctionv6.mexsg`

It is possible to place this executable in the directory `<home>/matlab/` also. Now HEAT can be coupled to a greater system. The ECS-system is an example of a very complex controller model. We can connect the subsystem to the complete model. The model is suitable for continuous solvers only, because it contains blocks which are solved with a continuous solver. HEAT is a discrete model. Coupling of both systems does not give problems. The total system is shown in Figure A.4.

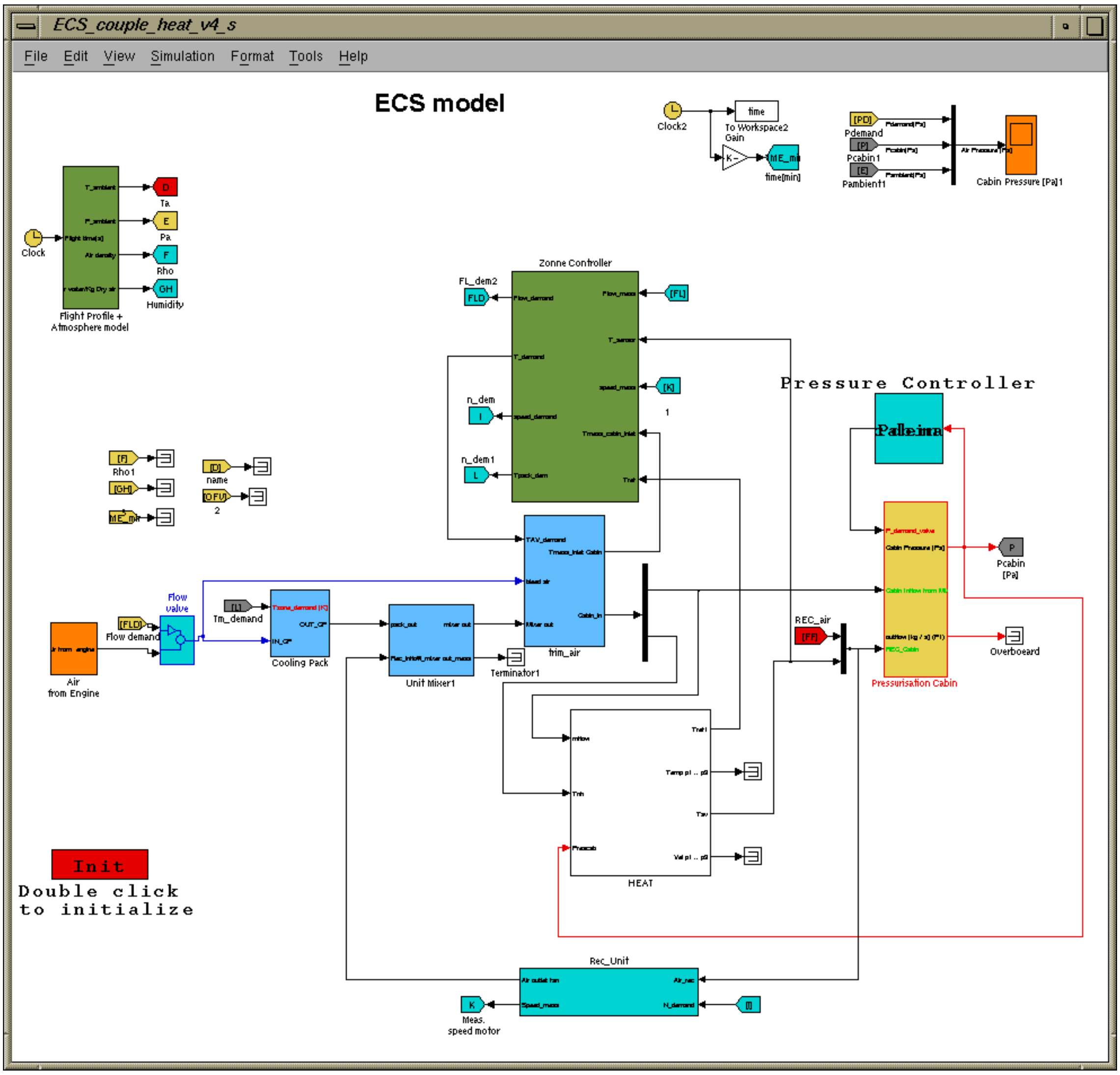


Figure A.4: The complete ECS model

Appendix B

Graphical results

This appendix contains the graphical results of the simulations with the sensor at p7, described in Section 5.3. The average temperature of the results found with HEAT are compared to the average temperature of the Thermal Cabin Model. The results are presented.

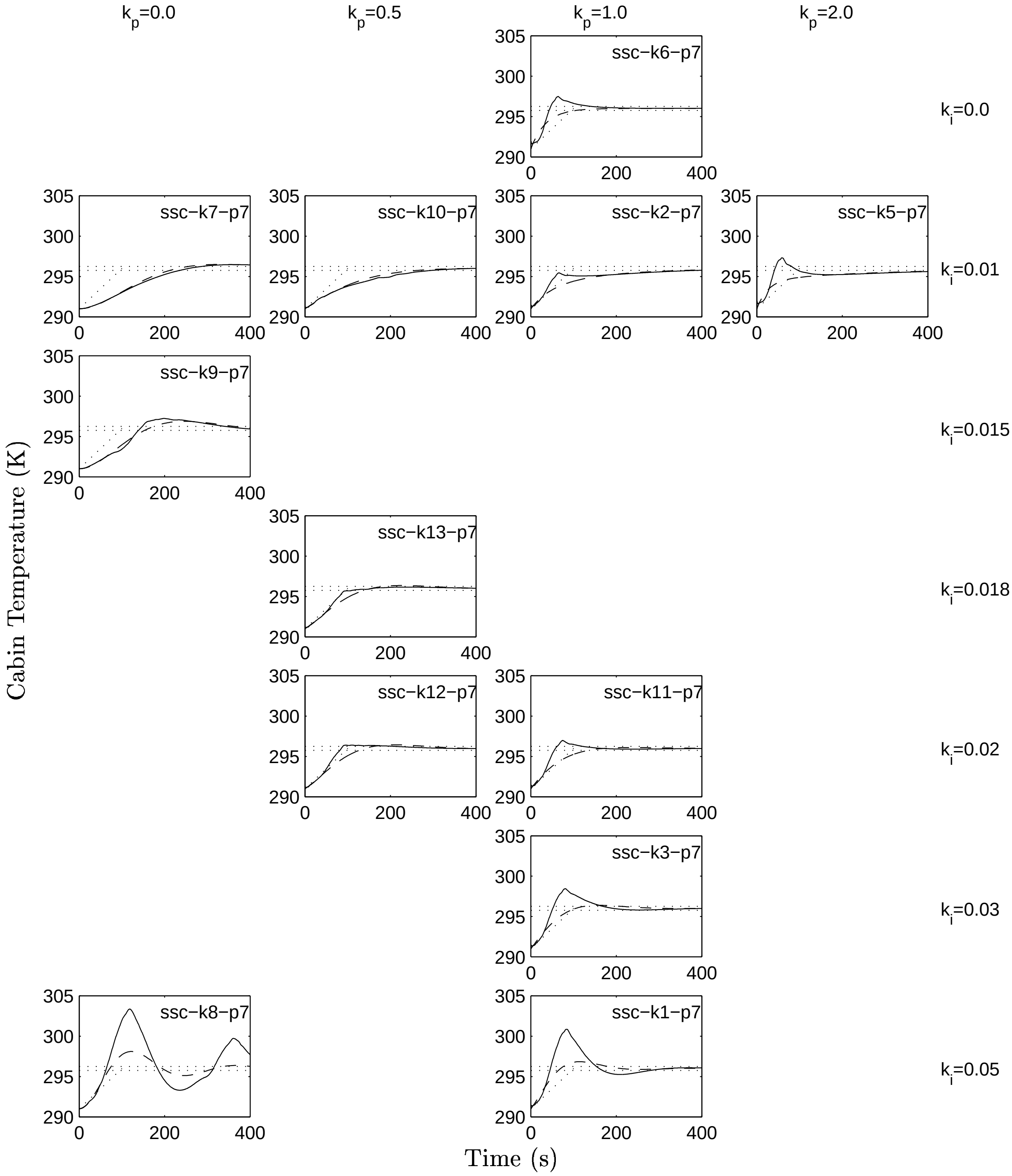


Figure B.1: The average temperature of HEAT compared to the average temperature of the Thermal Cabin Model of simulations with the sensor at p7. (– – Average temperature of the Thermal Cabin Model, – Temperature of HEAT as determined by taking the average at nine sensor locations, ··· Requirements for an optimal behavior of the temperature)

Appendix C

Accuracy of the coupling of HEAT with the PID-controller

Δt	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
$\lvert u\rvert_1$	0.0269	0.0171	0.0105	0.0063	0.0041	0.0023	0.0012	0.0006
$\lvert u\rvert_2$	0.0091	0.0092	0.0071	0.0050	0.0031	0.0018	0.0009	0.0004
$\lvert u\rvert_3$	0.0039	0.0054	0.0046	0.0030	0.0018	0.0009	0.0005	0.0002
$\lvert u\rvert_4$	0.0146	0.0099	0.0080	0.0052	0.0033	0.0018	0.0010	0.0005
$\lvert u\rvert_5$	0.0107	0.0137	0.0119	0.0078	0.0046	0.0024	0.0012	0.0006
$\lvert u\rvert_6$	0.0069	0.0045	0.0027	0.0016	0.0010	0.0005	0.0003	0.0001
$\lvert u\rvert_7$	0.0055	0.0057	0.0050	0.0033	0.0020	0.0011	0.0006	0.0003
$\lvert u\rvert_8$	0.0024	0.0032	0.0025	0.0015	0.0009	0.0005	0.0003	0.0001
$\lvert u\rvert_9$	0.0012	0.0007	0.0004	0.0002	0.0001	0.0001	0.0000	0.0000
T_1	0.4943	0.3237	0.2012	*0.1162*	*0.0639*	0.0338	0.0174	0.0083
T_2	0.3599	0.2917	0.2583	*0.1643*	*0.0872*	0.0425	0.0206	0.0099
T_3	0.2547	*0.1603*	*0.0946*	0.0543	0.0290	0.0153	0.0078	0.0042
T_4	0.4467	0.3638	0.2626	*0.1696*	*0.0952*	0.0516	0.0275	0.0128
T_5	0.3798	0.3856	0.2841	0.1726	*0.1119*	*0.0624*	0.0328	0.0158
T_6	0.2559	*0.1657*	*0.0983*	0.0564	0.0303	0.0159	0.0082	0.0039
T_7	0.3039	0.2338	*0.1609*	*0.0932*	0.0532	0.0292	0.0152	0.0073
T_8	0.2425	0.1934	*0.1362*	*0.0848*	0.0467	0.0252	0.0134	0.0062
T_9	0.2715	*0.1526*	*0.0956*	0.0555	0.0305	0.0163	0.0086	0.0042
T_{av}	0.2657	0.1766	*0.1089*	*0.0629*	0.0345	0.0183	0.0094	0.0047
T_{nh}	0.3072	0.1841	*0.1185*	*0.0721*	0.0406	0.0217	0.0111	0.0055

Table C.1: Difference between the results of simulations with different coupling time steps (Equation 6.2 at page 34). The results used in the last example in Section 6 have been printed in italics.