Modelling the rowing stroke

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RuG

Master's Thesis

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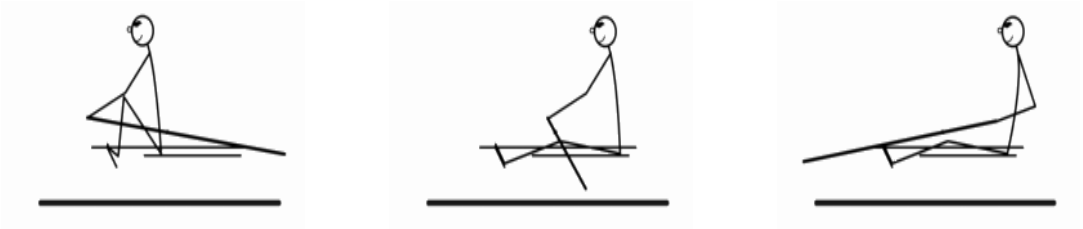
Introduction



It's very interesting to observe a rowing-match. The rowers try to get their boat, with only the help of a set of oars[27], to the finish as fast as possible. Relative to the shore, the rowing boat, or racing shell[38], moves entirely towards the finish, whereas relative to the boat the rowers move on their sliding seats both forwards and backwards.
For a matter of fact, first it's important to make an assumption about the meaning of the terms 'forward' and 'backward'. The word 'forward', or 'to the right' will be taken to mean the direction in which the boat is moving (in the direction of the finish) and 'backward' or 'to the left' will be taken to mean the opposite direction.
When the rowers slide towards the bow[6] of the boat, their oar blade(s) and a part of the oar itself are in the water. In this period, the rowers exert, to give the boat speed, forces to the handles through their hands and to the foot stretchers[18] through their feet. This phase in rowing is called the *power stroke*. The complete rowing stroke can be divided into four parts:

- The *Catch*: "the placement of the blade into the water"
- The *Drive* or *Power Stroke*: "the blades of the oars are in the water and the rowers pull on the oar handle(s) and straighten their legs, thus moving their bodies toward the bow on their sliding seats"
- The *Finish*: "the rower pushes down on the oar handle(s) to make it come out of the water"
- The *Recovery Phase*: "the blades are clear of the water and the rowers move stern-wards by bending their legs and leaning forward"

Still, rowing might not seem as complicated as it actually is. But the opposite is true; a great number of aspects can influence the way of rowing in one way or another. For example, adjustments on oars and rowing boat will have effect on the forward motion of the boat.

This research contains an analysis of the rowing stroke and in particular the power stroke. A two-dimensional mathematical model is set up to represent the consequences on several aspects of rowing by changing variables. Especially the effect of changing parameters regarding to oar and blade characteristics on, for example, the boat velocity has been investigated.

To create a mathematical model the following is done.
Firstly, to get an impression, a geometrical model of the orbit is given to describe the way a rowing boat and, in specific, the rowing oar and blade move through the water. The geometric model is described in chapter 1. In the second chapter, the hydrodynamical aspects of rowing are investigated. A presentation of the effects of the (hydrodynamic) forces which operate during the rowing of racing shells is given. Chapter 3 shows a differential equation, the equation of motion for rower and boat. With the help of this equation the boat velocity, as a function of time, can be obtained. The geometric model, together with the hydrodynamic model and the equation of motion describe the rowing stroke. Chapter 4 contains information about rowing *efficiency*. Terms like *energy*, *work* and *power* play an important role. The equations for the rowing stroke and efficiency form the mathematical model. A review of the mathematical model is given in chapter 5.
Of course, input data is needed for the model that is set up. Chapter 6 discusses the data that is required to produce a realistic model. Chapter 7 describes the numerical method which is used to solve the model.

A computer code is set up to find solutions of the numerical model. By the use of this computer code, it's possible to obtain, for example, the boat velocity. Besides this, also the effect of different input parameters on for example the blade velocity, the position of the boat and blade, the drag of the water on the boat and blade, the strength of a rower and other desired parameters can be obtained. The appendix contains a description of the computer program in detail.

Chapter 8 deals with test cases. The obtained results are discussed and conclusions are made (chapter 8, 9). On account of these conclusions, statements are made on what further research can be done to improve the model (chapter 9).

Used terms are explained in a glossary and a list of the applied variables is given. These can be found in the appendix.

As already said, many factors exert their influence on the motion of the boat in one way or another. For instance, adjustments in the type of oar and blade. Different blades and different oars lead to different output results. The sort of boat affects the movements as well. Also the number of rowers and with that the physical condition of the rowers have an important contribution. It is a matter of course, weather conditions are important too; the strength of the wind and next to this the strength of the water flow. With increasing water temperature, the shell speed seems to increase also [9].

It is obvious, not all these factors and others could be investigated and besides this, changes in a variable of interest cannot be evaluated, while all others are held constant. It is very complicated and hardly impossible to build a perfect model that exactly describes the real situation.

In the current research many of the parameters are considered to be given. The values of the parameters are found in literature, some of them acquired by experiments. In this way, an attempt has been made to set up a more or less realistic model. The main goal of this research is to give people a better insight in the complexity of rowing and besides this it might help coaches and other researchers to get a feeling on what adjustments they could make to improve the way of rowing, i.e. to give the boat maximal velocity with a minimum of rower power input.

Chapter 1

Geometric Model

The purpose of this chapter is to get an idea of the behaviour of the boat and especially the oar and the oar blade in the water. Equations are set up to describe their displacement as a function of time. As already said in the introduction, it's not easy to obtain correct equations, because many parameters strongly depend on each other. An important factor with regard to this is the movement of the water by moving the blade through the water. This disturbance of the water has great influence on the parameters.
Still, an attempt has been made to give a sufficient representation of the situation.
A more detailed description will be given in the report about this subject, *Het Geometrisch Model van de Roeibeweging* [8].

1.1 The Oar

To get an impression of the movements, consider this much-simplified representation of an oar[27] outside the boat (for those who are not familiar with rowing, see Fig. B.1 and Fig. B.2 of the appendix).

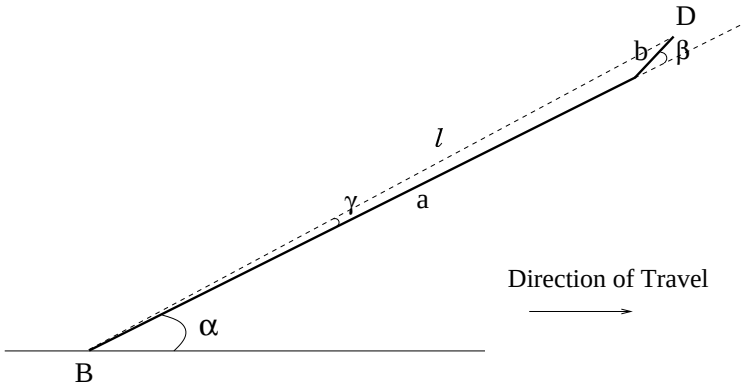


Figure 1.1: *Oar with Blade*

The oar is fixed to the boat at point B by an *oarlock*[28], a device which swivels around the rigger pin and holds the oar in place (see appendix, Fig. B.2). The distance from the oarlock to the place where the blade is fixed to the oar is defined as the *outboard length*[29] a. The

letter b denotes the length from the end of the oar to the so-called *center of pressure*[9]. This is a point along the blade in which the resultant of the combined pressure forces acts. The center of pressure is located in point D and its position is not on the tip of the blade but approximately one-third from the tip [18]. More about this point in the next chapter.
Usually, there's a (small) angle between the oarshaft axis and the blade chord. This angle, often about 6 degrees, is showed in Fig. 1.1 and is defined as the *blade cant angle*[5] β. This angle may be of significance to guide the flow in a particular way. The angle α denotes the angle between the boat and the oarshaft axis, which is defined as the *bow angle*[7].
Fig. 1.1 also shows a distance l, which denotes the absolute distance between B and D:

$$l = \sqrt{a^2 + b^2 + 2ab\cos\beta} \tag{1.1}$$

The angle γ is the angle between l and a:

$$\gamma = \arcsin\left(\frac{b\sin\beta}{l}\right) \tag{1.2}$$

In this simplified situation, the oar and blade are depicted by only two straight lines. In practice, one could imagine that the oar will bend a little during the rowing drive. According to Brearly, de Mestre and Watson [7], the effects of oar flexing on the conclusions are small and therefore oar flexing is neglected in this model. Naturally, the weight, thickness and shape of the blade and oar have their effect on parameters like the boat velocity and blade velocity also, but their effects will be omitted in this report. On the other hand, the length of the oar and the size of the blade will be examined. More about this in chapter 8.

1.2 Slip and Tractrix

Before anything can be said about oar and blade displacements, it's important to know something about an important factor, namely the so-called blade *slip*[39].
Slip is defined as the distance traversed in the water by the center of pressure of the blade parallel to the direction of the hull. When the rowers pull their blades through the water, the propulsion-force is applied to an unsteady medium (water). As a result, water molecules start to move and through that, energy (ch. 4) is lost. Because of this, the blade doesn't travel the path a rower really wants it to go. The traveled distance by the oar blade in forward direction, is much less than if there was no slip (see Fig. 1.2). In other words, now the velocity component of the center of pressure perpendicular to the blade is not zero. This becomes clear from Fig. 1.4 in §1.3.
In an optimal situation (no slip), the blade follows an ideal path, the *zero-slip path*. The zero-slip path is the path created under zero load at the handle, that is , the path followed by a flat-bladed oar dropped, in a moving boat, into the water at the catch angle and simply allowed freely to float or drift through to the release angle.
In reality however, due to the rower's effort at the handle, the blade slips continuously, increasingly departing from the zero-slip ideal path as the stroke progresses. The center of pressure of the blade traces a pattern as seen from above that looks somewhat like a comma. For a better understanding, take a look at the Fig. 1.2.

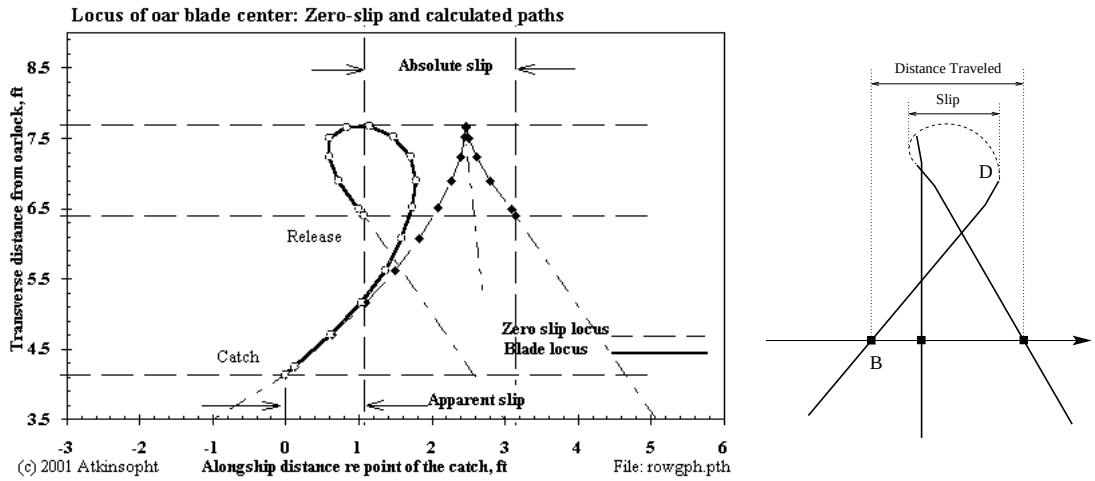


Figure 1.2: *Left: Zero-Slip Path and Slip Path (8+ Heavy Men, from Akinson [9]), Right: Oar and Blade with Slip Path*

The left figure shows the path point D traveled: the unreal optimal (zero-slip) path (black squares) and the path that matches with reality (white squares). The (longitudal) distance which a blade covered from the catch to the release (in Fig. 1.2 it is denoted by *apparant slip*) would be larger if the blade doesn't suffer a large and unseen *absolute slip*. The illustration on the right shows the real (slip) path of the center of pressure again, together with the oar at three successive moments.

The theoretical zero-slip path forms a so-called *tractrix*[49]. A tractrix (equitangential curve, tractory) is a curve, such that any tangent segment from the tangent point on the curve to the curves asymptote has constant length. Below an example of a tractrix. The tangent segment with constant length may be considered to be the oar.

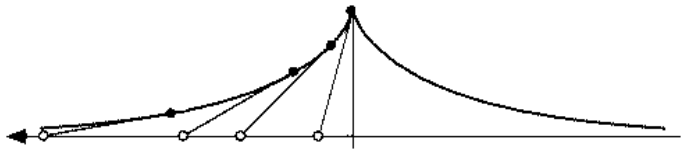


Figure 1.3: *Tractrix*

For more information about this mathematical phenomenom, see the bibliography [20] on page 67.

1.3 Angular Velocity, Blade Velocity and Slip Angle

If the blade slip is taken into account, one can consider the following.

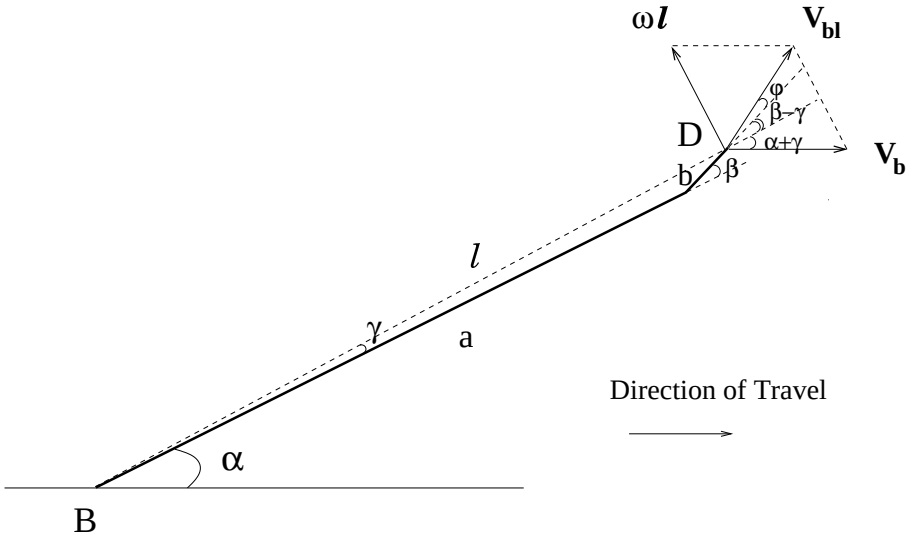


Figure 1.4: *Oar and Blade Displacements*

Fig. 1.4 shows a Greek letter φ, which denotes the angle between the plane of the blade and the direction of fluid motion, in other words the *slip angle* or *angle of attack*[2].
The figure also shows three vectors $\boldsymbol{V_b}$, $\boldsymbol{V_{bl}}$ and $\boldsymbol{\omega l}$. The boat and blade velocity are being denoted by $\boldsymbol{V_b}$ and $\boldsymbol{V_{bl}}$ respectively. The boat velocity points in forward direction and can be obtained with the help of a differential equation (see chapter 3). The blade velocity points in the direction of the fluid motion. It can be obtained by the vector sum of the boat velocity and the vector $\boldsymbol{\omega l}$. The Greek letter $\boldsymbol{\omega}$ denotes the angular velocity of the oar. In other words, the rate of change of angular position of the oar relative to the boat ($\boldsymbol{rad/s}$).
$\boldsymbol{\omega l}$ is the rate of change of the position of $\boldsymbol{D}$ perpendicular to $\boldsymbol{l}$. In other words, the center of pressure travels a distance $\boldsymbol{l d\alpha}$ in a time $\boldsymbol{dt}$.

With the help of Fig. 1.4 blade velocity, angular velocity and slip angle can be derived as follows. For the blade velocity holds

$$\cos(\beta - \gamma + \varphi) = \frac{V_b \cos(\alpha + \gamma)}{V_{bl}} \quad \Rightarrow \quad V_{bl} = \frac{V_b \cos(\alpha + \gamma)}{\cos(\beta - \gamma + \varphi)} \tag{1.3}$$

Since there's a situation at which the denominator becomes zero, which causes problems (ch. 7), the expression for the blade velocity is written as

$$V_{bl} = \omega l \sin(\beta + \varphi - \gamma) + V_b \cos(\alpha + \beta + \varphi) \tag{1.4}$$

as well.

The equation for the angular velocity holds

$$\begin{aligned} \omega l &= V_{bl}\sin(\beta-\gamma+\varphi)+V_b\sin(\alpha+\gamma) \quad \Rightarrow \\ \omega &= \frac{1}{l}(V_{bl}\sin(\beta-\gamma+\varphi)+V_b\sin(\alpha+\gamma)) \end{aligned} \tag{1.5}$$

Finally, the slip angle can be determined as follows

$$\varphi = \arctan\left(\frac{\omega l\cos(\alpha+\gamma)}{V_b-\omega l\sin(\alpha+\gamma)}\right)-\alpha-\beta \tag{1.6}$$

This chapter showed a few apects of boat, blade and oar movements. An important factor which affects these movements is blade slip. A typical mathematical aspect, the tractrix, has been treated. When the angular velocity increases, while the boat velocity is kept constant, an increase in blade velocity follows. As a consequence, the slip angle increases as well. Unfortunately, this slip causes the blade losses (see *blade efficiency* in chapter 4) and the advance of the shell per stroke. Thus, the contribution of slip should be as small as possible. In the next chapter, slip will be considered as a result of (hydrodynamic) forces.

Chapter 2

Hydrodynamic Model

2.1 Shell Forces

Drag Force
When a hull moves through the water, it will experience a retarding force known as resistance or *drag*. The major forces that resist the motion of the boat are the *hydrodynamic*[21] drag and the *aerodynamic*[1] drag, the former being responsable for about 90% of the total drag. The science of hydrodynamics is concerned with the behavior of fluids in motion.
The hydrodynamic drag is the sum of the so-called *wave* drag (due to energy lost in creating waves) and *viscous* drag. The latter is the sum of the *skin* drag (due to friction between the hull entraining water along with the hull) and *form* drag (due to turbulence created by the passage of the hull).

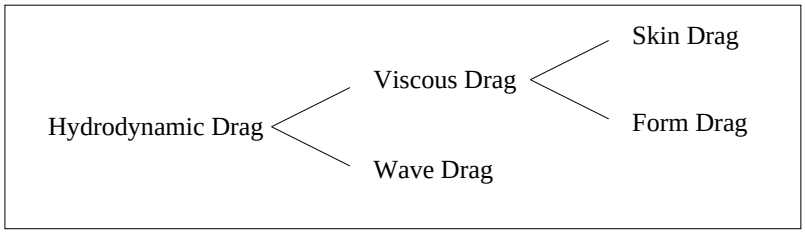


Figure 2.1: *Drag Forces*

For racing shells, the skin drag is the major source of resistance (about 80%). For most other craft wave drag dominates [12].

Air contributes to the total drag in similar ways. While the contribution from still air is only a few % of the water resistance, air velocity is much more variable; the contribution can rise to 10s of % in strong head winds [12]. However, since in general, this aerodynamic drag plays a much smaller part compared to the drag of the water, this contribution is neglected in the analysis.

Propulsive Force
The total force that drives the boat forward against the drag of the water is the sum of the

combined force exerted on the oarlocks and the force exerted by the rowers on their footrests[17]. In addition, the exerted force on the oarlock is the sum of the exerted force on the handle by the rower and the hydrodynamic force on the blade.
More information about the forces which contribute to the boat propulsion will be discussed in the next chapter. This chapter is primarily concerned with the (hydrodynamic) forces that act on an oar blade.

To get an idea of which propulsive force an oarblade exerts upon the water, look at the oar[27] with blade below.

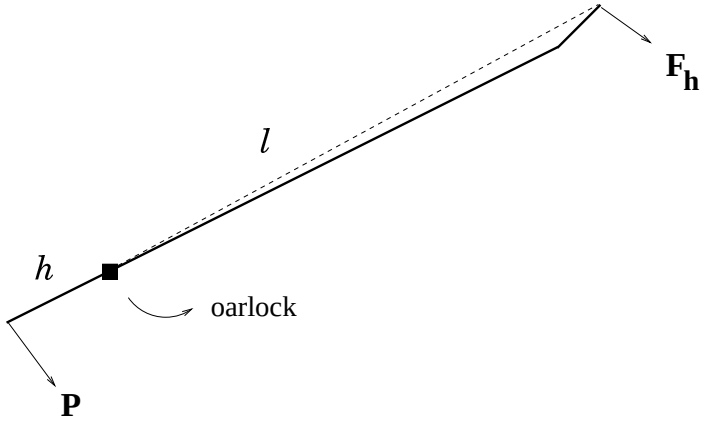


Figure 2.2: *Moments about the Oarlock*

In contrast to the oars in Fig. 1.1 and Fig. 1.4 in the previous chapter, now also that part of the oar, which is on the other side of the oarlock[28], is drawn. This distance is being denoted by h and is the distance from the oarlock to the center of the *oar handle*[20]. The latter is located at the end of the oar and is the place where the rower holds the oar in his hand (see Fig. B.1 and Fig. B.2 of the appendix). The oarhandle is a part of the so so-called *inboard oarlength*[22]. Distance l has been defined in section 1.1.
Two forces are depicted as well. $\boldsymbol{P}$ denotes the force applied by the rower on the oarhandle. The force which is exerted on the blade by the water is being denoted by $\boldsymbol{F_h}$, in other words the hydrodynamic force.
By considering moments[26] (torque) about the oarlock, the following equation holds.

$$\boldsymbol{h} \times \boldsymbol{P} = \boldsymbol{l} \times \boldsymbol{F_h} \tag{2.1}$$

Where $\boldsymbol{h}$ and $\boldsymbol{l}$ are considered to be position vectors, which act from a reference point, the oarlock, to the line of action of the forces $\boldsymbol{P}$ and $\boldsymbol{F_h}$ respectively. This relationship evidently holds throughout the power stroke, irrespective of the angle which the oar makes with the boat, since the oarblade has no appreciable acceleration with respect to the water. Through this, Newton's second law is applicable, which states that an object at rest or moving with constant velocity tends to remain at rest or in motion with constant velocity unless acted on by an outside force. So, the force $\boldsymbol{F_h}$ on the blade is uniquely determined by the oarhandle force $\boldsymbol{P}$ (and not by blade design).

Besides this *moment equilibrium*, there exists a *force equilibrium* (see page 20) for the boat and oar as well. This will be discussed in chapter 3.

2.2 Hydrodynamic Forces

The hydrodynamic force which acts on the oar blade can be split into two components, a so-called *lift*[25] (L) component which is perpendicular to the flow direction and a *drag*[12] (D) component which is in the direction of the flow.
When a plate is placed in a flow, one can consider the following.

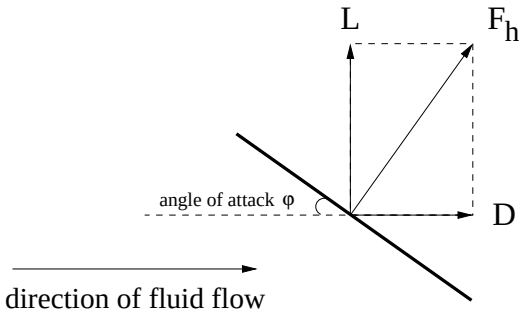


Figure 2.3: *Forces on a Flat Plate*

This is caused by Newton's third law, which states that '*for every action there is an equal and opposite reaction*'. In order to generate a lift force a blade must do something to the water. What the blade does to the water is the *action*, while lift is the *reaction*. In addition to the lift, the drag is a force directly opposing the motion of the blade through the water and is always present. The lift and drag forces may be considered to act at a fixed point with respect to the blade. This is the already mentioned *center of pressure*[9]. Its position moves with increasing angle of attack from the tip of the blade to approximately the center of the blade. Variation in surface area and curvature of the blade can change its position also. For simplicity, the position of the center of pressure is considered to be one-third from the tip of the blade [18].

Fig. 2.4 on the next page also shows the hydrodynamic force in the direction of motion of the boat, the propulsive force $F_{h,x}$.
It can be written as

$$F_{h,x} = L\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi) \tag{2.2}$$

F_h, the hydrodynamic resultant of the lift and drag forces, will always rise through slipping and satisfies eq. (2.1), in other words moment equilibrium. But if the lift and drag characteristics of the blade can be produced by a sophisticated blade design (of best coefficients), such that the propulsive force $F_{h,x}$ on the blade will be large, the slip velocity will be correspondingly small, i.e, more efficient; producing lower slip for a given applied force P. This is how lift and drag affect oarblades.

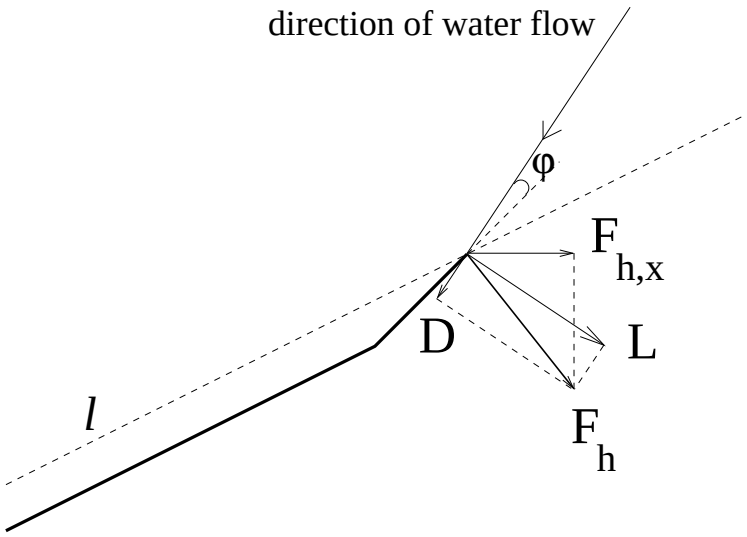


Figure 2.4: *Blade Forces*

For a given attitude of geometrically similar blades, the hydrodynamic forces L and D tend to vary directly with the water density (ρ), the blade surface area (A_{bl}) and the square of the blade speed (V_{bl}) [1].
It is accordingly convenient to express these forces in terms of nondimensional coefficients (c_L and c_D) that are functions primarily of the attitude of the blade. The lift and drag are given by the following expressions (note that the expression for L and D differ only in c_L and c_D).

$$L = \frac{1}{2}\rho V_{bl}^2 A_{bl} c_L \tag{2.3}$$

$$D = \frac{1}{2}\rho V_{bl}^2 A_{bl} c_D \tag{2.4}$$

A common way of describing the hydrodynamic characteristics of an oar blade is to plot the values of the nondimensional coefficients c_L an c_D against the angle of attack[2] φ (see Fig. 2.5), which was the angle between the plane of the blade and the direction of fluid motion (§1.2).
A problem arises because oar blade hydrodynamic characteristics have never been measured. Therefore, lift and drag coefficient data do not yet exist for oar blades. In airfoil theory, on the other hand, the aerodynamic characteristics of a wing have been measured, because this is quite easier. A reason for this is the hardly varying angle of attack. The airfoil is placed in an almost aligned, steady state homogeneous flow and therefore, the wing's attack angle is more or less the same, except in extreme cases such as 'mushing' in nose up for a landing. On the contrary, an oar blade is constrained by the oarlock and rower to follow a greatly varying path with respect to the water, manifesting attack angle changes through the entire 180 degree range. For this reason, data that comes from airfoil theory fails with respect to oar blades.
Because there's no data regarding to oarblades, data available for flat plates in water at various angles of attack is used. The data comes from Hoerner [9].

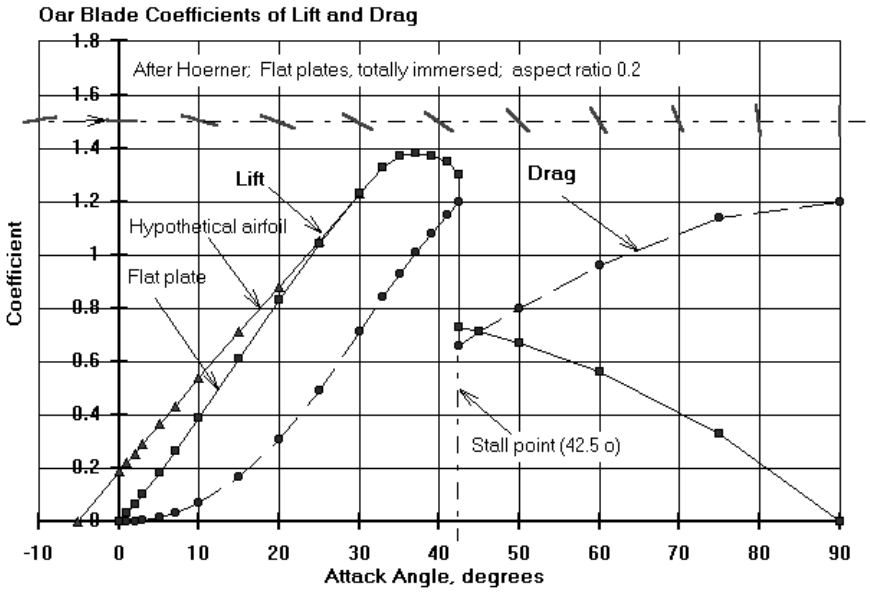


Figure 2.5: *Oar Blade Coefficients of Lift and Drag (from [9])*

Fig. 2.5 shows the plot of the available data for a typical flat plate in water. The lift coefficient c_L increases almost linearly with angle of attack until a maximum value is reached, whereupon the blade is said to *stall*[43]. At this stall point (the attack angle equals approximately 42.5°) the fluid flow is detached from the surface. These stall points for most airfoils are around fifteen degrees. To clear up this phenomenom, look at figure 2.6.

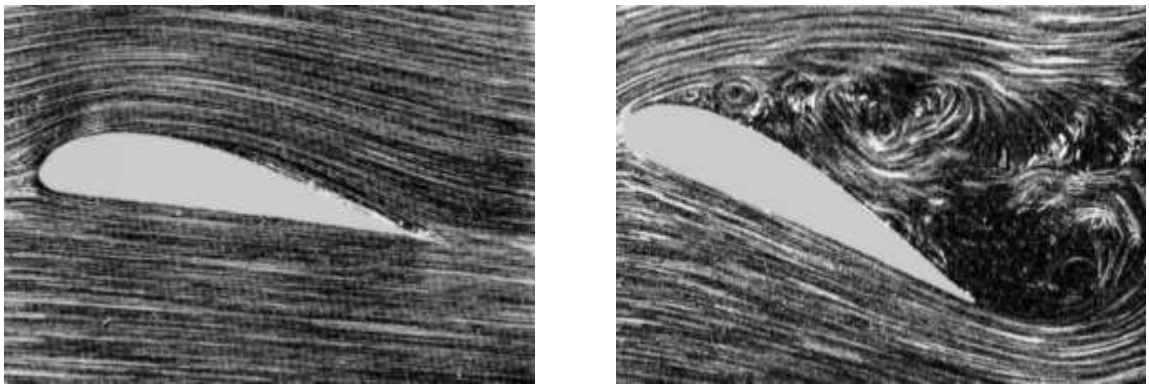


Figure 2.6: *Left: Attached flow around an airfoil at small angles of attack, Right: Separated flow around an airfoil at high angles of attack (stall)*

This figure shows the flow around an airfoil in two qualitatively different situations [21]. As usual, the flow comes from the left in both figures. The photograph on the left-hand side shows a fully attached flow. The flow field can follow the shape of the wing. A too large attack angle causes the situation showed in the photograph on the right-hand side. The inclination angle of the airfoil is increased, such that the flow around this airfoil is no longer able to follow the contour of the wing and the flow leaves the body contour. At this angle

the flow separates and the airfoil is said to stall. In consequence, a large region of separated flow forms behind the airfoil. Detailed information on this subject can be found in literature (e.g. [1], [21]).

Also the drag coefficient c_D is depicted in figure 2.5. It has a minimum value at a low lift coefficient and the shape of the curve is approximately parabolic at angles of attack below the stall. Only data for attack angles smaller than 90 degrees are depicted, while, as mentioned, the attack angle may reach values up to 180 degrees. This isn't a problem, because attack angles which vary from 90 till 180 degrees may also be considered to take values from -90 till 0 degrees. The coefficient data for c_L and c_D can be mirrored.

An additional remark has to be made on Fig. 2.5. Namely, there's another reason why lift and drag coefficient data from airfoil theory cannot be used in this case. The figure shows that the so-called *aspect ratio*[4] of the flate plate is taken at 0.2. It's important to take the aspect ratio into the consideration, since early wind-tunnel investigations of wing characteristics showed that the rates of change of the lift and drag coefficients with angle of attack were strongly affected by the aspect ratio of the model [1]. The aspect ratio is defined as the ratio of the span[41] squared to the blade area ($(span^2)/A_{bl}$), which reduces to the ratio of the span to chord[10] in the case of a rectangular blade ($span/chord$).
To make clear these terms, take a look at Fig. 2.7. Besides an simplified (rectangular) oar blade, also a top view of a (rectangular) wing is depicted.

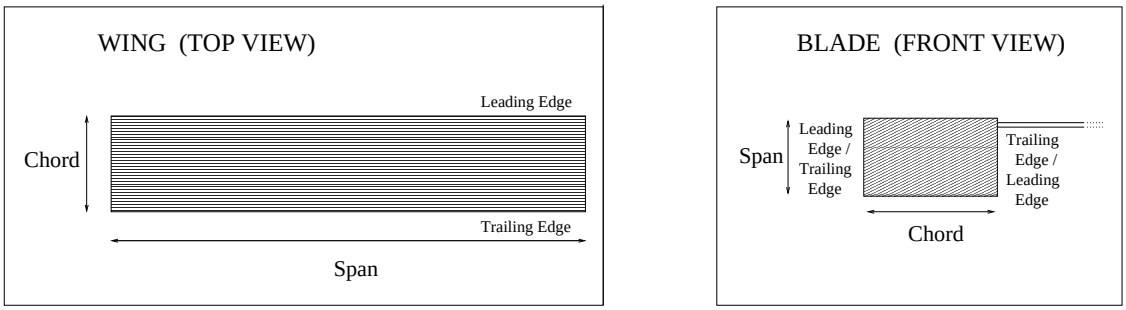


Figure 2.7: *Left: Top View of a Wing, Right: Front View of an Oar Blade*

The chord is the length from the leading edge to trailing edge. In the case of an oarblade, the leading edge (which is first, i.e. at the catch, defined to be at the edge at the tip of the blade) changes in the trailing edge (first located at the edge of the blade which is closest to the rower) and vice versa. This shift is due to the definition of the blade's attack angle. If the attack angle switches from $+90$ degrees to -90 degrees, the leading and trailing edge switch also. This often happens near the ninety-degree point of the oarshaft sweep.
From the illustration above, the conclusion can be drawn that, for a wing the chord is short relative to the span. Therefore, the aspect ratio of a wing is much larger than that of an oar blade. Most airfoil data are published for foils of infinite aspect ratio [1]. As a result, lift and drag coefficient data from airfoil theory is different and cannot be used in this case.
The flat plate ratio ($span/chord$) above is taken at 0.2. This value is comparable to the aspect ratio of an oar blade. Next to this, other data for a flate plat with small aspect ratio in water is available. These data comes from Beukelman [4]. Unfortunately, these data is

only usable for small angles of attack (till 20°).
Regrettably, the flat plate values of Hoerner are undoubtedly inaccurate either. Namely, tests were done for flat plates which were *totally* immersed. In reality, an oar blade is not "fully immersed", but it is hard to imagine how they could be better than those for the fully immersed condition.
Besides this, these tests can only be done in a *stationary* situation. Measurements were done while all factors (fluid flow rate, water temperature, etc.) have reached an equilibrium state. Therefore, the data can be different if the tests are done at another moment. In practice, the blade is constantly changing position and with that the water is continuously moving. As a consequence, real (non-stationary) data is very hard to measure.
In addition, experiments with flat plates will lead to other results than experiments with curved surfaces. From Fig. 2.5 it can be seen that the lift coefficient of the flat plate at zero angle of attack is zero. Also plotted here is the lift coefficient of a hypothetical blade with the zero attack angle characteristic of an airfoil. This hypothetical blade is not completely flat. Its lift coefficient at zero angle of attack is, in contrast to the flat plate, not zero. Maybe this is a benefit to rowers [5], [16], but that is not investigated in this report.

Chapter 3

Equations of Motion

A boat accelerates through the action/reaction principle (Newton's third law). The rowers try to move water one way with their oar(s) and the boat moves the other way. The *momentum* (mass $\times$ velocity) they put into the water will be equal and opposite to the momentum acquired by the boat.

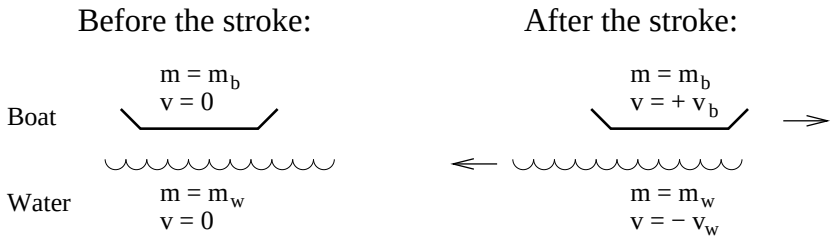


Figure 3.1: *Change in Momentum*

By considering a boat before and after a stroke, it is seen that before the stroke, the total momentum equals 0, since everything is at rest. After the stroke, the total momentum is $m_b v_b - m_w v_w = 0$ because the total momentum can't change. This is due to Newton's second law which state that the rate of change of momentum is proportional to the force applied. This conservation law enables to make an equation of motion. With vector notation

$$\frac{d}{dt}(m\mathbf{v}) = \mathbf{F}(\mathbf{x}, \mathbf{v}, t)$$

Where $\mathbf{F}$ denotes the applied force, which is a function of a position, say $\mathbf{x}$, velocity $\mathbf{v}$ and time t. The forces which play an important role in this are showed in Fig. 3.2. The forces are depicted for a single oar.

The forces which act on the boat itself are drawn with full arrows. These forces are $\boldsymbol{K}$, $\boldsymbol{Q}$ and $\boldsymbol{R}$. Where $\boldsymbol{K}$ denotes the backward force of a rower on his footrest and hence on the boat, $\boldsymbol{Q}$ is the force exerted on the boat at the oarlock and $\boldsymbol{R}$ denotes the drag of the water on the hull.

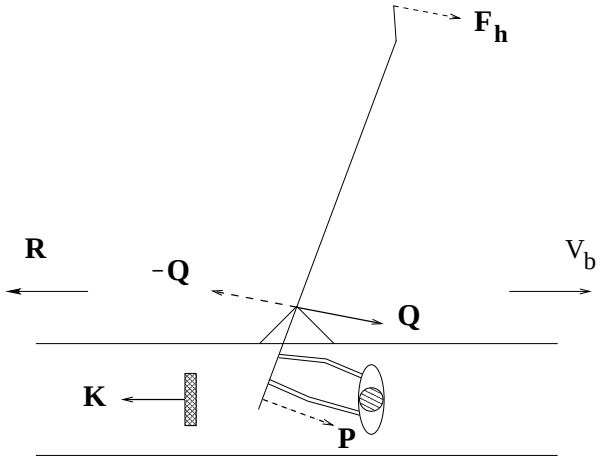


Figure 3.2: *Forces on Boat and Rower*

For the determination of the equations of motion, it is sufficient to consider only components *parallel* to the direction of travel, since force components perpendicular to the direction of travel will cancel for each pair of oars in opposite sides of the boat.

The theory of Brearley, de Mestre and Watson [7] is used to obtain the equations of motion for boat and rowers. This equation is obtained by adding the equation of motion of the rowers and the equation of motion of the boat. The water on which the boat travels may be regarded as a fixed reference frame.

Rowers
A rower exerts a forward force P_x on his oar(s) and a backward force K on his footrest. Due to Newton's third law, equal and opposite forces exert on the rower. Because of this, the equation of motion of the *rowers* in the forward direction can be written as

$$8K - 8P_x = m_r(H + \frac{dV_b}{dt}) \tag{3.1}$$

The '8' is placed, because the model is based on a men's eight. Therefore this force has to be multiplied by the number of rowers (see chapter 6). Consideration of an eight is representative of all possible combinations of rowers [7]. $8P_x$ denotes the combined forward force of the rowers on the oars, parallel to the direction of travel. V_b represents the velocity of the boat and dV_b/dt the acceleration of the boat. The combined mass of rowers is denoted by m_r.
The forward acceleration of the rowers' bodies relative to the boat is denoted by H. According to Brearley, de Mestre and Watson, the rowers begin and end their forward motion, relative to the boat, with zero velocity, and attain smoothly a maximum velocity at about the centre of their travel. This relative motion is such that it may reasonably be taken as half of a cycle of simple harmonic motion (SHM). This suggests writing the relative forward displacement of the rowers from the central point of their travel during the rowing stroke $0 \leq t \leq \tau_1$ as a negative cos-function,

$$x_1 = -a_1 \cos n_1 t \tag{3.2}$$

where the forward direction of the boat is taken as positive. The amplitude a_1 of this function is the amplitude averaged over all rowers of the SHM of the centres of mass of the rowers' bodies and is estimated to be 0.36 m. The circular frequency of the SHM is $n_1 = \pi/\tau_1$. τ_1 is the drive period. Because of this, the forward acceleration of the rowers, relative to the boat, H $(= \ddot{x}_1)$ satisfies

$$H = n_1^2 a_1 \cos n_1 t \tag{3.3}$$

Boat
The equation of motion of the *boat* can be written as

$$8Q_x - 8K - R = m_b \frac{dV_b}{dt} \tag{3.4}$$

Where m_b is the mass of the boat (including the cox[11], if present).
R denotes the drag of the water on the hull and is assumed to be a function of the boat velocity.

$$R = 24.93 - 11.22V_b + 13.05V_b^2 \tag{3.5}$$

Equation (3.5) contains a few constants that are obtained by experimental tests, especially tested on racing shells including eight rowers. These constants are calculable from data in Wellicome [7].

Rowers and Boat
Adding equations (3.1) and (3.4) leads to the following equation of motion of *rowers and boat* for the power stroke.

$$(m_r + m_b)\frac{dV_{boot}}{dt} = 8(Q_x - P_x) - m_r H - R \tag{3.6}$$

The term $8(Q_x - P_x)$ is the force in horizontal direction that drives the boat forward against the drag of the water during each power stroke. The meaning of this term can be explained by the following.

The total propulsive force on a shell, as mentioned in the previous chapter and shown in Fig. 3.2, is the sum of the combined force exerted on the oarlocks and the force exerted by the rowers on the footrests. As a result (for clarity, the factor 8 has been omitted), the following *force equilibrium* must be true.

$$\begin{aligned} \boldsymbol{F}_{boat} &= \boldsymbol{Q} - \boldsymbol{K} \\ &= \boldsymbol{Q} - \boldsymbol{P} \\ &= \boldsymbol{Q} - (\boldsymbol{Q} - \boldsymbol{F_h}) \\ &= \boldsymbol{F_h} \end{aligned} \tag{3.7}$$

Where $\boldsymbol{F_h}$ is the hydrodynamic force as explained in the previous chapter. Equation (3.7) shows that the propulsive force exerted on the boat is determined by the hydrodynamic force!

From the same equation it can be seen that the hydrodynamic force $\boldsymbol{F_h}$ must balance the force exerted on the oarlock $\boldsymbol{Q}$ and on the oarhandle $\boldsymbol{P}$. These three forces are exactly the forces which act on the oar. They are represented with dashed lines in Fig. 3.2.
Therefore, in horizontal direction

$$Q_x - P_x = F_{boat,x} = F_{h,x} \overset{eq.\ (2.2)}{=} L\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi) \tag{3.8}$$

The foregoing differential equation, in other words the *equation of motion of rowers and boat for the power stroke*, then becomes, by substituting equation (3.8) into (3.6)

$$(m_r + m_b)\frac{dV_{boot}}{dt} = 8(L\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi)) - m_r H - R \tag{3.9}$$

The equation of motion for the *recovery phase* can be written as

$$(m_r + m_b)\frac{dV_{boot}}{dt} = -m_r\tilde{H} - R \tag{3.10}$$

$$\tag{3.11}$$

During the recovery phase, the rowers do not exert any force on the oar handles and footrests and therefore $\boldsymbol{F_h}$ equals zero.
$\tilde{H}$ $(= n_2^2 a_1 \cos n_2 t)$ is not quite the same as the H in the power-stroke differential equation. The recovery phase period, denoted by τ_2, depends on the time that is needed to finish the stroke and equals $\tau_s - \tau_1$, where τ_s is written for the total stroke periode. The stroke period can be determined by the stroke rate[47] S, in other words the number of strokes per minute, by $\tau_s = 60/S$. The circular frequency of this SHM is $n_2 = \pi/\tau_2$ and $0 \leq t \leq \tau_2$.

Chapter 4

Propulsive Efficiency of Rowing

An important aim of the rowing investigation is to obtain some kind of rowing efficiency. In other words, that part of the effort, the rower really used to accelerate the boat. As soon as such an efficiency is obtained, gained results can be compared and with those results, conclusions can be made.
Rowing efficiency could be divided into two main parts: internal (muscle) and external (propulsive) efficiencies. The internal or muscle efficiency is determined mainly by the effectiveness of muscle contraction and is estimated for rowing to be in the range of 14-27%. The external or propulsive efficiency connected with hydrodynamics of the boat shell and oar blade is estimated to be in the range of 60-80%. Energy applied to the oar handle is the dividing point between these two energy transformation processes [14]. For the current model, the *propulsive efficiency of the oar blade* (§4.3) is considered.
But before proceding, it is necessary to explain a few words: *energy*, *work* and *power*.

4.1 Energy, Work and Power

To achieve a given increase in boat speed, it takes *energy*[14]. Energy is a measure of how much *work* can be done, or how long it takes to sustain the output of *power*.
Work[51] (W) is simply the application of a force ($\boldsymbol{F}$) over a distance ($\boldsymbol{d}$), with one catch. This distance only counts if it is in the same direction of the applied force.

$$W = \boldsymbol{F} \cdot \boldsymbol{d} \tag{4.1}$$

The SI unit for work, and for energy as well, is joule J ($= Nm$).
Power[31] (P) is a measure of how fast work can be done. Thus for a moving body with velocity $\boldsymbol{v}$ during a certain time dt.

$$P = \frac{dW}{dt} = \boldsymbol{F} \cdot \boldsymbol{v} \tag{4.2}$$

The SI unit for power is the watt W ($= J/s = Nm/s$).

4.2 Energy Conservation

One of the important conservation laws of mechanics, is *momentum conservation*, which is described in chapter 3. Besides this one, there's another important law which can also be derived from Newtonian theory: the *energy conservation law*. In general, this conservation law can be obtained by multiplying the second law of motion for a body of mass m, in one dimension

$$\frac{d}{dt}(mv) = F(x, v, t) \tag{4.3}$$

by v.
Since $v\ dv/dt = 1/2\ d/dt\ (v^2)$, this leads to

$$\frac{d}{dt}(\frac{1}{2}mv^2) = F(x, v, t)v \tag{4.4}$$

The left-hand side represents the change in kinetical energy[24] $1/2\ mv^2$. This theorem (eq. (4.4)) states that the work done by the force acting on m, or power, equals the change in kinetical energy.

In an optimal situation all the power applied by the rower is used to accelerate the boat and gain an optimal velocity. But in reality, not all the power applied by the rower can be used to accelerate the boat, because power is lost in two ways. Firstly, power is lost due to the movement of the blade through the water, in other words slip (§1.2) and secondly, because the boat-speed is not constant during one complete stroke cycle, extra power is also lost due to these velocity fluctuations. To show powers in rowing and energy conservation for this model, multiply both sides of the equation of motion (eq. (3.9)) of chapter 3 with the boat velocity V_b

$$V_b(m_r + m_b)\frac{dV_b}{dt} = (8F_{h,x} - m_rH - R)V_b, \tag{4.5}$$

where $V_bF_{h,x}$ can be rewritten as

$$
\begin{aligned}
V_b F_{h,x} &= V_b(L\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi)) \\
&\overset{1}{=} V_b\left(\frac{\frac{h}{l}P - D\sin(\beta+\varphi-\gamma)}{\cos(\beta+\varphi-\gamma)}\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi)\right) \\
&\overset{2}{=} Ph\frac{d\alpha}{dt} - V_b\frac{D\sin(\beta+\varphi-\gamma)}{\cos(\beta+\varphi-\gamma)}\sin(\alpha+\beta+\varphi) - V_b D\cos(\alpha+\beta+\varphi) \\
&\overset{(eq.\ (1.3))}{=} Ph\frac{d\alpha}{dt} - \frac{V_{bl}\cos(\beta+\varphi-\gamma)}{\cos(\alpha+\gamma)}D\left(\frac{\sin(\beta+\varphi-\gamma)\sin(\alpha+\beta+\varphi)}{\cos(\beta+\varphi-\gamma)} + \cos(\alpha+\beta+\varphi)\right) \\
&= Ph\frac{d\alpha}{dt} - V_{bl}D\left(\frac{\sin(\beta+\varphi-\gamma)\sin(\alpha+\beta+\varphi)}{\cos(\alpha+\gamma)} - \frac{\cos(\beta+\varphi-\gamma)\cos(\alpha+\beta+\varphi)}{\cos(\alpha+\gamma)}\right) \\
&= Ph\frac{d\alpha}{dt} - V_{bl}D \qquad (4.6)
\end{aligned}
$$

As a result, eq. (4.5) becomes

$$
\begin{aligned}
\frac{d}{dt}(m_r+m_b)V_b^2 &= 8(Ph\frac{d\alpha}{dt} - V_{bl}D) - m_r H V_b - RV_b \\
&= (8Ph\frac{d\alpha}{dt} - m_r H V_b) - (RV_b + 8V_{bl}D) \qquad (4.9)
\end{aligned}
$$

Or

$$
\frac{dE}{dt} = P_{rowers} - (P_{drag} + P_{blades}) \qquad (4.10)
$$

[1] Moments about the oarlock lead to $\boldsymbol{P}\times\boldsymbol{h} = \boldsymbol{F_h}\times\boldsymbol{l}$ (eq. (2.1)). Take a reference coordinate system for the left-hand side (x', y'), where x' is perpendicular to $\boldsymbol{h}$ and y' is parallel to $\boldsymbol{h}$. For the right-hand side take a reference coordinate system (x'', y''), where x'' is perpendicular to $\boldsymbol{l}$ and y'' is parallel to $\boldsymbol{l}$. The perpendicular component of $\boldsymbol{P}$ is denoted by the unbold version P.
Then,

$$
\begin{pmatrix} P \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ h \end{pmatrix} = \begin{pmatrix} F_{h,\perp l} \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ l \end{pmatrix} \iff Ph = (L\cos(\beta+\varphi-\gamma) + D\sin(\beta+\varphi-\gamma))l. \qquad (4.7)
$$

[2] From eq. (1.5),

$$
\omega = \frac{1}{l}(V_{bl}\sin(\beta-\gamma+\varphi) + V_b\sin(\alpha+\gamma)) = \frac{V_b\sin(\alpha+\beta+\varphi)}{l\ \cos(\beta-\gamma+\varphi)} \qquad (4.8)
$$

Here dE/dt represents the changes in kinetical energy, P_{blades} $(= 8V_{bl}D)$ the power lost due to moving water by the blade and P_{drag} $(= RV_b)$ represents the power to overcome drag (and thus move the boat) [10].
P_{rowers} $(= 8Ph\frac{d\alpha}{dt} - m_r H V_b)$ denotes the mechanical power generated by the rowers. It is written as the sum of two terms: $P_{handles}$ $(= 8Ph\frac{d\alpha}{dt})$, which denotes the power applied by the rowers to the handles and P_{bodies} $(= -m_r H V_b)$ denotes the power losses by moving the rowers' body, i.e.

$$P_{rower} = P_{handle} + P_{body} \tag{4.11}$$

Thus, if the boat with the rowers and oars are considered as one system, powers in rowing during the stroke-phase can be summarized by formula (4.10) above.

4.3 Propulsive Efficiency of the Blade

A way to describe efficiency is to obtain the *propulsive efficiency of the blade* Eff_{blade} [14].

$$Eff_{blade} = \frac{P_{handle} - P_{blade}}{P_{handle}} \qquad \left(= \frac{P_{hydro}}{P_{handle}}\right) \tag{4.12}$$

The numerator is the propulsive instantaneous power. This term also arises in equation (4.9). It is exactly that part of the power which is applied by the rower to the handle minus the power lost due moving water. It equals $Ph\frac{d\alpha}{dt} - V_{bl}D$ and it's the power that drives the boat forward against the drag of the water, or actually P_{hydro}. Thus, the propulsive efficiency of the blade is derived as a ratio of the handle power P_{handle} to the propulsive instantaneous power P_{hydro}. In the present report, *propulsive efficiency of the rowing blade* is often abbreviated to *blade efficiency*.

An improvement in blade efficiency is achieved if P_{blade} is decreased or P_{handle} is increased.
P_{blade} $(= DV_{bl})$ is the power lost due to moving water. If the slip is large, P_{blade} is large. So slip reduces the blade efficiency and the advance of shell per sweep.
P_{handle} $(= Ph\omega)$ can be increased by an increase of the angular velocity ω, i.e. an increase of the the stroke rate. The physical contribution of the rower will have influence on the power applied by the rower also. For this model, assumptions with regard to the rower's strength are made. The oarhandle force P is fixed. It is considered to be small at the beginning and end of the power stroke and has a maximum near the ninety-degree point of oarshaft sweep. More about this in chapter 6.
Factors which have negative and positive effects on P_{blade} and P_{handle} (for this report it only concerns boat and oar aspects), and with that on the blade efficiency and boat velocity, are discussed in chapter 8.

Chapter 5

Mathematical Model

In the previous chapters a geometric model (ch. 1), a hydrodynamic model (ch. 2) and a model which describes the motion of the boat (ch. 3), were obtained. Together, they describe the entire rowing stroke.
In chapter 4, a formula which describes propulsive efficiency of the blade has been obtained. All these equations form the mathematical model which is used to investigate a few aspects of rowing. Below a review of the mathematical model with the accompanying formulas (the explanation of the variables can be found on page 65).

- *Equations that describe the Rowing Stroke*
  - **Geometric Model**

Blade Velocity
$$V_{bl} = \frac{V_b \cos(\alpha+\gamma)}{\cos(\beta+\varphi-\gamma)} = \omega l \sin(\beta+\varphi-\gamma) + V_b \cos(\alpha+\beta+\varphi)$$

Angular Velocity
$$\omega = \frac{d\alpha}{dt} = \frac{1}{l}(V_{bl}\sin(\beta+\varphi-\gamma) + V_b \sin(\alpha+\gamma))$$

Slip Angle
$$\varphi = \arctan\left(\frac{\omega l \cos(\alpha+\gamma)}{V_b - \omega l \sin(\alpha+\gamma)}\right) - \alpha - \beta$$

  - **Hydrodynamic Model**

Moment Equilibrium
$$\boldsymbol{P} \times \boldsymbol{h} = \boldsymbol{F_h} \times \boldsymbol{l}$$

Hydrodynamic Force
$$F_{h,x} = L\sin(\alpha+\beta+\varphi) - D\cos(\alpha+\beta+\varphi)$$
$$L = cnst\ V_{bl}^2\ c_L$$
$$D = cnst\ V_{bl}^2\ c_D$$
$$(cnst = \frac{1}{2}\rho_w A_{bl})$$

– **Equation of Motion**

$$\begin{aligned}(m_r + m_b)\frac{dV_{boot}}{dt} &= 8F_{boat,x} - m_r H - R \\ &= 8F_{h,x} - m_r H - R\end{aligned}$$

– **Force Equilibrium**

$$\begin{aligned}\mathbf{F}_{boat} &= \mathbf{Q} - \mathbf{K} \\ &= \mathbf{Q} - \mathbf{P} \\ &= \mathbf{Q} - (\mathbf{Q} - \mathbf{F}_h) \\ &= \mathbf{F}_h\end{aligned}$$

- *Equation for the Energy Conservation and the Propulsive Efficiency of the Blade*

– **Energy Conservation**

$$\begin{aligned}\frac{dE}{dt} &= P_{rowers} - (P_{drag} + P_{blades}) \\ &= (8Ph\frac{d\alpha}{dt} - m_r H V_b) - (RV_b + 8V_{bl}D)\end{aligned}$$

Where,

$$P_{rower} = P_{handle} + P_{body}$$

$$P_{handle} = P\ h\ \omega\ = \frac{dW_{handle}}{dt} \qquad (W_{handle} = \int_{t=0}^{\tau_1} P\ h\ \omega\ dt) \tag{5.1}$$

$$P_{body} = -\frac{1}{8}m_r H V_b\ = \frac{dW_{body}}{dt} \qquad (W_{body} = -\int_{t=0}^{\tau_1} \frac{1}{8}m_r H V_b dt) \tag{5.2}$$

$$P_{blade} = D\ V_{bl}\ = \frac{dW_{blade}}{dt} \qquad (W_{blade} = \int_{t=0}^{\tau_1} D\ V_{bl}\ dt) \tag{5.3}$$

$$P_{drag} = R\ V_b\ = \frac{dW_{drag}}{dt} \qquad (W_{drag} = \int_{t=0}^{\tau_1} R\ V_b\ dt) \tag{5.4}$$

– **Propulsive Efficiency of the Blade**

$$Eff_{\,blade} = \frac{P_{handle} - P_{blade}}{P_{handle}} \qquad \left(= \frac{P_{hydro}}{P_{handle}}\right)$$

The computation of the above parameters can be done with the help of a computer program. The calling sequence in which the parameters are computed and a description of this computer program is given in the appendix A.
However, before this can be done, a numerical model has to be set up. This will be done in chapter 7.

Chapter 6

Discussion about the Input Data

Part of creating an accurate model is to have correct input data. Of course, the values of the parameters depend for a great part on the type of boat (thus rowers) which is used. For example, with regard to the size of the oar, one could imagine that a heavyweight man is more capable to handle a large oar than, on the other hand, a lightweight woman is.
And, if the stroke rate is an input parameter, one should take account of the difference between men and women. In general, women are shorter than men and have therefore, in general, shorter strokes. Depending on the rowers strength, 'optimal' values of, for instance, blade size, blade type, catch angle, spread[42] and inboard/outboard ratio (gearing[19]) exist [9], [17].

It's obvious that the input data can not be chosen arbitrarly. A literature study has to be done to find suitable data. Unfortunately, this literature study not always results in usable, i.e. realistic data. A part of the used input data has been obtained by measurements or experiments, but the other part of the input data is based on assumptions. A reason to make assumptions is that little published work has appeared on the study of instrumented shells on the water for determinations of velocity, acceleration, oarhandle pull, oarlock and footboard[16] forces, concurrent wind and current speeds and directions, etc. [9]
Sometimes, there's no data at all. As an example, data regarding the hydrodynamic characteristics of an oar blade have never been measured. Therefore, the best currently available data, those of a flat plate, totally immersed (ch. 2) have been used. In addition, one has to reckon the fact that data, which is available, might be different if experiments used to acquire these data, were done in other circumstances.

The model in this report is based on the assumption that the boat contains eight rowers, or more precisely, eight heavy men. Fours, pairs or lightweight women eights could, instead of this, also have been investigated, but the obtained results will not extremely differ from those that are acquired by the observation of a men's eight [7]. In this situation, appropriate data in terms of a heavyweight men's eight, is essential for the development of a more or less realistic model.

The input data can be divided into variable data and constant data. In the next sections a description of these two types of data will be given, together with the accompanying parameters, used in the present model.

6.1 Variable Input Data

Variable input data include parameters of which the values may change during the rowing stroke. They depend on the values of other parameters, but are fixed for these parameter values. For the current model, it concerns the following parameters.
The force on the oarhandle by the rower P, the drag of the water on the boat R, the acceleration of the rowers' body H and the blade lift coefficient c_L and drag coefficient c_D.
During the power stroke, the force P is assumed to begin and end with small magnitudes and attain smoothly a maximum near the center of this interval. Therefore, the features of P are adequately represented by a sin-function.

$$P = P_{max} \sin\left(\frac{\pi}{(\alpha_f - \alpha_0)}(\alpha - \alpha_0)\right) \tag{6.1}$$

The force P is a function of the catch bow angle α. The α_0 and α_f denote the catch bow angle at the beginning and the finish of the power stroke respectively. P_{max} is the maximum value of P and it reaches (in this model) a value of 650 (N). This can be shown with a plot, obtained with *MATLAB* (see appendix A).

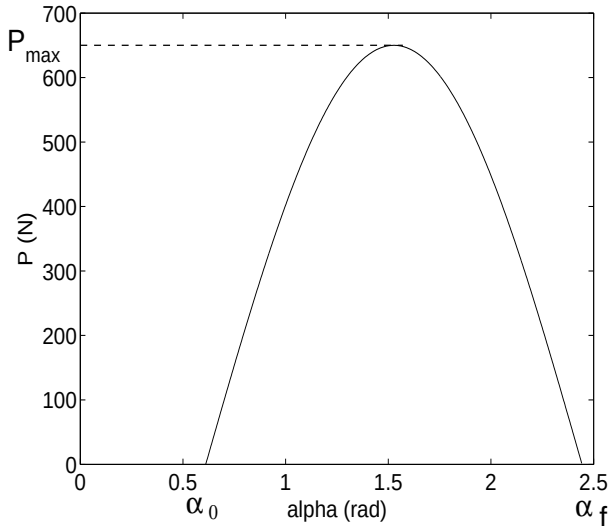


Figure 6.1: Force on the Oarhandle During the Power Stroke

A more accurate and realistic 'P' can be found in the biomechanic literature (see e.g. the biomechanics newsletters [15]).
The boat resistance R is considered to be a quadratic function of the boat velocity (eq. (3.5)) [7].
H, the acceleration of the rowers' body, is represented by a cos-function (eq. (3.3)) [7].
The blade lift coefficient c_L and drag coefficient c_D are prescribed functions of slip angle φ. Hoerner [9] described their dependence by a graph, see §2.2.

6.2 Constant Input Data

Constant input data include parameters which values are held constant during the entire rowing stroke. The input data of the present model that are held constant during the rowing stroke, are a, b, l, h, β, γ, A_{bl}, ρ_w, α_0, α_f, S and τ_s. S is being denoted for the stroke rate. To fix this stroke rate S, means to fix the stroke period τ_S $(60/S)$. A good national-level men's eight rowing a 2000 meter race, has a stroke rate S of approximately 37.5 per minute [5], [7]. The time for a complete stroke τ_s is therefore 1.6 sec. The definition of the other variables, and this one, can be found on page 65. The accompanying values are listed below.

$a = 2.10\ m$	$A_{bl} = 1100 \times 10^{-4}\ m^2$
$b = 0.37\ m$	$\rho_w = 1000\ kg/m^3$
$l = 2.47\ m$	$\alpha_0 = 35°$
$h = 1.00\ m$	$\alpha_f = 140°$
$\beta = 6°$	$S = 37.5\ str/min$
$\gamma = 0.0157°$	$\tau_s = 1.6\ s$

These values, conducted in terms of a men's eight, are mostly acquired from data of Steven Redgraves' book of rowing [19].

A modification of some input data of the model was done to investigate their effect on a few important output parameters. More precisely, the parameters a, α_0, α_f, β and A_{bl} were changed to investigate their effect on, among other things, the boat velocity, blade velocity, slip, work, power and blade efficiency (ch. 8).

A special case of the above-mentioned data is the *initial*[23] condition. An initial condition specifies the condition at $t = t_0$. The values of the parameters at $t = t_0$ are given on page 59.

Chapter 7

Numerical Model

7.1 Solving Variables

The mathematical model contains of 14 variables. These variables are
α, V_b, H, R, P, c_L, c_D, $\omega = \frac{d\alpha}{dt}$, φ, V_{bl}, L, D, F_h and $\frac{dV_b}{dt}$.

To obtain the values of these variables, 14 equations are needed, as well.
The first two variables, α and V_b, can be achieved by solving the differential equations described in the section 7.2.
The next five variables H, R, P, c_L and c_D are prescribed variables, in other words, the variable input data (ch. 6).
The two hydrodynamic forces L and D are obtained by equation (2.3) and (2.4). Their resultant is the hydrodynamic force F_h.
The boat acceleration $\frac{dV_b}{dt}$ can be achieved by equation (3.9).
Now, there are three variables left: ω, φ and V_{bl}. The way these variables are obtained, is described in section 7.3.

7.2 Discretization

The mathematical model contains two differential equations. One of these is the differential equation for the angular velocity ($\omega = d\alpha/dt$), the other one is the equation of motion. The most elementary time integration method, the *Forward Euler Method*, which is an *explicit* method, is used to solve the differential equations. A differential equation is explicit, if the highest derivative can be isolated (otherwise the differential equation is *implicit*). In this way, the value of a parameter at a particular moment can be obtained by that parameter, one time step before. The discretization of the first order derivative of α and V_b in time is shown here.

$$\frac{d\alpha}{dt} = \frac{\alpha_{n+1} - \alpha_n}{\delta t} = \omega(V_{b_n}, \alpha_n, \varphi_n) = \omega_n \tag{7.1}$$

$$\frac{dV_b}{dt} = \frac{V_{b,n+1} - V_{b,n}}{\delta t} = f(V_{b_n}, F_{h,n}, \alpha_n, \varphi_n, H_n, R_n) \tag{7.2}$$

Above equations can also be written as

$$\alpha_{n+1} = \alpha_n + \omega(V_{b_n}, \alpha_n, \varphi_n)\ \delta t \tag{7.3}$$

$$V_{b,n+1} = V_{b,n} + f(V_{b_n}, F_{h,n}, \alpha_n, \varphi_n, H_n, R_n)\ \delta t \tag{7.4}$$

Where δt represents a little computational time step, of which the value is determined by the number of time steps N and the duration of the stroke τ_s: $\delta t = \frac{\tau_s}{N}$. The subscript $n = 1, \cdots, N$ denotes the time step. The value of a parameter at time $t_n = n\delta t$ is denoted by that parameter together with the subscript n (likewise $n+1$).
If the initial conditions are fixed, all the values of the parameters at every moment can be obtained.

7.3 The Implicit Function Theorem

Since the oarhandle force P is given, it seems obvious to obtain the slip angle φ by solving equation (4.7). With the help of this calculated parameter, other parameters like V_{bl} and ω could be determined in terms of φ through eq. (1.4) and (1.5). Unfortunately, as a result of the implicit function theorem, this is not possible. This theorem [6] implies the following.

(Implicit Function Theorem) *Suppose that* $\mathbf{f} = (f_1, \cdots, f_n) : \mathbb{R}^{n+m} \Rightarrow \mathbb{R}^m$ *has continuous partial derivatives. Denoting points in* $\mathbb{R}^{n+m}$ *by* $(\mathbf{x}, \mathbf{z}) = (x_1, \cdots, x_n, z_1, \cdots, z_m)$, *where* $\mathbf{x} \in \mathbb{R}^n$ *and* $\mathbf{z} \in \mathbb{R}^m$. *Assume that* $(\mathbf{x}_0, \mathbf{z}_0)$ *satisfies* $\mathbf{f}(\mathbf{x}_0, \mathbf{z}_0) = 0$ *and that the* $m \times m$ *determinant of the matrix*

$$|D_j f_i(\mathbf{x}_0, \mathbf{z}_0)| \neq 0 \tag{7.5}$$

for $i, j = 1, \cdots, n$.
Then there is a ball U *containing* $\mathbf{x}_0$ *in* $\mathbb{R}^n$ *and a neighborhood* V *of* $\mathbf{z}_0$ *in* $\mathbb{R}^m$ *such that there is an unique function* $\mathbf{z} = \mathbf{g}(\mathbf{x})$ *defined for* $\mathbf{x}$ *in* U *and* $\mathbf{z}$ *in* V *that satisfies* $\mathbf{f}(\mathbf{x}, \mathbf{g}(\mathbf{x})) = 0$.
Moreover, if $\mathbf{x}$ *in* U *and* $\mathbf{z}$ *in* V *satisfy* $\mathbf{f}(\mathbf{x}, \mathbf{z}) = 0$, *then* $\mathbf{z} = \mathbf{g}(\mathbf{x})$.
Finally, $\mathbf{z} = \mathbf{g}(\mathbf{x})$ *is continuously differentiable.*

This time $n = 1$, $m = 2$ and for this example $x_1 = \varphi$, $z_1 = \omega$ and $z_2 = V_{bl}$ holds. Take

$$f_1 = f_1(\varphi, \omega, V_{bl}) = V_b \sin(\alpha + \beta + \varphi) - \omega l \cos(\beta + \varphi - \gamma) = 0 \tag{7.6}$$

$$f_2 = f_2(\varphi, \omega, V_{bl}) = V_{bl} - \omega l \sin(\beta + \varphi - \gamma) - V_b \cos(\alpha + \beta + \varphi) = 0 \tag{7.7}$$

which were obtained by rewriting de functions for ω (eq. (1.5)) and V_{bl} (eq. (1.4)) respectively. The determinant of the 2×2 matrix now yields the following

$$\begin{vmatrix} \frac{\partial f_1}{\partial \omega} & \frac{\partial f_1}{\partial V_{bl}} \\ \frac{\partial f_2}{\partial \omega} & \frac{\partial f_2}{\partial V_{bl}} \end{vmatrix} = \begin{vmatrix} -l\cos(\beta + \varphi - \gamma) & 0 \\ -l\sin(\beta + \varphi - \gamma) & 1 \end{vmatrix} = -l\cos(\beta + \varphi - \gamma) \tag{7.8}$$

The condition for solvability is that above determinant is nonzero.
Unfortunately, since $\cos(\beta+\varphi-\gamma)=0$ for $\beta+\varphi-\gamma=\pi/2$, one is not able to find a function $\mathbf{g}$ near this point, say $(\varphi_0,\omega_0,V_{bl,0})$, such that $g_1(\varphi_0)=\omega_0$ and $g_2(\varphi_0)=V_{bl,0}$.
As this is not a special (degenerate) point during the stroke, at which one could anticipate numerical problems, an alternative has been examined.
Another choice is to write φ and V_{bl} in terms of ω (now, $x_1=\omega, z_1=\varphi, z_2=V_{bl}$). Again eq. (7.6) and eq. (7.7) are used to obtain if there's a $\boldsymbol{g}$ such that $\boldsymbol{f}(\omega,\boldsymbol{g}(\omega))=0$ and $(\varphi,V_{bl})=\boldsymbol{g}(\omega)$. To obtain whether there's such a function, look at the following.

The determinant is given by

$$\begin{vmatrix} \frac{\partial f_1}{\partial \varphi} & \frac{\partial f_1}{\partial V_{bl}} \\ \frac{\partial f_2}{\partial \varphi} & \frac{\partial f_2}{\partial V_{bl}} \end{vmatrix} = \begin{vmatrix} V_b\cos(\alpha+\beta+\varphi)+\omega l\sin(\beta+\varphi-\gamma) & 0 \\ -\omega l\cos(\beta+\varphi-\gamma)+V_b\sin(\alpha+\beta+\varphi) & 1 \end{vmatrix}$$

$$= V_b\cos(\alpha+\beta+\varphi)+\omega l\sin(\beta+\varphi-\gamma) \tag{7.9}$$

$$\neq 0$$

The right-hand side of eq. (7.9) is exactly the expression for the blade velocity (eq. (1.4)). Therefore, to say that equation (7.9) should not equal zero, means the same as $V_{bl}\neq 0$.
Now, the position where $V_{bl}=0$ is indeed a degenerate point during the stroke. At this point, the vectors $\boldsymbol{\omega l}$ and $\boldsymbol{V_b}$ (see figure 1.4) are along the same line, have the same length and point in the opposite direction. As their resultant vector $\boldsymbol{V_{bl}}$ now has length zero, it is not possible to assign a direction to $\boldsymbol{V_{bl}}$. Hence it is not possible to define a slip angle φ now. The angle of attack is indefinite. However, this situation is unavoidable. Outside this point, V_{bl} and φ can, as a consequence of theorem 7.3, uniquely be solved in terms of ω.

If such an ω is known, V_{bl} and φ can be obtained through eq. (1.4) and eq. (1.6) respectively, but there should also be a way to obtain the ω. The method which is used to find ω is the *bisection method*. This method is described in section 7.4.

7.4 The Bisection Method

The bisection method is a simple procedure for iteratively converging on a solution of an equation which is known to lie inside some interval $[a,b]$. Bisection proceeds by evaluating the function in question at the midpoint of the original interval $x=(a+b)/2$ and testing to see in which of the subintervals $[a,(a+b)/2]$ or $[(a+b)/2,b]$ the solution lies. The procedure is then repeated with the new interval as often as needed to locate the solution to the desired accuracy.
This method is used, because the method has the important property that it always converges to a solution. A more detailed description of this method, together with the advantages and deficiencies of the algorithm, can be found in literature (for example in [2]).

For the current model, ω should satisfy the given equation for the oarhandle force P (eq. 6.1)). One begins the bisection algorithm with an interval for ω. Bisection chops this interval in half and with the help of this midpoint the parameters V_{bl} and φ are computed. The

computed parameters should satisfy the given P. Here-after, bisection discards the interval which does not contain a root. The iteration is repeated until the interval of ω is smaller than the requested epsilon. Then, the iteration stops and an ω is obtained.

Now, 14 variables and 14 equations are available. With the help of a computer code (explained in the appendix A), the computation of the variables can be done. Consequently, the rowing stroke can be simulated and tests can be carried out.
These tests and their results are described in the next chapter.

Chapter 8

Testing

8.1 Test Cases

As stated in the introduction, it is not feasible to investigate all the factors which influence the motion of the boat, oar and blade. For the current investigation, only a few parameters were changed to investigate their effect.
These parameters are

- the blade surface A_{bl};
- the outboard length a;
- the blade cant angle β;
- the catch bow angle α_0 (while the range of the bow angle $(\alpha_f - \alpha_0)$ was held constant).

Regarding to the parameters above, the following values (in terms of a men's eight) were taken as the base case:

$A_{bl} = 1100 \times 10^{-4}\ m^2$	$\beta = 6°$
$a = 2.10\ m$	$\alpha_0 = 35°$
$l = 2.47\ m$	$\alpha_f = 140°$

The next section shows the results which were obtained by a change of the above-mentioned parameters. A table is used to show the effects on a few important parameters, or exactly, on the average shell speed (m/s), the total rower work (W), the total rower power (J) , the total blade work (W), the total blade power (J) and the blade efficiency. The last five terms were explained in chapter 4.
The average shell speed is calculated from the moment that the average boat velocity is nearly constant, a so-called *steady-state*[44]. Work and power have been obtained from this moment as well. They were calculated for each power stroke. The *total* work and *total* power are averaged values over the number of strokes.

Graphs are used, to show the effect on the (average) shell speed and blade efficiency. To

make an easy comparison, the same scaling of the y-axis is used. The shell speed is plotted with values in the range of $6.35 - 6.7$ (m/s) and the blade efficiency with values in the range of $72 - 84\%$. The obtained results will be examined and the most remarkable aspects will be mentioned and discussed.

8.2 Test Results

- *Oar Blade Surface A_{bl}*

  Table 8.1 and Fig. 8.1 display the influence of increased surface A_{bl} on the aforementioned parameters. The base case and two blades (Mâcom, Big Blades), which are used in practice, are shown.

blade surface (cm^2)	743	929	1020 Maçom	1100 Base	1128 Big Blades	1301	1673	2200
average shell speed (m/s)	6.45	6.51	6.52	6.54	6.55	6.58	6.62	6.66
total rower work (J)	758	758	758	758	758	758	758	758
total rower power (W)	963	949	944	940	939	932	922	914
total blade work (J)	204	187	181	175	174	164	150	139
total blade power (W)	258	234	225	217	215	202	183	168
blade efficiency	0.73	0.75	0.76	0.77	0.77	0.78	0.80	0.82

Table 8.1: *Oar Blade Surface*

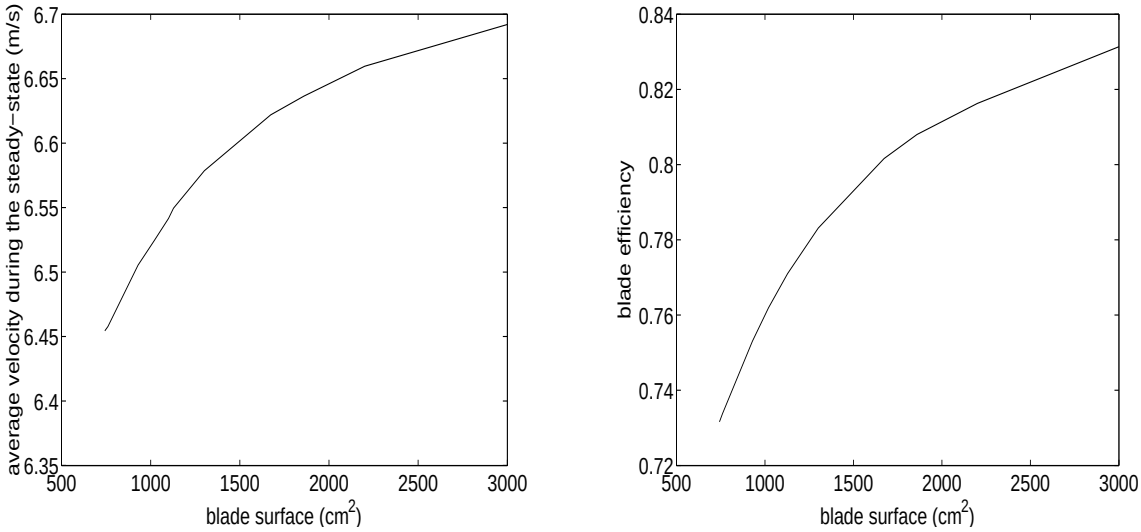


Figure 8.1: *Effect Oar Blade Surface on Boat Velocity and Blade Efficiency*

The most remarkable aspects

- An increase in oar blade surface leads to an increase in boat velocity and blade efficiency. This increase seems to level off.
- The work done by the rower remains the same for each blade surface. The work per sec, or power, applied by the rower decreases. Power losses by the blade decrease, but not as much as the rower's power decreases. Therefore, the blade efficiency increases.

Discussion

The model seems to indicate that increased blade surface has potential for improving blade efficiency and boat velocity. The surface should be as large as a rower can easily manage.

If A_{bl} is large, the rower is more able to reach his peak handle force and use all his strength to move the boat. On the contrary, if A_{bl} is small, the blade work between finish and catch is much faciliated. It's easy to pull the blade through the water. However, this is not effective, because a lot of the rower power is lost due to moving water. In other words, this causes slip. The blade efficiency is small.

In the current model, the contribution of the handle force as a function of α is fixed (ch. 6). Since h and the range of α (see ch. 4) are constants, the work done by the rower remains the same. The power applied by the rower is solely determined by the rate of change of α, the angular velocity $\omega = \frac{d\alpha}{dt}$. Therefore, the rower power increases if ω increases.

The aim in rowing is to transfer as much as possible of the rower's effort to the oarlock with the minimum of blade slip, i.e. a high blade efficiency. Blade slip is minimized by maximizing the hydrodynamic vector resultant of lift and drag, F_h. Near the catch and release high lift is useful, and near the ninety-degree point high drag is essential. The last one could be reached by a large surface area.

Because of this, the question may arise in what way the drag force D, and with that the lift force L, is distributed over a power stroke. The examination of three blade surfaces ($743cm^2$, $1128cm^2$ and $1859cm^2$) leads to the following. Note, the unit of the bow angle α is now rad ($= \frac{\pi}{180^\circ}\ degree$), instead of usually used unit $degree$.

The left figure of Fig. 8.2 shows the effect on the drag force D for an increase in blade surface. An increase in blade surface results in a drag profile that is narrower. This means that there's less drag at the beginning of the stroke. Since the force on the handle is kept constant, there's more lift at the beginning of the power stroke (eq. (4.7)). The increase in blade surface also leads to an increase in blade efficiency (the right plot), especially near the ninety-degree point of the sweep. However, near this point an abrupt decrease of the blade efficiency is also noticed. This is due to the change in slip angle. The slip angle reaches a value at which the blade stalls. The fluid is detached from the blade (§2.2). A lot of power is lost because of moving water molecules.

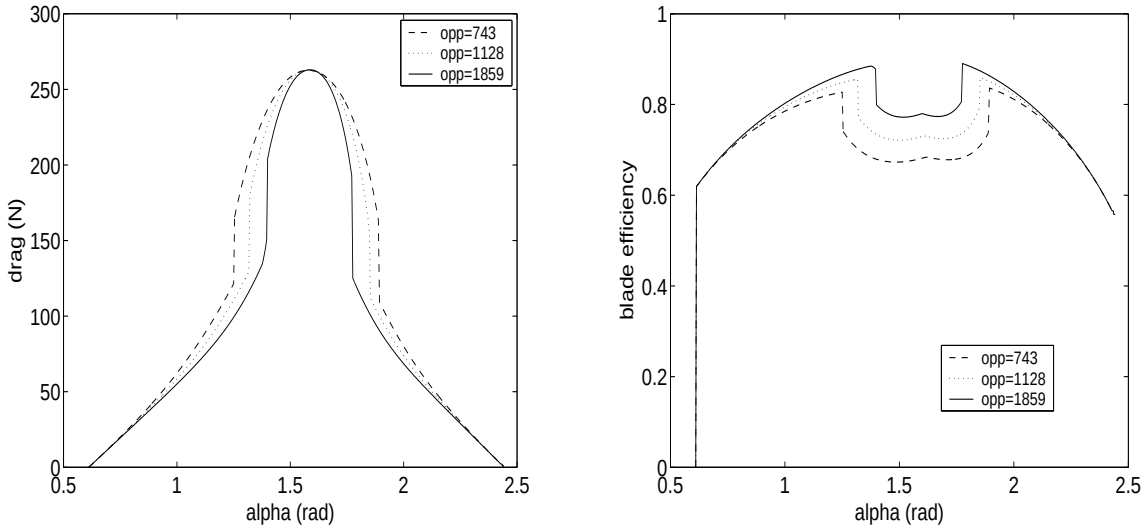


Figure 8.2: *Three Blade Sizes: Drag Force and Blade Efficiency*

Consequently, the question arises whether, at this moment, a decrease in rower handle force may cause less blade losses and therefore increase the blade efficiency. Tests indeed show an increase in blade efficiency around this ninety-degree point of oarshaft sweep. In practice, this is not easy to realize, as it's difficult to obtain at which moment the blade stalls.
What an increase in blade surface does to the slip angle is shown in Fig. 8.3.

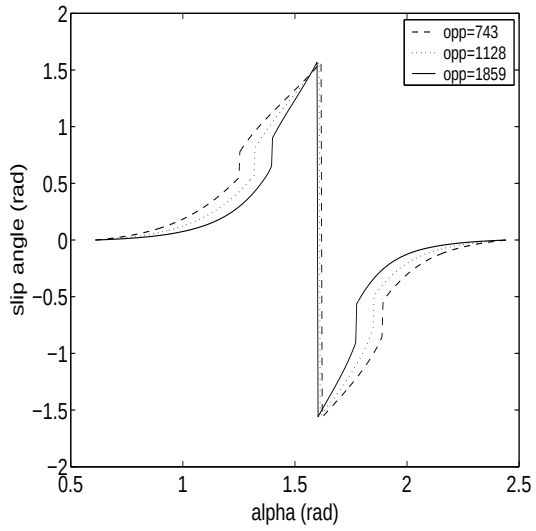


Figure 8.3: *Three Blade Sizes: Slip Angle*

An increase in blade surface reduces the slip angle before and after the ninety-degree point of oarshaft sweep. From this, the conclusion can be made that an increase in surface (drag) reduces slip thus increasing the advance of the shell per sweep.

The results of this investigation show that an increase in blade surface only have positive effects. The plots of Fig. 8.1 (and tests which were done with very large surfaces but not presented) show that both the velocity and efficiency level off, but still increase with blade surface. It's diserable to design a blade which has an extremely large surface. However in practice, a blade should not be too large. Increased surface is limited, of course, by the addition of weight and the ultimate impossibility of handling and feathering the result [9].

Also a smaller blade, with another blade form, may have more advantages in contrast to a large blade. Namely, the blade in the present model is just a flat plate. As mentioned in §2.2, maybe a curved blade is an advance. A curved form can be expected to decrease drag by guiding the flow [5].

- *Outboard Oarlength a*

Table 8.2 and Fig. 8.4 show the results for increased outboard oarlength a.

outboard length a (m)	1.85	2.10 Base	2.35	2.47	2.59	2.84
length l (m)	2.22	2.47	2.72	2.84	2.96	3.21
average shell speed (m/s)	6.55	6.54	6.53	6.54	6.53	6.51
total rower work (J)	758	758	758	758	758	758
total rower power (W)	1046	940	853	814	780	719
total blade work (J)	184	175	168	162	160	156
total blade power (W)	254	217	190	174	164	148
blade efficiency	0.76	0.77	0.78	0.79	0.79	0.79

Table 8.2: *Outboard Oarlength a*

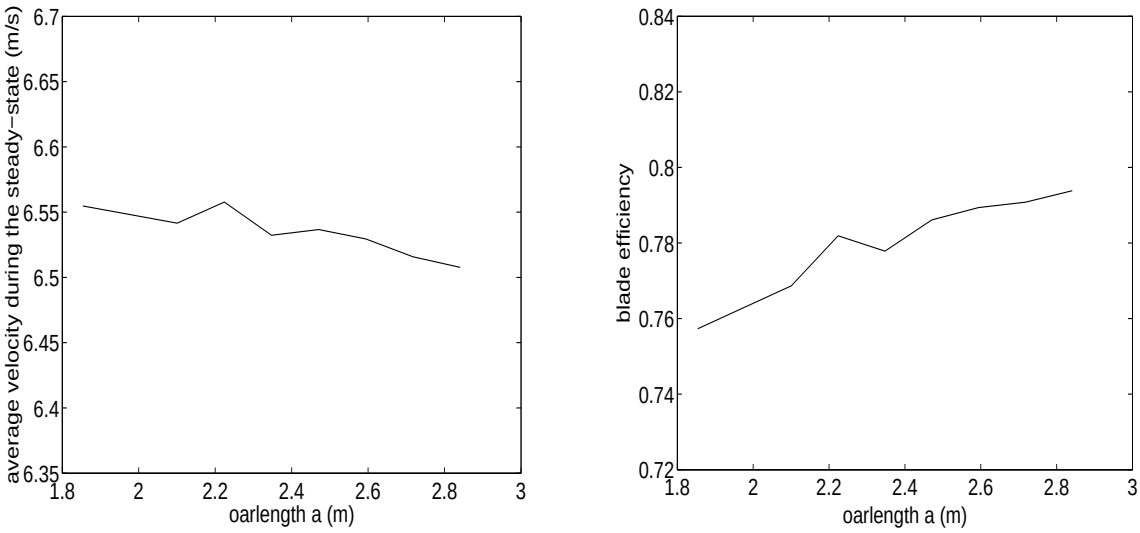


Figure 8.4: *Effect Oarlength on Boat Velocity and Blade Efficiency*

The most remarkable aspects

- An increase in outboard oarlength a leads to a small change in boat velocity and blade efficiency.
- A decrease in rower power and blade work and blade power is noticed.

Discussion
An enlargement of the blade surface experiences the rower as something heavier. This can also be reached by a shortened outboard length a. If this length is decreased, the propulsive force will be larger, but unfortunately the shaft transverse slip increases as well. Consequently, the blade efficiency decreases.
Below a graph of the increase in slip angle during the power stroke for different outboard lengths a ($1.85m$. $2.10m$ and $2.47m$). The right plot shows the drag force during the power stroke.

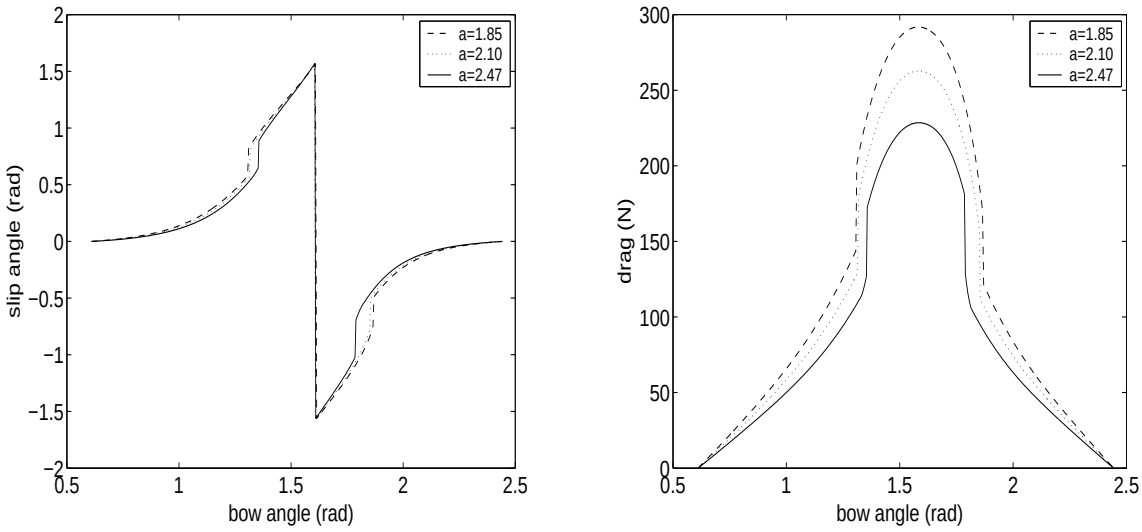


Figure 8.5: *Three Oarlengths: Slip Angle and Drag Force*

Fig. 8.5 shows the afore-mentioned remark. With equal handle force P and inboard length h, equation (2.1) shows that an enlargement of a, and with that l, results in a decrease of the hydrodynamic force F_h, The drag force decreases as well. This may seem undesirable, but since the propulsive force is small, the contribution of slip is small. As a consequence little improvement in blade efficiency can be noticed. The blade efficiency during the power stroke is shown in Fig. 8.6 on the next page.

Still, the results of this model do not show any striking improvement if the outboard length is enlarged or shortened.
Figure 8.4 shows (little) increase in blade efficiency, but at the same time, one can notice (little) decrease in boat velocity. This small difference between the boat velocity and blade efficiency is a result of variations in the movements of the rowers' body acceleration H. Power losses which are caused by the movements of the rowers' body can be shown by equation (4.9) in §4.2. Due to this variation in H, the boat velocity shows a small decrease.

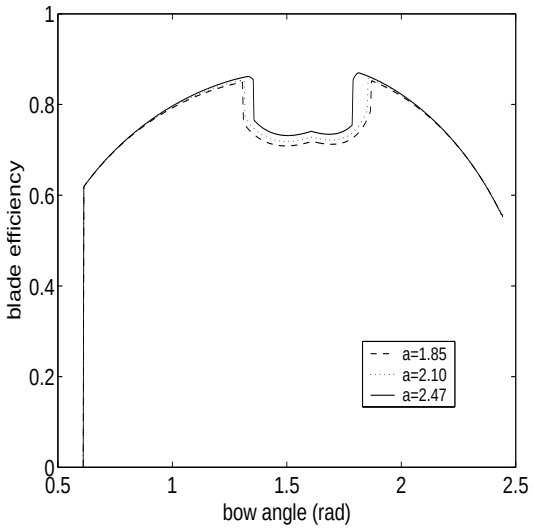


Figure 8.6: *Three Oarlengths: Blade Efficiency*

- *Blade Cant Angle β*

Table 8.3 and Fig. 8.7 show the results for an increase of the blade cant angle β. The tests are done, while the catch and finish bow angle were kept constant. The arc between catch and finish angle equals 105°.

blade cant angle (degrees)	-15	-9	0	3	6 Base	15	20
range alpha (degrees)					35-140		
average shell speed (m/s)	6.45	6.51	6.55	6.55	6.54	6.54	6.52
total rower work (J)	759	759	759	758	758	758	758
total rower power (W)	877	907	932	937	940	932	917
total blade work (J)	192	181	172	173	175	177	184
total blade power (W)	222	217	211	214	217	218	222
blade efficiency	0.75	0.76	0.77	0.77	0.77	0.77	0.76

Table 8.3: *Blade Cant Angle*

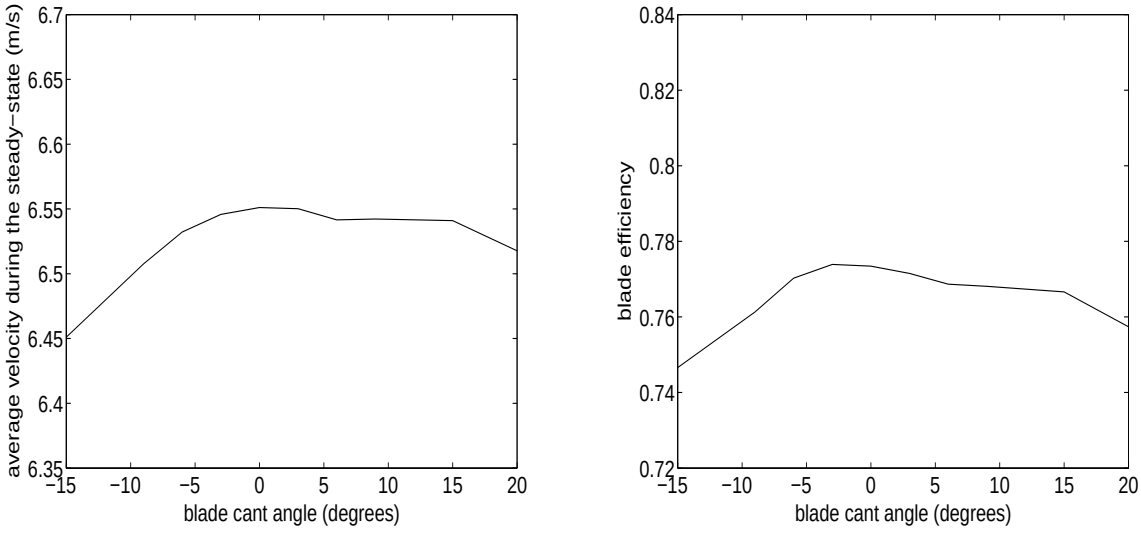


Figure 8.7: *Effect Blade Cant Angle on Boat Velocity and Blade Efficiency*

The most remarkable aspects

– First, an increase in boat velocity is noticeable. When a blade cant angle of 0° is reached, the boat velocity starts to decrease. A boat velocity of 6.55 (m/s) has been reached.

– The blade efficiency increases till $\beta = -3°$ is reached. A maximum of 77% is shown. After this angle, the blade efficiency decreases.

– A subsequent increase and decrease of rower power.

– A subsequent decrease and increase of blade power.

Discussion

The table and figures show that, till a blade cant angle β of approximately 0° is reached, increased blade cant angle has advantages. However, if this point has passed, the effect deteriorates. The small change of turning-point is due to variations in H (see discussion oarlength a).

Increased blade cant angle doesn't have any effect on the work done by a rower, but it does have effect on the blade work. Since drag and blade velocity determine the blade work (eq. (5.3)), it's worth looking at these terms.

If the angles α, γ and φ and the boat velocity V_b are held constant, an increase of β must result in an increase in blade velocity (eq. (1.4)). However, if the position of the blade is changed, it will effect φ. Therefore, it's worth looking at the effect on the slip angle also.

Below, four plots are presented: the drag force D, the slip angle φ, the blade velocity V_{bl} and the blade efficiency during the power stroke. Tests were done with three blade cant angles, $\beta = -9,\ 6$ and 15 degrees.

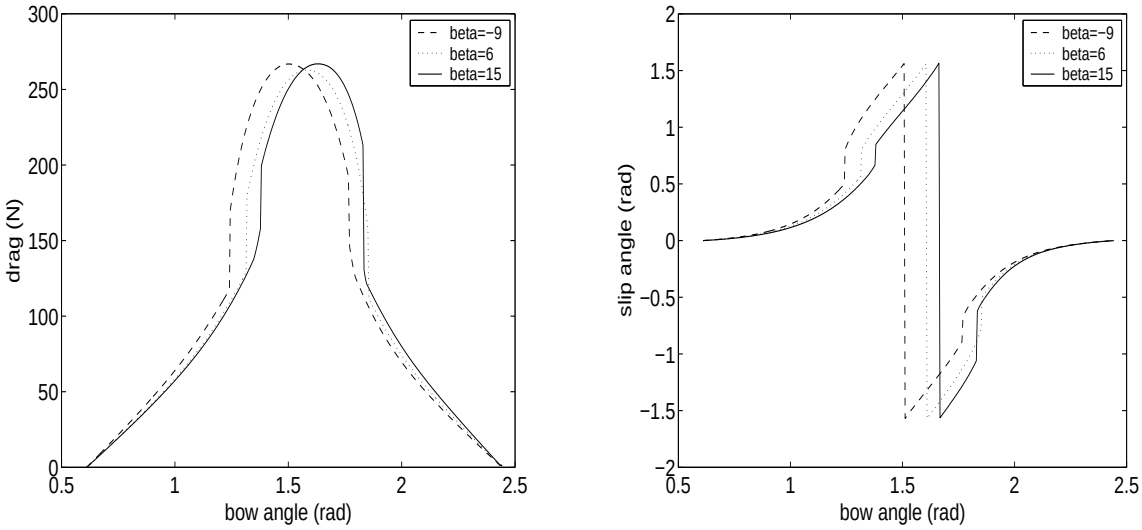


Figure 8.8: *Three Blade Cant Angles: Drag Force and Slip Angle*

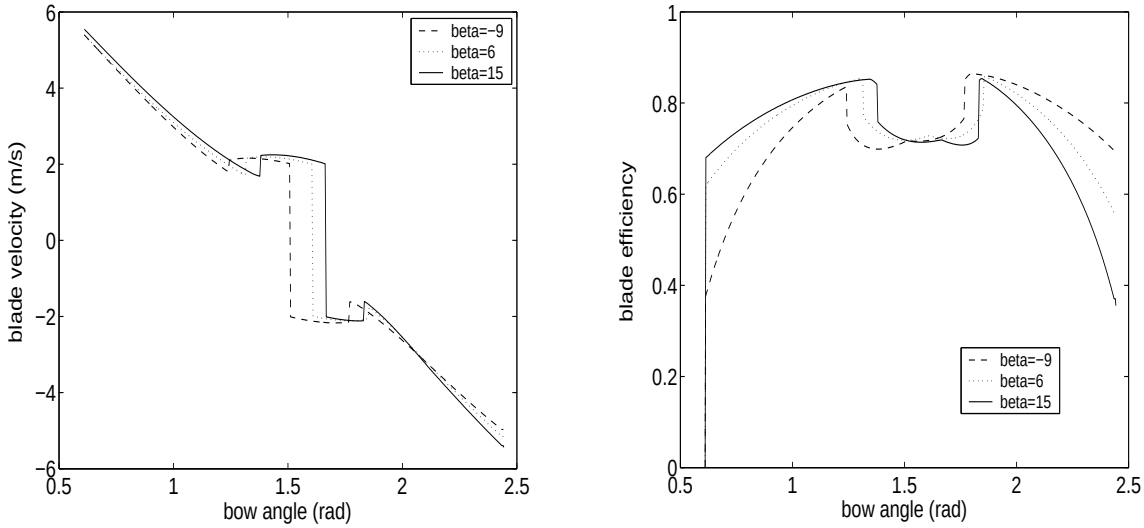


Figure 8.9: *Three Blade Cant Angles: Blad Velocity and Blade Efficiency*

The left plot of Fig. 8.9 indeed shows an increase in blade velocity, during (almost) the entire power stroke, if the blade cant angle is increased. Increased β leads to a smaller slip angle and less drag at the beginning. As a result, the blade efficiency is larger during the first part of the power stroke. At this part, a large (positive) blade cant angle is preferable. Here-after a large, but negative blade cant angle is very favourable. A negative blade cant angle causes less slip and the blade losses are small. Maybe another blade shape is able to realize both wishes [5], [16].

Nevertheless, if the entire power stroke is considered, there's an optimum blade cant angle. In the present model, the optimum cant angle β is approximately $0°$. However, a shift of this optimum can occur if other parameters are changed.

- *Catch Bow Angle α_0*

As a result of the previous test, the catch and finish bow angle were modified. Namely, if the blade position has influences on the results, the position of the oar might have some effect too.
Table 8.4 and Fig. 8.10 show the results of calculating average speed and blade efficiency for a change in catch and finish bow angle. The range of the bow angle, thus $\alpha_f - \alpha_0$ ($= 105°$) and the blade cant angle ($= 6°$) are fixed.

range alpha (degrees)	21-126	26-131	35-140 Base	38-143	41-146	47-152	56-161
blade cant angle (degrees)			6				
average shell speed (m/s)	6.51	6.54	6.54	6.54	6.53	6.49	6.36
total rower work (J)	758	759	758	758	758	758	758
total rower power (W)	897	926	940	933	920	874	754
total blade work (J)	178	174	175	177	179	186	209
total blade power (W)	210	212	217	218	217	214	208
blade efficiency	0.77	0.77	0.77	0.77	0.76	0.75	0.72

Table 8.4: *Catch Bow Angle and Finish Bow Angle*

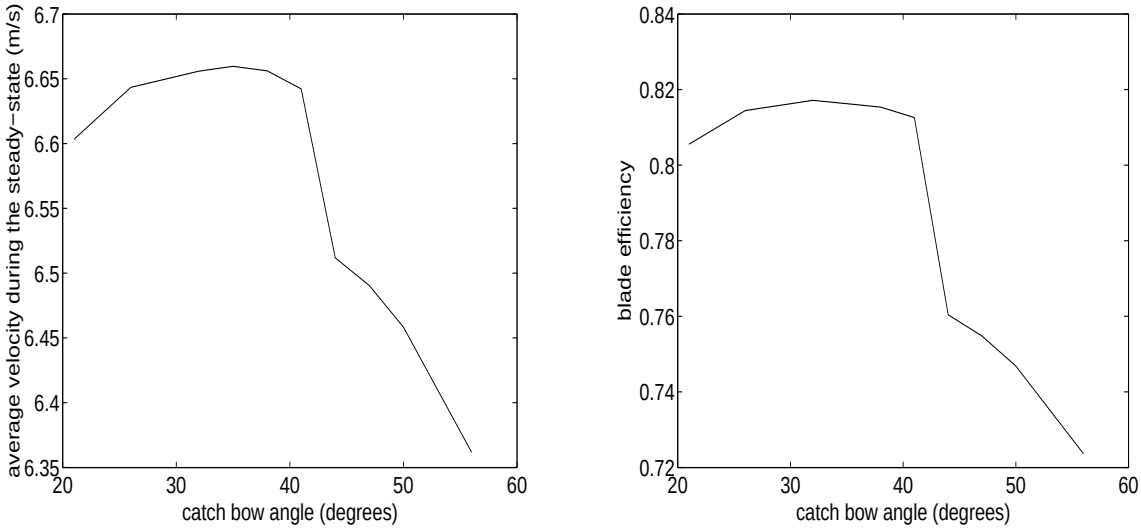


Figure 8.10: *Effect Catch Bow Angle on Boat Velocity and Blade Efficiency*

The most remarkable aspects

- These curves show a peak in velocity and blade efficiency near the 35-degree point of the oarshaft sweep. After this point, the parameters drop off quickly.
- A subsequent increase and decrease of rower power and blade power.

Discussion

Also this time, drag, slip and blade efficiency during the power stroke were observed. Now, 6 plots are displayed instead of 3. This is due to the shift in catch and finish angle. When plots are made as a function of the bow angle, it's difficult to make a comparison between the three different catch bow angles (26°, 35° and 50°). Important parts of the power stroke are not easy to recognize. Therefore, the results are plotted as a function of time (sec.) as well. Still, after these three plots, the results are presented as a function of α (rad) to show them for a particular position of oar and blade.

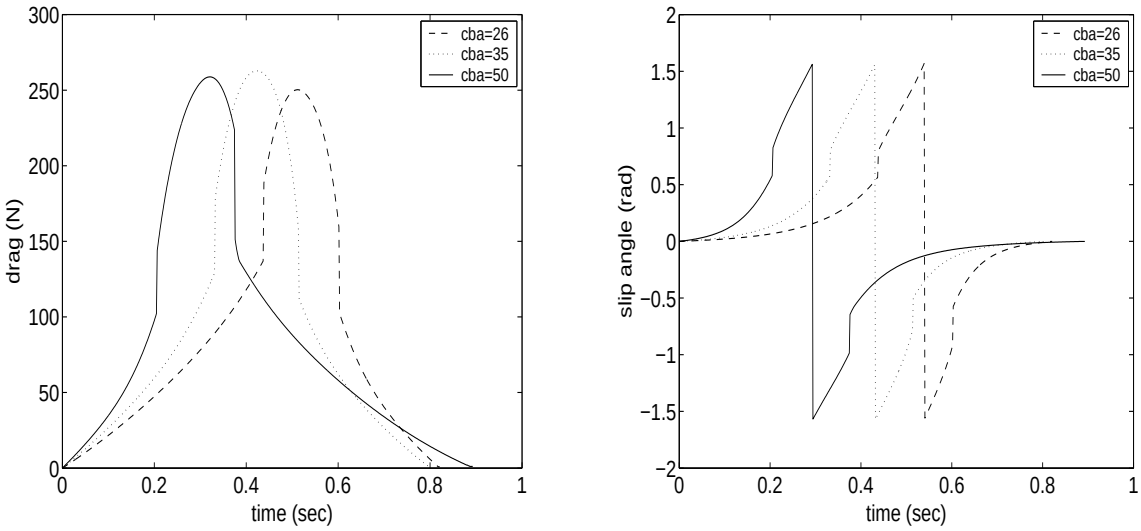


Figure 8.11: *Three Catch Bow Angles: Drag Force and Slip Angle*

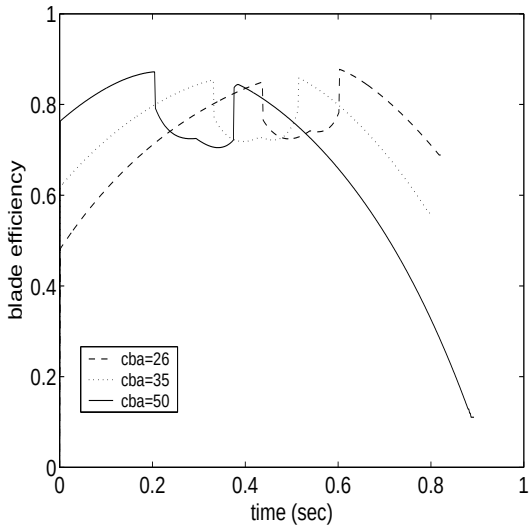


Figure 8.12: *Three Catch Bow Angles: Blade Efficiency*

What strikes for the most part is that a large catch bow angle causes an early increase in both drag force, slip angle and blade efficiency. In this situation, the direction of the oar with blade at the catch is almost transverse to the direction of the boat and water flow. The blade is more able to disturb the water in a short time which causes the high values of drag, slip and efficiency at the beginning.
What can be noticed also is that if a catch bow angle of 35 degrees has been taken, the power stroke is finished earlier. This is also the catch bow angle which shows a peak in boat velocity an blade efficiency in Fig. 8.10.

To look at the graphs plotted as a function of the bow angle (Fig. 8.13 and Fig. 8.14), one can notice, before the 90-degrees of oarshaft sweep, little drag and slip at the same position of the oar and blade (thus $\alpha + \beta$) with the boat. Little blade losses result in high blade efficiency. However, at the last part of the power stroke, a lot of water is moved. As a result, there's a lower blade efficiency than before.

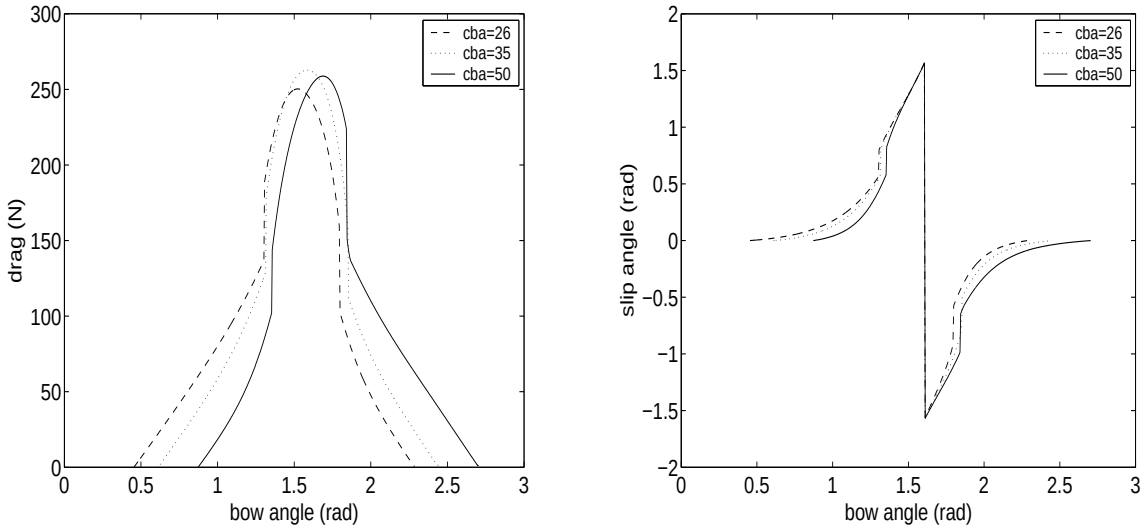


Figure 8.13: *Three Catch Bow Angles: Drag Force and Slip Angle*

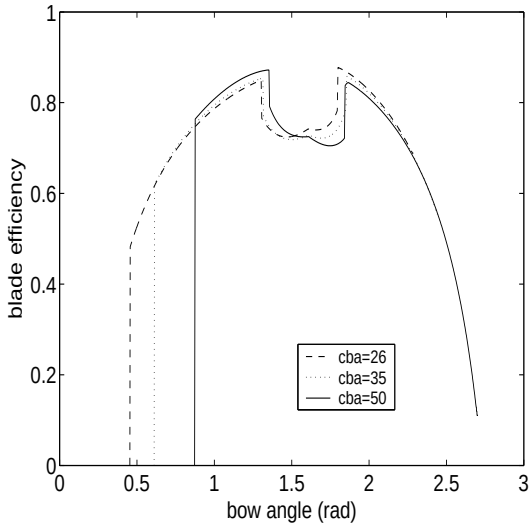


Figure 8.14: *Three Catch Bow Angles: Blade Efficiency*

If the entire stroke is considered, the tests show an optimum catch bow angle, namely $\alpha_0 = 35°$. Of course, one has to reckon with the fact that small catch bow angles require adjustments to the boat, otherwise the sweep oarhandle is awkwardly out of reach for the rower.

8.3 Validation of the Test Results

Not much is published about a parameter study with the aim to improve the way of rowing. Therefore, it's difficult to make statements about the results obtained with the current model. In the present study, the effect of a change in blade surface, outboard oarlength, blade cant angle and catch bow angle were investigated. Only Atkinson [9] gives a comprehensive description of the influences of these four factors on several aspects of rowing. He set up his own computer model and compared his results with those of the field trials of V. Kleshnev (see [9], [14]) of the Australian Institute of Sport (AIS). Atkinson's results agree well with the field results.
There are a few articles which discuss the effect of a change in oarlength [18] or another type of blade [16], [5], [11]. Especially with regard to the latter, there's a reasonable number of documentation. Blades with different shape, size, thickness, surface, edges etc. have been investigated. Still, it is not a real parameter study. Different types of blades have been compared, but information about equal blades with different blade sizes and different blade cant angles and a precise description of their influences is not available, or better: I didn't find it. Additionaly, information that is available is not always usable. As already mentioned a few times before, an analysis of an individual component may lead, when examined in other circumstances, to completely different results.
As a result, most of the gained results were compared with those of Atkinson. Below a validation of the results of the four tests. Of course, the results should be treated with any caution, since it's not completely obvious under which conditions the tests were done, what assumptions Atkinson has been made, the way he has defined the parameters etc.

- Blade Surface
  Atkinson showed the result of increased surface on the speed of a scull (single heavy man) for identical blade geometries, forces and total rower power. His model shows a strong and positive effect of increased surface on the boat velocity and blade efficiency (most probably defined in the same way as in this model [14]) as well.
  Atkinson only showed tests for a single man, but the remark was made that this benefit applies to all boats.
  Average shell speeds conducted to a men's eight are presented at Atkinson's site also. They all show a shell speed of approximately 6.5 m/s.
  Brearley, de Mestre and Watson [7] show an average speed of 6 m/s.
  The current model shows an average shell speed of approximately 6.5 m/s as well.
  At Atkinson's site, blade sizes in the range of 743 to 1859 cm^2 were examined. An improvement of 8% in blade efficiency was shown. The figures of the present model also show an improvement of approximately 8% for blade sizes in the same range.
  Atkinson shows an improvement of approximately 2.2% in shell speed; this model shows an improvement of approximately 2.8% for the same blade sizes.

- Outboard Oarlength
  In the present study, the oarhandle force applied by the rower is kept constant. The inboard length h is given as well. Next to others (e.g. [17], [18], [11]), Atkinson says that for a given rower's strength, there is, for every oarhandle length (inboard lever), an optimum outboard lever. If the outboard lever is long the propulsive reaction is weak; and if short the blade slip loss is high. There is therefore an optimum lever (or blade surface) which balances reaction and slip.

Atkinson shows that a strong rower wants a longer outboard lever. An optimum outboard lever for each rower.
Such as the results of the current model, the results of Atkinson do not show remarkable effects of an increase in outboard length either.
A situation where the inboard length is 99 cm, the blade surface is 1068 cm^2 and a rower that is capable to handle 635 Newton, has been investigated, since they are near the values of the same parameters in the base case of this model. The results both show a small decrease: Atkinson shows boat velocities from 6.3 to 6.2 m/s and the present model shows velocities from 6.54 to 6.51 m/s.

- Blade Cant Angle
  Some articles are concerned with blade shapes. Information about curved blades can be found in e.g. [5] and [16], however only Atkinson discusses the angle between blade chord and oarshaft axis. He shows the results of calculating average shell speed for a coxed (heavy men) eight with varying degrees of cant angle, but with equal overall sweep arcs, equal oarhandle force profiles, and at equal total rower power output. The results of Atkinson show an optimum cant bow angle for scullers (the results show that scullers might profit by eliminating cant from their blades), but the results do not show an optimum catch bow angle for sweepers. Atkinson prefers a large negative cant angle. Below a plot from Atkinson of the results gained with a cant angle study conducted to 8+ heavy men.

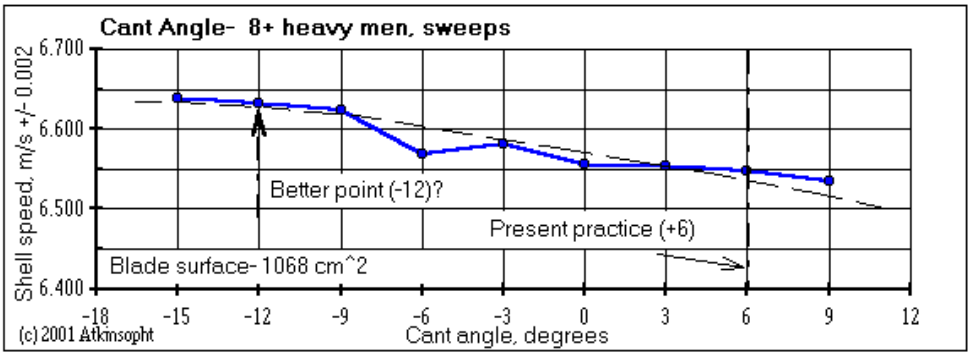


Figure 8.15: *Cant Angle Study 8+ heavy men (from Atkinson [9])*

- Catch Bow Angle
  The results of the model of this report show that smaller catch angles lead to higher shell speeds and blade efficiencies. Only, the catch bow angle should not be too small. Atkinson prefers small catch angles as well: the catch bow angle should not be larger than 35°. Atkinson also shows a fast decrease in shell speed.
  However, in contrast to the optimum catch bow angle of the current model, the results of Atkinon's model do not show an optimum catch bow angle. The catch angle should be as small as possible. Only, a difference between the model used in this research and the model of Atkinson is that the current model uses a constant stroke rate, while that of Atkinson uses a variable stroke rate. In the last case, at small catch angles, the rower is able to row at a lower stroke rate such that the time he needs to finish the stroke

effectively is longer. This could be benificial, but is not investigated in the current study.
The results of Atkinson are presented below.

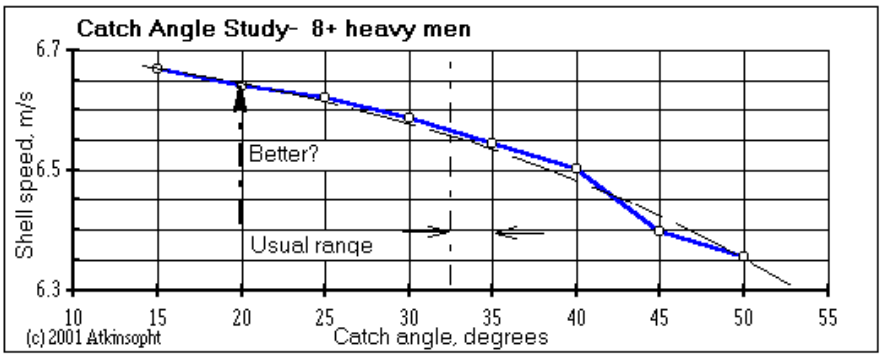


Figure 8.16: *Catch Angle Study 8+ heavy men (from Atkinson [9])*

In practice, as mentioned before, small catch angles require rigging[35] adjustments. Too small catch angles are awkwardly out of reach for the rower.
Basic Rigging Principles [17] discusses rigging arrangement. One of the aims of rigging is to co-ordinate the most efficient part of the rowing stroke with the most efficient position of the body.

Chapter 9

Conclusions and Future Work

An investigation of the rowing stroke and in particular the power stroke has been made. It will have become clear that the theory behind the rowing principle and with that the modelling of the rowing stroke is far from easy.
A small change in one or more parameters can have great influence on the way of rowing.
In the present research, it has been examined what consequences a change in oarlength, blade size, blade cant angle or catch bow angle has. The model seems to indicate that a change of blade cant angle, catch bow angle or blade size has significant effects. Especially an increase of the latter makes a positive difference: blade efficiencies over 83% are reached (an improvement of almost 10% in comparison to small blades!). Therefore, an increase in the blade surface area is very profitable. Furthermore, an early catch could be benificial, but when the catch angle is too small, the sweep oarhandle is awkwardly out of reach for the rower. In that case, adaptations to the boat have to be made. The blade cant angle seems to have an optimum value as well. The results of the tested blade cant angle, in the range of 15° negative to 20° positive, show a variation of 3% in blade efficiency.

Just a few parameters and the effects of their changes have been examined. In order to make sensible statements about possible adjustments to improve the rowing stroke, the effect of changes in other parameters have to be investigated also. But, as mentioned in the introduction, this can be very difficult, because a lot of the parameters strongly depend on each other.

Next to testing the effects of adjustments to the present model, improvements can be reached if other important factors are integrated into the model as well. One could think of the water temperature, the influence of the wind and so on. Such an extension is not always easy to make. As mentioned in chapter 6 there is, apart from the dependence of the parameters, little information available. Especially with regard to the hydrodynamical aspects of boat and blade, because these are hard (in the case of a moving blade almost impossible) to measure.

Even though, it's hard to extend the model, still the most important adjustments that should be made to improve the current model are listed below.

Adjustments to the Current Model

- Program Input Data
  Assumptions are made with regard to some parameters. This is explained in chapter 6. For example, the stroke rate is kept constant. However, if the effect of the stroke rate (together with the power applied by a rower) is investigated as well, more realistic results can be acquired (§8.3).
  More knowledge in the field of the biomechanics of rowing can help to adapt parameters as the oarhandle force P (only a simple sin-function in the present model) and the acceleration of the rowers' body H (assumed to be a cos-function) in such a way that they fit more or less with reality and with which better results can be achieved.

Extension of the Model

- Stationary/Instationary
  A stationary model, in other words time independent, for the lift and drag coefficient (flat plate data from Hoerner [9]) is used for the hydrodynamic model (§2.2). However, it's hard to obtain non-stationary information, since an oarblade is constantly changing orientation and velocity throughout the rowing stroke.

- 2D/3D
  The current model is a two-dimensional model. However, since a rowing boat moving in the water, and in addition the oar blade, will rotate and displace under the influence of waves and wind, an investigation of three-dimensional effects is important to make as well.

- Free Surface
  As mentioned in §2.2, flat plate data from Hoerner is obtained in a situation a which the blade is totally immersed. In practice, the blade, as well as the boat, are not continuously located in the water. Rotations and displacements (see first item) of boat and, together with the material, the weight of the boat and rowers affect the added mass, in other words the air and water resistance to the hull. Just as the assumptions which have been made with regard to the hydrodynamical characteristics of the blade, assumptions regarding the drag of the water (R) to the boat have been made.

Further Research

- Design Improvements
  Although, little is known about the hydrodynamic properties of the blade, one does know that lift and drag forces are generated by the blade. So adjustments to the blade can be made, in order to let the blade move through the water more efficiently. The blade size and blade cant angle have been examined, but one could also consider areas regarding the shape of the blade, thickness, edges and stiffness [5], [16].
  Of course, research to the boat itself (material, weight, stiffness, rigging arrangement etc.) can improve the way of rowing.

- Other Influences
  The influence of wind, air and water temperature, weight of the rower, the physical and psychological condition of the rower and much more, are factors which should be investigated also.

From the above, the conclusion can be drawn that there is still a lot of research to be done to improve the way of rowing. The research on the field of rowing is just at the beginning. As a consequence, the findings of the model are not perfect absolute results; varying or adding a variable may change the result, but they are relative results. On the ground of these relative results, adjustments can be made. In this respect, the findings provide a basis for further research.

Appendix A

Program

This appendix gives a more detailed discription of the program. The program is written in *MATLAB*, *MATLAB 6.1*. *MATLAB* is a high-performance, interactive software package for scientific and engineering numeric computation. *MATLAB* integrates numerical analysis, matrix computation, signal processing, and graphics in an easy-to-use environment without traditional programming. The name *MATLAB* stands for matrix laboratory. *MATLAB* is an interactive system whose basic data element is a matrix that does not require dimensioning. Furthermore, problem solutions are expressed in *MATLAB* almost exactly as they are written mathematically (read the program description of *MATLAB 6.1*).

A.1 Main Program

The main program consists of several subroutines (*MATLAB* macro files or *m*-files). Successively, each subroutine will be called. The main program (*matbaan.m*) is divided into the following parts:

- Calling the subroutine *data.m*: this subroutine consists of all input data;
- Calling the subroutine *init.m*: this subroutine defines the initial conditions;
- Calling the main loop: this loop calculates, for each time step, the values of the required parameters, which are needed to describe the rowing stroke. For example the bow angle, the angular velocity and boat velocity.
- Calling the subroutine *afdrukken.m*: displays a few disered output data;
- Calling the subroutine *visualisatie.m*: makes visualizations of the power stroke and a few plots.

A.2 Subroutines

The main loop calls the following subroutines or, in other words, m-files or functions.

falpha.m	*fbootsnelh.m*	*fversnroeier.m*
fbootweerstand.m	*fkrachthandle.m*	*fhoeksnelheidsformule.m*
fsliphoek.m	*fbladsnelh.m*	*fcl.m*

fcd.m	*lift.m*	*drag.m*
fnieuwekrachthandle.m	*fhydro.m*	*fversnboot.m*
positie.m	*fpower.m*	*fwork.m*
efficiency.m		

Functions have designated *input* and *output* variables. Any other variables used within a function are *local* variables, which do not remain after the function terminates. The main time loop is repeated a few times. How often the time loop will be executed depends on the number of strokes (the parameter *aantalhalen*) and the number of time steps. The number of time steps being denoted by N. In this way, each parameter has '*aantalhalen* $\times$ N' output values. The time (sec.) of one stroke depends on the given stroke rate.

Below, a short description of each function and their output variables as used in main loop. The functions are placed in calling sequence.

falpha.m:	Calculates the catch bow angle *(alpha)* by integrating the angular velocity.
fbootsnelh.m:	Calculates the boat velocity *(vboot)* by integrating the boat acceleration.
fversnroeier.m:	Determines the acceleration of the rowers' body *(H)* relative to the boat.
fbootweerstand.m:	Determines the drag by the water on the boat (R).
fkrachthandle.m:	The output *(P)* of this function describes the force on the handle by the rower. The assumption has been made that its typical force profile is more or less described by a sin-function (eq. (6.1)); little force is used at the beginning and the finish of the power stroke.
fhoeksnelheidsformule.m:	Determines the angular velocity *(omega)* by the use of the bisection method (§7.4). The angular velocity is calculated in such a way that it satisfies the equation for the oarhandle force P. On the base of this angular velocity, a new oarhandle force *nieuweP* (see below) is calculated to check if *nieuweP* agrees with P. Differences within a range of 5 N were allowed.
fsliphoek.m:	Calculates the angle of attack *(varphi)*.
fbladsnelh.m:	Calculates the blade velocity *(vblad)*.
fcl.m:	Determines the lift coefficient *(cl)* as a function of the angle of attack.
fcd.m:	Determines the drag coefficient *(cd)* as a function of the angle of attack.
flift.m	Calculates the lift force *(lift)*.
fdrag.m	Calculates the drag force *(drag)*.
fnieuwekrachthandle.m:	Calculates the force *(nieuweP)* exerted by the rowers on the oarhandle with the calculated angle of attack and blade velocity as input parameters.
fhydro.m:	Calculates the hydrodynamic force *(fh)* on the blade.
fversnboot.m:	Calculates the boat acceleration *(dvdt)*.

positie.m:	Calculates the position of the main parts of the boat and oar (such as the position of the oarlock, the center of pressure etc.) for each time step.
fwork.m:	Calculates the work done by the rower *(Wrower)*, the blade *(Wblade)* and the boat *(Wboat)*.
fpower.m:	Calculates the power delivered by the rower *(Prower)*, the blade *(Pblade)* and the boat *(Pboat)*.
efficiency.m:	Determines the blade efficiency *(eff)*.

A.3 Input

The input files of the program are *data.m* and *init.m*. The data file contains the following information. Tests can be done by modifying these parameters.

```
N=1000;                              %number of time steps
strokerate=37.5;                     %that is 37,5 strokes per minute
duurhaal=60/strokerate;              %1.6 sec required time for 1 stroke
dt=duurhaal/N;                       %time step
a=2.10;                              %outboard length
b=0.37;                              %blade length
h=1.00;                              %distance from centre of grip to
                                     %swivel
twopi=2*3.141592653589;
beta=6*twopi/360;                    %blade cant angle (in radians)
l=sqrt(a^2+b^2+2*a*b*cos(beta));     %distance from swivel to point of
                                     %pressure
gamma=asin(b*sin(beta)/l);           %angle between l and a
gamma=gamma*twopi/360;
oppblad=1100e-04;                    %blade surface
```

The initial conditions of the used parameters are described in the file *init.m*. These conditions are estimations for the initial values of the parameters at the beginning of a power stroke. The boat is already moving.

```
t(1)=0;
alpha(1)=35*twopi/360;
alphabegin=alpha(1);
alphafinish=140*twopi/360;
vboot(1)=5.000;                  %a low estimation.
H(1)=(twopi/(N*dt))^2*0.36;      %max amplitude acceleration rowers
R(1)=24.93-11.22*vboot(1)+13.05*(vboot(1))^2;
P(1)=0;
omega(1)=vboot(1)*sin(alphabegin+beta)/(l*cos(beta-gamma));
```

```
varphi(1)=0;
vblad(1)=fbladsnelh(t(1),vboot(1),alpha(1),varphi(1),omega(1));
cl(1)=0;
cd(1)=0;
lift(1)=0;
drag(1)=0;
nieuweP(1)=0;
fh(1)=0;
dvdt(1)=fversnboot(t(1),vboot(1),alpha(1),H(1),P(1),alphafinish);
```

A.4 Output

Data which is produced by the main program can be used to make visualizations. These visualizations are produced with the help of *MATLAB* as well. The function *visualisatie.m* contains all information to make the disered visualizations of the output data.
Data which has been acquired by calling the subroutine *positie.m* is now used to make a representation of the displacement of the oar and blade through the water. The slip path and zero-slip path can be shown.
Graphs of the output data are drawn with *visualisatie.m* as well. A modification of the input data may lead to different plots. Several conclusions can be drawn from these graphs (see ch. 8).

Appendix B

Glossary

1. **Aerodynamics**: the science that deals with the motion of air and other gaseous fluids and the forces acting on objects as a result of the relative motion between the air and the object.
2. **Angle of Attack (Slip Angle)**: (in the present study) the angle between the plane of the blade (wing) and the direction of fluid motion.
3. **Angular Velocity**: the angular velocity ω of a rigid body is the rate of change of its angular position.
4. **Aspect Ratio**: the ratio of the *span* squared to the blade (wing) area A: $span^2/A$, which reduces to the ratio of the span to *chord* in the case of a rectangular blade (wing) ($span/chord$) (see Fig. 2.7).
5. **Blade Cant Angle** β: the angle between the blade chord and the oarshaft axis.
6. **Bow**: the front of the boat.
7. **Bow Angle** α: the angle between the oarshaft and the longitudinal axis of the shell: zero at the bow[6], 90 degrees athwartship, and 180 degrees at the stern[45].
8. **Catch**: the part of the stroke when the blade is put in the water.
9. **Center of Pressure**: a point along the blade about which the moment due to the lift is zero, i.e., it is the point of action of the lift. The center of pressure will change its position when the angle of attack changes.
10. **Chord**: the length from leading edge to trailing edge of a blade (wing).
11. **Cox**: short for "coxswain", the person who steers the boat and gives commands to rowers.
12. **Drag**: the component of the hydrodynamic force in the direction of the fluid flow, called the drag force, D.
13. **Drive (Power Stroke)**: the part of the stroke between the catch and the finish.

14. **Energy**: a measure of how long one can sustain the output of power[31], or how much work[51] can be done. One common unit of energy is the kilowatt-hour (kWh, $1kWh = 3,600,000J$).
    There are two kinds of energy: *potential* and *kinetic*. Potential energy is waiting to be converted into power, kinetic energy is energy of motion.

15. **Equitangential Curve**: see 'Tractrix'.

16. **Footboard**: the adjustable wooden or plastic board which the shoes are attached to. The footboard is a part of a footstretcher[18], though the terms are sometimes used interchangeably.

17. **Footrest**: see 'Footboard'.

18. **Foot Stretcher**: an adjustable bracket in a shell to which rowers feet are secured.

19. **Gearing**: the gearing of a boat is the measure of how hard it is to complete each stroke [13]. Gearing is defined as the ratio

$$\frac{\text{Outboard Length}}{\text{Spread}}$$

20. **Handle**: the end of the oar a rower holds in his hand (see Fig. B.1).

21. **Hydrodynamics**: the science of hydrodynamics is concerned with the behavior of fluids in motion.

22. **Inboard Length**: the length of the oar[27] or scull[37] measured from the outside of the collar (see Fig. B.1 and B.2) to the handle.

23. **Initial Condition**: the condition at an initial time from which a given set of mathematical equations or physical system evolves.

24. **Kinetic Energy**: see 'Energy'.

25. **Lift**: the component of the hydrodynamic force normal to the direction of the fluid flow, called the lift force, L.

26. **Moment (Torque) τ of a Force**: a measure of the forces' tendency to produce rotation. Vector Formulation:

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}$$

    where $\boldsymbol{r}$ is a position vector from a reference point to a point along the line of action of a force $\boldsymbol{F}$.

27. **Oar**: a leaver by which the rower pulls at the handle against the oarlock to move the boat through the water (see Fig. B.1).

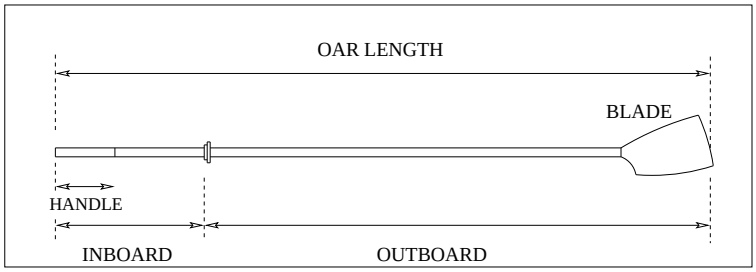


Figure B.1: *The Oar*

28. **Oarlock**: a device which swivels around the rigger pin and holds oar (see figure B.2).

29. **Outboard Length**: the length of oar or scull from the outside of the collar to the tip of the blade (see Fig. B.2).

30. **Potential Energy**: see 'Energy'.

31. **Power**: a measure of how quickly work[51] can be done. The SI unit for power is the Watt (W, $1W = 1Nm/s$).

32. **Power Stroke**: see 'Drive'.

33. **Rating (Stroke Rate)**: the number of strokes taken per minute.

34. **Recovery**: the part of the stroke cycle between the finish and the catch in which the oar is feathered and the seat is returned to the aft end of the slide.

35. **Rigging**: the adjustment and alteration of accessories (riggers, foot stretchers, oar,etc.) in and on the shell to maximize a particular rowers efficiency, based on their size and capabilities.

36. **Sculling**: the art of rowing by using two oars or sculls.

37. **Scull**: a short oar used in each hand for single, double, and quad sculling boats.

38. **Shell**: a rowing boat.

39. **Slip**: the distance traversed in the water by the center of pressure of the blade parallel to the direction of the hull.

40. **Slip Angle**: see 'Angle of Attack'.

41. **Span**: the length from the upside of the blade to the underside of the blade. longitudinal dimension of an oar blade).

42. **Spread**: (for sweeping[48]:) the distance from the centerline of the boat to the oarlock (Fig. B.2).

43. **Stall Point**: at this point, the fluid, which flows around a surface, can no longer follow that surface. The flow is no longer attached to the surface, and the upstream releases. At this moment a critical angle of φ is reached. At low angles of attack, the lift developed by the move of the blade will increase with an increase in angle of attack. However, there is a maximum angle of attack after which the lift will decrease instead of increase with increasing angle of attack. This is know as *stall* (see Fig. 2.6).
Knowing the stall angle of attack is extremely important in the airfoil or wing theory: for predicting the minimum landing and takeoff speeds of an airplane.

44. **Steady-State**: a steady-state is reached when the average boat velocity no longer changes with time.

45. **Stern**: the back of the boat.

46. **Stroke**: the complete cycle of moving the boat through the water using oars or sculls; the rower seated nearest the stern[45].

47. **Stroke Rate**: see 'Rating'.

48. **Sweeping**: the art of rowing with one oar held by both hands (see Fig. B.2).

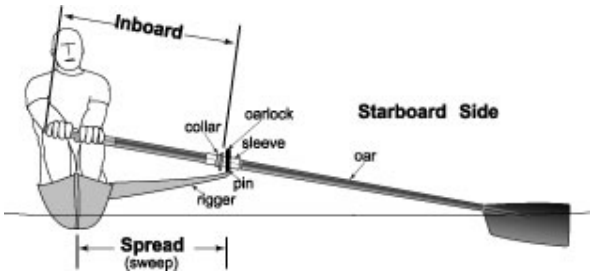


Figure B.2: *Sweep Diagram*

49. **Tractrix (Equitangential Curve, Tractory)**: a curve such that any tangent segment from the tangent point on the curve to the curve's asymptote have constant length (see Fig. 1.3). First studied by Huygens in 1692.

50. **Tractory**: see 'Tractrix'.

51. **Work**: the product of a force and a traveled distance caused by this force. The SI unit for power is the Joules (J, $1J = 1Nm$).

Appendix C

List of Variables

A_{bl}	blade surface (m^2)
a	outboard oarlength, the length from the oarlock to the place where the blade is fixed to the oar. (m)
α	bow angle (rad)
α_0	catch bow angle (rad)
α_f	finish bow angle (rad)
B	place where oar is fixed to the boat (m)
b	blade length from the end of the oar to the center of pressure (m)
β	blade cant angle (rad)
c_D	(nondimensional) drag coefficient
c_L	(nondimensional) lift coefficient
D	drag force (N), also denoted for the center of pressure (m)
F_h	hydrodynamic force (N)
H	acceleration of the rowers' body (m/s^2)
h	distance from the centre of the handle to the oarlock (m)
γ	angle between l and a (rad)
L	lift force (N)
l	length from the oarlock to the center of pressure (m)
m_b	mass of boat (including the cox, if present) (kg)
m_r	combined mass of rowers (kg)
ω	circular frequentie of the oar (rad)
P	force exerted at the oarhandle by the rower (N)
P_{rower}	power applied by the rower (W)
P_{handle}	power applied to the oarhandle by the rower (W)
P_{body}	power losses by moving the rower's body (W)
P_{blade}	power losses by moving the blade (W)
P_{drag}	power losses due to the movement of the boat (W)
R	drag of the water on the hull (N)
t	time from start of power stroke (s)
V_b	velocity of boat at time t (m/s)
V_{bl}	velocity of blade at time t (m/s)
dV_b/dt	acceleration of boat at time t (s)

φ	angle between the plane of the blade and the direction of fluid motion, in other words the slip angle or angle of attack (rad)
W_{rower}	work done by the rower (J)
W_{handle}	work done by the rower to move the handle (J)
W_{body}	work done by the rower to move his body (J)
W_{blade}	work done by the blade (J)
W_{drag}	work done by the boat (J)

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Photo Credit

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http://www.concept2.com/media/s6.gif
(A photo is one frame of an overhead video taken from a bridge. The boat is shown on the bottom and is moving from left to right. The red dots mark the tip of the blade at each frame of the video.)

Fig. 2.6:
http://www.fz-rossendorf.de/FWS/FWSH/EBLC/separation-control/airfoil-1.jpg
http://www.fz-rossendorf.de/FWS/FWSH/EBLC/separation-control/airfoil-2.jpg

Fig. 1.2:
http://www.atkinsopht.com/row/bladpth8.gif