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A time to share

Time discounting in resource
dilemma situations

Renate van der Kooij

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Voorwoord

Voor u ligt het verslag van het afstudeeronderzoek dat deel uitmaakte van mijn studie wiskunde aan de Rijks*universiteit* Groningen. De wiskundesectie heeft mij toegestaan af te studeren met de variant milieuwetenschappen. Daarom heeft het onderzoek plaatsgevonden in samenwerking met het Instituut Voor Energie en Milieukunde (IVEM) aan dezelfde universiteit.

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Renate van der Kooij,
juli 2000

Abstract

Time discounting makes cooperation in resource dilemma situations less probable. This paper develops a mathematical model of a general resource dilemma that incorporates the effects of time discounting. Several models for time discounting and some aspects of the resource dilemma are varied in the model to study how these characteristics influence cooperation in resource dilemma situations.

Contents

Summary

If a number of people share a renewable resource pool, this may result in a so-called resource dilemma situation. This is a situation in which individual needs are in conflict with collective interest. If a person overharvests from the resource pool, the pool is harmed, but this harm is spread out over all other users. On the other hand, the gain of the extra harvest is for the individual. However, if all individuals overharvest, their pay-offs are lower than if they had not.
This dilemma also has a temporal aspect to it. Overharvesting from the resource usually has a negative effect on future harvests. These later harvests are discounted with respect to the harvest now: the further away in time, the lower they are valued. This temporal dilemma increases the tendency to overharvest. Both psychologists and economists have studied time discounting and have proposed several functions and models that describe how people discount delayed outcomes.
Some researchers have studied time discounting in resource dilemma situations. Poortinga (1997) developed a mathematical model for a general renewable resource dilemma with a specific discount function. In the present study, we have attempted to extend and generalize Poortinga's model, to obtain predictions about the relations between various characteristics of the resource dilemma and the attractivity of overharvesting.
Compared to Poortinga's model, we made less restrictions in the choice options of players, we used different growth or replenishment functions, we implemented four different discount functions, and we allowed for different distributions of the effects of overconsumption across players.
Under certain assumptions, for different RD-situations, we obtained conditions for cooperation (i.e. refraining from overharvesting) that show how several RD-characteristics influenced the attractivity of cooperation. It was found that the choice whether or not to overharvest can depend on the number of participants, the discount rate of the participants, the replenishment rate of the resource, the initial state of the resource, the expected level of overharvesting by others and the vulnerability of the resource.
In all cases we were able to elaborate analytically, we found that a higher number of participants or a higher discount rate makes it more attractive to overharvest. Also, a higher replenishment rate makes it less attractive to overharvest. If the initial size of the resource pool is larger, it depends on other aspects of the dilemma situation whether this will have a positive or negative effect on the attractiveness of cooperation: in the case of logistic growth, if the initial resource pool is larger than its point of maximum growth, overharvesting is always more attractive than cooperation. The effect of the expected level of overharvesting by others on the attractivity of cooperation remained inconclusive. Finally, our model suggests that, in combination with other negative circumstances, viz. a small initial sustainable yield and a high expected level of overharvesting by others, a greater vulnerability of the resource pool will make overharvesting more attractive, according to our model.
In the last Chapter of this paper, some suggestions for helping to solve resource dilemmas are made from the obtained relations. Reducing the number of participants by, e.g., privatizing the resource, or reducing the discount rate of participants by supplying information about the long-term effects of overconsumption, would enhance the attractivity of cooperation. Characteristics of the resource, like growth rate, initial pool size and vulnerability, are determined by nature and can not be changed, but it is useful to be aware of their effects on the attractiveness of overharvesting and regard them in determining future policy concerning the exploitation of resources.

1 Introduction

The early Europeans in North America frequently commented on the huge numbers of blue, longtailed, fast and graceful pigeons in the country. From all over the New World came reports on thick flocks of these so-called passenger pigeons shadowing the sky for hours and hours as they flew by. The exact number of passenger pigeons in North America when the Europeans arrived is not known, but a good guess is 5 billion - about the same as the total number of birds to be found today in the United States.
The reason why the passenger pigeon existed in such numbers was, that it had few natural predators. It was, however, surprisingly vulnerable to human intervention. Each female laid only one egg per year, which made it difficult to replace any losses quickly. Its habit of nesting in vast colonies and migrating in huge flocks made it an easy prey. The birds were sought after for their meat as well as their feathers. It is doubtful whether the number of pigeons declined very much in the first couple of centuries of European settlement, considering the relatively small numbers of humans in the area. By the middle of the nineteenth century, the pigeon population was probably still several billions strong.
Then, in the early 1850s, railways were opened that linked the Great Lakes area with New York. In order to supply the developing cities of the east coast with a cheap sort of meat, large scale commercial hunting of the birds began. In the first years, hundreds of thousands of pigeons were sent to New York every year. In the 1860s and 1870s, the numbers of birds shipped east increased to millions per year.
The vast population of passenger pigeons might have withstood a yearly harvest of some hundreds of thousands. The huge flocks could have fed thousands of people to this very day. As it was, the hunters were not very much interested in preserving their source of income in the long run, but more in the short-term gain. They competed almost till the very last bird, knowing that every bird they would leave behind would be caught by another hunter. The last passenger pigeon, sole survivor of a species that had once numbered 5 billion, died in captivity in 1914. [Ponting, 1991]

What has happened to the passenger pigeon, has happened to countless species of birds, fishes, furred or edible animals. It still happens today to all the earth's riches. Many of the world's resources are exploited by more than one agent. Each agent tends to use too much of the resource because, while the benefit of overharvesting is for him alone, the damage done by it is shared by many others. Unilaterally refraining from overuse is therefore not in the individual's interest. On the other hand, refraining multilaterally from overharvesting would be in every agent's interest, since this would perpetuate the agents' source of income and, eventually, yield more profit than depleting the resource would do. This situation is called a resource dilemma (RD) [see e.g. Messick and Brewer, 1983]. Not only populations of animals are depleted for this reason. It also occurs to other resources, e.g. common pastures on which herds graze, or a river into which wastes are discharged, or a forest from which wood is harvested.

In our example of the passenger pigeon, we saw that in a resource dilemma, not only individual and collective interests are in conflict, but also short-term and long-term consequences. Since the negative consequences of overharvesting become apparent later than the profits, these consequences may be 'time-discounted'. Temporal distance can make the negative effects of overharvesting seem less important, because in human decision making, present outcomes usually weigh heavier than future outcomes. This phenomenon is called time discounting; see e.g. Roelofsma (1996). Despite the apparent importance

of the time dimension in the resource dilemma, this component has rarely been studied in the context of resource dilemmas.
We aim to study both phenomenons simultaneously, by constructing a descriptive mathematical model for people's behaviour in a resource dilemma, in which time discounting is incorporated. This model is used to predict relationships between several relevant characteristics of the resource dilemma and the choices people will make in the dilemma situation.

This paper is divided into four Chapters. The first Chapter is this introduction. The second Chapter deals with resource dilemmas and time discounting. Both topics are explained in some detail. Some models for time discounting found in literature are also presented and discussed. The final section of Chapter 2 presents a review of literature on combinations of models for resource dilemmas and time discounting, especially Poortinga's (1997) work, of which the present study is an extension.
In Chapter 3 a descriptive model for people's behaviour in a resource dilemma is constructed, in which time discounting is incorporated. The resource dilemma is considered as a game of two players: (1) one of the users of the resource, to whom we will refer as 'self' and (2) all other users of the resource. Each player has infinitely many moves to make, and in each move, every player has the choice whether to overuse the resource or to refrain from overuse. A similar approach was published by Muhsam (1973) for analyzing the tragedy of the commons. Into this model for a resource dilemma, models for time discounting that are discussed in section 2.2, are incorporated.
From this model, conclusions are drawn concerning the relations between characteristics of the modelled dilemma and choices made by the participants in the dilemma. Various model aspects (viz. the model for biological growth, the models for time discounting and the distribution of loss) are varied to see whether these variations affect the relations between dilemma characteristics and behaviour.
In Chapter 4 an inventory is made of the resulting relationships. These yield various suggestions for helping to solve resource dilemmas which, if left to themselves, tend to lead to disaster and destruction.

2 Literature Research

2.1 The resource dilemma

A social dilemma is a situation in which individual interests are in conflict with collective interests [Liebrand and Van Lange, 1989; Messick and Brewer, 1983]. The dilemma lies in the fact that an individual gain can be made through a non-cooperative choice, but if all individuals choose non-cooperatively, in the end, everybody is worse off than if all had chosen cooperatively. For instance, in the example mentioned in the introduction, each individual hunter gained by catching as many passenger pigeons as possible (non-cooperation or defection). On the other hand, if all hunters would have restrained their catches so that the species would not have extinguished (cooperation), the total yield would have been much greater.

Social dilemmas come in various kinds that differ with regard to their pay-off structure. They are called *Prisoner's dilemmas* if, irrespective of the choice of others, non-cooperation is most profitable. Table I shows that, in a two-person Prisoner's dilemma, the pay-off for person 1 is highest if person 1 defects, while person 2 cooperates. If person 2 defects, it will still be more profitable for person 1 to defect also, but they both would have been better off if they had chosen cooperatively.

Table I

PLAYER 2	PLAYER 1: C	PLAYER 1: D
C	3 / 3	4 / 1
D	1 / 4	2 / 2

Table II

PLAYER 2	PLAYER 1: C	PLAYER 1: D
C	3 / 3	4 / 2
D	2 / 4	1 / 1

Table III

PLAYER 2	PLAYER 1: C	PLAYER 1: D
C	4 / 4	3 / 1
D	1 / 3	2 / 2

<u>Table I</u>: Cooperation or Defection in a Prisoner's dilemma. <u>Table II</u>: Cooperation or Defection in a Chicken dilemma. <u>Table III</u>: Cooperation or Defection in a Trust dilemma. Pay-offs for player 1 above right; pay-offs for player 2 down left in every cell.

Deciding whether to go to work by car (defection) or by public transport (cooperation) is an example of an n-person Prisoner's dilemma. Defection is more comfortable, but if everybody defects, queues will become quite long.
In a *Chicken dilemma,* cooperative behaviour gets more attractive as more others are expected to choose non-cooperatively. See Table II for the pay-offs in a two-person Chicken dilemma. An example of an n-person Chicken dilemma is choosing whether to pay contribution to a much valued public good. If enough others contribute defection is (more) attractive since the contribution fee is saved, while the public good is still realised.
A person in a *Trust dilemma,* on the contrary, will prefer to cooperate as more others are also likely to be cooperative, as is seen in Table III which shows the pay-off matrix for a two-person Trust dilemma. The dilemma whether to hoard food is a Trust dilemma. Cooperation, i.e. deciding not to hoard, is more attractive when one expects few others to hoard.

A specific type of social dilemma is the so-called *resource dilemma.* Individuals share

a resource, for instance a meadow for cattle, a lake from which fish is harvested or, on a global scale, the oil supplies. It is each individual's private interest to take as much as possible from the resource, but if everybody does so, the resource will be depleted soon. For the resource to be used optimally, the individual use must be restricted.
The optimal use of a resource depends strongly on whether or not the resource is renewable. However, individual choices are not only determined by the properties of the resource, i.e. the character of the dilemma, but also by the individual's perception of the dilemma. Research into social dilemmas revealed several factors influencing the choice for or against cooperative behaviour in social dilemmas.

Expectations about other people's behaviour are an important factor. Depending on the kind of dilemma faced (Prisoner's, Chicken or Trust), the trust that agents have in other people's cooperation can either enhance or diminish the attractiveness of cooperation for themselves.
Other influential factors appeared to be social values and responsibility of participants, information about other people's behaviour, personal efficacy, ingroup identity and ingroup bias [Van Lange et al., 1992; Messick and Brewer, 1983].
The number of persons participating in the dilemma is also important. Generally, the more participation in the dilemma, the more attractive it is to defect. The reason for this is, that a larger number of participants causes diffusion of one's own responsibility, decrease of the perceived controllability of the dilemma and an increase of anonymity in the dilemma. Also, in a dilemma with a higher number of participants, the period after which the consequences of self-interested behaviour become visible tends to be longer [Liebrand and Van Lange, 1992].

The time at which a problem or loss will be felt, strongly influences the salience of the problem [e.g. Roelofsma, 1996]. In general, delayed outcomes are perceived as less important. This phenomenon is referred to as 'time discounting' [e.g. Loewenstein and Thaler, 1989; Kirby, 1997]. In dilemma situations, positive and negative effects of a certain behaviour usually occur at different moments in time, which causes a difference between the perceived importance of these effects. In resource dilemmas, the benefits are generally immediate, while the negative consequences become manifest at a later point in time. For instance, the profit made by putting too many heads of cattle on a commons (or fishing too much from a lake) is available this year, while the deterioration of the commons resp. the decrease in the amount of fish in the lake, is not felt till next or even a later year. The gain of exploiting oil sources is immediate, but increases in exploitation will lead to a lower pressure in the deposite, which makes future exploitation more costly. Vlek and Keren (1992) call this phenomenon a 'temporal dilemma'. The next section will discuss 'time discounting' more extensively.

2.2 Time discounting functions

Much research has been done in the field of intertemporal choice, i.e., the choice between outcomes occurring at different moments in time. In intertemporal choice, the phenomenon of time discounting we referred to in the former section, plays a crucial role. Time discounting is the effect of temporal distance on the subjective evaluation of outcomes; outcomes (gains or losses) that are delayed tend to be valued less than temporary nearer outcomes. Key questions in the studies were: how much do people discount, and what factors affect the way or extent to which people discount delayed outcomes?

This has resulted in several models that aim to describe how people choose between two outcome pairs (size, delay) of losses or gains. Or, phrased differently, how they value delayed outcomes. Generally, a model gives a decreasing function f of time t, where $f(0) = 1$. If at time t an outcome of size x_t will occur, the model states that, at time $t = 0$, an agent will attach the value $f(t)x_t$ to this outcome. This is called the present value of the outcome.

A commonly used model is the Discount Utility model (DU), first proposed by the economist Irving Fisher (1930). It assumes a constant percentual decrease in value per time unit, thus resulting in exponential decay of the perceived value over time. The corresponding discount function is

$$f(t) = \frac{1}{(1+r)^t}$$

where $r > 0$ is called the discount rate per time period. Usually the discount rate is assumed to be equal to the market rate of interest. Figure 1 shows the graph for this function.

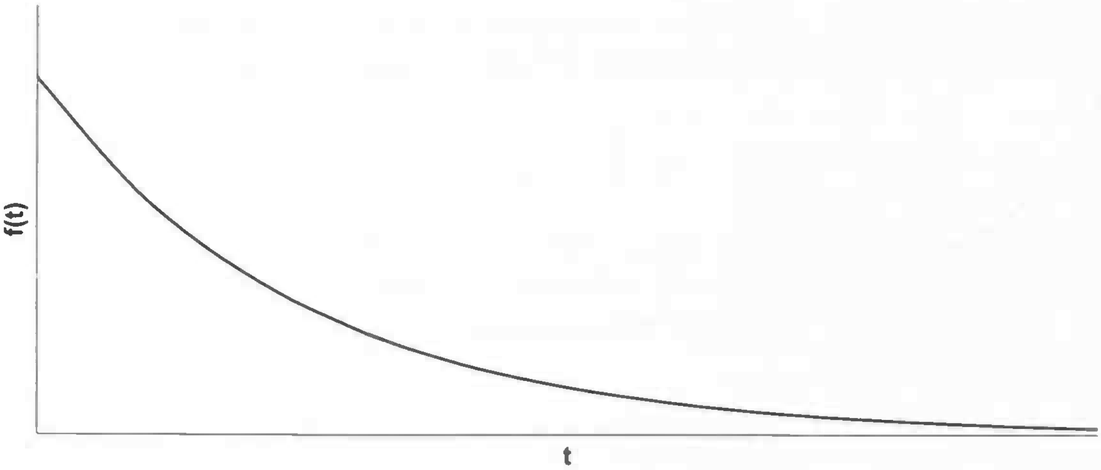

Figure 1: Graph of a DU discount function.

Roelofsma (1996) mentions several assumptions underlying this model. The most relevant one in this context is that DU predicts stationarity or time-consistency of choice. That is, discounting depends on time intervals, but is independent of time otherwise. More specifically, if subjects are indifferent between outcome pairs $(x, 0)$ and $(x + c, \tau)$, they will also be indifferent between (x, t) and $(x + c, t + \tau)$.
Another assumption of DU is that the nature of the outcome (size and sign) does not affect the discount rate: large and small outcomes, gains and losses are assumed to be discounted equally.
These assumptions are normative rather than realistic; they reflect a thoroughly rational concept of man. The model, therefore, is normative or prescriptive as opposed to descriptive. Empirical research into how people discount delayed outcomes has shown these assumptions not to be in accordance with reality [Loewenstein and Prelec (1992), Loewenstein and Thaler (1989), Roelofsma (1996)]. Discount rates were found to decline as the time at which the delay (τ) occurs is further away, causing shifts of preference. Large outcomes suffer less proportional discounting than smaller ones. And some studies

[e.g. Shelley (1993)] have shown that losses and gains are discounted differently.

Alternative models have been proposed to describe actual time discounting more adequately. Roelofsma (1996) lists three alternative approaches to DU, viz. Weber's Law, hyperbolic discounting and Prospect theory. Laibson (1997) adds a fourth: quasi-hyperbolic discounting.

The discount function derived from Weber's Law is

$$f(t) = \frac{1}{(1+r)^{c*ln(t+1)}}$$

Figure 2 shows the graph of a discount function according to Weber's Law.

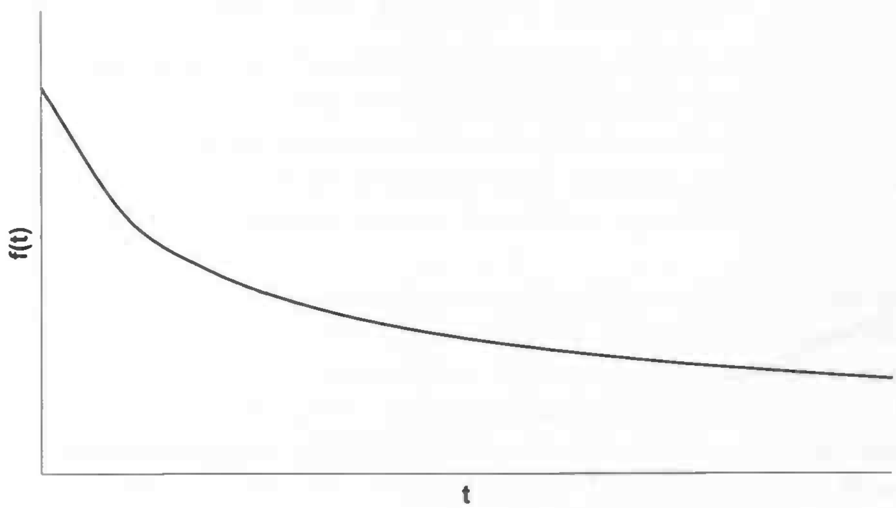

Figure 2: Graph of a Weber discount function.

This model assumes that, analogous to sound, time is perceived on a logarithmic scale. Because the logarithm is an increasing function that gets less steep as time processes, discounting does not only depend on time intervals, but also on time itself. As an outcome is delayed more, an extra delay does not produce as much extra discounting as when the outcome is near. This implies that preference shifts are possible. Size and sign of the outcome, however, are still not expected to influence the discount rate.

Some empirical findings are best fit by hyperbolic curves. It is then assumed that valuations are inversely related to a delay:

$$f(t) = \frac{1}{1+kt}$$

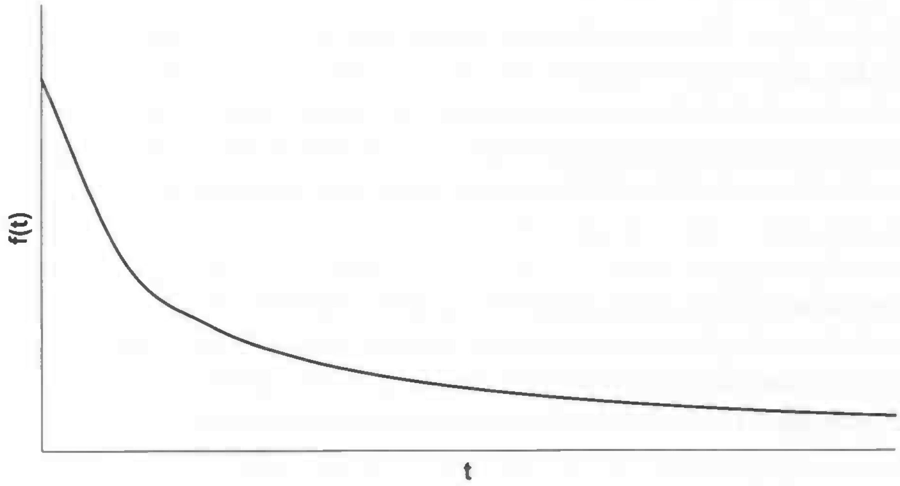

Figure 3: Graph of a hyperbolic discount function.

Hyperbolic functions may cross, thus accounting for possible preference shifts.

Quasi-hyperbolic discounting is somewhere in between Discounted Utility and hyperbolic discounting. The quasi-hyperbolic discount function is a discrete time function with values

$$\{1, \frac{\beta}{(1+r)}, \frac{\beta}{(1+r)^2}, \frac{\beta}{(1+r)^3}, \ldots, \} \qquad (0 < \beta < 1),$$

of which the plot (see Figure 4) is similar to the curve of a hyperbolic function, while it maintains most of the analytical tractability of the exponential function. This characteristic will appear to be very important in this study.

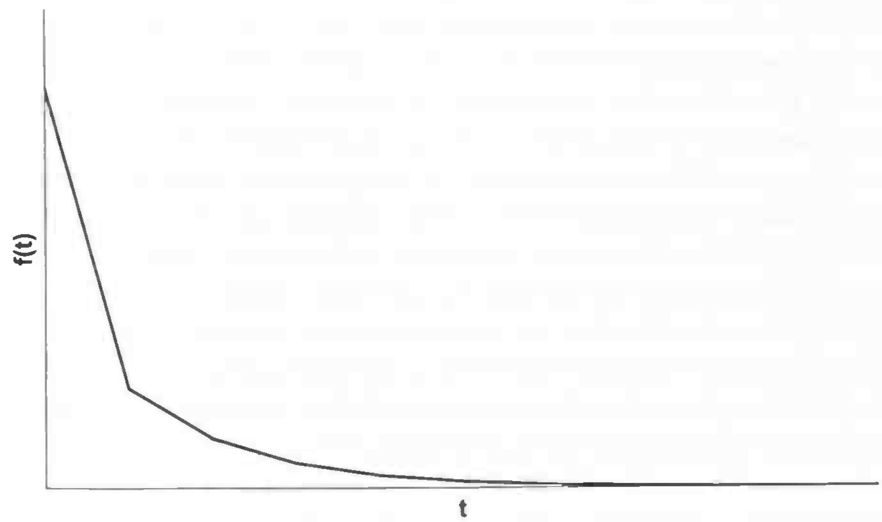

Figure 4: Graph of a quasi-hyperbolic discount function (points connected).

Neither hyperbolic nor quasi-hyperbolic discounting includes the assumption of time-consistency. Usually, however, it is assumed that the hyperbolic and quasi-hyperbolic functions are the same for both losses and gains, and the discount rate does not depend on the size of the outcome. So, these functions do account for the first, but not for the latter anomalies to DU.

The choice model based on Prospect Theory [Loewenstein and Prelec (1992)] does -in principle- account for all found anomalies to DU. It can not be written as $f(t)x_t$; the valuation of x_t contains probabilities of occurrence of this outcome and decision weights concerning the subject's treatment of probabilities. This makes the model so complicated that it will not be considered further in this study, despite its potential descriptive validity [Roelofsma,1996].

Concluding, various models for time discounting are available in litérature. The discounted utility model dominates as a normative model, but fails as a descriptive model. Several alternative models, viz. Weber's Law, hyperbolic and quasi-hyperbolic discounting, succeed in accounting for some observed discrepancies, but not all. The only candidate that may explain all discrepancies, the model based on Prospect theory, is too complex for present purposes.

2.3 Temporal Discounting in Resource Dilemmas: Literature Review and Research Questions

We mentioned in Chapter 1 that we expect that discounting of the value of delayed outcomes is an influential factor determining people's choice in a resource dilemma. Therefore, it should be useful to study the influence of temporal aspects on decisions in resource dilemmas. For this purpose one could combine mathematical models for resource dilemmas with the different models describing time discounting.

Few researchers have combined models of resource dilemmas with the discount functions. A related research area is the infinitely iterated Prisoner's dilemma. Many game theoretic and economic studies have addressed this particular dilemma [e.g. Taylor (1987), Bernheim and Dasgupta (1993), Stahl (1991), Stephens et al. (1995), Kalai et al. (1988)]. In some of these, effects of temporal discounting were incorporated into the model by using DU to assess the present value of delayed outcomes. However, these studies are not directly relevant to the present study: RD-models differ from iterated PD-models, since the former are time-dependent, whereas the latter are not. Essentially, time dependency means that the possible pay-offs at time t depend on the previous decisions of the players; a repeated dilemma is called 'time-independent' if the pay-off matrix remains unchanged across trials. In a resource dilemma, the possible pay-off depends, among other factors, on the size of the pool, and therefore on the players' previous decisions. As a consequence, a RD is time-dependent. In iterated PDs, on the other hand, the pay-off matrix does not change across trials and is time-independent. Models for time-independent dilemmas can not be used to describe time-dependent situations, since RD-models have to incorporate (the effects of) all previous decisions of the participants.

There are only a few papers addressing time-dependent dilemma situations and dealing explicitly with (the effects of) discounting.
An early paper on the subject is by Levhari and Mirman (1980) who studied a situation in which two agents harvest from a fish stock. The problem is formulated as an infinitely repeated, time-dependent game, like in the present study. These agents are assumed to discount future outcomes according to the DU-model. Levhari and Mirman compared the

harvest size that would occur in case of cooperation with the harvest size that would result if both agents maximized their discounted pay-offs. Their model implies that overfishing will occur, because in some situations, the latter will be larger than the former.

Fischer and Mirman (1996) expanded this model by introducing a second fish species, that is also fished by the two agents and that interacts with the first fish species. They observed that, depending on the type of biological relationship between the two fish species (mutual predation, symbiosis, or predator-prey interactions), these interactions can either enhance or mitigate the overfishing found by Levhari and Mirman.

Both papers deal with a resource dilemma situation with two agents, whereas we generalize our analysis to n agents. In addition, both studies used one specific discount function (DU) and one specific growth function (exponential), while we aim to generalize across discount and growth functions. However, the main reason that these studies are not directly relevant to our purposes is that these studies were aimed at describing one specific dilemma situation, while the present study aims to identify (temporal) RD characteristics that will generally influence choices.

Contrary to the former two studies, Hannesson's (1997) study does deal with n agents using a fish resource. He also modelled the dilemma as a repeated, time-dependent game of an infinite duration, where delayed outcomes are discounted according to the DU-model. The fish population was assumed to grow according to a logistic growth function. Hannesson implements a cost function, incorporating exploitation costs in the agents' pay-offs. It is assumed that the agents use a threat strategy to enforce cooperation: if an agent defects, the others will punish him by fishing down the stock to depletion (a so-called 'tit for tat'-strategy). According to Hannesson's model, the number of agents using the resource that can be forced to cooperate through this strategy, is limited; this is because the larger the number of agents, the more spread out are the costs of overfishing. Hannesson calculates how many agents can be forced to cooperation by this strategy, and studies how this number depends on the various parameters of the RD (discount rate, relative rate of growth, marginal cost of fish).

Hannesson's model describes a fish resource, whereas we aim to develop a model of behaviour in a more general resource dilemma situation. Some of the aspects of his model are specific for a fishing situation, e.g. the threat strategy, the cost function and migration of fish stocks across boundaries between agents, and will therefore not be included in our model. Also, we vary the functions for time discounting and fish growth in a further attempt to generalize, which Hannesson did not. Similar to Hannesson, we investigate how the various parameters influence cooperation.

Mason and Phillips (1997) build a model for designing a laboratory experiment with some 'firms' using a commons. They aim to find the relationship between the number of users and their harvesting behavior in a commons with only a static externality (what we called the resource dilemma) and in a commons with also a dynamic externality (the temporal dilemma). Their commons is assumed to grow logistically. The pay-offs of the users depend on the total harvest size and on the stock size, which correspond with the negative effects of crowding and depletion, respectively. They assume that the subjects in the experiment will discount according to the DU model.

All the above studies describe several specific resource dilemmas (usually, fishing situations) with different sets of characteristics. They seek to identify the circumstances under which overharvesting occurs. Poortinga (1997) does the same for a more general resource dilemma situation with very simple characteristics: exponential growth of the commons, costs of overharvesting uniformly distributed over all users, who discount their future pay-offs according to the DU-model. The pay-offs are assumed to be equal to the harvest size; varying market prices or harvest costs are not taken into account. Poortinga finds a relatively simple decision rule for deciding whether or not to overharvest. He shows that, under the above assumptions, agents will maximize their expected pay-offs by overharvesting if and only if the replenishment rate is smaller than the discount rate multiplied with the number of agents using the resource. His model has some severe restrictions: overharvesting is only possible in the first period, while in the following periods cooperation is compulsory. In addition, all other players are assumed to cooperate in Poortinga's model. We want to know whether this decision rule holds if the model is widened to a more general RD situation. Is it still valid without the restrictions Poortinga posed? And what decision rule applies if the commons is assumed to grow according to a different function; or if costs of overharvesting are assumed to be distributed proportionally to the use of the resource; or if pay-offs are discounted differently? In sum, for several basic types of resource dilemma, we want to find which decision rules will optimize the expected outcomes, and how these rules depend on various characteristics of the dilemma. In order to keep the model simple and as general as possible, we assume that the pay-offs to the users are equal to the size of the harvests, and we ignore possible effects of overharvesting on market prices and harvest costs. There are more, slighter shortcomings to Poortinga's model we might have dealt with, like possible biological interactions. The reasons why we do not do this, are, that we want to keep our analyses tractable to be able to arrive at clear decision rules; and we want to keep our model general, not making it applicable to only very specific resource dilemma situations.

3 Building a model

3.1 Introduction

In this Chapter, we develop a model for resource dilemmas, as encountered in fisheries, by using a game-theory approach (see also Muhsam (1973)). The resource dilemma is viewed as a two-person game. The two players are: (1) one of the users of the resource, to whom we will refer as 'self' and (2) all the other users. The unit of time is -arbitrarily- labeled as 'years'. Each player is assumed to make one move per year, which is: the decision how much to harvest from the resource. The players have infinitely many moves to make, to avoid having to deal with a 'last period', in which they can safely use up the whole resource.

The resource pool is assumed to replenish itself every year. If the players use so much of the resource that it can just replenish itself, the resource pool will remain constant. This level of total use is called the sustainable yield. If we assume that every user harvests the same amount from the resource, this means that every one can take the sustainable yield, divided by the number of users. If a user takes no more than that (and for the sake of simplicity we will assume that no user harvests less than that), we will call it 'cooperation'; otherwise it will be 'defection'. Every year each user has the choice between being cooperative or defective. If the users defect, the size of the resource pool will decrease and so will the sustainable yield for every year after that. In other words, from that year on the users will have to harvest less if they want to keep the resource pool at the same level. The problem is twofold: the loss in later years is shared by all users, while the gain in this year is exclusively for the overharvesting user; and the later losses are discounted, while the immediate gain is not.

In the following sections this base model will be expanded. Alternative functions for biological growth and time discounting will be implemented in the model and elaborated to see how they affect choices people make in a resource dilemma.

The example of fisheries is used to illustrate this RD-model. The model is a strong simplification of reality. It does not take into account a wide range of features. A cost function could be included, which would make harvesting more expensive as stock gets smaller, possibly making exploitation finally economically non-paying. Several biological features are disregarded, like migrations of fishes, interaction with other species, environmental influences and the age structure of the fish population. Full knowledge of the size of the fish stock and the rate of regeneration is assumed, which is usually not available.

On the other hand, the model is widely applicable. The dilemma of fisheries is representative of a whole class of resource dilemmas: the class of renewable, biological resources, where negative effects of overharvesting are equally shared by all users. Many resource dilemmas belong to this class. Examples are: harvesting wood from tropical forests and hunting for meat or fur.

In another class of biological resource dilemmas, closely related to the one in hand, the negative effects of overharvesting are not divided equally over all users, but proportional with the individual's use of the resource. Hardin's (1968) commons are a good example of this class. Overharvesting causes the quality of the grass to deteriorate, which in turn affects the production per cow, so that each user will have a decrease in income proportional to the number of cows owned. In section 3.5 an attempt will be made to adapt our model to this class of dilemmas.

3.2 Base model

In this base model for fisheries, we have n users harvesting from a fish population of size P_t, t denoting the year. We will assume a constant replenishment rate b of the fish population P_t in year t :

$$P_{t+1} = (1 + b)P_t.$$

The users discount future outcomes at a constant rate r, so the discount function is

$$f(t) = \frac{1}{(1 + r)^t}$$

A total catch level is said to constitute a sustainable yield if it leaves the fish population constant, so that this yield can be maintained forever. If we allow every user to harvest the same amount (m_t) of fish in year t, the sustainable yield is nm_t, which equals $P_t - P_t/(1+b)$. Indeed, demanding that $P_t = P_{t+1}$ gives us
$P_t = (1 + b)(P_t - nm_t)$,
which rewrites to
$P_t - (1 + b)P_t = -nm_t(1 + b)$,
so giving

$$nm_t = P_t - \frac{P_t}{1 + b}. \tag{1}$$

Overharvesting will reduce the size of the fish population, which will impair its capacity to replenish. Thus, overharvesting leads to a lower sustainable yield nm_{t+1} in year $t + 1$ than the sustainable yield nm_t in year t. We will establish that overharvesting (and taking lower later yields into the bargain) is only attractive if $b/n < r$.

Let q_t out of n users overharvest by an amount of y_{it} in year t, $i = 0, \ldots, q_t$, and let y_t be

$$y_t = \sum_{i=0}^{q_t} y_{it},$$

denoting the total number of fish harvested beyond the sustainable yield in year t. The fish population will decrease according to

$$P_t = P_0 - (1 + b) \sum_{j=0}^{t-1} y_j$$

Let Y_{t-1} denote

$$Y_{t-1} = \sum_{j=0}^{t-1} y_j$$

which is the total amount of overharvesting, across all users and all moments previous to t; then this equation for the fish population P_t in year t is written simply as

$$P_t = P_0 - (1 + b)Y_{t-1} \tag{2}$$

Indeed,

$$\begin{aligned}
P_t &= (1+b)(P_{t-1} - nm_{t-1} - y_{t-1}) \\
&= (1+b)(P_{t-1} - nm_{t-1}) - (1+b)y_{t-1} \\
&= P_{t-1} - (1+b)y_{t-1} \quad \{\text{as } nm_{t-1} \text{ is the sustainable yield in year } t-1\} \\
&= (1+b)(P_{t-2} - nm_{t-2} - y_{t-2}) - (1+b)y_{t-1} \\
&= P_{t-2} - (1+b)y_{t-2} - (1+b)y_{t-1} \\
&= \dots \\
&= P_0 - (1+b)(y_0 + y_1 + \dots + y_{t-1}) \\
&= P_0 - (1+b)Y_{t-1}
\end{aligned}$$

As a result, the sustainable yield will decrease during the course of years according to

$$nm_t = nm_0 - bY_{t-1} \tag{3}$$

for

$$\begin{aligned}
nm_t &= P_t - P_t/(1+b) \ \{\text{equation (1)}\} \\
&= P_0 - (1+b)Y_{t-1} - (P_0 - (1+b)Y_{t-1})/(1+\text{b}) \ \{\text{equation (2)}\} \\
&= P_0 - P_0/(1+b) - bY_{t-1} \\
&= nm_0 - bY_{t-1} \ \{\text{equation (1)}\}
\end{aligned}$$

As the size of the fish population can not be smaller than zero, we request that $P_0 - (1+b)Y_{t-1} > 0$ for all t, so that the total amount of fish overharvested must not exceed $P_0/(1+b)$. See also the note at the end of this section.

Let A represent a sequence of outcomes over a number of time periods for one of the n users:
$A = [w_0, w_1, w_2, \dots]$.
If this person is cooperative and does not harvest more than his equal share of the sustainable yield, his outcomes will be
$A_1 = [m_0, m_1, m_2, \dots]$.
If he is defective and overharvests by an amount of x_t in year t, his sequence of outcomes will be
$A_2 = [m'_0 + x_0, m'_1 + x_1, m'_2 + x_2, \dots]$
where $nm'_t = nm_0 - b(Y_{t-1} + X_{t-1})$,
X_{t-1} denoting

$$X_{t-1} = \sum_{j=0}^{t-1} x_j,$$

analogous to the reasoning above, as now $q_t + 1$ users out of n overharvest by a total amount of $y_t + x_t$ in year t. The size of the fish population will then decrease according to $P'_t = P_0 - (1+b)(X_{t-1} + Y_{t-1})$. This results in a sustainable yield of $nm'_t = nm_0 - b(X_{t-1} + Y_{t-1})$.

According to the idea of time discounting, this person does not attach equal importance to all the outcomes in a sequence. Using the Discount Utility function, we assume he perceives the present value or utility of an outcome series $A = [w_0, w_1, w_2, \dots]$ as

$$PU(A) = \sum_{t=0}^{\infty} \frac{w_t}{(1+r)^t}$$

The person prefers sequence A_1 over sequence A_2 if $PU(A_1) > PU(A_2)$, i.e. if

$$\sum_{t=0}^{\infty} \frac{m_t}{(1+r)^t} > \sum_{t=0}^{\infty} \frac{m'_t + x_t}{(1+r)^t}$$

or equivalently (see equation (3))

$$\sum_{t=0}^{\infty} \frac{m_0 - \frac{b}{n} Y_{t-1}}{(1+r)^t} > \sum_{t=0}^{\infty} \frac{m_0 - \frac{b}{n}(Y_{t-1} + X_{t-1})}{(1+r)^t} + \sum_{t=0}^{\infty} \frac{x_t}{(1+r)^t}$$

which rewrites to

$$0 > -\frac{b}{n} \sum_{t=0}^{\infty} \frac{X_{t-1}}{(1+r)^t} + \sum_{t=0}^{\infty} \frac{x_t}{(1+r)^t}$$

We have now

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} \frac{x_t}{(1+r)^t}}{\sum_{t=0}^{\infty} \frac{X_{t-1}}{(1+r)^t}} \tag{4}$$

This quotient of two series can not be made into one series as long as the $x_t s$ are unknown. If, however, we make the bold assumption that all $x_t s$ have the same value, namely x, which means that 'self' chooses to overharvest by the same amount every year, this inequality amounts to

$$\frac{b}{n} > \frac{x \sum_{t=0}^{\infty} \frac{1}{(1+r)^t}}{x \sum_{t=0}^{\infty} \frac{t}{(1+r)^t}}$$

and we get rid of the x altogether, while retaining two series in r with a known limit (see appendix). Inserting the limits, the inequality simplifies to

$$\frac{b}{n} > \frac{\frac{1+r}{r}}{\frac{1+r}{r^2}}$$

The right side is equal to r, so that the inequality becomes

$$\frac{b}{n} > r$$

quod erat demonstrandum. In other words, assuming that a particular user will overharvest by the same amount of fish at all moments (i.e. all $x_t s$ are equal), we have shown that he will prefer a cooperative choice (A_1) to a non-cooperative choice (A_2) if and only if $b/n > r$ (i.e. if the growth rate divided by the number of participants is larger than his discount rate).

From this result, we can conclude that cooperation in the dilemma is more reasonable as b/n is larger or r is smaller, i.e. as b is larger or n or r is smaller.

As b is the replenishment rate of the fish population, it follows from our model that the faster the resource replenishes itself, the larger the attractiveness of cooperation. Likewise, the model implies that the larger the number (n) of users involved, the more attractive overharvesting. And the larger the discount rate r, the less attractive cooperation.

This is quite a remarkable result, as apparently the attractiveness of cooperation in the resource dilemma does not depend on the (expected) behaviour of others, as the $y_t s$ do not show up in the inequality. Also, the value of x has no influence on cooperation, which means that the choice between cooperation and defection does not depend on the amount one might defect.

However, it stands to reason that without making the assumption that all $x_t s$ are equal, their value will stay in the inequality and therefore will have an impact on cooperation. On the other hand, it can be safely expected that, if we do not make the assumption that all $x_t s$ are equal, then the positive relationship between b and the attractivity of cooperation will still hold, just like the negative relationship between n or r and the attractivity of cooperation and the indifference to the values of the $y_t s$.

Note: we called the assumption that all $x_t s$ should have the same value, a 'bold assumption'. In fact, strictly speaking, this assumption is impossible.
We showed before that, in our model, the fish population in year t is $P_t = P_0 - (1 + b)(X_{t-1} + Y_{t-1})$. Since the fish population can not be less than zero, we must request that $P_0 > (1 + b)(X_{t-1} + Y_{t-1})$ for all t, which means that both series X_{t-1} and Y_{t-1} must necessarily converge to a constant. Thus, the sequence (x_t) must converge to zero as t approaches infinity, which makes it impossible for all $x_t s$ to be equal to a positive constant. Analytically, however, it appeared to be a very useful assumption.

Our base model is similar to Poortinga's (1997) work; in fact, it constitutes an extension in the sense that we allowed all users to overharvest every year, while Poortinga had only one of them ('self') overharvest in the first year.

3.3 An Alternative Function for Fish Growth

For small fish populations, the model of exponential growth of the fish population Poortinga used is quite realistic. Other authors have suggested alternative growth functions, e.g.Tietenberg (1992), who stated that the curve for biological growth as a function of the size of the population is as shown in Figure 5.

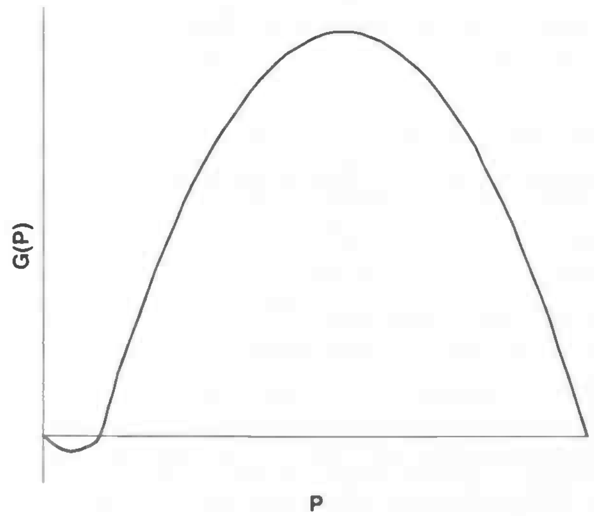

Figure 5: Growth of a population as a function of its size.

For very small fish populations, there is no growth, as the fishes can not find a mate. For large enough populations, growth increases with the population, till a certain maximum, where the fishes start to compete for food. Beyond that maximum, growth will decline till the fish population has reached the maximum size the environment can support; then growth is nil, the size of the population stays constant.

As an estimate, Tietenberg proposed the growth function $G(P) = 4P - 0.1P^2$.
We will use a more general form of this parabola, namely $G(P) = 2cMP - cP^2$ where $M, c > 0$. This is a parabola that opens downward, with $P = M$ in the middle and $G(M) = cM^2$, as shown in Figure 6. It corresponds to a fish population which, if left alone, grows almost logistically towards a maximum of $P = 2M$ (see Figure 7).

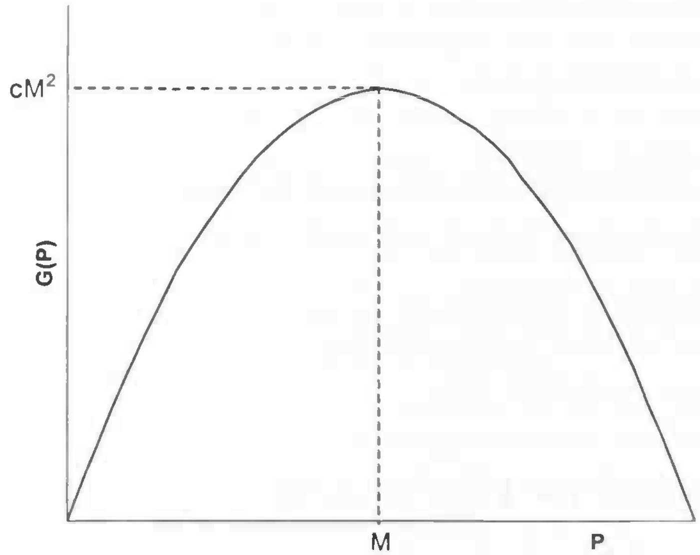

Figure 6: Graph of the growth function $G(P) = 2cMP - cP^2$.

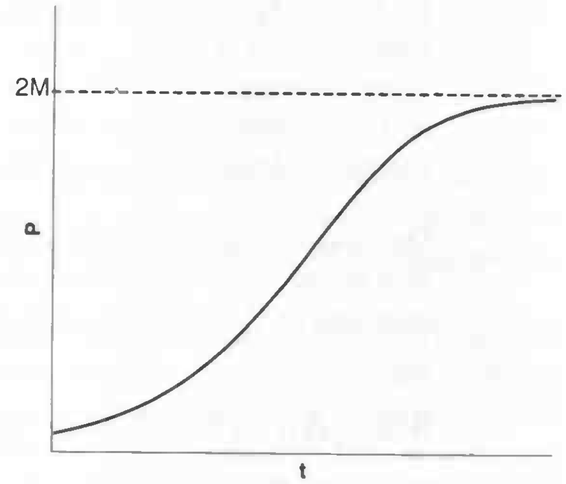

Figure 7: Development of a population that grows according to Figure 6.

However, the population will not be left alone. Each year, a sustainable yield can be obtained from it by fishing away an amount of fish, equal to that year's growth. In formula: $nm_t = G(P_t)$. We start out with a fish population of size P_0, which grows towards a size of $P_0 + G(P_0)$, so that the sustainable yield is $G(P_0)$. Some of the n fishermen will catch more than their share, say, in total up to an amount of y_0. This will result in a fish population of $P_0 - y_0$. The next growth will then be $G(P_0 - y_0)$, which is also the next sustainable yield: nm_1.
Keeping this up for a while, we find that $nm_t = G(P_0 - Y_{t-1})$ is the sustainable yield in year t, where Y_{t-1} denotes the sum of all y_js up to y_{t-1}, like in section 3.2. Inserting our new growth function, we find that

$$nm_t = 2cM(P_0 - Y_{t-1}) - c(P_0 - Y_{t-1})^2.$$

In every time period each of these fishermen has the choice between being cooperative or defective; that is, between a sequence of outcomes $[m_0, m_1, m_2, \ldots]$ or $[m'_0+x_0, m'_1+x_1, \ldots]$, where $nm'_t = G(P_0 - Y_{t-1} - X_{t-1})$, X_{t-1} being the sum of all x_js up to x_{t-1}.
If we name the first sequence A_1 and the other one A_2, a fisherman will prefer A_1 over A_2 if and only if $PU(A_1) > PU(A_2)$, where PU is the Discount Utility function indicated before. That is, if

$$\sum_{t=0}^{\infty} \frac{m_t}{(1+r)^t} > \sum_{t=0}^{\infty} \frac{m'_t + x_t}{(1+r)^t}$$

Inserting the expressions for m_t and $m'_t, m_t = \frac{1}{n}(2cM(P_0 - Y_{t-1}) - c(P_0 - Y_{t-1})^2)$ and $m'_t = \frac{1}{n}(2cM(P_0 - Y_{t-1} - X_{t-1}) - c(P_0 - Y_{t-1} - X_{t-1})^2)$, we find

$$\frac{1}{n}\sum_{t=0}^{\infty}\frac{2cM(P_0 - Y_{t-1}) - c(P_0 - Y_{t-1})^2}{(1+r)^t} >$$
$$\frac{1}{n}\sum_{t=0}^{\infty}\frac{2cM(P_0 - Y_{t-1} - X_{t-1}) - c(P_0 - Y_{t-1} - X_{t-1})^2}{(1+r)^t} + \sum_{t=0}^{\infty}\frac{x_t}{(1+r)^t}$$

After working out the squares and removing equal terms on both sides, this inequality becomes

$$\frac{1}{n}\sum_{t=0}^{\infty}\frac{2cMX_{t-1}}{(1+r)^t} + \frac{1}{n}\sum_{t=0}^{\infty}\frac{c(X_{t-1}^2 + 2X_{t-1}Y_{t-1} - 2P_0X_{t-1})}{(1+r)^t} > \sum_{t=0}^{\infty}\frac{x_t}{(1+r)^t}$$

To simplify this expression, we will again need the assumption that for all $t, x_t = x$. We will add a similar assumption now, that $y_t = y$ for all t. Both x and y will be very small compared to the total fish population in any year, since the fish population must not become lower than zero.

From these assumptions follows that $X_{t-1} = tx$ and $Y_{t-1} = ty$, so we have

$$\frac{c}{n}\left(2Mx\sum_{t=0}^{\infty}\frac{t}{(1+r)^t} + (x^2 + 2xy)\sum_{t=0}^{\infty}\frac{t^2}{(1+r)^t} - 2xP_0\sum_{t=0}^{\infty}\frac{t}{(1+r)^t}\right) > x\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}$$

In the appendix, the limits of these four series are given. Inserting them gives us

$$\frac{c}{n}\left(2Mx\frac{1+r}{r^2} + (x^2 + 2xy)\frac{(2+r)(1+r)}{r^3} - 2xP_0\frac{1+r}{r^2}\right) > x\frac{1+r}{r}$$

Dividing both sides by $x(1+r)/r^2$, and rearranging the terms, yields

$$\frac{c}{n}\left(2(M - P_0) + (x + 2y)\frac{2+r}{r}\right) > r$$

This is the condition under which cooperation is more attractive than defection in the case of logistic growth of the resource. As x and y should be very small in comparison to M and P_0, the left side of this inequality is negative if $P_0 > M$, approximately. So, if $P_0 > M$, the requirement for $PU(A_1) > PU(A_2)$ is impossible to fulfill, as r is always positive. This points to A_2 always being chosen if $P_0 > M$. That is, if the fish population we start with is on the right half of the parabola and therefore corresponds with a declining growth function, participants in the resource dilemma should always defect. It is no surprise that in that case overharvesting is attractive then, as a decline in population size leads to faster growth of the fish population.

If $P_0 < M$, i.e. if the fish population in year zero is somewhere on the left half of the parabola, so that growth increases if the population grows, the attractivity of cooperation is positively related to c, M, x and y, and negatively related to n, r and P_0. Again we see that the larger the number (n) of users, or the smaller the discount rate r, the more attractive overharvesting appears to be. Furthermore, cooperation becomes less attractive as the initial fish population P_0 is larger. Thirdly, if the parameters of the growth function,

c and M, are larger, indicating that the parabola is both broader and higher or, in other words, both the growth of the fish population at a given population size and the maximum size of the fish population is larger, defecting is less attractive. Finally, the amount of fish overharvested (x and y) correlates positively to the attractiveness of cooperation.
Like in section 3.2, we observe that the assumption of a constant yearly amount of fish overharvested ($x + y$) is one that will eventually lead to a negative fish population, but considering the simple, straightforward analysis we applied, there is no reason to doubt that the above conclusions about the relations between, on the one hand, the initial fish population, the number of users, the discount rate, the parameters of the growth function and the amount overharvested and, on the other hand, the level of cooperation, remain valid if we do not make that assumption.

3.4 Time Discounting Functions

In the preceding sections, we used the Discount Utility function in every model to determine the agents' preferred behaviour in the model. In section 2.2, however, we argued that this discount function does not appear to be a valid description of the way people actually discount delayed outcomes.
We also discussed some alternatives to the Discount Utility function: the hyperbolic and quasi-hyperbolic discount functions and the functions based on Weber's Law and Prospect theory (see section 2.2). In this chapter we will try to insert the three former ones into our model. The function based on Prospect Theory is too complicated for our purpose. We will find that only the quasi-hyperbolic discount function is sufficiently analytically tractable to produce useful results.

In general, inserting a discount function $f(t)$ into the model, so that a gain of x_t in t years from now will be valued as $f(t)x_t$, will not make so great a difference in the derivations of the model. Assuming again a linear fish growth function, we get the following condition for cooperation:

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} f(t)x_t}{\sum_{t=0}^{\infty} f(t)X_{t-1}} \tag{5}$$

Taking the parabolic fish growth function, this condition reads:

$$\frac{c}{n}\left(2M\sum_{t=0}^{\infty} f(t)X_{t-1} + \sum_{t=0}^{\infty} f(t)(X_{t-1}^2 + 2X_{t-1}Y_{t-1} - 2P_0X_{t-1})\right) > \sum_{t=0}^{\infty} f(t)x_t \tag{6}$$

Making the familiar assumption that $x_t = x$ and $y_t = y$ are constant, these simplify to, respectively:

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} f(t)}{\sum_{t=0}^{\infty} tf(t)} \tag{7}$$

and

$$\frac{c}{n}\left(2(M - P_0)\sum_{t=0}^{\infty} tf(t) + (x + 2y)\sum_{t=0}^{\infty} t^2 f(t)\right) > \sum_{t=0}^{\infty} f(t) \tag{8}$$

as $X_{t-1} = tx$ if all $x_t = x$ and $Y_{t-1} = ty$ if all $y_t = y$.
We will now fill in for $f(t)$ the different discount functions discussed before.

Hyperbolic discounting

The hyperbolic discount function is

$$f(t) = \frac{1}{1+kt}$$

We will now insert it in the four inequalities ((5), (6), (7) and (8)) that give the condition for cooperation in the RD under different circumstances.

with exponential growth of the fish population

Inserting the hyperbolic discount function in equation (5) produces the following requirement for cooperation in the RD:

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} \frac{x_t}{1+kt}}{\sum_{t=0}^{\infty} \frac{X_{t-1}}{1+kt}}$$

With the assumption that $x_t = x$ is constant, this simplifies to

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} \frac{1}{1+kt}}{\sum_{t=0}^{\infty} \frac{t}{1+kt}}$$

The right side is a quotient of two divergent series: the nominator is similar to the harmonic series ($\sum_{t=0}^{\infty} \frac{1}{t}$), which is known to diverge, and the denominator dominates[1] this series a.e. The quotient could be finite, however, so a computer simulation might produce a solution to this quotient of series.

with parabolic growth of the fish population

After inserting the hyperbolic discount function in equation (6) of the parabolic fish growth function, we see quickly from

$$\frac{c}{n}\left(2M\sum_{t=0}^{\infty}\frac{X_{t-1}}{1+kt} + \sum_{t=0}^{\infty}\frac{X_{t-1}^2 + 2X_{t-1}Y_{t-1} - 2P_0X_{t-1}}{1+kt}\right) > \sum_{t=0}^{\infty}\frac{x_t}{1+kt}$$

and, assuming that $x_t = x$ and $y_t = y$,

$$\frac{c}{n}\left(2(M-P_0)\sum_{t=0}^{\infty}\frac{t}{1+kt} + (x+2y)\sum_{t=0}^{\infty}\frac{t^2}{1+kt}\right) > \sum_{t=0}^{\infty}\frac{1}{1+kt}$$

that we get three terms with a divergent series here. We are not able to analytically elaborate this further. Again, numerical exploration may be useful.

Weber's Law Discounting

The Weber discount function is

$$f(t) = \frac{1}{(1+r)^{c*ln(t+1)}}$$

[1] i.e. the terms of the series in the denominator are even larger, so if the nominator diverges, the denominator certainly will too

We will insert this function in the conditions for cooperation we derived from $PU(A_1) > PU(A_2)$.

with exponential growth of the fish population

Inserting the Weber discount function in equation (5) supplies us with the condition

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} \frac{x_t}{(1+r)^{c \cdot ln(t+1)}}}{\sum_{t=0}^{\infty} \frac{X_{t-1}}{(1+r)^{c \cdot ln(t+1)}}}$$

If we make the familiar assumption that $x_t = x$ for all t, this can be simplified to

$$\frac{b}{n} > \frac{\sum_{t=0}^{\infty} \frac{1}{(1+r)^{c \cdot ln(t+1)}}}{\sum_{t=0}^{\infty} \frac{t}{(1+r)^{c \cdot ln(t+1)}}}$$

The two series in the right hand side of this inequality we can not solve analytically.

with parabolic growth of the fish population

The same problem applies to the case with the parabolic fish growth function. We assume again that, for all t, $x_t = x$. We also assume that $y_t = y$ for all t.

$$\frac{c}{n}\left(2(M - P_0)\sum_{t=0}^{\infty}\frac{t}{(1+r)^{k*ln(t+1)}} + (x+2y)\sum_{t=0}^{\infty}\frac{t^2}{(1+r)^{k*ln(t+1)}}\right) > \sum_{t=0}^{\infty}\frac{1}{(1+r)^{k*ln(t+1)}}$$

We see that simplifying this further involves solving the same two series as we had (and one more besides). We can not solve this problem analytically.

Quasi-hyperbolic discounting

The quasi-hyperbolic discount function is a discrete function with values $\{1, \frac{\beta}{1+r}, \frac{\beta}{(1+r)^2}, \frac{\beta}{(1+r)^3}, \ldots\}$, where $0 < \beta < 1$.
To find the condition for cooperation in a RD with quasi-hyperbolic discounting, we insert this function in expressions (5) to (8).

with exponential growth of the fish population

Assuming exponential growth of the fish population (expression (5)), we get

$$\frac{b}{n} > \frac{x_0 + \beta\sum_{t=1}^{\infty}\frac{x_t}{(1+r)^t}}{\beta\sum_{t=0}^{\infty}\frac{X_{t-1}}{(1+r)^t}}$$

Adding the assumption that $x_t = x$ for all t produces:

$$\frac{b}{n} > \frac{1 + \beta\sum_{t=1}^{\infty}\frac{1}{(1+r)^t}}{\beta\sum_{t=0}^{\infty}\frac{t}{(1+r)^t}}$$

In the appendix we derive the limits of these two series. Inserting them in the present equation gives

$$\frac{b}{n} > \frac{1 + \frac{\beta}{r}}{\beta\frac{1+r}{r^2}}$$

From which follows the condition for cooperation

$$\frac{b}{n} > \frac{\beta r + r^2}{\beta r + \beta}$$

From this, we find again a positive relationship of the amount of fish growth (b) and a negative relationship of the number of agents (n) with the attractiveness of cooperation. A larger β or a smaller r, both implying less time discounting, would also make cooperation more profitable.

with parabolic growth of the fish population

The parabolic function for biological growth provides us with the following condition for cooperation (see expression (6)):

$$\frac{c}{n}\left(2\beta M \sum_{t=0}^{\infty} \frac{X_{t-1}}{(1+r)^t} + \beta \sum_{t=0}^{\infty} \frac{X_{t-1}^2 + 2X_{t-1}Y_{t-1} - 2P_0X_{t-1}}{(1+r)^t}\right) > x_0 + \beta \sum_{t=1}^{\infty} \frac{x_t}{(1+r)^t}$$

Under the assumptions that $x_t = x$ and $y_t = y$ for all t, this inequality becomes:

$$\frac{c\beta}{n}\left(2(M - P_0)\sum_{t=0}^{\infty} \frac{t}{(1+r)^t} + (x+2y)\sum_{t=0}^{\infty} \frac{t^2}{(1+r)^t}\right) > 1 + \beta \sum_{t=1}^{\infty} \frac{1}{(1+r)^t}$$

Filling in the limits of these series (cf. appendix) gives us

$$\frac{c\beta}{n}\left(2(M - P_0)\frac{1+r}{r^2} + (x+2y)\frac{(2+r)(1+r)}{r^3}\right) > 1 + \frac{\beta}{r}$$

Multiplication with $r^2/(\beta(1+r))$ and rearranging the terms turns this inequality into:

$$\frac{c}{n}\left(2(M - P_0) + (x+2y)\frac{2+r}{r}\right) > \frac{\beta r + r^2}{\beta r + \beta}$$

The left side of this expression is the same we found in section 3.3 for the case of DU discounting and the parabolic fish growth function. We can, therefore, draw the same conclusions, showing that these results are fairly robust: if the initial fish population is large (roughly, $P_0 > M$), the participants in the resource dilemma should always defect. If $P_0 < M$, a larger number (n) of participants, makes overharvesting appear to be more attractive. Furthermore, cooperation becomes less attractive as the initial fish population P_0 is larger. Thirdly, if the parameters of the growth function, c and M, are larger, indicating that if the parabola is broader or higher or, in other words, if either the growth of the fish population at a given population size, or the maximum fish population size is larger, defecting is less attractive. The amount of fish overharvested (x and y) correlates positively with the attractiveness of cooperation. Finally, the discount rate r correlates negatively and, we can add here, the other parameter of the discount function, β, correlates positively with the attractivity of cooperation. Or, in other words, more time discounting should lead to less cooperation in the resource dilemma.

Like in section 3.2, we observe that the assumption of a constant yearly amount of fish overharvested ($x + y$) is one that will eventually lead to a negative fish population, but considering the simple, straightforward analysis we applied, we expect that the above conclusions regarding the relations between, on the one hand, the initial fish population size, the number of users, the parameters of the discount function and the growth function and the amount overharvested, and, on the other, the expected level of cooperation, remain valid if we do not make that assumption.

3.5 Hardin's commons

The most famous example of a resource dilemma must be Hardin's (1968) tragedy of the commons, where a number of herdsmen share a pasture to graze their herds on. In this example, when the carrying capacity of the commons is exceeded, the resource is not only depleted over the years, but in the present year the production per head of cattle also decreases. This latter loss is not distributed equally across all users, as it was in the fisheries example, but proportionally with the use of the resource, as explained in section 3.1. These two losses, the damage in the long run that is distributed equally across all users, and the acute damage that is distributed proportionally with the use of the resource, are treated separately in the model. A pasture can react strongly to overgrazing but recover quickly, or it may suffer hardly any acute damage from overgrazing but deteriorate badly over the years. Therefore, different parameters are used to model these two kinds of damage.

We model this dilemma using a linear function for biological growth and the Discount Utility function for time discounting. As a third assumption, we will use a linear loss function (i.e. the productivity loss is proportional to the amount of overgrazing). We will seek to deduce relations between the different variables in the model and the likelihood of cooperation.

We will call the maximum number of cattle at which the productivity of the pasture remains unchanged the 'carrying capacity' of the pasture. We assume that the loss of productivity in the present year is proportional to the number of cattle added beyond the carrying capacity (nm) of the pasture and spread evenly across all cattle: each head of cattle suffers a productivity loss of $0 < a < 1$ when one cow too much is added to the pasture. A larger a means a more acutely vulnerable resource. If X cows are added beyond the carrying capacity of the pasture, the loss of productivity for each head of cattle is aX. The total collective loss is then $aX(nm + X)$, which is not equally distributed over all persons, but proportional to the number of cattle each person owns.
The pay-off for a person in year t will be $m_t + x_t - a(x_t + y_t)(m_t + x_t)$, where x_t is the number of cattle this person added more than his share (m_t), and y_t is the total number of cattle all the other persons added beyond the carrying capacity of the pasture.

Instead of the fish population of the former example, we will consider the quality (Q) of the pasture as a state of the resource that deteriorates with overuse, which in turn affects the pay-off of the users. Q is the quantity that indicates the damage in the long run, while a has to do with the acute damage to the resource in case of overgrazing.

We assume that the pasture recovers itself over winter at a rate b, such that $Q_{t+1} = (1+b)Q_t, \quad 0 < b < 1$.
Analogous to the fish population in section 2.2, we find that

$$Q_t = Q_0 - (1+b)(X_{t-1} + Y_{t-1})$$

$$nm_t = nm_0 - bY_{t-1} \tag{9}$$

$$nm'_t = nm_0 - b(X_{t-1} + Y_{t-1}) \tag{10}$$

where $X_{t-1} = \sum_{j=0}^{t-1} x_j$ and $Y_{t-1} = \sum_{j=0}^{t-1} y_j$.

This person can choose between being cooperative or defective, i.e. a sequence of outcomes $A_1 = [m_0 - ay_0m_0, m_1 - ay_1m_1, m_2 - ay_2m_2, \ldots]$ or $A_2 = [m_0 + x_0 - a(x_0+y_0)(m_0+x_0), m'_1 + x_1 - a(x_1+y_1)(m'_1+x_1), \ldots]$. He will prefer being cooperative if $PU(A_1) > PU(A2)$. Using the Discount Utility function, we find that this inequality amounts to

$$\sum_{t=0}^{\infty} \frac{m_t - ay_tm_t}{(1+r)^t} > \sum_{t=0}^{\infty} \frac{m'_t + x_t - a(x_t+y_t)(m'_t+x_t)}{(1+r)^t}$$

Filling in the expressions (9) and (10) for m_t and m'_t gives us a long, complicated inequality:

$$\begin{aligned}
m_0\sum_{t=0}^{\infty}\frac{1}{(1+r)^t} - \frac{b}{n}\sum_{t=0}^{\infty}\frac{Y_{t-1}}{(1+r)^t} - am_0\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t} + \frac{ab}{n}\sum_{t=0}^{\infty}\frac{y_tY_{t-1}}{(1+r)^t} >\\
m_0\sum_{t=0}^{\infty}\frac{1}{(1+r)^t} - \frac{b}{n}\sum_{t=0}^{\infty}\frac{X_{t-1}}{(1+r)^t} - \frac{b}{n}\sum_{t=0}^{\infty}\frac{Y_{t-1}}{(1+r)^t} + \sum_{t=0}^{\infty}\frac{x_t}{(1+r)^t} - am_0\sum_{t=0}^{\infty}\frac{x_t}{(1+r)^t}\\
+\frac{ab}{n}\sum_{t=0}^{\infty}\frac{x_tX_{t-1}}{(1+r)^t} + \frac{ab}{n}\sum_{t=0}^{\infty}\frac{x_tY_{t-1}}{(1+r)^t} - am_0\sum_{t=0}^{\infty}\frac{y_t}{(1+r)^t} + \frac{ab}{n}\sum_{t=0}^{\infty}\frac{y_tX_{t-1}}{(1+r)^t}\\
+\frac{ab}{n}\sum_{t=0}^{\infty}\frac{y_tY_{t-1}}{(1+r)^t} - a\sum_{t=0}^{\infty}\frac{x_t^2}{(1+r)^t} - a\sum_{t=0}^{\infty}\frac{y_tx_t}{(1+r)^t}
\end{aligned}$$

Removing equal terms on both sides and rewriting, we obtain

$$\begin{aligned}
0 \quad > \quad & -\frac{b}{n}\sum_{t=0}^{\infty}\frac{X_{t-1}}{(1+r)^t} + (1-am_0)\sum_{t=0}^{\infty}\frac{x_t}{(1+r)^t} + \frac{ab}{n}\sum_{t=0}^{\infty}\frac{x_tX_{t-1} + x_tY_{t-1} + y_tY_{t-1}}{(1+r)^t}\\
& -a\sum_{t=0}^{\infty}\frac{x_t^2 + y_tx_t}{(1+r)^t}
\end{aligned}$$

To obtain an expression that will allow us to see how the different characteristics of the resource dilemma affect the attractiveness of cooperation in this resource dilemma, we will simplify this expression by assuming that x_t and y_t be constant for all $t : x_t = x$ and $y_t = y$.
In that case, the inequality becomes

$$\begin{aligned}
0 \quad > \quad & -\frac{bx}{n}\sum_{t=0}^{\infty}\frac{t}{(1+r)^t} + (1-am_0)x\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}\\
& +\frac{ab}{n}\left(x^2\sum_{t=0}^{\infty}\frac{t}{(1+r)^t} + xy\sum_{t=0}^{\infty}\frac{t}{(1+r)^t} + xy\sum_{t=0}^{\infty}\frac{t}{(1+r)^t}\right) - a(x^2+xy)\sum_{t=0}^{\infty}\frac{1}{(1+r)^t}
\end{aligned}$$

In the appendix we derive the limits of these series. After filling them in, the inequality becomes:

$$0 > -\frac{bx}{n}\frac{1+r}{r^2} + (1-am_0)x\frac{1+r}{r} + \frac{ab}{n}(x^2+2xy)\frac{1+r}{r^2} - a(x^2+xy)\frac{1+r}{r}$$

Dividing by $x(1+r)/r^2$ and rearranging the terms produces the following condition under which cooperation is more attractive than defection:

$$\frac{b}{n}(1 - a(x + 2y)) > (1 - a(m_0 + x + y))r$$

This is the condition under which cooperation is more attractive than defection in a resource dilemma of the same class as Hardin's commons. Once more we find a positive relationship between biological growth (b) and the attractiveness of cooperation and a negative relationship between the number of participants (n) in the resource dilemma and the attractiveness of cooperation. The initial sustainable yield (m_0) correlates positively with cooperation.

We can not really say whether the size of a, the linear loss parameter (i.e. the loss percentage per animal above the sustainable level), is related positively or negatively with cooperation. If $y > m_0$, or, in other words, if others are expected to defect a lot, or if the initial state of the resource is not very good, a larger a (a more vulnerable resource) reduces the inclination to cooperation. In this case x also correlates negatively with the attractivity of cooperation, which could be interpreted as follows: if others defect more, it becomes more attractive to defect by a large amount. This may be interpreted as reflecting a tendency to be less inclined to sacrifice oneself if the situation is more hopeless and if others are expected to be defective.

From the same reason as mentioned at the end of the former section, we expect that these findings still hold if we do not make the assumption of constant annual overgrazing.

4 Discussion

We have studied the influence of temporal aspects on choice behaviour in resource dilemmas. For this purpose, we have constructed a game theoretic model of a resource dilemma, in which a discount function is incorporated.
In a resource dilemma situation, each participant has the choice whether to cooperate or defect. Cooperation was defined as using the resource at one's equal share of the sustainable yield level, so that, if all participants act this way, the resource pool remains constant. Defection means: using the resource beyond one's share of the sustainable yield, so that the resource pool deteriorates, resulting in later lower yields.
These later yields are discounted. In the relevant literature several discount functions are given, viz. the Discount Utility function, the hyperbolic and the quasi-hyperbolic functions and the functions based on Weber's Law and Prospect Theory. We have implemented each of the first four functions into the discounted RD model; the last function is too complex for our purposes.
We have also varied the biological growth function: we have combined the four discount functions with two different growth functions, viz. a linear and a parabolic function, that correspond with exponential and logistic growth, respectively.
In all these combinations, we have assumed that the negative consequences of defective behaviour are equally distributed over all participants in the dilemma. Since this is not necessarily so in all resource dilemma situations, we have also adapted our model to a dilemma situation in which losses are distributed proportionally to a person's use of the resource. In this case we have contented ourselves with the DU-function and the linear growth function, since other combinations would probably make the calculations too complex.
By elaborating these combinations, we sought to determine how the various characteristics of the RD (viz. number of participants, rate of biological growth, discount rate, initial state of the resource, extent of defection of others and the linear loss parameter (see section 3.6, the way losses are assigned to participants)) influence the expected level of cooperation.

In all cases it appeared necessary to assume that the amount of overuse by 'self' is constant, in order to simplify the conditions for cooperation so far, that we could draw conclusions from them. In most cases, the additional assumption that the amount of overuse by 'others' was also constant, was necessary to arrive at meaningful results. One problem with these assumptions is that in an infinite time space, they will lead to a negative resource pool. However, mathematically these assumptions are not a very serious intervention, and our analyses are quite simple and straightforward. Therefore, we do not expect that our results are substantially affected by this problem.
Even with these assumptions, for many combinations of discount and growth functions, the models were not sufficiently analytically tractable to arrive at clear results. Two different discount functions, viz. the discounted utility function and the quasi-hyperbolic function, yielded clear (and, in fact, the same) conclusions (see next paragraphs); we can not exclude that the other discount functions would produce different results, so it is unclear whether the obtained results will be valid under other ways of discounting.

Our model yields the following conclusions about how RD-characteristics influence the choice whether or not to cooperate, assuming that people behave in a way aimed at optimizing the expected aggregate personal outcomes.

The number of participants in the resource dilemma is related negatively to the attractivity of cooperation. In section 2.1, some reasons for this are given: diffusion of responsibility, decrease of perceived controllability, increase of anonimity and longer delay of negative effects of overharvesting. Reducing the number of participants, for instance by privatizing commons, would therefore increase the expected level of cooperation. If there would be only one person exploiting the resource, only the temporal dilemma would be left.
In the case of exponential biological growth, faster growth is expected to lead to more cooperation. If the growth is logistic, defection is always more attractive than cooperation if the initial resource pool is larger than its point of maximum growth. If, at the start, the resource pool is smaller, again more growth enhances the attractivity of cooperation. In most realistic cases, these parameters are determined by nature and can not be influenced, but it is good to be aware of this relation, since it may mean that resources that recover more slowly from exploitation, could be more likely to be overexploited.
Inserting both the DU-function and the quasi-hyperbolic function revealed that a higher discount rate will make cooperation less attractive. This result is plausible, for if future outcomes are discounted more strongly, the losses due to defection are rated smaller, which makes defection more and cooperation less attractive. Reducing people's discount rates by e.g. presenting more information about the nature of resource dilemmas and the long term effects of current overconsumption, would promote cooperation.
In a resource dilemma situation with logistic growth, where losses are shared equally among participants, the initial size of the resource pool is related negatively to the attractivity of cooperation. This is no problem initially, since a large resource pool can stand a high level of defection. But the model also shows that if, due to overharvesting, the resource pool declines so much that it passes the point of maximum growth ($P < M$), cooperation can become an attractive strategy. We do suspect that a transition from a defective to a cooperative strategy will not be an easy one. One could conclude that, if a resource pool is large, people consider the consequences of defection as no big loss. However, in another type of RD, namely with exponential growth and a proportional loss distribution (Hardin's commons), we have obtained the opposite result: a larger initial sustainable yield makes cooperation more attractive, so the relation between the initial pool size and the attractivity of cooperation depends on the type of resource dilemma .
In this last type of RD, we have also found that if a small initial sustainable yield is combined with an expected high level of defection by others, cooperation becomes less likely as the linear loss parameter is larger, i.e. as the resource is more vulnerable. In this case the negative circumstances (a vulnerable resource, added to an already small pool, and the expectation that others will defect a lot) strengthen each other, leading to certain depletion of the resource. This suggests that in a fragile resource, it is not attractive to sacrifice oneself if one expects others to defect a lot, perhaps viewing the situation as hopeless.
In a RD-situation with an equal distribution of loss and logistic growth of the resource pool, more expected defection results in precisely the opposite: more cooperation. Apparently, according to the model, this case shows signs of a Chicken dilemma (see section 2.1). If growth of the resource pool in this type of dilemma is exponential, expectations about other people's choice behaviour appear to have no influence at all on choice, which is a quite remarkable result.

Obviously, a model does not include all aspects that influence choice in real resource dilemmas. The most important missing feature may be a cost function, that should bring

the costs of exploitation of the resource and prices of the product into the model. As we expect that the agents, in reality, are generally interested in financial gain instead of mere products, this would certainly affect the outcomes of the model in a way we can not predict off-hand. A problem discussed before, namely that the assumption that the amount of overuse is constant would lead to a negative resource pool, could be solved by introducing a cost function, since exploitation would cease if the pool became very small, assuming that the costs are related negatively to the pool size. Also, the biological connections are entirely disregarded, as growth of the resource stock only depends on the size of the stock and not on other intrinsic or external circumstances. Furthermore, we have assumed that all persons who are not defecting use the same amount of the resource. Differentiation among users is not included in the model, since this would make the definition of cooperation much more complex. Finally, full knowledge of all concerned variables, like the size of the fish stock and the rate of regeneration, is assumed, which is usually not available. Including these factors would definitely complicate the model. How it would affect our results is hard to predict.
In sum, the presented model represents an RD in a strongly simplified way. Several refinements are possible and might lead to different conclusions about the relationship between characteristics of the RD and the level of cooperation.

On the other hand, the model appeared to be very flexible and widely applicable. The dilemma of fisheries is representative of a whole class of similar resource dilemmas, and the model proved to be adaptable to other kinds of resource dilemmas. The many implementations of different functions into the model have yielded only partial results, because of the limited ability of analysis to cope with infinite series. Probably, numerical elaboration of these series would allow more definite statements about choice behaviour in resource dilemmas, which would be a recommendation for further research.
Another option for further study could be to test empirically whether the obtained relations and predictions can be demonstrated in actual choice behaviour. For instance, one could construct a game with characteristics analogous to the characteristics of the resource dilemma. These characteristics should be varied and the corresponding level of cooperation measured to ascertain whether the model describes human choice behaviour correctly. Several empirical studies have already observed the negative correlation of the number of participants to cooperation [Dawes (1980), Liebrand and Van Lange (1989), Messick and Brewer (1983), Mannix (1991), Mason and Phillips (1997)]. Mason and Phillips also noted that a smaller resource pool reduces harvests. This was a consequence of the way they linked costs to pool size; which is something we did not study. Very little research is available about the effect of temporal aspects on choice behaviour in the resource dilemma. An exception is Mannix (1991), whose experiments affirm our conclusion that a higher discount rate leads to less cooperation. Further empirical studies of the relations between cooperation and (temporal) characteristics of the RD would be worthwhile.

The model constructed and refined in this paper deals with an important, every-day problem of society. Therefore, we hope that other people will pick up and continue this line of research, and that this paper, in its tiny way, can bring the solutions to some perplexing resource dilemmas a little closer.

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APPENDIX

We want the limits of series a) $\sum_{t=0}^{\infty} \frac{1}{(1+r)^t}$

b) $\sum_{t=0}^{\infty} \frac{t}{(1+r)^t}$

c) $\sum_{t=0}^{\infty} \frac{t^2}{(1+r)^t}$

a) The first series is called the geometric series, $\sum_{t=0}^{\infty} a^t$, where $a = \frac{1}{1+r}$. Apostol (1967) calculates the limit as follows. Let s_n denote the nth partial sum of this series, so that

$$\begin{array}{rllr} s_n = & 1+ & a + a^2 + \cdots + a^{n-1} & \\ as_n = & & a + a^2 + \cdots + a^{n-1} & +a^n \quad - \\ \hline (1-a)s_n = & 1 & & -a^n \end{array}$$

Assuming that $a \neq 1$, we can divide by $1 - a$, from which follows $s_n = \frac{1-a^n}{1-a}$. This sequence converges for $|a| \leq 1$. We have

$$\sum_{t=0}^{\infty} a^t = \lim_{n\to\infty} s_n = \frac{1}{1-a}$$

Substituting $a = \frac{1}{1+r}$, we get

$$\sum_{t=0}^{\infty} \frac{1}{(1+r)^t} = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r}$$

Equivalently for $t = 0$, $\frac{1}{(1+r)^t} = 1$, so we find that

$$\sum_{t=1}^{\infty} \frac{1}{(1+r)^t} = \frac{1+r}{r} - 1 = \frac{1}{r}$$

. . . .

b) If we differentiate $\sum_{t=0}^{\infty} a^t = \frac{1}{1-a}$ to a, we get

$$\sum_{t=1}^{\infty} ta^{t-1} = \frac{1}{(1-a)^2}$$

Multiplying this by a and realizing that $ta^t = 0$ for $t = 0$ yields

$$\sum_{t=0}^{\infty} ta^t = \frac{a}{(1-a)^2} \tag{1}$$

And substituting $\frac{1}{1+r}$ for a, we obtain

$$\sum_{t=0}^{\infty} \frac{t}{(1+r)^t} = \frac{\frac{1}{1+r}}{\left(\frac{r}{1+r}\right)^2} = \frac{1+r}{r^2}$$

c) Equivalently, differentiating (1) to a yields

$$\sum_{t=1}^{\infty} t^2 a^{t-1} = \frac{1+a}{(1-a)^3}$$

Multiplication by a gives us

$$\sum_{t=0}^{\infty} t^2 a^t = \frac{a(1+a)}{(1-a)^3}$$

And if we substitute $\frac{1}{1+r}$ for a, we find that

$$\sum_{t=0}^{\infty} \frac{t^2}{(1+r)^t} = \frac{\frac{1}{1+r}\left(1+\frac{1}{1+r}\right)}{\left(1-\frac{1}{1+r}\right)^3} = \frac{(2+r)(1+r)}{r^3}.$$