Effects of Gravity and the Rotation of the Earth on Spin Dynamics

C.R. van Es

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Abstract

The effect of classical gravity on spin dynamics is studied. Normally, focusing magnets keep the particles from falling down. However, the magnetic fields from these focusing magnets influence the spin dynamics. This is calculated for EDM experiments. Also the gyroscopic effect of the rotation of the earth on spin dynamics is studied; a particle at rest on the earth actually finds itself in a rotating system and this has influence on the spin dynamics. An object moving with respect to the earth also experiences the so called Coriolis force. A corrected equation of motion for the intrinsic spin, which takes into account the gyroscopic effect and the Coriolis force, is obtained. Two other, less relevant, effects due to gravity and the fact that experiments take place on the rotating earth are also discussed. These are the centrifugal force and spin-gravity coupling.

1 Introduction

Many present-day experiments in particle physics are based on measurements of intrinsic spin of particles. Because the experiments get more and more precise, effects from the rotation of the earth and gravity could start to play a role within these experiments.

An example of such an experiment is the search for electric dipole moments (EDM's) of elementary particles. The Standard Model predicts EDM's to be very small, much smaller than one would be able to measure in the near future (e.g., it predicts the electron EDM to be $< 10^{-38}$ e $\cdot$ cm [1]). However, some new theories like *Multi-Higgs models*, *Left-Right Symmetric models* or *SUSY* claim the EDM's to be many orders of magnitude greater [2]. At the moment, it seems possible to measure EDM's as small as 10^{-29} e $\cdot$ cm [3], this is well within the range of the predictions of these new theories. Thus, if one would really measure an EDM at such a high level, it would be an indication for physics beyond the Standard Model.

In one group of EDM experiments [4, 5] one stores particles (e.g., deuterons or muons) in a storage ring for 10–1000 seconds. These particles rotate in the ring with the cyclotron frequency. The intrinsic spin precesses, due to the magnetic dipole moment, in the horizontal plane around the vertically directed magnetic field. Because the magnetic field transforms under a Lorentz transformation and the particles rotate with very high velocities, the particles experience a very high electric field. This electric field is higher than any directly applied electric field that would be practical to use. If the particles possess an EDM, there will be an additional precession around this radially directed electric field. This precession is very small, between 10^{-5} and 10^{-7} rad/s. During the time the particles are stored in the ring, this precession leads to a vertical polarization of intrinsic spin. This polarization increases linearly with time and is a measurement for the EDM. However, the intrinsic spin behaves just like a 'classical' gyroscope and thus also changes direction due to the rotation of the earth. In Section 3 this gyroscopic effect on this kind of experiments is discussed.

This article starts with a discussion of the effect of gravity on this kind of experiments. The particles don't just fall down when they are injected into the storage ring, because the particle beam is focussed by quadrupole magnets. However, this leads to a non-zero average radial magnetic field felt by the particles, which causes an additional precession in the vertical plane and thus gives a false EDM signal.

Because the experiment takes place in a noninertial frame, namely the rotating earth, the particles in the storage ring seem to be affected by some fictitious forces. These forces arise from the fact that Newton's second law $\mathbf{F} = m\mathbf{a}$ is only valid in an inertial frame. For a particle in a non-inertial frame this equation takes the form $\mathbf{F}_{\text{eff}} = m\mathbf{a}_f + \textit{non-inertial terms}$. Here $m\mathbf{a}_f$ is the 'real' force. To see what the non-inertial terms are, consider an arbitrary vector $\mathbf{G}$ in a rotating reference system. The time rate of change of this vector seen from a fixed reference frame is [6]

$$\left(\frac{d\mathbf{G}}{dt}\right)_f = \left(\frac{d\mathbf{G}}{dt}\right)_r + \mathbf{\Omega} \times \mathbf{G}, \tag{1}$$

where the first term on the right-hand side is the time rate of change in the rotating reference frame and $\mathbf{\Omega}$ is the angular velocity of the rotating frame.

The idea is to apply this equation first to the postion vector $\mathbf{r}$ and then another time to the velocity vector $\mathbf{v} = \frac{d\mathbf{r}}{dt}$. When this is done, the following equation for the acceleration in a frame fixed on earth arises (for a detailed calculation see [6])

$$\mathbf{a}_r = \mathbf{a}_f - 2\mathbf{\Omega}_e \times \mathbf{v} - \mathbf{\Omega}_e \times (\mathbf{\Omega}_e \times \mathbf{R}). \tag{2}$$

The (non-relativistic) force in this frame can be written as

$$\mathbf{F}_{\text{eff}} = m\mathbf{a}_f - 2m\mathbf{\Omega}_e \times \mathbf{v} - m\mathbf{\Omega}_e \times (\mathbf{\Omega}_e \times \mathbf{R}). \tag{3}$$

The second term on the left-hand side in this equation is the so called Coriolis force and the third term is the centrifugal force. Both these terms were first derived by the French mathematician Gaspard Gustave de Coriolis (1792-1843) [7]. In Section 3, where the particle and spin dynamics are studied, the Coriolis force is taken into account. As will be shown in Section 4.1 the centrifugal force can be neglected. The question may be raised whether the Coriolis force disturbs the experiments in the same way as gravity does. This is not the case, because the vertical component of the Coriolis force averages to zero over one orbit. The radial component of the Coriolis force leads to an extra vertically directed magnetic field felt by the particles and therefore only affects the precession in the horizontal plane.

Finally, in Section 4.2 an effect arising from general relativity, namely spin-gravity coupling, is discussed. As will be shown, this effect can be neglected for EDM experiments. This is not a surprise, since one is performing a large-scale experiment just to measure this effect anyway (see for example [8]).

2 Effects of classical gravity

Consider the motion of a moving charged particle in a vertically directed magnetic field B_z. This particle rotates, due to the Lorentz force $\mathbf{F}_L = q\mathbf{v} \times \mathbf{B}$, around the direction of B_z with the cyclotron frequency $\omega_c = -qB_z/\gamma m$. The radius R_0 of the particle's orbit can be obtained by setting the centripetal force equal to the Lorentz force:

$$\frac{\gamma m v^2}{R_0} = evB_z \longrightarrow R_0 = \frac{\gamma m v}{eB_z}. \tag{4}$$

For convenience, define a rotating coordinate system which origin moves along the orbit with radius R_0. Assume the particle is positively charged, so the system rotates clockwise. The vector $\hat{\boldsymbol{\rho}}$ points radially inward, the vector $\hat{\boldsymbol{\phi}}$ is parallel to the momentum and $\hat{\mathbf{z}}$ is in the vertical direction. In this frame the particle will be at rest if there are no disturbances. Therefore, this frame is called the particle's rest frame.

If the particle is out of the original orbit, i.e. ρ or z is non-zero, it will experience a magnetic field from the the quadrupole magnets in the storage ring. Because of this magnetic field an additional Lorentz force will act on the particle, which traps the particle back into the original orbit. In the particle's rest frame the Lorentz force is given by

$$\mathbf{F}_L = q(\mathbf{v} \times \mathbf{B}) = qv_\phi(B_{z,\text{trap}}\ \hat{\boldsymbol{\rho}} - B_{\rho,\text{trap}}\ \hat{\mathbf{z}}). \tag{5}$$

Gravity pulls the particle down and the particle gets to the place where the Lorentz force opposes the gravitational force:

$$\mathbf{F}_g + \mathbf{F}_L = -mg\hat{\mathbf{z}} + qv_\phi(B_{z,\mathrm{trap}}\ \hat{\boldsymbol{\rho}} - B_{\rho,\mathrm{trap}}\ \hat{\mathbf{z}}) = 0. \tag{6}$$

From this equation it can be seen that to hold the particle in a stable orbit, the average magnetic field the particle experiences must be

$$\langle B_{\rho,\mathrm{trap}} \rangle = -\frac{mg}{qv_\phi}. \tag{7}$$

Now the influence of this magnetic field on a deuteron EDM experiment will be discussed. Suppose the deuterons are injected with $v = v_\phi = 0.7c$. The deuteron mass $m_d = 3.34 \times 10^{-27}$ kg. Further $g = 9.81$ m/s^2 and $q = 1.60 \times 10^{-19}$ C. With eq. (7) it can be seen that the average radial magnetic field is

$$\langle B_{\rho,\mathrm{trap}} \rangle = -9.8 \times 10^{-16}\ \mathrm{T}. \tag{8}$$

The spin precession about the radial direction for a particle in an electromagnetic field is given by [5]

$$\omega_\rho = -\frac{e}{m}\left[aB_{\rho,\mathrm{trap}} + \frac{\eta}{2}(\boldsymbol{\beta} \times \mathbf{B})_\rho\right]. \tag{9}$$

The magnetic anomaly a for the deuteron is -0.143, so the first term between brackets is 1.4×10^{-16}. Suppose the magnetic field $\mathbf{B}$ is $B_z = 2$ T. This gives for the second term 0.7η. The electric dipole moment is given by [4]

$$\begin{aligned} d &= \eta\frac{e\hbar}{4mc} = \eta e \frac{1.05 \times 10^{-27}\ \mathrm{erg \cdot s}}{4 \times 3.34 \times 10^{-24}\ \mathrm{g} \times 3.0 \times 10^{10}\ \mathrm{cm/s}} \\ &= \eta \times 2.6 \times 10^{-15}\ \mathrm{e \cdot cm}. \end{aligned} \tag{10}$$

An EDM of 1.0×10^{-28} would give $\eta = 3.8 \times 10^{-14}$. Therefore the real EDM signal is about $(0.7 \times 3.8 \times 10^{-14})/(1.4 \times 10^{-16}) = 1.9 \times 10^2$ times greater than the false signal.

3 Effects of earth's rotation

3.1 Particle dynamics

The particles in the storage ring move under the influence of the Lorentz force. However, as seen in the introduction, the particles are also affected by the Coriolis acceleration $\mathbf{a}_C = -2\boldsymbol{\Omega}_e \times \mathbf{v}$. In classical mechanics the force $\mathbf{F}$ is just $m\mathbf{a}$. To get the relativistically correct Coriolis, the following relation between force and acceleration can be used

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \gamma m_0 \mathbf{a} + \gamma^3 m_0 \frac{\mathbf{v} \cdot \mathbf{a}}{c^2}\mathbf{v}. \tag{11}$$

The last term in this equation vanishes, because the Coriolis acceleration is perpendicular to the velocity. Therefore the Coriolis force is

$$\mathbf{F}_C = -2\gamma m \boldsymbol{\Omega}_e \times \mathbf{v}. \tag{12}$$

The earth rotates exactly one time around its own axis in one sidereal day, so

$$\Omega_e = \frac{2\pi}{23\text{h}56'} = 7.29 \times 10^{-5}\ \text{rad/s}. \tag{13}$$

In a Cartesian coordinate system (with $\hat{\mathbf{x}}$ in the longitudinal direction toward the equator, $\hat{\mathbf{y}}$ in the lattitudinal direction toward the east and $\hat{\mathbf{z}}$ perpendicular to the surface), placed at lattitude λ in the Northern Hemisphere, the components of the vector $\boldsymbol{\Omega}_e$ are

$$\begin{aligned} \Omega_x &= -\Omega_e \cos\lambda, \\ \Omega_y &= 0, \\ \Omega_z &= \Omega_e \sin\lambda. \end{aligned} \tag{14}$$

With the relativistic momentum $\mathbf{p} = \gamma m_0 \mathbf{v}$, the equation of motion can be written as

$$\begin{aligned} \frac{d\mathbf{p}}{dt} &= e\mathbf{v} \times \mathbf{B} - 2\gamma m \boldsymbol{\Omega}_e \times \mathbf{v} \\ &= \left[\frac{-e}{\gamma m}\mathbf{B} - 2\boldsymbol{\Omega}_e\right] \times \mathbf{p} \\ &= \boldsymbol{\omega}_c^{'} \times \mathbf{p}, \end{aligned} \tag{15}$$

where the effective cyclotron frequency

$$\boldsymbol{\omega}_c^{'} = \frac{-e}{\gamma m}\left[\mathbf{B} + \frac{2\gamma m}{e}\boldsymbol{\Omega}_e\right] = \frac{-e\mathbf{B}^{'}}{\gamma m} \tag{16}$$

is the frequency at which the particle rotates around the vector $\mathbf{B}^{'}$. The plane of the particle motion becomes tilted, making a small angle α with the horizontal plane. The angle $\alpha \approx \tan\alpha$ can be obtained by dividing the vertical by the horizontal component of $\mathbf{B}^{'}$:

$$\alpha \approx \frac{2\gamma m \Omega_e \cos\lambda}{eB_z + 2\gamma m \Omega_e \sin\lambda} \approx \frac{2\gamma m \Omega_e \cos\lambda}{eB_z}. \tag{17}$$

In these calculations trapping forces are not taken into account. When these are added to eq. (15), the following equation of motion is obtained:

$$\frac{d\mathbf{p}}{dt} = e\mathbf{v} \times \mathbf{B} - 2\gamma m \boldsymbol{\Omega}_e \times \mathbf{v} + e\mathbf{v} \times \mathbf{B}_{\text{trap}}. \tag{18}$$

However, $\mathbf{B}_{\text{trap}}$ is only known in the particle's rest frame. Therefore effects in that frame will be searched for. The magnetic fields from the quadrupole magnets, in the particle's rest frame, are given by $B_{\rho,\text{trap}} = kz$ and $B_{z,\text{trap}} = -k\rho$, where k is the magnetic gradient. Besides the effect of the radial component of the Coriolis force, the vector $\boldsymbol{\rho}$ is non-zero, because it is impossible to inject the particles with exactly the right momentum for the given configuration (and thus, from eq. (4), the orbital radius $R \neq R_0$). Because $\cos\alpha = 1$, neglecting second and higher order terms, the true orbital radius R can be found with:

$$\frac{\gamma m (v + \delta v)^2}{R} = e(v + \delta v)B_z + 2\gamma m \Omega_e \sin\lambda (v + \delta v) + e(v + \delta v)(B_{z,\text{trap}}). \tag{19}$$

With $\rho = R_0 - R$, and $R_0 = \gamma mv/eB_z$ from eq. (4), this equation can be written as a second order polynomial:

$$\Big[ek(v+\delta v)\Big]R^2 + \Big[e(v+\delta v)B_z + 2\gamma m\Omega_e \sin\lambda(v+\delta v) - e(v+\delta v)kR_0\Big]R \\ - \gamma m(v+\delta v)^2 = 0. \tag{20}$$

It can easily be seen that the change of the orbital radius R, due to $\delta v \sim 0.01v$, completely overwhelms the effect of the radial component of the Coriolis force.

To see the effect of the trapping force on the angle between the particle orbit and the vertical plane, consider the vertically directed forces in the particle's rest frame. To get the Coriolis force in the particle's rest frame, first calculate the components of this force in the frame fixed on earth. The velocity $\mathbf{v}$ in this frame can be approximated by (suppose at $t = 0$ the particle is at $x = R$, $y = 0$)

$$\begin{aligned} v_x &= -v\sin\omega t, \\ v_y &= -v\cos\omega t, \\ v_z &= 0, \end{aligned} \tag{21}$$

where $\omega = v/R$. With this expression for $\mathbf{v}$ and eqs. (12, 14), the Coriolis force in the frame fixed on earth can be calculated. When the transformation

$$\begin{aligned} \hat{\boldsymbol{\rho}} &= -\cos\omega t\hat{\mathbf{x}} + \sin\omega t\hat{\mathbf{y}}, \\ \hat{\boldsymbol{\phi}} &= -\sin\omega t\hat{\mathbf{x}} - \cos\omega t\hat{\mathbf{y}}, \\ \hat{\mathbf{z}} &= \hat{\mathbf{z}}, \end{aligned} \tag{22}$$

is used to switch from the frame fixed on earth to the particle's rest frame, the Coriolis force in the particle's rest frame is obtained:

$$\begin{aligned} F_\rho &= 2\gamma mv\Omega_e \sin\lambda, \\ F_\phi &= 0, \\ F_z &= -2\gamma mv\Omega_e \cos\lambda\cos\omega t. \end{aligned} \tag{23}$$

By taking the z-component of the Lorentz force eq. (5), the total force in the vertical direction becomes known. By dividing the force by γm, the following equation of motion in the vertical direction is obtained:

$$\ddot{z}(t) = -2v\Omega_e \cos\lambda\cos\left(\frac{vt}{R}\right) - \frac{evk}{\gamma m}z(t). \tag{24}$$

Because the trapping forces increases linearly with z, the particle's orbit isn't just tilted any more, but now it is flattened in some sense. However, suppose the orbit is just tilted, then the maximum value of z gives the tilt.

Now that the real radius and the real tilt are known, the equation of motion eq. (15) can be replaced by the real equation of motion. In this article the old equation of motion will be used (see eq. (32)).

As said in the introduction, the vertical component of the Coriolis force averages to zero over one orbit, this can now easily be seen from eq. (23).

3.2 Spin dynamics

The equation of motion for the intrinsic spin of a particle, in its rest frame, is given by [9]

$$\frac{d\mathbf{s}}{dt} = \frac{1}{\gamma}\mathbf{F}^{'} + \boldsymbol{\omega}_T \times \mathbf{s} + \boldsymbol{\omega}_{\text{edm}} \times \mathbf{s}. \tag{25}$$

Here the first two terms on the right-hand side involve the BMT-equation. The last term is a consequence of the particle's EDM. However, because of the rotation of the earth, an extra term has to be added to eq. (25) to get the rest-frame equation of motion for the spin as seen from the earth. As already seen in the introduction, the time rate of change of a vector $\mathbf{G}$ seen from a fixed reference frame is

$$\left(\frac{d\mathbf{G}}{dt}\right)_f = \left(\frac{d\mathbf{G}}{dt}\right)_r + \boldsymbol{\Omega} \times \mathbf{G}, \tag{26}$$

where the first term on the right-hand side is the time rate of change in the rotating reference frame and $\boldsymbol{\Omega}$ is the angular velocity of this rotating system. The spin vector $\mathbf{s}$ keeps the same direction seen from a fixed frame of reference:

$$\left(\frac{d\mathbf{s}}{dt}\right)_f = 0. \tag{27}$$

It is quite difficult, not to say impossible, to indicate an absolutely fixed frame of reference in the universe. By approximation, this could be a reference frame fixed with respect to the stars. The fact that the spin vector $\mathbf{s}$ remains the same in a fixed reference frame is completely analogous to the case of a 'classical' gyroscope.

A reference frame fixed on earth rotates, with respect to the 'fixed' reference frame, with an angular velocity of $\boldsymbol{\Omega}_e$. Therefore the time rate of change of the particle its intrinsic spin, in a reference frame fixed on earth, is

$$\left(\frac{d\mathbf{s}}{dt}\right)_r = -\boldsymbol{\Omega}_e \times \mathbf{s} = \mathbf{s} \times \boldsymbol{\Omega}_e \tag{28}$$

and the rest-frame equation of motion for the spin on earth seems to be

$$\frac{d\mathbf{s}}{dt} = \frac{1}{\gamma}\mathbf{F}^{'} + \boldsymbol{\omega}_T \times \mathbf{s} + \boldsymbol{\omega}_{\text{edm}} \times \mathbf{s} + \mathbf{s} \times \boldsymbol{\Omega}_e. \tag{29}$$

The first term on the right-hand side of this equation contains the transformation of the EM-fields, and is explicitely (for $\mathbf{E} = 0$)

$$\frac{1}{\gamma}\mathbf{F}^{'} = \frac{ge}{2m}\mathbf{s} \times \left[\mathbf{B} - \frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}\Big)\boldsymbol{\beta}\right]. \tag{30}$$

The second term is the Thomas precession, where the Thomas frequency $\boldsymbol{\omega}_T$ is given by

$$\boldsymbol{\omega}_T = \frac{\gamma^2}{\gamma+1}\left[\frac{d\boldsymbol{\beta}}{dt} \times \boldsymbol{\beta}\right]. \tag{31}$$

As can be seen from eq. (15) the acceleration of a moving charged particle in a uniform magnetic field is

$$\frac{d\boldsymbol{\beta}}{dt} = \frac{e}{\gamma m}\Big(\boldsymbol{\beta} \times \mathbf{B}^{'}\Big), \tag{32}$$

where

$$\mathbf{B}^{'} = \mathbf{B} + \frac{2\gamma m}{e}\boldsymbol{\Omega}_e. \tag{33}$$

Now the Thomas frequency can be written as

$$\begin{aligned}
\boldsymbol{\omega}_T &= \frac{\gamma^2}{\gamma+1}\left[\frac{e}{\gamma m}\Big(\boldsymbol{\beta}\times\mathbf{B}^{'}\Big)\times\boldsymbol{\beta}\right] \\
&= \frac{e\gamma}{m(\gamma+1)}\left[\boldsymbol{\beta}^2\mathbf{B}^{'} - \Big(\boldsymbol{\beta}\cdot\mathbf{B}^{'})\boldsymbol{\beta}\right] \\
&= \frac{e}{m}\left[\Big(1-\frac{1}{\gamma}\Big)\mathbf{B}^{'} - \frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}^{'}\Big)\boldsymbol{\beta}\right],
\end{aligned} \tag{34}$$

where in the second line use was made of the following relation from vector calculus

$$\mathbf{a}\times(\mathbf{b}\times\mathbf{c}) = (\mathbf{a}\cdot\mathbf{c})\mathbf{b} - (\mathbf{a}\cdot\mathbf{b})\mathbf{c} \tag{35}$$

and in the third line use was made of the relation

$$\frac{\gamma}{\gamma+1}\boldsymbol{\beta}^2 = 1 - \frac{1}{\gamma}. \tag{36}$$

The contribution of the particle's EDM is explicitely (again suppose $\mathbf{E}=0$)

$$\boldsymbol{\omega}_{\rm edm}\times\mathbf{s} = -\frac{e\eta}{2m}\Big(\boldsymbol{\beta}\times\mathbf{B}\Big)\times\mathbf{s}. \tag{37}$$

For the relation between η and the electric dipole moment d, see eq. (10). When eqs. (30, 34, 37) are inserted into eq. (29) the equation of motion for the rest-frame spin $\mathbf{s}$, as seen from the earth, becomes

$$\begin{aligned}
\frac{d\mathbf{s}}{dt} &= \frac{ge}{2m}\mathbf{s}\times\left[\mathbf{B} - \frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}\Big)\boldsymbol{\beta}\right] \\
&\quad - \frac{e}{m}\mathbf{s}\times\left[\Big(1-\frac{1}{\gamma}\Big)\mathbf{B}^{'} - \frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}^{'}\Big)\boldsymbol{\beta}\right] \\
&\quad + \mathbf{s}\times\boldsymbol{\Omega}_e + \boldsymbol{\omega}_{\rm edm}\times\mathbf{s} \\
&= \frac{e}{m}\mathbf{s}\times\left[\Big(\frac{g}{2}-1+\frac{1}{\gamma}\Big)\mathbf{B} - \Big(\frac{g}{2}-1\Big)\frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}\Big)\boldsymbol{\beta}\right] \\
&\quad - \frac{e}{m}\mathbf{s}\times\left[\frac{2\gamma m}{e}\Big(1-\frac{1}{\gamma}\Big)\boldsymbol{\Omega}_e - \frac{\gamma}{\gamma+1}\Big(\frac{2\gamma m}{e}\boldsymbol{\Omega}_e\cdot\boldsymbol{\beta}\Big)\boldsymbol{\beta}\right] \\
&\quad + \mathbf{s}\times\boldsymbol{\Omega}_e + \boldsymbol{\omega}_{\rm edm}\times\mathbf{s} \\
&= (\boldsymbol{\omega}_{\rm BMT} + \boldsymbol{\omega}_{\rm edm} + \boldsymbol{\omega}_{\rm earth})\times\mathbf{s},
\end{aligned} \tag{38}$$

where

$$\boldsymbol{\omega}_{\rm BMT} = -\frac{e}{m}\left[\Big(\frac{g}{2}-1+\frac{1}{\gamma}\Big)\mathbf{B} - \Big(\frac{g}{2}-1\Big)\frac{\gamma}{\gamma+1}\Big(\boldsymbol{\beta}\cdot\mathbf{B}\Big)\boldsymbol{\beta}\right] \tag{39}$$

and

$$\boldsymbol{\omega}_{\rm earth} = \left[\Big(2\gamma-3\Big)\boldsymbol{\Omega}_e - \frac{\gamma}{\gamma+1}\Big(2\gamma\boldsymbol{\Omega}_e\cdot\boldsymbol{\beta}\Big)\boldsymbol{\beta}\right]. \tag{40}$$

3.3 Combined particle and spin dynamics

The spin precession with respect to the particle its orbit can be obtained by substracting the effective cyclotron frequency eq. (16) from the apparent rest-frame spin precession (see eq. (38)):

$$\begin{aligned}\boldsymbol{\omega}_a^{'} &= (\boldsymbol{\omega}_{\mathrm{BMT}} + \boldsymbol{\omega}_{\mathrm{edm}} + \boldsymbol{\omega}_{\mathrm{earth}}) - \boldsymbol{\omega}_c^{'} \\ &= -\frac{e}{m}\left[\left(\frac{g}{2}-1\right)\mathbf{B} - \left(\frac{g}{2}-1\right)\frac{\gamma}{\gamma+1}\left(\boldsymbol{\beta}\cdot\mathbf{B}\right)\boldsymbol{\beta}\right] - \frac{e\eta}{2m}\left(\boldsymbol{\beta}\times\mathbf{B}\right) \\ &\quad + \left[\left(2\gamma-1\right)\boldsymbol{\Omega}_e - \frac{\gamma}{\gamma+1}\left(2\gamma\boldsymbol{\Omega}_e\cdot\boldsymbol{\beta}\right)\boldsymbol{\beta}\right]. \end{aligned} \tag{41}$$

4 Other effects

4.1 Centrifugal force

A particle on earth also experiences a centrifugal acceleration due to the earth's rotation. This accleration is given by (see eq. (2))

$$\mathbf{a}_c = -\boldsymbol{\Omega}_e \times (\boldsymbol{\Omega}_e \times \mathbf{R}) = \Omega_e^2 \mathbf{R}_\perp. \tag{42}$$

Here $\mathbf{R}$ is the position vector of the particle as seen from the center of the earth and $\mathbf{R}_\perp = |\mathbf{R}|\cos\lambda$ is the component of $\mathbf{R}$ perpendicular to the rotation axis. The vector $\boldsymbol{\Omega}_e$ is given by eq. (14). A relativistic centrifugal force can be obtained in the same manner as was done for the Coriolis force: $\mathbf{F}_c = \gamma m \Omega_e^2 \mathbf{R}_\perp$. This force splits up into a vertical component and a component longitudinally directed toward the equator:

$$\begin{aligned}\mathbf{F}_x &= \gamma m \Omega_e^2 \mathbf{R} \cos\lambda \sin\lambda, \\ \mathbf{F}_z &= \gamma m \Omega_e^2 \mathbf{R} \cos\lambda \cos\lambda.\end{aligned} \tag{43}$$

With $|\mathbf{R}| = 6,36 \times 10^6$ m, it can be seen that these forces are much smaller than gravity and the Coriolis force, so they can be neglected. In reality the x-component of the centrifugal force is even much smaller than these calculations show. The earth after all is not a perfect sphere, but an ellipsoid. Of course, this is due to the centrifugal force. Therefore gravity is not exactly directed perpendicular to the eart's surface any more, but there is also a small longitudinal component directed toward the poles. This component cancels the longitudinal component of the centrifugal force [7].

4.2 Spin-gravity coupling

A gravitomagnetic field is produced by a massive spinning body [10]. It is the same as a charged spinning body produces a magnetic field. Not surprisingly, a particle tries to align its spin with the gravitomagnetic field, just like it has the tendency to align its spin with the magnetic field [11]. This effect, also known as the Lense-Thirring effect, comes from general relativity. The gravitomagnetic field of the earth makes a reference frame on earth appear to be rotating with an additional frequency $-\boldsymbol{\Omega}_d$, this is called *frame-dragging*. Thus, in the same manner as was done for the earth rotation, a term $\boldsymbol{\Omega}_d \times \mathbf{s}$ should be added to eq. (29). However, $\Omega_d \sim 10^{-14}$ rad/s, this is much smaller than Ω_e, so the term $\boldsymbol{\Omega}_d \times \mathbf{s}$ can be neglected.

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