UHECR acceleration in a magnetar wind using the concept of Plasma Wakefield Acceleration

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February 14, 2011

Abstract

In this thesis is a model investigated for the acceleration of UHECR's in the magnetar wind. First, the configuration of the field lines of a magnetar is examined using pulsar models. Furthermore the concept of the plasma wakefield acceleration using Alfvén waves and what the conditions are for optimal acceleration is examined. Using the model we can derive a maximum energy, a power law spectrum and a flux. The maximum energy ranges from an energy of 10^{19} eV on a lower limit and of 10^{27} eV on a upper limit. We find a power law of ϵ^{-2} and a flux of $1.6 * 10^{-8}$ UHECR / m^2 /year for a particle with energies greater than 10^{20} eV. The power law is in line with observations. The flux number is also in line with observations. Estimates on the flux number are very rough, so we cannot make some final conclusions on that.

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Chapter 1

Introduction

In this thesis is a model investigated for the acceleration of UHECRs. The acceleration mechanism of Ultra High Energy Cosmic Rays is one of the biggest mysteries in the high energy astroparticle physics. The challenge for a viable model is to deal with the GZK-limit. This limits the distance a UHECR particle can travel before it collides with the cosmic microwave background to about 50 Mega Parsec. No current models can explain where the UHECRs come from that hit the earth since they must have been created rather nearby.

Furthermore there is a radical change in the powerlaw of the UHECR spectrum. This means that the α from ϵ^{α} goes from $\alpha = -3$ to an α that is much smaller at energies of $10^{18} - 10^{19}$. Why this happens, nobody knows, but it gives us an indication that UHECRs with energies above 10^{18} eV are from different sources than UHECRs with energies below 10^{18} eV.[1] This means a new theoretical model can be considered for energies higher than 10^{18} eV.

Currently the models that attempt to solve this problem can be divided in two categories, the top-down scenarios and the bottom-up scenarios. In the top-down scenarios, the solution is searched in exotic new particles. In the bottom-up scenarios solutions are looked for within the GZK-limit or with UHE neutrinos in stead of protons as the primary carriers of the energy. Both scenarios have their advantages. In the top-down scenario the main problem is the uncertainty a new particle gives. In the bottom-up scenarios the main challenge is to find an effective acceleration mechanism that can reach high enough energies and can deal with the GZK-limit.

In this thesis one of the bottom-up models is examined and refined. We look at plasma wakefield acceleration in the magnetar wind and check if it can explain the flux of UHECR within the restrictions that apply. This means wakefield acceleration must reach the Ultra High Energies and the flux of particles hitting the earth must be accelerated within the GZK-limit.

Chapter 2

Magnetars

In this chapter it is explained what a magnetar is, what parameters define them and how scarce they are. A star that has a mass between the 1.4 and 3 times that of the sun collapses to a neutron star. Below that, the star becomes a white dwarf (Chandrasekhar limit) and above the limit a star keeps on collapsing to form a black hole. As the name suggests the neutron star is composed almost entirely of neutrons. In a neutron star there is a balance between the gravitational force and the neutron degeneracy repulsion (the Pauli exclusion principle).

Most important parameters for a neutron star are it's rotation period, magnetic field and velocity. At first it was thought that all neutron stars looked more or less the same in this regard (a rotational period of tens of milliseconds and a magnetic field strength of 10^{11} to 10^{12} G). In 1995 [2] the first evidence that there could be neutron stars with a much larger magnetic field was found, namely Soft Gamma Repeaters. Soft Gamma Repeaters ("SGRs") are X-ray stars that emit bright, repeating flashes of soft (i.e. low-energy) gamma rays. SGRs sometimes spit out big bursts of energy. Because of these big bursts of energy, it is thought that SGRs should have a huge magnetic field. The magnetic fields should penetrate the curst (outer layer of atoms of the neutron star) and enormous magnetic forces pull at it, as the magnetic field lines diffuse through the curst. Sometimes the curst isn't stable anymore and the curst explodes throwing huge amounts of energy into outer space (the soft gamma ray bursts). The magnetic field could be well above the assumed critical magnetic field, $B_{cr} = 4.4 \times 10^{13}$ G, before this is likely to happen [3]. These things are called magnetars.

2.1 How is it possible that the magnetar has such high magnetic fields?

We can ask ourselves what mechanism causes the magnetars to have such high electro-magnetic fields. For this to be answered we need to know how the magnetar becomes a magnetar. Actually it starts when a bright star comes to an end and becomes a neutron star. A star usually has a weak electromagnetic field (due to currents inside the core). It is assumed [4] that the magnetic fields becomes larger simply due to contraction of the core. Due to the fact the magnetic field is frozen into the plasma, a magnetized object that shrinks in volume, will get a magnetic field that strengthens quadratically. Usually when the core of a star collapses into a neutronstar, it's volume shrinks by a factor 10^5 and the magnetic field in the core becomes a factor 10^{10} stronger.

This doesn't explain the high fields of the magnetar completely. We need to take into account another effect. The core of a star consists of ionized gasses. There are hotter parts en colder parts of ionized gas. The hotter parts rise to the surface of the star, the cooler parts sink to the core. Ions conduct electricity very well, so the magnetic field lines are dragged with the movement of the ions. [4]

2.2 How common are magnetars?

There are about ten known magnetars in the milky way. These ten magnetars, had a dimmed phase, where they could hardly be observed. Sometimes the magnetar lights up with a sudden burst of gamma rays. This phase can endure for hours, days or months until it gets into the dimmed phase again. It is likely that most of the magnetars are in this dimmed phase, so there may be far more magnetars (likely millions [3]) out there than we now know. Precise numbers are unfortunately currently unknown. [5]

Chapter 3

Magnetic Field

We will derive the magnetic field description of rapidly rotating magnetized stars (magnetars) with plasmas surrounding it (magnetars are such objects as far as we know). We will take the easiest case possible where the axis of rotation coincides with the magnetic multipole axis. The plasma around the star corotates with the star. If the plasma isn't at first corotating a current will flow due to the potential difference between the plasma and the rotating star. The current slows the star and accelerates the plasma, where eventually the plasma will corotate with the star. The plasma can't follow the rotation of the star anymore near the light cylinder. The light cylinder is a cylinder around a star, where the straight side is in the direction of the rotation axis (see Fig. 3.1). The radius of the cylinder is the point where a test particle corotating with the star, would travel with the speed of light. The light cylinder description is useful because usually it's a transition region. At this point the centrifugal force will take over the magnetic field force (which decreases with distance) and the magnetic field lines will be pulled open (see Fig. 3.2). Notice how a neutral current sheet is created, where the magnetic field is zero. In the case where the axis of rotation doesn't coincide with the magnetic multipole, things get a little complicate. In this case the neutral current sheet wraps up like a curtain (see Fig. 3.2). It gives an indication for how complex things become for a non co-rotating system. We will leave the subject as it is and will focus on the magnetic field equations for an axially co-rotating system.

Furthermore we will assume that the plasma inertia is negligibly small.

We have in a perfect plasma a force balance between the magnetic force and the electric force on a ion or electron. We can write

$$0 = \overrightarrow{F_e} = j_0\overrightarrow{E} + j_0\overrightarrow{v_\pm} \times \overrightarrow{B} \tag{3.1}$$

Here v_+ and v_- are the velocities of respectively the ions and the electrons in the plasma. We can reformulate this equation in the center-of-mass frame using the center of mass velocity

$$v = \frac{n_+v_+ + n_-v_-}{n_+ + n_-} \tag{3.2}$$

Here n_+ and n_- are the ion and electron density numbers respectively. The force balance equation (Eq. (3.1)) then becomes

$$\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B} = 0 \tag{3.3}$$

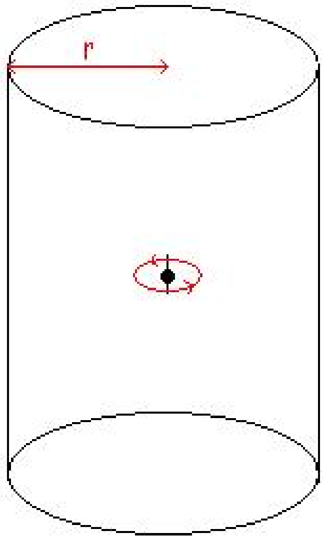

Figure 3.1: Schematic picture of the concept of a light cylinder. Here $r = c/\omega$, where c is speed of light and ω the rotationspeed of the magnetar.

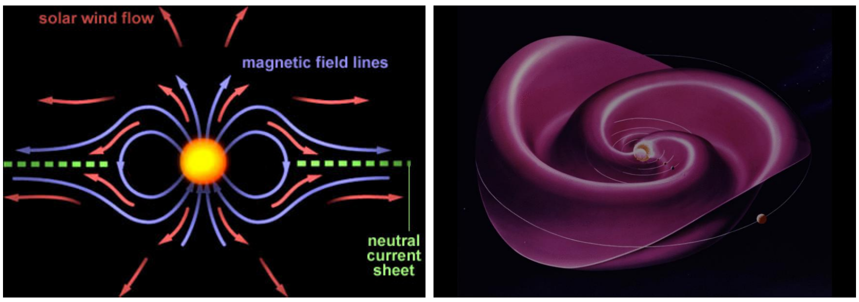

Figure 3.2: The left picture gives a very simplified drawing of the magnetic field of an axially corotating pulsar. The blue lines give the magnetic field lines and the red lines the magnetic wind flow. Notice how the current sheet is a plane through the equator of the pulsar. In the right picture can be seen what happens to the current sheet when magnetic and rotation axis doesn't coincide. It wraps up like a curtain.

We also have maxwells equations

$$\begin{aligned}
\overrightarrow{\nabla} \times \overrightarrow{B} &= n_+\overrightarrow{v_+} - n_-\overrightarrow{v_-} = 4\pi\overrightarrow{j} && (3.4)\\
\overrightarrow{\nabla} \cdot \overrightarrow{E} &= n_+ - n_- = 4\pi j_0 && (3.5)\\
\overrightarrow{\nabla} \cdot \overrightarrow{B} &= 0 && (3.6)\\
\overrightarrow{\nabla} \times \overrightarrow{E} &= 0 && (3.7)
\end{aligned}$$

And from the continuity of the magnetic field between the surface of the star and the base of the magnetosphere we get

$$\overrightarrow{v_\pm} = C(n_\pm)\overrightarrow{B} + \frac{\Omega r}{c}\widehat{\phi} \tag{3.8}$$

We can combine Eq. (3.5) and Eq. (3.6) with Eq. (3.2) and Eq. (3.3) to get

$$(\overrightarrow{\nabla} \times \overrightarrow{B}) \times \overrightarrow{B} + (\overrightarrow{\nabla} \cdot \overrightarrow{E})\overrightarrow{E} = 0 \tag{3.9}$$

This equation together with Eq. (3.3) forms the basis in which we will derive further equations, namely a single differential equation for the field lines. We know our system is axisymmetric so all our quantities are simplified to $F(r, \phi, z) = F(r, z)$. We know from Eq. (3.7) that $\overrightarrow{E}$ must take the following form

$$\begin{aligned}
\overrightarrow{E} &= \frac{\partial}{\partial r}u(r,z)\widehat{r} + \frac{\partial}{\partial \phi}u(r,z)\widehat{\phi} + \frac{\partial}{\partial z}u(r,z)\widehat{z}\\
&= \frac{\partial}{\partial r}u(r,z)\widehat{r} + \frac{\partial}{\partial z}u(r,z)\widehat{z} && (3.10)
\end{aligned}$$

Here $u(r, z)$ is the scalar potential for the electric field. This means that there is no electric field in the ϕ-direction. When we look at the ϕ-direction of Eq. (3.3), it can be seen that the poloidal part (that is the r and z components combined) of $\overrightarrow{v}$ is parallel to the poloidal part of $\overrightarrow{B}$. In other words

$$v_p = h(r, z)B_p \tag{3.11}$$

We know from Eq. (3.7) together with the axisymmetry condition that the magnetic field must look like

$$\overrightarrow{B} = B_\phi\widehat{\phi} + \overrightarrow{\nabla} \times (A_\phi\widehat{\phi}) \tag{3.12}$$

Here A_ϕ is the ϕ direction of the well known vector potential. We need to find an expression for B_ϕ in terms of A_ϕ. At first this looks impossible because $B = \nabla \times A$, but things are a bit more complicate when we look at the ϕ-direction of Eq. (3.9) [6]. We know that E_ϕ is zero, so Eq. (3.9) becomes

$$\begin{aligned}
0 &= [(\overrightarrow{\nabla} \times \overrightarrow{B}) \times \overrightarrow{B}]_\phi\\
&= (\overrightarrow{\nabla} \times \overrightarrow{B})_z B_r - (\overrightarrow{\nabla} \times \overrightarrow{B})_r B_z\\
&= (\frac{1}{r}\frac{\partial}{\partial r}rB_\phi)B_r - (\frac{\partial}{\partial z}B_\phi)B_z\\
&= (\frac{1}{r}\frac{\partial}{\partial r}rB_\phi)(-\frac{\partial A_\phi}{\partial z}) + (\frac{\partial}{\partial z}B_\phi)\frac{1}{r}\frac{\partial rA_\phi}{\partial r}
\end{aligned} \tag{3.13}$$

From this it follows through an easy calculation that

$$\frac{\frac{\partial r A_\phi}{\partial r}}{\frac{\partial r A_\phi}{\partial z}} = \frac{\frac{\partial}{\partial r} r B_\phi}{\frac{\partial}{\partial z} r B_\phi} \tag{3.14}$$

So we can say that rB_ϕ is functional dependent of rA_ϕ. In other words

$$B_\phi = \frac{1}{r} B(rA_\phi) \tag{3.15}$$

So we finally have a form for B_ϕ which we know is not zero and with which we can derive further quantities. To conclude we know the three components of the B-field as a function of A_ϕ:

$$B_r = -\frac{\partial}{\partial z} A_\phi = -\frac{1}{r}\frac{\partial}{\partial z} A \tag{3.16}$$
$$B_\phi = \frac{1}{r} B(rA_\phi) = \frac{1}{r} B(A) \tag{3.17}$$
$$B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\phi) = \frac{1}{r}\frac{\partial}{\partial r} A \tag{3.18}$$

Here $A \equiv rA_\phi$. With Eq. (3.3) and Eq. (3.8) we can find the equations for the electric field:

$$E_r = -\frac{\Omega r}{c} B_z = -\frac{\Omega}{c}\frac{\partial}{\partial r} A \tag{3.19}$$
$$E_\phi = 0 \tag{3.20}$$
$$E_z = \frac{\Omega r}{c} B_r = -\frac{\Omega}{c}\frac{\partial}{\partial z} A \tag{3.21}$$
$$\tag{3.22}$$

Furthermore we can derive from these equations the current- and charge density with the use of Maxwell's equations.

$$\rho = \overrightarrow{\nabla} \cdot \overrightarrow{E} = -\frac{\Omega}{4\pi c}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A}{\partial r}\right) + \frac{\partial^2}{\partial z^2} A\right) \tag{3.23}$$
$$j_r = \frac{1}{4\pi r} B'(A) \frac{\partial A}{\partial z} \tag{3.24}$$
$$j_\phi = -\frac{1}{4\pi}\left(\frac{1}{r}\frac{\partial^2}{\partial z^2} A + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r} A\right)\right) \tag{3.25}$$
$$j_z = \frac{1}{4\pi r} B'(A) \frac{\partial A}{\partial r} \tag{3.26}$$

We know A is constant along each field line because

$$\overrightarrow{B} \cdot \overrightarrow{\nabla} A = 0 \tag{3.27}$$

holds. We now formulate a partial differential equation for the magnetic field lines in using the r-direction of Eq. (3.9) and Maxwell's equation

$$\begin{aligned} 0 &= (\overrightarrow{\nabla} \times \overrightarrow{B})_\phi B_z - (\overrightarrow{\nabla} \times \overrightarrow{B})_z B_\phi + (\overrightarrow{\nabla} \cdot \overrightarrow{E}) E_r \\ &= j_\phi B_z - j_z B_\phi + \rho E_r \end{aligned}$$

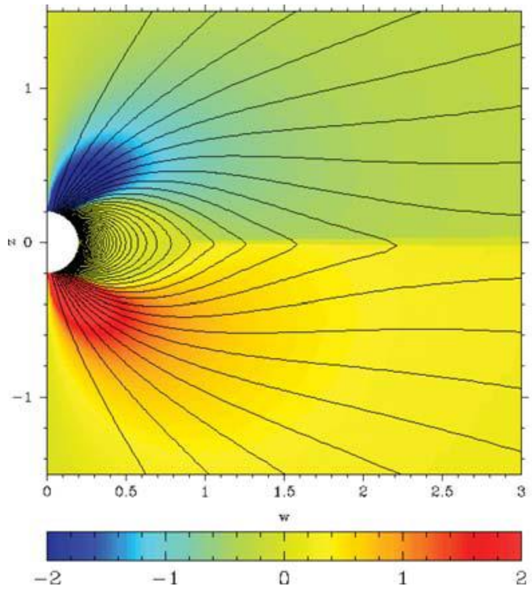

Figure 3.3: solutions to Eq. (3.28). The colors give the magnetic field in the ϕ direction. [7] On the x-axis the distance from the magnetar along the equator is put and on the y-axis the distance from the magnetar along the magnetic axis is put. The light cylinder is taken to be the normalization on the x-axis (so the light cylinder is at x=1)

$$
\begin{aligned}
&= -\frac{1}{4\pi}(\frac{1}{r}\frac{\partial^2}{\partial z^2}A + \frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}A))B_z - \frac{1}{4\pi r}B'(A)\frac{\partial A}{\partial r}B_\phi \\
&\quad -\frac{\Omega}{4\pi c}(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial A}{\partial r}) + \frac{\partial^2}{\partial z^2}A)E_r \\
&= -\frac{1}{4\pi}(\frac{1}{r}\frac{\partial^2}{\partial z^2}A + \frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}A))\frac{1}{r}\frac{\partial}{\partial r}A - \frac{1}{4\pi r}B'(A)\frac{\partial A}{\partial r}\frac{1}{r}B(A) \\
&\quad +\frac{\Omega}{4\pi c}(\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial A}{\partial r}) + \frac{\partial^2}{\partial z^2}A)\frac{\Omega}{c}\frac{\partial}{\partial r}A \\
&= \frac{1}{4\pi r^2}\frac{\partial A}{\partial r}[-(\frac{\partial^2}{\partial z^2}A + r\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}A)) - B'(A)B(A) \\
&\quad +\frac{\Omega^2}{c^2}(r\frac{\partial}{\partial r}(r\frac{\partial A}{\partial r}) + r^2\frac{\partial^2}{\partial z^2}A)] \\
&= \frac{1}{4\pi r^2}\frac{\partial A}{\partial r}[-(\frac{\partial^2}{\partial z^2}A + \frac{\partial^2}{\partial r^2}A) - B'(A)B(A) \\
&\quad +\frac{\Omega^2 r^2}{c^2}(\frac{\partial^2}{\partial r^2}A + \frac{\partial^2}{\partial z^2}A) + \frac{1}{r}\frac{\partial}{\partial r}A + \frac{\Omega^2 r^2}{c^2}\frac{1}{r}\frac{\partial}{\partial r}A] \\
0 &= (\frac{\Omega^2 r^2}{c^2} - 1)(\frac{\partial^2}{\partial z^2}A + \frac{\partial^2}{\partial r^2}A) \\
&\quad -B'(A)B(A) + (\frac{\Omega^2 r^2}{c^2} + 1)\frac{1}{r}\frac{\partial}{\partial r}A \qquad (3.28)
\end{aligned}
$$

On the basis of these equations, simulations have been made by [7] and they did come up with some nice pictures (see Fig. 3.3) .

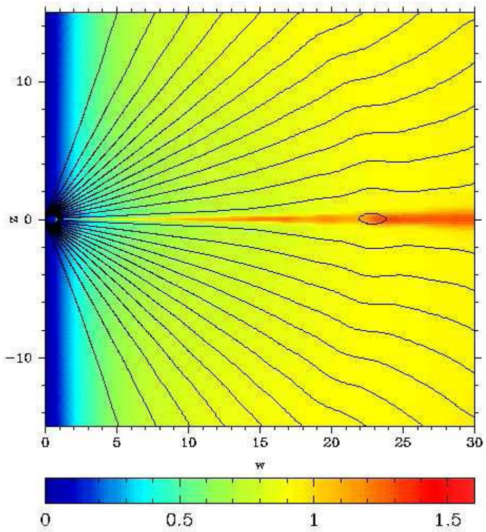

Figure 3.4: Simulation made by [7] on the far field approximations of the magnetic field. On the x-axis the distance from the magnetar along the equator is put and on the y-axis the distance from the magnetar along the magnetic axis is put. The light cylinder is taken to be the normalization on the x-axis (so the light cylinder is at x=1)

3.1 Far-field solutions

We now look at solutions of Eq. (3.28) far from the light cylinder ($r >> c/\Omega$). We physically expect that the fieldlines become more and more radial, when we go away from the light-cylinder. This can also be seen by the simulations from Fig. 3.4 We don't know this for certain, but we can at least check if there are solutions to this approximations. When the fieldlines become radial, it means that A depends only on

$$\mu = \frac{z}{\sqrt{z^2 + r^2}} = \cos\theta \tag{3.29}$$

And thus A becomes

$$A(r,z) = A(\mu) \tag{3.30}$$

Now we can simplify Eq. (3.28)

$$\begin{aligned}
0 &= (\frac{\Omega^2 r^2}{c^2} - 1)(\frac{\partial^2}{\partial z^2}A + \frac{\partial^2}{\partial r^2}A) - B'(A)B(A) + (\frac{\Omega^2 r^2}{c^2} + 1)\frac{1}{r}\frac{\partial}{\partial r}A \\
&= \frac{\Omega^2 r^2}{c^2}(\frac{\partial^2}{\partial z^2}A + \frac{\partial^2}{\partial r^2}A) - B'(A)B(A) + \frac{\Omega^2 r}{c^2}\frac{\partial}{\partial r}A \\
&= \frac{\Omega^2 r^2}{c^2}(\frac{\partial^2 A}{\partial \mu^2}(\frac{\partial \mu}{\partial z})^2 + \frac{\partial A}{\partial \mu}\frac{\partial^2 \mu}{\partial z^2} \\
&\qquad + \frac{\partial^2 A}{\partial \mu^2}(\frac{\partial \mu}{\partial r})^2 + \frac{\partial A}{\partial \mu}\frac{\partial^2 \mu}{\partial r^2}) - B'(A)B(A) + \frac{\Omega^2 r}{c^2}\frac{\partial A}{\partial \mu}\frac{\partial \mu}{\partial r} \\
&= \frac{\partial^2 A}{\partial \mu^2}\frac{\Omega^2 r^2}{c^2}((\frac{\partial \mu}{\partial z})^2 + (\frac{\partial \mu}{\partial r})^2)
\end{aligned}$$

$$
\begin{aligned}
& \quad + \frac{\Omega^2 r^2}{c^2} \frac{\partial A}{\partial \mu} (\frac{\partial^2 \mu}{\partial z^2} + \frac{\partial^2 \mu}{\partial r^2} + \frac{1}{r} \frac{\partial \mu}{\partial r}) - B'(A) B(A) \\
= & \quad \frac{\partial^2 A}{\partial \mu^2} \frac{\Omega^2 r^2}{c^2} (\frac{1}{r^2 + z^2} - \frac{z^2}{(z^2 + r^2)^2}) \\
& \quad + \frac{\Omega^2 r^2}{c^2} \frac{\partial A}{\partial \mu} (-2 \frac{z}{(z^2 + r^2)^{3/2}}) - B'(A) B(A) \\
= & \quad \frac{\partial^2 A}{\partial \mu^2} \frac{\Omega^2 r^2}{c^2} \frac{1}{r^2 + z^2} (1 - \mu^2) \\
& \quad + \frac{\Omega^2 r^2}{c^2} \frac{\partial A}{\partial \mu} (-2 \frac{1}{z^2 + r^2} \mu) - B'(A) B(A) \\
= & \quad \frac{\partial^2 A}{\partial \mu^2} \frac{\Omega^2}{c^2} (1 - \mu^2)^2 + \frac{\Omega^2}{c^2} \frac{\partial A}{\partial \mu} \mu (1 - \mu^2) - B'(A) B(A) = 0 \qquad (3.31)
\end{aligned}
$$

In this the assumption was explicitly made that $r >> c/\Omega$. We can now look at the boundary conditions on A. There are two boundaries to be specified, namely the poles (where $\mu = \pm 1$) and the equator (where $\mu = 0$). We have the freedom to choose one of the two zero. The following boundary conditions are chosen:

$$
\begin{aligned}
A(\mu) &= A(0) = A_0 \\
A(\mu) &= A(\pm 1) = 0 \qquad (3.32)
\end{aligned}
$$

In principle we can now numerically calculate the quantity A from Eq. (3.31) and the boundary conditions for Eq. (3.32). From [8] we know that a solution of Eq. (3.31) is also a solution from the following (much simpler) differential equation

$$
\frac{\partial A}{\partial \mu} = - \frac{B(A)}{1 - \mu^2} \qquad (3.33)
$$

This can be seen by simply substituting Eq. (3.33) into Eq. (3.31). In integral form Eq. (3.33) looks like

$$
\int \frac{dA}{B(A)} = - \int \frac{d\mu}{1 - \mu^2} \qquad (3.34)
$$

And thus we need a functional form of B(A) to solve this equation. We know from [8] that B(A) has the form

$$
B(A) = 4 \frac{A}{A_0} \sqrt{A_0^2 - A^2} \qquad (3.35)
$$

So Eq. (3.34) becomes

$$
\int \frac{dA A_0}{4A \sqrt{A_0^2 - A^2}} = - \int \frac{d\mu}{1 - \mu^2} \qquad (3.36)
$$

This gives for A the following solution

$$
A = A_0 \frac{(1 - \mu^2)^2}{1 + 6\mu^2 + \mu^4} \qquad (3.37)
$$

Chapter 4

Plasma Wakefield Acceleration

4.1 How does plasma wakefield acceleration work

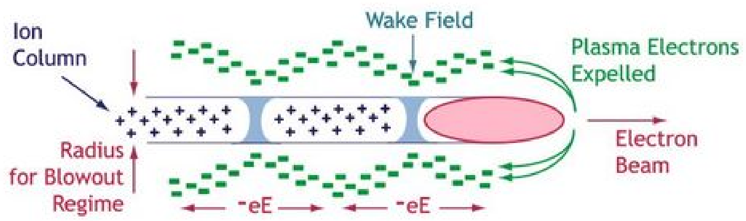

Figure 4.1: schematic picture of plasma wakefield acceleration [9]

First a qualitative description of a plasma wakefield will be given. The mechanism was first developed as an alternative for the standard linear colliders. The plasma wakefield accelerator can reach high energies within a much shorter length, making the accelerator much more compact. This is also the main advantage as to traditional colliders.

To accelerate a particle in a plasma we need to create a longitudinal electric field. This can be done by creating a wake behind a laser pulse which travels through the plasma. A plasma consists of positive and negative charged particles. The negative charged particles can move freely through the plasma, where the heavy ions barely move. We shoot a laser pulse through the plasma. The pulse travels through the plasma and separates the particles the same way, a static electric field would do. The positive charge is much heavier than the electrons so the ions remain in the center of the laser beam. The electrons are moved much farther away. When the laser pulse has passed, the electrons come back to the positively charged center. They build up speed and thus pass the center of the laser pulse again and get pulled back by the positive centre. In this way an oscillatory movement is created. The oscillation can break when the electrons get seperated too far from the ions. The electron then escapes from the ion potential and is free. The oscillatory movement breaks, which we call the cold wavebreaking limit.

Very close behind the laser pulse there is thus a very strong potential gradient. It is in this area where particles can be accelerated best. A positively charged particle gets accelerated by the pile up of negative charge. Eventually it can reach the speed of the wakefield. Even higher energies can be reached when the particle travels across the wakefield.

Consider a photon (laser pulse) that is injected in an under dense plasma. The photon in a plasma has a group velocity of

$$v_g = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{4.1}$$

where ω_p is the plasma frequency and ω the photon frequency. The photon creates an plasma wake which is excited by it's ponderomotive force (see the section about maximum energies) as is qualitatively described above. This plasma wake has a phase velocity which is the same as the group velocity of the photon

$$v_p = \frac{\omega_p}{k_p} = v_g^{photon} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{4.2}$$

Here k_p is the wave number of the plasmon. The wave packet (photon) as a whole is exciting the plasma wave, so it is obvious that the plasmon moves with the group velocity of the photon. From Fig. 4.2 it is clear that a plasmon is most effectively generated when the wavelength of the plasmon (the red line in Fig. 4.2) is two times the length of the wave packet or

$$L_{photon} = \frac{\lambda}{2} = \frac{\pi c}{\omega_p} \tag{4.3}$$

In our case of the plasma accelerator in space, a laser pulse is not a likely

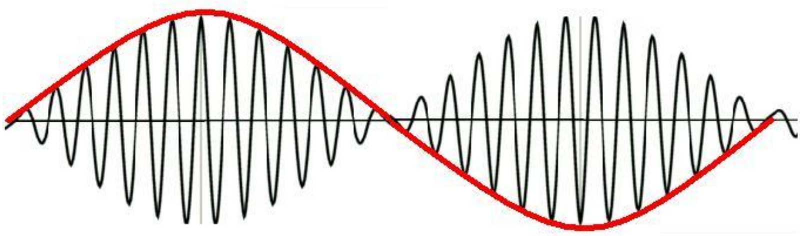

Figure 4.2: The photon wave packet and the plasmon

option. We can use Alfvén waves (see the section about Alfvén waves) which act as normal EM waves under the right circumstances. With two Alfvén waves superposed we get a beat like the laserpulse. In the most effective way the frequency difference should be

$$\Delta\omega = \omega_1 - \omega_2 = \omega_p \tag{4.4}$$

The mechanism on how the wake is generated will be explained. Consider the photon traveling in the x-direction with an oscillating electric field in the y-direction. Because of the electric field, the electrons in the plasma start to oscillate in the y-direction and thus pick up electric energy from the photon. In gaining energy the electrons must also pick up some of the momentum of the photon, $\Delta p_x = m_e \Delta v_x$. When the pulse passes an electron it is moved in the x-direction by $\Delta x = \Delta v_x \tau$, where τ is the length of the light pulse. When the light pulse passed, the electron is moved back by the empty space, which creates an effective positive charge. This is thus the plasma oscillation, which creates an longitudinal electric field in the x-direction. The protons can ride on this wave for a long time before they get out of phase, and thus gaining enormous amounts of energy.[10]

4.2 Riding the wave

In this section we will look deeper into the concept of wave riding. First will be looked at the linear regime. This means that all parameters are non-relativistic. Then the non-linear relativistic case will be explained.

4.2.1 linear regime

In this section the riding of the plasma wave by a proton is examined. As explained in the previous section the plasma oscillation creates a longitudinal electric field. The electric potential for this field takes the form [11]

$$\phi = \phi_0 \cos(k_p(x - v_p t)) \tag{4.5}$$

Here ϕ_0 is a constant related to the maximum field. We can see from this expression that the phase region $-\pi < k_p(x - v_p t) = \Psi < 0$ is accelerating.

Consider a proton in the plasma which is moving with $v_x < v_p$ at the phase $\Psi = 0$. This means that the proton is slipping behind on the plasma wave. After gaining energy from the plasma wave while moving backwards in phase space two things can happen at $\Psi = -\pi$. First the velocity of the proton doesn't reach the velocity of the plasma. The proton gets further behind in phase space and enters the decelerating region of $0 < \Psi < \pi$. It will thus lose energy.

The second case is the most interesting one. In this case the initial speed of the proton was high enough and the proton has a speed $v_x \geq v_p$ at $\Psi = -\pi$. The proton is moving up in phase space and thus moves again through the accelerating phase, gaining energy again. Eventually it will reach the point $\Psi = 0$ again and thus get into the decelerating phase $0 < \Psi < \pi$. It will starts to oscillate around $\Psi = 0$ and is thus trapped in the region of phase space $-\pi < \Psi < \pi$.

The initial speed of the proton is an important parameter to determine whether a proton gets trapped or not. We call v_{min} the minimum initial velocity to get a proton trapped in a plasma wave at $\Psi = 0$.

v_{max} is the maximum velocity gained by riding the plasma wave. The maximum value of the velocity of the proton is also gained at $\Psi = 0$.

4.2.2 non-linear regime

In the non-linear regime things get a little bit more complicated. First of all the electric potential ϕ is given by a differential equation

$$\frac{1}{k_p^2}\frac{\partial^2\phi}{\partial\zeta^2} = \gamma_p^2\beta_p\sqrt{1-\frac{1+a^2}{\gamma_p^2(1+\phi)^2}} - 1 \tag{4.6}$$

Here a is the ponderomotive vector potential and $\zeta = z - v_p t$. Furthermore we have the Hamiltonian for a proton that is accelerated in the non-linear plasma wave

$$H(\gamma,\psi) = \gamma(1-\beta\beta_p) - \phi(\psi) \tag{4.7}$$

In the coordinate system with generalized coordinates γ and ψ and $k_p x$ as the 'time' coordinate, we get for Hamilton's equations

$$\frac{d\gamma}{d(k_p z)} = \frac{\partial H}{\partial\psi} = \frac{\partial\phi}{\partial\psi} \tag{4.8}$$

$$\frac{d\psi}{d(k_p z)} = \frac{\partial H}{\partial\gamma} = \frac{\partial}{\partial\gamma}[\gamma(1-\sqrt{1-\frac{1}{\gamma^2}}\beta_p)] = 1 - \frac{\beta_p}{\beta} \tag{4.9}$$

We don't know the exact solution of ϕ but it can be argued that ϕ oscillates between a minimum value ϕ_{min} and a maximum value ϕ_{max}.[11] We can define for ϕ_{max}

$$\phi_{max} = (2\gamma_p^2 - 1)\frac{\epsilon}{\gamma_p} - 1 \tag{4.10}$$

Here ϵ gives a measure on how big the maximum longitudinal electric field will be. We can make plots for different values of ϵ in ψ,γ_p phase (see figure Fig. 4.3). For $\epsilon = 0.03$, $\gamma_{min} > 1$, which means that a particle with velocity $v_{min} > 0$ at $\Psi = 0$ will be trapped. For $\epsilon = 0.04$ the value for γ_{min} is one. This means that a particle at rest at $\Psi = 0$ will be trapped and accelerated. For higher values of ϵ particles traveling in the opposite direction will be trapped and accelerated. At even higher values of ϵ the electrons of the plasma wave will be trapped and the wave will break (the wavebreaking limit is reached). [11]

4.3 Conditions for particle acceleration

We should first ask ourselves what is needed to accelerate a charged particle. First of all we need of course a longitudinal electric field. With a longitudinal electric field the particle feels an electric force in the direction of the field. We have seen in the previous section that a longitudinal electric field is created by the plasma oscillation.

Second, the acceleration should be collision free. That means that the accelerated particle should not interact with other particles, otherwise it looses to much energy on the inelastic collisions. And last of all, the particle shouldn't be bend too much, because it then loses energy through synchrotron radiation. The plasma wakefield accelerators on earth promise to deliver on all these condition, so we are going to try to apply to the conditions in space (especially around magnetars). This means that we will consider the winds around the magnetars which are moving radially outward, and have a magnetic field parallel along that movement. [12]

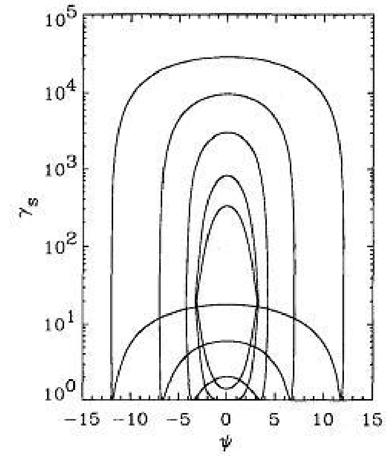

Figure 4.3: Plot of γ,Ψ space for different values of ϵ. The inner ring has the lowest value of ϵ, the outer ring the highest value (almost at the wavebreaking limit).[11]

4.3.1 The collision free process

We know that in a plasma, the collision process between an UHECR-proton and the background plasma,is dominated by proton-proton scattering. The proton-proton scattering cross-section can be approximated by a constant $\sigma_{pp} \sim \sigma_0 \sim 30$ mb in de ZeV regime of the UHECR protons. We can also make an approximation for the proton density as we know it is directly related to the plasma density. The plasma dilutes as it expands radially. So the plasma density goes like $1/r^2$. The formula for the plasma density n_p then becomes

$$n_p(r) = n_{p0}\frac{R_0^2}{r^2} \tag{4.11}$$

where n_{p0} is the plasma density at some reference distance R_0. The product of the plasma density and the proton-proton scattering cross-section gives us a collision probability. We can now construct the proton mean-free-path L_{mfp} (the distance a UHECR proton can travel freely) by integrating the collision probability up to unity

$$1 = \int_{R_0}^{R_0+L_{mfp}} \frac{\sigma_{pp} n_p(r)}{\Gamma_p} dr \tag{4.12}$$

Here Γ_p is the gamma factor for the speed of the plasma. What this says is that the probability that UHECR proton travelling from R_0 to $R_0 + L_{mfp}$ collides at least once is unity. We can work this out further by the use of Eq. (4.11)

$$1 = \int_{R_0}^{R_0+L_{mfp}} \frac{\sigma_{pp} n_{p0}}{\Gamma_p}\frac{R_0^2}{r^2} dr = \frac{\sigma_{pp} n_{p0} R_0}{\Gamma_p}[1 - \frac{R_0}{R_0 + L_{mfp}}] \tag{4.13}$$

It is obvious that R_0 and L_{mfp} are positive definite (> 1) so

$$0 < 1 - \frac{R_0}{R_0 + L_{mfp}} < 1 \tag{4.14}$$

This means that the coefficient in front of Eq. (4.13) should be larger that one. We want L_{mfp} to go to ∞ because then the path of the UHECR proton is essentially collision free. For this to happen Eq. (4.14) should approach unity and thus we can construct a threshold condition for the coefficient in front

$$\frac{\sigma_{pp} n_{p0} R_0}{\Gamma_p} = 1 \tag{4.15}$$

Below this threshold, the test particle will essentially be collision free. [12]

4.3.2 Cyclotron losses for the accelerating particle

The third requirement for acceleration is that the particle doesn't bend too much or otherwise the damping due to clyclotron radiation processes would be too big. The formula for cyclotron radiation is

$$-\frac{dE}{dt} = \frac{\sigma_t B^2 v_\perp^2}{c\mu_0} \tag{4.16}$$

Here σ_t is the Thomson crossection, B is the magnetic field and $v_\perp$ is the velocity perpendicular to the magnetic field.[9] We see from this formula that the cyclotron losses are zero when the movement is parallel to the magnetic field. We know from the section about Alfvén waves that they propagate along the magnetic field lines in the plasma most effectively. So this means that we assume that movement is mostly along the magnetic field lines and that the perpendicular velocity is very small compared to the total velocity. In practice the cyclotron losses should thus be very low compared to the total energy.

Chapter 5

Alfvén Waves

On earth getting a laser pulse through a plasma is relatively easy, but in space the story is a little different. We use in stead of a laser pulse an Alfvén wave. The Alfvén wave was discovered by Hannes Alfvén in the early 50's. It is an oscillation between the slowly moving ions and the magnetic field (see Fig. 5.1). The dispersion relation for an Alfvén wave in a plasma at rest and the wave equation for an Alfvén wave will be derived in this section. Furthermore the usefulness of Alfvén waves in plasma acceleration in space will be explained.

5.1 Dispersion relation

Consider a plasma at rest with density ρ_0 and uniform magnetic field $\overrightarrow{B_0}$. Now a perturbation creates a small flow $\overrightarrow{v_1}$ and a small perturbed magnetic field $\overrightarrow{B_1}$ so that

$$\begin{aligned} \overrightarrow{B} &= \overrightarrow{B_0} + \overrightarrow{B_1} && (5.1) \\ \overrightarrow{v} &= \overrightarrow{v_1} && (5.2) \end{aligned}$$

In this case $B_0 \gg B_1$. The momentum conservation equation in MHD theory gives

$$\rho \frac{d\overrightarrow{v}}{dt} = \overrightarrow{j} \times \overrightarrow{B} - \overrightarrow{\nabla} p \qquad (5.3)$$

In our simplified model we neglect the pressure term and assume an incompressible plasma (ergo $\overrightarrow{\nabla} \cdot \overrightarrow{v} = 0$). We use Ampère's law to get

$$\begin{aligned} \rho\mu_0 \frac{d\overrightarrow{v}}{dt} &= (\overrightarrow{\nabla} \times \overrightarrow{B}) \times \overrightarrow{B} \\ \rho_0\mu_0 \frac{\partial \overrightarrow{v_1}}{\partial t} &= (\overrightarrow{\nabla} \times \overrightarrow{B_1}) \times \overrightarrow{B_0} && (5.4) \end{aligned}$$

The last term is up to first order in $\overrightarrow{B}$. We can do the same for Faraday's Law

$$\begin{aligned} \overrightarrow{\nabla} \times \overrightarrow{E} &= -\frac{\partial \overrightarrow{B}}{\partial t} \\ \overrightarrow{\nabla} \times (\overrightarrow{v} \times \overrightarrow{B}) &= \frac{\partial \overrightarrow{B}}{\partial t} \\ \overrightarrow{\nabla} \times (\overrightarrow{v_1} \times \overrightarrow{B_0}) &= \frac{\partial \overrightarrow{B_1}}{\partial t} && (5.5) \end{aligned}$$

We now take for $\overrightarrow{v_1}$ and $\overrightarrow{B_1}$ plane wave solutions as

$$\overrightarrow{B_1}\overrightarrow{A}e^{i(-\omega t+\overrightarrow{k}\cdot\overrightarrow{x})} \tag{5.6}$$

Putting this into Eq. (5.4) and Eq. (5.5) we get

$$\begin{aligned} -\rho_0\mu_0\omega\overrightarrow{v_1} &= (\overrightarrow{k}\times\overrightarrow{B_1})\times\overrightarrow{B_0} \\ &= (\overrightarrow{k}\cdot\overrightarrow{B_0})\overrightarrow{B_1}-(\overrightarrow{B_1}\cdot\overrightarrow{B_0})\overrightarrow{k} \end{aligned} \tag{5.7}$$

and

$$\begin{aligned} -\omega\overrightarrow{B_1} &= \overrightarrow{k}\times(\overrightarrow{v_1}\times\overrightarrow{B_0}) \\ &= (\overrightarrow{k}\cdot\overrightarrow{B_0})\overrightarrow{v_1}-(\overrightarrow{k}\cdot\overrightarrow{v_1})\overrightarrow{B_0} \end{aligned} \tag{5.8}$$

We make use of the following equations concerning plasma incompressibility and Gauss' law for magnetism

$$\begin{aligned} \overrightarrow{\nabla}\cdot\overrightarrow{v_1} &= \overrightarrow{k}\cdot\overrightarrow{v}=0 \\ \overrightarrow{\nabla}\cdot\overrightarrow{B_1} &= \overrightarrow{k}\cdot\overrightarrow{B_1}=0 \end{aligned} \tag{5.9}$$

Taking the scalar product of k Eq. (5.7) becomes

$$\begin{aligned} -\rho_0\mu_0\omega(\overrightarrow{v_1}\cdot\overrightarrow{k}) &= (\overrightarrow{k}\cdot\overrightarrow{B_0})(\overrightarrow{B_1}\cdot\overrightarrow{k})-(\overrightarrow{B_1}\cdot\overrightarrow{B_0})(\overrightarrow{k}\cdot\overrightarrow{k}) \\ 0 &= (\overrightarrow{B_1}\cdot\overrightarrow{B_0}) \end{aligned} \tag{5.10}$$

So we get for Eq. (5.7) and Eq. (5.8)

$$-\rho_0\mu_0\omega\overrightarrow{v_1} = (\overrightarrow{k}\cdot\overrightarrow{B_0})\overrightarrow{B_1} \tag{5.11}$$

$$(\overrightarrow{k}\cdot\overrightarrow{B_0})\overrightarrow{v_1} = -\omega\overrightarrow{B_1} \tag{5.12}$$

Combining these two equations gives us the dispersion relation for an Alfvén in a plasma at rest

$$\begin{aligned} \omega^2 &= \frac{(\overrightarrow{k}\cdot\overrightarrow{B_0})^2}{\rho_0\mu_0} \\ &= \frac{(|k|^2|B_0|^2\cos^2(\theta)}{\rho_0\mu_0} \\ &= |k|^2 v_A^2\cos^2(\theta)=k_z^2 v_A^2 \end{aligned} \tag{5.13}$$

Here θ is the angle between the uniform magnetic field B_0 and the wave number k. v_A is given by

$$v_A=\frac{B_0}{\sqrt{\rho_0\mu_0}} \tag{5.14}$$

Thus the Alfvén wave travels like an ordinary wave most effectively along a field line and with a phase velocity of v_A. There is an alternative way of deriving the dispersion relation for an Alfvén wave by use of the equation of motion. The ion mass density provides the inertia and the magnetic field line tension provides the restoring force (see Fig. 5.1). This restoring force is due to the magnetic field line curvature and is of the form

$$\overrightarrow{F} = (\overrightarrow{\nabla} \times \overrightarrow{\nabla} \times (\zeta \times \overrightarrow{B_0})) \times \overrightarrow{B_0} \tag{5.15}$$

Here ζ is the small disturbance so that

$$\overrightarrow{\zeta} = \int v_1 dt \tag{5.16}$$

With this force we can work out an equation of motion and eventually get the same result as Eq. (5.13), which will be shown in the next section.

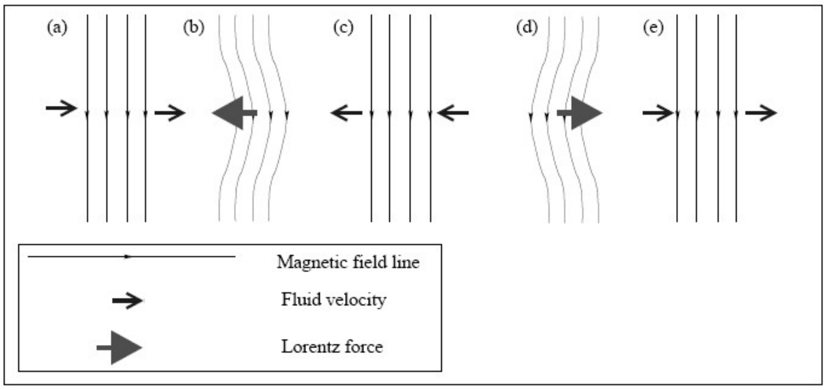

Figure 5.1: schematic picture of an Alfvén wave [9]

5.2 wave equation

We have an incompressible plasma in which a homogeneous magnetic field is embedded. We know from previous discussions that this magnetic field is practically frozen into the plasma. We take a coordinate system in such a way that this magnetic field is in the z-direction. We start with the fundamental Maxwell's equations from electromagnetism

$$\begin{aligned} \overrightarrow{\nabla} \times \overrightarrow{B} &= \mu_0 \overrightarrow{J} + \mu_0 \frac{\partial \overrightarrow{E}}{\partial t} \\ &\approx \mu_0 \overrightarrow{J} \end{aligned} \tag{5.17}$$

$$\overrightarrow{\nabla} \times \overrightarrow{E} = -\frac{\partial \overrightarrow{B}}{\partial t} \tag{5.18}$$

$$\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0 \tag{5.19}$$

In the first equation the time derivative on the $\overrightarrow{E}$ field may be neglected with respect to the current density $\overrightarrow{J}$. Furthermore we have a relation for the electrical conductivity

$$\overrightarrow{J} = \sigma(\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B}) \tag{5.20}$$

The hydrodynamic equation of motion is given by

$$\rho_0 \frac{d\overrightarrow{v}}{dt} = \rho\overrightarrow{G} + \overrightarrow{J} \times \overrightarrow{B} - \overrightarrow{\nabla} p \tag{5.21}$$

The first term on the right are non-electromagnetic forces, the second term is the magnetic force and the the third term gives us a pressure term. These are the basic equations in which we will further derive the Alfvén waves as a feature of plasma physics. We first make some assumptions on the conductivity σ and the density ρ. We assume that the ions and electrons can move freely inside the plasma so the conductivity becomes infinity. We next assume a constant mass distribution, so the density ρ_0 is constant. We also assume, as stated before that the plasma is a incompressible fluid. From fluid dynamics we know the continuity equation for a fluid [9]

$$\frac{\partial \rho_0}{\partial t} + \overrightarrow{\nabla} \cdot (\rho_0 \overrightarrow{v}) = 0 \tag{5.22}$$

For a constant density ρ_0 this equation simplifies to

$$\overrightarrow{\nabla} \cdot \overrightarrow{v} = 0 \tag{5.23}$$

We devide the magnetic field in a homogeneous term B_0 which corresponds to the magnetic field frozen into the plasma and a inhomogeneous term b caused by the current J.

$$\overrightarrow{B} = \overrightarrow{B_0} + \overrightarrow{b} \tag{5.24}$$

We know from the previous section that the wave travels in the direction of the magnetic field (and thus the z-direction). If we assume this we know that all parameters should only depend on z and on time. This leads to a pair of further simplifications. We know that $J_z = 0$, because of Eq. (5.18). From Eq. (5.18) we can see from the z-component that B_z is independent of time and from Eq. (5.19) we know that B_z is independent of z. So we say that

$$B_z = constant = B_0 \tag{5.25}$$

Furthermore we know for the velocity from Eq. (5.23) that $v_z = 0$, because the particle is at rest initially. We still have one dimension of freedom to choose our coordinate system (we can rotate it around the z-axis). Let's choose the coordinate system in such a way that $J_y = 0$. We then get the following pair of equations from Eq. (5.18)

$$J_x = -\frac{1}{\mu_0}\frac{\partial b_y}{\partial z} \tag{5.26}$$

$$J_y = J_z = 0 \tag{5.27}$$

From the last equation we get $\frac{\partial b_x}{\partial z} = \mu_0 J_y = 0$ and this gives

$$b_x = constant = 0 \tag{5.28}$$

We can put these assumptions into Eq. (5.21), where we ignore the non electromagnetic forces. We also know that the x- and y-direction of the pressure term are assumed to be 0 (no dependence of x and y on all variables). We get

$$\frac{\partial v_x}{\partial t} = 0 \quad \Rightarrow \quad v_x = constant = 0 \tag{5.29}$$

$$\frac{\partial v_y}{\partial t} = -\frac{1}{\rho} J_x B_z = \frac{1}{\rho_0 \mu_0} \frac{\partial b_y}{\partial z} B_0 \tag{5.30}$$

$$v_z = 0 \tag{5.31}$$

With $v_z = 0$ we can deduce from Eq. (5.21) that the following equation for the pressure must hold

$$\frac{\partial p}{\partial z} = J_x B_y = -\frac{1}{\mu_0} \frac{\partial b_y}{\partial z} b_y = -\frac{1}{2\mu_0} \frac{\partial h_y^2}{\partial z} \tag{5.32}$$

For the electric field we find with the help of Eq. (5.21) that

$$E_x = \frac{i_x}{\sigma} - \mu \frac{v_y}{c} H_0 \tag{5.33}$$

$$E_y = 0 \tag{5.34}$$

$$E_z = 0 \tag{5.35}$$

We find from Eq. (5.18) the following differential equation

$$\frac{\partial b_y}{\partial t} = -\frac{\partial E_x}{\partial z} \tag{5.36}$$

With the help of Eq. (5.33), Eq. (5.26) and Eq. (5.30) we get

$$\begin{aligned} \frac{\partial^2 b_y}{\partial t^2} &= -\frac{\partial^2 E_x}{\partial z \partial t} \\ &= \frac{\partial^2}{\partial z \partial t}\left(-\frac{J_x}{\sigma} + v_y B_0\right) \\ &= \frac{\partial}{\partial z}\left(\frac{1}{\sigma\mu_0} \frac{\partial^2 b_y}{\partial z \partial t} + \frac{B_0^2}{\rho_0 \mu_0} \frac{\partial b_y}{\partial z}\right) \\ &= \frac{1}{\mu_0 \sigma} \frac{\partial^3 h_y}{\partial z^2 \partial t} + \frac{B_0^2}{\rho_0 \mu_0} \frac{\partial^2 b_y}{\partial z^2} \end{aligned} \tag{5.37}$$

In our assumptions we said that in an ideal plasma the conductivity σ is infinity. The first term on the right disappears then and Eq. (5.38) becomes

$$\frac{\partial^2 b_y}{\partial t^2} = \frac{B_0^2}{\mu_0 \rho_0} \frac{\partial^2 b_y}{\partial z^2} \tag{5.38}$$

We see that this is the common known traveling wave equation with the earlier derived velocity v_A

$$v_A = B_0 \frac{1}{\sqrt{\rho_0 \mu_0}} \tag{5.39}$$

So we see that the wave propagates in the direction of the magnetic field. The motion of the ions and the perturbation of the magnetic field are in the same direction and transverse to the direction of propagation (see Fig. 5.1). Normally the flow of ions is parallel to the magnetic field lines. There needs to be some kind of disturbance in the ion flow to create an Alfvén wave.

5.3 Alfvén Wave in space

We have seen from the previous sections that an Alfvén wave in a plasma at rest is a low-frequency traveling oscillation of the ions and the magnetic field. We, however, are not interested in a plasma at rest. Nearby magnetars, the plasma is moving with very fast relativistic speed. The dispersion relation then becomes (Eq. (5.13))

$$\frac{k_z^2 c^2}{\omega^2} = 1 - \frac{1}{\Gamma}\frac{(\omega_{pi}^2 + \omega_{pe}^2)(1 - \frac{vk}{\omega})}{(\omega - vk \pm \frac{\omega_{Bi}}{\Gamma})(\omega - vk \mp \frac{\omega_{Be}}{\Gamma})} \tag{5.40}$$

Here ω_{pe}, ω_{pi}, ω_{Be}, ω_{Bi} are respectively the plasma frequency for electrons, the plasma frequency for ions, the cyclotron frequency for electrons and the cyclotron frequency for ions. More important are v, the bulk velocity of the plasma, and Γ, the corresponding gamma factor. We can also say something about the electric and magnetic field ratio of a Alfvén-wave:

$$|\frac{E}{B}| \sim v \tag{5.41}$$

This is implied by Ohm's Law in ideal MHD theory

$$\overrightarrow{E} + \overrightarrow{v} \times \overrightarrow{B} = 0 \tag{5.42}$$

When the plasma as a whole is moving relativistically (that is $V_p \to c$, $\Gamma_p \to \infty$) the dispersion Eq. (5.40) becomes

$$\frac{k_z^2 c^2}{\omega^2} = 1 \tag{5.43}$$

which is the dispersion relation for an ordinary EM wave moving in the z-direction. The electromagnetic field ratio becomes

$$\frac{E}{B} \sim V_p \sim c \tag{5.44}$$

This is also implied for an EM wave by Maxwell's equation (Eq. (5.18)) and the plane wave solution. So when an Alfvén wave is traveling inside a plasma which is moving relativistically, it behaves like a normal EM wave. The Alfvén wave can thus excite the plasma just like a laser pulse does.

Chapter 6

Maximum energies

In this chapter a method for finding the maximum energy will be examined. First the concept of the ponderomotive force will be explained, which is useful to derive the maximum energy. A ponderomotive force is a non-linear force that a charged particle experiences in an inhomogeneous oscillating electromagnetic field. This is what occurs when a laser pulse (or relativistic alfven wave) travels through a plasma, as we have seen previously. The ponderomotive force is given by

$$F_p = -\frac{e^2}{4m\omega^2}\nabla E^2 \tag{6.1}$$

where m is the mass and e is the charge of the particle where the force exerts on. ω is the frequency of the oscillating field and E the amplitude of the field. The derivation of this can be easily done. We see that E^2 is a measure of the amplitude of the field and acts like an envelope around the pulse (the dashed line in Fig. 6.1). The gradient of the envelope is a measure for how strong the ponderomotive force becomes. Thus a steep pulse creates a stronger ponderomotive force then a flat pulse. The minus sign indicates that the ponderomotive force on the particle is always in the direction of a decrease in the electric field. In the case of a laser pulse in a plasma, the ponderomotive force is the driving force of the plasma wakefield. One can construct a ponderomotive vector

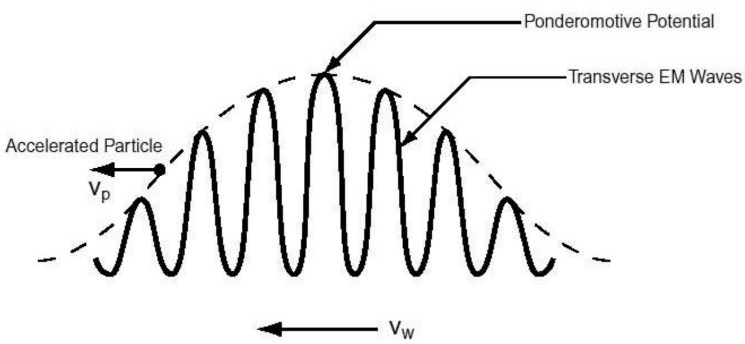

Figure 6.1: schematic picture of ponderomotive potential. [9]

potential a_0

$$a_0 = \frac{eE}{mc\omega} \tag{6.2}$$

When this parameter exceeds unity the nonlinearity is strong. The linear regime means that the oscillation length of the plasmon wake should not exceed the wavelength (else we speak of cold wave breaking [13] [14]). Wave breaking occurs when oscillations get so big that the electrons get into each other region. This causes a breaking in the ordening and the wave isn't single valued anymore at every point x (see also an explanation at the section about plasma wakefield acceleration). The normal models don't apply anymore in this case. We will first discuss the linear regime, and how a particle gains energy in this regime.

6.1 linear regime

Consider the rest frame of the plasma wake. We see from Eq. (4.2) that

$$\beta \equiv \frac{v_p}{c} = \sqrt{1 - \frac{\omega_p^2}{\omega^2}} \tag{6.3}$$

and

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{\omega}{\omega_p} \tag{6.4}$$

The rest frame of the plasmon is also the rest frame of the photon. This means that the photon has no momentum. We can Lorentz transform the momentum four vector of the photons from the laboratory frame to the wave frame (rest frame of the plasmon)

$$\begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \frac{\omega}{c} \\ k_x \end{pmatrix} = \begin{pmatrix} \frac{\omega_p}{c} \\ 0 \end{pmatrix} \tag{6.5}$$

Here k_x is de wave number for the photon in the laboratory frame. The right hand side refers to the wave frame. As mentioned before, it is obvious that the photon is standing still in the wave frame. We can do the same Lorentz transformation for the plasmon and get

$$\begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \frac{\omega_p}{c} \\ k_p \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{k_p}{\gamma} \end{pmatrix} \tag{6.6}$$

We make use in these transformations of the well known dispersion relation for a photon in a plasma

$$\omega^2 = \omega_p^2 + k_x^2 c^2 \tag{6.7}$$

This Lorentz transformation gives us an invariant longitudinal electric field E_x. This can be seen with help of the electromagnetic tensor

$$F_{\mu\nu} = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & B_z & -B_y \\ E_y & -B_z & 0 & B_x \\ E_z & B_y & -B_x & 0 \end{pmatrix} \tag{6.8}$$

and expand the Lorentz transformation for 3D to

$$R^\mu_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{6.9}$$

The Lorentz transformation becomes

$$\begin{aligned} E'_x &= F'_{10} = R^\mu_1 R^\nu_0 F_{\mu\nu} = R^0_1 R^1_0 F_{01} + R^1_1 R^0_0 F_{10} \\ &= \beta^2\gamma^2 F_{01} + \gamma^2 F_{10} = \gamma^2(1-\beta^2)F_{10} = F_{10} = E_x \end{aligned} \tag{6.10}$$

So E_x is indeed invariant. De maximum electric field is determined by the cold wave-breaking limit. This means that the oscillation length of the plasmon k_p should not exceed the wavelength $x_L = \frac{eE_L}{m\omega_p^2}$. This implies that $k_p * x_L \approx 1$. We can now find that

$$eE_l = \frac{m\omega_p^2}{k_p} = \frac{m\omega_p^2 v_p}{\omega_p} = m\omega_p v_p \tag{6.11}$$

so that the maximum amplitude for the field is

$$eE_l^{max} = m\omega_p c \tag{6.12}$$

The electric wave potential ϕ for the plasmon moving in the x-direction is by definition

$$\nabla_x \phi^{wave} = E_l^{wave} \tag{6.13}$$

Filling in a plane wave solution for ϕ gives

$$k_p^{wave}\phi^{wave} = E_l^{wave} \tag{6.14}$$

We now make use of the Lorentz invariance of E_l and the Lorentz transformation for k_p given by Eq. (6.6) to get

$$e\phi^{wave} = e\frac{E_l^{wave}}{k_p^{wave}} = e\frac{E_l\gamma}{k_p} = \frac{m\omega_p v_p\gamma}{k_p} = mv_p^2\gamma \tag{6.15}$$

for the maximum amplitude we get

$$e\phi^{wave} = \gamma mc^2 \tag{6.16}$$

In the linear regime, the accelerated particle gets it's maximum energy when the acceleration in the wave frame reverses. This means that the accelerated particle surpasses the plasmon. It cannot be accelerated more. We conclude from this that all the potential energy of the plasmon from Eq. (6.16) is transformed to the accelerated particle. In transforming energy and momentum back to the laboratory frame we find for this energy $W_{max} = \gamma_p mc^2$

$$\begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} E \\ p_x c \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix}\begin{pmatrix} \gamma mc^2 \\ \gamma\beta c^2 \end{pmatrix} = \begin{pmatrix} \gamma^2 mc^2(1+\beta^2) \\ \gamma^2\beta mc^2 \end{pmatrix} \tag{6.17}$$

So the maximum energy in the laboratory frame becomes

$$W_{max} = \gamma^2 mc^2(1+\beta^2) \approx 2\gamma^2 mc^2 \tag{6.18}$$

6.2 nonlinear regime

The parameter a_0 determines how steep a pulse increases and decreases and is a measure of how strong the ponderomotive force (and thus the plasma wakefield) will be. In the linear regime the maximum field amplitude is given by

$$E_{max} = E_{wb} = \frac{m_e c \omega_p}{e} \tag{6.19}$$

This limit is completely determined by the cold wave breaking limit. In the non-linear regime this maximum field is enhanced by a factor $\sqrt{2}(\gamma_p - 1)^{1/2}$ [15]. When this factor is exceeded fluid oscillations cannot occur anymore. For a cold plasma, this singularity corresponds to a peak amplitude of the phase velocity of the wave, which implies a strong wave-particle resonance. [15] What it means is that beyond this limit the cold plasma wave equations cannot hold and new equations have to be developed for a warm plasma. The equation for the maximum field will thus be

$$E_{max} = \sqrt{2}\sqrt{\gamma_p - 1} E_{wb} \tag{6.20}$$

For a maximum energy we need to find the length L_d on which a proton can ride the wave. We know from the discussion on wakefield acceleration that the acceleration takes place between $-\pi < \Psi < 0$ in Ψ,γ-space so that $\Delta\Psi$ becomes π. We also know from that section the expression for Ψ (Eq. (4.5)

$$\begin{aligned} \Delta\Psi &= k_p(z_d - v_p t_d) \qquad (6.21) \\ &= k_p v_p(\frac{z_d}{v_p t_d} - 1)t_d \\ &= \omega_p(\frac{v_{proton}}{v_p} - 1)t_d = \pi \end{aligned}$$

Here t_d is the total acceleration time and z_d the total acceleration distance. The total acceleration length z_d is given by

$$\begin{aligned} z_d = v_{proton} t_d &= \frac{v_{proton}}{c} \frac{\pi c}{\omega_p(\frac{v_{proton}}{v_p} - 1)} \\ &= \frac{v_{proton}}{c} \frac{\lambda_p}{2(\frac{v_{proton}}{v_p} - 1)} \qquad (6.22) \end{aligned}$$

When assuming that the speed of the accelerated proton reaches the speed of light we can derive

$$\begin{aligned} z_d &= \frac{v_{proton}}{c} \frac{\lambda_p}{2(\frac{c}{v_p} - 1)} = \frac{\lambda_p \frac{v_p}{c}}{2(1 - \frac{v_p}{c})} \\ &= \frac{v_{proton}}{c} \lambda_p \frac{v_p}{c} \frac{1}{2} \frac{1 + \frac{v_p}{c}}{(1 + \frac{v_p}{c})(1 - \frac{v_p}{c})} \\ &= \frac{v_{proton}}{c} \lambda_p \frac{v_p}{c} \frac{1}{2} \frac{1 + \frac{v_p}{c}}{1 - \frac{v_p^2}{c^2}} \end{aligned} \tag{6.23}$$

In the ultra relativistic limit where $v_p -> c$ we find for this expression

$$z_d = \frac{\lambda_p}{1-\frac{v_p^2}{c^2}} = \lambda_p \gamma_p^2 \tag{6.24}$$

Here γ_p is the usual gammafactor for the plasma. We can find the maximal energy, through the electric potential

$$E_{max} = \frac{\partial}{\partial x}\phi \tag{6.25}$$

Here ϕ is the electric potential and a function of the longitudinal field direction x. The electric energy is defined by

$$W = e\phi \tag{6.26}$$

Combining these two equations we get for the maximum energy

$$W_{max} = e\int_0^{z_d} E_{max} dx = eE_{max} z_d \tag{6.27}$$

The last step in this equation assumes that E_{max} is not dependent on x. We can fill in the different parts for E_{max} and z_d and get

$$\begin{aligned} W_{max} &= eE_{max} z_d = e\frac{E_{max}}{E_{wb}} E_{wb} \lambda_p \gamma_p^2 \\ &= \frac{E_{max}}{E_{wb}} mc\omega_p \frac{2\pi c}{\omega_p}\gamma_p^2 = 2\pi \frac{E_{max}}{E_{wb}} mc^2 \gamma_p^2 \end{aligned} \tag{6.28}$$

So in the non-linear regime the maximum energy is enhanced by a factor $\pi\frac{E_{max}}{E_{wb}}$.

6.3 estimates on highest energies

We can see from Eq. (6.28) that if we can make good guesses for γ_p and the ratio E_{max}/E_{wb} (which are linked through Eq. (6.20)), we can find an estimate on the highest energy possible with this acceleration mechanism. [11] gives us an estimate on the highest possible ratio $E_{max}/E_{wb} \approx 6.2$. This corresponds with the use of Eq. (6.20) to

$$\sqrt{2}\sqrt{\gamma_p - 1} \approx 6.2 \tag{6.29}$$

and

$$\gamma_p \approx 20 \tag{6.30}$$

Filling these values in, into Eq. (6.28) we get

$$\begin{aligned} W_{max} &= 2\pi\frac{E_{max}}{E_{wb}} mc^2\gamma_p^2 \\ &= 2\pi * 6.2 * 1.7 * 10^{-27} * 9 * 10^{16} * 20^2 = 1.4 * 10^{13} eV \end{aligned} \tag{6.31}$$

According to [16] the Lorentz factor of a magnetar wind has a lower bound of $\gamma_p = 10^4$ and can take up to $\gamma_p = 10^7$. This gives us an energy of

$$\begin{aligned} W_{max} &= 2\pi\frac{E_{max}}{E_{wb}} mc^2\gamma_p^2 \\ &= 2\pi * \sqrt{2}\sqrt{10^4 - 1} * 1.7 * 10^{-27} * 9 * 10^{16} * 10^8 = 8.4 * 10^{19} eV \end{aligned} \tag{6.32}$$

and an upper bound of

$$
\begin{aligned}
W_{max} &= 2\pi \frac{E_{max}}{E_{wb}} mc^2 \gamma_p^2 \\
&= 2\pi * \sqrt{2}\sqrt{10^7 - 1} * 1.7 * 10^{-27} * 9 * 10^{16} * 10^1 4 = 2.6 * 10^{27} eV
\end{aligned} \tag{6.33}
$$

This seems like a lot of energy and gets into the territory of the UHECR's. According to [11] however, wave breaking occurs after the highest possible ratio $E_{max}/E_{wb} \approx 6.2$. This means that with electric fields higher than this value, the acceleration mechanism of riding the wave can't work anymore. This means that if the γ_p is as high in magnetars as [16] says it is, other models should apply for the plasma wakefield acceleration and this model cannot hold anymore.

Chapter 7

Energy Spectrum

Of course the wakefield excited by a Alfvén is not as smooth as one excited with as laser pulse on earth. You can optimize the circumstances on earth by making the pulse as smooth and short as possible. In a astrophysical setting this is however not the case. We assume that the acceleration and deceleration of particles is random. We can describe this phenomenon by a stochastic process. In a stochastic process there is a distribution function $f(\epsilon, t)$. This function gives the amount of particles with energy ϵ at a time t. There is a famous equation that explains the evolution in time of this distribution function [17]:

$$\begin{aligned}\frac{\partial}{\partial t}f(\varepsilon,t) &= \int_{-\infty}^{\infty} d(\Delta\epsilon)W(\epsilon-\Delta\epsilon,\Delta\epsilon)f(\epsilon-\Delta\epsilon,t) \\ &\quad - \int_{-\infty}^{\infty} d(\Delta\epsilon)W(\epsilon,\Delta\epsilon)f(\epsilon,t) - \nu(\epsilon)f(\epsilon,t) \qquad (7.1)\end{aligned}$$

This equation is called the Chapman-Kolmogorov equation. Here $W(\epsilon, \Delta\epsilon)$ is a probability function for a particle to move from an energy ϵ to an energy $\epsilon + \Delta\epsilon$. The first term in Eq. (7.1) gives how many particles move from some energy to the energy ϵ and the second term gives how many particles move from an energy ϵ to a random energy. It is clear that there needs to be a minus sign in the second term because it gives a negative effect to the amount of particles with energy ϵ. The last term in Eq. (7.1) tells us how much particles loose their energy ϵ due to collision or radiation losses. In practice this term can be neglected because we assume that the UHECR particles are accelerated below the threshold condition and are thus essentially collision-free. For radiation we can say that the acceleration is mostly linear so cyclotron radiation losses are negliable.

The assumption that ϵ is much larger than $\Delta\epsilon$ can be made. It is a reasonable assumption because usually the energy gain (or loss) goes in small steps while the final energy is very high. With this assumption we can Taylor-expand $W(\epsilon - \Delta\epsilon, \Delta\epsilon)f(\epsilon - \Delta\epsilon, t)$ around $W(\epsilon, \Delta\epsilon)f(\epsilon, t)$ in the first term. Eq. (7.1) then becomes

$$\begin{aligned}\frac{\partial}{\partial t}f(\varepsilon,t) &= \frac{\partial}{\partial\epsilon}\int_{-\infty}^{\infty} d(\Delta\epsilon)\Delta\epsilon W(\epsilon,\Delta\epsilon)f(\epsilon,t) \\ &\quad + \frac{\partial^2}{\partial\epsilon^2}\int_{-\infty}^{\infty} d(\Delta\epsilon)\frac{\Delta\epsilon^2}{2}W(\epsilon,\Delta\epsilon)f(\epsilon,t) \qquad (7.2)\end{aligned}$$

Here the terms of order $\Delta\epsilon^3$ and higher are neglected.

We can now make some further assumptions on the probability function $W(\epsilon, \Delta\epsilon)$ in a purely stochastic proces. We say that W is an even function. This means that gaining energy has the same probability as loosing energy. This is the case in a plasma wakefield (see the discussion about the ponderomotive force). Secondly we say that W is independent of ϵ. This means that the probability distribution for gaining or loosing energy is not dependent of the initial energy ϵ.

Our last assumption is that W is independent of $\Delta\epsilon$. So the probability of gaining an amount of energy $\Delta\epsilon$ does not depend on $\Delta\epsilon$. All $\Delta\epsilon$ are equally probably. This is the case in a stochastic white noise process. A remark must be made that $\Delta\epsilon$ is still chosen small (remember our assumption for the Taylor expansion).

Let us look at a stationary energy distribution, i.e. $\partial f/\partial t = 0$. The first term in Eq. (7.2) becomes zero then. W is an even function and so the integration is over an uneven function. An integration with even limits over an uneven function gives zero. We can also say that the total energy should not be less than zero. This means that $\Delta\epsilon$ must be larger than $-\epsilon$ and smaller than ϵ. We can change the integration limits from $(-\infty, +\infty)$ to $[-\epsilon, +\epsilon]$.

Eq. (7.2) then becomes

$$\frac{\partial^2}{\partial\epsilon^2}\int_{-\epsilon}^{\epsilon} d(\Delta\epsilon)\frac{\Delta\epsilon^2}{2}W(\epsilon,\Delta\epsilon)f(\epsilon,t) = W(\epsilon,\Delta\epsilon)\frac{\partial^2}{\partial\epsilon^2}\int_{-\epsilon}^{\epsilon} d(\Delta\epsilon)\frac{\Delta\epsilon^2}{2}f(\epsilon,t) = 0 \tag{7.3}$$

The integration can now be easily done. We find a power-law scaling for the energy distribution function

$$f(\epsilon) = \frac{\epsilon_0}{\epsilon^2} \tag{7.4}$$

Here ϵ_0 is taken to be some normalization factor, usually the typical background proton energy. This is something that we more or less expect from the UHECR spectrum after the 'ankle'.

The power law index observed is a little sharper, but this may be due to the travel from the acceleration site to the observers site (the earth).

Chapter 8

A second approach on the maximum energy

With the energy spectrum found in the previous section, it looks like there is no restriction on the maximum energy, there are just fewer particles. In principle, this is the case. The only restriction comes from the conservation of energy and of course the energy distribution f which we discussed before. So there is an amount of energy in the Alfvén wave that gets transferred into the plasma wakefield which accelerates particles. The transfer happens efficiently so we assume that all the energy of the Alfvén is loaded into the plasma wakefield. We also assume that all the plasma wakefield energy is used to accelerate UHE protons via the stochastic process. We can then make an estimate to the maximum energy a UHECR proton can have

$$\eta_a \epsilon_{\text{wind}} \sim \epsilon_{\text{alfven}} \sim \epsilon_{UHE} \sim N_{UHE} \int_{\epsilon_0}^{\epsilon_m} \epsilon f(\epsilon) d\epsilon \tag{8.1}$$

Here ϵ_m is the maximum energy, ϵ_0 is the average energy of an plasma proton and N_{UHE} is the amount of plasma protons that are accelerated. η_a is the efficiency at which the magnetar wind energy is loaded into an Alfvén shockwave. We can work the integral out using Eq. (7.3) for the energy distribution f,

$$\epsilon_m = \epsilon_0 \exp(\frac{\eta_a \epsilon_{\text{wind}}}{N_{UHE} \epsilon_0}) \tag{8.2}$$

We can look a little further into this by stating that η_b of the magnetar wind energy is needed for the plasma flow (plasma protons rest kinetic energy), i.e.

$$\eta_b \epsilon_{\text{wind}} \sim N_p \Gamma_p m_p c^2 \sim N_p \epsilon_0 \tag{8.3}$$

Here N_p are the amount of plasma protons present in the wind. We further say that η_c of the plasma protons gets accelerated into UHE particles, i.e.

$$N_{UHE} \sim \eta_c N_p \tag{8.4}$$

Filling all this into Eq. (8.2), we find

$$\epsilon_m = \epsilon_0 \exp(\frac{\eta_a}{\eta_b \eta_c}) \tag{8.5}$$

With guesses for the efficiencies η_a,η_b and η_c we can thus find how much higher the energy of the UHE proton can become with respect to it's normal energy.

8.1 estimates on η_a, η_b, η_c

It is difficult to make a reasonable guess for the efficiencies of η_a, η_b and η_c. The circumstances are very unpredictable and several complications can occur. Still we can try to estimate the order of magnitude of the efficiencies.

In η_a we find how much of the Wind energy is used in the Alfvén wave. It is reasonable to assume this is about one percent so $\eta_a \approx 10^{-2}$. Furthermore we approximate $\eta_b \approx 10^{-2}$. So one percent of the wind energy is taken to be the kinetic energy of the average protons in the bulk plasma flow. And for η_c we again take about 10^-2. So one of the hunderd plasma protons is accelerated to Ultra high energy protons. All these assumptions are based on the numbers for the acceleration in a Gamma Ray Burst discussed in [12]. It is not unlikely that these numbers are (way) off, but currently they are all we have. When combining the three estimates we get

$$\frac{\eta_a}{\eta_b \eta_c} \approx O(10^2) \tag{8.6}$$

When we fill in this value in Eq. (8.5), we find that the maximum energy is essentially unbound. So this approach gives us no limit on the maximum energy.

Chapter 9

Flux of UHECR particles

From previous chapters we know that there are Magnetars found within the GZK-limit. We can find the flux of UHE particles found on our earth using the following formula

$$\Phi_{UHECR} = f_{\text{magnetar}} N_{UHE} \frac{1}{4\pi R_{gzk}^2} \tag{9.1}$$

Here f_{magnetar} is the total number of active magnetars per year found within the GZK-limit. The N_{UHE} in this formula gives us the amount of UHE particles (so particles with energies higher than 10^{20} eV) per magnetar. R_{gzk} is the GZK-radius. The formula is easily understandable. The amount of magnetars within the GZK-limit together with the amount of UHE particles per magnetar gives us the total amount of UHECR's. Of course not all of these cosmic rays will hit the earth. That is why we have to divide by the area of a sphere around the earth with a radius of the GZK-radius. With estimates on f_{magnetar}, N_{UHE} and R_{gzk} we can find the flux of UHE particles hitting the earth.

We know from the theory of Greisen, Zatsepin and Kuzmin, that UHE particles cannot travel a distance larger than 50 Mega Parsec (about 150 million lightyears) or they will react with cosmic microwave background. We can thus take for R_{gzk}

$$R_{gzk} \approx 100Mpc \approx 3 * 10^{24} m \tag{9.2}$$

There is a lot of discussion about the amount of magnetars found within the GZK limit. Only a few magnetars have been found since their known existance but speculation has begun that this is only the tip of the iceberg. Thus far there have only been found a dozen Magnetars, but because of their short bright status it is possible that there are hundreds of Million Magnetars out there, which we can't see [4]. One other source suggests ([5] that there can be up to 100 million magnetars within our own galaxy. For the wakefield acceleration to work, we however need active magnetars. A magnetar is active for about 10,000 years and one magnetar is formed every 1,000 year in a galaxy. [18] states that there are about 30,000 galaxies within the GZK limit. So we can estimate the number of magnetars active at any time within the GZK limit is

$$f_{\text{magnetar}} = \frac{N_g * N_{am}}{t_g} = 30,000 * 10 = 300,000 \approx 10^5 \tag{9.3}$$

Here N_g is the amount of galaxies within the GZK limit and N_{am} the amount of active galaxies per galaxy at one instant. This may seem a low number but unfortunately, currently not much is known about the number of magnetars. In the future this number can get a lot higher. For the amount of UHE Protons N_{UHE} we can look at Eq. (8.4) and Eq. (8.3) and find

$$N_{UHE} = \eta_b \eta_c \frac{\epsilon_{wind}}{\epsilon_0} \tag{9.4}$$

Here η_b and η_c have been given in the previous section. If we find a good estimate for the energy of the magnetar wind ϵ_{wind} we can find the number of UHE protons in a magnetar wind. If we make an assumption on the total internal energy reservoir of a magnetar and assume that that is roughly the same as the wind energy. We can make this assumption because a magnetar looses more than 50 percent of it's energy through the magnetar wind. From [18] and [19], we know that the internal energy of a magnetar is about $\epsilon_{mg} = 10^{39} J = 6.2 * 10^{57}$ eV. We need to divide this number by the amount of active years of a magnetar (t_{al}. Then we know how much energy is accessible per year (if we assume a constant energy flow to the magnetar wind). So we get for ϵ_{wind}

$$\epsilon_{wind} = \frac{\epsilon_{mg}}{t_{al}} = \frac{6.2 * 10^{57}}{1000} = 6.2 * 10^{54} \text{eV/year} \tag{9.5}$$

For ϵ_0, the value for a typical plasma proton is about 10^13 eV. Filling in these values for ϵ_{wind} and ϵ_0 into Eq. (9.4) we find

$$N_{UHE} = 10^{-2} 10^{-2} \frac{6.2 * 10^{54}}{10^{13}} \approx 10^{38} \tag{9.6}$$

We now have all the quantities to find the UHECR flux. Filling in the numbers found into Eq. (9.1) we get

$$\begin{aligned} \Phi_{UHECR} &= f_{magnetar} N_{UHE} \frac{1}{4\pi R_{gzk}^2} = 3 * 10^5 * 10^{38} \frac{1}{4\pi (3 * 10^{24})^2} \\ &\approx 1.6 * 10^{-8} \text{ UHECR / m}^2 \text{ /year} \end{aligned} \tag{9.7}$$

The value of the flux for particles with energies higher than 10^20 eV measured by the Auger laboratory is as follows

$$\Phi_{UHECR} \approx 10^{-8} \text{ UHECR / m}^2 \text{ /year} \tag{9.8}$$

We can say that the predicted value for the flux of 10^{20} eV or higher is within one order of magnitude the same as the measured value, so that the flux could be well predicted by the assumptions made. A few remarks must be made however. The estimates made here are very rough. The efficiencies for the plasmawakefield acceleration to accelerate to the UHE-regimes could be a few orders lower or higher. I have assumed that all the magnetars are on a sphere of the GZK distance away from us. This is clearly not the most realistic situation. The estimate found here, is a nice first guess, but further research should proof that it's the right answer.

Chapter 10

Conclusions

10.1 estimates on the amount of magnetars

Magnetars have a bright phase of about ten thousand years. In this phase, they are relatively easy recognized by the bright, repeating flashes of soft gamma rays. We know that about ten magnetars per galaxy are in this bright phase simultaneously. Unfortunately the magnetar more or less extinguishes after this phase. This means it can hardly be observed. That makes the estimate on the amount of magnetars per galaxy very hard. The amount of magnetars in this phase could be far beyond the million, but we simply do not know. The amount of magnetars in the bright phase however, are the most important for our discussion on the acceleration of UHECR in the magnetar wind, which occurs in the bright phase.

10.2 estimates on the highest possible energy

Two methods of obtaining the highest energies have been discussed.

In the first case, the value of $1.4 * 10^{13}$ eV has been found for the maximum energie taking into account the wave breaking limit. When not taking into account the wave breaking limit, and thus getting in the warm plasma regime, we get values for the maximum energy of $8.4{*}10^{19}$ eV in the lower limit and $2.6{*}10^{27}$ eV in the higher. It is highly uncertain if the model holds in the wave breaking regime and that other models for warm plasma's should be investigated.

In the second case is looked at the spectrum for the UHECR's. We found a power law of the energy spectrum of $\epsilon^{-}2$ for the UHE regime with the help of the famous Chapman-Kolmogorov equation. This is in agreement with the observed UHECR's. Furthermore we have made guesses for the efficiencies in which the UHECR's are accelerated. This gives us an essentially unbound maximum energy. The efficiencies are very rough guesses so the value for the maximum energie is much more uncertain than the values found in the first case.

10.3 estimates on the flux of UHECR particles

In the last chapter we have tried to explain the flux of UHECR particles using the earlier found spectrum for the UHECR. Guesses have been made for the energy of the wind, the amount of UHE protons in a magnetar wind and the GZK distance a UHE proton can travel. We have found a number for the flux of UHE particles dependent on the energy ϵ of about $1.6 * 10^{-8}$ UHECR / m^2 /year. This number is in line with the measurements of the Auger observatory. The estimates made here however, are very rough, so further research is necessary to check if the first guess found here is an accurate one.

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