SET Learning - A study of top-down learning in a bottom-up perceptual task

(Bachelor's thesis)

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Abstract

A lot of research has been done at looking at the difference between novice and practiced players in learning tasks. One of these tasks is SET. The SET game provides opportunity to study both perceptual and cognitive processes in a single task. In this research we look at how players transition from novice to expert level, and what roles perceptual and cognitive processes play in such transition. Both types of processes are studied as integral parts of a single task. To do this, we asked subjects to play SET against a computer opponent while logging their data. Strategies and preferences which influence performance are examined and we will emphasize on how these aspects change over time.

1 Introduction

Often a complex task is performed more easily after extensive training, and in some cases these tasks are then executed almost effortlessly, without consciously having to focus on it. A famous example of this is the Stroop test (Stroop, 1935). Many people are used to reading words day in day out. Therefore, for many it becomes an automatic process. In the Stroop test subjects were asked to name the color in which words were written, what is clearly a perceptual task. Therefore, the process of reading words needed to be suppressed, requiring more top-down cognitive processes. Without any practice it is a quite difficult task to do and you will need to consciously suppress the process of reading the words. After some training subjects get better at this task, and the suppression of reading words becomes relatively easy. The difference between the novice and practiced subjects has been the topic of many studies. However, the way that subjects learn and change strategies over time is a process that requires further study.

A platform that many researchers have used is the card game SET[1]. This visual perception game is very suitable for research because it gives insight into both perceptual and cognitive processes (Pomerantz, 2006; Schyns, Bonnar, and Gosselin, 2002). In addition, it is an easy game to analyse because of its relative simple rules.

1.1 The game SET

The card game SET contains 81 unique cards. These cards can differ on four different attributes: color (red, green or blue), shape (oval, rectangle or wiggle), filling (open, solid or textured) and number (one, two or three). The goal of this game is to find a combination of three cards, called a set, in the twelve cards that are dealt open, whereby each attribute on these cards need to be all the same or all different. The amount of different attributes determines the difficulty of the set, further referred to as

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[1]SET is a game developed by Marsha Falco in 1974 (Set Enterprises; www.setgame.com)

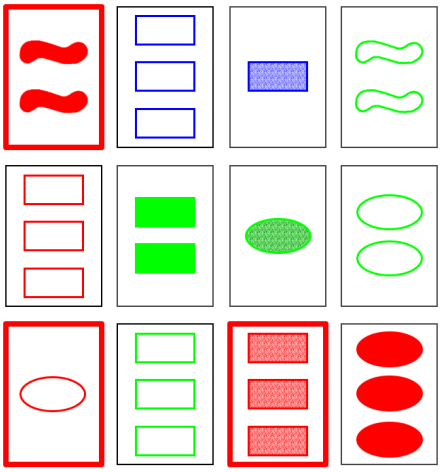

Figure 1: An example situation in the game SET. The cards marked with red form a level three set, only the attribute color is the same. There are five other sets that can be found in this example.

the level of the set. A set where only one attribute differs is called a level one set. When two different attributes differ, it is called a level two set. The same applies to three and four different attributes. An example situation of a level three set is shown in Figure 1.

The first player that finds a set in the 12 cards takes the three cards that make up the set. These three cards are then replaced by three new cards from the deck. The game ends if the deck is exhausted. The winner is the player with the most sets found.

1.2 Related works on SET

In a study by Taatgen, van Oploo, Braaksma, and Niemantsverdriet (2003) the authors used questionnaires and reaction times (RT) to gain more insight into player's behavior. They used these findings to construct a model based on the ACT-R cognitive architecture (Anderson, 2007). They suggested that finding lower level sets required mainly perceptual processes, these perceptual obvious sets can be easily indentified because they 'pop up'. This would explain why novices and experts do not differ much in performance on the lower level sets, as was stated by the authors.

Taatgen et al. found that the RT of finding sets which contained more similar cards, the lower level sets, were shorter than higher level sets. The bias to more perceptual similar cards would explain why players need less time to find lower level sets than higher level sets.

Jacob and Hochstein (2008) also found that players prefer to look at perceptually similar cards by utilizing mainly perceptual processes such as similarity detection.

Jacob and Hochstein proposed a strategy based on the most abundant value (MAV). The MAV refers to the attribute value that occurs most. Note that there can be more than one MAV at the same time. In Figure 1 the MAV would be the shape 'rectangle' and the filling 'open', because these values are the most common ones, each of them appear six times. The group of cards that contain the MAV is called the most abundant value group (MAVG). Jacob and Hochstein found that subjects prefer the sets in the MAVG to the sets outside the MAVG. Also the RT of found sets in the MAVG were shorter than those outside. When the MAVG was large, the RT decreased if the set found was part of the MAVG. When there were no sets within MAVG, the MAVG became more of a distraction resulting in a larger RT for sets found outside of MAVG.

Another aspect Jacob and Hochstein looked at was the influence of the number of sets present on the RT. They found that the larger the number of sets present, the shorter the RTs. They used a horse race model (Miller, 1982, 1986; Raab, 1962; Townsend and Ashby, 1983) to get a relation between the total number of sets on the table and the RT. The decrease in RT does not follow the increasing chance probability of finding sets with increasing number of available sets, but a more logaritmic decrease. The predictions of their model closely fits the actual results. They concluded that there is an independence of the processes of finding each set when there are more sets present.

Jacob and Hochstein also did an experiment on learning and generalization. They found that with more training subjects had a gradual improvement in speed of playing the SET game. They also found that the RT of lower level sets were lower and had more stabilization than the

RT of higher level sets. They tested their subjects with variations of the game SET with different stimulus values. The subjects performed equally well on these variations, suggesting a high-level learning effect.

Nyamsuren and Taatgen (2011) improved the original ACT-R model of Taatgen et al. (2003) by gaining more insight using an eye tracker experiment. They studied the roles of the perceptual and cognitive processes. Their model consisted of two parallel processes to reflect both internal planning and reaction to perceptual stimulus from the environment. They have shown that both processes are used in decision making, and that there is a complex interaction between them. The adjusted model performed better when it was learning on top-down level and found the optimal balance between the bottom-up and top-down processes.

1.3 Research objectives

While a lot of attention has been given to describing the difference between novice and expert players, little attention has been given to the learning part.

Jacob and Hochstein (2008) dit look at learning, but only looked at the RTs. They used the RTs in combination with two different versions of the game to see whether learning the SET task is high or low level. They did not look at specific strategies over time. We hope to get more insight into what kind of strategies novice and expert players use, and especially in how these used strategies change over time. The learning experiments of Jacob and Hochstein included only 9-12 games. They stated that the training generalization may have resulted from the fact that the subjects did not yet reach a stabilized automatic level. In our learning experiments, subjects play for about five hours, which will result in playing approximately 40 games.

Our assumption is that there is some kind of shift in dependence from the bottom-up features of the game to more top-down control (explicit strategy), and think novice players will use more lower-level, perceptual strategies while expert players rely more on the higher-level, cognitive strategies. As was stated by Taatgen et al., novices and experts do not differ much in performance on the lower level sets, while experts mainly excel at the higher level sets. Because finding higher level sets needs top-down processes, we would expect a top-down learning effect in SET.

Are expert players using other strategies than novice players? Will there be more preference for specific attributes as an expert? And will the MAV be used more as a novice than as an expert?

Summarizing these sort of questions results in our main question: What are the top-down learning effects in a bottom-up perceptual task?

To answer this, we asked the subjects to play the SET game against a modified version of the ACT-R model developed originally by Taatgen et al. and later improved by Nyamsuren and Taatgen. With this model we can look at different strategies, such as the most abundant value. We will also look at level preference, what will be explained further on. Next, we look at the dimensional salience like Jacob and Hochstein did, but we will look at it within the context of learning. We will also look at the ratio of found sets of different levels and finally at the learning rates of the different levels of sets. The latter might tell us if players learn more at the lower or higher level sets indicating more learning at the bottom-up or top-down level of the task. With these results, we hope to gain more insight in the bottom-up and top-down changes in the learning task.

2 Method

In total, 12 students of the University of Groningen (Groningen, the Netherlands) have participated in the experiment for either course credits or were provided with financial incentive (five female; the age of the subjects ranged from 20 to 30 years). All the subjects were novice players having played the game once or never before. If a subject did not know the rules, the rules were explained to him and a few examples were given. The subjects were asked to play the SET game for at least five hours spread over a few sessions. Each session lasted at least one hour to a maximum of two and a half hours. This limit was

set because after such time the subjects could suffer from fatigue. The different sessions had to be played at most five days apart so there would be no loss of learning. The five hours resulted in approximately 40 games per subject and a total of 6300 sets found.

2.1 The ACT-R model

The model we used is a slightly modified version of the model of Nyamsuren and Taatgen. This version will play the standard version of the game. It will enable us to not only record the RT, but record every action of the player, as well as the actions and parameters of the model and every detail of the environment such as the cards on the table.

At each round, 12 cards are displayed, always including at least one set. The subjects have to select (via mouse clicks) the three cards that make up a set. They can deselect, by just selecting the same card again, or select three cards that did not form a set without penalty. After a correct set is found, the three cards that formed the set are excluded from the game and replaced by three new cards from the deck. The game continues right away. If an incorrect set is found, a continue button appears along with a message why the three selected cards did not form a set. After pressing the continue button, the game continues. If the model finds a set, again a continue button appears. Upon pressing the continue button, the three cards that formed the set are excluded and replaced by three new cards from the deck. If the deck is exhausted, the player has completed the game and can start a new one. Figure 2 shows the user interface of the model.

The original model by Nyamsuren and Taatgen was implemented with a novice and expert mode. The experience of the model defined how the model performed visual search and comparison. In our version, the model has two parameters, a speed and an expert parameter. The speed parameter defines how fast the model is in finding any set. The speed parameter is increased when the player finds a set, and decreased if the model finds a set. The parameter decreases a little bit more than it will increase if a set is found, resulting in the player having a slight advantage over the model, creating a more fun and still challenging opponent. The expert parameter defines how good the model is at finding higher level sets, and will mainly react to the higher level sets. If the player finds a set of level three or four, the parameter increases. If the model finds a level three or four set, the paramter decreases. The same for the speed parameter also applies here. In this way the model will ensure that the player stays stimulated to get better throughout the game.

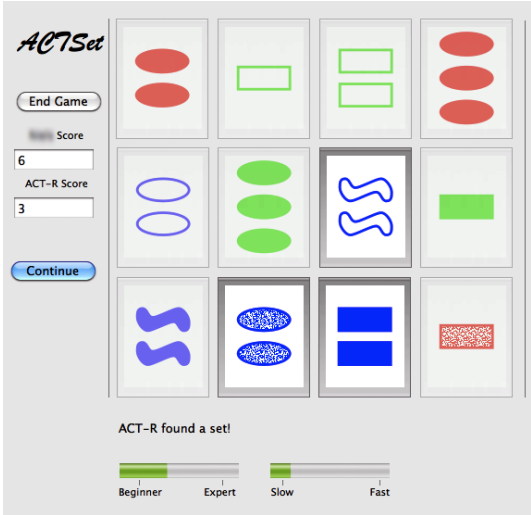

Figure 2: The user interface of the model. The model found a level two set.

R was used for data collection and analysis. RT was measured from the moment the cards were dealt on the table to the moment the third card that formed a set was clicked and the set had been verified.

3 Analysis and Results

We can look at learning in a few different ways. One of them is looking at the RT of the subjects. If the RT decreases, the subject becomes faster at recognizing sets. This learning effect can be explained for instance by learning new strategies or by having a better understanding of the rules. Figure 3 shows how the mean RT for different levels over all subjects changes over time. Time is binned into ten different intervals. The intervals we used were 0 to 0.5 hour, 0.5 to

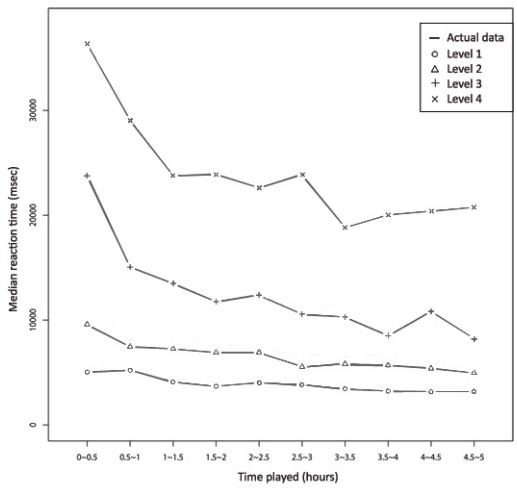

Figure 3: RT of different levels over time according to actual data. The variable time is binned into ten different intervals each containing the median RT of that bin.

1 hour, ..., 4.5 to 5 hours. The value of the RT at a bin is the median of the RTs belonging to that bin. In this way, outliers won't have much influence. The graph shows that there is some learning at every level. We will look at different strategies and try to combine the different results with a mixed-effect regression analysis (Baayen, Davidson, and Bates, 2008). With this analysis we can then create a model and get the most significant variables needed for a good fit of the found data.

3.1 Most Abundant Value

In this research, we looked at different strategies over time, some of them mentioned earlier. One aspect we looked at is how subjects used the Most Abundant Value (MAV) and how this changed over time. Jacob and Hochstein looked at how the MAV influences which set is chosen and how it changes the RT for finding it. We looked at the MAV within the context of learning.

The MAV can be deduced from the 12 cards on the table. A player finds a set that is part of the MAVG if the MAV is present in all three cards that make up the set. In case of multiple MAVs, if the three cards that formed the found set all contains one of the MAVs then it is considered to be part of the MAVG. Only the situations where there are sets in and outside the MAVG will be taken into consideration. If there are only sets in or outside the MAVG, the player would not had the possibilty to choose a set in or outside the MAVG. The chance is also taken into consideration. In terms of the MAV, the value a set gets (the MAV dependency, or MAVD) is given in formula 3.1. If a set within MAVG is found, the MAVD for that set will be calculated by dividing the total sets available on the table (totalSets) by the total number of sets within MAVG (setsInMAVG). MAVD will be 0 if a set is found outside the MAVG.

$$MAVD = \begin{cases} \dfrac{totalSets}{setsInMAVG} & \text{set} \in \text{MAVG} \\ 0 & \text{otherwise} \end{cases} \tag{3.1}$$

If the average MAV dependency equals one, it means that sets are found within the MAVG as many times as we would expect beforehand by chance. Values above one will indicate that there are more sets found within MAVG than expected by chance.

In an example situation where there are three sets available within the 12 cards and where one set is part of the MAVG, we would expect that the chance of finding the set within the MAVG to be one third. So in three trials, the chance of finding the set within the MAVG should be statistically one. Using formula 3.1 in the situation where a player finds the set within the MAVG, we get a MAVD of three. The other situations will be scored 0. Averaging the three possible situations, the mean MAVD will be one, exactly what we wanted.

As shown in Figure 4 the mean MAV dependence stays above one, indicating that subjects use the MAV more than expected by chance. Jacob and Hochstein also reached this conclusion, but what we can see here is that the use of the MAV does not change much over time, indicating that subjects do not learn regarding the use of the MAV.

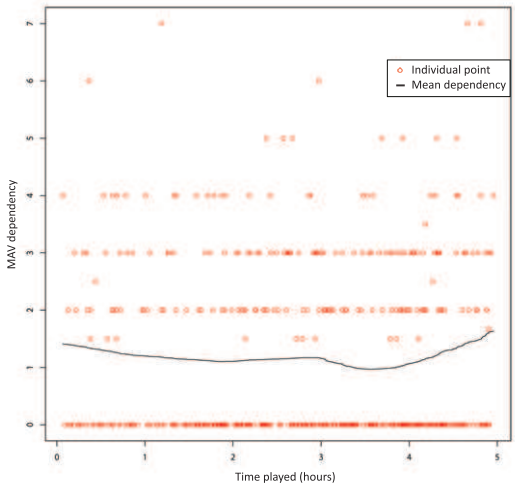

Figure 4: The graph depicts the mean MAV dependency averaged over all subjects over time. Only situations where there is a minimum of one inside and one outside the MAVG has been taken into consideration.

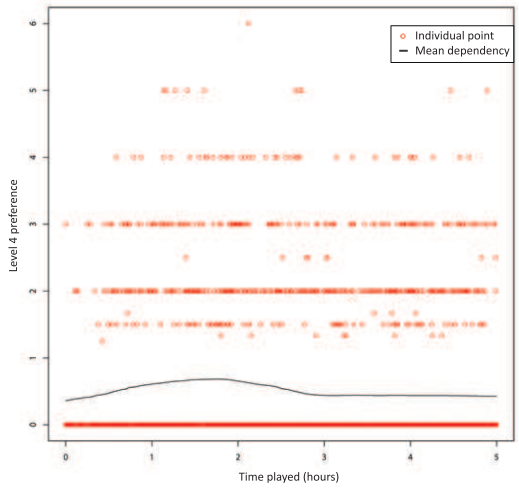

Figure 5: Mean level four preference averaged over all subjects over time. Only situations where there is a minimum of one level four and one other level set has been taken into consideration.

3.2 Level preference

Another aspect we looked at is level preference. Level preference tells us how often a player preferres a particular level set over other available sets on the table. We were mainly interested in level four preference, meaning how often a level four set is preferred over other lower level sets. Only situations where a player can choose a level four set above other lower level sets have been taken into consideration. A similar experiment was done by Jacob and Hochstein where they looked at the mean level preference. We are studying the potential change in level preference over time.

The formula for level four preference as shown in 3.2 is similar to that of MAVD (3.1). The chance is, as with the MAVD, also taken into consideration. When a level four sets is found, the preference for that level four set (l4preference) is calculated by dividing the total sets on the table (totalSets) by the total number of level four sets on the table (totalL4Sets). When a lower level set is found, level four preference will be 0.

$$l4preference = \begin{cases} \dfrac{totalSets}{totalL4Sets} & \text{set level} = 4 \\ 0 & \text{otherwise} \end{cases} \tag{3.2}$$

When we average the level four preference over all subjects we get the result as shown in Figure 5. Level four preference stays below one, indicating that players prefer level four sets over lower level sets less than expected by chance. Also a little bump at the beginning of the graph can be seen where players had a higher preference for level four sets. Around two hours of playing, this preference dampens. Generally, the level four preference does not change much over time, indicating no change in preference over time.

We also looked at the preference of level three sets over other lower level sets. Only situations where there were level three and lower level sets were taken into consideration. We applied the same analysis here as we have done for the level four preference. When looking at the individual graphs, subjects showed different patterns of preference. We can not draw a general con-

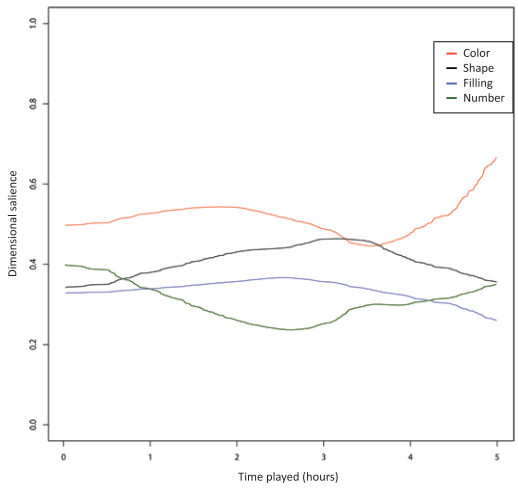

Figure 6: Dimensional salience. Sets which have the same value for color for all three cards are found the most.

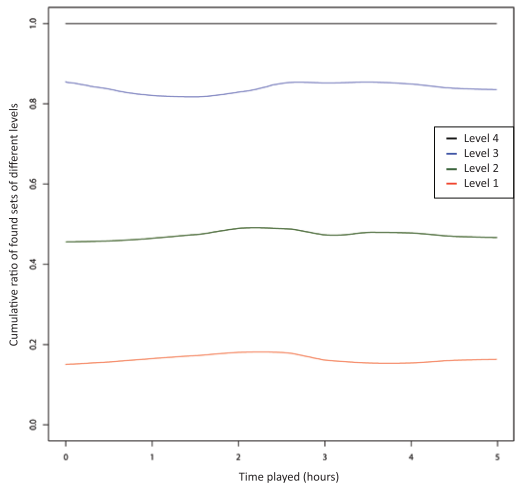

Figure 7: Cumulative ratio of found sets of different levels. The difference between a level and the level below it depicts the total ratio of that level. Example: the ratio of level three sets can be found by subtracting the level two ratio from the level three ratio.

clusion from this, but as with level four preference most subjects showed no preference for level three sets.

3.3 Dimensional salience

Jacob and Hochstein did an experiment on dimensional preference, they found that subjects did have a preference for one dimension but that there were different preference orderings for different subjects. We looked at whether this preference changed over time.

Of each found set the attributes that were the same in all three cards were given the value one, zero otherwise. Figure 6 depicts the mean dimensional preference over all subjects over time. Although the preference varies a bit over time, color is clearly the most preferred by the subjects. This is consistent with the claims of Nyamsuren and Taatgen.

3.4 Level ratio

Players prefer to look at perceptually similar cards (Jacob and Hochstein), the lower level sets. There might be a change in the ratio of the found sets of different levels over time. The ratio for the different levels are calculated as follows: Every time a set is found, the found level scores one, the other levels score zero. When fitting a line for every level, we get the ratio line for the different levels. The cumulative ratio for the different levels is shown in Figure 7. As can be seen in the graph, level two and level three sets are found more often than level one and level four sets.

Table 1 shows how many sets of each level are expected (through combinatorics by Jacob and Hochstein) and the data we found. The general equation for the total number of sets of level i, with d dimensions and three values, by Jacob and Hochstein is shown in Equation 3.3. The total number of sets summed over i is 1080. When applying this equation for the four different levels, we can then calculate the ratio of occurrence for the different levels. Applying the ratio of occurance to the total number of sets found by the subjects in our experiment, we get the expected number of sets for each level. The column 'difference' depicts the ratio between the expected and actual found sets.

Table 1: Expected and actual number of sets found per level

	Expected	Found	Difference
Level 1	630	1034	1.64
Level 2	1890	1965	1.04
Level 3	2520	2290	0.91
Level 4	1260	1011	0.80

Expected sets are based on a total of 6300 found sets over all subjects and combinatorics by Jacob and Hochstein. The difference is the found data divided by the expected data.

$$nSets = \frac{3^d \cdot \binom{d}{i} \cdot 2^i}{3!} \tag{3.3}$$

The lower level sets are found more often than expected by chance, whereas higher level sets are found less often than expected by chance. So there is a clear preference for the lower level sets, confirming the conclusions of Jacob and Hochstein. In the context of learning, we can see that the ratio of found sets of different levels does not seem to change over time. The player thus finds lower level sets as a novice as often as an expert does.

The bump that we saw in Figure 5 can also be seen here if we look at the difference between the level four and three line. This indicates that players find more level four sets when they have not much experience. As in Figure 5, also here the bump decreases at about two hours of playing.

3.5 LME analysis and model

We analyzed the performance of subjects with a mixed-effect regression analysis. With this analysis we created a model to get the most significant variables needed for a good fit of the found data. This model apart from fixed effects also allows nested random effects. The random effects are shown in Table 2, *subj* stands for the different subjects in the experiment. We used an analysis of variance to compare different models.

Table 2: Random effects of the LME model

Groups	Variance	Std. Deviation
subj	0.085	0.291
Residual	0.528	0.727

The dependent variable in the regression model is the reaction time on logaritmic scale. We used logarithmic scale because outliers will then not have much influence. Predictors that significantly contributed to the model's prediction are shown in Table 3.

time represents the experience of the subject, the time that subjects played the SET game in hours. This predictor is significant and has a negative coefficient value which indicates that RT decreases with increasing value of time. Variable *time* provides statistical evidence that there is indeed a significant learning effect.

totalSets is the total number of sets on the table. The horse race model by Jacob and Hochstein showed a logarithmic curve, so we used the logarithmic scale here as well. We tested the two variations, with and without the logarithmic transformation, with an analysis of variance and got a better result with the logartimic transformation as expected. *totalSets* is significant and has a negative coefficient value which indicates that RT decreases with increasing number of sets on table.

The variable *Color* is used when the attribute color is the same on all three cards of the found set. The other attributes (number, shape and filling) did not show any significant effect in model prediction. The variable *Color* is significant and shows a negative coefficient, indicating that RT decreases when the attribute color is the same on all three cards that make up the found set. Variable *Color* provides statistical evidence that the attribute color is preferred in a set and provides faster recognition.

The variables *Level*2, *Level*3 and *Level*4 indicate the level of the set found. As can be seen in Table 3, these variables are significant and show a positive coefficient with increasing size as the level goes up. The higher the level, the higher the RT, which is constistent with the claims of Jacob and Hochstein.

The interesting part is the interaction between the time and the level of the set. The model predicted better with this interaction, providing statistical evidence that there are different learning rates for different set levels. The interaction variables $(Log(time) : Levelx)$ show a negative coefficient which indicates that RT decreases with increasing value of time. The most learning takes place within the higher level sets, but there is also learning at the lower levels.

Figure 8 shows the found and the predicted RTs of the different levels. We reused Figure 3 and added the model's predictions. As can be seen, the model we used closely predicts the found data.

4 Discussion

Our assumption was that there would be some kind of shift in dependence from the bottom-up features of the game to more top-down control. The main question focuses on the top-down learning effects in a bottom-up perceptual task.

We have seen that the MAV is not more or less used as a novice than as an expert, indicating that there is no learning regarding the MAV. Although more sets were found inside the MAVG, in our regression model the MAV was not significant for predicting RT. Because the MAVG is a group of cards that are similar in a certain value, we can conclude that players use similarity as a novice as much as they would as an expert.

As we have seen, the RTs for higher level sets is larger than for the lower level sets. So lower level sets should be found more easily than the higher ones. When looking at the level four preference, the level four sets are not preferred over other lower level sets, as we would expect. When looking at the learning aspect, we see no change in preference. Together with the results of the ratio of sets of different levels found, we can conclude that there is no shift from finding less higher level sets to finding more higher level sets.

The bump around one hour of playing in Figure 5 indicates that players are focusing more on level four sets when they have almost no experience. When looking at the different ratios of the level of sets in that time region, we also see an increase of level four sets found, supporting our assumption of focusing on level four sets. A possible explanation for this is that players overlearn the level four sets. When having almost no experience with the SET game, their opponent, the model, finds level four sets more often that the player will. The player will therefor focus more on the level four sets, other level sets are thereby less frequently found. Eventually, the model will beat the player at the lower level sets. When the player realizes that the model is finding more sets and more sets can be found when focusing on the lower level sets, the preference for level four sets drops. Further research is needed to confirm this assumption.

Color is the attribute that was clearly the most preferred under all subjects. The presence of the same color on all three cards of the set also contributed to the model's RT prediction. There is a little increase in color preference after about four hours of playing. Further research on this aspect could be done to check wether this is just coincidence or if other aspects are involved in this change.

Combining previous results resulted in our regression model that closely predicts the RTs of the different level of sets. We saw that there are different learning rates for the different levels. As was stated in studies by Jacob and Hochstein and Nyamsuren and Taatgen, in the game SET players use perceptual abilities to find lower level sets. However, it is also impossible to find higher level sets without cognitive skills. So SET is a perceptual and cognitive task, and we can clearly say when perceptual skills are used and when cognitive skills are used. With the higher learning rates for the higher level sets, we can conclude that the major learning comes from the cognitive part and not from the perceptual part. This emphasizes the importance of cognitive skills even in a perceptual task as SET.

With these results we can answer the questions that were raised while formulating our main research question. The strategies that we have looked at did not show a significant change between novice and expert players, indicating that experts do not use other strategies than novices. The preference for higher level sets and

Table 3: The fixed effects' coefficients, t and p values

Fixed effects	Coefficients	t value	p value
Intercept	9.290	100.54	0
Log(time)	-0.103	-4.09	0
Log(totalSets)	-0.586	-31.57	0
Color	-0.198	-9.26	0
Level 2	0.511	13.12	0
Level 3	0.962	25.17	0
Level 4	1.310	27.53	0
Log(time):Level 2	-0.113	-3.47	0
Log(time):Level 3	-0.174	-5.55	0
Log(time):Level 4	-0.097	-2.53	0.01

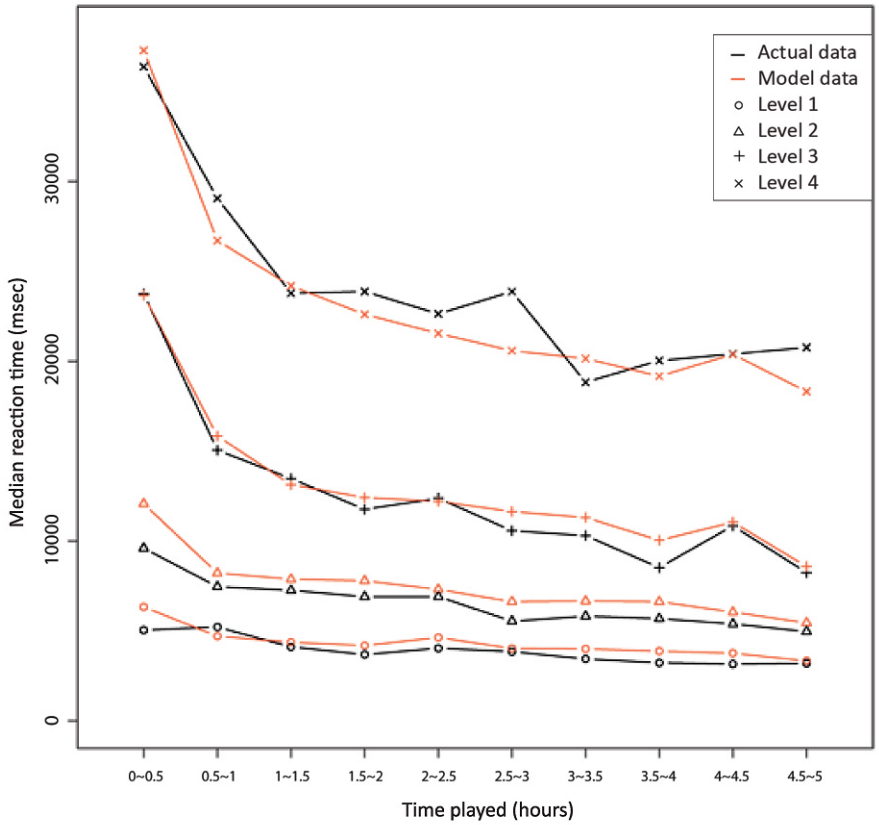

Figure 8: RT of different levels over time according to actual and model data. The variable time is binned into ten different intervals each containing the median RT of that bin.

the preference for specific attributes also does not change significantly over time, indicating that there is no change in preference between novices and experts.

5 Conclusion

In this paper we looked at how players transition from novice to expert level in a complex task requiring both perceptual and cognitive processes. Our experiment shows that the major learning comes from the cognitive part, the top-down features of the task, as we can conclude from the different learning rates. But also at the perceptual level there is some learning. This might as well be the case in other categorization tasks. The strategies and preferences mentioned in this paper did not show any significant change over time.

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